## Multimodality in Macro-Financial Dynamics

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The views expressed here are the authors' and are not representative of the views of the International Monetary Fund, the Federal Reserve Bank of New York or of the Federal Reserve System



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## Outline

#### 1. Introduction

2. Data

- 3. Methodology
- 4. Empirical Results
- 5. Simulated pseudo out-of-sample forecast evaluation

6. Euro area

7. Economics

8. Density Impulse Response Function



#### **Motivation**

- Macro-financial interactions are likely to be nonlinear
- Nonlinearities are pervasive feature in firms
  - Narrative and anecdotal evidence, especially at the time of financial crises
  - Macro-finance "structural" models, often featuring occasionally binding constraints for intermediaries, households and firms
- Need for descriptive and robust facts that can inform and validate theories
  - Many models of nonlinearities, each with his own mechanisms
  - No consensus has emerged in the literature



#### February 10, 2009

"Last Friday we learned that the economy had lost three million jobs last year, and an additional 600,000 just last month...Instead of catalyzing recovery, the financial system is working against recovery. And at the same time, the recession is putting greater pressure on banks.

This is a dangerous dynamic, and we need to arrest it...Today, as Congress moves to pass an Economic Recovery Plan that will help create jobs and lay a foundation for stronger economic future, we [at the US Treasury] are outlining a new Financial Stability Plan...Our plan will help restart the flow of credit, clean up and strengthen our banks, and provide critical aid for homeowners and for small businesses...

We believe that action has to be sustained until recovery is firmly established. In the United States in the 30s, Japan in the 90s, and in other cases around the world, previous crises lasted longer and caused greater damage because governments applied the brakes too early. We cannot make that mistake."

#### Treasury Secretary Geithner Introduces Financial Stability Plan

## **Macro-Finance Literature and Nonlinearity**

- He Krishnamurthy (2013) and Brunnermeier Sannikov (2014)
  - Present continuous time theories of macro-financial amplification with occasionally binding constraints
  - All shocks are conditionally gaussian, but vol and drift depend on state variables in highly nonlinear ways
- Adrian Boyarchenko (2016) and Adrian Duarte (2018)
  - present macro-financial theories with value-at-risk constraints
  - VaR constraints directly tie conditional means and vols together, even when constraints always bind
- Gertler Kiyotaki Prestipino (2016, 2019)
  - Present macro-finance models with multiplicity



In a "run" equilibrium banks stop financial intermediation

#### From linear to nonlinear reduced form models

#### Vector Autoregressive models are very flexible and powerful:

- Forecasting, scenario and structural analysis
- Establish stylized fact to guide and validate economic (behavioral) modeling
- The only limit is linearity

#### Extensions to nonlinear setting, several approaches:

- Smooth Transition Models: Terasvirta and Anderson (1992); van Dijk, Terasvirta, and Franses (2002); Kilian and Taylor (2003)
- Time Varying parameters VAR and Stochastic Volatility: Cogley and Sargent (2005); Primiceri (2005); Cogley and Sbordone (2008); D'Agostino, Gambetti, and Giannone (2013); Del Negro and Primiceri (2015); Carriero, Clark, and Marcellino (2018)
- Thresholded VAR: Altissimo and Violante (2001); Aikman, Lehnert, Liang, and Modugno (2016)
- Markov switching: Hamilton (1989); Sims and Zha (2006); Chang, Choi, and Park (2017); Hubrich and Tetlow (2015)
- Quadratic autoregressions and pruning: Aruoba, Bocola, and Schorfheide (2017)
- For a survey see Kilian and Lutkepohl (2018).

#### This paper:

- Estimation the one-step ahead predictive density p(yt|yt-1) based on Kernel smoothing
- Montecarlo simulation for multistep predictions



#### What we do

- Estimate the one step ahead predictive density using Kernel smoothing
- Discretize (and bound) the support and simulate predictive paths
- Assess forecasting performances
  - Compare in-sample and out-of-sample estimates
  - Evaluate accuracy (mean square forecast errors and predictive scores) and calibration (PIT)
- Density Impulse Responses: perturb initial conditions and track the dynamic effects on the distribution of all the possible outcomes



#### What we find

- Out-of-sample forecasts are reasonably accurate and calibrated
   ⇒ The empirical model is able to capture in a parsimonious way the
   salient features of a nonlinear dynamic economy
- 2. Evidence of multiple modes
  - Bi-modal distributions when financial conditions are tight
- 3. Density impulse responses are nonlinear
  - Policies affecting financial conditions can make the economy coordinate on good (or bad) equilibria



