Can the cure kill the patient?
Corporate credit interventions and debt overhang

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Motivation

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New policy tool: **Business Funding Programs (BFPs)** [BFPs in the US]

- Corporate Credit Facilities (Fed)
- Main Street Lending Program (Fed)
- Paycheck Protection Program (Treasury+Fed)

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Q2 How should BFPs be designed?

untargeted or targeted? loans vs. grants/debt forbearance/equity?

Overview

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Structural model:

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Roadmap

1. Model and estimation

2. Business Funding Programs when financial markets function normally

3. Business Funding Programs during sudden stops

1. Model and estimation

Model building blocks

[math]

- *ak* production with convex adjustment cost function Φ (Hayashi, 1982)

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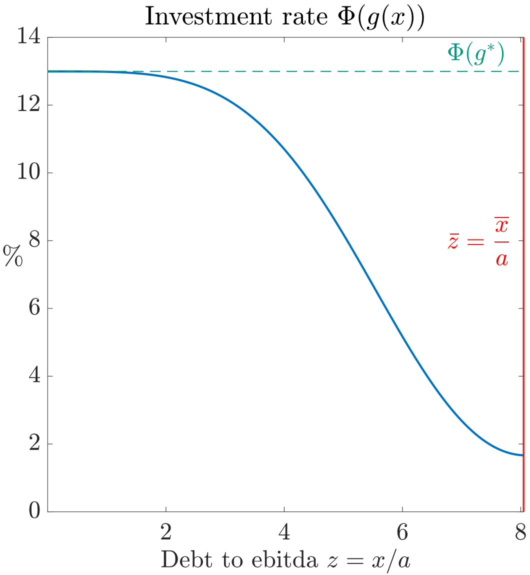
- leverage $x := b/k$ sufficient statistic for a given firm

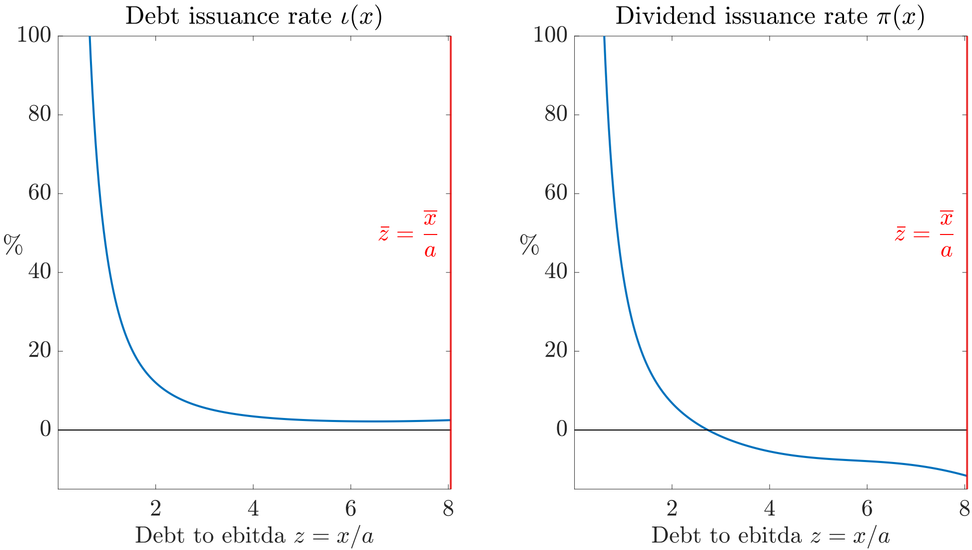
Investment and debt overhang

[math]



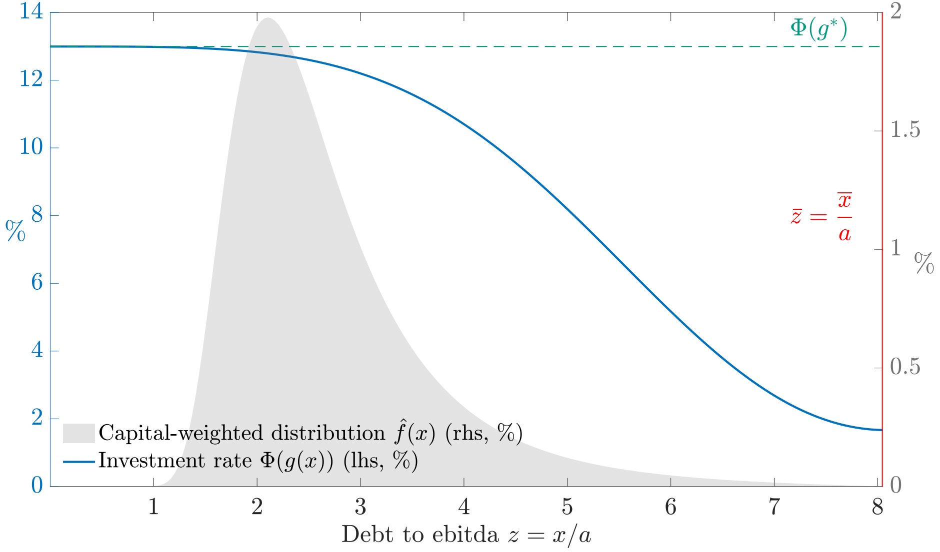
Financing policies

[math]



