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Abstract

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I. Introduction

The placement of Fannie Mae and Freddie Mac into conservatorship on September 7, 2008 represented a historic milestone in the history of both government sponsored enterprises, making explicit the long-understood implicit government guarantee backing the financial operations of each institution. Given that the public sector in the U.S. and in other countries provides a number of explicit or implicit guarantees to financial institutions and public and quasi-public sector enterprises, policymakers and market participants are acutely interested in forming timely and accurate estimates of the likelihood that the contingent liability of the government will be triggered and of the estimated market value of the guarantee in the event of default.

The purpose of this paper is to estimate the market value of the implicit government guarantee to Fannie Mae and Freddie Mac prior to their placement into conservatorship using modern contingent claims analysis. Contingent claims analysis has been applied extensively in private sector risk assessment (Crosbie and Bohn, 2003) and is commonly referred to as the "Merton Model" when used to measure corporate default risk (Merton, 1974, 1977, 1992, 1998). The framework has also been extended to estimate sovereign risk (Gapen et al., 2008) and economywide risk (Gray and Malone, 2007; Gray, Merton, and Bodie, 2007; Gapen et al. 2004). Contingent claims analysis has been shown to be an effective tool for risk management because it incorporates uncertainty inherent in balance sheet components and translates this uncertainty into quantifiable indicators of financial risk. The incorporation of uncertainty is important in risk analysis because uncertain changes in future asset value relative to promised payments on debt ultimately drive default risk, the triggering of implicit or explicit guarantees, and the transfer of risk across balance sheets. Traditional macroeconomic models have not been successful in studying financial fragility since they largely linear in design, abstract from default, and often provide a trivial role for financial intermediaries. The structural changes in financial markets over the past several decades, beginning with the fall of the Bretton Woods system and continuing through the liberalization of capital account flows, have led to large increases in volatility and a greater need to understand and manage this risk. The continuous-time mathematics of stochastic processes permits a richer interaction between time and uncertainty than found in traditional macroeconomic models, and the extension of option pricing to the pricing of corporate liabilities makes modern contingent claims analysis better suited to estimate discontinuous, nonlinear default risk.

The paper develops a set of key market-based capital and credit risk indicators to measure financial fragility in Fannie Mae and Freddie Mac. The main result of the paper is that the estimated market value of the guarantees on senior debt of each GSE using contingent claims analysis broadly corresponded to the potential size of capital injections under the Treasury Preferred Stock Purchase Agreement at the time of their placement in conservatorship on September 7, 2008. Over the five trading days before placement into conservatorship, the average expected market value of the guarantee was $95 billion for Fannie Mae and $40 billion for Freddie Mac. However, Monte Carlo simulations reveal that the total cost to recapitalize the GSEs may be significantly higher than provided for under the existing terms of the agreement. VaR measures on the day each was placed into conservatorship indicate that expected market value of the guarantee remained below US$310 billion for Fannie and US$133 billion for Freddie in 95 percent of simulations.

Consequently, the $100 billion size of the Treasury Preferred Stock Purchase Agreement comes much closer to a Basel-based standard of holding capital to cover unexpected losses at a specified confidence interval for Freddie Mac, while falling well short of this standard for Fannie Mae.

A second main result of the paper is to illustrate how similar measures of financial risk could easily be applied to banking and non-bank financial institutions. Financial risk indicators such as the distance to minimum capital, probability of falling below minimum capital, and expected market value losses on capital can complement the traditional credit risk indicators from the Merton Model to fully inform the setting of appropriate levels of capital and to provide early warning indicators of the need for prompt corrective action on the part of regulators. In this paper, Monte Carlo simulations of the contingent claim risk indicators clearly showed that the existing levels of capital in Fannie Mae and Freddie Mac were well below that needed to maintain financial soundness in the face of building market turbulence, and suggested that significant financial risk was present as early as November 2007, or nearly a year before each was placed into conservatorship.

The note is organized as follows. Section II first details how contingent claims analysis can be applied to each government sponsored enterprise when the balance sheet is comprised of multiple liabilities. In this case, each GSE is modeled with four layers of liabilities: common equity, preferred equity, subordinated debt, and senior debt. The second part of this section illustrates how the approach can be used to generate the expected market value of the guarantee and expected market value losses on capital. Section III applies the framework to Fannie Mae and Freddie Mac using available mrket data and balance sheet information from publicly available financial statements.

II. Modeling Risk Transfer with Contingent Claims

The contingent claims framework outlined in this section provides an explicit measure of probability of default and estimates of the expected loss on senior debt, or the value of the government guarantee to each GSE. The framework is flexible enough to provide forward looking estimates of the likelihood that the contingent liability of the federal government is triggered and, if so, the expected market value of the guarantee in the event of default.

