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Recent developments in quantitative models of sovereign default

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Abstract:

The current crisis and discussions, in the euro area in particular, show that sovereign debt crises/defaults are no longer restricted to developing economies. After crises in many Latin American countries, the literature on quantitative dynamic macro-models of sovereign default has been advancing. Current debate should take notice of the findings from this literature – an extensive overview of which has been provided in this paper. This paper also discusses the difficulties involved in, but also possibilities of, integrating this type of model in standard business cycle models (RBC and DSGE models). This is likely to be particularly helpful when using models to analyse upcoming issues in the euro area, such as a suitable (sovereign) insolvency law or the assumption of joint liability.

Keywords: Sovereign Debt, Default Risk, Endogenous Borrowing Constraints, Small Open Economy

JEL code: F34, F41, E21, E32, G10

Non-technical summary

The current crisis and discussions, in the euro area in particular, show that sovereign debt crises/defaults are no longer restricted to developing economies. After crises in many Latin American countries, the literature on quantitative dynamic macro-models of sovereign default has been advancing. Current debate should take notice of the findings from this literature – an extensive overview of which has been provided in this paper.

The literature on quantitative dynamic macro-models of sovereign default shows that (1) despite the absence of international insolvency legislation, sovereigns have an interest in repaying debt owing to reputational damage and negative output effects; (2) in the event of transitory shocks, debt level is the key factor determining the probability of default; in the event of trend shocks, the output level also plays a decisive role; (3) the political stability of a country reduces its probability of default; (4) bailouts or the possibility of borrowing from international financial institutions such as, for example, the IMF (can) increase the probability of default; (5) fiscal policy acts procyclically if it takes the probability of default and the impact on interest rates into account; (6) taking private sector debt into account does not change the above-mentioned qualitative statements but can increase a government's propensity to borrow (in relative terms); (7) endogenising negotiations about debt recovery rates tends to increase the interest rates on sovereign debt but a government's propensity to borrow can still rise; (8) it can be efficient for borrowers and creditors to delay negotiations over restructuring sovereign debt, although this is heavily dependent on the assumed bargaining game; (9) long-term bonds usually have a higher rate of interest than short-term ones; (10) contagion effects owing to risk-averse investors as well as information shocks affecting investors that impact on the interest rate can even cause fundamentally sound governments to default; and (11) the formal method of solving these models also plays a role in quantitative terms. All of these aspects and the relevant effects that lead to these conclusions are explained in detail in this paper.

Nicht-technische Zusammenfassung

Die gegenwärtige Krise und die Diskussionen insbesondere im Euroraum zeigen, dass Staatskrisen/-insolvenzen nicht mehr nur ein sich auf entwickelnde Ökonomien beschränktes Thema sind. Nach den Krisen vieler lateinamerikanischer Länder entwickelte sich die Literatur zum Sovereign Default in quantitativen dynamischen Makromodellen. Die aus diesem Literaturstrang gewonnenen Erkenntnisse – über die in diesem Papier ein ausführlicher Überblick gegeben wird – sollten auch in der gegenwärtigen Diskussion Beachtung finden.

In aller Kürze lässt sich aus der Literatur zum Sovereign Default in quantitativen dynamischen Makromodellen ableiten, dass 1.) Staaten trotz Nichtvorhandensein eines internationalen Insolvenzrechts wegen Reputationsverlusten und negativen Outputeffekten ein Interesse an Schuldenrückzahlung haben; 2.) bei transitorischen Schocks der Schuldenstand, bei Trendschocks auch das Outputniveau die determinierende Größe zur Bestimmung der Insolvenzwahrscheinlichkeit ist; 3.) politische Stabilität eines Landes die Insolvenzwahrscheinlichkeit negativ beeinflusst; 4.) Bailouts oder die Möglichkeit, sich bei internationalen Finanzinstitutionen (z.B. IWF) zu verschulden, die Insolvenzwahrscheinlichkeit erhöhen (können); 5.) Fiskalpolitik bei Beachtung der Insolvenzwahrscheinlichkeit und den Auswirkungen auf die Verzinsung prozyklisch agiert; 6.) die Berücksichtigung der Verschuldung des Privatsektors die zuvor genannten qualitativen Aussagen nicht verändern, sich aber die Verschuldungsneigung des Staates (relativ) erhöhen kann; 7.) die Endogenisierung der Verhandlungen über Rückerstattungsraten tendenziell die Verzinsung der Staatsschuld erhöht, aber trotzdem die Verschuldungsneigung des Staates steigen kann; 8.) es für Schuldner und Gläubiger effizient sein kann, Verhandlungen über Restrukturierung der Staatsschuld zu verzögern, dies aber stark vom unterstellten Verhandlungsspiel abhängig ist; 9.) langfristige Anleihen in der Regel höher verzinst werden als kurzfristige; 10.) Ansteckungseffekte aufgrund risikoaverser Investoren sowie den Zinssatz beeinflussende Informationsschocks auf Investorenseite auch zu Insolvenzen grundsätzlich solider Staaten führen können; und 11.) die formale Methode zur Lösung dieser Modelle quantitativ eine Rolle spielt. All diese Aspekte und die entsprechenden Effekte, die zu den Schlussfolgerungen führen, werden in diesem Papier ausführlicher erläutert.

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1 Introduction

The current crisis has reawakened the (public) interest in sovereign defaults in general – but also in stabilisation measures aimed at preventing the occurrence of such events in particular. Debate in the euro area shows that such issues are no longer restricted to developing countries only but are also becoming more relevant for developed economies. Topics such as contagion, suitable insolvency legislation or a workable – at least partial – assumption of joint liability, for example as part of the European Stability Mechanism (ESM), the European Financial Stability Facility (EFSF) or even Eurobonds, have made it on to the agenda (see Arghyrou and Kontonikas, 2011). Business cycle models (ie RBC or DSGE models) are generally a suitable theory-based analysis tool for evaluating/simulating various (fiscal) policies. Many major (international) institutions are also currently using them in this way to analyse a wide range of crisis-related issues.[2] However – owing to understandable technical reasons – they are rather tacit when it comes to issues related to sovereign defaults.

This paper provides an overview of recent developments in the literature on quantitative dynamic macro-models of sovereign default and a brief explanation of how this model class could be more closely integrated into standard RBC/DSGE models. Integrating sovereign default into such economic models is likely to prove quite helpful for a micro-based discussion of the issues mentioned above. However, the findings from quantitative models of sovereign default gained to date should also be considered in the current debate. To the author's knowledge, no extensive overview of this literature yet exists. The main section of this paper has deliberately been written intuitively to make the analysis of this model class accessible to non-theoreticians, too. A simple base model has been included, however.

The literature on quantitative dynamic macro-models of sovereign default primarily evolved in the wake of government crises in many Latin American countries over the last two decades. In the literature, a sovereign's decision about

[1] Author: Nikolai Stähler, Deutsche Bundesbank, email: nikolai.staehler@bundesbank.de. I would like to thank Gerrit Köster, Michael Krause, Maria Longson, Bernhard Manzke, Christoph Moser, Christoph Priesmeier and Karsten Wendorff for their helpful comments. The opinions expressed in this paper do not necessarily reflect the opinions of the Bundesbank or its staff. Any remaining errors are the author's alone.

[2] The following is a non-exhaustive list of fiscal policy issues that can be cited as examples: the effectiveness of fiscal stimulus programmes (see inter alia Almeida et al, 2010; Coenen et al, 2010a, 2010b; Hebous, 2010), a cost-benefit analysis of various consolidation measures (see inter alia Freedman, 2009; Stähler and Thomas, 2011) or an evaluation of structural changes to tax legislation (see inter alia Coenen et al, 2008).

whether to service its debt *or* default is endogenised. In a way, this means an analysis of the aspects of being "willing to repay debts" rather than those of being "able to repay debts". However, this should not be seen as a limitation. Many sovereign defaults were or are brought about by a loss of public support for measures to restructure the government budget that would probably enable debts to be repaid. Even in the current European context, this point is likely to be important.

The quantitative models of sovereign debt that are primarily examined in this paper endogenise the sovereign's default decision by comparing present value functions of whether to meet its payment obligations. Governments borrow from abroad. International investors fix interest rates based on their expectations of the government's repayment behaviour. The models are able to explain interest rate developments and sovereign default using a country's output fluctuations. However, in the majority of cases, the output process is assumed to be exogenous. Repercussions of interest rate developments on the productivity of the country in question are usually not shown endogenously. Another branch of the literature that endogenises the output process in the sense of RBC models but assumes interest rate developments to be exogenous shows that there is a negative correlation between the two quantities.[3] However, combining these two approaches may be a suitable way of addressing the shortcoming in each individual branch of the literature (see also Mendoza and Yue, 2010). We will revisit this point at the end of the paper once we have integrated the models into the general literature on sovereign defaults and summarised the present findings gained from the literature.

In brief, the literature on quantitative dynamic macro-models of sovereign default shows that (1) despite the absence of international insolvency legislation, sovereigns have an interest in repaying debt owing to reputational damage and negative output effects; (2) in the event of transitory shocks, debt level is the key factor determining the probability of default; in the event of trend shocks, the output level also plays a decisive role; (3) the political stability of a country reduces its probability of default; (4) bailouts or the possibility of borrowing from international financial institutions such as, for example, the IMF (can) increase the probability of default; (5) fiscal policy acts procyclically if it takes the probability of default and the impact on interest rates into account; (6) taking private sector debt into account does not change the above-mentioned qualitative statements but can increase a government's propensity to borrow (in relative terms); (7) endogenising negotiations about debt recovery rates tends to increase the interest rates on sovereign debt but a government's propensity to borrow can still rise; (8) it can be efficient for borrowers and creditors to delay negotiations over restructuring sovereign debt,

[3]This literature and the findings thereof are examined in depth in Mendoza (2006, 2010) and are therefore not described in detail in this paper.

although this is heavily dependent on the assumed bargaining game; (9) long-term bonds usually have a higher rate of interest than short-term ones; (10) contagion effects owing to risk-averse investors as well as information shocks affecting investors that impact on the interest rate can even cause fundamentally sound governments to default; and (11) the formal method of solving these models also plays a role in quantitative terms. All of these aspects and the relevant effects that lead to these conclusions are explained in detail below.

