

# Real-time Forecasting of Inflation and Output growth in the Presence of Data Revisions

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## Abstract

We compare the predictability of post WWII US output growth and inflation in a pseudo out-of-sample setting and in a real-time forecasting exercise. We analyse two different ways of conducting the real-time exercise: the traditional method which uses end-of-sample vintage data, and using ‘real-time vintages’ at each point in the estimation sample. Assuming the observed data for a given vintage can be decomposed into ‘true data’ plus news and noise components, we show that forecasts computed only using data from the latest vintage to estimate the model may be dominated by a strategy that uses real-time vintages. The empirical exercises show gains to the use of real-time vintages for autoregressive forecasting models, but not for Phillips Curve inflation forecasting models and ADL models of output growth.

Keywords: real-time data, news and noise revisions, optimal forecasts.

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# 1 Introduction

There has been much interest in the recent literature regarding the effects of different data vintages on model specification and forecast evaluation, and in the use of ‘real-time data’ in assessing predictability, as opposed to using ‘final-revised’ data, based on concerns that the use of final-revised data may exaggerate the predictive power of explanatory variables relative to what could actually have been achieved at the time using the then available data (see, e.g., Diebold and Rudebusch (1991b, 1991a), Robertson and Tallman (1998), Orphanides (2001), Croushore and Stark (2001, 2003), Stark and Croushore (2002), Faust, Rogers and Wright (2003) and Orphanides and van Norden (2005)). In this paper, we carry out a detailed assessment of the predictability of US output growth and inflation in real-time, and compare the results with those obtained from a pseudo out-of-sample forecasting exercise. By pseudo out-of-sample is meant that at each point in time the forecasting models are specified and the parameters estimated using only data for time periods up to that point in time, but that the vintages of data used are not restricted to those which would have been available at that time. Forecasts are ‘real-time’ if only information that would have been available at the time the forecast was made is used to produce that forecast. We consider forecasting using the sorts of models that are popular in the literature, various forecasting schemes, and examine the extent to which the findings are robust to the forecast period.

Our other main interest concerns how best to account for data revisions in a real-time forecasting exercise. The majority of the literature uses the ‘traditional approach’ to real-time forecasting when there are data revisions, and this will be one of the focuses of our approach to real-time forecasting. The traditional approach takes at each point in time the values of all the observations from the latest available vintage of data and uses these to estimate the forecasting model. Hence this is known as the end-of-sample vintage approach, or EOS, following Koenig, Dolmas and Piger (2003). To the extent that later estimates of a data point are more accurate or reliable than earlier estimates, this strategy uses the ‘best’ estimates of the data which are available at the time the forecast is made. However, it implies that a large part of the data used in model estimation has been revised many times, while the forecast is conditioned on data that has been just released or only revised a few times. In the context of autoregressive models, we show the traditional way of using real-time data for forecasting does not minimise the expected squared forecast error in population. One of our main methodological contributions is to show that the use of ‘real-time vintage’ (RTV) can overcome the deficiencies of using EOS data.<sup>1</sup> When RTV data is used, the data vintages on the

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<sup>1</sup>Harrison, Kapetanios and Yates (2005) consider the use of EOS data when there are measurement errors, and suggest that the most recent observations might be downweighted.

observations that are available in real-time are organized in such a way that the data vintages used in model estimation are of a similar maturity to the vintage of data on which the forecast is conditioned.

Some have attempted to make use of the multiple estimates of the same observation that are typically available, possibly using state-space models, or some other way of modelling the revisions process along with the process for the ‘true’ data: see, for example, Harvey, McKenzie, Blake and Desia (1983), Howrey (1984), Patterson (1995, 2003), Jacobs and van Norden (2007) and Cunningham, Eklund, Jeffery, Kapetanios and Labhard (2007). A related approach is that of Kishor and Koenig (2005) who build on earlier contributions by Howrey (1978, 1984) and Sargent (1989), and aims at obtaining estimates of the ‘current truth’ from the first-release data, which is then used in the forecasting model. However, in a recent review of forecasting with real-time data, Croushore (2006) concludes that the results of forecasting with state-space models that incorporate data revisions are mixed, compared to simply ignoring data revisions. Moreover, our interest is in the best way of organizing the available data vintages to estimate and forecast using autoregressions (AR) and autoregressive distributed lag models (ADL), as models of this sort dominate the forecasting literature.

We derive analytical results that relate the properties of the forecasts generated by the use of EOS and RTV data to the properties of data revisions for an assumed data generation process that characterises data revisions as ‘news’ or ‘noise’, in the Mankiw and Shapiro (1986) sense. We use the Jacobs and van Norden (2007) statistical framework to model the ‘regular’ rounds of revisions<sup>2</sup> which are made to the data at a level of detail that allows us to delineate between first and subsequent revisions, as the sizes of the variances of the first and subsequent revisions are found to affect the relative performance of RTV and EOS. It might be argued that data revisions are irregular and not amenable to modelling as a stationary process. This is likely to be true of benchmark revisions but the evidence presented by Croushore (2006) suggests that growth rates - the focus of our analysis - will be affected to a much lesser degree than the levels of variables.<sup>3</sup> We assume stationarity of revisions in order to be able to characterize the effects of revisions on

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<sup>2</sup>The Bureau of Economic Analysis (BEA) releases an ‘advance’, ‘preliminary’ and ‘final’ estimate of real GDP growth at about one month, two months, and three months after the end of the quarter. We observe the ‘advance’, which we call the first release, and then the next quarter the ‘final’ estimate. The data are then subject to three annual revisions which occur in the July of each year, as described by, e.g., Fixler and Grimm (2005, 2008) and Landefeld, Seskin and Fraumeni (2008).

<sup>3</sup>Siklos (2008) identifies eight benchmark revisions in 1966, 1971, 1976, 1981, 1986, 1992, 1996 and 2001, all occurring in the data vintage of the first quarter of the year - so, for example, the 1981:1 data set has data up to 1980:4 calculated on a different basis or definition to the 1980:4 vintage data set. The way which the national accounts data are calculated then remains unchanged until the 1986:1 data set. Base year changes occurred in 1976, 1985 and 1991.

the forecasts of models constructed using RTV and EOS. The analytics we present are for an  $AR(p)$ , although the logic underlying our approach suggests that, at least in population, with the maintained assumption of the stationarity of revisions, RTV should also dominate for autoregressive models with additional explanatory variables, namely autoregressive-distributed lag (ADL) models.

For AR models we are able to derive analytical results that relate the population properties of the estimator, RTV or EOS, back to the properties of the hypothesized data generation process, including the properties of the revisions process, which yields additional insight when we can clearly categorize a series in terms of news or noise revisions. In models with explanatory variables it will not in general be possible to obtain the direction of any bias of the estimator, without making a range of further assumptions, including specifying the covariances between the revisions to the series being forecast and the revisions to the explanatory variables. Hence our analytical results are for the AR, as this allows for sharper predictions, although as mentioned, the general arguments that support RTV in the case of AR models are also applicable to ADL models.<sup>4</sup>

Although our analytical results (based on a large estimation sample relative to the number of rounds of revisions) show that it is better in principle to use RTV data instead of EOS data, of interest is whether gains will be observed once an allowance is made for parameter estimation uncertainty. Hence we extend the analytical results with a Monte Carlo. The results suggest that the gains to using RTV data relative to EOS data are likely to be modest, of the order of around 2-3% on mean squared error for reasonably large samples. For small samples, larger gains of around 3-8% might occur in the case of revisions which add news. These results are based on what might be regarded as reasonably typical, empirically-calibrated data generation processes. Nevertheless, the Monte Carlo focuses on the role of data uncertainty by holding fixed other factors that might be relevant empirically. Chief amongst these are the assumed constancy of the underlying models of the variables and their revisions over time. Of interest will be the usefulness of these results as a guide to empirical outcomes.

In our empirical forecasting exercises we compare EOS and RTV as competing approaches to real-time forecasting, and look at the extent to which the empirical findings are consistent with our analysis. Our analytical results are directly applicable to AR models, but we also consider the Phillips Curve models of inflation of Stock and Watson (2008) as well as ADL models of output growth.

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<sup>4</sup>Koenig *et al.* (2003) derive expressions for the bias of the estimator of distributed lag (DL) models for various ways of using real-time vintage data, but do not explicitly model the revisions process. They simply specify the first and final estimates, and hence have a single revision. We model the sequence of revisions, and as noted, find that the properties of the estimators depend on the relative variances of the successive rounds of revision.

The plan of the rest of the paper is as follows. Section 2 presents the statistical framework that we use to model data revisions. This is essentially the Jacobs and van Norden (2007) state-space model, but modified to allow for non-zero mean revisions, because this appears to be a feature of some US macroeconomic aggregates we consider. Section 3 presents one of the main results of the paper, that the traditional way of using real-time data (EOS) to estimate autoregressive models generates forecasts that are not optimal, and are biased when data revisions are non-zero mean. Section 4 shows how the use of RTV data to estimate the autoregressive model generates optimal, unbiased forecasts. Section 5 presents the Monte Carlo investigation of the small-sample relevance of our analytical results, and section 6 is the empirical forecast comparison. Section 7 offers some concluding remarks. The derivations of the main results are confined to an appendix.

## 2 Statistical framework

Generally, the basic statistical framework for modelling data revisions relates a data vintage estimate to the true value plus an error or errors, where the errors are typically unobserved. So the period  $t + s$  vintage estimate of the value of  $y$  in period  $t$ , denoted  $y_t^{t+s}$ , where  $s = 1, \dots, l$ , consists of the true value  $\tilde{y}_t$ , as well as (in the general case) news and noise components,  $v_t^{t+s}$  and  $\varepsilon_t^{t+s}$ , so that  $y_t^{t+s} = \tilde{y}_t + v_t^{t+s} + \varepsilon_t^{t+s}$ . Data revisions are news when initially released data are optimal forecasts of later data, so news revisions are not correlated with the new released data,  $Cov(v_t^{t+s}, y_t^{t+s}) = 0$ . Data revisions are noise when each new release of the data is equal to the true value of  $y_t$ , denoted  $\tilde{y}_t$ , plus noise, so that noise revisions are not correlated with the truth,  $Cov(\varepsilon_t^{t+s}, \tilde{y}_t) = 0$ . We adopt the framework of Jacobs and van Norden (2007) which stacks the  $l$  different vintage estimates of  $y_t$ , namely,  $y_t^{t+1}, \dots, y_t^{t+l}$  in the vector  $\mathbf{y}_t = (y_t^{t+1}, \dots, y_t^{t+l})'$ , and similarly  $\boldsymbol{\varepsilon}_t = (\varepsilon_t^{t+1}, \dots, \varepsilon_t^{t+l})'$  and  $\mathbf{v}_t = (v_t^{t+1}, \dots, v_t^{t+l})'$ , so that:

$$\mathbf{y}_t = \mathbf{i}\tilde{y}_t + \mathbf{v}_t + \boldsymbol{\varepsilon}_t \quad (1)$$

where  $\mathbf{i}$  is a  $l$ -vector of ones. One way of defining a revisions process with the required characteristics is to assume a process for  $\tilde{y}_t$ , for example, an  $AR(p)$  with iid disturbances  $\eta_{1t}$ , plus a sum of iid disturbances  $\eta_{2t}$ :

$$\tilde{y}_t = \rho_0 + \sum_{i=1}^p \rho_i \tilde{y}_{t-i} + R_1 \eta_{1t} + \sum_{i=1}^l \sigma_{v_i} \eta_{2t,i}, \quad (2)$$

and to specify:

$$\mathbf{v}_t = - \begin{bmatrix} \sigma_{v_1} & \sigma_{v_2} & \cdots & \sigma_{v_l} \\ & \sigma_{v_2} & & \\ & & \ddots & \\ 0 & & & \sigma_{v_l} \end{bmatrix} \begin{bmatrix} \eta_{2t,1} \\ \eta_{2t,2} \\ \\ \eta_{2t,l} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \sigma_{\varepsilon_1} \eta_{3t,1} \\ \sigma_{\varepsilon_2} \eta_{3t,2} \\ \\ \sigma_{\varepsilon_l} \eta_{3t,l} \end{bmatrix}. \quad (3)$$

For (2), we let  $\rho(L) = \sum_{i=1}^p \rho_i L^i$  and assume that the roots of  $(1 - \rho(L)) = 0$  lie outside the unit circle, so that  $\tilde{y}_t$  is a stationary process. In (3) we have  $\boldsymbol{\eta}_t = [\eta_{1t}, \boldsymbol{\eta}'_{2t}, \boldsymbol{\eta}'_{3t}]$  is iid,  $E(\boldsymbol{\eta}_t) = 0$ , with  $E(\boldsymbol{\eta}_t \boldsymbol{\eta}'_t) = I$ . Thus the  $\sigma_{v_1}, \dots, \sigma_{v_l}$  are the standard deviations of the  $\eta_{2t,1}, \dots, \eta_{2t,l}$  processes,  $R_1 = \sigma_{\eta_1}$  is the standard deviation of the disturbances of the underlying AR( $p$ ) process for the true values, and  $\sigma_{\varepsilon_1}, \dots, \sigma_{\varepsilon_l}$  are the standard deviations of the  $\eta_{3t,1}, \dots, \eta_{3t,l}$  processes. Therefore, the first estimate of  $y_t$ ,  $y_t^{t+1}$ , estimates  $\tilde{y}_t$  with noise ( $\sigma_{\varepsilon_1} \eta_{3t,1}$ ) and a news term consisting of  $l$  separate components ( $-\sum_{i=1}^l \sigma_{v_i} \eta_{2t,i}$ ). Later estimates are also characterised by noise, but fewer news components and therefore provide more accurate estimates of  $\tilde{y}_t$ . If  $\sigma_{v_l} = 0$  and  $\sigma_{\varepsilon_l} = 0$  the  $l$ -vintage value is the true value,  $y_t^{t+l} = \tilde{y}_t$ . The assumption that  $\tilde{y}_t$  is a stationary process ensures that  $\mathbf{y}_t$  is a stationary process from (1), as both the news and noise terms are stationary.

By construction, when revisions are **pure noise** ( $\boldsymbol{\nu}_t = 0$ ),  $\mathbf{y}_t - \mathbf{i}\tilde{y}_t = \boldsymbol{\varepsilon}_t$ , and with  $E(\boldsymbol{\eta}_t) = 0$ ,  $E(\boldsymbol{\varepsilon}_t \tilde{y}_t) = 0$ . When revisions are **pure news**,  $E(\boldsymbol{\nu}_t^{t+s} y_t^{t+s}) = 0$ , as required.

The assumption that  $E(\boldsymbol{\eta}_t) = 0$  implies that the noise and news revisions are zero mean, so that the unconditional mean of the underlying series  $\{\tilde{y}_t\}$  and the observed data  $\{\mathbf{y}_t\}$  are equal at  $\rho_0(1 - \rho(1))^{-1}$ . However, there is evidence that the revisions to some US macroeconomic data are non-zero mean. For example, consider the revisions to quarterly output growth and (GDP deflator) inflation using data vintages from 1965:Q4 to 2009:Q1.<sup>5</sup> Denoting by  $y_t^{t+1}$  the first-released data,  $y_t^{t+12}$  the data revised after 12 quarters, and by  $y_t^{2009:1}$  the latest available vintage, we calculate revisions as  $y_t^{t+12} - y_t^{t+1}$  and  $y_t^{2009:1} - y_t^{t+1}$ . When output growth and inflation are expressed as percentages at annual rates, the mean revisions defined in this way are 0.20 and 0.48 for output growth, and 0.23 and 0.16 for inflation.<sup>6</sup> To account for this characteristic of the revisions process, we consider a modified version of the statistical model that allows data revisions to affect the mean

<sup>5</sup> All real-time data employed in this paper are from the Real Time Data Set of the Philadelphia Fed available at the Philadelphia Fed webpage,

<http://www.phil.frb.org/research-and-data/real-time-center/real-time-data/>. See Croushore and Stark (2001).

