

Pandemic Recessions and Contact Tracing

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Introduction

- The COVID-19 pandemic set off a worldwide health and economic crisis
- Progress to reach herd immunity against the coronavirus seems to languish
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 - Low global vaccination rates and breakthrough infections
 - Emergence of new variants of the coronavirus
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 - Emergence of new variants of the coronavirus
 - Important to understand tools that can contrast this long-running pandemic
- ⇒ This paper: The efficacy of **contact tracing** to combat a pandemic crisis
- Testing strategy based on tracing and testing the contacts of confirmed infected cases
 - Rests on **reconstructing the network of interactions and infection chain**

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A macro-epidemiological model with [asymptomatic transmission](#) and [contact tracing](#)

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 - Contact tracing aims to reconstruct the interactions of confirmed positive cases
- Agents' decisions have an **externality on the number of subjects to be traced**
 - This externality can cause **the tracing and testing system to become overburdened**
- The collapse of the system can be averted by
 - A sufficiently comprehensive tracing technology
 - A complementary lockdown aimed at buying time to expand the tracing and testing scale

The Importance of Reconstructing the Infection Chains

A typical challenge of uninformed or random testing

- At the onset of an epidemic or a new variant of the virus, spreaders are only a few
- Detecting and isolating enough spreaders to prevent flare-ups is challenging

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- Contact tracing can prevent flare-ups of infections if tracing externality is mitigated

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 - The resulting infection rate is isomorphic to the one used in other studies
- Our approach is general and can be extended to a broad set of epi-mac models

Overview of the Model

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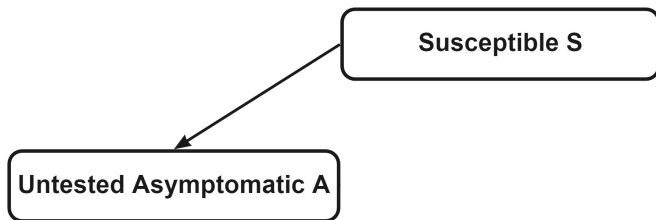
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- The govt administers tests, quarantine infected agents, and can enact lockdowns