The rest of the paper is structured as follows. Section 2 integrates the model class we focus on here in the general literature on government debt. Section 3 presents the underlying articles that led to the development of newer quantitative models of sovereign default. Furthermore, it contains a stylised (and simplified) base model and describes the solution method. Section 4 describes recent developments in this literature. Section 5 outlines the approaches to improving the integration of this literature with RBC models. A summary can be found in Section 6.

2 Integrating quantitative models of sovereign default into the general literature on government debt

In order to be able to improve the understanding of the model class discussed below, it appears appropriate to first of all integrate it into the literature on government debt. It is particularly important to examine why governments borrow, what motivates them to repay their debt and which problems arise should they default.

2.1 Why governments borrow

In the political economy there is, in principle, little doubt that governments have a propensity to borrow (for more details, see Eslava, 2010). This is triggered by a certain degree of fiscal illusion or by “common pool” problems. In both cases there are clear points in favour of debt-financed spending for a certain group, where the financing costs are distributed over a larger base (the common pool) or put off until a future date (see, for example, von Hagen, 1992; and Velasco, 2000). Furthermore, a government uses debt financing inter alia to increase its chance of being reelected or – if it has reason to believe it will be voted out of office – to limit the new government’s discretionary leeway (Alesina and Tabellini, 1990). In conventional politico-economic (usually one-country) models, long-term equilibrium is established by corresponding changes on the capital market (for instance, risk premiums and a corresponding decline in private investment) and in fiscal policy (in some cases, rule-bound), which thus bring spending into line with potential revenue (see Woo, 2005; or Stähler, 2009). The exclusion of Ponzi games generally results in a mechanism that prevents bankruptcy.

The (New) Keynesian theory can (at least, if there is a prolonged sequence of unfavourable shocks) also be used to justify government borrowing as countercyclical fiscal policy – in particular when aided by automatic stabilisers – smoothes the consumption path, which is usually a welcome development from a welfare perspective (see, for example, Schmitt-Grohe and Uribe, 2007). However, in this theory, in periods of upswing debt should be repaid and replaced by a formation of assets, as is shown, for example, by Fatás and Mihov (2002), Galí and Perotti (2003) and Wyplosz (2006).

The literature discussed below is nevertheless based on this argument of government debt. In contrast to the literature mentioned above, quantitative models of sovereign default rationalise the reason why sovereigns default in such an environment – that is if, in terms of (discounted) welfare, the financing costs of government debt are higher than the fines for a defaulting sovereign. Thus these models examine the "willingness" rather than the "ability" to repay debts. It should be noted in this context that the model class discussed below is limited to real foreign debt. Aspects of domestic debt and hence the potential monetisation of government debt are not examined.[4]

2.2 Why governments service their debt

In contrast to insolvencies in the private sector, it is impossible – or at least greatly difficult – to force independent sovereigns to repay outstanding debts, especially to foreign creditors. According to politico-economic arguments, sovereigns are nevertheless keen to repay debts if no distinction can be drawn between domestic and foreign creditors and if the incumbent government can maximise the utility for domestic households or if it is concerned that there may be domestic unrest and/or it may be voted out of office (see Broner et al, 2006; or Bornsztein and Panizza, 2009). However, the models discussed below focus on foreign debt, thus leaving almost no place for these arguments.

The potential (partial) exclusion from the capital market as an important incentive to repay debts was identified at an early stage in the literature. It is assumed that foreign debt presents an opportunity for governments to insure households against fluctuations in output (and the resulting fluctuations in utility or production). Eaton and Gersovitz (1981) assume that debtor countries are excluded from the capital market if they do not service their debt. This is sufficient grounds to

[4]For information on these aspects, see the discussions in Sargent and Wallace (1981), Kocherlakota and Phelan (1999), Leeper and Yun (2006), and McCallum and Nelson (2006).

always service outstanding debt – at least up to a certain limit. This paper is discussed in greater detail in Section 3.1. Even if the extreme assumption of permanent exclusion and approval of other insurance options for sovereigns is relaxed, situations may arise in which a (partial) exclusion from the capital market represents a serious threat (see, for example, Kletzer and Wright, 2000; or Wright, 2002). However, from an empirical perspective, this threat does not appear to be the main incentive to service debts as any exclusion from the capital market (in particular, in the recent past) is of a rather limited duration and cyclical fluctuations on credit markets themselves seem to have a much larger impact on individual sovereigns' access to the capital market (see Sandleris et al, 2004; and Richmond and Dias, 2009). The increased price of borrowing options following a loss of reputation also seems to be rather short lived as corresponding risk premiums no longer show up in the statistics three years later (see Borensztein and Panizza, 2009).[5]

It seems plausible that a country's refusal to repay its debts could have (negative growth) consequences within that country, too. For example, domestic and foreign economic agents may see sovereign default as a sign of a country's (structural) situation being worse than anticipated, which could result in a correction of expectations about future fiscal policy, capital outflows, lower investment and potential banking crises (see, for example, Sandleris, 2008; Catão and Kapur, 2006; as well as Catão et al, 2007). For instance, Sturzenegger and Zettelmeyer (2007) and Borensztein and Panizza (2009) find relatively clear evidence of a statistical link between sovereign default and banking crises. Sturzenegger (2004), de Paoli et al (2006) and Panizza et al (2009) find a negative correlation between bankruptcy and growth, while Levy-Yeyati and Panizza (2006) cannot confirm such a finding. The causality here does not appear to be one hundred percent clear (for a discussion, see Panizza et al, 2009).

In summary it can be said that, from an empirical perspective, a sovereign's incentives to repay its debt are primarily explained by the domestic cost of sovereign default, while uncertainty about the form and the extent of debt restructuring can intensify the effects mentioned above (Panizza et al, 2009). The literature discussed below assumes a mix of domestic unrest and a temporary exclusion from

[5] If capital market effects play only a limited role in debt repayment, the debtor country may fear the imposition of sanctions by the creditor countries. These include political – or even military – pressure, imminent or actual seizure of assets as well as constraints on trade relations. While military actions were in fact carried out in the past, these do not seem likely today – at least in developed economies. Likewise, the threat of (sovereign) assets being seized is not expected – at least directly – to be a great incentive to repay debts (for a discussion of these aspects, see Mitchener and Weidenmier, 2005; and Tomz, 2007). However, a negative impact on trading has actually been identified – albeit not always for long periods of time (see Rose, 2005; Lanau, 2008; and Borensztein and Panizza, 2010). Whether constraints on trade relationships per se are a reason to repay debts remains to be seen. Yet a debtor country's export-oriented companies are especially affected, a fact which could also be included in the domestic effects discussed below.

the capital market following sovereign default.

2.3 Problems in the event of sovereign default

The fact that there are no legal provisions to force a country to repay outstanding government debt also means that there are no provisions on what to do should such a situation actually arise. In this case, an unequal distribution of rights between creditors and debtors can result in considerable inefficiencies. For the most part, current literature reduces this issue to collective action problems which can be summarised into three categories (for what follows, see Sturzenegger and Zettelmeyer, 2007; Bernhard and Kellermann, 2008; Berensmann and Herzberg, 2009; Rogoff and Zettelmeyer, 2002; and Roubini, 2002 for more details).

The first category is "holding out" for original claims (free-rider effects). It is very difficult to establish an evolved restructuring procedure that could be advantageous for a majority of creditors if a minority of creditors can circumvent the procedure and may thus not have to participate in the burden of restructuring. Instead the minority counts on full repayment of the claims once restructuring has been completed. This problem is intensified in situations where there is no perfect information and thus uncertainty about the behaviour of other creditors.

The second category is "rush to the exit". As soon as there is the risk of a debt crisis in a debtor country (even in the case of a short-term reconcilable liquidity crisis), there is a tendency for creditors to engage in disorderly and immediate ("panic") sales or redemptions of their claims. This can put a stop to new bond issues and inflate risk premiums and interest rates. Ultimately, it may even lead to a self-fulfilling liquidity and debt crisis.

The third category is "rush to the courthouse". If a debtor offers to restructure the debt burden, the creditor can refuse and instead try to sue to enforce its claims. The debt is then repaid from any assets that the debtor still has. This can result in a collective action problem if the debtor's assets are limited. The first creditor, whose legal claim is met in full, then leaves a smaller insolvency estate for any other remaining creditors. As soon as there is an indication of a debtor's insolvency, the courts may be stormed with every creditor insisting on re-payment in full.

Authors such as Roubini (2002) or Rogoff and Zettelmeyer (2002) have identified another key problem area for sovereign default on the part of the debtor, namely a potential "rush to default". This could be caused by a debtor country's potential immunity, in particular in the case of claims on physical assets by creditors. If costs of default for a debtor country are low from the start, then the moral hazard on the part of the debtor increases the incentive to de-fault opportunistically. Merely observing a "rush to default" in a debtor country often automatically leads to higher opportunity costs in the form of rising interest rates. Berensmann and Herzberg (2009), Häseler (2009) and Panizza et al (2009), to name but a few, describe and compare various existing proposals – culminating in an international insolvency law for states – to solve these problems. To date, with one exception, the literature outlined below refrains from analysing such rules. The author believes that there is great potential for extending the literature on this subject.