<sup>6</sup> See Aruoba (2008) and Corradi, Fernandez and Swanson (2009) for recent analyses of the properties of data revisions. Calculating the revision between the first estimate ( $y_t^{t+1}$ ) and  $y_t^{t+12}$  should lessen the impact of benchmark revisions, relative to using the 2009:1 data vintage ( $y_t^{2009:1}$ ).

of the  $\{\tilde{y}_t, \mathbf{y}_t\}$ :

$$\tilde{y}_t = \left[ \rho_0 + \sum_{i=1}^l \mu_{\eta_{2,i}} \right] + \sum_{i=1}^p \rho_i \tilde{y}_{t-i} + R_1 \eta_{1t} + \sum_{i=1}^l \sigma_{v_i} \eta_{2t,i}, \quad (4)$$

and

$$\mathbf{v}_t = - \begin{bmatrix} \sum_{i=1}^l \mu_{\eta_{2,i}} \\ \sum_{i=2}^l \mu_{\eta_{2,i}} \\ \vdots \\ \mu_{\eta_{2,l}} \end{bmatrix} - \begin{bmatrix} \sigma_{v_1} & \sigma_{v_2} & \cdots & \sigma_{v_l} \\ & \sigma_{v_2} & & \\ & & \ddots & \\ 0 & & & \sigma_{v_l} \end{bmatrix} \begin{bmatrix} \eta_{2t,1} \\ \eta_{2t,2} \\ \vdots \\ \eta_{2t,l} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_t = - \begin{bmatrix} \mu_{\eta_{3,1}} \\ \mu_{\eta_{3,2}} \\ \vdots \\ \mu_{\eta_{3,l}} \end{bmatrix} + \begin{bmatrix} \sigma_{\varepsilon_1} \eta_{3t,1} \\ \sigma_{\varepsilon_2} \eta_{3t,2} \\ \vdots \\ \sigma_{\varepsilon_l} \eta_{3t,l} \end{bmatrix}. \quad (5)$$

Hence we allow news revisions to have a non-zero mean provided  $Cov(v_t^{t+s}, y_t^{t+s}) = 0$ , and similarly, noise revisions may be non-zero mean. Hence news revisions may be predictable in the limited sense that the early estimate is expected to be revised up or down by a fixed amount, but the expected revision is not allowed to depend on the magnitude of the early estimate.

The statistical model can be cast in state-space form (SSF) and the parameters can be estimated by maximum likelihood using the Kalman Filter, as described by Jacobs and van Norden (2007), amongst others.

### 3 Estimating and Forecasting with AR models using EOS data

In this section we derive the values of the  $AR(p)$  parameters that minimise the real-time expected squared forecast error assuming the model of data revisions described in the last section. For ease of exposition, we first assume that revisions are zero mean. We then show that the traditional way of using real-time data for forecasting does not generate forecasts that minimise the real-time loss function. Finally, we assess the impact of non-zero mean revisions on the traditional way of forecasting with AR models in real-time.

#### 3.1 Optimal AR parameters in population

Consider how forecasters normally use real-time data to compute forecasts with an autoregressive model. At time  $T + 1$ , the  $T + 1$  vintage of data contains data up to period  $T$ , so that the  $T + 1$  vintage is used to estimate the  $AR(p)$  model, and the forecasts are obtained by conditioning on the model estimates and the lagged values of  $y$  from the latest data vintage, namely,  $\mathbf{y}_T^{T+1} = (y_T^{T+1}, \dots, y_{T-p+1}^{T+1})'$ . But does least squares estimation of the AR model using the  $T + 1$  data vintage minimise the expected squared forecast error? To answer this question, we first obtain the

population value of the parameter vector that minimizes the real-time squared forecast error for a forecast conditioned on  $\mathbf{y}_T^{T+1}$ , when the data are subject to revisions as described in section 2. For a  $p^{th}$ -order autoregression, the forecast is given by the combination  $\phi_0 + \boldsymbol{\phi}'\mathbf{y}_T^{T+1}$ , where  $\phi_0$  is the intercept and  $\boldsymbol{\phi}' = (\phi_1, \dots, \phi_p)$  contains the slope parameters. The optimal parameter values  $(\phi_0^*, \boldsymbol{\phi}^*)$ , in terms of minimizing the real-time squared-error loss, are the solution to:

$$(\phi_0^*, \boldsymbol{\phi}^*) = \arg \min_{\phi_0, \boldsymbol{\phi}} E(L(\phi_0, \boldsymbol{\phi}))$$

where:

$$L(\phi_0, \boldsymbol{\phi}) = \left( y_{T+1}^{T+1+f} - \phi_0 - \boldsymbol{\phi}'\mathbf{y}_T^{T+1} \right)^2. \quad (6)$$

In (6), the vintage of data employed to compute the forecast errors is that available at  $T+1+f$ , so that  $f=1$  indicates first-release data is used. The following proposition is derived in the Appendix.

**Proposition 1** *The optimal parameters  $(\phi_0^*, \boldsymbol{\phi}^*)$  when the data generating process is given by (1)–(3), are*

$$\begin{aligned} \boldsymbol{\phi}^* &= \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\mathbf{v}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \right)^{-1} \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} \right) \boldsymbol{\rho} \\ \phi_0^* &= (1 - \boldsymbol{\phi}^{*\prime}\mathbf{i}) \mu_{\tilde{\mathbf{y}}}. \end{aligned} \quad (7)$$

The slope parameter simplifies to  $\boldsymbol{\phi}_{news}^* = \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\mathbf{v}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} \right)^{-1} \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} \right) \boldsymbol{\rho}$ , if data revisions add news, and is  $\boldsymbol{\phi}_{noise}^* = \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \right)^{-1} \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}}\boldsymbol{\rho}$  if data revisions only reduce noise, where the second moment matrices  $\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}}$ ,  $\boldsymbol{\Sigma}_{\mathbf{v}}$ ,  $\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}}$ , and  $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$  are defined in the Appendix,  $\mathbf{i}$  is a  $p$ -vector of 1's, and  $\mu_{\tilde{\mathbf{y}}} \equiv E(\tilde{y}_t)$ . The optimal values hold for all  $f \geq 1$ , i.e., irrespective of whether the goal is to forecast the first-released value ( $y_{T+1}^{T+2}$ ) or the latest available estimate ( $y_{T+1}^{T+l}$ ).

For the special case of an AR(1) model, for general revisions that are a combination of news and noise:

$$\phi_1^* = \frac{\rho_1 \left( \sigma_{\tilde{y}}^2 - \sigma_v^2 \right)}{\sigma_{\tilde{y}}^2 - \sigma_v^2 + \sigma_{\varepsilon_1}^2}$$

where  $\sigma_v^2 \equiv \sum_{i=1}^l \sigma_{v_i}^2$ , and  $\sigma_{\tilde{y}}^2 = Var(\tilde{y}_t)$ . As a consequence, for pure news ( $\sigma_{\varepsilon_1}^2 = 0$ ):

$$\phi_{1,news}^* = \rho_1, \quad \phi_{0,news}^* = \rho_0.$$

Note that  $\phi_{1,news}^* = \rho_1$  only holds for  $p=1$ : in general when there are news revisions the parameter vector of the underlying process  $\tilde{y}_t$  (i.e.,  $\boldsymbol{\rho}$ ) is not optimal from a forecasting perspective when the

forecasts are conditioned on early estimates, as is typically the case in a real-time forecasting exercise.

For pure noise ( $\sigma_v^2 = 0$ ):

$$\phi_{1,noise}^* = \frac{\rho_1 \sigma_{\tilde{y}}^2}{\sigma_{\tilde{y}}^2 + \sigma_{\varepsilon_1}^2} \quad (8)$$

so that  $\phi_{1,noise}^* < \rho_1$  provided  $\sigma_{\varepsilon_1}^2 \neq 0$ .

### 3.2 Estimating AR forecasting models using EOS data

As noted, when forecasting with AR models using real-time data, the standard approach is to replace the model estimation data with the latest estimates of all the past observations given in the data vintage available at the forecast origin. So, for example, at time  $T + 1$ , the  $T + 1$  vintage of data contains data up to  $T$ , and is used for estimation, while at  $T + 2$ , the  $T + 2$  vintage is used for estimation. We call this use of real-time data ‘*end-of-sample*’ vintage data (EOS).

For forecasting  $y_{T+1}^{T+1+f}$  the AR forecasting model is given by:

$$y_t^{T+1} = \alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i}^{T+1} + e_{t,EOS} \quad (9)$$

where  $t = p + 1, \dots, T$ , assuming the latest data vintage is dated  $T + 1$ . In matrix notation:

$$\mathbf{Y}^{T+1} = \mathbf{i}\alpha_0 + \mathbf{Y}_{-1}\boldsymbol{\alpha} + error$$

where  $\mathbf{Y}_{-1} = [\mathbf{Y}_{-1}^{T+1}, \dots, \mathbf{Y}_{-p}^{T+1}]$ ,  $\mathbf{i}$  is a  $T - p$  vectors of 1’s, and the vectors of observations  $\mathbf{Y}^{T+1}$  and  $\mathbf{Y}_{-i}^{T+1}$ ,  $i = 1, \dots, p$ , are:

$$\mathbf{Y}^{T+1} = [y_{p+1}^{T+1}, \dots, y_{T-1}^{T+1}, y_T^{T+1}]', \quad \mathbf{Y}_{-i}^{T+1} = [y_{p+1-i}^{T+1}, \dots, y_{T-i-1}^{T+1}, y_{T-i}^{T+1}]'$$

for  $i = 1, \dots, p$ . Notice that the more recent  $y$ ’s will therefore have been revised fewer than  $l$  times.

The main result is summarized in the following proposition, which is derived in the appendix.

**Proposition 2** *The population (asymptotic) value of the least-squares estimator of the parameter vector in the autoregressive model using EOS data, when the data are generated by (1)–(3), are given by:*

$$\begin{aligned} \boldsymbol{\alpha}^* &= \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\mathbf{v}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \right)^{-1} \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}'_{\tilde{\mathbf{y}}\mathbf{v}} \right) \boldsymbol{\rho} \\ \alpha_0^* &= (1 - \boldsymbol{\alpha}^{*'} \mathbf{i}) \mu_{\tilde{\mathbf{y}}}, \end{aligned} \quad (10)$$

where  $\Sigma_{\mathbf{y}}$  and  $\Sigma_{\boldsymbol{\varepsilon}}$  are second moment matrices of the news and noise components, and  $\Sigma_{\tilde{\mathbf{y}}\mathbf{y}}$  is the second moment matrix between the news and the underlying process,  $\tilde{\mathbf{y}}_t$ , and  $\mu_{\tilde{\mathbf{y}}} \equiv E(\tilde{\mathbf{y}}_t)$ .

A comparison of (10) and (7) shows that the conventional use of real-time data (EOS) for estimation of the AR( $p$ ) model does not deliver the optimal population parameters when there are data revisions. That is,

$$\boldsymbol{\alpha}^* \neq \boldsymbol{\phi}^* \text{ and } \alpha_0^* \neq \phi_0^*,$$

so that the forecasts of  $\tau + 1$  computed using  $\alpha_0^* + \boldsymbol{\alpha}'^* \mathbf{y}_\tau^{\tau+1}$  (for a set of forecast origins  $\tau = T, T + 1, T + 2, \dots$ ), where recall that  $\mathbf{y}_\tau^{\tau+1} = (y_\tau^{\tau+1}, \dots, y_{\tau-p+1}^{\tau+1})'$ , are not optimal in a squared-error loss sense. Intuitively, when the sample is large, the use of EOS data amounts to mainly using fully-revised data (i.e., data from the  $y_t^{t+l}$  vintage) whilst optimal forecasts are obtained by relating the first estimates of the LHS variable to early estimates of the RHS variables. The finding of the lack of optimality of EOS forecasts holds for news and noise revisions, although forecasts are unbiased when the data revisions are zero mean, as described by the following remark, derived in the Appendix.

**Remark 1** *Forecasts computed using EOS data using the AR model with  $(\alpha_0^*, \boldsymbol{\alpha}^*)$  are unbiased when data revisions are described by (1) – (3).*

Consider the special case of an AR(1). When revisions are news, we can show that the EOS estimator simplifies such that  $\alpha_1^* = \rho_1$ , matching the optimal value, but this is true only for the special case of  $p = 1$ .

Under noise:

$$\alpha_1^* = \frac{\rho_1 \sigma_y^2}{\sigma_y^2 + \sigma_{\varepsilon_l}^2} \quad (11)$$

An immediate implication is that  $|\alpha_1^*| > |\phi_1^*|$  if earlier revisions are larger than later revisions (compare (11) to (8) when  $\sigma_{\varepsilon_1}^2 > \sigma_{\varepsilon_l}^2$ ). Note that if  $\sigma_{\varepsilon_l}^2 = 0$ , so that the truth is eventually revealed when there is noise, then  $\alpha_1^* = \rho_1$  for a large estimation sample. Even so,  $\rho_1$  is not the parameter vector that minimizes the real-time squared forecast loss ( $\phi_1^* \neq \rho_1$ ).

### 3.3 Optimal AR coefficients in population when revisions are non-zero mean

In this section we allow for non-zero mean revisions. Non-zero mean revisions indicate that in general  $E(y_t^{t+r}) \neq E(y_t^{t+s})$ , so that forecasts may be biased for some vintage estimates of the actuals but not for others. We need to take a stance on the vintage, i.e.,  $f$ , in  $y_{T+1}^{T+1+f}$ . Suppose

the objective is to predict the first-released value, namely,  $y_{T+1}^{T+2}$ , so the optimal values  $(\phi_0^*, \phi^*)$  are defined as the solution of (6) with  $f = 1$ .

**Proposition 3** *The optimal parameters  $(\phi_0^*, \phi^*)$  when the data are generated by (1)-(4) and (5), for the loss function given by (6) with  $f = 1$ , are equal to  $\phi^*$ , as derived in Proposition 1, and*

$$\phi_0^* = (1 - \phi^{*'} \mathbf{i}) \mu_{\tilde{y}} + \mu_{v_1} + \mu_{\varepsilon_1} - \phi^{*'} \boldsymbol{\mu}_\varepsilon - \phi^{*'} \boldsymbol{\mu}_v, \quad (12)$$

where  $\mu_{\tilde{y}} = (1 - \rho(1))^{-1} [\rho_0 - \mu_{v_1}]$ ,  $\mu_{\varepsilon_1} \equiv E(\varepsilon_t^{t+1}) = -\mu_{\eta_{3,1}}$ ,  $\mu_{v_1} \equiv E(v_t^{t+1}) = -\sum_{i=1}^l \mu_{\eta_{2,i}}$ , and  $\boldsymbol{\mu}_\varepsilon$  and  $\boldsymbol{\mu}_v$  are  $p \times 1$  vectors of the means of noise and news components.

### 3.4 Estimating AR forecasting models using EOS data when revisions are non-zero mean

In the Appendix we derive the following proposition:

**Proposition 4** *The population (asymptotic) value of the least-squares estimator of the parameter vector in the autoregressive model using EOS data (eq. (9)), when the data are generated by (1), (4) and (5), is equal to the slope value of  $\boldsymbol{\alpha}^*$  given in Proposition 2, and the intercept is given by*

$$\alpha_0^* = (1 - \boldsymbol{\alpha}^{*'} \mathbf{i}) (\mu_{\tilde{y}} + \mu_{v_l} + \mu_{\varepsilon_l}), \quad (13)$$

where  $\mu_{\tilde{y}} = (1 - \rho(1))^{-1} [\rho_0 - \mu_{v_1}]$ ,  $\mu_{\varepsilon_l} \equiv E(\varepsilon_t^{t+l}) = -\mu_{\eta_{3,l}}$  and  $\mu_{v_l} \equiv E(v_t^{t+l}) = -\mu_{\eta_{2,l}}$ .

Comparing (12) with (13), we have that  $\alpha_0^* \neq \phi_0^*$ , that is, the EOS estimation will in general yield biased forecasts when revisions are non-zero mean.

**Remark 2** *Forecasts of the first-released value computed using the AR model with parameter vector  $(\alpha_0^*, \boldsymbol{\alpha}^*)$ , as under EOS, are biased when data revisions are described by (1), (4) and (5), with bias of  $(\mu_{v_1} - \mu_{v_l}) + (\mu_{\varepsilon_1} - \mu_{\varepsilon_l}) - \boldsymbol{\alpha}^{*'} [(\boldsymbol{\mu}_v - \mathbf{i}\mu_{v_l}) + (\boldsymbol{\mu}_\varepsilon - \mathbf{i}\mu_{\varepsilon_l})]$ .*

Summarizing, the use of EOS data to estimate AR models for forecasting in real-time when data are subject to news and noise revisions delivers predictions that are not optimal, that is, the resulting forecasts do not minimise expected quadratic forecast loss. We show in the following section that alternative forecasts conditioned on the same information set deliver a smaller squared-error loss in population. When in addition data revisions are non-zero mean, forecasts using EOS are generally biased for the first-release of the target variable, whereas unbiased forecasts are easily obtained from the AR model by organizing the data as described in the next section.