Long run size-weighted leverage distribution

[math]



Estimation

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- Calibrate 6 parameters:

$$r = \kappa = 5\%, \quad \delta = 10\%, \quad \Theta = 35\%, \quad 1/m = 10 \text{ years}, \quad 1 - \alpha_b = 85\%, \quad 1 - \alpha_k = 67\%.$$

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  σ (TFP shock vol.)
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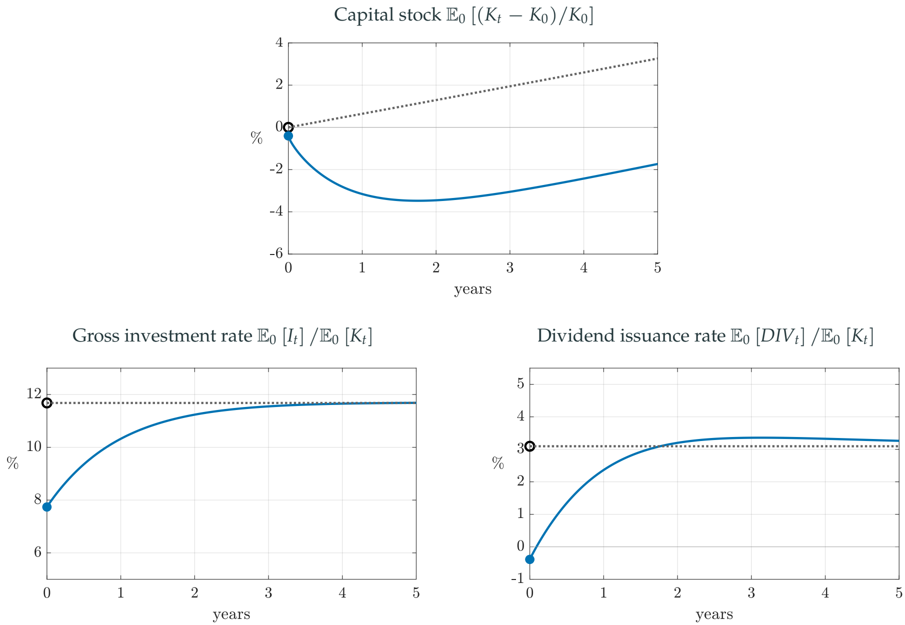
Moment	Description	Targeted?	Data	Model
$\hat{\Phi}$	average investment rate	✓	9.48	9.47
$\hat{z}$	average debt/ebitda	✓	2.71	2.71
$\frac{cov(\Phi(x),z(x))}{var(z(x))}$	slope of inv. w.r.t debt/ebitda	✓	−3.66	−3.66

Calibration of crisis state

- Transient aggregate shock with exponentially distributed length (1 year)
- Productivity drop and risk-price increase
- Outcomes of focus $\mathbb{E}_0 [K_t], \mathbb{E}_0 [Y_t]$

	EBITDA jump $100 \cdot (Y_{0+}/Y_0 - 1)$	Stock price jump $100 \cdot (E_{0+}/E_0 - 1)$	Credit spread jump $100 \cdot (\overline{cs}_{0+} - \overline{cs}_0)$
A. Data			
Before Fed announcement	-25.0	-34.0	3.57 / 7.30
B. Model			
Cash flow shock	-25.0†	-5.2	0.12
Cash flow + risk premium shock	-25.0†	-34.0†	3.23

† = targeted moment.



Capital stock $\mathbb{E}_0\left[(K_t - K_0)/K_0\right]$
%
years
Gross investment rate $\mathbb{E}_0\left[I_t\right]/\mathbb{E}_0\left[K_t\right]$
%
years
Dividend issuance rate $\mathbb{E}_0\left[DIV_t\right]/\mathbb{E}_0\left[K_t\right]$
%
years

2. BFPs when financial markets function normally

Result 1: irrelevance theorem

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Result

Suppose that, in the crisis state

(a) financial markets continue to function normally

(b) the government offers extra funding to firms at market prices

(c) the intervention does not change investors' SDFs

Then, relative to the laissez-faire, all outcomes are unchanged.

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Funding program can

- consist of debt, equity, any hybrid instrument
- be implemented via (fairly priced) government-backed credit guarantees
- be unconditional or conditional on leverage

Result 2: subsidized loan programs depress investment

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Suppose that, in the crisis state

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Then, relative to the laissez-faire, future debt issuance is higher and future investment is lower.

