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**Why do banks bear interest rate risk?**

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# Non-technical summary

## Research Questions

Interest rate risks in banks arise if the fixed interest periods of their assets differ from the ones of their liabilities. This paper is about the question of why banks bear interest rate risk in the first place, because banks could completely hedge this risk with suitable derivatives.

## Contribution

It is often said that the bearing of interest rate risk is a part of the banking business and that – at least in the past – credit and interest rate risk could not be completely separated when granting loans. In this paper, two additional reasons are given, namely a long optimization horizon of a part of the banks and the hedging of earning risks arising from a decline of the interest rate level. In an empirical study for the banks in Germany, we try to check the validity of the two additional reasons.

## Results

In the study, we find empirical evidence that banks with a longer optimization horizon bear more interest rate risk. Moreover, banks seem to bear interest rate risk in order to hedge earnings risk resulting from a decline in the interest rate level. The stronger a bank is exposed to the earnings risk, the higher its exposure to interest rate risk is.

# Nichttechnische Zusammenfassung

## Fragestellung

Zinsänderungsrisiken bei Banken entstehen, wenn die Zinsbindungsfristen ihrer Vermögensgegenstände von denen ihrer Verbindlichkeiten abweichen. Dieses Papier widmet sich der Frage, warum Banken überhaupt Zinsänderungsrisiken eingehen, denn sie könnten ja auch dieses Risiko durch geeignete Finanzderivate vollständig vermeiden.

## Beitrag

Es wird häufig gesagt, dass das Eingehen von Zinsänderungsrisiken zum Bankgeschäft gehört und dass – zumindest in der Vergangenheit – bei der Kreditvergabe das Kredit- und das Zinsänderungsrisiko nicht vollständig getrennt werden konnten. In dem Papier werden zwei weitere Gründe für das Eingehen der Zinsänderungsrisiken aufgeführt, nämlich ein langer Optimierungshorizonts eines Teils der Banken und das Begrenzen von Ertragsrisiken aus einem Fallen des Zinsniveaus. In einer Studie für die Banken in Deutschland wird die Stichhaltigkeit der zwei zusätzlichen Gründe überprüft.

## Ergebnisse

In unserer Studie finden wir empirische Belege dafür, dass eine Bank mit mutmaßlich langem Planungshorizont höhere Zinsänderungsrisiken eingeht als eine Bank mit mutmaßlich kurzem Optimierungshorizont. Auch scheinen Banken mit Hilfe der Zinsänderungsrisiken das Ertragsrisiko aus einem Sinken des Zinsniveaus zu begrenzen: Je stärker eine Bank diesem Ertragsrisiko ausgesetzt ist, desto höher sind ihre Zinsänderungsrisiken.

# Why do banks bear interest rate risk?\*

Christoph Memmel<sup>†</sup>

## Abstract

This paper investigates determinants of banks' structural exposure to interest rate risk in their banking book. Using bank-level data for German banks, we find evidence that a bank's exposure to interest rate risk depends on its presumed optimization horizon. The longer the presumed optimization horizon is, the more the bank is exposed to interest rate risk in its banking book. Moreover, there is evidence that banks hedge their earnings risk resulting from falling interest levels with exposure to interest rate risk. The more a bank is exposed to the risk of a decline in the interest rate level, the higher its exposure to interest rate risk.

**Keywords:** Interest rate risk, banks' business model, hedging

**JEL classification:** G21.

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# 1 Introduction

As a rule, the fixed interest periods of banks' assets do not exactly match those of their liabilities; the fixed interest periods on the asset side are usually longer than on the liability side. This mismatch exposes the banks to interest rate risk. Empirically, we find that interest rate risk is a material risk for many banks, but some banks have very little exposure to it.<sup>1</sup> For instance in [Deutsche Bundesbank \(2015\)](#), it is shown that the exposure to interest rate risk for an average savings or cooperative bank in Germany is many times higher than the average exposure of other banks in Germany.

In the past, credit risk could not be easily separated from interest rate risk so that exposure to this risk, given the long fixed interest periods of housing loans in Germany, was a byproduct of the loan granting business. Today, since financial instruments to hedge the interest rate risk are wide-spread, it may be that banks bear this risk because it is customary, because hedging with interest rate swaps is costly, or in order to avoid adjustments to their business models and earn the risk premium from term transformation. [Busch and Memmel \(2016\)](#) show that this risk premium amounted to roughly one third of the net interest income for the average German bank in 2012 and 2013.

In this paper, two additional reasons for banks' exposure to interest are investigated. First, interest rate risk differs from other risks, especially in the manner in which it depends on the horizon: A ten-year government zero-bond is risk-free – as long as the investment horizon is exactly ten years. If this horizon is, say, a month or one year, the ten-year government zero-bond is risky, not because of its credit risk, but due to its interest rate risk. Accordingly, a bank may appear to be exposed to interest rate risk if one looks at the current present value changes of its equity, but from a perspective of a horizon of several years, it may be exposed to less or even zero interest rate risk. If a bank has a long optimization horizon, it does not perceive current present value changes of its equity as a substantial risk. This paper argues that a bank's business model determines its optimization horizon: Banks with a strong capital market orientation are supposed to have a short optimization horizon whereas banks with a traditional business model are expected to have a longer optimization horizon.

Second, interest rate risk may serve a bank for hedging purposes. In the event of a fall in interest rates, a bank's immediate net interest margins tend to increase, mirroring present value gains of its equity. However, in the long run, this fall in interest rates is believed to lead to lower net interest margins. Hence, in the event of a fall in interest rates, a bank gains as immediate net interest margins rise, but also loses as its long-term net interest margins deteriorate. A bank can change its exposure to interest rate risk relatively easily and thereby the extent to which a decrease in the interest rate level benefits its immediate net interest margins. By contrast, the long-term net interest margin depends on its asset and liability composition which cannot be adjusted without changing the bank's business model. If the aim is to stabilize its net interest margin in the mid-term, a bank may choose a level of interest rate risk such that the positive effect on the mid-term net interest margins of a fall in the interest rates and the negative effect tend to cancel each other out, leading to a reduction in the variance of the bank's mid-term net interest

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<sup>1</sup>Note that whenever we refer to the banks' interest rate risk exposure in this paper we have their interest rate risk in the banking book in mind, i.e. the interest rate risk resulting from the banks' traditional business and not from their trading activities.

margin. In the current low interest rate environment, where the interest rate level has been falling for the last years, this argument is echoed. It has been suggested that the banks in Germany have benefited from loans with relatively high coupons granted in the past and that these loans, which are maturing now or will do so in the near future, have delayed the impact of the low interest environment.

The aim of this paper is twofold: First, the argument according to which a bank's optimization horizon determines its exposure to interest rate risk is theoretically grounded and, second, empirical evidence is sought concerning the banks' structural exposure to interest rate risk.

In our empirical study for the German banks for the period 2011Q4 to 2016Q1, we find evidence that a bank's perception of and exposure to interest rate risk depends on the presumed optimization horizon. The longer the presumed optimization horizon is, the more the bank is exposed to interest rate risk in its banking book. Moreover, there is evidence that banks hedge the earnings risk resulting from falling interest levels with exposure to interest rate risk: The more a bank is exposed to the risk of a decline in the interest rate level, the higher its exposure to interest rate risk.