## 4 Estimating AR forecasting models using RTV data

In this section we consider a way of using real-time data, motivated by the approach suggested by Koenig *et al.* (2003) in the context of distributed lag models, that delivers optimal estimators of the forecasting model in population. We adapt their approach to the estimation of AR models by regressing the period  $t + 1$  vintage value of  $y_t$  on the  $t$ -vintage data values of the lags,  $y_{t-i}$ ,  $i = 1, \dots, p$ , for  $t = p + 1, \dots, T$ . The reason for using the  $t$ -vintage value of the lags (rather than the  $t + 1$ -vintage value) is that when the model is used for forecasting, say, the  $T + 2$  vintage value of  $y_{T+1}$ , the forecast will be conditioned on the previous  $T + 1$ -vintage values of the explanatory variables. This use of vintage data is called ‘*real-time-vintage*’ data (RTV). The model is:

$$y_t^{t+1} = \beta_0 + \sum_{i=1}^p \beta_i y_{t-i}^t + e_{t,RTV} \quad (14)$$

where  $t = p + 1, \dots, T$ . In matrix notation:

$$\mathbf{Y}^t = \mathbf{i}\beta_0 + \mathbf{Y}_{-1}^t \boldsymbol{\beta} + error$$

where  $\mathbf{Y}^t$  and  $\mathbf{Y}_{-1}^t = [Y_{-1}^t, \dots, Y_{-p}^t]$  are given by:

$$\mathbf{Y}^t = [y_{p+1}^{p+2}, \dots, y_{T-1}^T, y_T^{T+1}]', \quad \mathbf{Y}_{-i}^t = [y_{p+1-i}^{p+1}, \dots, y_{T-i-1}^{T-1}, y_{T-i}^T]', \quad i = 1, \dots, p.$$

Note that  $\mathbf{Y}^t$  and  $\mathbf{Y}_{-i}^t$  contain early vintages of data relative to the period  $T + 1$ -vintage: these observations are not replaced with the latest available ( $T + 1$ -vintage) values for these observations. The fact that later data revisions are not incorporated in the model estimation sample proves to be beneficial from a forecasting perspective, as we show below.

Consider estimating equation (14) by OLS. A typical observation on the LHS and RHS variables is  $\{y_t^{t+1}, \mathbf{y}_{t-1}^t = (y_{t-1}^t \dots y_{t-p}^t)'\}$ . This is a covariance stationary process, so we can calculate the population values of the OLS estimators as the values that satisfy:

$$(\beta_0^*, \boldsymbol{\beta}^*) = \arg \min_{\beta_0, \boldsymbol{\beta}} E (y_t^{t+1} - \beta_0 - \boldsymbol{\beta} \mathbf{y}_{t-1}^t)^2.$$

This estimation loss function is identical to the real-time forecast loss function (6), when  $f = 1$ , so that the solutions to the two coincide. Hence the use of RTV data to estimate the AR model

delivers optimal forecasts, that is,

$$\beta_0^* = \phi^*; \boldsymbol{\beta}^* = \phi_0^*.$$

The unbiasedness of using RTV data to compute forecasts follows directly because  $(\beta_0^*, \boldsymbol{\beta}^*)$  satisfy  $E(y_t^{t+1} - \beta_0 - \boldsymbol{\beta} \mathbf{y}_{t-1}^t) = 0$  (which is one of the FOCs), and so by stationarity  $E(y_{T+1}^{T+2} - \beta_0^* - \boldsymbol{\beta}^* \mathbf{y}_T^{T+1}) = 0$ . The unbiasedness of the forecasts holds irrespective of whether or not the revisions are zero mean, provided that  $f = 1$  when revisions are non-zero mean. Suppose the goal were instead to forecast  $y_{T+1}^{T+1+f}$ , when  $f > 1$ , then the estimation and out-of-sample loss criteria no longer match, and RTV estimation would yield a systematic forecast error if  $E(y_{T+1}^{T+1+f} - y_{T+1}^{T+2}) \neq 0$ .<sup>7</sup> However, a simple solution suggests itself - the forecast should be corrected by an estimate of the difference between the vintage we wish to forecast and the first release, e.g., the sample mean of  $y_t^{t+f} - y_t^{t+1}$ ,  $t = 1, \dots, T + 1 - f$ .

Summarizing, the use of RTV data to estimate and forecast with AR( $p$ ) models in real time delivers forecasts that minimise the real-time expected loss (optimal forecasts) and unbiased forecasts of  $y_{T+1}^{T+2}$ . Unbiasedness also holds for  $f > 1$  if data revisions are zero mean, but in the event of non-zero mean revisions a simple correction can be applied to the intercept.

## 5 Measuring the impact of using EOS and RTV data

In section 3, we showed that the use of EOS data to compute estimates of the AR model does not generate optimal forecasts. The differences between the optimal parameter values  $(\phi_0^*, \boldsymbol{\phi}^*)$  in (7) and (12) and those obtained using EOS data  $(\alpha_0^*, \boldsymbol{\alpha}^*)$  in (10) and (13)) depend on the sizes of the means and variances of the various vintages of revisions, as well as on the properties of the data generating process for the underlying true data (e.g.,  $\mu_{\tilde{y}}$  and  $\sigma_{\tilde{y}}^2$ , and the correlation structure). In this section we firstly evaluate the analytical formulae numerically for empirically relevant values of the key parameters, to assess the potential importance of using RTV versus EOS for forecasting with AR models. Secondly, because the results derived in sections 3 and 4 were based on large sample approximations, we also assess by Monte Carlo the relevance of the analytical predictions for the ‘small samples’ that are likely to be used in practice.

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<sup>7</sup>The expected forecast error is  $E(y_{T+1}^{T+f+1} - y_{T+1}^{T+2}) = E(v_{T+1}^{T+f+1} - v_{T+1}^{T+2} + \varepsilon_{T+1}^{T+f+1} - \varepsilon_{T+1}^{T+2}) = -\sum_{i=f}^l \mu_{\eta_{2i}} + \sum_{i=1}^l \mu_{\eta_{2i}} + (\mu_{\varepsilon_f} - \mu_{\varepsilon_1}) = -\sum_{i=1}^{f-1} \mu_{\eta_{2i}} + (\mu_{\varepsilon_f} - \mu_{\varepsilon_1})$ , where  $\mu_{\varepsilon_i} \equiv E(\varepsilon_t^{t+i})$ , and  $E(v_t^{t+1}) = -\sum_{i=1}^l \mu_{\eta_{2i}}$  and  $E(v_t^{t+l}) = -\mu_{\eta_{2,l}}$ , and so would be non-zero if either  $\mu_{\varepsilon_f} \neq \mu_{\varepsilon_1}$  under noise revisions or if  $\sum_{i=1}^{f-1} \mu_{\eta_{2i}} \neq 0$  when revisions are news.

## 5.1 Calibrating the key parameters

Because we want to assess the potential size of the forecast losses from using EOS data for typical macroeconomic aggregates subject to revisions, Figure 1 displays some of the key characteristics of the revisions processes for real output growth and two measures of inflation (GDP deflator, PCE deflator). All three series are expressed in quarterly percentage differences. In the first panel, we plot the mean of each revision as a proportion of the mean of the first-released data. The figure plots the sample averages of revisions defined as  $r_t^{(i)} = y_t^{t+1+i} - y_t^{t+i}$ , for  $i = 1, \dots, 12$ . It is apparent that the first revision tends to increase the mean of the first-released data by around 4% for output growth and GDP inflation. After the first revision, subsequent revisions tend to be smaller in terms of first-moment effects. In the second panel, we present the standard deviation of each revision as a proportion of the standard deviation of the first released data. For output growth and GDP inflation, the first-revision has a variance that is almost 40% of the first-release data. An important feature is that the standard error of revisions tends to decrease in  $i$ , although the variance of the revision between the 12th and 11th vintage estimates is still in excess of 10% of the standard deviation of the first released data. The variance of the second revision is also on average markedly smaller than the first.

Based on these estimates of the typical sizes of revisions, we construct eight sets of parameters for the statistical model in (1), (4) and (5), assuming throughout that  $p = 2$  and  $l = 12$ . Firstly, consider the AR coefficients of the model for  $\tilde{y}_t$ . The first four sets of parameters in Table 1 have autoregressive coefficients that sum up to 0.4, while those of the last four sets of parameters sum to 0.8. The first block is more typical of a process such as output growth which exhibits moderate persistence, while the second is typical of a more persistent process such as inflation. Comparisons between these two blocks will therefore be informative about whether the relative forecast accuracy of EOS and RTV estimation depends upon the persistence of the underlying process.

For all sets of parameters, we assume that the means of the first and the fifth revisions are non-zero, as suggested by Figure 1. The mean values were chosen in conjunction with the values of the AR coefficients to give ratios of revision means to first-released data of 4% and 2%, as suggested by Figure 1. Within each of the two blocks we also vary the sizes of the standard deviations of the revisions. The first set of values within each block has a relatively large first revision variance ( $\sigma_{r_1}/\sigma_{y_t^{t+1}} = .4$ ), followed by equal-sized revisions of smaller variance ( $\sigma_{r_i}/\sigma_{y_t^{t+1}} = .2$ , for  $i = 2, \dots, 11$ ), with a small final revision ( $\sigma_{r_{12}}/\sigma_{y_t^{t+1}} = .1$ ). This decay is a stylized representation

of the results in the second panel of Figure 1.<sup>8</sup> The second set of revision standard errors has proportionally larger revision standard deviations (50% larger) than the first set. The third one is useful to check the effect of no decay in the standard errors of the revisions. Finally, the fourth one assumes that the last revision reveals the true data ( $\sigma_{r_{12}} = 0$ ).

The parameter values set out in Table 1 will be applied under the assumption that revisions are news, and under the assumption of noise. This will allow us to determine whether the news versus noise issue is relevant to the relative forecast accuracy of EOS and RTV estimation of AR models for forecasting, both in large samples and in small samples when we allow for parameter estimation uncertainty in the Monte Carlo simulations. Because we have carefully calibrated our design parameters to reproduce the sorts of patterns we observe in the revisions of output growth and inflation, we would hope that our results might be informative about empirical outcomes.

## 5.2 Numerical quantification of gains to RTV versus EOS estimation: large sample results

In this section we evaluate the impact of data revisions on estimating and forecasting with an AR model assuming a large sample, such that the analytical results hold. For each set of parameter value in Table 1, we allow for revisions to be either pure news ( $\sigma_{r_i} = \sigma_{v_i}$ ) or pure noise ( $\sigma_{r_i} = \sigma_{\varepsilon_i}$ ). We compute the population values ( $\phi_0^*, \phi_1^*, \phi_2^*$ ) and ( $\alpha_0^*, \alpha_1^*, \alpha_2^*$ ) using equations (7)-(12), and (10)-(13), respectively. We then compute the population value of the loss function, (6), with  $f = 1$ , at these parameter values, and also calculate the bias of EOS estimation (from Remark 2). The results are recorded in Table 2. We find that the impact of data revisions is larger for more persistent data (compare DGPs 5 to 8 versus 1 to 4), and that there are marked changes in individual autoregressive coefficients (e.g., under news, for DGP 7  $\phi_1^* = 0.69$  and  $\alpha_1^* = 0.51$ ), but that the sum of the autoregressive parameters changes less. The size of the forecast bias under EOS estimation is commensurate with the sizes of the revision means, while the loss in terms of MSFE from EOS estimation in comparison with optimal forecasting is in the 2-3% range for the more persistent data (DGPs 5 to 8) but it is smaller for the less persistent data.

In terms of the effect of the pattern of the variances of the revisions on the relative performance of EOS versus RTV estimation, we find that: i) under noise, EOS is more heavily penalized when the variance of the first revision is large relative to subsequent revisions (DGP 6); and ii) under news revisions, EOS fares relatively poorly in the same scenario, but is worst affected when there is no variance decay so that  $\sigma_{v_i} = 0.3$  for  $i = 1, \dots, l$  (DGP 7). When the last vintage reveals the

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<sup>8</sup>Specifically, for the purpose of computing the DGP parameters, we use  $\sigma_{y_t^{t+1}}^2 = \frac{R_1^2(1-\rho_2)}{(1+\rho_2)[(1-\rho_2)^2-\rho_1^2]}$ .

true data (DGPs 4 and 8), our main results still hold, so that even when EOS estimation is based on the true data (for all but the last 3-years of observations) one would still do better to use RTV estimation.

### 5.3 Monte Carlo estimates of small sample effects

In this section we assess by Monte Carlo simulation whether the large-sample results provide a useful guide to small-sample outcomes. Specifically, we assess whether: i) the values  $(\phi_0^*, \phi_1^*, \phi_2^*)$  and  $(\alpha_0^*, \alpha_1^*, \alpha_2^*)$  are good approximations to the estimates obtained using RTV and EOS data with small samples; and ii) whether greater or smaller losses to using EOS estimation relative to RTV estimation data are realised in small samples. We again use the set of parameters detailed in Table 1, and assuming that revisions are either news or noise, we simulate data using the statistical model described in section 2, and forecast from the AR model estimated by both EOS and RTV.

Table 3 presents the average bias across replications of estimating the autoregressive parameters using RTV  $((\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$  versus  $(\phi_0^*, \phi_1^*, \phi_2^*)$ ) and EOS data  $((\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2)$  versus  $(\alpha_0^*, \alpha_1^*, \alpha_2^*)$ ). The entries are computed with samples of size  $T = 50, 100, 200$  and  $500$ , and employ 10,000 replications. The asymptotic results are seen to provide a reasonable approximation to the small-sample estimates of the slope parameters with samples of size 200, but larger samples are required for the intercepts.

Table 4 presents the differences of the absolute value of the forecast biases between forecasts using EOS and RTV data for four different sample sizes. The first column records the biases computed in Table 2 for ease of comparison. Table 4 also shows the MSFE of forecasts computed with RTV data as a ratio of the MSFE of forecasts computed with EOS data. Finally, using the same estimates employed to compute one-step-ahead forecasts, we compute four-step-ahead forecasts by iteration. The right panel of Table 4 presents MSFE ratios of these forecasts, where the actuals are again the first-release values (that is,  $y_{T+4}^{T+1+4}$ ). Our analytical formulae generally work well for the bias differences and MSFE ratios when the sample is large ( $T = 500$ ). An important finding for practical forecasting is that the losses to using EOS instead of RTV data are markedly larger for small samples when revisions are news, so that the analytical results downplay the likely empirical relevance of RTV estimation. For example, consider the relative performance of EOS and RTV estimation in terms of MSFE for DGP 6 when there are news revisions. For  $T = 50$  the gains to RTV are in excess of 8%, while the large sample results indicate gains closer to 4%. The results for forecasting four-steps ahead suggests even larger gains, of 9 to 12% for the more persistent processes (DGPs 5 to 8) when there are news revisions, for  $T = 50$ .

In summary, we would expect that the forecast loss of using EOS data instead of RTV data would be higher when i) the estimation sample is relatively small, ii) the process is reasonably persistent, and iii) revisions primarily add news.

## 6 Forecasting US output growth and inflation

Our first empirical forecasting exercise compares the forecast performance of RTV and EOS for simple autoregressive models for predicting quarterly output growth and inflation 1 and 4-steps ahead. We begin with these models (rather than ADL models) because the analysis in sections 3 and 4 that relates forecast performance to the properties of revisions is more directly applicable.

We then present a forecasting exercise based on the pseudo out-of-sample forecasting exercise of Stock and Watson (2008). They are interested in whether incorporating activity variables improves upon the forecast performance of simple univariate benchmark models of US inflation, and they use a large number of different measures of economic activity to test the claim by Atkeson and Ohanian (2001) that the Phillips curve is not useful for forecasting inflation from 1985 onwards. Despite the comprehensive nature of the assessment of Phillips Curve inflation forecasting models by Stock and Watson (2008), it is a *pseudo* out-of-sample assessment of forecast performance, because although at each point in time the forecasting models are specified and the parameters estimated using only data for time periods up to that point in time, the observations are drawn from the final vintage of data, and so use is made of data that would not have been available at that time. Given that our interest is in the effects of data revisions on forecast performance, we consider whether i) their findings hold up in real-time, and ii) whether the theoretical advantages to the use of RTV over EOS in real-time forecasting are realized in practice. We will use only a small subset of the activity variables considered by Stock and Watson (2008), in part because of the availability of the real-time data vintages, but are reassured by their finding that the results are not overly sensitive to the particular measure of activity used.

We also report on a similar forecasting exercise for output growth - we follow Stock and Watson (2003) and compare the accuracy of ADL models using economic indicators to autoregressive models.