3 Base model of sovereign default

The literature on quantitative models of sovereign default discussed below analyses default in a micro-based model following Eaton and Gersovitz (1981) as well as Grossman and van Huyck (1988). The first part of this section outlines these two articles. In the second part we present a stylised quantitative (base) model and give a brief overview of the solution method used. The latter seems helpful to better understand the findings of the literature described in Section 4.

3.1 The foundations

The analysis by Eaton and Gersovitz (1981) and by Grossman and van Huyck (1988) can, in principle, be seen as a theoretical basis for the dynamic (stochastic) macro models that endeavour to portray government crises using numerical solutions. Sovereigns cannot be forced to repay outstanding government debt and there is no international insolvency law governing debt restructuring. Sovereign default is thus interpreted as a contingent claim. Both Eaton and Gersovitz (1981) and Grossman and van Huyck (1988) formalise the reasons why sovereigns usually choose to service their debt from a theoretical point of view. The models assume a sovereign that maximises households' intertemporal utility resulting from (the path of) consumption. Owing to budget constraints, the government has to finance public spending and interest on outstanding government debt with revenue and, possibly, borrowing (debt roll-over). Eaton and Gersovitz (1981) assume that debtor countries which default once are permanently excluded from the capital markets and thus cannot roll over debt in the future. As, due to its utility function, households have a preference for smoothing future consumption, this provides a strong incentive to service debts. By not servicing present government debts, the potential for current consumption can be increased. However, following a default, future consumption can no longer be "brought forward" and the effects of income fluctuations on consumption can no longer be smoothed.

When deciding whether to default, the sovereign compares the present value of the utility with and without the option of smoothing consumption. If the latter is higher, the outstanding debt is no longer serviced. Due to this (theoretical) possibility of lenders being left holding their claims, these lenders will (endogenously) set an upper credit ceiling depending on how high they expect the government's utility loss resulting from an exclusion from the credit market to be. The model's structure implies that debts are serviced in a state of equilibrium. Grossman and Huyck (1988) relax the assumption of a permanent exclusion from the capital markets and assume that the exclusion is only temporary which, as we explained in Section 2.2, is a more realistic scenario. They show that a sovereign may default in a state of equilibrium. Because in this model lenders form expectations about the trustworthiness of a sovereign and also differentiate between excusable default and unjustifiable repudiation, the sovereign in question is keen to maintain a good reputation and to restrict its non-servicing of debts to "bad times" only. Lenders consider a sovereign to be trustworthy if it borrows only what it can pay back in "normal times" (the debt level can be calculated in the model). If a sovereign still defaults in normal or even in good times, then it loses its trustworthy reputation and the costs of default rise relatively sharply or lenders are no longer willing to provide loans. This also means that in this model (1) defaults in a state of equilibrium occur only in "bad times" and (2) debt is not built up in excess of that which can be paid back in normal times. Furthermore, default is usually only partial. From a lender's perspective, only the worst state of the world (ie the lowest level of productivity possible in the model) can produce total default.

3.2 A base model: Setup and solution method

Model setup and equilibrium

The literature generally considers a small open economy that receives a stochastic stream of income, y. The sovereign trades bonds, d, with risk neutral competitive foreign investors at price $q(d, y)$, but debt contracts are not enforceable. Every period, the sovereign is, therefore, in one of two states: default or non-default, D and N, respectively. Its default decision can be represented by

$$V(d, y) = \max_{D} \left\{ V^D(y), V^N(d, y) \right\}, \tag{1}$$

where D is the binary choice to default ($D = 1$) or not ($D = 0$), output y follows a stochastic process explained below and d denotes the asset stock of the sovereign. Hence, for an indebted country it holds that $d < 0$. The sovereign maximises the economy's utility by choosing whether to default or not. Default, $D = 1$ is chosen only if $V^D > V^N$, where

$$V^D(y) = u(y^{def}) + \beta \int_{y'} \left[\theta V(0, y') + (1 - \theta) V^D(y') \right] f(y', y) dy \tag{2}$$

is the present value function of being in the default state, while

$$V^N(d,y) = \max_{d'} \left\{ u\left(y - q(d',y)d' + d\right) + \beta \int_{y'} V(d',y')f(y',y)dy \right\} \tag{3}$$

is the present value function of being in the non-default state. A prime indicates (expected) next period's values of the corresponding variable. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 0$, is a CRRA per-period utility function of consumption, c, β is the discount factor and θ the exogenously determined probability of re-entry to the credit market when in the default state.

In equation (2), it is assumed that $c = y^{def}$, ie households consume all income y^{def} available in default states, with

$$y^{def} = \begin{cases} y & \text{if } y < \bar{y} \\ \hat{y} = \psi \cdot \bar{y} & \text{if } y > \bar{y} \end{cases},$$

where $\psi \in (0,1)$ are output costs of default and $\bar{y} = E\{y\}$ is expected output. This implies that, in autarky (ie when the country is excluded from the capital market), the output process is truncated because it is assumed that default entails some direct output cost. The main motivation for doing this is to bring the default probability implied by the model in line with the data (see Arellano, 2008, and section 2.2 for a discussion).

In equation (3), ie in the non-default state, $c = y - q(d',y)d' + d$. The sovereign can borrow from international investors by selling one-period bonds d' at price $q(d',y)$, which is determined below, and promises to repay this by giving up one unit of consumption in the next period. Hence, d is the repayment due to previous borrowing (remember that $d < 0$ for indebted countries). Note that the lower is the price $q(d',y)$, the higher is the interest on debt.

Creditors are generally assumed to be risk-neutral. Whenever they lend to the sovereign in the current period, they buy bonds d' at price $q(d',y)$ from the government. Next period period, the creditors may receive the face value of the bond depending on whether the government defaults or not. $\delta(d',y) = Pr\left[V^D(y) > V^N(d,y)\right]$ is the default probability. Therefore, with this probability, the creditors get nothing, while, with probability $1 - \delta(d',y)$, they get the face value d'. Assuming a risk-free interest rate r, free market entry and zero-profit on the creditor' side, the bond price thus equals

$$q(d',y) = \frac{1 - E\{\delta(d',y)\}}{1+r}. \tag{4}$$

We see that it depends on the default probability, $\delta(d',y)$, the current output state y, current asset holding in the economy d as well as the amount of new bonds d'

sold by the government.

These equations are sufficient to characterize a simple default model. The *recursive equilibrium* of this economy is characterized by a set of value functions $\{V(d,y), V^D(y), V^N(d,y)\}$, a set of policy functions for households' consumption $c(d,y)$ as well as policy functions for the government's default decision D and optimal asset holdings d' such that

a. Taking as given the government policies, households' consumption $c(d,y)$ satisfies the budget constraints (ie $c = y^{def}$ in autarky and $c = y - q(d', y)d' + d$ otherwise).

b. Taking as given the bond price function $q(d'y)$, the governments policy functions d' (ie the desired amount of new borrowing per period) and default sets D satisfy the government's optimisation problem.

c. Bond prices $q(d'y)$ reflect the government's default probabilities and are consistent with the creditors' expected zero profit condition.

Numerical solution

The model is then solved numerically. Parameters are generally based on existing literature or calibrated to match specific facts. The output process is usually estimated according to the economy to be matched. Here, we assume that it is given by $y = exp(z)$, where $z_t = \rho z_{t-1} + \epsilon_t$ and $\epsilon_t \sim N(0, \sigma_z^2)$. Our parameter choices (and the corresponding sources) are given in Table 1.

Table 1: Parameter values

What	Symbol	Value	Source
Risk aversion	σ	2.000	Arellano (2008)
Discount factor	β	0.953	Arellano (2008)
Risk-free interest rate	r	0.017	Arellano (2008)
Re-entry probability	θ	0.252	Cuadra and Sapriza (2008)
Default output costs	ψ	0.950	Arellano (2008)
Output shock	ρ	0.850	Cuadra and Sapriza (2008)
	σ_z^2	0.025	Cuadra and Sapriza (2008)

Note: Values are taken from the literature without performing an own empirical analysis because the presented model is meant as an illustrative example and not to replicate actual/specific data.

Because the sovereign uses present value functions to calculate the utility of not-servicing or servicing its debt (which are generally utility functions with non-linear equations) and one has to calculate the default probability endogenously, the solution of this maximisation problem also has to use non-linear techniques. The standard solution method used is the *discrete space technique*. It works as follows. After having defined the functional forms as well as the parameters, we need to define a state space for the sovereign's assets/debt with a lower and an upper bound. We define these bounds to be $\underline{d} = -0.25$ and $\bar{d} = 0.05$. A certain number of evenly spaced grid points are then taken from this space to be used as a proxy for all possible debt levels. As described in the literature overview below, it can be shown that this number of grid points has to be relatively large to avoid incorrect interest rate movements in the model.

In addition, a discrete state space has to be generated for output. We also need a matrix for transitional probabilities. For this purpose, it is usually assumed – as we did, too – that output follows an autoregressive process and a shock can move this level up or down. This shock, originating from a certain distribution function, can be converted to an n-state Markov chain using a quadrature-based method (see Tauchen and Hussey, 1991). This Markov chain (with dimension $n \times n$) can be interpreted as a "*matrix of transitional probabilities*". At this juncture it should be noted again that if a sovereign defaults, output costs occur which lower the level described here further. For the illustrative example model here, we assume 22 output states and 200 asset states. Once these preparations have been made, the numerical solution of the model can begin.