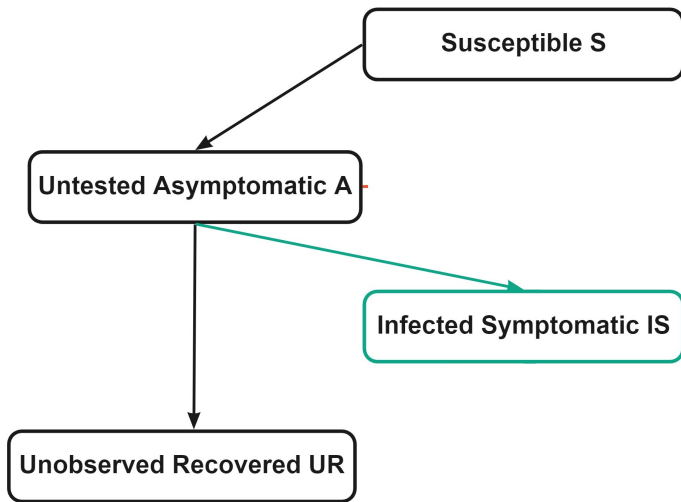
Change in Health Status

Susceptible S

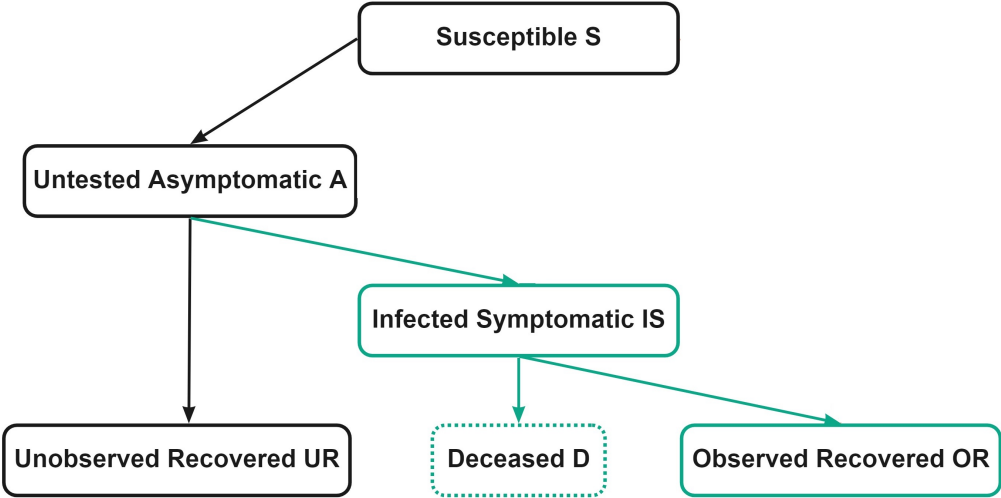
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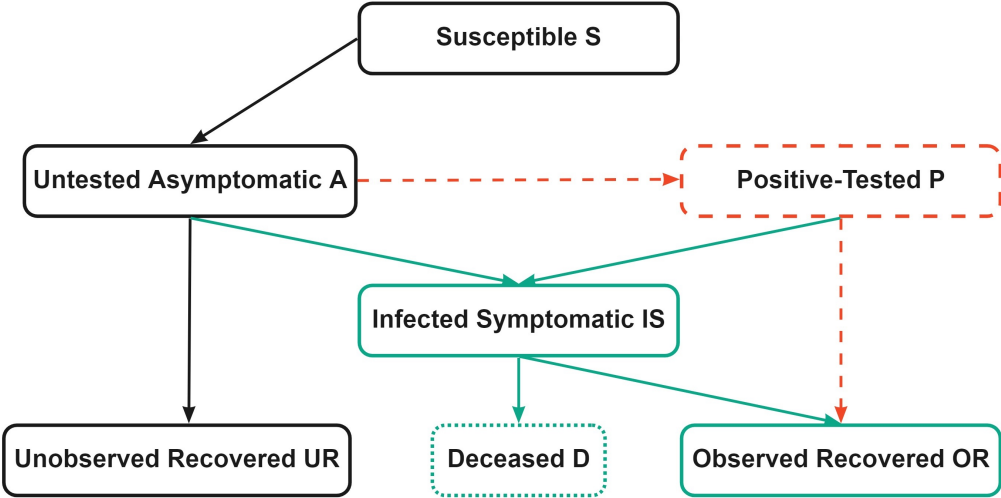
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Observability of Health Status, Tracing, and Testing

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- Tests deliver a binary outcome: positive or negative (can be false negative)

Infection and Testing Probabilities

To close the model we need to characterize

1. The probability of becoming infected
2. The probability of being traced and tested
 - Endogenous network of interactions characterizes these probabilities

The Probability of Random Meetings

- The probability for an agent to randomly meet with k asymptomatic agents when consuming is given by the Binomial distribution \mathcal{B}

$$f_{c,t}(k) \equiv \mathcal{B}\left(k, \overbrace{\varphi_C(c_t^S)}^n, \underbrace{\frac{C_t^A}{C_t}}_p\right) = \binom{\varphi_C(c_t^S)}{k} \left(\frac{C_t^A}{C_t}\right)^k \left(1 - \frac{C_t^A}{C_t}\right)^{\varphi_C(c_t^S) - k}$$

- Similarly defined probabilities for labor interactions and other interactions

Probability of Becoming Infected

- If the agent is susceptible, the probability of becoming infected in one meeting is τ
- The probability of becoming infected for a susceptible agent that chooses c_t^s and n_t^s

$$\tau_t = \sum_{k_c=0}^{\varphi_C(c_t^s)} \sum_{k_n=0}^{\varphi_N(n_t^s)} \sum_{k_o=0}^{\varphi_O} \left[1 - (1 - \tau)^{k_c+k_n+k_o} \right] f_t(k_c, k_n, k_o),$$

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- Linearized version is **isomorphic to SIR** and **Macro-SIR models** [Details](#)

$$\tau_t \approx \Xi \left[\varphi_C \cdot c_t^S \left(\frac{C_t^A}{C_t} \right) + \varphi_N \cdot n_t^S \left(\frac{N_t^A}{N_t} \right) + \varphi_O \left(\frac{A_t}{Pop_t} \right) \right],$$

Testing Probabilities

- The probability for an infected agent to test positive:
 1. Probability of tracing infected agents
 2. Testing capacity Y_t relative to number of traceable people E_t
 3. Accuracy of tests due to false negative outcomes with probability π_F

$$\pi_{P,t}^i = \pi_{C,t}^i \cdot \min \left\{ \frac{Y_t}{E_t}, 1 \right\} \cdot (1 - \pi_F)$$

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- This is also the fraction of asymptomatic spreaders quarantined in period t

The Importance of Reconstructing the Infection Chain

- The probability of being traced, $\pi_{C,t}^i$ captures the information resulting from **ex-post** reconstructing the network of interactions of newly symptomatic cases Example

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- The probability of being traced, $\pi_{C,t}^i$ captures the information resulting from **ex-post reconstructing the network of interactions of newly symptomatic cases** Example
- This network contains the **the infection chain** – **chain of interactions that led a newly symptomatic case to become infected or to infect other agents**
- **The reconstruction of the infection chain improves the efficacy of testing**
 1. Exploiting the infection chain raises the chance of detecting asymptomatic agents
 2. Random meetings between asymptomatic agents of different infection chains are rare