A. Contingent Claim Analysis with Multiple Layers of Liabilities

Contingent claims analysis uses three main elements to characterize the risk of default: the value of firm assets, asset volatility, and leverage; where asset value is defined as the market value of firm assets, asset volatility captures the inherent uncertainty over future asset value, and leverage measures the size of the firm's contractual liabilities. Contractual liabilities are measured in book value terms since these are the amounts that the firm is obligated to pay. The liability side of the balance sheets of Fannie Mae and Freddie Mac are transparent and are made publicly available through quarterly financial statements. Each of these GSEs has a liability structure composed primarily of senior debt and common equity, augmented by subordinated debt and preferred equity which act as capital buffers similar to traditional banking institutions. This analysis proceeds with these four layers of liabilities and their order of seniority in the capital structure.[1] As shown below in Figure 1, the government

[1] There are small amounts of "other liabilities" present on each balance sheet, but these can be ignored or subsumed into one of the remaining four liabilities without loss of generality.

guarantee is simultaneously a contingent liability of the federal government and a contingent asset of each GSE.

Since asset value and volatility are unobservable, contingent claims analysis uses the capital structure of the GSEs described above to derive implied values. In particular, the value of common equity, or market capitalization, is viewed as a call option on GSE assets with remaining contractual liabilities serving as distress and default barriers. If GSE assets exceed the amount necessary to service bondholders and preferred equity shareholders, then common equity has value. Financial distress increases as the market value of assets approaches the book value of liabilities from above. A sufficiently large decline in the market value of assets affects the least senior liabilities first, namely preferred equity and subordinated debt, leading to a potential breach of minimum capital adequacy standards and the triggering of prompt corrective action measures on the part of the regulator (Kupiec, 2005, 2007; Chan-Lau and Sy, 2006).[2] Default ultimately occurs when the market value of assets falls below promised payments on senior debt. Contractual liabilities and their promised payments therefore constitute two important distress points for GSEs and other financial institutions. Risk of prompt corrective action and default risk are measured by the relationship between GSE assets and these distress points.[3,4]

[2] The level of capital that causes the firm to seek additional capital or causes the regulator to exercise prompt corrective action is somewhat arbitrary and is likely higher than the level of capital that constitutes the regulatory minimum (see Chan-Lau and Sy (2006) for additional discussion). For the purposes of this exercise, the discounted distress barrier composed of preferred equity and subordinated debt is assumed to meet each of these definitions simultaneously.

[3] Fannie Mae and Freddie Mac are regulated by the Office of Federal Housing Enterprise Oversight.

[4] The market value of firm assets can trade below the book value of total liabilities for significant periods of time if the majority of liabilities are long-term, allowing the firm to continue servicing interest payments while undertaking steps to improve its financial health. Analysis of corporate defaults indicates that the market value of firm assets that triggers default lies somewhere between the book value of total liabilities and short-term liabilities. In this note I follow the approach of Moody's KMV: the distress and default

Using the Black-Scholes option pricing formula, equity as a call option on GSE assets is,

$$V_E = V_A N\left(d_1\left(DB_3\right)\right) - DB_3 e^{-r_f \tau} N\left(d_2\left(DB_3\right)\right), \tag{1}$$

where V_E is the value of equity, V_A is the value of firm assets, DB_3 is the distress barrier comprised of senior debt, subordinated debt, and preferred equity, r_f is the risk-free rate of interest, and τ is the time to maturity on a default-free bond in years. $N(d)$ is the cumulative probability distribution function for a standard normal variable where,

$$d_1\left(DB_3\right) = \frac{\ln\left(\dfrac{V_A}{DB_3}\right) + \left(r_f + \dfrac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\sqrt{\tau}}, \quad d_2\left(DB_3\right) = d_1\left(DB_3\right) - \sigma_A\sqrt{\tau}, \tag{2}$$

and σ_A is the standard deviation of return on firm assets. The Black-Scholes formula in equation (1) contains two unknowns, the value of GSE assets and asset volatility. The relationship between asset value and volatility of equity is given by,

$$V_E = \frac{\sigma_A}{\sigma_E} \mathrm{V}_\mathrm{A} N\left(d_1\right), \tag{3}$$

where σ_E is the standard deviation of equity.[5] However, the main implication is that equations (1) and (3) can be solved simultaneously for the implied value of GSE assets and asset volatility using a limited set of observable market data as inputs (market capitalization, implied equity volatility, existing liabilities, the risk-free rate, and time).

The market value of assets in relation to the distress and default barriers is illustrated in Figure 2. The uncertainty in future asset value is represented by a

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barriers are constructed using short-term debt plus half of long-term debt, plus interest payments due over the one-year ahead time horizon.

[5] See Hull (1993, p. 38). Here, $N(d_1)$ is the option *delta*.

probability distribution at the time horizon. At the end of the period, the value of assets may be above the distress and default barriers, indicating that debt service can be made and capital is preserved, or below one or both of the distress and default barriers, leading to a need to raise additional capital, prompt corrective action on the part of the regulator or default. The probability that assets will fall below either the distress or default barrier is simply the area of the distribution that lies below the barrier. Any decline in expected asset value or increase in asset volatility means more of the probability distribution lies below the unchanged barrier and a higher probability of corrective action or default.

