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Discussion of „Efficient Estimation of Forecast Uncertainty“

Discussion of “Efficient Estimation of Forecast Uncertainty” by Malte Knüppel

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The paper

- Estimate the forecast uncertainty of multi-step ahead forecasts.
  - Typically we report the MSFE (or MAFE) for each forecast horizon.
  - Using information from different horizons together can improve our estimate of the forecast uncertainty.
- Great paper.
  - Topic that is becoming increasingly noticed but not much investigated yet.
  - Focuses on forecast errors only, independent from the underlying model.
  - In small samples efficiency gains can be obtained.

General comments

- The paper contains too much.
    - Different data structures: RT, FM, FR.
    - Different estimators: GLS, SUR, FGLS.
    - Different data generating processes: AR(1), AR(2), VAR(2), Markov-Switching, non-invertible MA(19), ARCH(1,1), AR(1) with break in σ^2.
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- Split or shorten the paper.
  - Improve notation: Two different notations for the RT structure, with the second one being much easier to understand.
  - Give the reader more intuition.
  - Focus on applicability of the method.

Data structures

- Why are the efficiency gains for different data structures so different?
- Example with H=2:
  - h = 1:
    - no efficiency gain for RT,
    - increasing efficiency gain with $|b_1|$ for FR,
    - decreasing efficiency gain with $|b_1|$ for FM.
  - h = 2:
    - Non-monotonous efficiency gain for RT,
    - increasing efficiency gain with $|b_1|$ for FR,
    - decreasing efficiency gain with $|b_1|$ for FM.
- Intuition?

Efficiency gains for AR(1)

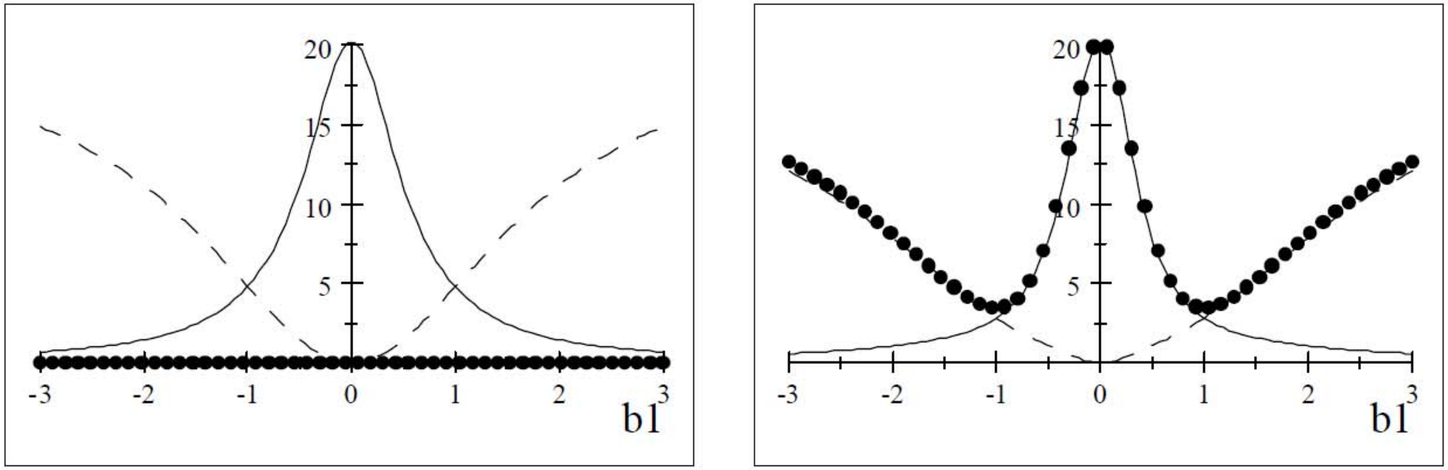


Figure 1: Efficiency gains φ_1 (left panel) and φ_2 (right panel) as functions of b_1 for the FR data structure ($N = 2, H = 2$, dashed line), the FM data structure ($N = 2, H = 2$, solid line) and the RT data structure ($N = 3, H = 2$, dots), always with $\alpha = 3$.