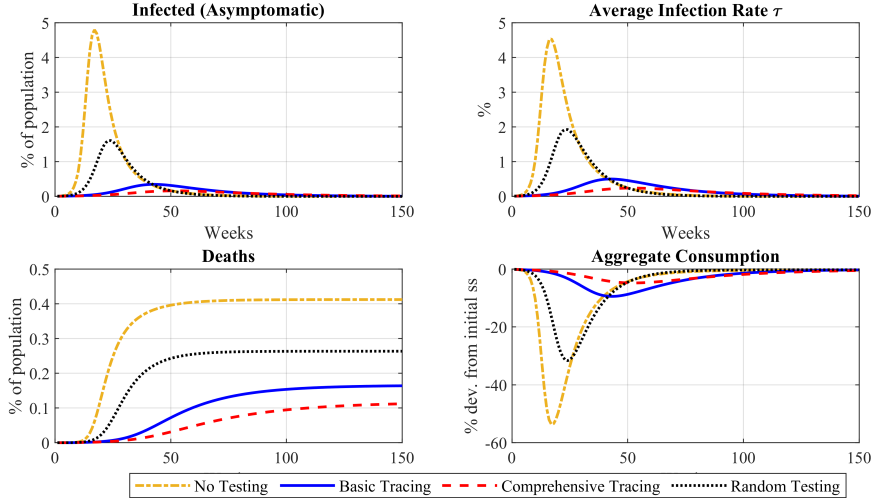
Model Solution and Calibration

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 - Initial surprise shock that infects tiny share of population
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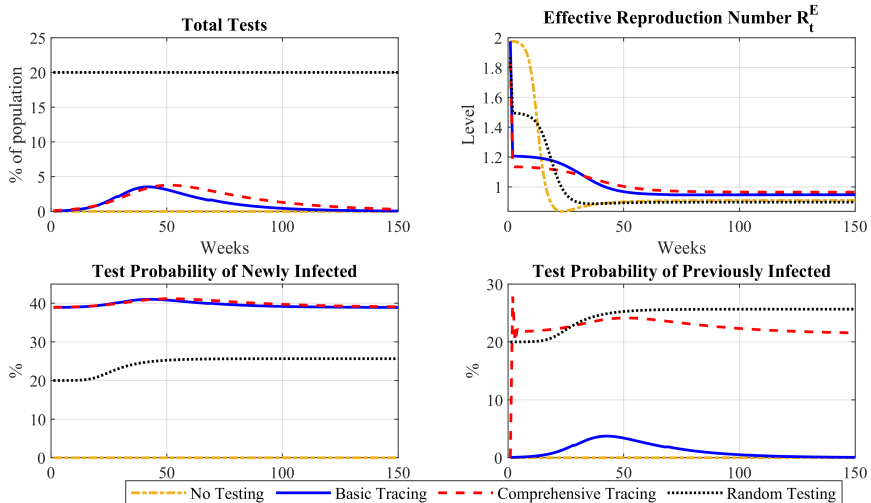
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- Calibration
 - Economic parameters are set in line with literature
 - Probability that interaction results in infection τ is 5% (WHO, 2020)
 - Share of different transmission (consumption, labor, other) is 1/3 (Ferguson et al. 2006)
 - Basic Reproduction number is 2 (e.g. Zhang et al, 2020)
 - Share of infected agents with symptoms is 50% (e.g. Baqae et al., 2020)
 - Agents recover after 18 days on average (WHO, 2020)
 - Infection fatality rate of 0.3% (Hortascu, Liu, Schweg, 2020)
 - False negative outcome $\pi_F = 0$

Contact Tracing with Unconstrained Testing I



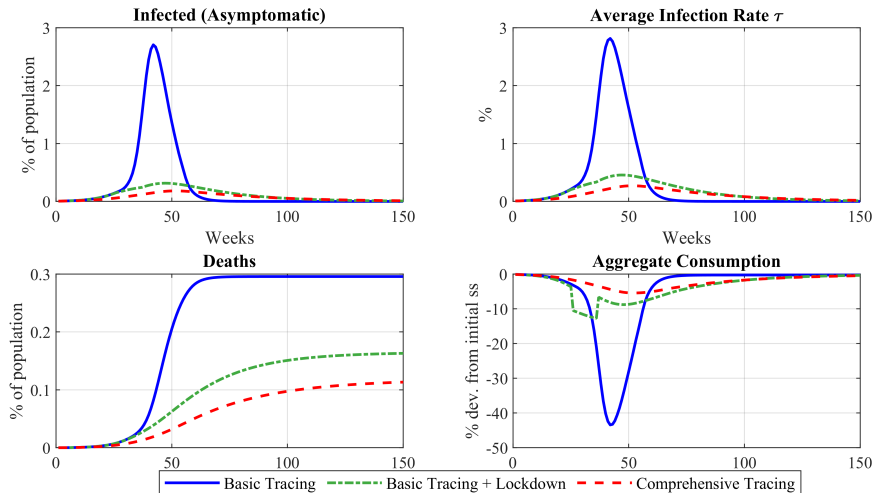
Contact Tracing with Unconstrained Testing II