Distance to minimum capital, *D2MC*, measures the number of standard deviations implied asset value is above the distress barrier on senior, subordinated, and preferred equity. Distance to default, *D2D*, is the number of standard deviations implied asset value is above the default barrier on senior debt. The precise value of each of these measures comes directly from equation (1), or

$$D2MC = d_2\left(DB_3\right) = \frac{\ln\left(V_A * \exp\left(\left(r_f - \frac{1}{2}\sigma_A^2\right)\tau\right)\right) - \left(\ln DB_3\right)}{\sigma_A\sqrt{\tau}} , \qquad (4)$$

and,

$$D2D = d_2\left(DB_1\right) = \frac{\ln\left(V_A * \exp\left(\left(r_f - \frac{1}{2}\sigma_A^2\right)\tau\right)\right) - \left(\ln DB_1\right)}{\sigma_A\sqrt{\tau}} , \qquad (5)$$

where DB_1 represents the distress barrier comprised of obligated payments to holders senior debt. Lower implied value of assets, higher levels of senior debt, and higher levels of asset volatility all reduce the distance to minimum capital and default.

Distance to minimum capital and distance to default can be translated into risk-neutral measures of probability. The Merton option pricing framework assumes that future asset value is distributed log-normally and the risk-neutral probability of default is the portion of the distribution that that lies below the distress or default barrier. The risk-neutral probability of breaching minimum capital, *RNMC*, and the risk-neutral default probability on senior liabilities, $RNDP_{SL}$, are,

$$RNMC = N\left(-d_2\left(DB_3\right)\right). \qquad (6)$$

and,

$$RNDP_{SL} = N\left(-d_2\left(DB_1\right)\right). \qquad (7)$$

"Risk-neutral" or "risk-adjusted" valuation is an important factor underlying the derivation of the Black-Scholes option pricing formulas whereby the value of the option can be derived by forming a riskless hedge portfolio. As a result, option values do not depend on the investor's attitude toward risk. However, market practitioners frequently adjust the risk-neutral risk indicators from the Merton Model to estimate "actual" or "real-world" risk indicators. The procedure used to translate risk-neutral probabilities into estimated actual probabilities follows that outlined in Gray and Malone (2007) and Gray, Merton, and Bodie (2007). For example, the risk-neutral distance to default has an asset drift of the risk-free rate, r_f, and a corresponding risk-neutral default probability of $N(-d_2)$, while the "actual" distance to default has asset drift of μ_A , which includes the level of investor risk aversion, and "actual" default probability of $N(-d_{2,\mu})$. These two risk indicators are related by the market price of risk, λ, according to,

$$N\left(-d_{2,\mu}\right) = N\left(-d_2 - \lambda\sqrt{\tau}\right), \qquad (8)$$

A common approach to estimating the market price of risk is to use the capital asset pricing model. In this framework, the market price of risk is estimated by,

$$\lambda = \rho_{A,M} SR \,, \tag{9}$$

where *SR* is the market Sharpe Ratio and,

$$\frac{\mu_A - r_f}{\sigma_A} = \lambda \quad \text{or} \quad \mu_{\mathrm{A}} = r_f + \rho_{A,M} SR \sigma_A \,. \tag{10}$$

Here, $\rho_{A,M}$ is the correlation of the implied asset return with the market.

B. Estimating the Market Value of the Guarantee

Senior liabilities derive their value from two sources: pure default-free value and the bearing of default risk of the issuer. If the issuer does not default over the life of the bond, the bondholder receives the stated interest and principal payments. In the contingent claims framework this default-free value is equal to the book value of debt as approximated by the default barrier, discounted to the present. In the event of default, senior bondholders receive less than the default-free value after assuming control of GSE assets. Any guarantee against default would need to offset this loss. The market value of the guarantee of GSE debt at any point in time, therefore, is equal to the difference between the default-free value of debt and its value in the presence of default risk. Letting '*risky debt*' define the value of debt that contains risk of default,

$$\textit{Implicit guarantee} = \textit{default-free debt} - \textit{risky debt}. \tag{11}$$

This can be re-written to solve for the market value of risky senior liabilities as,

$$\textit{Risky debt} = \textit{default-free debt} - \textit{implicit guarantee}. \tag{12}$$

The value of the implicit guarantee behaves like a put option on firm assets with strike price equal to the default barrier. As firm assets approach the default barrier from above, the value of equity declines, probability of default increases, and the expected value of the guarantee rises. Thus, in the contingent claims framework, risky debt and the value of the implicit guarantee—like equity—derive their value from stochastic assets. Inserting the value of default-free senior debt, $DB_1 e^{-r_f \tau}$, and the put option formula for the implicit guarantee into equation (12), the value of risky senior liabilities, V_{SL}, is,

$$V_{SL} = DB_1 e^{-r_f \tau} - \left[DB_1 e^{-r_f \tau} N\left(-d_2\left(DB_1\right)\right) - V_A N\left(-d_1\left(DB_1\right)\right)\right]. \qquad (13)$$