### 6.1 AR models of output growth and inflation: RTV versus EOS.

In this subsection we assess the empirical relevance of the analytical and Monte Carlo results for forecasts from autoregressive models of output growth and the two measures of inflation. The

variables are defined as (one hundred times) the quarterly difference of the log of the level. We compute descriptive statistics of the different vintages of data and the revisions between them as follows. We consider first-released data  $y_t^{t+1}$ , data available three years later  $y_t^{t+12}$ , as well as latest-available, which in our case is from the 2009:Q1 vintage dataset, denoted  $y_t^{09:1}$ . Table 5 presents means, standard deviations and first-order autocorrelations for the three data series, as well as  $p$ -values of tests for whether revisions ( $y_t^{t+12} - y_t^{t+1}$  and  $y_t^{09:1} - y_t^{t+1}$ ) are noise, or add news, and whether they are zero-mean. These are all calculated for the forecast period 1985:Q3 to 2006Q4, and separately for the estimation period 1965:Q3 to 1985:Q2.<sup>9</sup> Recall that revisions are defined as noise if the initial estimate is an observation on the final series but measured with error, so that the revisions are uncorrelated with final value, but are correlated with data available when the initial estimate was made. Hence noisy revisions are predictable. Alternatively, revisions are news if the initial estimate is an efficient forecast of the final value, such that the revision is unpredictable from information available at the time the initial estimate was made. We test for news and noise revisions using, respectively, the following auxiliary regressions:

$$\begin{aligned} y_t^{t+l} - y_t^{t+1} &= \alpha + \beta y_t^{t+1} + \omega_t \\ y_t^{t+l} - y_t^{t+1} &= \alpha + \beta y_t^{t+l} + \omega_t \end{aligned}$$

where the null hypothesis is that  $\alpha = \beta = 0$  in both cases. In place of  $y_t^{t+l}$ , we use both  $y_t^{t+12}$  and  $y_t^{09:1}$ . We also tests separately whether revisions are zero mean ( $H_0: \alpha = 0$  in  $y_t^{t+l} - y_t^{t+1} = \alpha + \omega_t$ ).

Data revisions to output growth and inflation are seen to have different characteristics, and show some variation across the forecast and estimation periods. For output growth we can reject the noise hypothesis for both periods using the 2009:1 data vintage, and there is no evidence against the news hypothesis using the  $t + 12$  data vintage (the latter matching the findings of Mankiw and Shapiro (1986)). For both inflation measures the only clear finding is that over the forecast period the revisions relative to the 2009:1 vintage can be assumed to be noise, whereas over the estimation period we do not reject the news or noise hypotheses at the 1% level. There is also evidence that the 2009:1 revisions to output growth have been significantly upward, as have the  $t + 12$  revisions to both inflation rates over the forecast period.

The results of sections 3 and 4 suggest that we should expect improvements in forecast accuracy from using RTV instead of EOS data when there are news revisions, especially when estimation sample sizes are short, as well as when there are noise revisions for more highly persistent processes

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<sup>9</sup>Real-time data are available starting with the 1965:Q4 vintage.

(as is true of inflation compared to output growth). Our out-of-sample period for forecast comparison is such that the initial estimation sample has around 100 observations, and as we adopt a recursive forecasting scheme, this increases to around 194 by the end.

We compare recursive forecasts computed with RTV and EOS data for the three series, using autoregressive models with fixed specifications - the lag length is either  $p = 1, 2$  or  $4$ . Recall that both RTV and EOS condition the forecasts on exactly the same information set  $(y_T^{T+1}, \dots, y_{T-p+1}^{T+1})$  for  $T = 1985:Q3, \dots, 2008:Q4$ . The forecasts will differ to the extent that the parameters of the AR models estimated by EOS and RTV differ. As the forecast origin is moved through the data, for EOS estimation the AR model will be estimated using the full set of data from the new vintage that becomes available at that origin. By way of contrast, for RTV estimation, we add only the estimate of the most recent period from the new vintage, as described in section 4.

We evaluate forecasts by computing forecast errors using first-released data ( $y_{T+1}^{T+2}$ ), data after three years of revision ( $y_{T+1}^{T+13}$ ) and data from the last available vintage ( $y_{T+1}^{09Q1}$ ). Table 6 presents ratios of MSFEs of forecasting using RTV and EOS data such that values smaller than one favour RTV. We give results for one-step-ahead forecasts, and results for four-step-ahead forecasts (computed by iteration of the one-step forecasts).

In general there are gains to RTV over EOS for all three series, and some of these are sizeable. There are gains to RTV for forecasting the early release as well as the revised data, and they are generally larger for four-step-ahead forecasts. Bias-correction of the RTV forecasts (as discussed in section 4) is not effective, as might have been anticipated from the differences in the mean revisions between the estimation and forecast periods. Nevertheless, there is promising evidence in favour of RTV over EOS.

## 6.2 Real-Time data and Forecasting Inflation with the Phillips Curve

Following Stock and Watson (2008), the form of the Phillips Curve forecasting model is given by the autoregressive-distributed lag (ADL) model :

$$\pi_t^h = \pi_{t-h} + \mu + \alpha(L)\Delta\pi_{t-h} + \beta(L)x_{t-h} + v_t \quad (15)$$

where  $\pi_t^h = h^{-1} \sum_{i=0}^{h-1} \pi_{t-i}$  and  $\pi_t = 400 \ln(P_t/P_{t-1})$  for a price index  $P_t$ , and where  $x_t$  is an ‘activity variable’. We set  $h = 4$  and so consider forecasts of the (logarithmic approximation) to the annual rate of inflation a year ahead,  $\pi_{t+4}^4 \equiv 100 \ln(P_{t+4} - P_t)$ , using an autoregression in the

quarterly percentage inflation rate (annualised) with a unit root in  $\pi_t$  imposed,<sup>10</sup> and a distributed lag in  $x_t$ . By lagging the right-hand-side (RHS) variables in (15) by four quarters the forecasting model can be used directly to produce 4-quarter forecasts, as is now standard practice.<sup>11</sup>

If we use a single vintage of data, such as the latest available at the time the exercise is performed, then the pseudo out-of-sample forecasting exercise proceeds as follows. For the first forecast origin, say  $t_1$ , estimate (15) on date up to  $t = t_1$ , having determined the orders  $p_1$  and  $p_2$  of the lag polynomials  $\alpha(L) = \alpha_0 + \alpha_1 L + \dots + \alpha_{p_1} L^{p_1}$  and  $\beta(L) = \beta_0 + \beta_1 L + \dots + \beta_{p_2} L^{p_2}$  by some criterion. Then use the estimated model coefficients and the values of the RHS variables dated period  $t_1$  (and earlier) to forecast  $\pi_{t_1+4}^4$ . Then repeat for  $t = t_1 + 1$ , and so on. Forecast errors are calculated by comparing the forecasts to the latest-available vintage estimates of  $\pi_{t_1+4}^4$ ,  $\pi_{t_1+1+4}^4$ , etc. For recursive forecasting, the estimation sample runs from the first available observation to the forecast origin, and so increases each time a forecast is made. A rolling forecasting scheme keeps the estimation sample fixed in size, so for an estimation sample of size  $n$ , the first forecast is based on a model estimated on observations  $t_1 - n + 1$  to  $t_1$ , the second on  $t_1 - n + 2$  to  $t_1 + 1$ , and so on.

In a real-time exercise, we need to keep track of the vintage as well as the time period of the observation. We let  $\pi_{t+4}^{4,t+4+s} \equiv 100 \ln(P_{t+4}^{t+4+s} - P_t^{t+s})$  for  $s = 1, 2, \dots$  denote the annual rate of inflation between  $t + 4$  and  $t$  in the  $t + 4 + s$  data vintage:  $\pi_{t+4}^{4,t+4+1}$  is the first estimate. Data are observed with a lag of one quarter and the inflation rate is based on same vintage (here, vintage  $t + 4 + 1$ ) observations on the price level. Similarly for  $\pi_t$ , namely,  $\pi_t^{t+s} = 400 \ln(P_t^{t+s} / P_{t-1}^{t+s})$ . The traditional way of conducting a real-time forecasting exercise (EOS) is to use the data vintage available at each forecast origin. So at the first forecast origin  $t_1$ , we assume we have data vintage  $t_1 + 1$  (so that we observe data for period  $t_1$ ). Then the model is estimated on observations for  $t = 6 + \max(p_1, p_2)$  to  $t_1$ ,<sup>12</sup> all drawn from data vintage  $t_1 + 1$ :

$$\pi_t^{4,t_1+1} = \pi_{t-4}^{t_1+1} + \mu + \alpha(L)\Delta\pi_{t-4}^{t_1+1} + \beta(L)x_{t-4}^{t_1+1} + v_t \quad (16)$$

and the forecast of  $\pi_{t_1+4}^h$  is conditioned on  $\pi_{t_1}^{t_1+1}, \dots, \pi_{t_1-p_1-1}^{t_1+1}$  and  $x_{t_1}^{t_1+1}, \dots, x_{t_1-p_2-1}^{t_1+1}$ . The alternative way of using real-time data (RTV) instead involves estimating:

$$\pi_t^{4,t+1} = \pi_{t-4}^{t-4+1} + \mu + \alpha(L)\Delta\pi_{t-4}^{t-4+1} + \beta(L)x_{t-4}^{t-4+1} + v_t \quad (17)$$

<sup>10</sup>When the equivalent of (15) is used to forecast output growth, the activity variable is omitted, and the unit root in the autoregressive dynamics is not imposed.

<sup>11</sup>See, for example, Clements and Hendry (1996) and Marcellino, Stock and Watson (2006), inter alia.

<sup>12</sup>The start date is calculated assuming  $t = 1$  is the first time period, and accomodates the calculation of lags implied by  $\alpha(L)$  and  $\beta(L)$  and the 4-quarter lag of the RHS variables relative to the dependent variable.

for the same time-period, namely, for  $t = 6 + \max(p_1, p_2)$  to  $t_1$ . Note that the lag operator acts on the time index (the subscript) and not the vintage (superscript) so for example  $\alpha(L)\Delta\pi_{t-4}^{t-4+1} = \alpha_0\Delta\pi_{t-4}^{t-4+1} + \alpha_1\Delta\pi_{t-5}^{t-4+1} + \dots$ . Hence the model parameters are estimated using first estimates (and early estimates, in the case of lagged RHS variables) of the LHS and RHS variables, matching the first-estimates of the data on which the forecasts are conditioned (which is the same data as for EOS). This is in essence the approach described in section 4 adapted to the direct forecasting of the annual rate of inflation one-year ahead.

A variant to this method is to estimate the model as:

$$\pi_t^{4,t+1} = \pi_{t-4}^{t+1} + \mu + \alpha(L)\Delta\pi_{t-4}^{t+1} + \beta(L)x_{t-4}^{t+1} + v_t \quad (18)$$

for  $t = 6 + \max(p_1, p_2)$  to  $t_1$ , so that the vintage for the LHS and RHS matches for each ‘row’ of the regression. We refer to this as  $\text{RTV}_v$ .  $\text{RTV}_v$  is an alternative implementation of the assertion in Koenig *et al.* (2003) that ‘right-side variables ought to be the most up-to-date estimates available at that time’ (although these authors do not consider autoregressive models).

The activity variables we use are output growth (GDP/GNP), the output gap, the unemployment rate, employment and industrial production.<sup>13</sup> We use two inflation measures, the GDP/GNP deflator and the consumers’ expenditure deflator, as vintages were available for these two variables back to 1965, whereas for the other price series considered by Stock and Watson (2008) vintages are only available from 1992. As well as purely autoregressive models (i.e., equation (15) without the activity variable  $x_t$ ) we also take as a benchmark that used by Atkeson and Ohanian (2001), whereby the forecast of the annual percentage change is the (logarithmic approximation) to the annual percentage change at the forecast origin. For a forecast origin  $t_1$ , the forecast is simply  $\pi_{t_1}^{4,t_1+1}$ . This is often referred to as a ‘no change’ forecast, and is an attractive comparator in our framework as it is the same for both EOS and RTV (but will of course differ for final vintage data). Following Stock and Watson, this is referred to as AO.

### 6.2.1 Results of a pseudo out-of-sample exercise

The results of our pseudo out-of-sample exercise (the data vintage was 2009Q1) were consistent with those of Stock and Watson. Calculating forecast accuracy measures separately for the four periods 1977:1 – 1984:4, 1985:1 – 1992:4, 1993:1 – 2000:4 and 2001:1 – 2008:4, we found that the ‘best’ ADL model with output growth as an activity variable had RMSEs relative to AO of around

<sup>13</sup>In the case of employment and industrial production, we use quarterly vintages by considering the data vintage of the month at the middle of the quarter. Data vintages are available since 1965:Q4.

0.74 and 0.72, for PCE and GDP deflator inflation, respectively, for the first sub-period using a recursive forecasting scheme: see Table 7. The rows labelled ‘Final’ relate to the pseudo exercise using the 2009Q1 data vintage throughout. We define the ‘best’ as the forecasts with the smallest MSFE out of a set of models consisting of: a model selected by AIC (from a maximum of 4 lags), a model selected by BIC (from a maximum of 4 lags), and a fixed-specification model with  $p_1 = 4$  and  $p_2 = 1$ . If instead of output growth we used an alternative activity variable (the output gap, unemployment, employment or industrial production) the gains to the ADL were typically less but generally in excess of 10% (the exception is for GDP inflation with the output gap as the activity variable).

Table 7 also reports equivalent comparisons for a rolling scheme, using a data window of 69 observations. The direction of results favouring the ADL for the period up to 1984:4 is reproduced, but the gains to the ADL are generally lower relative to the recursive scheme.

For the subsequent three sub-periods the gains to the Phillips Curve are either negative (ratio to AO benchmark in excess of one), much reduced, or dependent on the period, price and activity measure. Certain indicators work well at particular times (e.g., using output growth as the activity variable results in a RMSFE of around 0.95 for PCE inflation for 2001:1 – 2008:4 for the rolling scheme) but the ‘across the board’ gains regardless of the inflation measure and activity variable that characterize the pre-1985 period are no longer found.

### **6.2.2 Results of pseudo out-of-sample exercise versus a standard real-time forecasting exercise**

The results of the real-time forecasting exercise using EOS are also recorded in Table 7 (the rows labelled ‘EOS’). It is sometimes argued that pseudo forecasting exercises overstate the size of the gains that are achievable in real-time. In the case of Phillips Curve inflation forecasts just the opposite appears to be the case during the first period. For instance, the pseudo-exercise dramatically under-estimates the gains achievable in real-time from using the output gap as the predictor for forecasting up to 1985.<sup>14</sup> Nevertheless, the overall picture is unchanged between the pseudo and real-time exercises. There are marked improvements over AO for both inflation measures pre-1985, but for later periods the picture is much more mixed.

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<sup>14</sup>In both exercises we calculate forecast errors using the final vintage actual values.

### 6.2.3 Real-time forecasting: EOS versus RTV

We consider the relative accuracy of two approaches to real-time forecasting: the use of EOS data, and the use of RTV. Table 8A reports ‘best’ ADL model RMSFE relative to the AO RMSFE, for EOS and RTV (and  $RTV_v$ ) where the actuals are 2009Q1-vintage actuals as well as first-release actuals. Table 8A is based on a recursive forecasting exercise. Equivalent results for a rolling scheme are presented in table 8B. By and large, the performance of RTV (and  $RTV_v$ ) follows that of EOS. We conclude that the forecast performance of the Phillips Curve in real-time is not sensitive to the way in which the real-time data vintages are used (i.e., EOS versus RTV). This is true whether forecast errors are calculated using early vintage actuals or latest available data.

### 6.3 Real-time data and Forecasting Output Growth with Economic Indicators

When forecasting inflation, Stock and Watson (2008) suggest that the ‘no change’ forecast is a good benchmark that does not require the estimation of unknown parameters. In the case of output growth, Stock and Watson (2003) and our own preliminary results suggest an autoregressive model as a benchmark. The positive impact of the use of RTV data on the forecasting accuracy of autoregressive models of output growth indicates that the use of real-time data could affect the conclusions one draws concerning the value of indicator variables as predictors. The relative ranking of the AR and ADL might depend on whether RTV or EOS is used.

When estimating ADL models for output growth, the model is modified to:

$$Y_t^h = \mu + \alpha(L)Y_{t-h} + \beta(L)x_{t-h} + v_t,$$

where  $Y_t = 400 \ln(Q_t/Q_{t-1})$ ,  $Q_t$  is real GDP (or GNP), and  $Y_t^h = h^{-1} \sum_{i=0}^{h-1} Y_{t-i}$ . We evaluate forecasts from four economic indicators ( $x$ ) chosen due to the availability of real-time data: industrial production, employment, weekly hours on manufacturing, and housing starts.<sup>15</sup> The last two indicators are included in the Conference Board leading indicator composite index. As for the ADL models of inflation, we report forecasts based on final data, EOS, RTV and  $RTV_v$ , and for both rolling and recursive estimation windows.

The benchmark model is an autoregressive model estimated for direct forecasting of next year output growth:

$$Y_t^4 = \mu + \alpha(L)Y_{t-4} + v_t.$$

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<sup>15</sup>As before, we use quarterly vintages (month at the middle of the quarter) of industrial production and employment since 1965:Q4. For hours and housing starts, data vintages are available since 1971:Q4. Estimation period starts in 1959:Q1.

