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Money growth and consumer price inflation in the euro area: a wavelet analysis

Martin Mandler
Michael Scharnagl

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Research Question

The existence of a stable and reliable relationship between money growth and inflation with money growth as a leading indicator for future inflation is a central underpinning of the Eurosystem's monetary policy strategy. Our aim is to investigate the strength and stability of the relationship between broad money growth and consumer price inflation in the euro area over the period from 1970 to 2012 with a particular focus on changes over time.

Contribution

We study the relationship between money growth and inflation using wavelet analysis. Wavelet analysis allows investigating both changes in the relationship between these two variables across fluctuations with different frequencies as well as across time. We analyze the extent of comovements between money growth and inflation at different frequencies and different points in time as well as the lead-lag relationship between the variables. Our main focus is on fluctuations with periods of more than eight years which allows a meaningful analysis of time variation.

Results

Using the wavelet transform we estimate the local correlation (coherency) as a measure of the extent of comovements between the growth rate of the broad monetary aggregate M3 and HICP inflation at different frequencies and different points in time. We also estimate phase differences which show the lead-lag relationship between money growth and inflation and present results on the strength of the level-relationship between innovations in both series. Our results indicate strong comovements close to a one-to-one relationship between the very long-run fluctuations (24-32 years) in M3 money growth and HICP inflation with money growth leading inflation by about two to three years. Concerning possible time-variation in the relationship, we find evidence for a weakening at medium-to-long run fluctuations (8-16 years) after the mid-1990s which is in contrast to most of the previous literature using more conventional techniques in the frequency domain. Various modifications to the analysis, such as correcting the money growth series for real GDP growth or replacing the broad monetary aggregate M3 with the narrow monetary aggregate M1 lead to similar results.

Nichttechnische Zusammenfassung

Fragestellung

Das Vorliegen einer stabilen Beziehung zwischen der Wachstumsrate der Geldmenge und der Inflationsrate mit einem Vorlauf des Geldmengenwachstums vor der Inflation ist eine der Grundlagen der geldpolitischen Strategie des Eurosystems. In diesem Beitrag untersuchen wir die Stärke und Stabilität der Beziehung zwischen Geldmengenwachstum und Verbraucherpreisinflation im Euroraum zwischen 1970 und 2012. Einen Schwerpunkt legen wir dabei auf die Analyse möglicher Veränderungen im Zeitablauf.

Beitrag

Wir untersuchen die Beziehung zwischen Geldmengenwachstum und Verbraucherpreisinflation mit Hilfe einer Wavelet-Analyse. Wavelet-Analysen erlauben es, Veränderungen in der Beziehung zwischen diesen Variablen sowohl in Abhängigkeit von der Länge der betrachteten Schwingungen als auch im Zeitverlauf zu untersuchen. Damit sind wir in der Lage, den Grad der Gemeinsamkeit von Schwingungen von Geldmengenwachstum und Inflation auf unterschiedlichen Frequenzen und zu unterschiedlichen Zeitpunkten zu analysieren und Aussagen über den Vorlauf bzw. Nachlauf der beiden Zeitreihen im Verhältnis zueinander zu machen. Neben sehr langen Schwingungen konzentrieren wir uns dabei auch auf mittlere bis lange Schwingungen, da für diese die Wavelet-Analyse besonders geeignet ist, um Veränderungen im Zeitablauf aufzuzeigen.

Ergebnisse

Mittels einer Wavelet-Zerlegung schätzen wir die Stärke des Zusammenhangs zwischen der Wachstumsrate des weit gefassten Geldmengenaggregates M3 und der Inflationsrate des harmonisierten Verbraucherpreisindex (HVPI) im Euro-Währungsgebiet. Dazu schätzen wir die lokale Korrelation (Kohärenz) als Maß für die Stärke des Zusammenhangs zwischen Fluktuationen beider Zeitreihen auf verschiedenen Frequenzen und für verschiedene Zeitpunkte. Wir messen die Vorlauf-/Nachlaufeigenschaften der beiden Zeitreihen relativ zueinander mit Hilfe von Phasendifferenzen und präsentieren Ergebnisse über die Stärke der Niveaubeziehung zwischen Veränderungen der beiden Zeitreihen. Unsere Ergebnisse zeigen einen engen, nahezu eins-zu-eins-Zusammenhang zwischen M3-Wachstum und HVPI-Inflation für sehr langfristige Schwingungen, wobei das Geldmengenwachstum einen Vorlauf von zwei bis drei Jahren aufweist. Für die Zeitvariabilität für mittlere bis lange Schwingungen (8-16 Jahre) deuten unsere Ergebnisse, im Unterschied zu den meisten anderen Studien, die sich auf Standard-Analyseverfahren im Frequenzbereich stützen, auf eine Abschwächung des Zusammenhangs ab der Mitte der 1990er Jahre hin. Verschiedene Modifikationen, wie die Korrektur der Geldmengenwachstumsrate um die Wachstumsrate des realen BIP oder das Ersetzen der weit gefassten Geldmenge M3 durch die enge Geldmenge M1 führen zu ähnlichen Ergebnissen.