Data structures for H=2

From/for period	1	2	3	4	5
0	e_{10}	e_{20}			
1		e_{21}	e_{31}		
2			e_{32}	e_{42}	
3				e_{43}	e_{53}
4					e_{54}

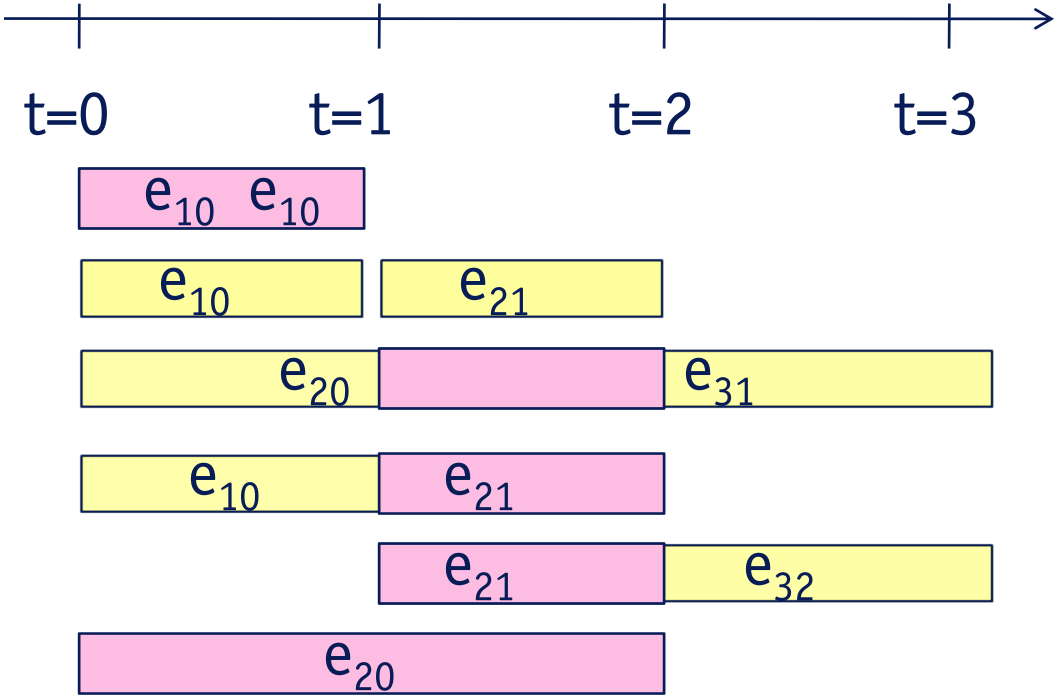
RT: All forecast errors: 5 observations for h=1, 4 for h=2.

FM: Forecast errors **from** a certain time span (without e_{54}).

FR: Forecast errors **for** a certain time span (without e_{10}).

Using all available information is always most efficient.

The covariances (ignoring $2\sigma^4$)