In order to find the equilibrium allocations and the bond price schedule for debt, the following algorithm is generally used:

1. Discretise the state space as described above.
2. Start with a guess for the bond price schedule such that $q^0(d, y) = 1/(1+r)$ for all d and y.
3. Use $q^0(d, y)$ to solve the sovereign's problem recursively using value function iteration by taking the assumed price schedule for bonds as given and obtain the optimal policy functions for consumption $c(d, y)$, asset holdings $d'(d, y)$ and default sets D.
4. Given these policy functions, compute the probability of default $\delta(d', y)$.
5. Update the price of bonds using equation (4).
6. Use the updated price of the bond $q^1(d, y)$ to repeat steps 2 to 5 until the convergence criterion, $\max\{q^0(d, y) - q^1(d, y)\} < \Delta$, where Δ is a (very) small number, is met.

It is obvious that this solution method can be rather time-consuming and computationally intensive even in small models. Given 22 output and 200 asset states, there are already 4,400 states/combinations for which the solution just described has to solve the model. Hence, models with many state spaces as well as larger models may stretch even modern computers to their limits. We also see that linear-approximative solutions, which are commonly used in larger (ultimately linearised) DSGE models, cannot be applied here. Linearisation means that they are not precise enough to calculate risk premiums effectively. Therefore, the main reasons why this idea has not yet been included in larger models are likely to be the inaccuracy of linear solution methods as well as, in many cases, the complexity of many of the DSGE models used.[6] As will be seen at the end of this paper, this should not stand in the way of continuing to develop the literature described below as it has great potential to provide answers to future questions.

Results

Merely all papers on this issue present the resulting bond price schedule as a function of assets for two values of the productivity shock (high and low), which we also do in Figure 1.

As the figure shows, the bond price is an increasing function of foreign assets, which can be explained as follows. It is evident from the model description that the utility for the representative household increases as consumption rises. If the sovereign repays debt, the utility for the representative household falls with outstanding government debt *ceteris paribus*. If the sovereign decides against repaying outstanding debt, then the utility for the representative household in that period increases. Hence, if investors were then to buy more sovereign bonds (without considering the default risk), this would be an optimal time for the government to default in any period (see also Grossman and van Huyck, 1988). Of course, this does not happen. Thus it is assumed that a sovereign default entails costs for the government: (1.) Access to the capital markets is temporarily lost, meaning that the possibility of consumption smoothing is limited. In addition, it is assumed that if a sovereign defaults there will (2.) also be unrest within the country which would lower productivity and therefore income. Just how long the government is

[6] At this juncture it appears appropriate to note that there are now alternative approaches to the endogenisation of risk premiums in larger dynamic macro models. However, these are less concerned with the sovereign's decision that it is not willing to repay debt but assume that creditors believe that governments are no longer able to repay debt as of a certain debt ratio. As of this debt level, households are no longer willing to purchase government debt. The key word here is fiscal limits. Basically in this branch of the literature it is assumed that the sovereign (credibly) defines a certain path of primary surpluses which then calculates the default probability to ensure that the transversality condition continues to hold. The default probability then generally determines how much of the outstanding government debt is repaid and can also be interpreted as an "endogenous haircut". A fiscal rule still ensures that long-term equilibrium is guaranteed, but now also taking the default probability into consideration. As this model class is not the main focus of the present paper, see Bi et al (2010) or Juessen et al (2010) for more details.

Figure 1: Bond price schedule

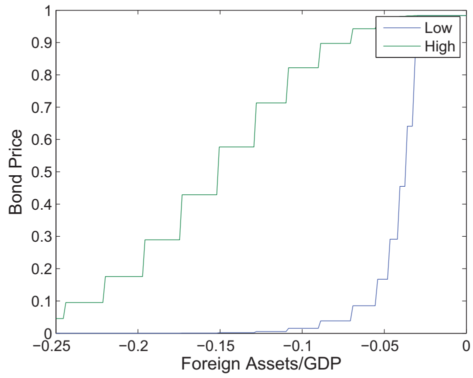
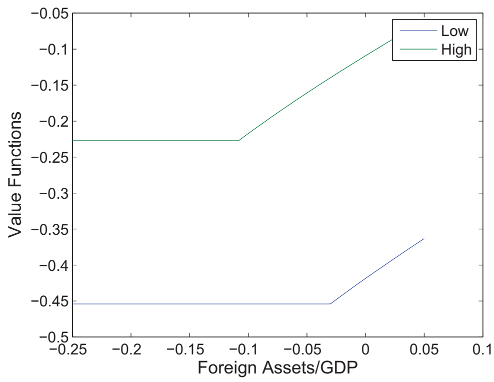


Figure 2: Value function



excluded from the international capital markets is shown by an exogenous probability of regaining access.

The sovereign therefore has to make the following decision. If it pays down the debt, it lowers households' utility today but can continue to borrow and, in general, can expect higher output. If it chooses not to pay, it has to cope with productivity losses and a delay in smoothing consumption owing to exclusion from the capital market. The sovereign compares the expected present value of these two options in that period and decides whether to default. Figure 2 shows the value of the option to default or repay as a function of assets-to-GDP for a high and a low productivity level. We observe that, for a given output realisation, default is chosen for all levels of assets below a certain threshold, ie when the value function become horizontal. In this case, the autarky is better than staying in the contract because financing costs become too high. When setting prices for bonds, international investors take the sovereign's decision into consideration. The more likely they believe the sovereign is to default on its debt, the less they are prepared to pay for a bond. Hence, as is shown, the price of bonds falls (ie the interest payments on government debt rise) the larger the outstanding government debt and the lower the (expected) productivity of the country in question. In extreme cases, the price may fall to zero (and interest payments thus rise infinitely). This would mean that no-one wants to buy sovereign bonds of the country in question.

Furthermore, we can present some simulation results and the statistical properties of the model. In Table 2, we show the business cycle moments of selected macroeconomic variables for the simulated economy. These statistics are average values over 1000 simulations of 100 realizations each, drawn from a stationary distribution. The simulated series are logged and filtered. Figure 3 shows how the interest rate spread increases as output contracts in the economy.

Table 2: Simulation results

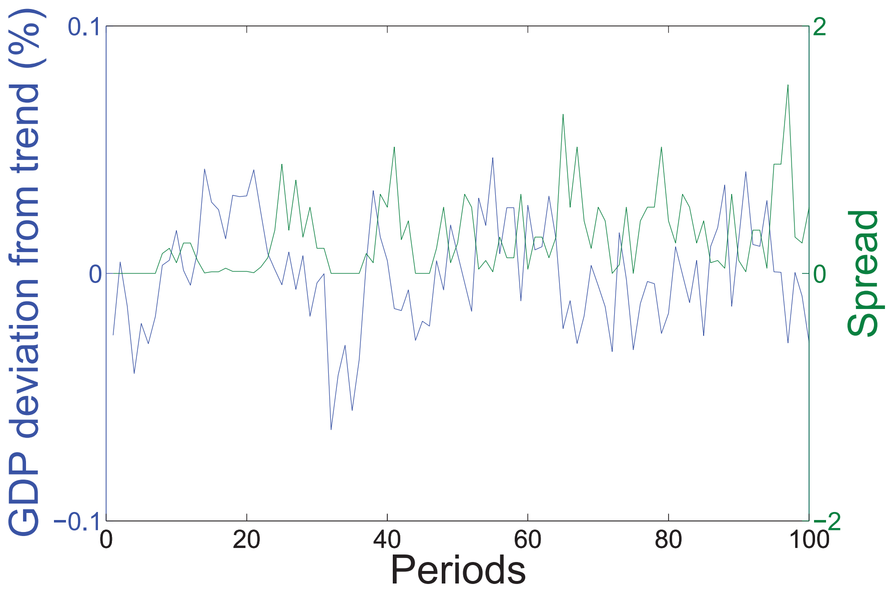
	Argentinean data	Arellano (2008)	Model results
Mean bond spread	10.25	3.58	3.65
Default probability	3.00	3.00	3.55
Mean debt-to-GDP ratio	48.79	5.95	4.70
Correlation (Spread-GDP)	-0.88	-0.29	-0.20
Correlation (Trade balance-GDP)	-0.64	-0.25	-0.32
Correlation (Consumption-GDP)	0.98	0.97	0.88

Note: The table shows selected simulated moments of the models and compares them to the "base model" of Arellano (2008) as well as Argentinean data. The latter is obtained from Cuadra and Sapriza (2008).

We see that this model class is able to match several stylised facts that can be observed in the data (at least in developing countries). These facts include, among others, that incentives to default are higher for more indebted countries and default risk and interest rate spreads move countercyclical (see Figures 1, 2 and 3). It can also be seen that private consumption is positively correlated with output, while the trade balance is negatively correlated (see Table 2). However, we also observe some drawbacks of this model class. For example, they have difficulties replicating the magnitude of the mean bond spreads and fare quite bad in replicating realistically high debt-to-GDP ratios. In order to show that this is not a feature of the simple model at hand, we also present the results of Arellano (2008) to compare our model results with hers. The data for Argentina is taken from Cuadra and Sapriza (2008). Despite those drawbacks, this class of models is, nevertheless, able to qualitatively fit the data as well as highlight some interesting aspects and mechanisms.

In the next section, we will describe recent developments in the literature of this model class in more detail. We will primarily focus on the qualitative findings of the literature and point out where there may be options for extensions.

Figure 3: Dynamics of GDP and spread



Notes: GDP is shown in percentage deviations from its trend. The spread is defined as $s = 1/q(d', y) - 1 - r$.