The paper is structured as follows: In Section 2, a brief overview of the literature in this field is given. The theoretical and empirical models are described in Section 3. In Section 4, the data that is used is explained and, in Section 5, the empirical results are given. Section 6 concludes.

## 2 Literature

This paper contributes to the literature of the banks' strategic choice of their interest rate risk exposure. To our knowledge, we are the first to theoretically show that the length of a bank's optimization horizon has an impact on the bank's perception of the risk in its interest rate risk positions, and, in our study for the German banks, we find empirical evidence that banks with a presumably longer optimization horizon structurally bear more interest rate risk, measured by the present value changes in its assets and liabilities. Moreover, [Schrand and Unal \(1998\)](#) find that US banks have an internal risk budget that they allocate to the various risks, especially to credit and interest rate risk. In our empirical study, we find for German Banks as well that they act as if they have an internal risk budget.

If, for banks, interest rate risk were nothing other than a tradeable market risk, they would – according to the framework of [Froot and Stein \(1998\)](#) – completely hedge this risk. However, the literature on the effects of an interest rate shift on a bank's net interest margin casts doubt on the assumption that banks' interest rate risk is a completely tradeable market risk. Especially, the long-run effects of a parallel shift in the term structure on a bank's net interest margin would be zero if interest rate risk were solely a tradeable market risk for banks. [Alessandri and Nelson \(2015\)](#) find in a theoretical model – and confirm their theoretical results in an empirical study for UK banks – that, in the long run, a bank's net interest margin positively depends on the level of interest rates. In the short run, however, a rise in interest rates compresses the net interest margin. This compression is also found in the theoretical model developed by [Dell'Araccia, Laeven, and Marquez \(2014\)](#), where a less than complete pass-through to the loan rates is responsible for the decline in the net interest margin as a consequence of a rise in the interest rate.

In his study for ten developed countries, [English \(2002\)](#) finds mixed results concerning changes in the short and long-term interest rate. By contrast, [Albertazzi and Gambacorta \(2009\)](#) and [Bolt, de Haan, Hoeberichts, Oordt, and Swank \(2012\)](#) carry out empirical studies for banks in industrialized countries and find a positive relationship between long-term interest rates and the banks' net interest income. [Claessens, Coleman, and Donnelly \(2017\)](#) find in their study for 47 countries that the banks' net interest margins increase by 8 basis points per 100 basis points increase in the interest rate level. In their empirical study for the German banking system, [Busch and Memmel \(2017\)](#) also find that, in the long run, an increase in the interest rate level is beneficial for the banks' net interest margin. In addition, they find that, in the short run, this increase compresses the net interest margin. They estimate that the critical horizon where the effect of a change in the interest rate level is zero is around 1.5 years. All in all, there is theoretical and empirical evidence that the effect of changes in the interest rate level depends on the horizon: In the short run, an increase in the interest rate level compresses the net interest margin, mirroring net present value losses, and in the long run, an increase in the interest rate level leads to a higher net interest margin, presumably due to a higher long-run pass-through on the asset side than on the liability side. In our empirical study, we find evidence that German banks use these differing effects for hedging purposes, meaning that they hedge the risk of a decline in their far future net interest margin by exposure to interest rate risk. To our knowledge, we are the first to investigate this issue in an empirical study.

The issue of the banks' exposure to interest rate risk has a structural and a more tactical aspect. Most of the empirical studies in this field use the time dimension which has a strong connection to the tactical aspect (an example of an exception is [Angbazo \(1997\)](#)). In our paper, we focus on the structural differences between banks, i.e. the variation in the cross-section of the banks. We do so because it becomes apparent that, for many bank characteristics, most of the variation is between the banks and not in the time series.

In this paper, we try to be as exact as possible concerning the notions of interest rate risk on the one hand and the risk from maturity transformation on the other hand. Whereas interest rate risk is due to mismatches between the fixed interest periods on a bank's asset and liability side, the risk from maturity transformation arises from mismatches in the capital commitments on the respective sides. Interest rate risk and the risk from maturity transformation are often bundled together, especially where the fixed interest periods correspond to the periods of capital commitment. But even then, these two risks can be separated so that banks with little interest rate risk can be strongly engaged in maturity transformation as described by [Allen and Gale \(1997\)](#).<sup>2</sup>

## 3 Modeling

### 3.1 General remarks

The general setting of our empirical analyses is a regression to explain a bank's interest rate risk, where we measure the exposure to this risk using the Basel interest rate

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<sup>2</sup>See [Vuillemeys \(2016\)](#) for an overview of the banks' bearing of interest rate risk from a macroeconomic point of view.



coefficient, *bic*. This coefficient can be seen as a bank’s present value losses due to a standardized interest rate shock, normalized with the bank’s equity (see Section 4 for a detailed description). In the paper, we show that the present value changes are not the only relevant measure for the banks’ interest rate risk exposure. Nevertheless, we opted for this measure for the following reasons: First, the notion of risk implies that a shock has an immediate impact, rather than in the distant future as would be the case with other measures, for instance the long-run change of a bank’s net interest margin. Second, it is objective in the sense that only the current positions of a bank, not its possible future margins, are accounted for.<sup>3</sup> Third, this notion of interest rate risk is prevalent in the financial markets because, for instance, this risk can be hedged by standard financial instruments like interest rate swaps. Fourth, the risk measured like this is relevant because the taking of this risk is remunerated (see Memmel (2011) and Busch and Memmel (2016)).

To account for the fact that the exposure to interest rate risk may be determined by slowly changing, structural characteristics, we run a cross-sectional regression. Accordingly, as the explanatory variables mainly concern structural bank characteristics and therefore have their main variation in the cross section ( $i = 1, \dots, N$ ) and very little or no variation in the time series ( $t = 1, \dots, T$ ), we use the following regression:

$$bic_i = \alpha + \beta_1 \cdot x_{1,i} + \dots + \beta_N \cdot x_{N,i} + \gamma_1 \cdot y_{1,i} + \dots + \gamma_M \cdot y_{M,i} + \varepsilon_i \quad (1)$$

where  $x_{1,i}, \dots, x_{N,i}$  are bank-specific explanatory variables that will be characterized in the following subsections, and  $y_{1,i}, \dots, y_{M,i}$  are control variables.