Based on a preliminary forecasting exercise with  $p_1 = 1, \dots, 4$ , the autoregressive order is set to 1 for the benchmark model. Following the forecasting exercise of inflation, Tables 9 and 10 present results for the ‘best’ ADL model chosen to minimise RMSFE from the following set of models: a model selected by AIC (maximum of 4 lags), a model selected by BIC (maximum of 4 lags), and a fixed-specification model with  $p_1 = 1$  and  $p_2 = 1$ . Forecast errors are computed using the last vintage (2009Q1) in Table 9, and the first-vintage in Table 10.

The economic variables considered are generally not able to improve on the accuracy of the autoregressive benchmark in a pseudo-out-of-sample exercise (i.e., using final data; Table 9), with the exception of housing starts in the initial and last periods. In a real-time exercise with EOS data, gains with respect to the benchmark are slightly larger. Table 10 reports on three different ways of organizing real-time data for the ADL, versus the AR benchmark estimated with EOS data, as well as giving results for the AR estimated by EOS, and by RTV. Confirming the results in Table 6, the use of RTV data improves forecasts of the AR model for all periods except one (1993-2000). However, as for inflation, these gains are not observed when forecasting with ADL models with one exception. Gains in forecasting accuracy are observed for forecasting the last vintage of output growth with housing starts in the 2001-2008 period.

Interestingly, all four indicators outperform the benchmark when the models are estimated with RTV data, at least for forecasting final vintage output growth in the 2001-2008 using rolling samples. However, when using the AR model estimated with RTV data as the benchmark, these gains are generally not large enough to imply that the indicators have predictive power.

## 7 Conclusions

In recent times there has been a growing appreciation of the effects of data revisions on various aspects of macro-modelling (such as the calculation of output gaps and conduct of monetary policy: e.g., Orphanides (2001) and Orphanides and van Norden (2005)), as well as the relevance of data revisions for forecasting (as reviewed by Croushore (2006)). We have tackled one aspect of forecasting when there are data revisions: namely, does the way in which the real-time data set is employed to estimate ADL and AR models matter when forecasting in real time? We have considered two ways of constructing the estimation sample - the conventional approach of using end-of-sample data, and the use of real-time-vintage data. Our analytical results show that the use of EOS data will lead to AR model forecasts which are not optimal relative to the so-called RTV approach in population. Moreover, the use of EOS may result in biased forecasts when data

revisions are non-zero mean, whereas forecasts remain unbiased when estimation is by RTV when the aim is to forecast the first-release data. If the goal is to forecast the revised data, the logic of the RTV approach suggests a simple bias-correction can be applied to the forecasts.

We observe the gains to using RTV predicted by our analytical results and the Monte Carlo simulations when we use AR models to forecast. This is true for forecasting the quarterly rate of inflation and output growth both 1 and 4-steps ahead. AR model forecasts are of interest in their own right but are also typically used as benchmark forecasts against which forecasts from other models may be assessed. These models might contain leading indicators, whose predictive ability we wish to assess for real-time forecasting (see, for example, Clements and Galvão (2008, 2009)). Because the gains to including the putative predictor variables relative to the AR model forecasts are typically relatively small, and of a similar magnitude to the difference between estimating the AR model by RTV versus EOS, care is required in ensuring that the finding of predictability of an indicator variable is not simply due to data vintage effects. An example of this forecasting performance measurement issue is provided when comparing models of economic indicators for forecasting output growth with an autoregressive benchmark.

The results of the empirical forecast comparisons suggest that the conclusions we draw regarding the predictability of inflation using Phillips curve models is largely unchanged when the exercise is performed in real time compared to the pseudo out-of-sample exercise: there are marked improvements relative to a benchmark model that excludes an ‘activity variable’ pre-1985, but not post-1985. However, the forecast performance of the Phillips Curve in real-time is not sensitive to the way in which the real-time data vintages are used (i.e., EOS versus RTV). Similarly, when forecasting output growth with measures of economic activity, the organization of the real-time data set does not affect the forecasting performance of the indicators. However, the use of RTV data to estimate the benchmark autoregressive model may affect the relative forecast performance of economic indicators.

There are a number of avenues to pursue. These include the reasons for the failure of RTV to improve on EOS when using economic indicators subject to data revisions in forecasting models. Although our analytical results are for the AR, we have argued that RTV might also be expected to beat EOS for models such as ADL models which have explanatory variables in addition to lags. This is borne out by the distributed lag models of Koenig *et al.* (2003). In addition, in future research one might ask whether approaches that explicitly model data revisions fare better for the key macro variables of output growth and inflation. It is not clear that more elaborate, complicated models that simultaneously model the true process and the revisions process would yield more accurate

forecasts, given the findings of the recent empirical forecast comparison literature that ‘simple’ models generate competitive forecasts relative to more sophisticated models,<sup>16</sup> but this would be an interesting topic to explore.

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<sup>16</sup>See, for example, Fildes and Makridakis (1995), Clements and Hendry (1999) and Makridakis and Hibon (2000).

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## A Proofs

### Proof. Proposition 1: Optimal AR population parameter.

The expected squared forecast error is given by:

$$E \left( y_{T+1}^{T+1+f} - \phi' \mathbf{y}_T^{T+1} \right)^2 = E \left( \rho_0 + \boldsymbol{\rho}' \tilde{\mathbf{y}}_T + R_1 \eta_{1T+1} + \hat{v}_{T+1}^{T+1+f} + \varepsilon_{T+1}^{T+1+f} - \phi_0 - \phi' \tilde{\mathbf{y}}_T - \phi' \mathbf{v}_T^{T+1} - \phi' \boldsymbol{\varepsilon}_T^{T+1} \right)^2$$

where  $\mathbf{y}_T^{T+1} = \tilde{\mathbf{y}}_T + \mathbf{v}_T^{T+1} + \boldsymbol{\varepsilon}_T^{T+1}$ ,  $\hat{v}_{T+1}^{T+1+f} = -v_{T+1}^{T+2} + v_{T+1}^{T+1+f}$ , with  $\mathbf{y}_T^{T+1'} = \left( y_T^{T+1}, \dots, y_{T-p+1}^{T+1} \right)$ ,  $\tilde{\mathbf{y}}_T' = \left( \tilde{y}_T, \dots, \tilde{y}_{T-p+1} \right)$ ,  $\mathbf{v}_T^{T+1'} = \left( v_T^{T+1}, \dots, v_{T-p+1}^{T+1} \right)$ , with typical element  $v_{T-j}^{T+1} = -\sum_{i=j+1}^l \sigma_{v_i} \eta_{2,T-j,i}$ , for  $j < l$ , and  $v_{T-j}^{T+1} = \sigma_{v_i} \eta_{2,T-j,l}$ , for  $j \geq l$ , and  $\boldsymbol{\varepsilon}_T^{T+1'} = \left( \varepsilon_T^{T+1}, \dots, \varepsilon_{T-p+1}^{T+1} \right)$ . Note that  $\mathbf{y}_{T+1}^{T+1+f} = \tilde{\mathbf{y}}_{T+1} + \mathbf{v}_{T+1}^{T+1+f} + \boldsymbol{\varepsilon}_{T+1}^{T+1+f} = \rho_0 + \boldsymbol{\rho}' \tilde{\mathbf{y}}_T + R_1 \eta_{1T+1} - \mathbf{v}_{T+1}^{T+2} + v_{T+1}^{T+1+f} + \varepsilon_{T+1}^{T+1+f}$ , so that  $\hat{v}_{T+1}^{T+1+f} = -v_{T+1}^{T+2} + v_{T+1}^{T+1+f} = 0$  when  $f = 1$ . We let  $E(\tilde{\mathbf{y}}_T \tilde{\mathbf{y}}_T') = \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\mu}_{\tilde{\mathbf{y}}} \boldsymbol{\mu}_{\tilde{\mathbf{y}}}'$ , where  $\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} = \text{Var}(\tilde{\mathbf{y}}_T)$ ,  $\boldsymbol{\mu}_{\tilde{\mathbf{y}}} = \mathbf{i} \boldsymbol{\mu}_{\tilde{\mathbf{y}}} = E(\tilde{\mathbf{y}}_T)$ ,  $\mathbf{i}$  a  $p$ -dimensional vector of 1's;  $\boldsymbol{\Sigma}_{\mathbf{v}} \equiv \text{Var}(\mathbf{v}_T^{T+1} \mathbf{v}_T^{T+1'}) = E(\mathbf{v}_T^{T+1} \mathbf{v}_T^{T+1'}) = \text{diag}(\sum_{i=1}^l \sigma_{v_i}^2, \dots, \sum_{i=p}^l \sigma_{v_i}^2)$  for  $p \leq l$ , with terms of  $\sigma_{v_i}^2$  on the diagonal for  $p > l$ ;  $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \equiv \text{Var}(\boldsymbol{\varepsilon}_T^{T+1} \boldsymbol{\varepsilon}_T^{T+1'}) = E(\boldsymbol{\varepsilon}_T^{T+1} \boldsymbol{\varepsilon}_T^{T+1'}) = \text{diag}(\sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2, \dots, \sigma_{\varepsilon_p}^2)$ , with  $\sigma_{\varepsilon_s}^2 = \sigma_{\varepsilon_l}^2$  for  $s > l$ , and  $\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}} \equiv \text{Cov}(\tilde{\mathbf{y}}_T \mathbf{v}_T^{T+1'}) = E(\tilde{\mathbf{y}}_T \mathbf{v}_T^{T+1'})$ . Solving the first-order conditions  $\partial E(y_{T+1}^{T+f} - \phi' \mathbf{y}_T^{T+1})^2 / \partial \phi = \mathbf{0}$  and  $\partial E(y_{T+1}^{T+f} - \phi' \mathbf{y}_T^{T+1})^2 / \partial \phi_0 = \mathbf{0}$  gives:

$$\begin{aligned} \phi^* &= \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\mathbf{v}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}}' + \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \right)^{-1} \left( \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}} + \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}\mathbf{v}}' \right) \boldsymbol{\rho} \\ \phi_0^* &= (1 - \phi^{*'} \mathbf{i}) \boldsymbol{\mu}_{\tilde{\mathbf{y}}}. \end{aligned}$$

■

### Proof. Proposition 2: Estimation of AR(p) with EOS data (zero-mean data revisions).

1. If we allow  $T \rightarrow \infty$  for a fixed  $l$ , then OLS estimator of eq(14) is asymptotically equivalent to the estimator obtained from a regression using only fully-revised data. Denote this data by  $\{y_t, y_{t-1} \dots y_{t-p}\}$ , where  $y_t = \tilde{y}_t + v_t^{t+l} + \varepsilon_t^{t+l}$ , and  $y_t = \tilde{y}_t$  if the true data are eventually revealed. As the data  $\{y_t, \mathbf{y}_{t-1} = (y_{t-1} \dots y_{t-p})'\}$  are covariance stationary we can use the population moments (assuming that  $T \rightarrow \infty$ ) to compute the OLS estimators as values that satisfy

$$(\alpha_0^*, \boldsymbol{\alpha}^*) = \arg \min_{\alpha_0, \boldsymbol{\alpha}} E (y_t - \alpha_0 - \boldsymbol{\alpha} \mathbf{y}_{t-1})^2,$$

which are:

$$\alpha_0^* = E(y_t) - \boldsymbol{\alpha} E(\mathbf{y}_{t-1}) \quad (19)$$

and

$$E(\mathbf{y}_{t-1} \mathbf{y}_{t-1}') \boldsymbol{\alpha}^* + \alpha_0^* E(\mathbf{y}_{t-1}) - E(y_t \mathbf{y}_{t-1}) = 0. \quad (20)$$

Combining (19) and (20) gives the standard FOC for  $\boldsymbol{\alpha}^*$ ,

$$\text{Cov}(\mathbf{y}_{t-1} \mathbf{y}_{t-1}') \boldsymbol{\alpha}^* = \text{Cov}(y_t \mathbf{y}_{t-1}) \quad (21)$$

where  $Cov(\mathbf{y}_{t-1}\mathbf{y}'_{t-1}) = E(\mathbf{y}_{t-1}\mathbf{y}'_{t-1}) - E(\mathbf{y}_{t-1})E(\mathbf{y}'_{t-1})$ ,  $Cov(y_t\mathbf{y}_{t-1}) = E(y_t\mathbf{y}_{t-1}) - E(y_t)E(\mathbf{y}_{t-1})$ .

2. The moments in (19) are obtained from  $E(y_t) = E(\tilde{y}_t + v_t^{t+l} + \varepsilon_t^{t+l}) = \mu_{\tilde{y}}$ , and  $E(\mathbf{y}_{t-1}) = \mathbf{i}\mu_{\tilde{y}}$ , since  $\mathbf{y}_{t-1} = \tilde{\mathbf{y}}_{t-1} + \mathbf{v}_{t-1}^{t+l} + \boldsymbol{\varepsilon}_{t-1}^{t+l}$ , with  $\tilde{\mathbf{y}}_{t-1} = [\tilde{y}_{t-1}, \dots, \tilde{y}_{t-p}]'$ ,  $\mathbf{v}_{t-1}^{t+l} = [v_{t-1}^{t+l}, \dots, v_{t-p}^{t+l}]'$ ,  $\boldsymbol{\varepsilon}_{t-1}^{t+l} = [\varepsilon_{t-1}^{t+l}, \dots, \varepsilon_{t-p}^{t+l}]'$ , so that:

$$\alpha_0^* = (1 - \boldsymbol{\alpha}^* \mathbf{i}) \mu_{\tilde{y}}. \quad (22)$$

For (21) we obtain:

$$Cov(\mathbf{y}_{t-1}\mathbf{y}'_{t-1}) = \Sigma_{\tilde{y}} + \Sigma_{\mathbf{v}} + \Sigma_{\boldsymbol{\varepsilon}} + \Sigma_{\tilde{y}v_l} + \Sigma'_{\tilde{y}v_l} \quad (23)$$

where  $\Sigma_{\tilde{y}v_l} \equiv E\left[(\tilde{\mathbf{y}}_{t-1} - E(\tilde{\mathbf{y}}_{t-1}))(\mathbf{v}_{t-1}^{t+l} - E(\mathbf{v}_{t-1}^{t+l}))'\right]$ ,  $\Sigma_{\mathbf{v}} = E\left[\mathbf{v}_{t-1}^{t+l} - E(\mathbf{v}_{t-1}^{t+l})(\mathbf{v}_{t-1}^{t+l} - E(\mathbf{v}_{t-1}^{t+l}))'\right]$ ,  $\sigma_{v_l}^2 I_p$ ,  $\Sigma_{\boldsymbol{\varepsilon}} = E\left[\boldsymbol{\varepsilon}_{t-1}^{t+l} - E(\boldsymbol{\varepsilon}_{t-1}^{t+l})(\boldsymbol{\varepsilon}_{t-1}^{t+l} - E(\boldsymbol{\varepsilon}_{t-1}^{t+l}))'\right]$ . Note that  $\Sigma_{\tilde{y}v_l}$  is upper diagonal, and its diagonal is (minus) the diagonal of  $\Sigma_{\mathbf{v}}$ . Also:

$$Cov(y_t\mathbf{y}_{t-1}) = V(\tilde{\mathbf{y}}_{t-1}\tilde{\mathbf{y}}'_{t-1})\rho + V(\mathbf{v}_{t-1}^{t+l}\tilde{\mathbf{y}}'_{t-1})\rho = \Sigma_{\tilde{y}}\rho + \Sigma'_{\tilde{y}v_l}\rho \quad (24)$$

Substituting (23) and (24) into (21) gives:

$$\boldsymbol{\alpha}^* = \left(\Sigma_{\tilde{y}} + \Sigma_{\mathbf{v}} + \Sigma_{\boldsymbol{\varepsilon}} + \Sigma_{\tilde{y}v_l} + \Sigma'_{\tilde{y}v_l}\right)^{-1} \left(\Sigma_{\tilde{y}} + \Sigma'_{\tilde{y}v_l}\right)\rho. \quad (25)$$

Recall that  $\Sigma_{\boldsymbol{\varepsilon}} \neq \Sigma_{\boldsymbol{\varepsilon}} = \text{diag}\{\sigma_{\varepsilon_1}^2, \dots, \sigma_{\varepsilon_p}^2\}$ ,  $\Sigma_{\tilde{y}v_l} \neq \Sigma_{\tilde{y}\mathbf{v}}$  and  $\Sigma_{\mathbf{v}} \neq \Sigma_{\mathbf{v}}$  from the assumption that in the large samples the use of EOS data implies the use of data from the  $t+l$  vintage.

■

**Proof. Remark 1:** Forecasts obtained using EOS data to estimate the AR(p) model are unbiased when data revisions follow (1)-(3).