The value of the implicit put option, which estimates the market value of the implicit government guarantee, can be computed directly after solving equations (1) and (3) simultaneously and inserting these values into equation (13). While adjustments are needed to translate risk-neutral probability of default into actual probability of default, no such adjustments are needed for valuation purposes. Thus, risk-neutral properties of Black-Scholes option pricing are sufficient for valuing the estimated market value of the guarantee and risk transfer.[6]

A similar procedure can be implemented to compute the expected market value of capital losses. Under the assumption that subordinated debt and preferred equity constitute the level of capital, summing over the implicit put option on each yields a measure of expected reductions in the market value of capital within each GSE.

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[6] Adjustments to the standard Black-Scholes option pricing equation to account for fat tails and non-normal distributions are discussed in Gray and Malone (2007), who use mixtures of lognormals consistent with a jump diffusion process. Hull et al. (2004) study the relationship between implied volatility implied by equity options and CDS spreads. Finally, the Merton Model can easily be extended to include stochastic interest rates as shown in Shimko, Tejima and Van Deventer (1993) and Longstaff and Schwartz (1995).

Alternatively, this measure could be viewed as estimating the market value of additional capital needed to preserve existing capital ratios. The value of subordinated debt, V_{SUB}, is,

$$\begin{aligned} V_{SUB} = {} & DB_2 e^{-r_f \tau} - \left[DB_2 e^{-r_f \tau} N\left(-d_2\left(DB_2\right)\right) - V_A N\left(-d_1\left(DB_2\right)\right) \right] \\ & - \left\{ DB_1 e^{-r_f \tau} - \left[DB_1 e^{-r_f \tau} N\left(-d_2\left(DB_1\right)\right) - V_A N\left(-d_1\left(DB_1\right)\right) \right] \right\}, \end{aligned} \qquad (14)$$

where DB_2 is computed using the promised payments on senior and subordinated debt. The corresponding value of preferred equity, V_{PRE}, is,

$$\begin{aligned} V_{PRE} = {} & DB_3 e^{-r_f \tau} - \left[DB_3 e^{-r_f \tau} N\left(-d_2\left(DB_3\right)\right) - V_A N\left(-d_1\left(DB_3\right)\right) \right] \\ & - \left\{ DB_2 e^{-r_f \tau} - \left[DB_2 e^{-r_f \tau} N\left(-d_2\left(DB_2\right)\right) - V_A N\left(-d_1\left(DB_2\right)\right) \right] \right\}. \end{aligned} \qquad (15)$$

As with the case of senior debt, the value of subordinated debt and preferred equity is the default-free value of promised payments minus a guarantee against default, though a guarantee need not exist in the case of preferred equity or subordinated debt. In this case, the guarantee against default is somewhat equivalent to the expectation that capital charges will be taken to offset losses. The expected loss is modeled as an implicit put option with strike price equal to DB_2 or DB_3. However, the reduction in the expected market value of capital on subordinated debt is capped at the discounted book value of these liabilities. Continued declines in the value of implied assets will eventually exceed the level of capital and eventually lead to expected losses on senior debt which increases the value of the guarantee. Therefore, the value of risky senior liabilities and its implicit put option must be subtracted off. A similar correction is made for preferred equity, where lower implied assets will eventually create losses in senior and subordinated debt.

In the Merton framework, expected loss can be used to derive estimated implied credit default swap spreads. The protection buyer in a CDS contract pays a series of payments of spread until maturity or a credit event, whichever comes first, while the protection seller is obligated to make a contingent payment of par minus recovery following a credit event. To derive an estimated implied CDS spread on senior liabilities, $EICDS_{SL}$, first begin with the yield to maturity, y, on the value of risky senior debt,

$$y = \frac{\ln\left(DB_1 e^{-r_f \tau} / V_{SL}\right)}{\tau}. \qquad (16)$$

where this can be rearranged to show $V_{SL} = DB_1 e^{-y\tau}$. Assuming EL_{SL} is the value of the implicit put option in equation (13), the credit default swap spread on senior debt can be written as,

$$EICDS_{SL} = y - r_f = \frac{\ln\left(DB_1 e^{-r_f \tau} / V_{SL}\right)}{\tau} - r_f = -\frac{1}{\tau}\ln\left(1 - \frac{EL_{SL}}{DB_1 e^{-r_f \tau}}\right). \qquad (17)$$

III. Case Study: Fannie Mae and Freddie Mac

The contingent claims framework outlined above was used to derive credit risk indicators and an estimate of the market value of the government guarantee to Fannie Mae and Freddie Mac prior to their placement into conservatorship on September 7, 2008. Data on market capitalization and implied equity volatility was obtained from Bloomberg, and the distress and default barriers were constructed using debt statistics from publicly-available GSE financial statements between end-2006 and the second quarter of 2008. The risk-free rate was assumed to be 3 percent and the time horizon is equal to one year.[7,8]

[7] An alternative approach is to use the actual yield from one-year Treasury bills. However, this series may be distorted due to the flight to quality stemming from the unwinding of the subprime mortgage market. Estimations using this series as an input were conducted and do not change the thrust of the exercise.