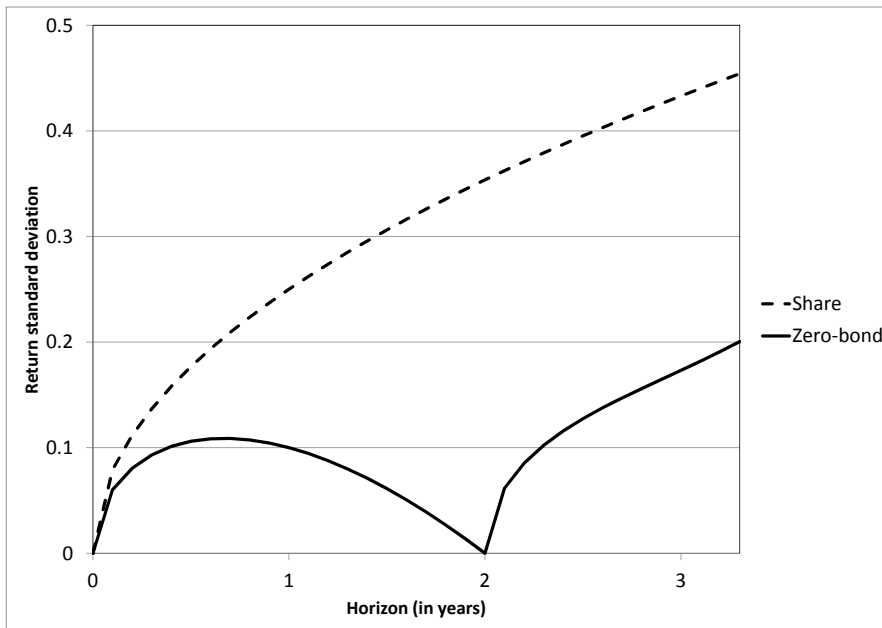
In our empirical study, we especially focus on the two additional determinants mentioned before, namely the optimization horizon and the hedging of earnings risks. We additionally include a measure for the extent of credit risk in a bank’s assets ( $x_{1,i} = rwa\_ta_i$ ). The assumption is that banks have an internal risk budget that they allocate to either credit risk or interest rate risk, neglecting the fact that banks are exposed to other risks as well. Therefore, we expect the variable  $x_{1,i} = rwa\_ta_i$  to have a negative association with the exposure to interest rate risk. As control variables, we use a bank’s capital ratio ( $y_{1,i} = CR_i$ ), its size ( $y_{2,i} = S_i$ ) and its usage of interest rate swaps ( $y_{3,i} = swap_i$ ).

We do not claim to find a causal relationship to explain the level of the banks’ interest rate risk exposure, as we do not have an experiment or other truly exogenous events at our disposal. Nevertheless, we believe that the explanatory variables are more or less exogenous (for instance the bank being a savings or cooperative bank), depend on a bank’s location (for instance the bank’s credit risk in its assets) or on its structural business model (for instance the bank products’ long-run pass-through and having a trading book) or seem in other ways more difficult to change than the exposure to interest rate risk (*bic*), so that the exposure to interest rate risk is rightly the left-hand variable in the regression (1).

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<sup>3</sup>In addition, English (2002) argues that, according to theory, the change in economic (or present) value should be equal to the present value of the changes in the stream of future net interest incomes.

Figure 1: Risk and Horizon



In this figure, the log return standard deviations of a share and a zero-bond in dependence of the horizon  $t$  are depicted. The share price  $S_t$  follows a geometric Brownian motion:  $\log(S_t/S_0) = \mu t + \sigma_S \sqrt{t} \varepsilon$ , where  $\mu$  is a drift term,  $\sigma_S = 0.25$  is the annualized return volatility and  $\varepsilon$  is standard normally distributed. The zero-bond (with price  $P_t$ ) has a maturity of  $T = 2$  years and the term structure of interest rates is assumed to be and to remain flat, but the level  $r_t$  (with annualized volatility  $\sigma_r = 0.1$ ) changes in the course of time:  $\log(P_t/P_0) = r_0 t - \sigma_r (T - t) \sqrt{t} \varepsilon$ . For horizons  $t$  that exceed the maturity  $T = 2$ : We assume a reinvestment in a new zero-bond with the same maturity  $T = 2$ .

### 3.2 Optimization Horizon

Interest rate risk has a special feature that makes it different to other risks, such as credit risk or stock market risk, namely that the riskiness of an asset non-monotonously depends on the horizon under consideration: A zero-bond (issued by a debtor of the highest creditworthiness) with a residual maturity of, say, two years is risk-free if a horizon of two years is considered, but risky if the horizon is different from two years.<sup>4</sup> By contrast, the risk of a share usually monotonically increase if the horizon becomes longer. An example of this reasoning is depicted in Figure 1. To some extent, interest rate risk shares features with currency risk, where an amount in Swiss francs is risky for all investors except the Swiss ones.

In the following model, we aim to theoretically show that the length of the optimization horizon impacts the exposure to interest rate risk. In the model, we assume that an investor can only invest into zero-bonds of different maturities. For the ease of exposition, we resort to those basic instruments; but the model can be easily extended to portfolios of bonds so that the asset-liability-structure of banks can be captured. However, for the reason of keeping the model tractable, we neglect the side constraints that a bank has to

<sup>4</sup>The concept of the so-called “pull-to-par” effect (in the context of bonds) is closely related to the reasoning above.

be able to service its debts resulting from the deposits at any point in time.

Let  $B_t(s)$  be the value of a (credit) risk-free zero-bond in time  $t$  that pays in time  $s$  one euro. We consider two zero-bonds, namely  $B_t(1)$  and  $B_t(2)$ . In  $t = 0$ , investor  $i$  distributes his initial wealth  $W_{0,i}$  on the two zero-bonds, where  $k_i$  is his holding in the short-term zero-bond  $B_0(1)$ . Accordingly, his wealth  $W_{1,i}$  in  $t = 1$  is (with  $B_1(1) \equiv 1$ )

$$W_{1,i} = k_i + \left( \frac{W_{0,i} - k_i \cdot B_0(1)}{B_0(2)} \right) \cdot B_1(2) \quad (2)$$

and in  $t = 2$  (with  $B_2(2) \equiv 1$ )

$$W_{2,i} = \frac{k_i}{B_1(2)} + \left( \frac{W_{0,i} - k_i \cdot B_0(1)}{B_0(2)} \right). \quad (3)$$

Unlike [Diamond and Dybvig \(1983\)](#), the investor is already certain in  $t = 0$  when he will consume. Therefore, he optimizes the expected utility of either the wealth in  $t = 1$  or in  $t = 2$ .<sup>5</sup> We assume a utility function with constant absolute risk aversion  $\gamma_i$  for investor  $i$ :

$$U_i = -\exp(-\gamma_i \cdot W_{t,i}) \quad (4)$$

where – under the assumption of normally distributed wealth  $W_{t,i}$  – the expected utility can be obtained in a closed form solution; the function  $\phi_i^t$  with  $t = 1, 2$  describes the same preference ordering (see [Freund \(1956\)](#)):

$$\phi_i^t = E(W_{t,i}) - \frac{\gamma_i}{2} \cdot \text{var}(W_{t,i}) \quad (5)$$

Let us assume that  $B_1(2)$  is normally distributed:

$$B_1(2) \sim N(E(B_1(2)), \sigma_B^2) \quad (6)$$

Using the following approximation

$$\frac{1}{a} \approx 2 - a \quad (7)$$

which is relatively accurate for values of  $a$  of around 1,<sup>6</sup> we replace the term  $1/B_1(2)$  by the term  $2 - B_1(2)$  in Equation (3), consequently the wealth  $W_{2,i}$  in  $t = 2$  is also normally distributed.