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#### Measuring economic and financial conditions

- Chicago Fed National Activity Index (CFNAI)
  - Common factor extracted from 85 indicators of economic activity
    - 1. Production and income
    - 2. Employment, unemployment, and hours
    - 3. Personal consumption and housing
    - 4. Sales, orders, and inventories
  - Principal component (Stock and Watson, 1999)
- National Financial Conditions Index (NFCI)
  - Common factor extracted from 105 financial indicators
    - 1. Money markets
    - 2. Debt markets
    - 3. Equity markets
    - 4. Traditional and shadow banking
  - Quasi Maximum Likelihood (Doz, Giannone, and Reichlin, 2012)



Quarterly averages since 1973Q1

#### Measuring economic and financial conditions







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#### The estimator

• Data: 
$$y_t = (y_{1,t}, ..., y_{n,t})', x_t = (y'_{t-1}, ..., y'_{t-p})'$$

Estimator:

$$\hat{\rho}(y|x) = \frac{\frac{1}{T-\rho}\sum_{t=\rho+1}^{T} K_{\omega_{y}}^{y}(y-y_{t}) K_{\omega_{x}}^{x}(x-x_{t})}{\frac{1}{T-\rho}\sum_{t=\rho+1}^{T} K_{\omega_{x}}^{x}(x-x_{t})}$$

Kernels: multivariate (independent) normal

$$\mathcal{K}_{\omega_{y}}^{y}(y-y_{t}) = \prod_{i=1}^{n} \frac{1}{\omega_{y_{i}}} \varphi\left(\frac{y_{i}-y_{i,t}}{\omega_{y_{i}}}\right) \qquad \mathcal{K}_{\omega_{x}}^{x}(x-x_{t}) = \prod_{j=1}^{np} \frac{1}{\omega_{x_{j}}} \varphi\left(\frac{x_{j}-x_{j,t}}{\omega_{x_{j}}}\right)$$

Bandwidths: 
$$\omega_{y_i} = c_{0,i}\sigma_{y_i}, \ \omega_{y_i} = c_{1,j}\sigma_{x_j}$$

- Set  $c_{0,i} = c_{1,j} = c$  for all i, j
- Estimate c it by maximizing out-of-sample predictive accuracy

#### Discuss alternatives:

- Splines: Gallant, Rossi, and Tauchen (1993)
- Bayesian non-parametric: Norets and Pati (2017), Sims (2000)
- Quantile or distributional regression: Koenker, Leorato, and Peracchi (2013), Adrian, Boyarchenko, and Giannone (2019)



## **Density Impulse Responses**

- Comparison of a baseline forecast with a counterfactual forecast
- Perturb initial conditions and track the dynamic effects

Gallant, Rossi, and Tauchen (1993) Koop, Pesaran, and Potter (1996) Potter (2000)



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#### **US: Bandwidth selection**









## U.S.: Marginal quantiles, 1 quarter ahead







### U.S.: Marginal quantiles, 1 quarter ahead







## U.S.: Marginal quantiles, 1 quarter ahead







#### U.S.: Marginal quantiles, 2 quarters ahead







#### U.S.: Marginal quantiles, 2 quarters ahead







### U.S.: Marginal quantiles, 2 quarters ahead







#### U.S.: Marginal quantiles, 4 quarters ahead







#### U.S.: Marginal quantiles, 4 quarters ahead







#### U.S.: Marginal quantiles, 4 quarters ahead







#### U.S.: Marginal quantiles, 8 quarters ahead







#### U.S.: Marginal quantiles, 8 quarters ahead







#### U.S.: Marginal quantiles, 8 quarters ahead



























## **U.S.: Joint distribution of CFNAI and NFCI**





## **U.S.: Joint distribution of CFNAI and NFCI, OOS**





## **U.S.: Joint distribution of CFNAI and NFCI**




# **U.S.: Joint distribution of CFNAI and NFCI, OOS**





# **U.S.: Joint distribution of CFNAI and NFCI**





# **U.S.: Joint distribution of CFNAI and NFCI, OOS**





# **U.S.: Joint distribution of CFNAI and NFCI**







# **U.S.: Joint distribution of CFNAI and NFCI, OOS**





















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#### **Forecasting the Great Recession**