[8] The one-year time horizon follows Moody's KMV, who conducted extensive empirical studies to match the one-year time horizon, estimated distress barrier, incidents of corporate default, and the mapping of

As shown in Figure 3, the negative outlook for the mortgage market caused the market capitalization of Fannie Mae to decline significantly beginning in October 2007, and implied volatility measured from equity options to spike upward. By the third week in November, Fannie Mae had lost $37 billion in market value. Using contingent claims analysis, falling equity prices translated into a lower implied value for GSE assets, higher implied asset volatility, and increased credit risk. Distance to minimum capital, which stood at 4 standard deviations in mid-2007, fell to around 1 standard deviation in November. Distance to default followed a similar pattern, falling from nearly 6 standard deviations to below 2 standard deviations. By late November 2007, the estimated actual probability of breaching minimum capital spiked briefly to 16 percent, while estimated actual probability of default on senior debt rose above 10 percent. The increase in credit risk, however, was not severe enough to substantially increase the expected market value of the guarantee, but did produce around a combined $4 billion of expected market-value capital losses on preferred equity and subordinated debt. A similar story for Freddie Mac is shown in Figure 4, though the market appeared to view Freddie Mac as being slightly more fragile than Fannie Mae by late 2007. A halving of the market capitalization and increased equity volatility caused distance to minimum capital and default to fall below 1 standard deviation, and estimated actual probabilities of breaching minimum capital and default on senior debt spiked to around 20 percent. Finally, the expected market value of capital losses were estimated at a combined $3 billion in November 2007.

Consequently, contingent claims analysis revealed significant expected weakness in the financial soundness of both institutions by late 2007, suggesting additional

risk-neutral default rates into estimated default probability. However, the time period can be varied to examine default over any horizon. Adjustments to remaining variables would be needed for consistency.

measures to raise capital would have been warranted at this time. Indeed, Freddie Mac issued preferred equity shares in three separate offerings between July and November of 2007. On July 24 and September 24, 2007, Freddie Mac conducted two $500 million offerings with a dividend of 6.02 percent and 6.55 percent, respectively. These offerings were followed by a substantial $6 billion dollar fixed-to-floating preferred offering at 8.375 percent on November 29, 2007. The increasing cost of capital at these offerings was consistent with the increased credit risk from contingent claims framework.

When the views of market participants took another step downward in June 2008, the value of the guarantee began to rise sharply. The reductions in market capitalization to below $10 billion for each GSE and the significant increase in implied volatility dramatically increased the estimated actual probabilities of falling below minimum capital and default. This increase in the likelihood of default was also confirmed by the CDS market, as CDS spreads on both Fannie Mae and Freddie Mac jumped, though not as high as suggested by the estimated implied CDS spreads from the Merton Model.[9] The estimated market value of the guarantee for both institutions peaked in July, with the value of the implicit government guarantee rising to $226 billion for Freddie Mac on July 1st, and $166 billion for Fannie Mae on July 18.[10] Between July and September 5, 2008, when the two GSEs were placed into conservatorship, the average expected value of the guarantee was $59 billion for Fannie Mae and $90 billion for Freddie Mac. Over the five

[9] That the actual market CDS spreads remained below the model-implied CDS spreads in mid- to late-2008 is likely due to market participants assumptions that the government would make good on its implicit guarantee to both institutions, thereby reducing the likelihood of default and expected loss given default. The model does not include this judgment and, consequently, yields a higher implied CDS spread.

[10] As assets decline, $N(d_1)$ and $N(d_2)$ in equations (13)–(15) approach zero and equity becomes increasingly worthless. At the same time, however, the implicit put option from the guarantee on debt becomes more valuable since $N(-d_1)$ and $N(-d_2)$ both approach 1. As implied asset volatility rises for a given level of assets, the implicit put option on debt becomes more valuable since the probability that assets fall below the default barrier increases.

trading days before placement into conservatorship, the average expected value of the guarantee was $95 and $40 billion, respectively. These values correspond closely to the potential size of the capital injections under the Senior Preferred Stock Purchase Agreement with the Treasury; the plan provides for up to $100 billion in capital infusions into each GSE.[11] Thus, contingent claims analysis on each GSE indicates that this amount should be sufficient to offset expected losses on senior debt and maintain financial solvency.[12]