Combining the expressions (2), (3), (6) and (5), we obtain

$$\phi_i^1 = k_i + \left( \frac{W_{0,i} - k_i \cdot B_0(1)}{B_0(2)} \right) \cdot E(B_1(2)) - \frac{\gamma_i}{2} \cdot \left( \frac{W_{0,i} - k_i \cdot B_0(1)}{B_0(2)} \right)^2 \cdot \sigma_B^2 \quad (8)$$

and

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<sup>5</sup>In Appendix 1, we show how a bank optimizes its exposure to interest rate risk if the optimization horizon is continuous, and not discrete with only two possible values as in the case above. However, as we have only dummy variables in the empirical study to characterize the optimization horizon, we concentrate on the model above.

<sup>6</sup>This approximation corresponds to the first-order Taylor approximation of the term  $1/a$  at  $a = 1$ .

$$\phi_i^2 = k_i \cdot (2 - E(B_1(2))) + \frac{W_{0,i} - k_i \cdot B_0(1)}{B_0(2)} - \frac{\gamma_i}{2} \cdot k_i^2 \cdot \sigma_B^2, \quad (9)$$

depending on whether investor  $i$  maximizes the expected utility of his wealth in  $t = 1$  or  $t = 2$ . The demand for the short-term asset zero-bond  $k_i^{d,t}$  is

$$k_i^{d,1} = \frac{1 - \frac{B_0(1) \cdot E(B_1(2))}{B_0(2)}}{\gamma_i \cdot \sigma_B^2 \cdot \left(\frac{B_0(1)}{B_0(2)}\right)^2} + \frac{W_{0,i}}{B_0(1)} \quad (10)$$

for an investor maximizing the expected utility of his wealth in  $t = 1$  and

$$k_i^{d,2} = \frac{E(B_1(2)) \cdot \left(1 - \frac{B_0(1) \cdot E(B_1(2))}{B_0(2)}\right)}{\gamma_i \sigma_B^2}. \quad (11)$$

for those maximizing their wealth in  $t = 2$ . In Equation (11), we apply again the approximation of Equation (7); this time, however, the other way around.

Several statements can be derived from Equations (10) and (11) where we interpret a higher value of  $k_i$  as having a lower exposure to interest rate risk. Note that the Basel interest rate coefficient *bic* is approximately proportional to the duration of the bank's equity (see Appendix 2). One can show that the duration  $D_i$  of the zero-bond portfolio from above can be written as  $D_i = a - b \cdot k_i$  where  $a = 2/(1+r)$  and  $b = B_0(1)/(W_{0,i}(1+r))$  are positive constants. First, the demand for short-term bonds for investors maximizing the expected utility from their wealth in  $t = 1$  can be divided into a speculation component (namely the first summand) and a hedging component ( $k_i^{d,1,h} = W_{0,i}/B_0(1)$ ).<sup>7</sup> Second, the speculation component is negative, i.e. investment into interest rate risk takes place, if and only if the expected yield of the roll-over from one period to the next is lower than the investment in the zero-bond with a maturity of two periods, i.e.  $B_0(1) \cdot E(B_1(2)) > B_0(2)$  which approximately corresponds to the inequality  $(1+r_0(1)) \cdot (1+E(r_1(2))) < (1+r_0(2))^2$ , where  $r_t(s)$  is the interest rate (per year) in  $t$  until  $s$ , i.e.  $B_t(s) = (1+r_t(s))^{-(s-t)}$ . If the implicit interest rate  $r_1^{im}(2) = (1+r_0(2))^2/(1+r_0(1))$  corresponds on average to the future spot rate  $r_1(2)$ , the speculation component vanishes and only the hedging component remains. Third, the demand for short-term bonds goes up whenever the roll-over strategy increases its attractiveness compared to the long-term investment, irrespective of the individual's investment horizons. This raises the question of who in the economy provides the additional supply of short-term bonds in that event, meaning who tactically bears the interest rate risk. Fourth, the hedging component corresponds to the present value of the initial wealth, but transferred in  $t = 0$  to  $t = 1$ . Apart from reasons of speculation, individuals optimizing their positions with respect to the wealth after the first period invest all their money in short-term bonds.

According to the model above, the structural characteristics that determine an investor's demand for short-term and long-term bonds and thereby the exposure to interest rate risk are the bank's degree of risk aversion  $\gamma_i$  and the investment horizon (here: whether the investor maximizes the expected utility of wealth in  $t = 1$  or  $t = 2$ ).

A bank's optimization horizon cannot be directly observed. In this paper, we use

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<sup>7</sup>The hedging component in the case of investors maximizing their wealth  $t = 2$  is zero.

empirically observable variables to proxy this horizon, namely the existence of a trading book and the belonging to certain banking groups. We introduce the variable  $trade_i$  as  $x_{2,i}$  into Equation (1). This variable indicates whether bank  $i$  has (in the period under consideration on average) a trading book. The presumption is that banks with a trading book are more short-term orientated whereas banks without a trading book are more engaged in (the less market-based) traditional banking business. From a theoretical viewpoint, a bank with complete market orientation would maximize today's market value of its future cash flows. As hedging interest rate risk – for instance, the present value of an interest rate swap is zero when the contract is concluded – does not create or destroy any market value, such a bank would be indifferent towards its exposure to interest rate risk. In the case of indifference, we make the assumption that banks completely get rid of the risks.

In addition, we have indicator variables for membership of the group of savings banks ( $svbk_i$ ) or cooperative banks ( $coop_i$ ), which would be  $x_{3,i}$  and  $x_{4,i}$ , respectively, as further variables to map a bank's optimization horizon. It can be presumed that savings and cooperative banks have a business model with longer optimization horizons than private and capital market orientated banks. However, membership of the savings banks or the cooperative banks sector means much more than having a certain optimization horizon. To account for the dominating effect of these two variables, we split the sample into savings banks and cooperative banks on the one side and the remaining banks on the other side as a robustness check (see Subsection 5.2).

In the literature (see, for instance, [McShane and Sharpe \(1985\)](#)), a bank's degree of risk aversion (in the model above  $\gamma_i$ ) is sometimes proxied by its capital ratio  $CR_{t,i}$  ( $CR_i$  is the time series average for bank  $i$ ). However, we include this variable merely as a control variable  $y_{1,i}$  in Equation (1), because this variable may represent other economic effects as well, for instance capital buffers to cover interest rate risks.

### 3.3 Hedging

After a fall in the interest rate level, two direct effects occur concerning a bank's net interest margin: First, due to the usually longer fixed interest periods on the asset side, the fraction of assets that are adjusted in a given time span to the new interest rates is smaller than the fraction of liabilities. For instance – under the assumption of a constant balance sheet –, if a bank revolvingly grants loans with a 10-year maturity (with a fix coupon), only 10 per cent of these loans are newly granted in one year. By contrast, if the same bank revolvingly collects one-year term deposits, the entire deposits are renewed within one year. After a fall in the term structure, this effect will lead to a higher net interest margin, because – at least in the first immediate years – the fraction of assets that are renewed and accordingly adjusted to the lower interest rate level is smaller than the fraction of the liabilities, leading to a positive net effect on the net interest margins. Second, the interest rate pass-through  $\theta$  on the asset side tends to be larger than on the liability side. The extreme case would be a central bank: on the asset side, there are loans to banks, geared towards the (short-term) interest rate level and, on the liability side, there are banknotes without remuneration regardless of the interest rate level. That means, on the asset side, there would be a complete pass-through whereas on the liability side, there would not even be a partial pass-through. For normal banks (and not the

stylized central bank), the differences in the pass-through would be less extreme. For instance, for the German banks in the period 1968-2013, [Busch and Memmel \(2017\)](#) find a long-run pass-through on the asset side of 78% and on the liability side of 71%, yielding an effect on the net interest margin of about 7 basis-points per 100 basis point parallel shift in the term structure.