4 Developments in quantitative models of sovereign default

4.1 The base models

To the author's knowledge, Alfaro and Kanczuk (2005) were the first to integrate the purely theoretical studies on sovereign default into a quantitative model.[7] In their model, it can be seen that, in addition to exclusion from the capital markets, following a sovereign default in the country in question further (negative) output effects themselves are a necessary condition for generating equilibria that contain both repayment and default situations at debt ratios that are seen as being more realistic (yet constant). The model presented in Alfaro and Kanczuk (2005) contains a "good" and a "bad" sovereign. A "good" sovereign is one that attributes relatively high importance to future consumption (and discounts it at the same rate as households), whereas a "bad" sovereign is one that attributes a great deal of importance to current consumption and in the model always decides not to service its debt. After a default, lenders cannot verify which category the sovereign falls into and have to form expectations as to whether the sovereign is good or bad when purchasing

[7]Of course these issues were also pursued non-quantitatively (see, for example, Bulow and Rogoff, 1989, as well as Cole et al, 1995, and Cole and Kehoe, 1998, 2000 who also include the possibility of sunspot equilibria). However, as this paper focuses on newer (quantitative) models, we do not discuss this point in detail.

and pricing new government bonds. Based on this assumption, the model is able to explain an empirical observation (at least as far as Latin American countries are concerned) as to why sovereigns do not default until they have experienced a series of negative shocks (ie not until after a sharp recession). A good sovereign uses a delay of this kind to signal its "goodness" and thus to obtain better conditions on the capital markets in the future. In other words, the decision to default is not made until it is clear that there is no alternative (the authors call this phenomenon "muddling-through equilibrium"). To use Grossman and Huyck's (1988) terminology, one would speak of a (very) excusable default. Aided by numerical analysis, the authors also demonstrate that welfare is higher in all equilibria in which there is no default in any productivity state (measured as consumption equivalents as in Lucas, 1987).[8]

Alfaro and Kanczuk's (2005) model is relatively simple in the sense that it envisages only three productivity states, that the transitional matrix for these states is stipulated exogenously (ie it is not based on a "full stochastic" distribution function) and that the (equilibrium) debt level is assumed to be exogenous and constant (the sovereign thus decides only whether to pay interest or not). By contrast, Cuadra and Sapriza (2008) endogenise government debt and assume that output follows a stochastic, normally distributed process. Furthermore, they assume that the economy under consideration has two different types of household, each of which derives utility from different (public) goods. These two types of household are represented by two different political parties (for instance, left-wing or conservative governments) which – when in office – give preferential treatment to their own supporters. Unlike in the literature discussed in Section 2.1, both parties are not in power at the same time. One party governs but is at risk of being replaced by the opposition. Should this happen, the incumbent party becomes the opposition and the other party takes office. The transition probability is set exogenously and thus closely corresponds to Alfaro and Kanczuk's (2005) probability of being a good or bad sovereign. As the political parties have different preferences regarding the structure of public goods and as they know that this composition will change as soon as the other party takes office, the incentive for the incumbent party to give preferential treatment to its supporters and to finance this by borrowing increases. From the incumbent government's perspective, the costs of sovereign default fall in relative terms as it is fairly certain that they will have to be borne by the other party (one can speak of a quasi-finite time horizon). This model structure improves the models' quantitative fit with the data and explains the interaction between po-

[8]Here, consumption equivalents as defined by Lucas (1987) would be interpreted as the percentage of steady-state consumption that households are prepared to surrender to enter a non-default equilibrium.

litical risk, government debt and interest rate spreads. Hatchondo et al (2009), who differentiate between patient and impatient governments but not between public goods themselves, and D'Erasmo (2010), who additionally integrates debt renegotiation explicitly, come to similar conclusions. It can be demonstrated empirically, for example by Moser (2007), that political risk can actually impact on bond spreads and default probabilities.

Aguiar and Gopinath (2006) use a similar model, however without distinguishing between types of sovereign, to isolate transitory shocks from shocks to trend growth. They show that if transitory shocks exist, it is primarily the debt level which determines whether a sovereign defaults, and that, in model simulations, output shocks at a given (relatively low original) debt level result in only a very low default rate. As outlined above, sovereigns decide to default if the discounted utility of the capital market autarky (ie following partial exclusion from the capital markets) is larger than the discounted utility of servicing debts. The difference between these two discounted utilities is comparatively constant in the case of transitory shocks (as such a shock has roughly the same effect on both utility functions), making the debt level the driving factor. However, the presence of trend shocks causes a greater response by the values of both discounted utilities as well as the difference between them so that, in this scenario, shocks have a much larger impact. This implies that, given the same initial level of debt, the insolvency rate is affected perceptibly more after a trend shock than compared to transitory one. A positive trend shock means that income rises today but rises even further tomorrow. By making it easier to access government debt, tomorrow's consumption can be brought forward to the present and the discounted utility of servicing debt thus rises at a relatively faster pace than in capital market autarky. The opposite is true in the case of negative trend shocks. Hence a trend shock has a much greater impact on the propensity to default than a transitory shock. From this we can derive that (regular) economic fluctuations should have virtually no impact on a sovereign's default decision, whereas (long-term) growth prospects are quite relevant.

An interesting aspect in this analysis is the finding that bail-outs increase the probability of default significantly. Aguiar and Gopinath (2006) model bailouts as a transfer from a (unmodelled) third party to the country in question under the assumption that this third party guarantees part of the outstanding debt should the sovereign default. Lenders can thus purchase sovereign bonds up to this limit without incurring default risk as only debt in excess of this limit is subject to the risk of losses. This lowers the average interest rate on outstanding government debt and increases the incentive to borrow excessively because financing costs decrease. In comparison to a situation without bailouts, if corresponding output shocks exist, the risk of a country defaulting rises. This is a very simplified description of bailouts, which are usually ad hoc measures and not initially integrated into the system (see also Section 2.3). However, this could form the basis for integrating the analysis into a more realistic environment and also for modelling the third (bailout) party; for this, see also Boz (2011) and Roch and Uhlig (2011), which we will discuss in more detail below.

Arellano (2008) – whose working paper version of the article published in the *American Economic Review* in 2008 can probably be viewed as the basis for the other literature on this type of model (some of which is already discussed above) – shows that the model is able to replicate (most) economic facts and the scenarios that led to the crisis in Argentina rather well. Arellano (2008) illustrates another important aspect of this type of model. Since lenders take default probabilities into consideration, the interest rate they require rises, *ceteris paribus*, along with the level of public debt. This creates an endogenous borrowing limit, which the sovereign (voluntarily) does not exceed. In principle, the government would have an interest in increasing its borrowing so as to raise the current level of consumption, and would be able to do so (subject to rising interest rates). However, the additional credit available on the market is comparatively expensive (the price at which the sovereign can sell its bonds is low, hence the interest rate is high), meaning that additional current consumption can be funded only through very high losses in future consumption (price and volume effects moving in the opposite direction). Consequently, the government will not wish to borrow above the endogenous borrowing limit; this is essentially a kind of "Laffer curve" effect.

In a model where investors additionally obtain information on the future output situation of an economy, Durdu et al (2010) show that sovereign defaults can also occur in good times. This happens when investors view negative information obtained about an economy as credible and therefore raise the interest rates on government bonds, which means that it is no longer worthwhile for the government to service its debt. The information is modelled as a shock process that is correlated with the output process. Since this correlation is imperfect, however, the information obtained by investors does not necessarily reflect actual developments. Durdu et al (2010) show that a corresponding calibration of the shock process for information can be used to replicate spread developments in industrial countries.

4.2 Risk aversion and contagion

Arellano (2008) also shows that more realistic interest rate spreads can be generated within the model framework if risk-neutral international investors include a stochastic discount factor in their calculations of the bond price, flexibly interpreted as somewhat similar to stochastic news shocks. However, this could also be derived endogenously in the case of risk-averse investors. Lizarazo (2010) explicitly models these risk-averse investors using a framework that is otherwise largely based on the model in Arellano (2008); as she illustrates, this allows realistic debt ratios to be calibrated much more easily than in the standard model and her model can better replicate the movements in yield spreads. In this framework, investors can decide whether to invest in T-Bills (which are assumed to be risk free) or in risky government bonds. The bond price (ie the interest rate that a sovereign has to pay on its debt) now no longer depends solely on the economy's fundamentals but also on the financial wealth and risk aversion of the lender. This implies that the bond price can now be broken down into two components: a base premium that compensates the investors for the actual probability of default and the associated loss of income, and an *excess premium* that compensates them for taking the risk of default in the first place. The wealthier the investors, or the lower their risk aversion, the cheaper it is for a sovereign to borrow, because the excess premium, at least, falls.

Lizarazo (2009) explicitly integrates contagion of government crises from one country to others based on the model in Lizarazo (2010). This is not possible in Arellano's (2008) base model, which replicates risk-neutral investors. If investors are assumed to be risk-averse, however, it is. In Lizarazo (2009), it is assumed that a (representative) risk-averse investor can purchase the bonds of several countries. The wealth of the investor now depends on the country composition of his bond portfolio and the respective default probabilities. If one of the countries defaults, the investor's wealth decreases. If he is risk-averse, this will reduce his willingness to expose himself to other risky investments and, all other things being equal, he will invest more heavily in riskless T-Bills. A country whose fundamentals are essentially unchanged must now, ceteris paribus, pay a higher risk premium (excess premium) as a result of the reduced wealth of the investor in order to convince him to purchase its bonds. A sovereign default (or an elevated risk thereof) in one country thus diminishes other countries' borrowing possibilities – even if the financial situation of these countries can essentially be regarded as secure/sound. Lizarazo (2009) describes this as the "income" effect. Higher funding burdens in one country can thus also increase other countries' default probabilities. However, a substitution effect also occurs. The investor wishes to restructure his portfolio because of the income effect. He can do this both via riskless (but relatively low-yield) T-Bills and the (slightly higher-yield) government bonds of other countries whose financial situation he considers to be "sound enough". Depending on his degree of risk aversion, he will hence wish to shift his bond portfolio away from "unsound" and towards "sound enough" countries, leading to a disproportionately sharp rise in the financing burdens of "unsound" countries. Their default probabilities then increase as a result of this additional burden.[9] For countries with fundamentals that are strong (or regarded as such by the investor), the restructuring of the portfolio (ie the substitution effect) can then trigger a fall in their financing burden, thus enhancing their borrowing possibilities. This effect could also be described as a *flight to quality*. Which of the effects actually occurs depends to a large extent on the parameters used in the model. It is not possible to formally derive reliable theoretical relationships governing the threshold above which an investor considers a country to be sound or how strongly this and the other results are affected by increased risk aversion among investors (see also Lizarazo, 2010). In principle, however, it can be assumed that the substitution effect (in addition to the income effect) has a greater negative impact on a country's financial burdens the worse the country's fundamentals, the greater the investor's risk aversion, the lower his wealth and the more exposed his (existing) portfolio is to the "problem country" (which may choose/chooses to default) and to other "unsound" countries. Simulations show that the model can successfully replicate the contagion effects of the Argentine crisis on Uruguay.