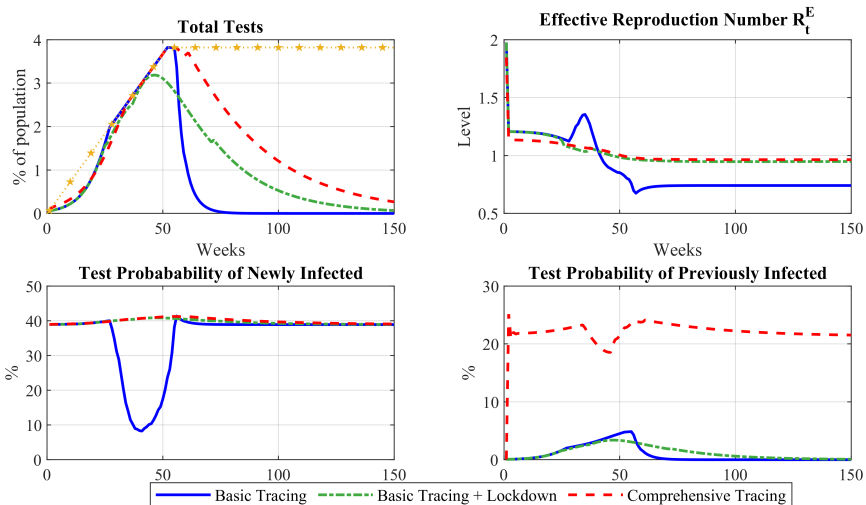
Contact Tracing with Unconstrained Testing: Summary

- Contact tracing does considerably better than random testing
 - Random testing does not leverage the existence of infection chains
 - Contact tracing leads to a sudden, rapid fall in the reproduction number, averting the flare-up of infections
- Basic and comprehensive contact tracing technologies lead to comparable outcomes
 - Similar efficacy in detecting the newly infected
 - effective reproduction number is much more sensitive to catching newly infected than agents who were infected in previous periods [Details](#)

Contact Tracing with Constrained Testing I



Contact Tracing with Constrained Testing II



Contact Tracing with Constrained Testing: Summary

- The comprehensive tracing technology delivers the best outcome
 - Agents infected in period $t - 1$ can be traced using the reconstructed infection chains
 - Early on, more spreaders are quarantined, preventing E_t from getting ahead of Y_t
 - Eventually testing capacity Y_t becomes constrained, lowering the ability of detecting previously infected agents. But the reproduction number hardly budes

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 - Early on, more spreaders are quarantined, preventing E_t from getting ahead of Y_t
 - Eventually testing capacity Y_t becomes constrained, lowering the ability of detecting previously infected agents. But the reproduction number hardly budes
- The basic contact tracing technology alone cannot avert the flare-up of infections
 - Tracing externality causes the testing capacity to become constrained
 - A complementary lockdown, timed to avoid the testing capacity from becoming constrained, averts the collapse of the tracing system and the ensuing deep recession

Concluding Remarks

- Contact tracing is a valuable tool to keep long-lasting epidemics under control
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Concluding Remarks

- Contact tracing is a valuable tool to keep long-lasting epidemics under control
 - Key to its success is the exploiting of the infection chain to trace and isolate asymptomatic spreaders
- However, tracing externality combined with critical bottlenecks of the tracing and testing system may require to complement this tool with a well-timed lockdown
- A general methodology to characterize the network of interactions and to study contact tracing in large set of epi-mac models

Agents with Unknown Health Status

- Susceptible S , untested asymptomatic A and unobserved recovered UR individuals **do not know their health status**
 - Assumption: These agents believe that they are susceptible
 - Conditional this belief, agents compute model-consistent probabilities

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- Agents choose consumption c_t^S and labor n_t^S to maximize utility V_t^S

$$V_t^S = \max_{c_t^S, n_t^S} u(c_t^S, n_t^S) + \beta \left[(1 - \tau_t) V_{t+1}^S + \tau_t \left\{ \pi_{P,t}^T V_{t+1}^P + (1 - \pi_{P,t}^T) V_{t+1}^A \right\} \right]$$
$$\text{s.t.} \quad (1 + \mu_{c,t}^L) c_t^S = w_t^S n_t^S + \Gamma_t^L$$

- Agents expect to be **newly infected with τ_t**
- Newly infected agents get **tested positive with $\pi_{P,t}^T$**
- $\mu_{c,t}$ denotes a tax on consumption (proxy for lockdown) that is rebated Γ_t^L

Agents with Unknown Health Status (cont'd)