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$$\tilde{\iota}(x) = \underbrace{\frac{\Theta\kappa}{-d'(x)}}_{\text{tax motive}} + \underbrace{\frac{\left(\tilde{R}_d(x) - R_d(x)\right) d(x)}{-d'(x)}}_{\text{arbitrage motive}} > \iota(x)$$

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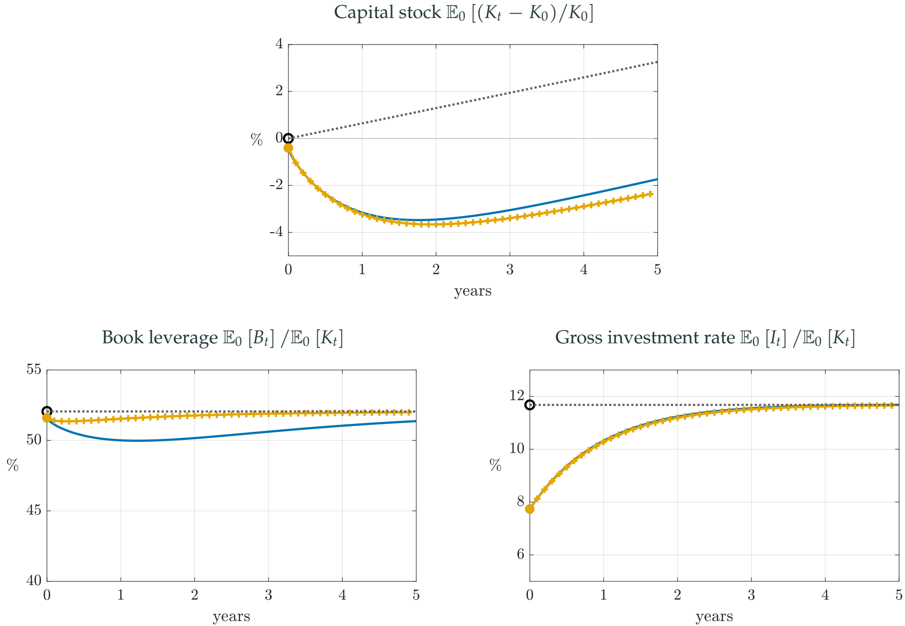
More issuance $\implies$ distribution $\hat{f}_t(x)$ shifts right $\implies$ lower investment


Capital stock $\mathbb{E}_0\left[(K_t - K_0)/K_0\right]$
%
4
2
0
-2
-4
0
1
2
3
4
5
years
Book leverage $\mathbb{E}_0\left[B_t\right]/\mathbb{E}_0\left[K_t\right]$
%
55
50
45
40
0
1
2
3
4
5
years
Gross investment rate $\mathbb{E}_0\left[I_t\right]/\mathbb{E}_0\left[K_t\right]$
%
12
10
8
6
0
1
2
3
4
5
years

Result 3: expansionary announcement effects

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Suppose that, in the crisis state

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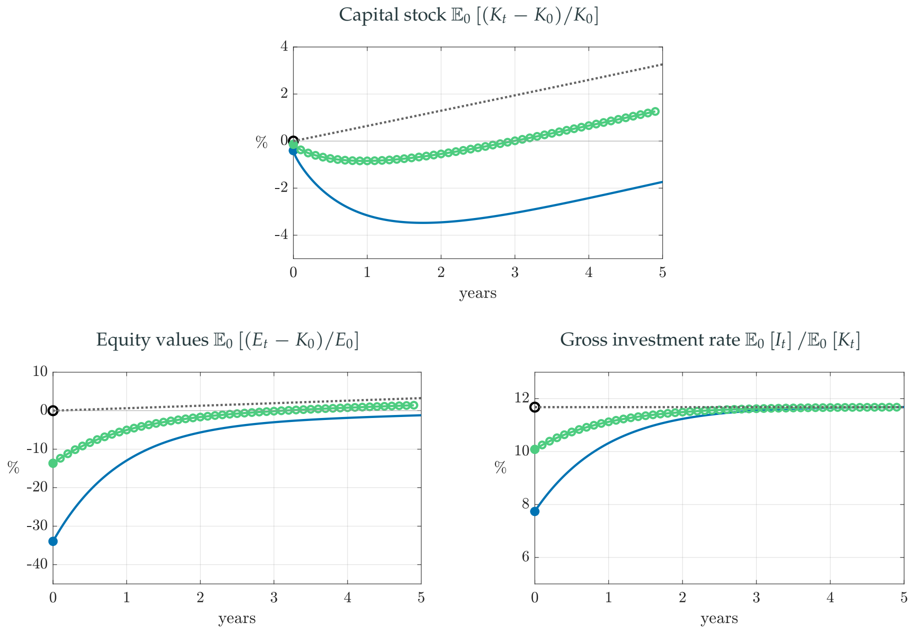
Caveat: with segmented markets, if intervention also leads to $\uparrow$ in $\tilde{R}_d(x) - R_d(x)$ ...