After a change in the interest rate level, all future net interest margins of a bank are subject to these two effects described above. Whereas the first effect is pronounced for the immediate net interest margins, the second effect has more of an impact on the long-term net interest margins. [Busch and Memmel \(2017\)](#) find that after a year and a half, both effects cancel each other out. In [Figure 2](#), this relationship between the change in the net interest margin and the time since the shift in the term structure is depicted for a bank where we look at three different cases concerning the bank’s interest rate exposures ( $bic_i$ ), namely a low exposure to interest rate risk (“low bic”), the level of exposure to interest rate risk that was derived from the parameter values of [Busch and Memmel \(2017\)](#) (“base”), and a high exposure to interest rate risk (“high bic”). We assume that the long-run pass-through is determined by a bank’s business model and that it is far easier for this bank to change its exposure to interest rate risk ( $bic_i$ ) than the long-run pass through of its assets and liabilities ( $\theta_i$ ).<sup>8</sup> Therefore, irrespective of the level of interest rate risk, the long-run pass-through of the net interest margin converges to the same value.

We assume that a bank’s aim is it to stabilize its mid-term net interest margin, i.e. the aim is to reduce the impact of today’s movements in the term structure on the bank’s net interest margin in the near future. The bank can do this by choosing an appropriate exposure to interest rate risk. For instance, in the example in [Figure 2](#), if the goal is to immunize the bank’s net interest margin in two years time against today’s changes in the interest rate level, the bank will be well advised to choose the high interest rate risk exposure (“high bic”). By contrast, if the bank chooses the low interest rate risk exposure (“low bic”) or the exposure as in the base case, the net interest margin in two years time will change by 5 and 3 bp, respectively, provided a 100 bp downward shift of the term structure occurs. In [Appendix 2](#), we show that the change in a Bank  $i$ ’s net interest margin  $NIM$  due to an interest rate shock  $\Delta R$  at the beginning of the year under consideration can be expressed as

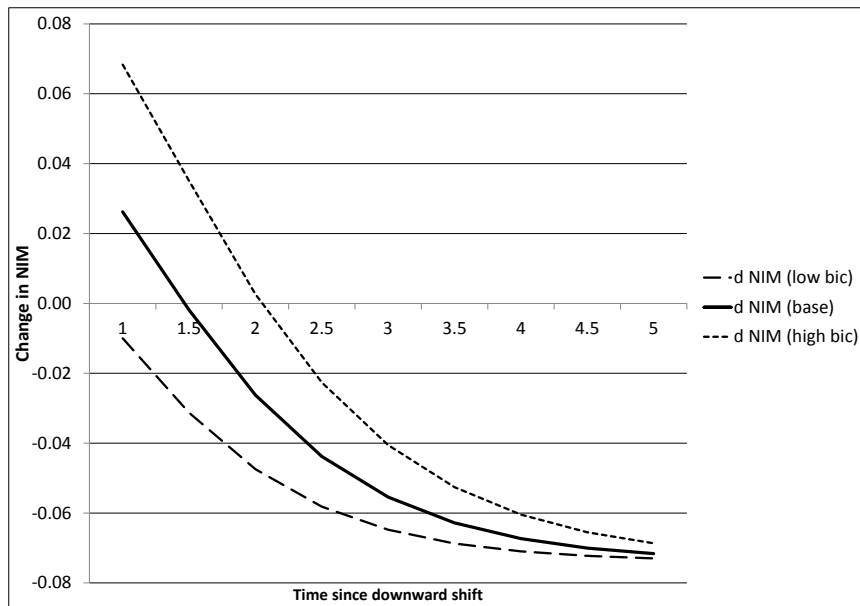
$$\Delta NIM_i = (\beta \cdot \theta_i - \gamma \cdot bic_i) \cdot \Delta R + \varepsilon_i, \tag{12}$$

where  $\beta$  and  $\gamma$  take on positive values and  $\varepsilon_i$  is an unexplained residual. From [Equation \(12\)](#) in combination with the assumption of a fix (and exogenous) long-run pass-through, we see that minimizing the variance of the (change in the) current net interest margin requires us to choose an exposure to interest rate risk  $bic_i$  proportional to the long-run pass-through  $\theta_i$ . So, if the hypotheses hold that banks aim at stabilizing their mid-term net interest margins and that the long-run pass-through is rather outside the control of the banks, than there should be a positive relationship between a bank’s exposure to interest rate risk ( $bic_i$ ) and its long-run pass-through ( $\theta_i$ ).

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<sup>8</sup>In [Table 2](#) in [Section 4](#), it is shown that the share of the serial variation of the variable  $bic$  is much higher than the share of the variable  $\theta$ , namely 24% (=100%-76%) vs. 6% (=100%-94%). This finding supports the assumption that the exposure to interest rate risk ( $bic$ ) is much easier to alter than the long-run pass-through  $\theta$  of the bank’s assets and liabilities.

Figure 2: Net interest margin: Impact of a downward shift of the interest rate level



In this figure, the impact on a bank's net interest margin (NIM) of a 100 bp downward shift of the term structure is qualitatively displayed. The change in NIM is given in percentage points, the time since the downward shift is in years. The base case is based on the parameter values of [Busch and Memmel \(2017\)](#); the cases of low and high interest rate risk ("low bic" and "high bic") are constructed from the base case, keeping the long-run pass-through constant, but assuming different exposure to interest rate risk.

The long-run pass-through  $\theta$  is not directly observable because banks' interest margins are a combination of past and current changes in interest rates. Especially in the long run, an increase in the interest rate level might be expected to lead to a higher future net interest margin – and this is what is found in the empirical literature (see Section 2) – because the first effect mentioned above, i.e. the decrease due to less new business on the asset side than on the liability side, vanishes. The second effect mentioned above is especially pronounced for banks engaged in traditional banking business where the pass-through on the asset side is much higher in long-run, for instance due to mortgage loans, than the pass-through on the liability side with its large proportion of money-like deposits. By contrast, banks relying on wholesale funding will see a much smaller effect on their long-term net interest margin.<sup>9</sup> In our paper, we measure the long-term pass-through  $\theta$  for each bank as follows:

$$\theta_{t,i} = \sum_{j=1}^{J_a} w_{t,i,j}^a \cdot pt_j^a - \sum_{j=1}^{J_l} w_{t,i,j}^l \cdot pt_j^l \quad (13)$$

where  $w_{t,i,j}^a$  is bank  $i$ 's balance sheet weight in time  $t$  for asset  $j = 1, \dots, J_a$  and  $pt_j^a$  gives the long-run pass-through of asset position  $j$  (see Section 4), the respective variables for the liability positions are indexed with  $l$ . As the long-run pass-through, we take the time series average of the variable  $\theta_{t,i}$  and include it in Equation (1) as  $x_{5,i} = \theta_i$ .