4.3 More complex fiscal policies

Cuadra et al (2010) integrate more complex production and fiscal sectors into their model. They assume that households derive utility from private and public consumption goods and, at the same time, make a consumption-leisure choice, and that output is produced using labour. The state generates revenue by taxing consumption, sets the level of government expenditure and (new) borrowing and decides whether or not to default. This significantly extends the sovereign's room for manoeuvre, and its decisions have more complex repercussions on the rest of the economy. With respect to the default decision, bond prices and risk spreads, the model can replicate the stylised facts just as well as the aforementioned models. Additionally, fiscal policy can be shown to act procyclically in this kind of model framework (fiscal agents raise expenditure and cut taxes in good times). This is, at least for developing countries, a stylised fact, but it tends to be counterproductive from the point of view of wealth and consumption smoothing. During recessions, a government's debt financing conditions deteriorate owing to a higher risk of default and the associated rise in risk premiums. As households derive utility from the provision of public goods, the government then increasingly finances this expenditure via taxes, since debt financing is now (relatively) more expensive. How-

[9] This is also due to the fact that – as described in the base model (Lizarazo, 2010) – the incentives of a country with low output and/or a high level of debt to default rise more sharply than that of a higher-output, lower-debt country because the utility function of the private sector is concave. This behaviour is therefore rational from the investor's point of view.

ever, higher taxes reduce the consumption of private goods (which causes utility losses) and lead, at the same time, to lower output. This is because higher taxes also induce house-holds to reduce their supply of labour due to the consumption-leisure choice (owing to the fact that the relative price ratio of consumption and leisure must correspond to the ratio of the marginal utilities). To (partly) compensate for the negative labour incentive, the government now reduces expenditure during the recession, too, so that taxes do not have to be raised as sharply.

In a two-period model, Cuadra et al (2010) additionally investigate whether the results change if it is assumed that the private sector – in this case households – can likewise borrow on foreign credit markets and choose to default. When setting the interest rate on government bonds, international investors now take account of both public and private sector debt. The same applies to interest rates on private debt instruments. The conclusions of the models in which only the sovereign can become indebted are, it transpires, essentially confirmed. However, the government's propensity to borrow can increase. If the economy is hit by a series of adverse shocks, it becomes more difficult for the sovereign to borrow because the default probability for the private sector (and thus the overall probability of default) also rises. As outlined above, this increases the government's incentive to raise taxes. As the private sector can now, in principle, borrow abroad and can therefore carry out consumption smoothing itself, this incentive is actually strengthened. On the other hand, the private sector faces similar funding constraints as the government (or even has to pay higher risk premiums), which means that, for the private sector, borrowing is not necessarily the best instrument for smoothing consumption. A government seeking to maximise household utility may be inclined to raise taxes less sharply and take on more debt. The model's simulations show that the magnitude of the economic (utility) loss that a country faces following a default depends on whether it is the private sector, the government or both that decide not to fulfil the credit obligations. This initial – comparatively simple and rather stylised – step towards integrating the interactions between private and public sector debt into dynamic macro models could form the basis for more detailed analyses.

4.4 Negotiating to regain capital market access

Yue (2010) and D'Erasmo (2010) endogenise the probability of a country regaining access to the capital markets following a default – something which had previously been regarded as exogenous. They assume that a country explicitly negotiates its re-entry into the capital markets and debt repayment. This takes the form of a (one-shot) Nash bargaining game, in which it is assumed that the debtor and the creditor negotiate how much of the outstanding debt is to be repaid. If this debt renegotiation fails, the creditor gets nothing and the country remains in permanent autarky. It is only once the negotiated repayment has been made that the defaulting country can regain access to the capital markets. These models thus combine both elements: the inclusion of endogenous default probabilities (and thus also endogenous risk premiums) and endogenous debt recovery rates. The models can be shown to replicate the empirical facts of the other models while also being capable of reflecting the observable haircuts.

The incorporation of endogenous debt recovery rates influences the sovereign's default decision *ex ante*, too. As in the other models, the default probability increases, ceteris paribus, with rising government debt. However, the debt recovery rate following a default falls as debt rises.[10] When setting the bond price, creditors not only take the default probability into consideration but also form expectations regarding the debt recovery rate in the event of a default. The lower the (expected) recovery rate, the lower the price they are willing to pay for purchasing government bonds (ie the higher the required interest rates). The sovereign's incentives to borrow are now influenced by two counterveiling effects. On the one hand, the incentive to borrow rises because the government might regain access to the capital markets in the event of a default comparatively cheaper subject to a rather large haircut; on the other, it falls because this is taken into account by the investors when purchasing government bonds. In the baseline calibration of the model in Yue (2010), the former effect appears to be predominant, as it generates higher government debt. The same applies to D'Erasmo (2010), who, by integrating a reputation effect into a very similar model, is able to generate even higher debt ratios.

The delays in debt restructuring negotiations (which, in the case of Argentina, took as long as 40 months) found in empirical data are generally regarded as evidence of inefficiencies (see also section 2.3). Nonetheless, building on the literature discussed in the previous paragraphs and using a model in which government debt restructuring and the probability of a government re-entering the international capital markets are also negotiated following a default, Bi (2008) shows that it can be beneficial for both the defaulting country and the debt holders to delay the negotiations (ie "waiting" can be an efficient equilibrium outcome). Based on Merlo and Wilson's (1995, 1998) bargaining game, in each negotiation period borrowers or creditors are randomly selected to make a proposal regarding the size of the debt recovery rate. If the selected player decides to make a proposal, the other player

[10] The higher the outstanding sovereign debt of a defaulting country, the greater the loss that creditors, as a whole, will suffer in the event of a total default. As they negotiate repayment with the defaulting sovereign, they accept a lower debt recovery rate when government debt is higher, although the recovery amount itself increases with rising debt. Ultimately, this is a composition effect resulting, among other things, from the Nash bargaining solution.

can choose to accept it. The government then repays according to the agreement, regains access to the capital markets, and the negative output effects in the country "disappear". In all other cases (ie the selected player passes or the other player rejects the proposal), the game is repeated in the following period.[11] Depending on the volume of outstanding government debt and the output shock, it can be beneficial to both the defaulting country and the creditors to delay negotiations (ie to reject or pass). For the same reason as in the Nash bargaining games described above, the recovery rate falls when the debt-to-GDP ratio rises (ie, the higher the ratio, the lower the recovery in relative terms); in addition, it is easier for the country to make a higher repayment in good times. Given a negative (temporary but fairly long-lasting) output shock, it can be advantageous for both parties to wait (the creditor can hold out for a larger repayment because the size of the "cake" will increase; the defaulting country can lessen the relative costs of repayment because the utility losses incurred by foregoing consumption have less of an impact in good times). Interestingly, the debt recovery rate is affected by the identity of the proposer in the actual debt recovery period. If it is the defaulting country, debt recovery is lower than when the creditor makes the proposal (which is accepted in equilibrium). The debtor (creditor) will make a proposal that the creditor (debtor) is just about willing to accept. Although the creditor (debtor) would like to receive more (pay less), the expected probability of the player in question being able to improve the offer appears too low.

In a comprehensive paper, Benjamin and Wright (2009) present a new database covering 90 defaults by 73 countries that were settled in the period from 1989 to 2006 and containing extensive data on the relevant restructuring negotiations (and their outcomes). They too embed the restructuring negotiations in the type of sovereign default model discussed above. Structurally, the negotiations are modelled in a similar way as in Bi (2008), ie one player is randomly selected in each negotiation period to be the proposer, who is allowed to make an offer that the other player can either accept or reject. Unlike in Bi (2008), however, the players do not negotiate the debt recovery rate; instead, the selected player proposes a transfer of resources from the debtor to the creditor. This transfer (also when accepted by the creditor) can exceed the resources currently available to the debtor, requiring expected future income to be transferred as well. As this is not so simple to achieve, such proposals involve the sovereign borrowing new funds, which must be provided by the creditor. There is therefore a risk that the debtor country will default

[11] Merlo and Wilson (1995, 1998) use a game with complete information. It can be shown that, for this reason, a stationary subgame perfect equilibrium implies that a proposal is only submitted if the other player accepts it. The formal solution is thus provided by the fixed point of a system of equations consisting of the relevant present value functions of propose/pass and accept/reject and of the recovery rates (selected as a proposal) resulting from maximisation.

on this new government debt in a future period. In this structure, the creditor has two strategies: (1) he can demand a transfer and offer to lend (limited) new funds or (2) he can demand full repayment of the government debt. The debtor country can (1) repay the debt in full, (2) offer a transfer and request to borrow new funds or (3) offer nothing (and remain in autarky). The other player can either accept or reject whichever proposal is made. If the proposal is rejected, the next round of bargaining begins. If a proposal is accepted, the debtor country is allowed to re-access the capital markets. Creditors have an interest in maximising the payments they receive, whereas the debtor country wants to keep them as low as possible. It therefore tends to prefer the option of funding a transfer through new borrowing over full repayment of the sovereign debt. Creditors, on the other hand, cannot be entirely confident that the debtor country will not default on this new debt. It can therefore be beneficial for both parties to wait for better times, when the probability of a default on the new debt falls and the (expected) payments to the creditor rise. The model is able to replicate the stylised facts calculated by Benjamin and Wright (2009) using the new database and explains why some debtor countries are more highly indebted when they exit such negotiations than when they entered them.