- on impact, always $R_e(x) \downarrow$, $q \uparrow$, investment $\uparrow$
- but over time, because $\tilde{R}_d(x) \downarrow$, corporate leverage $\uparrow$, investment $\downarrow$.



Capital stock $\mathbb{E}_0\left[(K_t - K_0)/K_0\right]$
%
years
Equity values $\mathbb{E}_0\left[(E_t - K_0)/E_0\right]$
%
years
Gross investment rate $\mathbb{E}_0\left[I_t\right]/\mathbb{E}_0\left[K_t\right]$
%
years

4. BFPs during a sudden stop

A recession with a sudden stop

- While the economy is in the crisis state, assume that:

$$\text{dividends } \pi_t \geq 0, \quad \text{debt issuance } \iota_t \leq 0$$

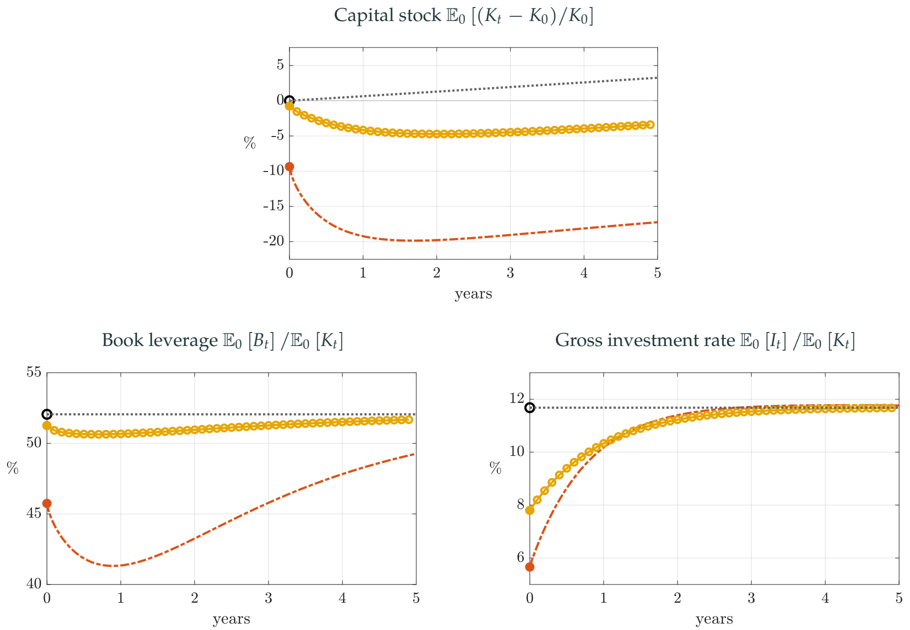
- Compared to normally functioning capital markets

  investment is cash flow constrained

  default boundary $\bar{x} \downarrow \implies$ wave of defaults on impact

- **Loan programs** in the crisis state become unambiguously beneficial for investment

  despite their debt overhang effects


Capital stock $\mathbb{E}_0[(K_t - K_0)/K_0]$
%
5
0
-5
-10
-15
-20
0 1 2 3 4 5
years
Book leverage $\mathbb{E}_0[B_t]/\mathbb{E}_0[K_t]$
%
55
50
45
40
0 1 2 3 4 5
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12
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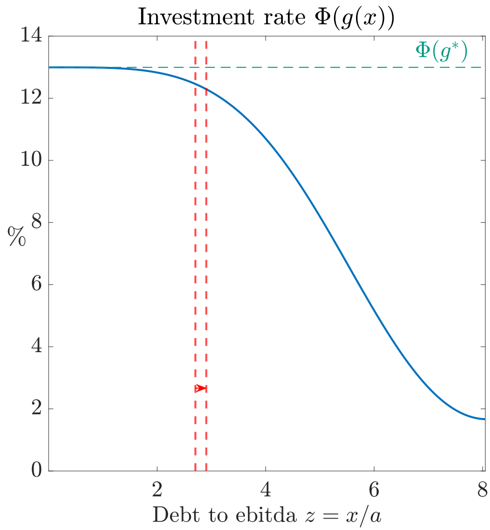
years

Why are BFPs' debt overhang effects not larger?



BFP loans move the debt/ebitda ratio:

$$z_t = \frac{b_t}{ak_t} \quad \rightarrow \quad z'_t = \frac{b_t + \overbrace{(1/\chi)\,(a - \underline{a})\,k_t}^{\text{amount borrowed}}}{ak_t}$$

$$\approx 2.20 \qquad\qquad = z_t + \frac{1}{\chi}\left(1 - \frac{\underline{a}}{a}\right)$$

$$= z_t + 0.25 \approx 2.45$$

Small move, in a region where the slope of investment is not steep.