Our hypothesis is that banks hedge the risk of a decline in the interest rate level (which the banks cannot control) with their exposure to interest rate risk (which the banks can control). If this hypothesis holds, we expect that the greater the long-term difference in the pass-through  $\theta_i$ , the higher the bank's exposure to interest rate risk  $bic_i$  will be.

## 4 Data

All data in the paper is provided by the Deutsche Bundesbank. It comes from regular supervisory reports.

In every quarter since end-2011, each bank in Germany has had to report its exposure to interest rate risk in its banking book. In doing so, the bank has to determine the changes in present values of the assets and liabilities in its banking book as a consequence of interest rate shocks. The shocks consist of parallel overnight shifts of the entire term structure by +200 bp and by -200 basis points, respectively. The more adverse of the two outcomes is chosen and normalized with the bank's regulatory capital, known as the Basel interest rate coefficient ( $bic_{t,i}$ ). For more than 95% of the banks, the more adverse scenario is the one involving the increase in the interest rate level; in our empirical study, we therefore use only this scenario.

The variable  $rwa\_ta$ , which measures the credit risk included in the assets, is calculated as the ratio of a bank's risk weighted assets ( $RWA$ ) and to its total assets ( $TA$ ).

The variables  $svbk_i$  and  $coop_i$  are dummy variables that take on the value of 1 if bank  $i$  in time  $t$  belongs to the savings and cooperative banks sector, respectively. As the

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<sup>9</sup>Busch and Memmel (2017) find a long-run effect on the net interest margin for the small and medium size banks in Germany of 8 basis points (bp) per 100 bp shift in the term structure while the effect for the more wholesale oriented large banks amounts to 3 bp.



Table 1: Long-run pass-through

Fixed interest period	No split	Short-term	Medium	Long-term
Domestic private households				
Consumer loans		0.322	0.224	0.330
Housing loans		0.847	0.820	0.802
Other loans		0.799	0.696	0.820
Sight deposits	0.380			
Term deposits		0.731	0.755	0.522
Firms, public sector, other non-financials				
Loans		0.805	0.679	0.819
Sight deposits	0.510			
Term deposits		0.891	0.893	0.999
All non-financials				
Savings account	0.555			
Savings account (ext. notice p.)	0.721			

This table shows the estimated long-run pass-through for various balance sheet positions; monthly data; period January 2003 - March 2016; “ext. notice p.” means extended notice period, i.e. notice period of more than three months.

savings and cooperative banks have not changed their sectors (also called pillars), there is no variation in the time series of these two variables.

To construct the long-run pass-through, measured by the variable  $\theta_i$  in Equation (13), we apply the method described in [Busch and Memmel \(2017\)](#) according to which the time series of various bank products are replicated by portfolios of government bonds of different maturities and by an investment with timely constant return. We interpret the fraction of the replicating portfolios that is invested in government bonds as the extent of the long-run pass-through  $pt_j$  of the product  $j$  under consideration. Using the German contribution to the MFI statistics, we estimate the replication portfolio for each product for the period January 2003 to March 2016. The results for the share of the long-run pass-through are shown in Table 1. We see that the extent of the pass-through varies across the product categories as also found by [De Graeve, De Jonghe, and Vennet \(2007\)](#) and [Kleimeier and Sander \(2006\)](#). Especially sight deposits of domestic private households and consumer loans show a small long-run pass-through of less than 40%. For wholesale positions (for instance interbank loans and deposits, bonds), we assume a complete pass-through; for non-interest-bearing positions (for instance real estate, shares, capital, cash reserve), we assume a pass-through of zero.

As mentioned before, we use a bank’s Tier 1 capital ratio  $CR$ , its size  $S$ , measured by the logarithm of its total assets, and the time-series average of the dummy variable  $swap_{t,i}$  as control variables. The dummy variable  $swap_{t,i}$  takes on the value of one if in time  $t$  the notional amount of interest rate or currency swaps is strictly greater than zero;  $swap_i$  is the corresponding time series average.

We apply a mild outlier treatment by removing the first and 99th percentile of the (non-dummy) variables  $bic$ ,  $rwa\_ta$  and  $\theta$ .

In the paper, we focus on the structural exposure to interest rate risk. To substantiate the importance of this focus, we show that most of the determinants’ variation is in

Table 2: Summary Statistics

Variable	Observations	Mean	Standard dev.	Share of cross-sec. var.
<i>bic</i>	28,913	18.790	7.697	76.1%
<i>rwa_ta</i>	28,913	0.561	0.124	92.2%
<i>trade</i>	8,018	0.080	0.271	92.1%
<i>svbk</i>	28,913	0.256	0.436	100.0%
<i>coop</i>	28,913	0.652	0.476	100.0%
$\theta$	28,913	0.234	0.100	94.1%
<i>CR</i>	28,913	0.145	0.067	89.4%
<i>S</i>	28,913	13.323	1.470	99.6%
<i>swap</i>	28,913	0.470	0.499	89.0%

This table shows summary statistics for the variables *bic*, which measures the interest rate risk exposure, *rwa\_ta*, which gives a bank’s credit risk per total assets, *trade*, which is a dummy variable indicating that a bank has a trading book, *svbk* and *coop*, which are dummy variables indicating that a belongs bank to the savings and cooperative banks, respectively,  $\theta$ , which is the long-run pass-through, *CR*, which is the capital ratio and *S*, which is the natural logarithm of a bank’s total assets. “Share of cross-sec. var.” is the abbreviation for the share of the cross-sectional variation according to Equation (14). Quarterly data except for the variable *trade*, which is only available annually, period 2011Q4-2016Q1.

the cross section, i.e. between the banks, and not in the time dimension. The total variation  $\sigma_x^2$  of a variable  $x_{t,i}$  where  $t = 1, \dots, T$  describes the time-series dimension and  $i = 1, \dots, N$  its cross section is split up into its cross-sectional variance  $\sigma_{c,x}^2$  and its time series variance  $\sigma_{T,x}^2$ . The empirical equivalents are  $\hat{\sigma}_x^2 = \frac{1}{T \cdot N} \sum_{t=1}^T \sum_{i=1}^N (x_{t,i} - \bar{x})^2$ ,  $\hat{\sigma}_{c,x}^2 = \frac{1}{N} \sum_{i=1}^N (\bar{x}_i - \bar{x})^2$  and  $\hat{\sigma}_{T,x,i}^2 = \frac{1}{T} \sum_{t=1}^T (x_{t,i} - \bar{x}_i)^2$  where  $\bar{x}_i = 1/T \sum_{t=1}^T x_{t,i}$  and  $\bar{x} = 1/N \sum_{i=1}^N \bar{x}_i$ . It can be shown that

$$\hat{\sigma}_x^2 = \hat{\sigma}_{c,x}^2 + \bar{\sigma}_{T,x}^2 \quad (14)$$

with  $\bar{\sigma}_{T,x}^2 = \frac{1}{N} \sum_{i=1}^N \hat{\sigma}_{T,x,i}^2$ .<sup>10</sup> For the implementation, we multiply the empirical cross-sectional variance  $\hat{\sigma}_{c,x}^2$  by  $N/(N-1)$  and the empirical average serial variation  $\bar{\sigma}_{T,x}^2$  by  $T/(T-1)$  to account for the respective biases. The variance decomposition can lead to minor distortions if – as in our case – the panel is unbalanced.