- Continuation value conditional of becoming asymptomatic V_t^A :

$$V_t^A = u(\tilde{c}_t^s, \tilde{n}_t^s) + \beta \left[\pi_{IS} V_{t+1}^{IS} + \pi_R V_{t+1}^{UR} + (1 - \pi_{IS} - \pi_R) \left(\pi_{P,t}^A V_{t+1}^P + (1 - \pi_{P,t}^A) V_{t+1}^A \right) \right]$$

- π_{IS} is the probability to get infected-symptomatic
 - π_R is the probability to become unobserved recovered
 - $\pi_{P,t}^A$ is the probability to test positive conditionally on staying asymptomatic
- Continuation value conditional of becoming an unobserved recovered agent V_t^{UR} :

$$V_t^{UR} = u(\tilde{c}_t^s, \tilde{n}_t^s) + \beta V_{t+1}^{UR}.$$

Agents with Known Health Status

- The utility function of tested-positive Agents P

$$V_t^P = \max_{c_t^P, n_t^P} u(c_t^P, n_t^P) + \beta \left[\pi_{IS} V_{t+1}^{IS} + \pi_R V_{t+1}^{OR} + (1 - \pi_{IS} - \pi_R) V_{t+1}^P \right]$$

s.t. $(1 + \mu_c^Q + \alpha \mu_{c,t}^L) c_t^P = w_t^P n_t^P + \Gamma_t^Q,$

- μ_c^Q proxies the effects of imposing a quarantine on individuals' decisions

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- μ_c^Q proxies the effects of imposing a quarantine on individuals' decisions
- Infected symptomatic agents IS

$$V_t^{IS} = \max_{c_t^{IS}, n_t^{IS}} u(c_t^{IS}, n_t^{IS}) + \beta \left[\pi_R V_{t+1}^{OR} + (1 - \pi_R - \pi_D) V_{t+1}^{IS} \right],$$

- Similar budget constraint but penalty on labor $\phi < 1$

Agents with Known Health Status (cont'd)

- Observed recovered agents OR

$$V_t^{OR} = \max_{c_t^{OR}, n_t^{OR}} u(c_t^{OR}, n_t^{OR}) + \beta V_{t+1}^{OR}$$

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⇒ To close the model, we need calculate following key objects Law of Motions for Types

- τ_t : Average probability of getting infected
- Probabilities of testing positive for newly infected $\pi_{P,t}^T$ and previously infected asymptomatic $\pi_{A,t}^T$

Dynamics of Agents' Types I

- The law of motion for the share susceptible agents reads

$$S_{t+1} = S_t - T_t$$

- Newly infected subject in period t

$$T_t = \tau_t \cdot S_t$$

- Untested asymptomatic agents evolves according to the law of motion

$$I_{t+1}^A = (1 - \pi_{P,t}^T) T_t + (1 - \pi_{P,t}^A)(1 - \pi_{IS} - \pi_R) I_t^A$$

Dynamic of Agents' Types II

- The pool of tested positive subjects is given by

$$P_{t+1} = (1 - \pi_{IS} - \pi_R)P_t + \pi_{P,t}^T T_t + \pi_{P,t}^A (1 - \pi_{IS} - \pi_R)I_t^A$$

- The pool of infected symptomatic people evolves as follows:

$$I_{t+1}^S = (1 - \pi_R - \pi_D)I_t^S + \pi_{IS}(I_t^A + P_t)$$

Back

Microfoundation of SIR and Macro-SIR Models

- Average probability of getting infected τ_t for a susceptible individual is as follows:

$$\tau_t = \sum_{k=0}^{\varphi_C(c_t^S)} \underbrace{\left[1 - (1 - \tau)^k \right]}_{\text{Prob. of getting infected cond. on } k \text{ interactions}} \times \underbrace{f_{t,c}(k)}_{\text{Prob. of } k \text{ interactions}}$$

- Linearized version is isomorphic to SIR and Macro-SIR models

$$\tau_t \approx \mathbb{E} \left[\varphi_C c_t^S \left(C_t^A / C_t \right) \right]$$

- Extending this expression with labor and other interactions nests this to the formulation of Eichenbaum, Rebelo and Trabandt (2020) [Back](#)

Effective Reproduction Number and Contact Tracing

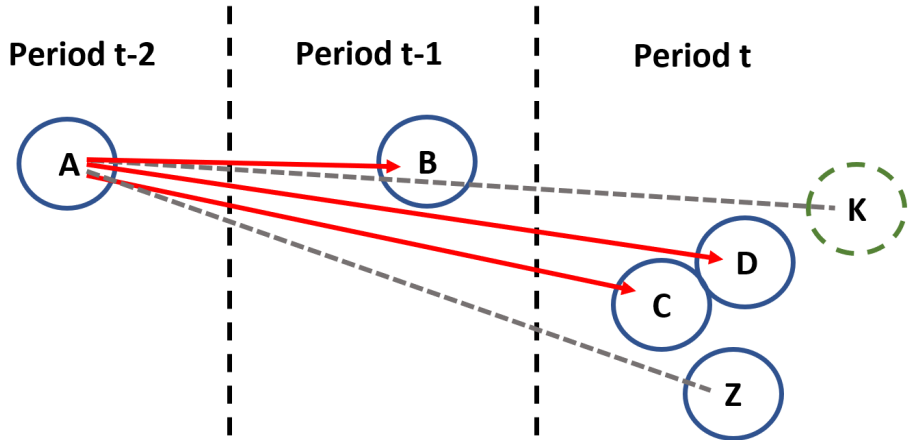
- Key epidemiological number: **Effective Reproduction Number**