Alternative Program Designs

- Targeted loan programs
  - implemented via loans extended at a fixed price (selection effect)
  - significant improvement in program efficiency
  - eliminates incentive to over-issue for low-leverage firms

- Loans with dividends/share buy-back restrictions
  - limits dynamic commitment problem
  - moderate improvement in program efficiency (since few firms are constrained)

- Grants program
  - similar to PPP
  - much lower return on tax payer dollars than subsidized loans

Key take-aways

Conclusion

Novel policy tool: **Business Funding Programs (BFPs).**

Q1 What is the effect of BFPs on corporate financing, default, and investment decisions?

perfect financial markets

ambiguous effects

subsidized funding $\rightarrow$ investment $\downarrow$ — but small quantitative effects

sudden stop

in the short-run, investment $\uparrow\uparrow$

in the long-run, investment $\downarrow$ — but small quantitative effects

Q2 How should BFPs be designed?

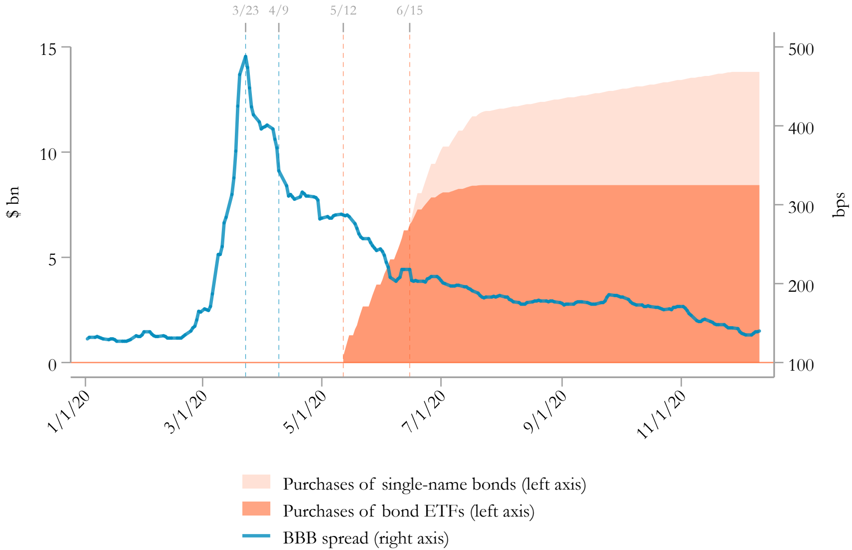
targeting high-leverage firms improves "bang for the buck"

grants with much lower returns per tax-payer dollar than loans

More

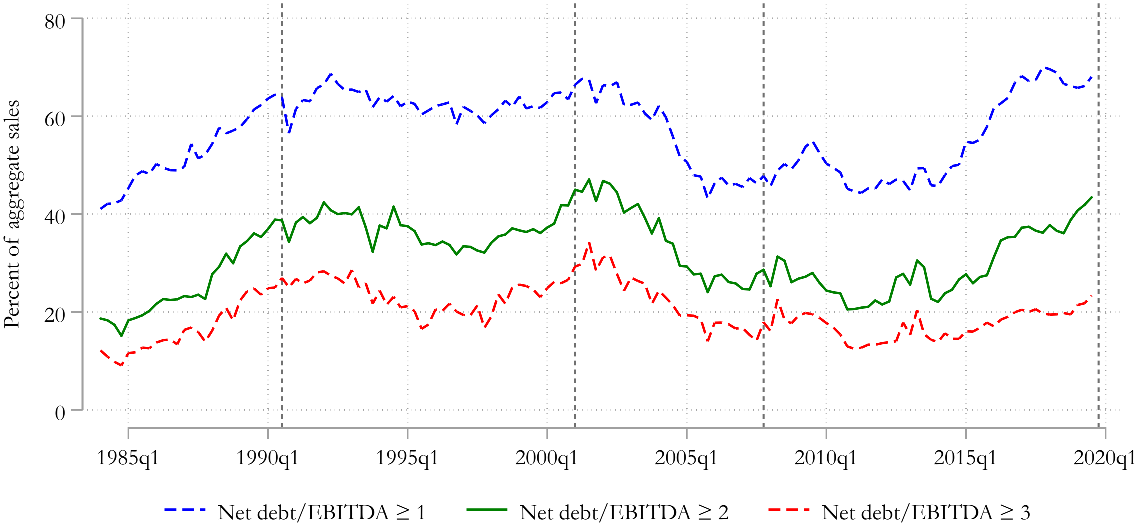
Secondary market corporate credit facilities' ("SMCCF") purchases

[Back]