Delays in restructuring negotiations can thus represent an equilibrium which, depending on the interpretation, may be efficient because, in the underlying bargaining structure, both parties may be able to minimise/maximise their future loss/gain in utility. This is particularly true of Bi (2008), but such cases can also occur in Benjamin and Wright (2009). Nonetheless, the models described above do not take the coordination problems among creditors mentioned in section 2.3 into account. The model in Pitchford and Wright (2010), for example, shows that when creditors are uncoordinated the negotiations can be (inefficiently) delayed, as individual investors hold out for better payoffs, or that individual investors free ride on negotiation costs. Bai and Zhang (2010) show that the structure of sovereign debt also plays a role. Before the 1990s, it was usually banks that acted as creditors, which made creditor coordination during government debt restructuring comparatively simple; since then, it has become much more difficult to coordinate creditors because governments issue bonds that can be traded on the secondary market. This delays restructuring and the government's re-entry into the capital markets. However, these studies often have more of a focus on game theory and tend to be more concerned with restructuring negotiations than the macro modelling of sovereign defaults; they are therefore not discussed in more detail in this article.

4.5 Debt maturity and debt dilution

All of the above-mentioned papers assume that sovereigns issue one-period bonds that must be fully refinanced in each period. In reality, however, governments generally issue bonds with different maturities and also incur long-term debt. Building on the framework in Alfaro and Kanczuk (2005), which assumes a constant debt level, Alfaro and Kanczuk (2007) integrate one and two-period bonds into their model. A country which can issue only one-period maturity bonds is compared with an otherwise identical country which can issue two-period maturity bonds. As outlined in more detail above, in Alfaro and Kanczuk (2005, 2007), investors form expectations regarding the type of government (good or bad) they are confronted with and set the interest rates accordingly. The likelihood of the government type changing is mapped using a Markov chain. Information on the current type of government is thus comparatively indicative of the government type in the following period. However, uncertainty regarding the government type increases for periods in the more distant future. If investors consider the current government type to be "good", it is relatively likely that this will still be the case in the following period. Nonetheless, the likelihood of this remaining so in later periods is lower. Alfaro and Kanczuk (2007) use this argument regarding uncertainty to explain why empirical data show that the interest rates paid on longer-term bonds are usually higher than on short-term bonds. The opposite is true, however, if investors consider the current government to be "bad". Longer bond maturities lead to a fall in the volume of sovereign debt to be refinanced per period and (at least in bad times) in the financial burden of future sovereign debt due to the (opposite) uncertainty argument just described. This leads, on the one hand, to a decline in the utility of a government default; on the other, the costs of such a default also fall. The latter can be explained by the fact that, in the model, the sovereign is not excluded from the capital markets after a government default but instead faces "only" a higher interest rate, and the rise in interest rates is smaller for long-term bonds than for short-term ones because of the (opposite) uncertainty argument. It is impossible to tell which of the two effects is predominant from a theoretical point of view. For most of the calibrations in Alfaro and Kanczuk (2007), however, it is the latter, meaning that the propensity to borrow increases when bonds have long maturities.

The simplifying assumption of a constant debt level given longer-term debt – as in Alfaro and Kanczuk (2007) – is relaxed by Hatchondo and Martinez (2009), who incorporate long-term debt and varying debt levels into Arellano's (2008) "baseline framework". A logical starting point is the assumption that the repayment of bonds issued in the current period (t) will mature in T periods. Since, given endogenous new borrowing, new bonds can be issued in each period, however, such an assumption means that the (debt) state space consists of the vector $(b_0, b_1, b_2, ..., b_{T-1})$, where b_t denotes the volume of debt that must be repaid in t periods in the future. Because the solution method to solve these models is non-linear, the computational time and effort is immense (or even unfeasible) even for small T values, as there are at least T state variables that can themselves take on many different values. Hatchondo and Martinez (2009) therefore assume that the government issues bonds that promise an infinite stream of coupons which decreases at a constant rate. Depending on this rate, the model can replicate one-period bonds. This is the case if the rate is equal to one.[12]

In this model, the maturity structure of the outstanding bonds is exogenous and constant, but the level of government debt is variable. It is only possible to study the different effects that arise given different exogenously determined maturities. If the sovereign declares a default, it (permanently) discontinues payment of the concerned coupons to the investors, but can issue new coupons. Hatchondo and Martinez (2009) then look at the differences that arise when one-period and four-year bonds are issued. They, too, show that the interest rates on four-year bonds are higher, ceteris paribus, than those on one-period bonds. The argument is similar to that of the uncertainty argument in Alfaro and Kanczuk (2007). In addition, the paper addresses the fact that, unlike in the case of one-period bonds, holders of old debt face *debt dilution*. The longer the maturity, the less a government can commit to servicing the debt in the future. Furthermore, new borrowing increases a sovereign's probability of default – even for already/still outstanding government debt. These factors combined reduce the (expected) value of the government debt for those who own old bonds, leading to debt dilution. The interest rate spread therefore rises (even) more strongly than can be justified by the uncertainty argument alone. In good times, and given a low level of sovereign debt, governments issuing one-period bonds within these models can borrow at almost the zero-profit price (ie at the risk-free rate). The longer the maturity of the bond, the less possible this becomes. The welfare gains generated through long-term debt – resulting from the lower debt rollover (and thus higher consumption) per period[13] – are reduced by this debt dilution argument. The model's simulations show that, although one-period bonds do not appear to be optimal from an ex ante welfare perspective, to maximise welfare, the maturity of the bonds should be kept relatively short.

[12] This can also be interpreted as if the debt issued by the government consisted of a portfolio of zero-coupon bonds of different maturities, where the portfolio weights decline geometrically with maturity. If the rate is equal to one, the entire weight is on the one-period bond.

[13] Relatively higher consumption in "bad times" results because the financial burdens rise disproportionately if debt is short-term. In good times, the generally higher interest rate on long-term bonds can lead to a decline in consumption. Owing to falling marginal returns for consumption, however, higher consumption in bad times – at least up to a certain threshold for the maturity of the bonds – is the dominant concern from a wealth perspective.

Chatterjee and Eyigungor (2009) incorporate long-term bonds into their model using a very similar approach. They assume that bonds with a certain maturity can be issued in each period, but that unit bonds are infinitesimally small. These assumptions also prevent a disproportionately large rise in the state space, since the authors are able to calculate a sort of law of motion for bonds and thus still have only "one" (debt) state space. The qualitative results are similar to those of Hatchondo and Martinez (2009).

As a thought experiment, Hatchondo et al (2011) extend the model in Hatchondo and Martinez (2009). They assume the existence of a rule which requires that the government receive the consent of existing bondholders before issuing (new) debt. This could be viewed as an expanded form of the collective action clause. In order that the existing bondholders consent to the government issuing debt, they must be compensated for the loss resulting from debt dilution (in the model: through a transfer). This reduces the interest rate spread required by creditors by lessening the possible future loss of income. While this is a very stylised analysis, such techniques could have a role to play in the design of adequate stabilisation mechanisms – albeit, perhaps, in a different form.

4.6 Choosing the debt composition and where to borrow

Arellano and Ramanarayanan (2010) examine the question of governments' preferred maturity structure when they have bonds with different maturities at their disposal. Based on the model in Hatchondo and Martinez (2009), who compare the impact of differing (exogenously assumed) maturities, they endogenise the government's decision regarding the maturity structure if it can issue debt using (two) different bonds with different maturities. The government faces the following tradeoff: by issuing debt in the form of long-term bonds, it can hedge against (shorter-term) fluctuations in the bond price (and thus in the refinancing costs) because (1) it must roll over only part of the outstanding government debt in each period and (2) the interest rate on the bonds is fixed when they are issued. By issuing short-term bonds, the government can, in a given period, obtain more liquidity at a cheaper interest rate because it avoids the above-mentioned commitment and debt dilution involved in long-term debt. However, owing to the rapid debt rollover, the financing costs in the following period are higher (and the discounted utility loss resulting from lower consumption in the near future falls). In good times (ie given comparatively low spreads), the government issues more long-term bonds, since its motives for hedging are more pressing. In bad times, it tends to issue more short-term debt, as additional liquidity is so important in these periods that it outweighs the near-term increase in financing costs. This continues to hold – at least up to a certain threshold – even when the interest rates on short-term bonds rise more sharply than those on long-term bonds owing to the increased short-term default risk. Owing to the concave utility function, additional current consumption provides a stronger justification for future consumption losses in such periods because the marginal utility of each additional unit of consumption is relatively high. This mechanism can (at least partly) explain why governments take on more short-term debt in bad times (including shortly before a default) even though the spreads (over the risk-free yield) on short-term bonds rise more sharply than those on long-term bonds.