$$\begin{aligned} R_t^E &= (1 - \pi_{t-1}^T) [\tau_t + (1 - \pi_{IS} - \pi_R) (1 - \pi_t^A) \tau_{t+1} + \\ &\quad (1 - \pi_{IS} - \pi_R)^2 (1 - \pi_t^A) (1 - \pi_{t+1}^A) \tau_{t+2} + \dots] \\ &= (1 - \pi_{P,t-1}^T) \sum_{j=0}^{\infty} \left(\tau_{t+j} (1 - \pi_{IS} - \pi_R)^j \prod_{k=0}^j (1 - \pi_{P,t+k}^A) \right) \end{aligned}$$

- Testing infrastructure affects R_t^E directly via **testing newly infected** π_{t-1}^T and **testing asymptomatic infected earlier** π_t^A
- Basic technology operates mostly over π_{t-1}^T , while comprehensive relies also on π_t^A
- Lockdowns lower the reproduction number via the infection rate τ_t [Back](#)

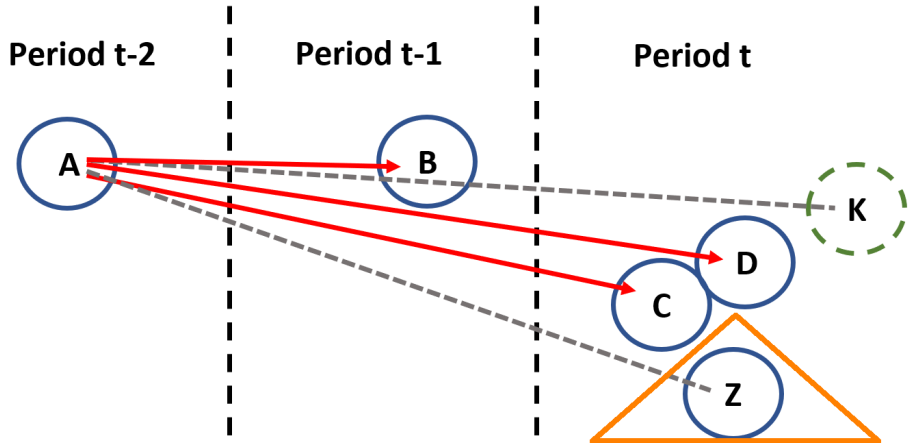
Example of a Network of Interactions

- Network of interactions and infection chain of Agent A



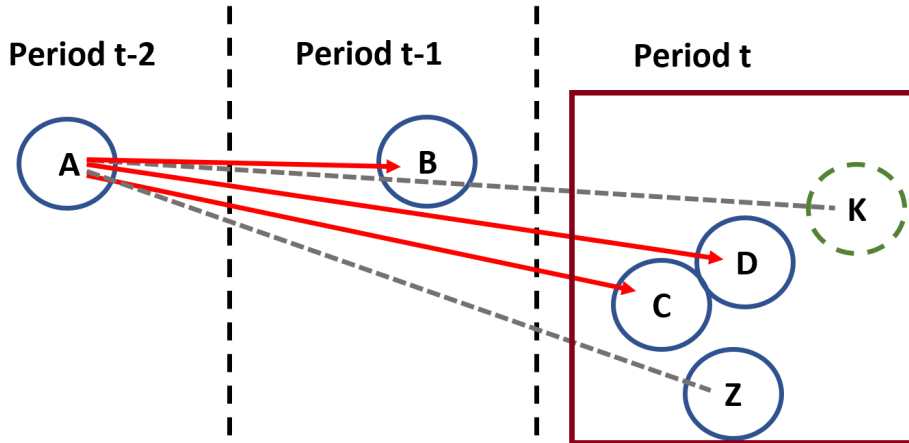
Example of a Network of Interactions

- **Random meetings** with asymptomatic agents from different infection chain [Back](#)