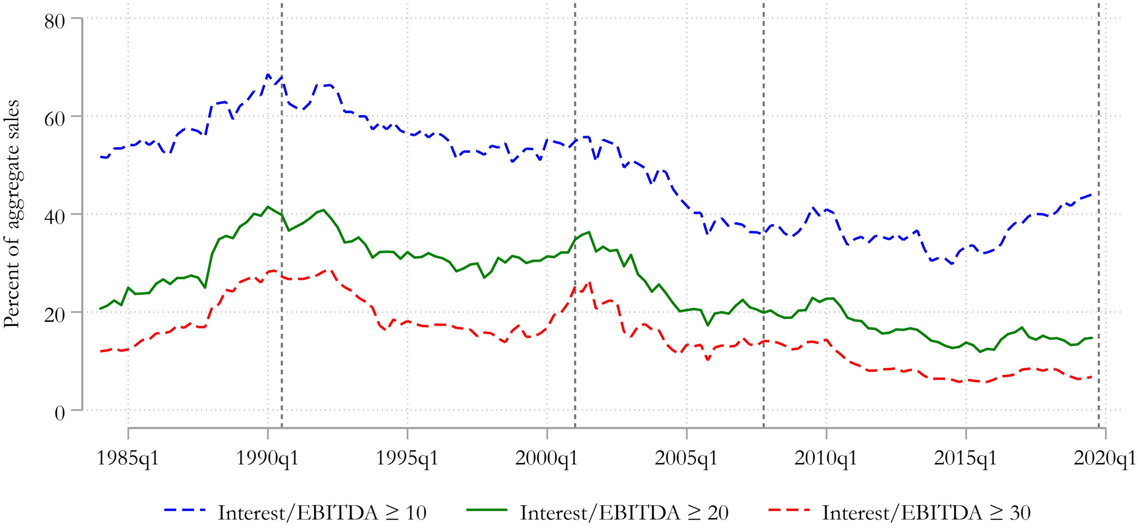
Leverage in the run-up to the crisis: net debt

[Back]



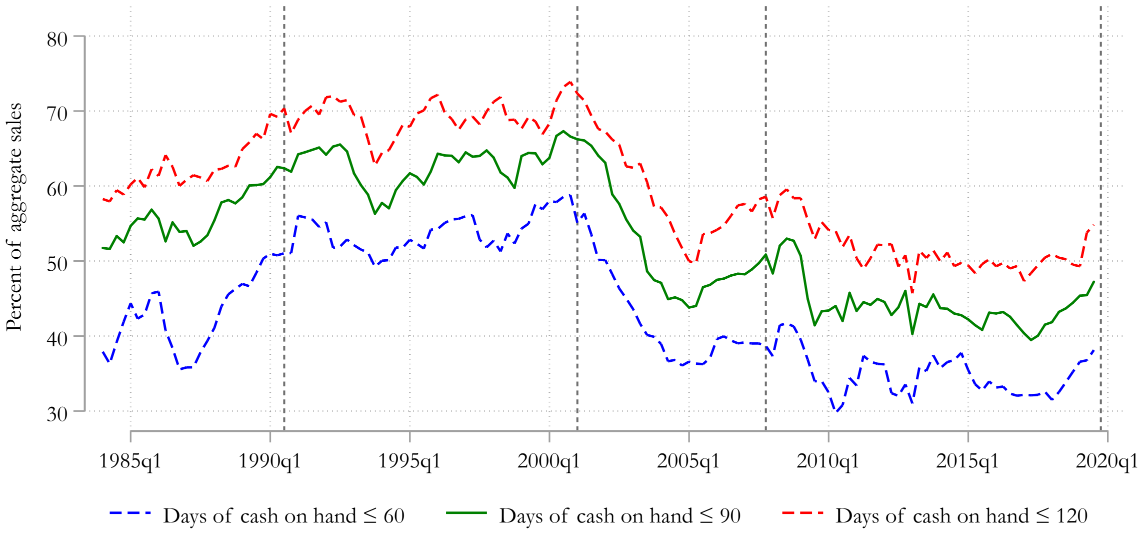
Interest coverage ratios in the run-up to the crisis

[Back]