# **U.S.: Inspecting the mechanism**







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## **Scores (Joint)**









## Log scores, difference from NL-VAR(1)

Model	h=1	h=2	h=3	h=4	h=8
NL-VAR(1)	-2.27	-1.66	-1.61	-1.45	-1.55
(L-)VAR(1) - NL-VAR(1)	0.935	-0.225	-0.733	-0.979	-1.11
SE	(0.983)	(0.146)	(0.223)	(0.287)	(0.281)
(L-)VAR(2) - NL-VAR(1)	1.10	-0.168	-0.571	-0.907	-1.02
SE	(1.092)	(0.131)	(0.126)	(0.273)	(0.236)
NL-VAR(2) - NL-VAR(1)	1.08	0.053	0.031	-0.363	-0.209
SE	(1.276)	(0.133)	(0.106)	(0.309)	(0.133)



## **Scores (CFNAI)**









## **Scores (NFCI)**









## **Joint CRPS**









## Mean joint CRPS, difference from NL-VAR(1)

Model	h=1	h=2	h=3	h=4	h=8
NL-VAR(1)	0.328	0.412	0.439	0.454	0.488
(L-)VAR(1)	-0.003	0.033	0.075	0.098	0.127
SE	(0.02)	(0.017)	(0.008)	(0.01)	(0.023)
(L-)VAR(2)	-0.028	0.019	0.063	0.082	0.102
SE	(0.031)	(0.019)	(0.008)	(0.009)	(0.015)
NL-VAR(2)	-0.003	0.005	0.004	-0.001	-0.012
SE	(0.017)	(0.013)	(0.01)	(0.01)	(0.007)



# **CRPS (CFNAI)**









# **CRPS (NFCI)**









# **PITS (CFNAI)**







CFNAL h+2

Collins.

# PITS (NFCI)







CFNAI, h+2

0.5 0.6 0.7 0.8 0.9

PIT

0. NAPCT

NL-VARO

## Median forecasts (CFNAI)









## Median forecasts (NFCI)



# **Squared errors (CFNAI)**









# **Squared errors (NFCI)**



1.1150

# MSFE of median forecast (CFNAI), difference from NL-VAR(1)

Model	h=1	h=2	h=3	h=4	h=8
NL-VAR(1)	0.299	0.487	0.508	0.548	0.599
(L-)VAR(1) - NL-VAR(1)	-0.084	-0.035	0.076	0.085	0.035
SE	(0.081)	(0.081)	(0.03)	(0.037)	(0.029)
(L-)VAR(2) - NL-VAR(1)	-0.138	-0.106	0.019	0.035	0.025
SE	(0.093)	(0.093)	(0.025)	(0.025)	(0.02)
NL-VAR(2) - NL-VAR(1)	-0.111	-0.040	0.016	0.003	-0.014
SE	(0.076)	(0.074)	(0.034)	(0.011)	(0.013)



# MSFE of median forecast (NFCI), difference from NL-VAR(1)

Model	h=1	h=2	h=3	h=4	h=8
NL-VAR(1)	0.138	0.201	0.223	0.246	0.275
(L-)VAR(1) - NL-VAR(1)	-0.063	-0.044	-0.009	0.007	0.135
SE	(0.042)	(0.034)	(0.028)	(0.047)	(0.095)
(L-)VAR(2) - NL-VAR(1)	-0.066	-0.041	-0.011	-0.001	0.081
SE	(0.044)	(0.029)	(0.023)	(0.033)	(0.049)
NL-VAR(2) - NL-VAR(1)	-0.007	0.012	0.002	-0.011	-0.013
SE	(0.018)	(0.02)	(0.011)	(0.009)	(0.007)



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Measuring economic and financial conditions: Euro area

- Economic activity: GDP growth
- Financial Conditions: Composite Indicator of Systemic Stress (CISS)
- Quarterly data since 1999



## Euro Area: Marginal quantiles, 1 quarter ahead







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#### Euro Area: Marginal quantiles, 8 quarter ahead







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#### Euro Area: Joint distribution, 1-8 quarters ahead







#### Euro Area: Joint distribution, 1-8 quarters ahead







#### Euro Area: Joint distribution, 1-8 quarters ahead












































































































































































































































































































































































































































































































































































































































































































































































































































































































































































































































































































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# **Multiplicity in Macroeconomics**