Money growth and consumer price inflation in the euro area: a wavelet analysis*

Martin Mandler
Deutsche Bundesbank

Michael Scharnagl
Deutsche Bundesbank

Abstract

Our paper studies the relationship between money growth and consumer price inflation in the euro area using wavelet analysis. Wavelet analysis allows to account for variations in the money growth-inflation relationship both across the frequency spectrum and across time. We find evidence of strong comovements between money growth and inflation at low frequencies with money growth as the leading variable. However, our analysis of time variation at medium-to-long-run frequencies indicates a weakening of the relationship after the mid 1990s which also reflects in a deterioration of the leading indicator property and a decline in the cross wavelet gain. In contrast, most of the literature, by failing to account for the effects of time variation, estimated stable long-run relationships between money growth and inflation well into the 2000s.

Keywords: money growth, inflation, wavelet analysis

JEL classification: C30, E31, E40.

*Contact address: Deutsche Bundesbank, Wilhelm-Epstein-Str. 14, D-60431 Frankfurt am Main, Germany. E-Mail: martin.mandler@bundesbank.de, michael.scharnagl@bundesbank.de. This paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or its staff. We are indebted to Luca Benati, Marco Gallegati, Mathias Hoffmann, Martin Kliem, seminar participants at the Deutsche Bundesbank, participants of the 2013 International Conference on Macroeconomic Analysis and International Finance, Rethymnon, Crete, of the 45th Annual Conference of the Money, Macro, and Finance Research Group, London and of the 2013 International Conference on Computational and Financial Econometrics, London, for helpful comments.

1 Introduction

A central underpinning of the Eurosystem's two-pillar monetary policy strategy is a stable medium to long-run relationship between the growth rates of monetary aggregates and consumer price inflation. As argued, for example, by Drudi et al. (2010) and von Landesberger and Westermann (2010), this relationship is most clearly visible in the comovements in low frequency components of growth rates of the broad monetary aggregate M3 and consumer price inflation in the euro area. This claim is supported by various empirical studies analyzing the explanatory or predictive power of low frequency components of money growth for inflation.

Our paper adds to this literature by using wavelet analysis to study the relationship between money growth and inflation in the euro area. Wavelet analysis allows to account for variability in the money growth-inflation relationship both across the frequency spectrum and across time. Thus, for a given frequency band, we can study whether comovements in money growth and inflation have become stronger or weaker through time and whether there have been changes in the lead-lag patterns between the two variables.

Our results show that while there is evidence for strong comovements in money growth and inflation at low frequencies with money growth as the leading variable and an estimated gain that indicates approximately a one-to-one relationship, the picture is much different if we look at medium-to-long run frequencies for which our methodology allows a thorough investigation into the time-variation in the money growth-inflation relationship. Although we still find evidence for strong comovements between both time series at these frequencies up to the mid 1990s there are signs of a weakening of the relationship afterwards. This weakening does not only show in a decline in the coherency between both series but also in a deterioration of the leading property of money growth before inflation as well as in a decline in the cross wavelet gain in the later part of the sample. In contrast, most of the empirical literature, by failing to account for the effects of time variation, presents evidence of stable long-run relationships between money growth and inflation in the euro area well into the 2000s. Various robustness checks, such as using a narrow monetary aggregate (M1) instead of a broad one, adjusting the money growth series for real GDP growth, and controlling for short-term interest rates leave the results qualitatively unchanged.