In Table 2, we report summary statistics of the variables and the share of cross-sectional variation (see Equation (14)). What we see is that, in general, around 90% of the variation is in the cross section, i.e. between the banks, not in the time dimension, apart from the variable *bic*, where this share is 76.1%. In our sample, the average interest risk exposure is nearly 19 (per cent losses in present value relative to the bank capital); the average riskiness of the assets, measured by the RWA, is 56.1%, 8% of the observations are of banks with a trading book, 25.6% and 65.2% of the observations belong to savings and cooperative banks, respectively, and in 47% of the observations banks use interest rate derivatives. Concerning the net interest margin, 23.4% of an interest rate shock is

<sup>10</sup>Note the resemblance to the theoretical concept of the variance decomposition (see, for instance, Greene (2012)):  $var(x) = var(E(x|y)) + E(var(x|y))$  where  $y$  contains the bank-specific information.

finally passed through (on the asset side: 76.9%, on the liability side: 53.5%). [Busch and Memmel \(2017\)](#) estimate this net pass-through as 7.3%. Possible explanations for these differing estimates are as follows. First, the estimates for the long-run pass-through are based on different models and on different periods. Second, the net pass-through in this paper is under the implicit assumption of constant balance sheet weights whereas the net pass-through in [Busch and Memmel \(2017\)](#) estimates the total effect, also taking into account possible balance sheet adjustments, which should dampen the effect of an interest rate shock.

## 5 Results

### 5.1 Baseline Results

In [Table 3](#), we show the results of the regression (1). We obtain empirical evidence that the lower the credit risk per total assets, the higher the exposure to interest rate risk (highly significant negative coefficient for *rwa\_ta*). This is in line with the notion of [Schrand and Unal \(1998\)](#) that banks have an internal risk budget that they divide up into credit risk or interest rate risk. These findings help also to explain why [Mommel \(2011\)](#) and [Busch and Memmel \(2016\)](#) find contrary results when they investigate the relationship between a bank’s net interest margin and its earnings from term transformation. [Mommel \(2011\)](#), who finds only little empirical evidence for any relationship between the time series average of a bank’s net interest margin and its serial average exposure to interest rate risk, does not control for the exposure to credit risk. By contrast, [Busch and Memmel \(2016\)](#) find a positive relationship between a bank’s net interest income and its exposure to interest rate risk. Unlike [Mommel \(2011\)](#), they look at single years, namely 2012 and 2013, and, what is more, they control for the bank’s credit risk exposure.

Concerning the optimization horizon, we find – in line with the predictions from the theoretical model and the proxies for the length of this horizon – that not having a trading book (significant negative coefficient *trade*) and being a savings or cooperative bank (highly significant coefficients *svbk* and *coop*) is associated with higher exposure to interest rate risk.

We find highly significant support for our hedging hypothesis, according to which banks use interest rate risk to hedge the risk of a decline in the interest rate level (see [Subsection 3.3](#)). The higher the long-run pass-through (coefficient  $\theta$ ), the higher the exposure to interest rate risk.

The results for the control variables (capital ratio (*CR*), the size (*S*) and the usage of swaps (*swap*)) suggest that lowly capitalized, large banks which use interest rate swaps tend to structurally bear more interest rate risk.

Concerning the economic importance of these effects, which we measure as the cross-sectional standard deviation of the explanatory variable multiplied by the absolute coefficient, we see that being a savings or cooperative bank has by far the greatest impact (5.75 and 6.30), followed by the long-run pass-through (0.75), the riskiness of the assets (0.48) and the existence of a trading book (0.32). This compares with the economic importance of the control variables which are 0.70 for the capital ratio (*CR*), 0.33 for size (*S*) and 0.27 for the usage of swaps (*swap*).

Table 3: Results: Exposure to Interest Rate Risk

Variable	<i>bic</i>	<i>bic</i>	<i>bic</i>
<i>rwa_ta</i>	-3.927***	-5.808***	-2.933
	(1.225)	(1.584)	(2.358)
<i>trade</i>	-1.220**	-0.553	-2.594**
	(0.560)	(0.654)	(1.108)
<i>svbk</i>	13.349***		
	(0.501)		
<i>coop</i>	13.156***		
	(0.511)		
$\theta$	7.445***	7.889***	4.972
	(1.515)	(1.880)	(3.064)
<i>CR</i>	-9.488***	-13.120***	-9.745***
	(2.044)	(4.290)	(2.531)
<i>S</i>	0.217*	0.359***	-0.133
	(0.129)	(0.135)	(0.288)
<i>swap</i>	0.577*	0.328	1.376
	(0.327)	(0.351)	(1.104)
<i>constant</i>	4.964**	17.825***	9.928*
	(2.263)	(2.332)	(5.108)
R-squared	41.2%	3.6%	11.4%
Number of banks	1,718	1,529	189
Sample	All banks	Only savings and cooperative banks	Banks excluding savings and cooperative banks

This table shows the results of the regression (1). The dependent variable *bic* measures the interest rate risk exposure, *rwa\_ta* gives a bank's credit risk per total assets, *trade* indicates the existence of a trading book, *svbk* and *coop* are dummy variables indicating that a bank belongs to the savings and cooperative banks, respectively,  $\theta$  is the long-run pass-through, *CR* is the capital ratio, *S* is the natural logarithm of a bank's total assets and *swap* indicates the usage of interest rate swaps. Standard errors in brackets. \*\*\*, \*\*, \* denote significance at the 1%, 5% and 10%-level, respectively.

## 5.2 Robustness Checks

German banks' exposure to interest rate risk greatly depends on whether or not the bank belongs to the savings banks or cooperative banks sector. As a robustness check, we split the sample into banks that are savings or cooperative banks and banks that are other banks. In Table 3 (second and third columns), the results of these two subsamples are displayed. The strong decline in the R-squared (nearly 42% in the first column; less than 4% and just above 11% in the second and third column, respectively) and the high economic importance of the variables *subk* and *coop* (compared to the other variables; see Subsection 5.1) show that much variation is due to the membership of these banking groups. The results qualitatively remain the same. However, in the sample of savings and cooperative banks, the variable *trade* is no longer significant; in the other sample, the variables *rwa\_ta* and  $\theta$  become insignificant.

Another robustness check for the cross-sectional analysis is performed by looking at a certain point in time, and not at time series averages. When we look at year-end 2013, which is approximately in the middle of our sample period, the cross-sectional results qualitatively remain the same; the variable *trade*, however, becomes insignificant.

As a further robustness check, other control variables are introduced in regression (1), namely the share (with respect to total assets) of bonds (on the asset side and on the liability side) and of interbank funding. These variables turn out to be insignificant. When the capital ratio is replaced by the leverage ratio, this variable qualitatively shows the same results as the variable capital ratio.