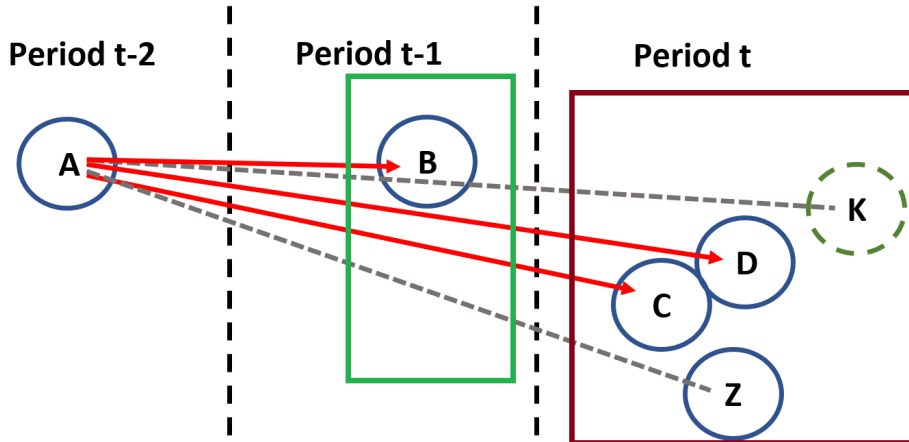
Example of a Network of Interactions

- **Basic tracing:** Current week contacts [Back](#)



Example of a Network of Interactions

- **Comprehensive tracing**: Current week contacts and previous week contacts [Back](#)



What Type of Lockdowns?

- Lockdowns are typically enacted in response to flare-ups of infection – often to prevent hospitals from becoming overburdened.
- We suggest a different strategy: moderate lockdowns as preemptive tools
 1. These lockdowns are generally **less stringent**
 2. The timing of these lockdowns is chosen so as to **move ahead of the infection curve**
 3. The objective is to **keep the testing system viable while policymakers ramp up the testing capacity**

Back

Tracing Probabilities - Basic Tracing

- Agents get traced if at least one of their k asymptomatic contacts becomes symptomatic: $1 - (1 - \pi_{IS})^k$
- Tracing probability for **previously infected asymptomatic agents** $\pi_{C,t}^A$

$$\pi_{C,t}^A = \sum_{k=0}^{K(c_t^S, n_t^S)} \underbrace{\left[1 - (1 - \pi_{IS})^k \right]}_{\text{Prob. of contact getting symptomatic cond. on } k \text{ contacts}} \times \underbrace{f_t(k)}_{\text{Prob. of } k \text{ contacts}}$$

Tracing Probabilities - Basic Tracing (cont'd)

- Tracing probability for a newly infected agent T is different $\pi_{C,t}^A$

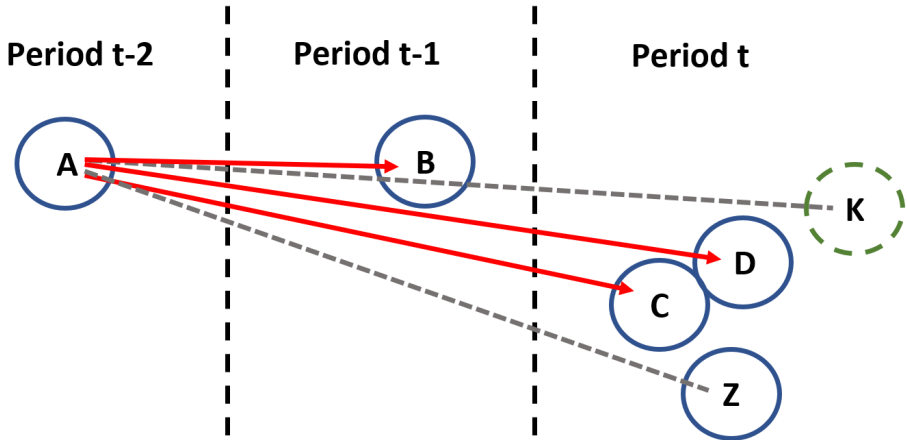
$$f_t^T(k) = \frac{f_t(k)\tilde{\tau}(k)}{\tau_t} = \frac{f_t(k) \overbrace{\left[1 - (1 - \tau)^k\right]}^{\text{Probability to get at least 1 infectious contact}}}{\tau_t}$$

- Characterization of the probability for a newly infected individual to be traced

$$\pi_{C,t}^T = \sum_{k=0}^{K(c_t^S, n_t^S)} \underbrace{\left[1 - (1 - \pi_{IS})^k\right]}_{\text{Prob. of contact getting symptomatic cond. on k contacts}} \times \underbrace{f_t^T(k)}_{\text{Prob. of k contacts for a newly infected}}$$