By construction,  $\{\alpha_0^*, \boldsymbol{\alpha}^*\}$  satisfy  $E(y_{T+1} - \alpha_0 - \boldsymbol{\alpha}'\mathbf{y}_T) = 0$ , where  $y_{T+1}$  and  $\mathbf{y}_T = (y_T, \dots, y_{T-p+1})'$  are ‘final data’. But the forecasting exercise is to forecast  $y_{T+1}$  using the latest available data,  $\mathbf{y}_T^{T+1} = (y_T^{T+1}, \dots, y_{T-p+1}^{T+1})'$ . Consider the expected value of the forecast error  $E(y_{T+1}^{T+2} - \alpha_0 - \boldsymbol{\alpha}'\mathbf{y}_T^{T+1})$ , where we substitute  $y_{T+1}^{T+2} = y_{T+1} - (y_{T+1} - y_{T+1}^{T+2})$  and  $\mathbf{y}_T^{T+1} = \mathbf{y}_T - (\mathbf{y}_T - \mathbf{y}_T^{T+1})$  to give:

$$E(y_{T+1}^{T+2} - \alpha_0 - \boldsymbol{\alpha}'\mathbf{y}_T^{T+1}) = E(y_{T+1}^{T+2} - y_{T+1} - \boldsymbol{\alpha}'(\mathbf{y}_T^{T+1} - \mathbf{y}_T)) = 0$$

because  $E(y_{T+1}^{T+2}) = E(y_{T+1}) = \mu_{\tilde{y}}$  and  $E(\mathbf{y}_T^{T+1}) = E(\mathbf{y}_T) = \mathbf{i}\mu_{\tilde{y}}$ . ■

**Proof. Proposition 3: Optimal AR population parameter when data revisions are non-zero mean.**

The optimal parameters solve  $E(y_{T+1}^{T+2}) = \phi_0 + E(\boldsymbol{\phi}'\mathbf{y}_T^{T+1})$ . The optimal slope parameters

$\phi^*$  only depend on (centred) second moment matrices, so are the same whether the data revision process is (1)-(4)-(5) or (1)-(2)-(3). Under the data revision process (1)-(4)-(5), we have

$$E\left(y_{T+1}^{T+2}\right) = E\left(\tilde{y}_t + v_t^{t+1} + \varepsilon_t^{t+1}\right) = \mu_{\tilde{y}} + \mu_{v_1} + \mu_{\varepsilon_1},$$

where  $\mu_{\varepsilon_1} \equiv E\left(\varepsilon_t^{t+1}\right) = -\mu_{\eta_{3,1}}$ ,  $\mu_{v_1} \equiv E\left(v_t^{t+1}\right) = -\sum_{i=1}^l \mu_{\eta_{2,i}}$ , and  $E\left(\phi' \mathbf{y}_{t-1}^t\right) = \phi' \left(\mathbf{i} \mu_{\tilde{y}} + \boldsymbol{\mu}_\varepsilon + \boldsymbol{\mu}_v\right)$ , where  $\mathbf{i} \mu_{\tilde{y}} = E\left([\tilde{\mathbf{y}}_{t-1}]\right) = E\left([\tilde{y}_{t-1}, \dots, \tilde{y}_{t-p}]'\right)$ ,  $\mathbf{i}$  a  $p$ -dimensional vector of 1's, and  $\boldsymbol{\mu}_\varepsilon$  and  $\boldsymbol{\mu}_v$  are the means of the noise and news revisions,  $\boldsymbol{\mu}_\varepsilon \equiv E\left([\varepsilon_{t-1}^t, \dots, \varepsilon_{t-p}^t]\right) = \left[-\mu_{\eta_{3,1}}, \dots, -\mu_{\eta_{3,p}}\right]$ ,  $\boldsymbol{\mu}_v \equiv E\left([v_{t-1}^t, \dots, v_{t-p}^t]\right) = \left[-\sum_{i=1}^l \mu_{\eta_{2,i}}, \dots, -\sum_{i=p}^l \mu_{\eta_{2,i}}\right]$ . Finally,  $\mu_{\tilde{y}} = (1 - \rho(1))^{-1} [\rho_0 - \mu_{v_1}]$ . Using the previous results, it is straightforward to show that:

$$\phi_0^* = (1 - \phi^* \mathbf{i}) \mu_{\tilde{y}} + \mu_{v_1} + \mu_{\varepsilon_1} - \phi^* \boldsymbol{\mu}_\varepsilon - \phi^* \boldsymbol{\mu}_v.$$

■

**Proof. Proposition 4: Estimation of AR(p) with EOS data (zero-mean data revisions).**

Allowing for non-zero mean revisions, in place of  $E(y_t) = \mu_{\tilde{y}}$  and  $E(\mathbf{y}_{t-1}) = \mathbf{i} \mu_{\tilde{y}}$  in Proposition 2, we now have  $E(y_t) = E\left(\tilde{y}_t + v_t^{t+l} + \varepsilon_t^{t+l}\right) = \mu_{\tilde{y}} + \mu_{v_l} + \mu_{\varepsilon_l}$ , where  $\mu_{v_l} \equiv E\left(v_t^{t+l}\right) = -\mu_{\eta_{2,l}}$ ,  $\mu_{\varepsilon_l} \equiv E\left(\varepsilon_t^{t+l}\right) = -\mu_{\eta_{3,l}}$ , and  $E(\mathbf{y}_{t-1}) = \mathbf{i} \mu_{\tilde{y}} + \mathbf{i} \mu_{v_l} + \mathbf{i} \mu_{\varepsilon_l}$ .  $\boldsymbol{\alpha}^*$  is still given by (25), but (22) becomes:

$$\alpha_0^* = (1 - \boldsymbol{\alpha}^* \mathbf{i}) (\mu_{\tilde{y}} + \mu_{v_l} + \mu_{\varepsilon_l}).$$

■

**Proof. Remark 2:** Forecasts computed using EOS data to estimate AR(p) model are biased when data revisions are described by (1)-(4)-(5).

Consider the first moment properties of the forecast errors when an AR(p) is estimated by EOS in the presence of non-zero mean data revisions. Suppose the aim is to forecast the first vintage estimate  $y_{T+1}^{T+2}$ . To see that EOS population forecasts are generally biased, note that by construction,  $\{\alpha_0^*, \boldsymbol{\alpha}^*\}$  satisfy  $E(y_{T+1} - \alpha_0 - \boldsymbol{\alpha}' \mathbf{y}_T) = 0$ , where  $y_{T+1}$  and  $\mathbf{y}_T = (y_T, \dots, y_{T-p+1})$  are 'final data'. But forecasts are of necessity conditioned on  $\mathbf{y}_T^{T+1} = (y_T^{T+1}, \dots, y_{T-p+1}^{T+1})$ , so of interest is the expected value of the forecast error  $E\left(y_{T+1}^{T+2} - \alpha_0^* - \boldsymbol{\alpha}^* \mathbf{y}_T^{T+1}\right)$ . Substituting  $y_{T+1}^{T+2} = y_{T+1} - (y_{T+1} - y_{T+1}^{T+2})$  and  $\mathbf{y}_T^{T+1} = \mathbf{y}_T - (\mathbf{y}_T - \mathbf{y}_T^{T+1})$  gives:

$$\begin{aligned} E\left(y_{T+1}^{T+2} - \alpha_0^* - \boldsymbol{\alpha}^* \mathbf{y}_T^{T+1}\right) &= E\left(y_{T+1}^{T+2} - y_{T+1} - \boldsymbol{\alpha}^* (\mathbf{y}_T^{T+1} - \mathbf{y}_T)\right) \\ &= (\mu_{v_1} - \mu_{v_l}) + (\mu_{\varepsilon_1} - \mu_{\varepsilon_l}) - \boldsymbol{\alpha}^* [(\boldsymbol{\mu}_v - \mathbf{i} \mu_{v_l}) + (\boldsymbol{\mu}_\varepsilon - \mathbf{i} \mu_{\varepsilon_l})] \end{aligned}$$

since  $E\left(y_{T+1}^{T+2}\right) = \mu_{\tilde{y}} + E\left(v_{T+1}^{T+2}\right) + E\left(\varepsilon_{T+1}^{T+2}\right) = \mu_{\tilde{y}} + \mu_{v_1} + \mu_{\varepsilon_1}$ , and  $E(y_{T+1}) = \mu_{\tilde{y}} + E\left(v_{T+1}^{T+1+l}\right) + E\left(\varepsilon_{T+1}^{T+1+l}\right) = \mu_{\tilde{y}} + \mu_{v_l} + \mu_{\varepsilon_l}$ , and similarly for  $E\left(\mathbf{y}_T^{T+1}\right)$  and  $E(\mathbf{y}_T)$ . Recall that  $\mu_{v_1} = -\sum_{i=1}^l \mu_{\eta_{2,i}}$ ,  $\mu_{v_l} = -\mu_{\eta_{2,l}}$ ,  $\mu_{\varepsilon_1} = -\mu_{\eta_{3,1}}$ ,  $\mu_{\varepsilon_l} = -\mu_{\eta_{3,l}}$  and  $\mu_{\tilde{y}} = (1 - \rho(1))^{-1} [\rho_0 - \mu_{v_1}]$ . The expression for this bias will not equal zero if the mean of the revisions are not zero. ■

Figure 1: The mean and standard deviation of the revisions relative to the first-released data.

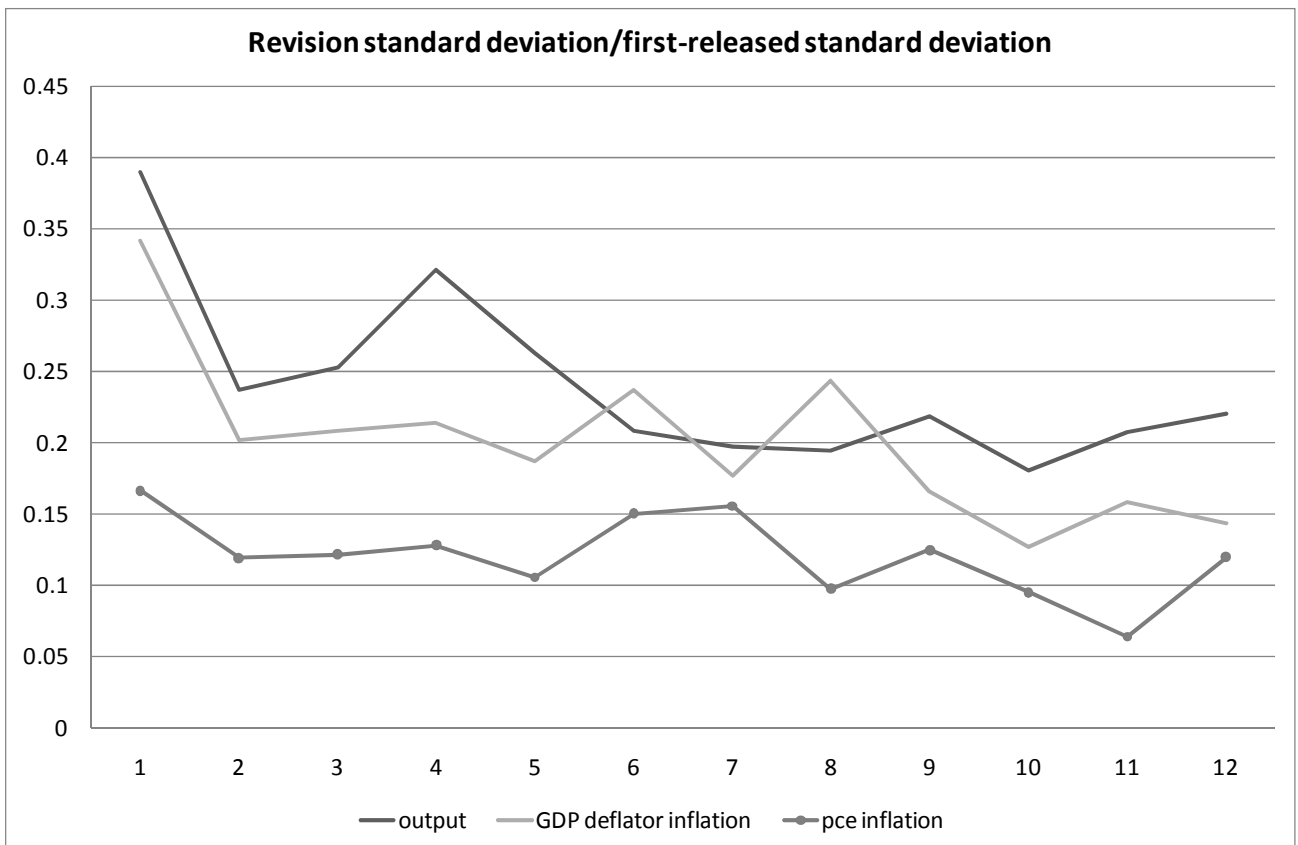
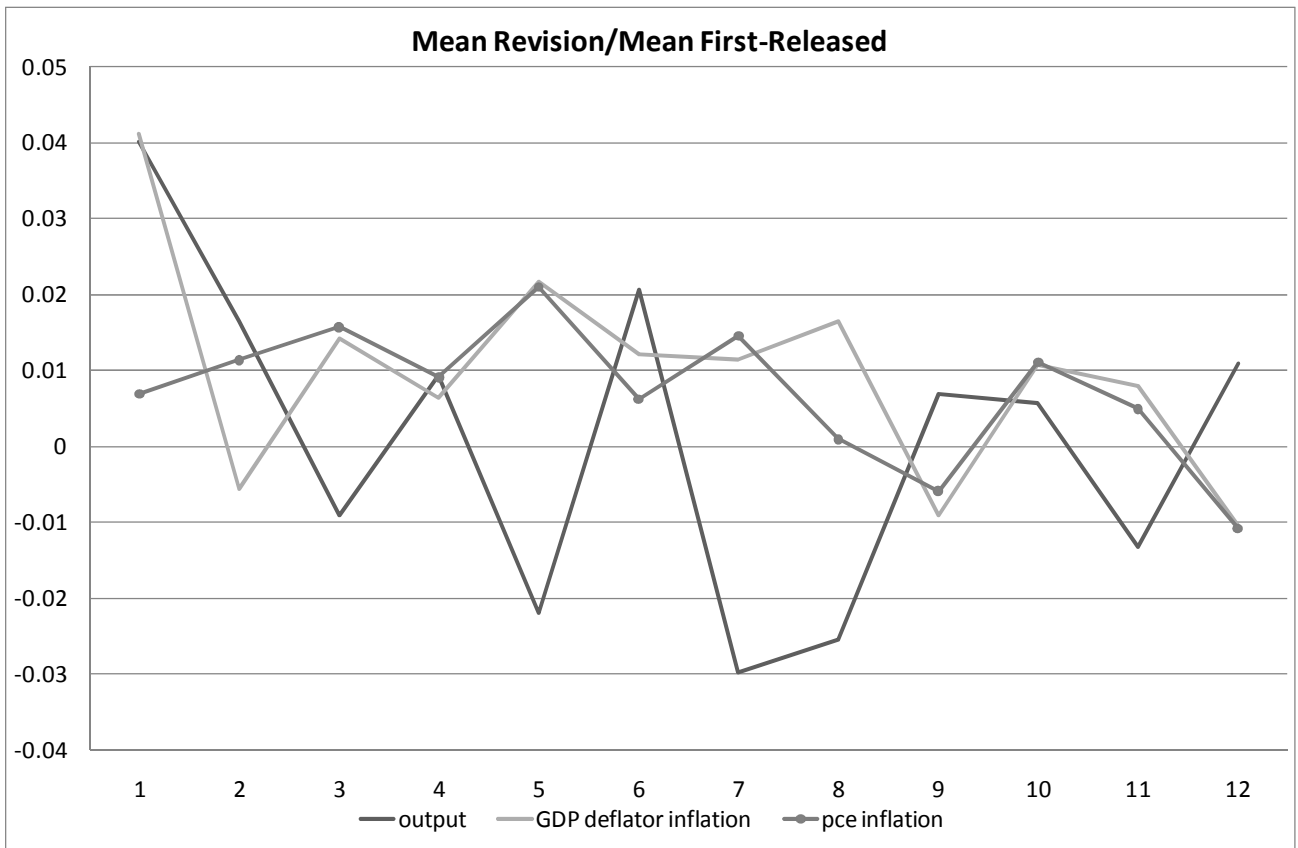


Table 1: Characteristics of the Data Generating Processes for the Simulated Data.