The relationship between low frequency movements in money growth and inflation has been analyzed in various empirical studies. For example, Hofmann (2006) extracts the low frequency component of the growth rate of euro area M3 using the Hodrick-Prescott (HP) filter and shows out-of-sample forecasts of consumer price inflation (1999Q1-2005Q4) based on trend money growth to be superior to those based on unadjusted money growth over medium and long forecast horizons. However, the quality of these forecast deteriorates after 2003. Neumann and Greiber (2004) derive a measure of core money growth for the euro area based on HP filtered growth rates of M3 adjusted for the trend growth of real output. Estimates for the sample period 1980Q3-2004Q2 show a robust one-to-one relationship between their core money growth measure and HICP inflation. For higher frequency components of money growth they find no evidence of a significant relation to consumer price inflation. Similarly, Carstensen (2007) shows that low frequency components of M3 growth provide good predictions for HICP inflation for forecast horizons of eight to twelve quarters. Regressing the low frequency component of consumer price inflation on the low frequency component of M3 growth yields a coefficient not significantly different from one. However, using a rolling sample, he finds that the explanatory power of the regression tends to deteriorate after 2004. According to Gerlach (2004), based on the sample period 1971Q1-2003Q3, for the euro area smoothed money growth (exponential filter) has predictive content for consumer price inflation one quarter ahead. Assenmacher-Wesche and Gerlach (2007) extend this approach by regressing HICP inflation on low frequency components of M3 growth and obtain a regression coefficient insignificantly different from one for the sample period 1970Q1-2004Q4. Haug and Dewald (2004) analyse the correlation structure between low frequency components of money growth and consumer price inflation or GDP deflator inflation for a large set of countries. They find high cross-correlation coefficients for most of the countries with trend money growth leading trend inflation for a subset of countries while for most other countries the strongest correlation between both series is contemporaneous.

These results are derived from extracting low frequency or trend components from money growth in order to explain inflation and use conventional regression techniques. An alternative approach is the application of spectral analysis to investigate comovements in money growth and inflation: Jaeger (2003) studies the relationship between the growth rate of broad monetary aggregates and consumer price inflation for various later EMU countries over the time period from 1981-98. The cross-spectral coherency of these two variables turns out to be highest for low frequencies, independent from the actual level of inflation. Similar results for a set of industrialized countries are presented by Haug and Dewald (2004). Assenmacher-Wesche and Gerlach (2007) use band spectrum regressions to estimate the effects of low frequency movements in the growth rate of euro area M3 on consumer price inflation for the period from 1970-2004. For frequencies with periods exceeding two years the coefficient on money growth is not significantly different from one while it is much smaller if higher frequency components are included.

Benati (2009) uses a very extensive dataset of 40 countries (including the euro area) with some time series going back into the 19th century. For each individual country he estimates the spectral coherency of money growth and inflation at frequency zero, i.e. for permanent innovations, and the cross spectral gain for periods of 30 years and longer. For most of the countries the cross-spectral coherency at frequency zero is close to one while the spectral gain is significantly less than one. This implies, that while innovations in money growth account for almost all of the long-run variance of inflation, there is no one-to-one relationship between these two variables. Benati tries to incorporate time-variation in the money growth-inflation relationship by using rolling windows of 25 years and shows that the cross spectral gain varies strongly through time for most of the countries and often is much less than one, while the cross spectral coherency tends to remain very stable and close to one. His interpretation of these results is that, in times of low inflation, the money growth-inflation relationship is obscured by velocity shocks and that the relationship is only uncovered in these rare episodes in which surges in inflation and money growth occur. While Benati (2009) accounts for time-variation in a relatively crude way by performing spectral analyses on a moving window of observations, Sargent and Surico (2011) present evidence for time-variation in the money growth-inflation relationship in the U.S. using time-varying vector autoregressions and show the cross spectral gain at frequency zero to have moved far below one after the early 1980s.

Most closely related to our paper is the study by Rua (2012a) who uses wavelet analysis to investigate the relationship between euro area M3 growth and HICP inflation based on monthly data from 1970 to 2007. He finds significant cross-spectral coherencies at low frequencies (periods of 12 to 16 years) throughout his sample period with evidence for a weakening of this relationship after 2000. In contrast to our study, Rua does not use the official euro area inflation data but relies for most of his sample on self-constructed data which leads to much different results compared to using the official HICP series. Apart from using the official series for consumer prices our study extends his analysis by investigating the effects of various corrections to the inflation and money growth series, such as correcting money growth for real output growth as in Assenmacher-Wesche and Gerlach (2007) and in Teles and Uhlig (2010) as shifts in trend output growth might obscure the money growth-inflation relationship.