As shown in Figures 3 and 4, however, day-to-day fluctuations in the value of the guarantee are large, which stems from the fact that option price sensitivities (e.g. the "greeks") are largest when implied assets are in the neighborhood of the distress and default barriers. Small changes in implied asset value or volatility in the vicinity of default can therefore translate into large value changes in the implicit put option on risky debt. Indeed, simulations indicate that the distribution of market value of the guarantee was quite wide, with value-at-risk (VaR) measures suggesting that the total cost to recapitalize the GSEs may be significantly higher than provided for under the existing terms of the agreement. Figure 5, for example, shows the distribution of the expected market value of the guarantee under Monte Carlo simulations for September 5, 2008, the last trading day prior to the placement of Fannie Mae and Freddie Mac into conservatorship. VaR measures for this day indicate that value of the guarantee remained below US$310 billion for Fannie Mae and US$133 billion for Freddie Mac in 95 percent of simulations. An average of the VaR measures from simulations performed on

[11] In addition to the preferred stock purchase agreement, the Treasury also provided for a Government Sponsored Enterprise Credit Facility and a GSE mortgage-backed securities purchase program.
[12] This conclusion should not be interpreted as indicating that the level of capital in each GSE with the Preferred Stock Purchase Arrangement is adequate from a Basel-based standard of covering unexpected losses. The simulation exercise is intended to examine this issue more closely.

contingent claim balance sheets for the week of September 2–5, 2008 reveals a similar picture. The average 95 percent VaR for Fannie Mae and Freddie Mac during this period was $320 billion and $140 billion, respectively. Although contingent claims analysis reveals that the size of the Treasury Preferred stock Purchase Agreement is sufficient to cover the expected market value of the guarantee on the day the GSEs were placed into conservatorship, this amount could be far short of what may eventually be needed should financial conditions of the GSEs worsen. The $100 billion size of the Preferred Stock Purchase Agreement comes much closer to a Basel-based standard of holding capital to cover unexpected losses in Freddie Mac, while falling well short of this standard for Fannie Mae.

IV. Conclusion

Modern contingent claims analysis, with its underpinnings in continuous-time mathematics of stochastic processes, is a superior method for assessing important indicators of capital and credit risk in financial institutions. These indicators provide a useful barometer in the risk management process for setting optimal levels of capital in a structurally different and volatile financial market environment. By relying on market prices and the capital structure of the balance sheet as inputs, contingent claims analysis yields a structural model with forward-looking estimates of risk, thereby overcoming an important criticism of widely used financial soundness indicators, or FSIs, that rely on historical accounting balance sheet data. These forward estimates of risk provided accurate early warning signals of building financial risk in Fannie Mae and Freddie Mac as early as November 2007, a framework for translating building default risk into estimates of the market value of guarantees on senior debt, and risk-neutral distributions of the value of the implicit government guarantee to each institution prior to their placement into conservatorship. Finally, the application of contingent claims analysis to Fannie Mae and Freddie Mac also highlights the flexibility of the framework to handle multiple liabilities and seniority structures which may be present on the balance sheets of corporates, banks, and non-bank financial entities.

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Figure 1. The Implicit Government Guarantee