$\mathrm{cov} = 1$

$\mathrm{cov} = 0$

$\mathrm{cov} = b_1^2$

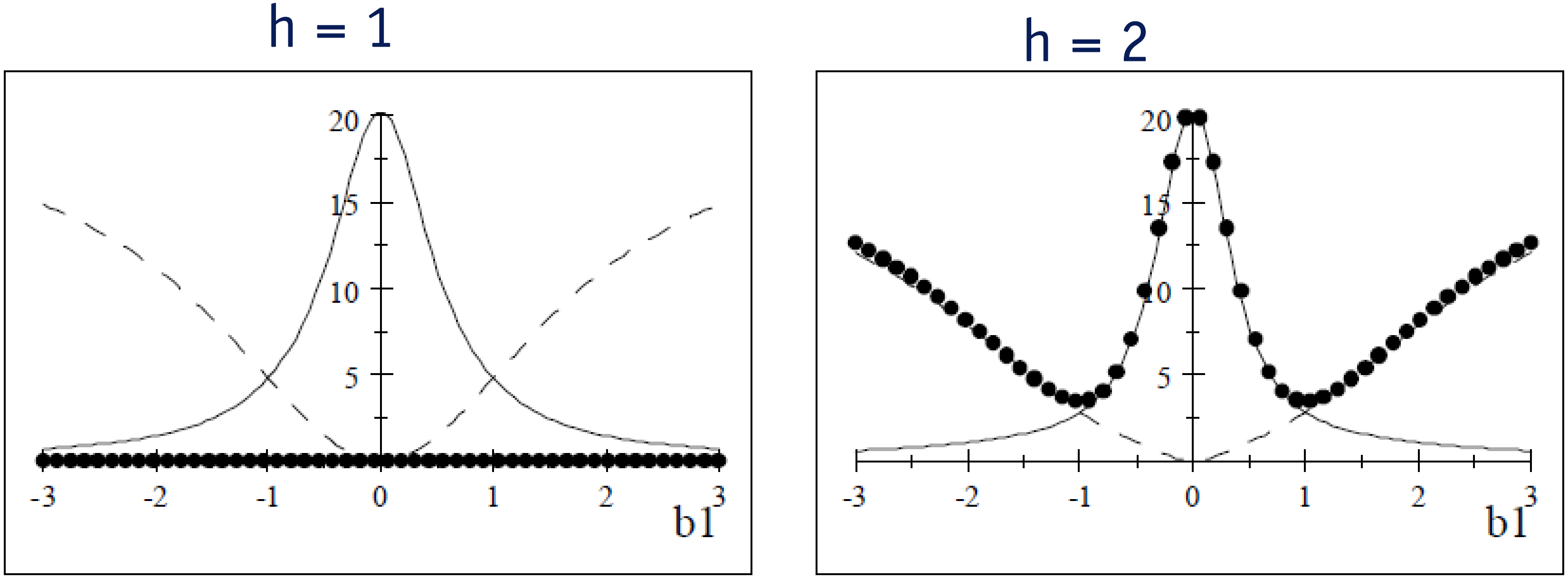
$\mathrm{cov} = b_1^2$

$\mathrm{cov} = b_1^2$

$\mathrm{cov} = (b_1^2+1)^2$

Overlap between forecasts determines the covariance.

Efficiency gains for AR(1)



Intuition: Substitute one observation with cov = 1 in FR structure by one observation with cov = b_1^2 in FM structure.

Intuition

$$\mathbf{\Omega}_{FR,2,2} = 2\sigma^4 \begin{array}{c} \begin{array}{cccc} e_{21} & e_{20} & e_{32} & e_{31} \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 0 & b_1^2 \\ 1 & (b_1^2+1)^2 & 0 & b_1^2 \\ 0 & 0 & 1 & \enclose{circle}{1} \\ b_1^2 & b_1^2 & \enclose{circle}{1} & (b_1^2+1)^2 \end{array}\right] \end{array} \begin{array}{c} \\ e_{21} \\ e_{20} \\ e_{32} \\ e_{31} \end{array}$$

$$\mathbf{\Omega}_{FM,2,2} = 2\sigma^4 \begin{array}{c} \begin{array}{cccc} e_{10} & e_{21} & e_{20} & e_{31} \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & \enclose{circle}{b_1^2} & 0 \\ 0 & 1 & 1 & b_1^2 \\ \enclose{circle}{b_1^2} & 1 & (b_1^2+1)^2 & b_1^2 \\ 0 & b_1^2 & b_1^2 & (b_1^2+1)^2 \end{array}\right] \end{array} \begin{array}{c} \\ e_{10} \\ e_{21} \\ e_{20} \\ e_{31} \end{array}$$

What do we learn from this?

- Small sample aspect of a small sample problem – take any observation you can get.
- For RT structure: Efficiency gains only at longer horizons which have less observations (known from SUR literature).
- **RT is still most efficient at h = 1!**
- But note:
  - Differences between data structures will increase with H because more observations are cut off.
  - Adjusting *N* to have same number of forecasts to evaluate is possible in simulations but not quite realistic.

Feasible GLS

- True SUR (block-diagonal) covariance matrix instead of true covariance matrix still more efficient than OLS.
  - Remark:
    - $\rho = 1.5$ and $\rho = 2.0$ do not seem to be relevant in practice.
    - Derivation of covariance matrices assumes stationarity at the beginning.
- For SUR estimation with RT a certain structure of the covariance matrix is sufficient: No need to estimate any parameters.
- Intuition: We exploit the fact that forecast errors are MA(h-1) and that we have more observations at $h < H$.

A muddle of results for non-optimal forecasts

- Almost optimal forecasts with N=4, H = 2:
    - Estimating **b** by the Kalman filter is almost as efficient as GLS.
    - Direct estimation of Ω less efficient than OLS.
    - **Why?** Neglecting information about the structure of Ω?
- Strongly non-optimal forecasts
    - „The GLS estimator used here is *not* the true GLS estimator, but it is based on the AR(1)-model used for forecasting.“
    - Seems to be a deviation from the original intention of presenting a method that relies on the forecast errors only.
- Is this relevant if we can use SUR that does not depend on any estimated parameters?

What should the applied researcher do?

- Use all forecast errors you can get (RT structure).
- Don‘t care about the underlying model.
- Use SUR which takes account of the MA(h-1) structure of forecast errrors.