2 Wavelet analysis

2.1 Intuition

Wavelet analysis is an extension of spectral analysis, i.e. a tool for analysis in the frequency domain.[1] Spectral analysis measures the contribution of periodic cycles of specific frequencies to the variance of an individual time series or the relationship between cycles of specific frequencies in multiple time series. However, while Fourier analysis, the standard tool for spectral analysis, allows to distinguish between changes across the frequency spectrum in the importance of cycles within time series and their relationships it has no resolution in time, i.e. it does not convey information about their changes over time. If, for example, an AR(1) process exhibits a structural break in its persistence implying a change in the dominant frequency of the time series, the estimated spectrum will highlight both the dominant frequencies before and after the break without being able to assign the two dominant frequencies to their respective subsamples. Furthermore, Fourier analysis requires the time series to be stationary, an assumption that is often violated in macroeconomics.

One proposed solution to the lack of time resolution in standard Fourier analysis is to use the short time or windowed Fourier transformation, which applies the Fourier transformation not to the full time series but to rolling subsamples. The choice of window length implies a trade-off: narrow windows provide good localization in time, i.e. result in more precision as far as the detection of changes in time is concerned while wider windows provide a better frequency resolution (e.g. Rua, 2012b, p. 73). The short time Fourier analysis that relies on rolling windows with length independent of frequencies results in a suboptimal choice within this trade-off, since the optimal window length depends on the frequency under investigation.

An alternative tool to uncover changes in the periodic behaviour of time series and in the relations between multiple time series is wavelet analysis (Aguiar-Conraria et al., 2008). Basically, wavelet analysis captures periodic cycles of different frequencies in time

[1] This section draws heavily on Aguiar-Conraria and Soares (2014). For an introduction to wavelet analysis, see also Rua (2012b). Wavelet analysis was initially proposed for applications in economics and finance by Ramsey and Lampart (1998) and Ramsey (2002). See also Crowley (2005).

series using periodic functions with only finite length which can be stretched to approximate lower or compressed to approximate higher frequencies. Stretching and compressing the wavelet can be thought of as corresponding to the choice of a flexible window length depending on the frequency, with wider windows being used when moving to lower frequencies. As a result, wavelet analysis allows for an improved time resolution for high frequency fluctuations and and improved frequency resolution for low frequencies compared to the short time Fourier transform.

2.2 Univariate, bivariate and multivariate wavelet analysis

The starting point is a so-called mother wavelet ψ from which, by scaling and translating a variety of wavelets can be generated

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \tag{1}$$

where $s \neq 0$ is a scaling or dilation factor which controls the width of the wavelet (an increase in s stretches the wavelet in time). High frequency fluctuations correspond to low values of the scaling factor while low frequency fluctuations correspond to high values for s. The translation parameter τ controls the location of the wavelet, i.e. changes in τ shift the wavelet in time. The function ψ has to fullfil some requirements in order to have the properties of wavelets.[2]

The continuous wavelet transform (CWT) of a time series $x(t)$ with respect to the wavelet ψ is obtained by projecting $x(t)$ on a family of wavelets $\{\psi_{\tau,s}\}$ and is defined as

$$\begin{aligned} W_x(\tau,s) &= \langle x, \psi_{\tau,s}\rangle \\ &= \int_{-\infty}^{\infty} x(t)\,\frac{1}{\sqrt{|s|}}\psi^*\left(\frac{t-\tau}{s}\right)dt, \end{aligned} \tag{2}$$

where * denotes the complex conjugate. The CWT may also be represented in the frequency domain as

$$W_x(\tau,s) = \frac{\sqrt{|s|}}{2\pi}\int_{-\infty}^{\infty}\psi^*(s\omega)\,X(\omega)\,e^{i\omega\tau}d\omega,$$

where $X(\omega)$ denotes the Fourier transform of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-i\omega t}dt,$$

and ω is the angular frequency.

There exists a variety of different wavelet functions. The one most widely used in

[2] For details see, for example Percival and Walden (2002).

applications in economics and which will also be used in this paper is the Morlet wavelet

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{3}$$

with parameter ω_0. The Morlet wavelet is complex valued allowing for an analysis of phase differences, i.e. lead-lag relations between time series. The most common choice is $\omega_0 = 6$ which results in some desirable properties of the Morlet wavelet (e.g. Aguiar-Conraria and Soares, 2014, p. 352). In particular, this specific choice yields a simple relation between scale (s) and frequency ω, $\omega \approx \frac{1}{s}$ and implies an optimal joint time-frequency resolution (e.g. Aguiar-Conraria and Soares, 2014, p. 352).