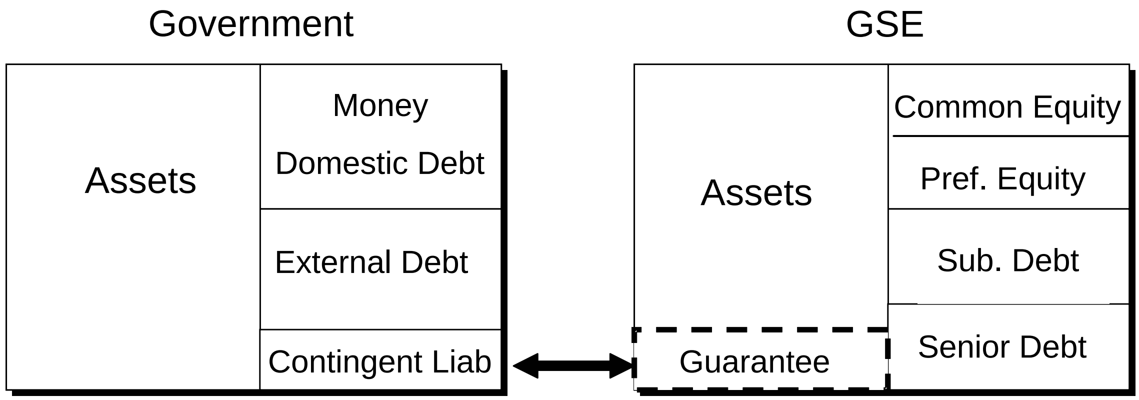

Figure 2. Distribution of Implied Assets and the Distress and Default Barriers

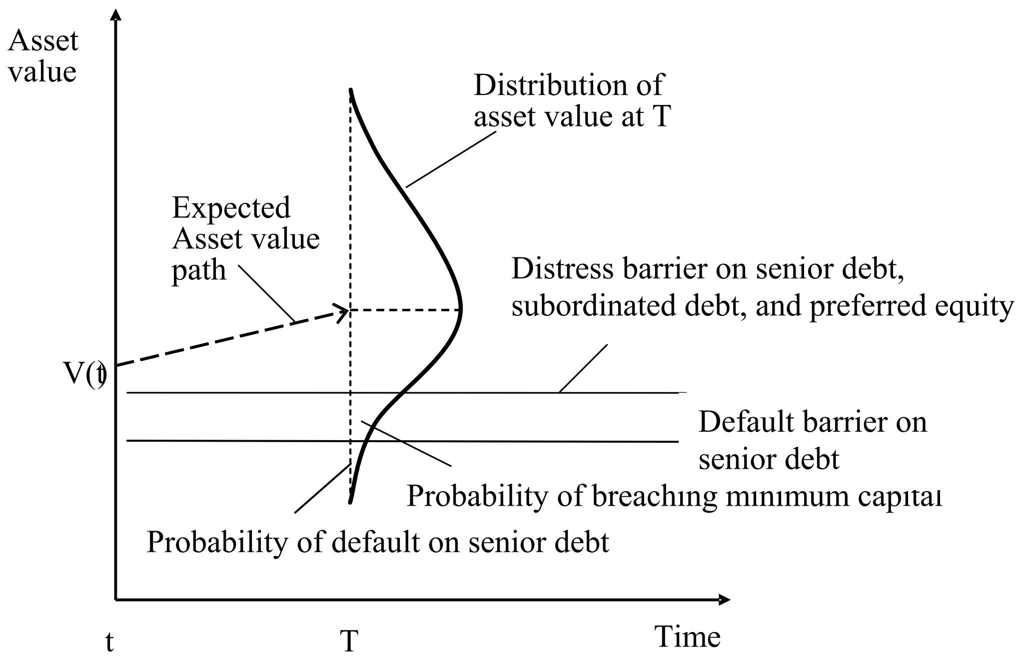

Figure 3. Estimating the Value of the Implicit Guarantee to Fannie Mae