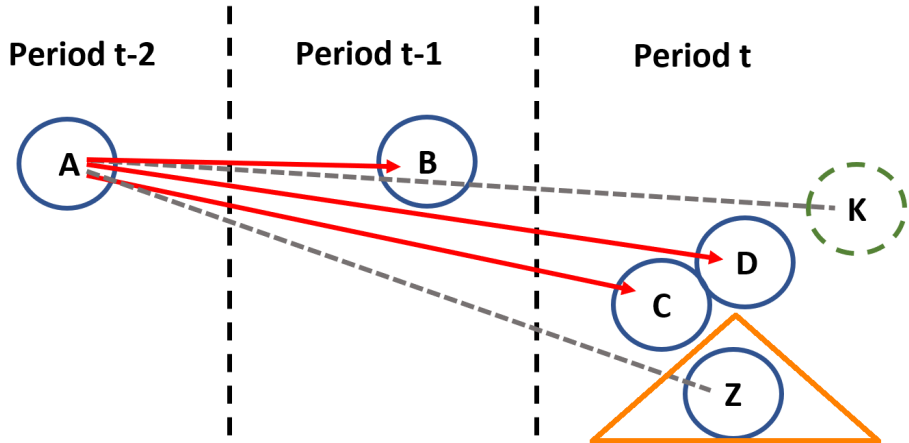
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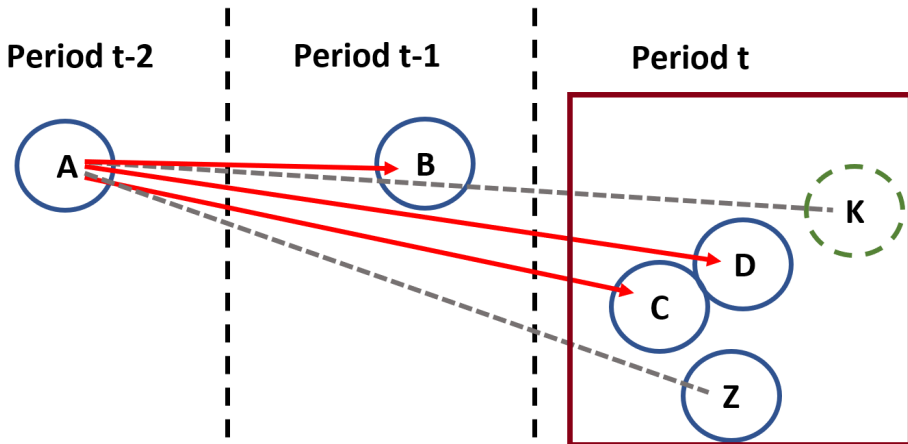
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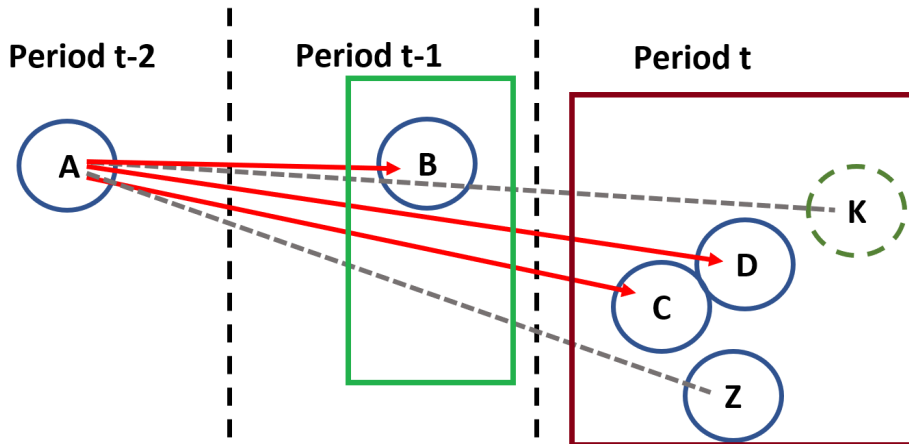
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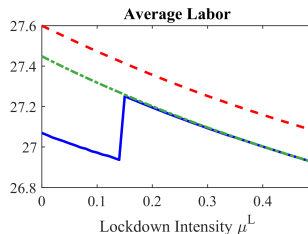
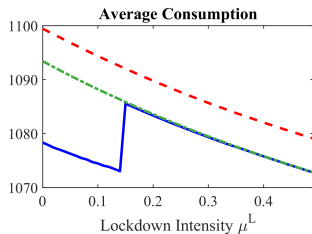
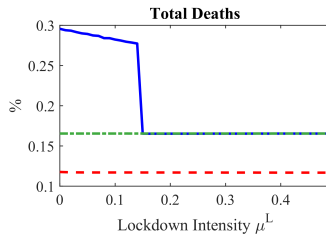
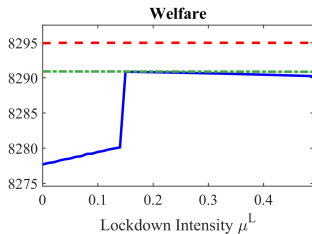


Example of a Network of Interactions

- **Comprehensive tracing**: Current week contacts and previous week contacts



Optimal Stringency of Lockdowns



Random Meetings are Rare