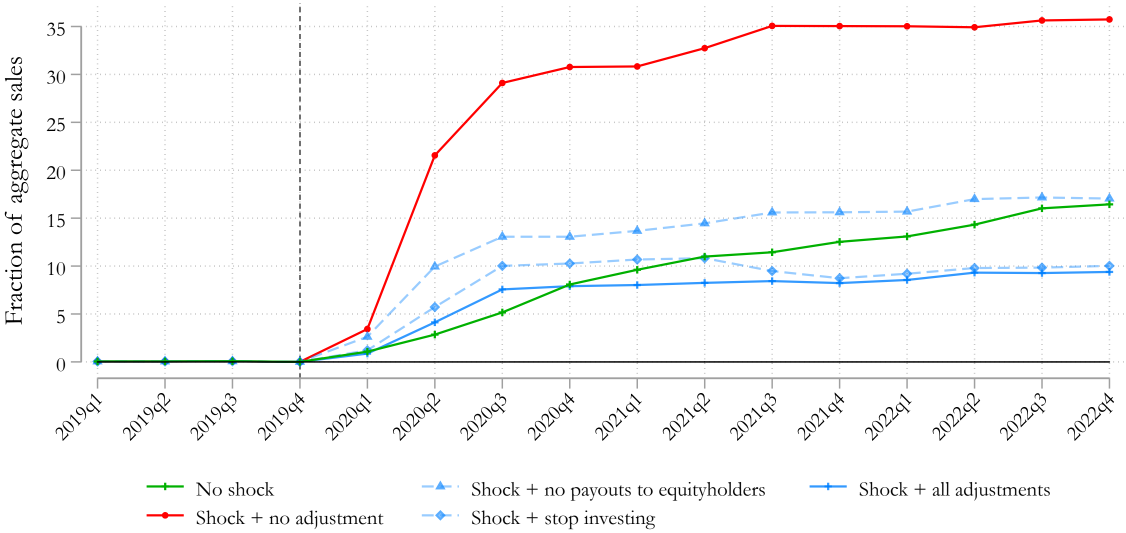
Days of cash on hand in the run-up to the crisis

[Back]



Projected firms with zero cash

[Back]



Model of the firm

[Back]

- Technology with adjustment costs: $\Phi\left(g_t\right) k_t dt$ spent allows capital to grow by $g_t k_t dt$

$$dk_t^{(j)} = k_{t-}^{(j)} \left[g_{t-}^{(j)} dt + \sigma \left(\rho dZ_t + \sqrt{1-\rho^2} dZ_t^{(j)} \right) + \left(\alpha_k - 1\right) dN_t^{(j)} \right]$$

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- Financing via long term debt with notional $b_t^{(j)}$ that satisfies: $db_t^{(j)} = \left(\iota_t^{(j)} k_t^{(j)} - m b_t^{(j)} \right) dt$

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- Dividends to shareholders of firm j

$$\pi_t^{(j)} k_t^{(j)} := \overbrace{a k_t^{(j)} - \Phi\left(g_t^{(j)}\right) k_t^{(j)}}^{\text{ebitda} - \text{capex}} + \underbrace{\iota_t^{(j)} k_t^{(j)} D_t^{(j)} - (\kappa + m)\, b_t^{(j)}}_{\text{net debt issuance}} - \overbrace{\Theta\left(a k_t^{(j)} - \kappa b_t^{(j)}\right)}^{\text{taxes}}$$

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- Investor n ($n \in \{e, d\}$) with SDF $\xi_{n,t}$ that satisfies $\frac{d\xi_{n,t}}{\xi_{n,t}} = -r_n dt - \nu_n dZ_t$

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[Back]

- Technology with adjustment costs: $\Phi(g_t)\,k_t dt$ spent allows capital to grow by $g_t k_t dt$

$$dk_t^{(j)} = k_{t-}^{(j)}\left[g_{t-}^{(j)}dt + \sigma\left(\rho dZ_t + \sqrt{1-\rho^2}dZ_t^{(j)}\right) + (\alpha_k - 1)\,dN_t^{(j)}\right]$$

- Financing via long term debt with notional $b_t^{(j)}$ that satisfies: $db_t^{(j)} = \left(\iota_t^{(j)}k_t^{(j)} - mb_t^{(j)}\right)dt$
- Dividends to shareholders of firm j

$$\pi_t^{(j)}k_t^{(j)} := \overbrace{ak_t^{(j)} - \Phi\left(g_t^{(j)}\right)k_t^{(j)}}^{\text{ebitda} - \text{capex}} + \underbrace{\iota_t^{(j)}k_t^{(j)}D_t^{(j)} - (\kappa+m)\,b_t^{(j)}}_{\text{net debt issuance}} - \overbrace{\Theta\left(ak_t^{(j)} - \kappa b_t^{(j)}\right)}^{\text{taxes}}$$

- Investor n ($n \in \{e,d\}$) with SDF $\xi_{n,t}$ that satisfies $\frac{d\xi_{n,t}}{\xi_{n,t}} = -r_n dt - \nu_n dZ_t$
- Shareholder problem and debt valuation

$$E(k_t,b_t) = \sup_{g,\iota,\boldsymbol{\tau}} \mathbb{E}^{\mathbb{Q}_e}\left[\int_t^{+\infty} e^{-r_e(s-t)}\pi_s k_s ds\right] \qquad D(k_t,b_t) = \mathbb{E}^{\mathbb{Q}_d}\left[\int_t^{+\infty} e^{-(r_d+m)(s-t)}\alpha_b^{N_t}(\kappa+m)ds\right]$$