- Much research in the 80s and 90s
  - Diamond (1982): thick market externalities
  - Bryant (1983): technological complementarities
  - Diamond and Dybvig (1983): bank runs
  - Murphy, Shleifer, and Vishny (1989): demand spillovers
- But multiplicity has never been shown empirically
  - Morris and Shin (2000, 2003) propose unique equilibrium via imperfect knowledge while preserving amplification
  - hugely impactful literature
- Our evidence suggests a reconsideration of multiple equilibria


#### Is Bimodality Evidence of Multiplicity?

- Recently, policy has aggressively counteracted "bad" equilibria
- But bad policy can lead to the bad equilibrium, as history has shown
- However, bimodality could be an expression of nonlinearity



#### **Macro-Finance Literature and Nonlinearity**

- He Krishnamurthy (2013) and Brunnermeier Sannikov (2014)
  - Present continuous time theories of macro-financial amplification with occasionally binding constraints
  - All shocks are conditionally gaussian, but vol and drift depend on state variables in highly nonlinear ways
- Adrian Boyarchenko (2016) and Adrian Duarte (2018)
  - present macro-financial theories with value-at-risk constraints
  - VaR constraints directly tie conditional means and vols together, even when constraints always bind
- Gertler Kiyotaki Prestipino (2016, 2019)
  - Present macro-finance models with multiplicity
  - In a "run" equilibrium banks stop financial intermediation
- All these theories produce multimodal distributions, but only the latter relies on multiplicity

#### **Implications for Macro-Finance**

- We only detect multiplicity when financial conditions are included
- This suggests that macro-financial dynamics are key
- However, the data says that multimodality only occurs in recessions, in normal times the conditional density is unimodal, and it is unimodal in the long run



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#### **Forecasting the Great Recession**









Control and



























### **CFNAI Density Impulse Responses: Selected** quantiles











### NFCI Density Impulse Responses: Selected Quantiles





### Density Impulse Responses: Expected shortfall and longrise











Contro was





















### **CFNAI Density Impulse Responses: Selected** quantiles (shifted)











### NFCI Density Impulse Responses: Selected Quantiles (shifted)









### Density Impulse Responses: Expected shortfall and longrise (shifted)







#### **Drawbacks and further research**

- Beyond two dimensions: Curses of dimensionality
  - Accuracy of the smoothing Kernel estimates deteriorates quickly with the dimension
  - Direct sampling becomes computationally too demanding
- Solutions
  - Bayesian estimation of conditional densities + Shrinkage priors
  - Penalized quantile regression or distributional regression + smoothing
    - Recursive models (marginal, joint)
    - Conditionally specified models combined with a "pseudo" Gibbs sampler
    - Combination/ensemble across different permutations



#### Conclusion

- We present a parsimonious and computationally straightforward method to estimate nonlinear system dynamics
- We document that the joint density of economic growth and financial conditions features bimodality in bad times
- We argue that economic theory has to consider macro-financial models that feature multiple equilibria in bad times














































































































































































































































































































































































## Alternative FCI: T10Y-T3M





### **Alternative FCI: AAA-T10Y**





#### Alternative FCI: BAA - AAA





## **Alternative FCI: Goldman Sachs FCI**





## Alternative FCI: GZ Excess Bond Premium





## Alternative FCI: GZ spread





## Alternative FCI: IMF Global FCI





## Alternative FCI: IMF US FCI





## Alternative FCI: JLN, 1 month





## Alternative FCI: JLN 3 months





## Alternative FCI: JLN 12 months





## Alternative FCI: Kansas City Fed FCI




## Alternative FCI: St. Louis FCI





## Alternative FCI: T10Y - T2Y





## **Alternative FCI: VXO**





## References

- ADRIAN, T., N. BOYARCHENKO, AND D. GIANNONE (2019): "Vulnerable Growth," *American Economic Review*, 109(4), 1263–1289.
- AIKMAN, D., A. LEHNERT, J. N. LIANG, AND M. MODUGNO (2016): "Financial Vulnerabilities, Macroeconomic Dynamics, and Monetary Policy," Finance and Economics Discussion Series 2016-055, Board of Governors of the Federal Reserve System (US).
- ALTISSIMO, F., AND G. L. VIOLANTE (2001): "The non-linear dynamics of output and unemployment in the U.S," *Journal of Applied Econometrics*, 16(4), 461–486.
- ARUOBA, S. B., L. BOCOLA, AND F. SCHORFHEIDE (2017): "Assessing DSGE model nonlinearities," *Journal of Economic Dynamics and Control*, 83(C), 34–54.