Boz (2011) analyses governments' decisions to borrow from private sector creditors or international financial institutions (IFIs) such as the International Monetary Fund (IMF). This borrowing takes the form of one-period bonds. Private sector lending is modelled, in principle, following the approach in Arellano (2008): the interest rate is determined endogenously according to the expected default probabilities. Loans from IFIs differ from private sector loans in three ways. First, IFI loans are enforceable. The stated reason for this is effective monitoring and the fact that the IFI has a preferred creditor status. Second, IFIs require an interest rate calculated from the sum of the risk-free rate and a charge that increases with the amount borrowed from the IFI. An IFI sets the interest rate purely on the basis of the outstanding level of debt owed to it, as this is sure to be serviced. Third, loans to a sovereign from an IFI are attached to conditionality arrangements regarding budgetary discipline. This conditionality is approximated by assuming that the sovereign has to switch to a higher discount factor (lower discount rate) when it is indebted to an IFI. It is thus assumed that fiscal policy becomes more "patient" as soon as it borrows from an IFI. In a sense, Boz (2011) thus endogenises the decision of the government to be a "good" or "bad" fiscal authority as defined by Alfaro and Kanczu (2005), Hatchondo et al (2009) or D'Erasmo (2010). If the country decides to borrow from an IFI, it decides to adopt a "good" fiscal policy (since future consumption is valued more highly, budgetary policy automatically becomes "more restrictive", making a default less likely). The model is able to explain why borrowing on commercial credit markets is procyclical, whereas IFI lending is countercyclical. In good times, sovereigns prefer to borrow commercially because the low default probabilities keep the costs low. In bad times, financing conditions on commercial capital markets deteriorate. Unlike the private sector, which takes total indebtedness into account, an IFI will set interest rates purely on the basis of the debt owed to it; this makes it more worthwhile for a sovereign to borrow some of the funds that it requires from an IFI when times are bad. In relative terms, IFI lending becomes cheaper for the sovereign. From a theoretical perspective, it is unclear what effects the possibility of borrowing from IFIs has on a country's default probability. On the one hand, it can be expected to rise given that this debt, too, must be redeemed, reducing future consumption possibilities and increasing the incentive to default outlined above. On the other, the incentive to default falls because the sovereign becomes more "patient" and places a higher value on future consumption (because the discount factor is assumed to rise). In simulations that are calibrated to developing countries, the former effect appears dominant.

As the debate in the euro area shows, these issues are no longer relevant solely to developing countries and this model allows room for expansions, eg including the funding of such organisations. Going in this direction, a similar modelling approach as in Boz (2011) is taken by Roch and Uhlig (2011) in their (still quite preliminary) paper. They (want to) address contagion and assistance mechanisms in a monetary union setting by merging the modelling approaches of Arellano (2008) and Cole and Kehoe (2000); so far, however, their focus is still more on the theoretical properties of the model and results are only preliminary.

5 Integration of the approaches into a RBC model

To compare present value functions in the above-mentioned models – which inform the government's decision on whether to default – non-linear solution methods are needed; it is therefore difficult or (in large models) impossible to incorporate these decisions into the conventional (often larger) DSGE models used by many institutions. Even where non-linear methods can be used, the specific solution method applied plays a role – at least from a quantitative point of view. This is illustrated by Hatchondo et al (2010) in a rather technical analysis. The most commonly used solution method is the discrete state space technique (see section 3.2), although this requires a large (predetermined) state space to be assumed in order to avoid erroneous interest rate movements. In simulations, however, this technique is comparatively inefficient, time-intensive and – very importantly – prone to parametrisation. Other solution methods (such as Chebyshev polynomials or cubic splines) seem less vulnerable to parametrisation or the choice of state space and, in some cases, allow more rapid calculation. As the discussion of these aspects is very technical, readers who would like more detailed information should refer to Hatchondo et (2010) and Zhu and Xie (2011).

Although it is difficult to transfer the modelling of a government's endogenous decision to conventional DSGE models, it does appear necessary – particularly in the current situation – to develop more complex default models so as to be able to replicate relevant current and future issues (eg restructuring as compared with an assumption of joint liability, adequacy of borrowing limits,...). In particular, it would appear desirable to model the repercussions of a sovereign default on the domestic economy, which are, in most cases, built into the aforementioned models as an exogenous decline in domestic output. Mendoza and Yue (2010) present a model that can be viewed as an initial step in this direction. This model transcends the separation between RBC models, in which default risk is assumed to be an exogenous process, and the sovereign default models described here, which assume an exogenous output process. They combine the literature on “sudden stops” – in which certain events can occur within a RBC model that lead to a sudden stop in the external financing possibilities of a country/sovereign – with the type of model discussed above, in which the sovereign decides whether or not to default.[14]

In their model, Mendoza and Yue (2010) assume that firms producing final goods do so using working capital and labour. Working capital consists of a combination of domestic and imported intermediate goods, which are imperfect substitutes. Imported intermediate goods must be paid for before production begins; this is funded, at least in part, through (foreign) loans. In the event of a sovereign default, it is assumed that the production sector also experiences financing difficulties on (ie loses access to) the capital markets, meaning that it can no longer purchase imported intermediate goods. Although imported intermediate goods can be replaced with domestic ones, they are imperfect substitutes, and this leads to a loss in efficiency. Unlike in the literature discussed above, the default probability and the level of domestic productivity therefore interact. Lower productivity affects default probability as in the aforementioned models (positively) and influences financing conditions (negatively, ie higher spreads, including for the private sector). This, in turn, has negative repercussions for domestic productivity, (more sharply) increasing the incentives for a sovereign default. Because the intermediate products are modelled as Armington CES aggregators, the loss in output following a government default in this model is an endogenous, sharply rising and convex function of the productivity shock. Ultimately, this means that larger shocks have a greater impact on the default probability than small shocks and that output dynamics and default risk are determined jointly in this model. All of the previously discussed literature either assumes that productivity shocks have symmetric effects on output (and thus, implicitly, quasi-symmetric effects on default probability) or installs ad hoc asymmetries which are not explained endogenously to the model.

Mendoza and Yue’s (2010) model is thus able to capture the feedback effects between default and output (losses) more explicitly. More specifically, it is able to illustrate three stylised facts: (1) the V-shaped dynamics of output around default

[14]For more information on the basic structure of these models, see Mendoza (2010). Mendoza (2006,2010) provides an overview of the findings of this literature. As these models essentially fall into a different category, a detailed explanation of their workings would extend far beyond the scope of this paper; readers interested in learning more should refer to the aforementioned articles for an initial grounding in the subject.

events, (2) the negative correlation between interest rates on sovereign debt and output and (3) the disproportionately high government debt ratio when defaults take place. Conventional RBC/DSGE models are usually unable to explain points (1) and (3) given that sovereign defaults (if included at all) follow an exogenous process, whereas most of the literature discussed in this article cannot explain point (2) because the output process is modelled exogenously. Mendoza and Yue (2010) is therefore a promising approach to expanding such models so that they encompass both approaches.

6 Conclusions

Discussions about sovereign defaults will probably take on new dimensions in the course of the current crisis and are unlikely to remain restricted to developing countries. Expanding (theory-based) analytical tools such as RBC or DSGE models, which are used in large (international) institutions to investigate economic relationships or simulate certain political measures, so that they can model sovereign defaults would therefore be a welcome development. Formalising aspects such as contagion effects, assumptions of joint liability, suitable insolvency legislation etc could provide a useful contribution to the likely debate on these issues.

Sovereign default literature – whose findings should at least be acknowledged in the current situation – came into being, above all, in response to government crises in a number of Latin American countries. The modelling techniques developed in this context could be helpful for refining the models that have been used until now.

From the literature discussed in this paper, it can be concluded that (1) despite the absence of international insolvency legislation, sovereigns have an interest in repaying debt owing to reputational damage and negative output effects; (2) in the event of transitory shocks, debt level is the key factor determining the probability of default; in the event of trend shocks, the output level also plays a decisive role; (3) the political stability of a country reduces its probability of default; (4) bailouts or the possibility of borrowing from international financial institutions such as, for example, the IMF (can) increase the probability of default; (5) fiscal policy acts procyclically if it takes the probability of default and the impact on interest rates into account; (6) taking private sector debt into account does not change the above-mentioned qualitative statements but can increase a government's propensity to borrow (in relative terms); (7) endogenising negotiations about debt recovery rates tends to increase the interest rates on sovereign debt but a government's propensity to borrow can still rise; (8) it can be efficient for borrowers and creditors to delay negotiations over restructuring sovereign debt, although this is heavily dependent on the assumed bargaining game; (9) long-term bonds usually have a higher rate of interest than short-term ones; (10) contagion effects owing to risk-averse investors as well as information shocks affecting investors that impact on the interest rate can even cause fundamentally sound governments to default; and (11) the formal method of solving these models also plays a role in quantitative terms.

With respect to expanding the models used by large (international) institutions to include the aspect of sovereign defaults and approaches from the literature described above, the work of Mendoza and Yue (2010), in particular, lends itself to combining the RBC models with the default structure discussed in this paper. If their model could be expanded to include a multi-country setup as in Lizarazo (2009), this would doubtless produce interesting findings regarding contagion effects or the design of a stabilisation mechanism. Incorporating an information shock into such a model, as in Durdu et al (2010), could also make the model easier to calibrate to the euro area. Financial markets then also look at other information alongside economies' fundamentals when setting interest rate spreads. Aguiar and Gopinath (2006) show how bailouts can be integrated into such systems, Hatchondo et al (2010) incorporate a (very stylised) type of collective action clause, Boz (2011) allows for international financing institutions (such as, for example, the IMF) lending and aspects of debt restructuring can be analysed using the proposed modelling approaches in Bi (2008), Benjamin and Wright (2009), D'Erasmo (2010) or Yue (2010). All in all, there should be a number of options for expanding this type of model so that it can be applied to the issues currently being debated in the euro area.

The extent to which all of this is actually feasible remains an open question, however, because the necessary solution methods are complex. The main difficulties are caused by the fact that the solution mechanisms used to date in DSGE models are linearised approximations of mostly very complex models which cannot really model a (voluntary) default. All in all, smaller models would probably need to be developed initially in order to analyse specific issues. The integration of nominal frictions, as is usual in DSGE models, could also cause complications. Nonetheless, the author considers it worthwhile to pursue this course of action, as the sovereign default literature provides an informed basis for incorporating sovereign defaults into dynamic macro models.

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