DGP n.	$\rho_0$	$\rho_1$	$\rho_2$	$R_1$	$\mu_{r1}, \mu_{r5}$	$\sigma_{y_t^{t+1}}$	$\sigma_{r_1} / \sigma_{y_t^{t+1}}$	$\sigma_{r_2, \dots, r_{l-1}} / \sigma_{y_t^{t+1}}$	$\sigma_{r_l} / \sigma_{y_t^{t+1}}$
1	.4	.2	.2	.5	.06, .03	.589	.4	.2	.1
2	.4	.2	.2	.5	.06, .03	.589	.6	.3	.15
3	.4	.2	.2	.5	.06, .03	.589	.3	.3	.3
4	.4	.2	.2	.5	.06, .03	.589	.4	.2	0
5	.4	.5	.3	.5	.12, .06	.749	.4	.2	.1
6	.4	.5	.3	.5	.12, .06	.749	.6	.3	.15
7	.4	.5	.3	.5	.12, .06	.749	.3	.3	.3
8	.4	.5	.3	.5	.12, .06	.749	.4	.2	0

Table 2: Analytical values of the estimators, forecast biases and mean squared forecast errors for the DGPs of Table 1.

DGP n.	AR parameters								Forecasting $y_{T+1}^{T+1}$					
	$\phi_0^*$	$\alpha_0^*$	$\phi_1^*$	$\alpha_1^*$	$\phi_2^*$	$\alpha_2^*$	$\phi_1^* + \phi_2^*$	$\alpha_1^* + \alpha_2^*$	Optimal bias	EOS bias	Bias Diff	Optimal MSFE	EOS MSFE	MSFE Ratio
<b>Under Noise</b>														
1	0.39	0.40	0.17	0.20	0.20	0.20	0.37	0.40	0.000	-0.032	-0.032	0.296	0.298	0.993
2	0.40	0.41	0.15	0.20	0.20	0.20	0.34	0.39	0.000	-0.032	-0.032	0.354	0.356	0.994
3	0.39	0.42	0.19	0.19	0.19	0.19	0.37	0.37	0.000	-0.033	-0.033	0.277	0.278	0.996
4	0.39	0.40	0.17	0.20	0.20	0.20	0.37	0.40	0.000	-0.032	-0.032	0.296	0.298	0.993
5	0.42	0.41	0.39	0.49	0.36	0.30	0.76	0.80	0.000	-0.061	-0.061	0.360	0.367	0.981
6	0.49	0.42	0.32	0.49	0.39	0.30	0.71	0.79	0.000	-0.061	-0.061	0.490	0.508	0.965
7	0.42	0.48	0.46	0.46	0.30	0.30	0.76	0.76	0.000	-0.065	-0.065	0.317	0.321	0.988
8	0.42	0.40	0.39	0.50	0.36	0.30	0.76	0.80	0.000	-0.060	-0.060	0.360	0.368	0.978
<b>Under News</b>														
1	0.41	0.46	0.22	0.20	0.20	0.20	0.41	0.40	0.000	-0.044	-0.044	0.261	0.263	0.992
2	0.40	0.46	0.24	0.20	0.19	0.20	0.43	0.40	0.000	-0.044	-0.044	0.274	0.276	0.993
3	0.40	0.46	0.24	0.20	0.19	0.20	0.43	0.40	0.000	-0.044	-0.044	0.273	0.275	0.993
4	0.41	0.46	0.22	0.20	0.20	0.20	0.41	0.40	0.000	-0.044	-0.044	0.261	0.263	0.992
5	0.45	0.58	0.60	0.50	0.23	0.30	0.82	0.80	0.000	-0.072	-0.072	0.347	0.356	0.975
6	0.42	0.58	0.68	0.50	0.16	0.30	0.84	0.80	0.000	-0.072	-0.072	0.463	0.481	0.963
7	0.42	0.57	0.69	0.51	0.15	0.29	0.84	0.80	0.000	-0.071	-0.071	0.436	0.455	0.958
8	0.45	0.58	0.60	0.50	0.23	0.30	0.82	0.80	0.000	-0.072	-0.072	0.345	0.354	0.975

Table 3: Small Sample Biases: AR model parameters

DGP n.	$\hat{\beta}_0 - \phi_0^*$				$\hat{\alpha}_0 - \alpha_0^*$				$(\hat{\beta}_1 + \hat{\beta}_2) - (\phi_1^* + \phi_2^*)$				$(\hat{\alpha}_1 + \hat{\alpha}_2) - (\alpha_1^* + \alpha_2^*)$			
	T=50	T=100	T=200	T=500	T=50	T=100	T=200	T=500	T=50	T=100	T=200	T=500	T=50	T=100	T=200	T=500
<b>Under Noise</b>																
1	0.06	0.03	0.01	0.01	0.06	0.03	0.01	0.01	-0.09	-0.04	-0.02	-0.01	-0.09	-0.04	-0.02	-0.01
2	0.06	0.03	0.01	0.01	0.06	0.03	0.01	0.01	-0.09	-0.04	-0.02	-0.01	-0.09	-0.05	-0.02	-0.01
3	0.06	0.03	0.01	0.01	0.06	0.03	0.01	0.01	-0.09	-0.04	-0.02	-0.01	-0.09	-0.04	-0.02	-0.01
4	0.06	0.03	0.01	0.01	0.06	0.03	0.01	0.01	-0.09	-0.04	-0.02	-0.01	-0.09	-0.05	-0.02	-0.01
5	0.26	0.12	0.06	0.03	0.24	0.11	0.06	0.02	-0.13	-0.06	-0.03	-0.01	-0.12	-0.06	-0.03	-0.01
6	0.28	0.13	0.07	0.03	0.25	0.12	0.06	0.02	-0.14	-0.07	-0.03	-0.01	-0.13	-0.06	-0.03	-0.01
7	0.25	0.12	0.06	0.02	0.25	0.12	0.06	0.02	-0.13	-0.06	-0.03	-0.01	-0.13	-0.06	-0.03	-0.01
8	0.26	0.12	0.06	0.03	0.24	0.11	0.06	0.02	-0.13	-0.06	-0.03	-0.01	-0.12	-0.06	-0.03	-0.01
<b>Under News</b>																
1	0.06	0.03	0.01	0.01	0.07	0.03	0.02	0.01	-0.08	-0.04	-0.02	-0.01	-0.09	-0.04	-0.02	-0.01
2	0.05	0.03	0.01	0.01	0.07	0.03	0.02	0.01	-0.07	-0.04	-0.02	-0.01	-0.09	-0.04	-0.02	-0.01
3	0.06	0.03	0.01	0.01	0.07	0.03	0.02	0.01	-0.09	-0.04	-0.02	-0.01	-0.09	-0.05	-0.02	-0.01
4	0.06	0.03	0.01	0.01	0.07	0.03	0.02	0.01	-0.08	-0.04	-0.02	-0.01	-0.09	-0.05	-0.02	-0.01
5	0.27	0.13	0.06	0.03	0.33	0.15	0.08	0.03	-0.10	-0.05	-0.02	-0.01	-0.11	-0.05	-0.03	-0.01
6	0.24	0.11	0.06	0.02	0.32	0.15	0.07	0.03	-0.09	-0.04	-0.02	-0.01	-0.12	-0.05	-0.03	-0.01
7	0.26	0.12	0.06	0.02	0.32	0.15	0.07	0.03	-0.09	-0.04	-0.02	-0.01	-0.11	-0.05	-0.03	-0.01
8	0.27	0.13	0.06	0.03	0.33	0.15	0.08	0.03	-0.10	-0.05	-0.02	-0.01	-0.12	-0.06	-0.03	-0.01

Table 4: Small Sample Forecast Accuracy Findings: Biases and MSFEs.

DGP n.	Forecasting $y_{T+1}^{T+1}$										Forecasting $y_{T+4}^{T+4}$ , MSFE ratio			
	Bias Diff.	T=50	T=100	T=200	T=500	MSFE Ratio	T=50	T=100	T=200	T=500	T=50	T=100	T=200	T=500
<b>Under Noise</b>														
1	-0.032	-0.032	-0.021	-0.033	-0.032	0.993	1.001	1.001	0.995	0.995	1.002	1.003	0.998	0.998
2	-0.032	-0.032	-0.020	-0.033	-0.031	0.994	1.004	1.002	0.994	0.995	1.006	1.006	1.001	1.000
3	-0.033	-0.032	-0.023	-0.033	-0.032	0.996	0.994	0.999	0.993	0.995	0.996	0.999	0.994	0.994
4	-0.032	-0.032	-0.021	-0.033	-0.032	0.993	1.002	1.000	0.995	0.995	1.003	1.003	0.999	0.998
5	-0.061	-0.062	-0.051	-0.062	-0.059	0.981	0.990	0.986	0.982	0.981	1.028	1.037	1.031	1.033
6	-0.061	-0.064	-0.050	-0.063	-0.059	0.965	0.981	0.974	0.970	0.968	1.041	1.046	1.041	1.043
7	-0.065	-0.067	-0.057	-0.066	-0.065	0.988	0.981	0.990	0.983	0.985	0.989	0.997	0.989	0.995
8	-0.060	-0.062	-0.050	-0.061	-0.059	0.978	0.991	0.984	0.982	0.980	1.032	1.041	1.037	1.038
<b>Under News</b>														
1	-0.044	-0.044	-0.034	-0.043	-0.044	0.992	0.969	0.985	0.988	0.991	0.970	0.983	0.981	0.986
2	-0.044	-0.044	-0.032	-0.043	-0.044	0.993	0.949	0.974	0.984	0.990	0.950	0.972	0.975	0.983
3	-0.044	-0.044	-0.033	-0.044	-0.044	0.993	0.959	0.980	0.985	0.991	0.957	0.975	0.977	0.984
4	-0.044	-0.044	-0.034	-0.043	-0.044	0.992	0.969	0.985	0.988	0.992	0.969	0.983	0.982	0.986
5	-0.072	-0.073	-0.058	-0.072	-0.073	0.975	0.943	0.966	0.969	0.974	0.916	0.934	0.940	0.950
6	-0.072	-0.071	-0.055	-0.072	-0.073	0.963	0.915	0.944	0.952	0.958	0.882	0.900	0.912	0.924
7	-0.071	-0.069	-0.058	-0.071	-0.071	0.958	0.921	0.945	0.948	0.956	0.912	0.918	0.925	0.934
8	-0.072	-0.073	-0.058	-0.072	-0.073	0.975	0.942	0.966	0.969	0.974	0.915	0.935	0.940	0.950

Note: "Bias Diff" are differences between the absolute bias of RTV and the absolute bias of EOS: negative values indicate that RTV reduces bias. "MSFE ratio" are ratios of MSFE: RTV/EOS; values smaller than one indicate that RTV reduces the MSFE. The first columns under "Bias Diff" and "MSFE ratio" report the asymptotic values (of Table 2) for ease of comparison.

Table 5: Characteristics of Data Vintages during the Estimation and out-of-sample Forecast Periods.

		Mean			Standard Deviation			Autocorrelation (1st)			H <sub>0</sub> : Mean = 0		H <sub>0</sub> : News		H <sub>0</sub> :Noise	
		$y_t^{t+1}$	$y_t^{t+12}$	$y_t^{09Q1}$	$y_t^{t+1}$	$y_t^{t+12}$	$y_t^{09Q1}$	$y_t^{t+1}$	$y_t^{t+12}$	$y_t^{09Q1}$	$r_{12}$	$r_{09Q1}$	$r_{12}$	$r_{09Q1}$	$r_{12}$	$r_{09Q1}$
Output growth	1965Q3-1985Q2	0.76	0.78	0.81	1.08	1.06	1.04	0.28	0.28	0.29	[.14]	[.01]	[.22]	[.02]	[.31]	[.01]
	1985Q3-2006:Q4	0.68	0.68	0.75	0.43	0.53	0.50	0.33	0.42	0.23	[1.0]	[.06]	[.72]	[.03]	[.00]	[.00]
GDP Deflator	1965Q3-1985Q2	1.45	1.44	1.41	0.59	0.59	0.59	0.70	0.75	0.80	[.44]	[.03]	[.48]	[.13]	[.17]	[.03]
	1985Q3-2006:Q4	0.58	0.66	0.60	0.28	0.28	0.23	0.57	0.62	0.56	[.00]	[.30]	[.00]	[.00]	[.00]	[.47]
PCE Deflator	1965Q3-1985Q2	1.40	1.40	1.38	0.63	0.64	0.65	0.83	0.83	0.84	[.65]	[.13]	[.65]	[.23]	[.15]	[.04]
	1985Q3-2006:Q4	0.64	0.70	0.64	0.38	0.35	0.30	0.49	0.61	0.56	[.00]	[.76]	[.00]	[.00]	[.00]	[.14]

Note:  $p$ -values are computed for  $F$ -statistics (news/noise) and  $t$ -statistics (mean) using Newey-West standard errors, and are displayed in [].

The revisions are defined as  $r_{12} = y_t^{t+12} - y_t^{t+1}$  and  $r_{09Q1} = y_t^{09Q1} - y_t^{t+1}$ .

Table 6: Comparing the RMSFEs from RTV and EOS in a Recursive out-of-sample Forecasting Exercise (for the period 1985:Q3-2008:Q4;  $n = 94$  quarters).

6A. One-step-ahead forecasts

	p = 1			p = 2			p = 4		
Forecasting:	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$
Output growth	<b>0.976</b>	<b>0.978</b> 0.991	0.999 0.994	0.994	0.989 1.006	1.002 1.001	0.989	0.983 0.996	1.000 0.995
Inflation (GDP defl.)	0.989	0.996 1.007	<b>0.977</b> 1.004	<b>0.979</b>	0.992 0.998	<b>0.975</b> 0.999	<b>0.961</b>	<b>0.971</b> <b>0.972</b>	<b>0.959</b> 0.980
Inflation (PCE defl.)	0.986	0.983 0.986	0.982 0.993	0.988	0.986 0.990	0.984 0.994	<b>0.979</b>	0.983 0.987	<b>0.977</b> 0.986

6B. Four-step-ahead forecasts

	p = 1			p = 2			p = 4		
Forecasting:	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$	$y_{T+1}^{T+1+1}$	$y_{T+1}^{T+13}$	$y_{T+1}^{09Q1}$
Output growth	<b>0.956</b>	<b>0.959</b> <b>0.978</b>	<b>0.972</b> <b>0.975</b>	<b>0.965</b>	<b>0.964</b> 0.991	<b>0.971</b> 0.981	<b>0.958</b>	<b>0.961</b> 0.981	<b>0.974</b> <b>0.977</b>
Inflation (GDP defl.)	<b>0.948</b>	<b>0.938</b> 1.006	<b>0.920</b> 0.989	<b>0.965</b>	<b>0.962</b> 1.013	<b>0.948</b> 1.001	<b>0.959</b>	<b>0.969</b> 0.997	<b>0.953</b> 0.995
Inflation (PCE defl.)	1.002	0.993 1.013	0.993 1.017	0.992	0.983 0.997	0.984 1.002	<b>0.977</b>	<b>0.974</b> 0.985	<b>0.969</b> 0.984

Note: Entries with values smaller than one indicate that RTV reduces the RMSFE relative to EOS. Entries in the second line are for RTV bias-corrected forecasts (relative to EOS). The correction is based on the difference between the unconditional means of  $y_t^{t+12}$  and  $y_t^{t+1}$  with data up to the forecast origin. Emboldened entries indicate RMSFE reductions larger than 2%. Multiple step-ahead forecasts are computed by iteration. Models are estimated with increasing windows of data. In-sample period starts in 1959:Q1.

Table 7: Comparison of ADL models with activity variables (‘Phillips Curve models’) to AO for forecasting next year’s inflation rate: final and end-of-sample data. Actuals from the Final vintage (2009Q1).