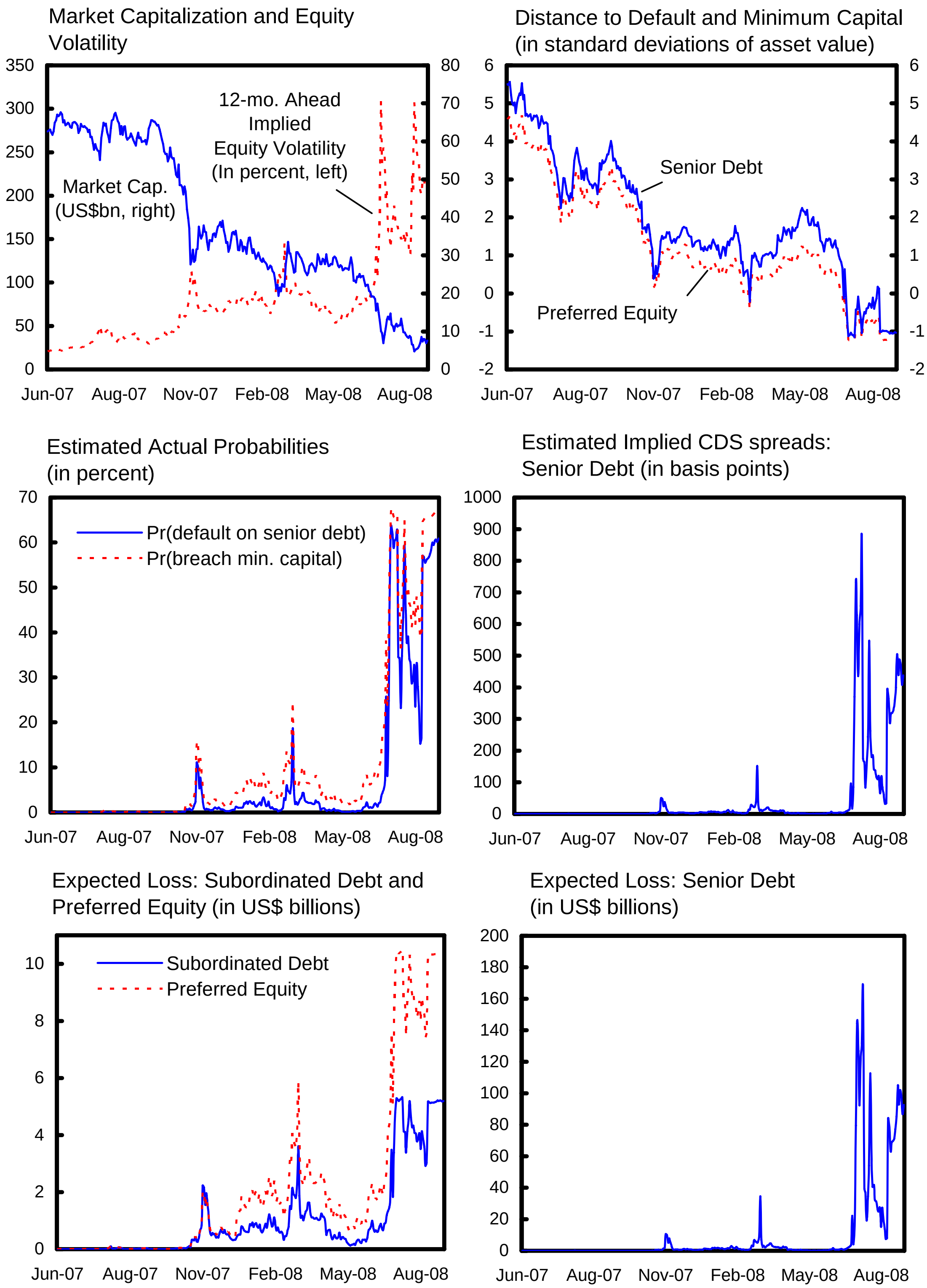

Figure 4. Estimating the Value of the Implicit Guarantee to Freddie Mac

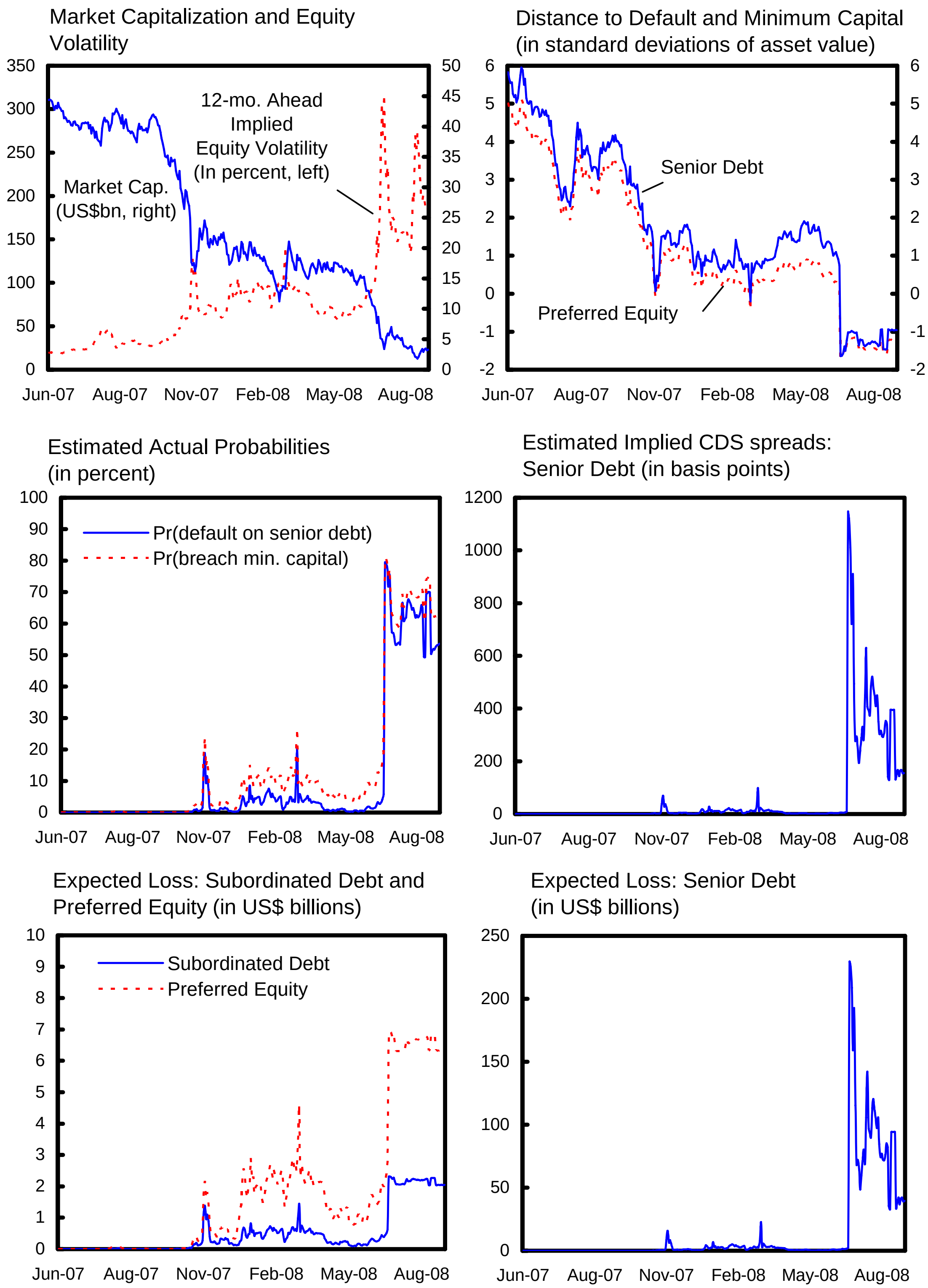

Figure 5. Distribution of the Value of the Implicit Guarantee at Placement into Conservatorship on Sept. 5, 2008