Key model outcomes

[Back]

- leverage $x := b/k$ sufficient statistic for a given firm

$$E(k,b) = ke(x) \qquad D(k,b) = d(x) \qquad G(k,b) = kg(x) \qquad I(k,b) = k\iota(x)$$

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- debt overhang: $g'(x) < 0$ and $g(x) < g^*$
- debt issuance rate (per unit of capital): trade-off theory with a twist

$$\iota(x) = \underbrace{\frac{\Theta\kappa}{-d'(x)}}_{\text{tax motive}} + \underbrace{\frac{\left(\tilde{R}_d(x) - R_d(x)\right) d(x)}{-d'(x)}}_{\text{arbitrage motive}}$$

- $\tilde{R}_d(x) - R_d(x)$: debt expected return wedge (between equity and credit market investors)

Mathematical derivations

[Back]

- HJB equation for shareholders

$$0 = \max_{\iota, g} \Big[-(r-g)e(x) + a - \Phi(g) - (\kappa + m)x + \iota d(x) - \Theta(a - \kappa x) + [\iota - (g+m)x] e'(x) + \frac{\sigma^2}{2} x^2 e''(x) \Big]$$

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$$(r+m)d(x) = \kappa + m + \left[\iota(x) - \left(g(x) + m - \sigma^2\right)x\right] d'(x) + \frac{\sigma^2}{2}x^2 d''(x).$$

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- Expected debt returns (R_d and $\tilde{R}_d$) and equity returns (R_e)

$$R_d(x) = r_d - \rho\nu_d\sigma\frac{xd'(x)}{d(x)}, \qquad \tilde{R}_d(x) = r_e - \rho\nu_e\sigma\frac{xd'(x)}{d(x)}, \qquad R_e(x) = r_e - \rho\nu_e\sigma\left[1 - \frac{xe'(x)}{e(x)}\right]$$

Aggregation

[Back]

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- Aggregate capital-share-weighted moments

Default rate $\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2\bar{x}^2\partial_x\hat{f}_t(\bar{x})$ Average growth $\hat{g}_t = \int g\left(x\right)\hat{f}_t(x)dx$

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- Aggregate growth $\mu_{K,t} := \hat{g}_t - (1-\alpha_k)\hat{\lambda}_t$ and aggregate capital stock dynamics

$$dK_t = \mu_{K,t} K_t dt + \rho\sigma K_t dZ_t$$

GMM (exactly identified case)

Parameter	**Description**	**Point estimate**	**Standard error**	[5, 95] **normal CI**
a	average product of capital	0.223	0.001	[0.231, 0.235]
σ	volatility of idiosyncratic shock	0.236	0.010	[0.219, 0.253]
γ	curvature of capital adjustment cost	2.550	0.643	[1.493, 3.608]

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GMM (exactly identified case)

Moment	Description	Targeted?	Data	Model
$100 \times \hat{\Phi}$	average investment rate	✓	9.48	9.47
$\hat{z}$	average debt-to-ebitda	✓	2.71	2.71
$100 \times \frac{cov(\Phi(x),z(x))}{var(z(x))}$	slope of inv. w.r.t debt-to-ebitda	✓	−3.66	−3.66
$100 \times \kappa\hat{z}$	average (inverse) interest coverage ratio	✗	11.61	13.53
$100 \times \hat{\pi}$	average dividend issuance rate	✗	3.32	3.49
$100 \times \hat{\iota}$	average gross debt issuance rate	✗	10.21	7.38
$100 \times (\hat{\iota} - m\hat{x})$	average net debt issuance rate	✗	0.96	1.06
$var(z(x))$	variance of debt-to-ebitda	✗	3.08	0.90
$var(100 \times \Phi(x))$	variance of investment rate	✗	23.36	13.32
$100 \times \hat{F}(z(x) \leq 1)$	total asset share, debt-to-ebitda ≤ 1	✗	9.21	0.00
$100 \times \hat{F}(z(x) \leq 2)$	total asset share, debt-to-ebitda ≤ 2	✗	43.00	19.89
$100 \times \hat{F}(z(x) \leq 3)$	total asset share, debt-to-ebitda ≤ 3	✗	67.47	77.94

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The strength of the debt overhang channel

Average growth:

$$\text{Growth rate of all-equity firm} = 2.8\%$$
$$\text{Aggregate growth rate of } K_t = 0.9\%$$

Marginal effects:

$\partial(i/k)_t/\partial x_t$	$(i/k)_t =$ Gross investment	$(i/k)_t =$ Net investment
Model	-0.094	-0.106
Lang, Ofek, Stulz (1996)		-0.105
An, Denis, Denis (2006)		-0.086
Cai, Zhang (2011)	-0.038	
Wittry (2020)	-0.038	

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