## 6 Conclusion

This paper investigates the question of why banks bear interest rate risk in the first place, especially the reasons for their structural exposure are in the focus. We theoretically show that banks with different optimization horizons choose different exposures to interest rate risk. In our empirical study for the banks in Germany, we find that a bank's presumed optimization horizon, proxied by its membership of certain banking groups and its degree of capital market orientation, determines the structural exposure to interest rate risk. The longer this horizon is, the higher is the bank's exposure to interest rate risk in its banking book. Moreover, we find that banks with higher exposure to the risk of declining interest rate levels have higher exposure to interest rate risk. This supports the view that banks hedge this earnings risk with exposure to interest rate risk.

Most of the variation in the bank characteristics appears to be between the banks and not over the course of time. Therefore, the investigation in this paper with its focus on the cross section is an important complement to the existing literature, where the focus is mainly on the differences over the course of time.

## Appendix

### Appendix 1: Optimal Interest Rate Risk Exposure

Let there be the following relationship between the initial wealth  $W_0$  and the wealth in time  $t, W_t$ :

$$W_t = W_0 \cdot e^{R \cdot t} \quad (15)$$

where  $R$  is the relevant (annualized) interest rate.

Using the first order Taylor-approximation at  $t = 0$ , we obtain

$$W_t = W_0 \cdot (1 + R \cdot t). \quad (16)$$

We assume that there is an overnight parallel shift of the term structure in  $t = 0$ . This assumed shock is permanent and no further interest rate shocks will take place. Therefore, the initial wealth  $W_0$  is exposed to interest rate risk. If the interest rate changes, the present value of the asset changes as well where its modified duration  $D$  gives the sensitivity ( $\bar{R}$  and  $\bar{W}$  are the expectations of  $R$  and  $W$ , respectively):<sup>11</sup>

$$W_0 = \bar{W} \cdot (1 + D \cdot (\bar{R} - R)) \quad (17)$$

Combining (16) and (17) and neglecting terms of second-order importance, we obtain

$$\begin{aligned} W_t &= \bar{W} \cdot (1 + D \cdot (\bar{R} - R) + R \cdot t + D \cdot t \cdot (\bar{R} - R) \cdot R) \\ &\approx \bar{W} \cdot (1 + D \cdot (\bar{R} - R) + R \cdot t) \\ &= \bar{W} \cdot (1 + D \cdot \bar{R} + R \cdot (t - D)). \end{aligned} \quad (18)$$

We assume that the relevant interest rate risk  $R$  is composed of the interest rate level  $r$  and a term premium that is proportional to the duration, i.e.

$$R = r + c \cdot D \quad (19)$$

where  $c$  is the term premium per year of duration  $D$ ; in case of a normal term structure, it is positive.

If we assume in addition that the interest rate level  $r$  is normally distributed with  $E(r) = \bar{r}$  and  $var(r) = \sigma_r^2$ , then the wealth in  $t$ ,  $W_t$ , is normally distributed as well (see Equation (18)) and we can, assuming constant absolute risk aversion, state the preferences as follows (see Subsection 3.2), where  $t = T_i$  is the exogenous optimization horizon for bank  $i$ :

$$\begin{aligned} \phi_{T_i} &= E(W_{T_i}) - \frac{\gamma}{2} \cdot var(W_{T_i}) \\ &= \bar{W} \cdot (1 + T_i \cdot (\bar{r} + c \cdot D_i)) - \frac{\gamma}{2} \cdot \bar{W}^2 \cdot (T_i - D_i)^2 \cdot \sigma_r^2 \end{aligned} \quad (20)$$

Accordingly, the optimal modified duration  $D_i^*$  for bank  $i$  is

$$D_i^* = \frac{T_i \cdot c}{\gamma \cdot \bar{W} \cdot \sigma_r^2} + T_i. \quad (21)$$

If the term structure is normal (and  $c$  therefore positive), the optimal duration for bank  $i$  is greater than its optimization horizon  $T_i$ .

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<sup>11</sup>Note that the duration is defined as a positive number.

## Appendix 2: Hedging

We approximate bank  $i$ 's balance sheet as follows: On the asset side, there are two types of assets, namely loans (share:  $\theta_{A,i}$ ) and cash (share:  $1 - \theta_{A,i}$ ). The loans are handed out revolvingly, are free of credit risk, have an original maturity of  $M_{A,i}$  and their coupons correspond to the prevailing interest rate at the time the loans were granted. Assume that, at the beginning of the year under consideration, there is a parallel shift of the entire term structure by  $\Delta R$ . The change in the bank's net interest margin  $\Delta NIM_i$  of this year is

$$\Delta NIM_i = (\varphi_{A,i} \cdot \theta_{A,i} - \varphi_{L,i} \cdot \theta_{L,i}) \cdot \Delta R + \varepsilon_i, \quad (22)$$

where  $\varepsilon_i$  is some bank-specific noise and  $\varphi_{A,i}$  is the fraction of the bank's loans that have become due in the year after the shock, weighted by the period of time when – within the year – they matured (see [Busch and Memmel \(2017\)](#)):

$$\varphi_{A,i} = \frac{1}{2 \cdot M_{A,i}} \quad (23)$$

The modified duration of this bank's loan portfolio is approximately

$$D_{A,i} = \frac{1}{2} M_{A,i}. \quad (24)$$

The variables  $\theta_{L,i}$ ,  $\varphi_{L,i}$  and  $M_{L,i}$  are the corresponding variables on the bank's liability side.

Rearranging Equation (22) gives

$$\Delta NIM_i = (\varphi_{L,i} \cdot \theta_i + (\varphi_{A,i} - \varphi_{L,i}) \cdot \theta_{A,i}) \cdot \Delta R + \varepsilon_i, \quad (25)$$

where  $\theta_i := \theta_{A,i} - \theta_{L,i}$  is bank  $i$ 's (net) long-term pass-through.

Combining Equations (24) and (23) and setting the result into Equation (25), we obtain

$$\Delta NIM_i = \left( \varphi_{L,i} \cdot \theta_i + \left( \frac{1}{D_{A,i}} - \frac{1}{D_{L,i}} \right) \cdot \theta_{A,i} \right) \cdot \Delta R + \varepsilon_i \quad (26)$$

With  $\beta = \varphi_{L,i} > 0$  and  $\gamma = 50 \cdot E_i / (A_i \cdot D_{A,i} \cdot D_{L,i}) > 0$ , where  $E_i := A_i - L_i$  denotes the bank  $i$ 's equity, Equation (26) turns into Equation (12). To obtain the expression for  $\gamma$ , two additional assumptions need to hold. First, we assume that bank  $i$ 's interest bearing assets  $\theta_{A,i} \cdot A_i$  correspond to its interest bearing liabilities  $\theta_{L,i} \cdot L_i$ ,<sup>12</sup> so that the duration  $D_i$  of the bank's equity  $E_i$  can be written as

$$D_i = \frac{A_i \cdot \theta_{A,i}}{E_i} \cdot (D_{A,i} - D_{L,i}). \quad (27)$$

Second, we assume that the relevant of the two scenarios for the Basel interest coefficient is the increase in the interest rate level, so that  $bic_i = D_i \cdot 0.02$ .

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<sup>12</sup>[Mommel and Schertler \(2013\)](#) find for the German banks not using derivatives that, for the median bank, 92.1% of the assets and 89.8% of the liabilities (including equity) are interest bearing.

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