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
Forecasting PCE inflation										
model	predictor	data	Recursive estimates				Rolling estimates			
AO	--	Final	1.699	0.886	0.669	0.887	1.699	0.886	0.669	0.887
		EOS	<b>1.679</b>	1.033	0.758	0.971	1.679	1.033	0.758	0.971
Best ADL	Output growth	Final	0.743	1.322	1.086	1.162	0.796	1.250	1.074	0.949
		EOS	<b>0.706</b>	<b>1.206</b>	<b>1.032</b>	<b>1.137</b>	<b>0.720</b>	1.272	<b>1.064</b>	0.957
Best ADL	Output gap	Final	0.901	1.149	0.875	1.206	0.930	1.089	1.026	0.985
		EOS	<b>0.790</b>	<b>1.103</b>	<b>0.824</b>	<b>1.168</b>	<b>0.769</b>	1.119	1.024	1.071
Best ADL	Unemp.	Final	0.862	1.110	0.918	1.207	0.920	1.189	0.976	1.052
		EOS	<b>0.859</b>	1.207	<b>0.917</b>	<b>1.150</b>	0.952	1.368	1.015	1.061
Best ADL	Employ.	Final	0.774	1.109	0.956	1.311	0.817	1.165	1.022	1.113
		EOS	0.781	<b>1.038</b>	0.985	<b>1.307</b>	<b>0.791</b>	<b>1.067</b>	1.085	1.154
Best ADL	Industrial production	Final	0.770	1.126	1.062	1.182	0.760	1.204	1.052	0.969
		EOS	0.791	1.164	<b>1.032</b>	1.182	0.810	1.266	1.075	1.005
Forecasting GDP inflation										
model	predictor	data	Recursive estimates				Rolling estimates			
AO	--	Final	1.506	0.764	0.383	0.560	1.506	0.764	0.383	0.560
		EOS	1.489	0.889	0.564	0.818	1.489	0.889	0.564	0.818
Best ADL	Output growth	Final	0.718	0.997	0.937	1.073	0.835	1.014	1.111	1.018
		EOS	0.744	0.999	<b>0.924</b>	1.076	<b>0.747</b>	1.033	<b>1.060</b>	1.052
Best ADL	Output gap	Final	0.955	1.064	0.872	1.234	0.989	0.928	1.275	1.103
		EOS	<b>0.795</b>	<b>0.995</b>	<b>0.790</b>	<b>1.131</b>	<b>0.760</b>	0.932	<b>1.054</b>	1.134
Best ADL	Unemp.	Final	0.908	0.913	1.294	1.345	0.956	1.095	1.105	1.089
		EOS	0.912	0.954	<b>1.064</b>	<b>1.135</b>	0.975	1.176	<b>1.096</b>	1.135
Best ADL	Employ.	Final	0.799	1.033	1.165	1.380	0.834	1.111	1.197	1.224
		EOS	<b>0.771</b>	<b>1.028</b>	<b>1.038</b>	1.396	<b>0.807</b>	1.079	<b>1.165</b>	1.315
Best ADL	Industrial production	Final	0.847	0.958	1.167	1.085	0.829	1.001	1.214	1.040
		EOS	<b>0.842</b>	1.020	<b>1.046</b>	1.122	0.888	1.054	<b>1.123</b>	1.116

Note: Entries for the benchmark models (AO) are RMSFEs. Remaining entries are Ratios of RMSFE with respect to the benchmark. Ratios are computed for the same choice of data (Final or EOS). Values in bold indicate that ratio with respect to AO is smaller with EOS data than with final data. In-sample period starts in 1959:Q1. Rolling estimation uses windows of 69 observations

Table 8: Comparison of ADL models with activity variables (‘Phillips Curve models’) to AO for forecasting next year’s inflation rate: end-of-sample and real-time vintage data.

8.A. Using recursive estimation

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
Forecasting PCE Deflator inflation										
model	predictor	data	Forecasting final vintage (2009Q1)				Forecasting first release data			
AO	--	EOS	1.679	1.033	0.758	0.971	1.610	1.202	0.710	0.965
Best ADL	Output growth	EOS	0.706	1.206	1.032	1.137	0.717	1.185	0.964	1.134
		RTV	0.707	1.236	1.036	<b>1.086</b>	<b>0.715</b>	1.217	1.047	<b>1.089</b>
		RTV <sub>v</sub>	0.749	1.296	1.023	1.121	0.738	1.272	1.038	1.112
Best ADL	Output gap	EOS	0.790	1.103	0.824	1.168	0.769	1.070	0.875	1.168
		RTV	<b>0.729</b>	1.205	0.871	1.188	0.773	1.124	0.972	1.201
		RTV <sub>v</sub>	0.807	1.178	0.861	1.168	0.851	1.129	0.912	1.168
Best ADL	Unemp.	EOS	0.859	1.207	0.917	1.150	0.832	1.146	0.958	1.177
		RTV	0.890	1.254	0.926	<b>1.120</b>	0.853	1.161	0.972	<b>1.156</b>
		RTV <sub>v</sub>	0.924	1.294	0.921	1.132	0.901	1.204	0.966	1.156
Best ADL	Employ.	EOS	0.781	1.038	0.985	1.307	0.762	1.043	1.017	1.253
		RTV	<b>0.778</b>	1.128	1.034	1.315	<b>0.758</b>	1.109	1.086	1.254
		RTV <sub>v</sub>	0.789	1.105	1.046	1.357	0.763	1.099	1.099	1.291
Best ADL	Indust. Prod.	EOS	0.791	1.164	1.032	1.182	0.814	1.132	1.033	1.166
		RTV	0.795	1.261	1.078	<b>1.153</b>	0.819	1.209	1.080	<b>1.135</b>
		RTV <sub>v</sub>	0.807	1.274	1.083	1.188	0.820	1.224	1.093	1.161
Forecasting GDP Deflator inflation										
			Forecasting final vintage (2009Q1)				Forecasting first release data			
AO	--	EOS	1.489	0.889	0.564	0.818	1.502	0.800	0.553	0.643
Best ADL	Output growth	EOS	0.744	0.999	0.924	1.076	0.758	0.999	1.011	1.089
		RTV	0.744	1.132	0.979	<b>1.044</b>	0.778	1.172	1.154	1.078
		RTV <sub>v</sub>	0.739	1.058	0.957	1.102	0.781	1.123	1.132	1.065
Best ADL	Output gap	EOS	0.795	0.995	0.790	1.131	0.752	0.954	0.905	1.125
		RTV	<b>0.770</b>	1.192	0.915	<b>1.115</b>	<b>0.743</b>	1.130	1.061	1.174
		RTV <sub>v</sub>	0.707	0.988	0.853	1.128	0.706	0.989	0.986	1.052
Best ADL	Unemp.	EOS	0.912	0.954	1.064	1.135	0.868	0.978	1.115	1.319
		RTV	0.947	0.999	<b>1.014</b>	<b>1.126</b>	0.885	1.064	1.100	1.326
		RTV <sub>v</sub>	0.988	0.971	0.947	1.129	0.937	1.060	1.040	1.242
Best ADL	Employ.	EOS	0.771	1.028	1.038	1.396	0.741	0.940	1.134	1.325
		RTV	0.780	1.137	1.136	1.417	0.746	1.065	1.268	1.354
		RTV <sub>v</sub>	0.776	1.072	1.155	1.527	0.739	1.009	1.293	1.435
Best ADL	Indust. Prod.	EOS	0.842	1.020	1.046	1.122	0.846	1.001	1.111	1.088
		RTV	0.863	1.110	1.116	1.126	0.881	1.121	1.218	1.103
		RTV <sub>v</sub>	0.859	1.061	1.124	1.188	0.880	1.098	1.235	1.103

8B. Using rolling estimation

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
Forecasting PCE inflation										
model	predictor	data	Forecasting final vintage (2009Q1)				Forecasting first release data			
AO	--	EOS	1.679	1.033	0.758	0.971	1.610	1.202	0.710	0.965
Best ADL	output growth	EOS	0.720	1.272	1.064	0.957	0.737	1.236	1.099	0.945
		RTV	0.722	1.341	1.075	<b>0.933</b>	0.741	1.311	1.094	<b>0.926</b>
		RTV <sub>v</sub>	0.773	1.312	1.082	0.995	0.766	1.291	1.105	0.970
Best ADL	output gap	EOS	0.769	1.119	1.024	1.071	0.784	1.108	1.078	1.051
		RTV	<b>0.742</b>	1.227	1.061	1.097	0.783	1.164	1.179	1.152
		RTV <sub>v</sub>	0.836	1.267	0.953	1.095	0.892	1.219	1.025	1.130
Best ADL	Unemp.	EOS	0.952	1.368	1.015	1.061	0.936	1.226	1.065	1.067
		RTV	0.965	1.417	1.053	1.061	0.958	1.253	1.085	<b>1.057</b>
		RTV <sub>v</sub>	1.017	1.360	1.030	1.127	0.998	1.262	1.066	1.117
Best ADL	Employ.	EOS	0.791	1.067	1.085	1.154	0.765	1.087	1.151	1.101
		RTV	0.828	1.169	1.077	1.194	0.798	1.159	<b>1.135</b>	1.137
		RTV <sub>v</sub>	0.819	1.102	1.121	1.290	0.784	1.120	1.178	1.222
Best ADL	Indust. Prod.	EOS	0.810	1.266	1.075	1.005	0.842	1.226	1.117	0.977
		RTV	0.863	1.363	<b>1.041</b>	1.014	0.902	1.302	<b>1.062</b>	0.987
		RTV <sub>v</sub>	0.849	1.357	1.097	1.050	0.872	1.301	1.119	1.038
Forecasting GDP deflator inflation										
			Forecasting final vintage (2009Q1)				Forecasting first release data			
AO	--	EOS	1.489	0.889	0.564	0.818	1.502	0.800	0.553	0.643
Best ADL	output growth	EOS	0.747	1.033	1.060	1.052	0.806	1.091	1.215	0.969
		RTV	0.747	1.215	1.007	<b>0.985</b>	0.820	1.242	<b>1.160</b>	<b>0.942</b>
		RTV <sub>v</sub>	0.779	1.098	1.061	1.076	0.831	1.165	1.183	0.962
Best ADL	output gap	EOS	0.760	0.932	1.054	1.134	0.711	0.952	1.223	1.054
		RTV	0.803	1.173	1.212	<b>1.091</b>	<b>0.769</b>	1.113	1.403	1.064
		RTV <sub>v</sub>	0.722	0.994	1.067	1.242	0.728	1.040	1.144	1.157
Best ADL	Unemp.	EOS	0.975	1.176	1.096	1.135	0.943	1.146	1.200	1.162
		RTV	0.985	<b>1.165</b>	1.005	<b>1.120</b>	0.956	1.130	<b>1.067</b>	1.193
		RTV <sub>v</sub>	1.006	1.005	0.989	1.200	0.984	1.009	1.053	1.203
Best ADL	Employ.	EOS	0.807	1.079	1.165	1.315	0.795	1.027	1.317	1.228
		RTV	0.825	1.211	<b>1.076</b>	<b>1.251</b>	0.798	1.128	<b>1.243</b>	<b>1.189</b>
		RTV <sub>v</sub>	0.853	1.128	1.160	1.416	0.827	1.057	1.303	1.303
Best ADL	Indust. Prod.	EOS	0.888	1.054	1.123	1.116	0.910	1.089	1.265	1.037
		RTV	0.943	1.143	<b>1.050</b>	<b>1.061</b>	0.960	1.165	<b>1.180</b>	<b>1.018</b>
		RTV <sub>v</sub>	0.917	1.089	1.115	1.161	0.939	1.149	1.243	1.042

Notes: Entries for the benchmark model (AO) are RMSFEs. Remaining entries are ratios of RMSFE with respect to the benchmark. Values in bold: RTV is more accurate than EOS. In-sample period starts in 1959:Q1. Rolling estimation uses windows of 69 observations.

Table 9: Comparing ADL models with indicator variables to AR(1) model direct forecasts of next year's output growth: final and end-of-sample data. Forecasting final vintage (2009Q1)

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
model	predictor	data	Using Recursive Estimates				Using Rolling Estimates			
AR(1)	--	Final	3.171	1.490	0.934	1.509	3.190	1.496	0.977	1.386
		EOS	3.024	1.377	1.014	1.568	3.071	1.501	1.259	1.453
Best ADL	Industrial Production	Final	1.003	1.001	0.999	0.994	1.001	1.013	1.030	0.965
		EOS	<b>0.995</b>	<b>0.999</b>	1.001	0.989	1.002	<b>0.997</b>	<b>1.024</b>	0.979
Best ADL	Employ.	Final	1.045	1.073	1.056	1.091	1.042	1.069	0.997	0.971
		EOS	<b>0.997</b>	<b>1.041</b>	<b>0.998</b>	1.098	<b>0.999</b>	1.091	1.001	<b>0.947</b>
Best ADL	Hours Manufac.	Final	1.005	1.036	1.050	1.029	1.004	1.025	1.003	1.004
		EOS	<b>0.976</b>	<b>1.033</b>	<b>1.020</b>	1.034	<b>0.979</b>	1.043	1.021	<b>0.976</b>
Best ADL	Housing Starts	Final	0.880	1.038	1.065	0.968	0.815	1.080	1.036	0.982
		EOS	0.907	1.104	<b>1.048</b>	<b>0.936</b>	0.859	1.159	<b>1.032</b>	<b>0.974</b>

Note: Entries for the benchmark models (AR(1)) are RMSFEs. Remaining entries are Ratios of RMSFE with respect to the benchmark. Ratios are computed for the same choice of data (Final or EOS). Values in bold indicate that ratio with respect to AR is smaller with EOS data than with final data. In-sample period starts in 1959:Q1. Rolling estimation uses windows of 69 observations.

Table 10: Comparing ADL models with indicator variables to AR(1) model direct forecasts of next year's output growth: end-of-sample and real-time vintage data.

10.A. Using recursive estimation

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
model	predictor	data	Forecasting final vintage (2009Q1)				Forecasting first release data			
AR(1)	--	EOS	3.024	1.377	1.014	1.568	2.787	1.519	0.951	1.380
		RTV	<b>0.989</b>	<b>0.987</b>	1.140	<b>0.860</b>	<b>0.981</b>	<b>0.927</b>	1.136	<b>0.920</b>
Best ADL	Industrial Production	EOS	0.995	0.999	1.001	0.989	0.996	0.992	1.007	0.994
		RTV	1.056	1.098	1.221	<b>0.983</b>	1.051	<b>0.920</b>	1.244	1.008
		RTV <sub>v</sub>	1.035	1.057	1.207	0.933	1.028	0.903	1.223	0.965
Best ADL	Employ.	EOS	0.997	1.041	0.998	1.098	0.985	1.014	1.019	1.072
		RTV	<b>0.981</b>	1.199	1.244	1.178	<b>0.969</b>	1.049	1.256	1.153
		RTV <sub>v</sub>	0.982	1.149	1.216	1.082	0.962	1.000	1.222	1.065
Best ADL	Hours Manufac.	EOS	0.976	1.033	1.020	1.034	0.961	0.992	1.021	1.028
		RTV	0.995	1.157	1.228	1.105	<b>0.940</b>	<b>0.988</b>	1.220	1.154
		RTV <sub>v</sub>	0.974	1.096	1.208	0.988	0.929	0.933	1.218	1.042
Best ADL	Housing Starts	EOS	0.907	1.104	1.048	0.936	0.948	1.051	1.081	0.872
		RTV	0.947	<b>1.076</b>	1.177	<b>0.754</b>	0.975	<b>0.962</b>	1.189	<b>0.729</b>
		RTV <sub>v</sub>	0.951	1.122	1.217	0.757	0.984	0.983	1.238	0.742

10.B. Using rolling estimation

			1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4	1977Q1 1984Q4	1985Q1 1992Q4	1993Q1 2000Q4	2001Q1 2008Q4
model	predictor	data	Forecasting final vintage (2009Q1)				Forecasting first release data			
AR(1)	--	EOS	3.071	1.501	1.259	1.453	2.780	1.384	1.169	1.349
		RTV	1.006	1.060	1.090	<b>0.940</b>	<b>0.994</b>	<b>0.921</b>	1.003	<b>0.997</b>
Best ADL	Industrial Production	EOS	1.002	0.997	1.024	0.979	1.006	1.003	1.027	1.037
		RTV	1.130	1.141	1.136	<b>0.943</b>	1.142	1.051	1.143	1.000
		RTV <sub>v</sub>	1.089	1.115	1.148	0.928	1.092	1.017	1.151	0.985
Best ADL	Employ.	EOS	0.999	1.091	1.001	0.947	0.996	1.010	1.023	1.007
		RTV	<b>0.979</b>	1.358	1.120	<b>0.931</b>	<b>0.974</b>	1.172	1.140	1.016
		RTV <sub>v</sub>	0.988	1.314	1.181	0.914	0.975	1.089	1.179	1.010
Best ADL	Hours Manufac.	EOS	0.979	1.043	1.021	0.976	0.957	1.018	1.050	1.006
		RTV	1.009	1.202	1.112	<b>0.964</b>	0.980	1.080	1.062	1.020
		RTV <sub>v</sub>	1.009	1.169	1.165	0.937	0.972	0.985	1.177	1.000
Best ADL	Housing Starts	EOS	0.859	1.159	1.032	0.974	0.910	1.127	1.056	0.960
		RTV	<b>0.840</b>	1.174	1.083	<b>0.927</b>	<b>0.883</b>	<b>1.007</b>	<b>1.042</b>	0.979
		RTV <sub>v</sub>	0.966	1.234	1.092	0.934	0.999	1.070	1.055	0.990

Notes; Entries for the benchmark model (AR(1) with EOS data) are RMSFEs. Remaining entries are Ratios of RMSFE with respect to the benchmark. Values in bold: RTV is more accurate than EOS. In-sample period starts in 1959:Q1. Rolling estimation uses windows of 69 observations