Using the CWT, the wavelet power spectrum is defined as

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2. \tag{4}$$

At each time and frequency, the wavelet power spectrum can be interpreted as the local variance of the time series $x(t)$. In the case of a complex-valued wavelet, the corresponding wavelet transform is also complex–valued and can be decomposed into a real part, the amplitude, $|W_x(\tau, s)|$, and its imaginary part, the phase, $|W_x(\tau, s)|\ e^{i\phi_x(\tau,s)}$. The phase-angle $\phi_x(\tau, s)$ is

$$\phi_x(\tau, s) = \arctan\left(\frac{\Im\{W_x(\tau, s)\}}{\Re\{W_x(\tau, s)\}}\right), \tag{5}$$

where $\Im$ denotes the imaginary part and $\Re$ the real part of the wavelet power spectrum. Consequently, the phase angle is only defined for complex-valued wavelets.

Our analysis focusses on the relationship between two time series. For two time series $x(t)$ and $y(t)$ the cross wavelet transform is defined as

$$W_{xy} = W_x W_y^*, \tag{6}$$

where * , as before, denotes the complex conjugate.

From the cross wavelet transform of $x(t)$ and $y(t)$ and the wavelet power spectra of both time series the wavelet coherency can be derived as

$$R_{xy}(s) = \frac{|S(s^{-1}W_{xy}(s))|}{\sqrt{S(s^{-1}|W_x|^2)}\sqrt{S(s^{-1}|W_y|^2)}}. \tag{7}$$

The wavelet coherency can be interpreted as local correlation between the two time series, similar to a correlation coefficient. S is a smoothing operator with respect to time and scale. Without smoothing the wavelet coherency would be equal to one across all times and scales. If the wavelet coherency is complex valued it can be decomposed into its real and imaginary parts $\Re$ and $\Im$, and the wavelet phase difference can be computed as

$$\phi_{x,y}(s, \tau) = \arctan\left(\frac{\Im\{W_{xy}(\tau, s)\}}{\Re\{W_{xy}(\tau, s)\}}\right), \tag{8}$$

with $\phi_{x,y}(s, \tau) = \phi_x(s, \tau) - \phi_y(s, \tau)$. The phase difference $\phi_{x,y}(s, \tau) \in [-\pi, \pi]$ provides information about the lead-lag relationships between the two time series. If $\phi_{x,y}(s, \tau) = 0$, the series x and y move together at the given scale and time. If $\phi_{x,y}(s, \tau) \in \left(0, \frac{\pi}{2}\right)$, series

x leads y and if $\phi_{x,y}(s,\tau) \in \left(-\frac{\pi}{2}, 0\right)$, y leads x.[3] Using the phase difference, the time lag or time difference, which gives the lead or lag of the series in the time domain can be calculated as

$$\Delta T_{x,y}(s,\tau) = \frac{\phi_{x,y}(s,\tau)}{\omega}. \tag{9}$$

Finally, the cross wavelet gain is defined as

$$G_{yx}(s) = \frac{\left|S\left(s^{-1}W_{xy}(s)\right)\right|}{S\left(s^{-1}\left|W_x\right|^2\right)}. \tag{10}$$

It can be interpreted as a regression coefficient in the regression of y on x.

When investigating the relationship between two variables x and y it might also be of interest to control for their correlation with other variables when calculating coherency and phase differences. Given p time series $x_1, x_2, ..., x_p$ define L as the $p \times p$ matrix of all smoothed cross-wavelet power spectra $S_{ij} = S(W_{ij})$

$$L = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \vdots & \vdots & & \vdots \\ S_{p1} & S_{p2} & \cdots & S_{pp} \end{bmatrix}.$$

Denote L^d_{ij} as the cofactor of the (i,j) element of L, $L^d_{ij} = (-1)^{(i+j)} \det L^j_i$,where L^j_i represents the sub-matrix obtained from L by deleting the ith row and the jth column and $L^d = \det L$.

The complex partial wavelet coherency of x_1 and x_j $(2 \leq j \leq p)$ is the wavelet coherency between x_1 and x_j given all the other series and it is defined as

$$\rho_{1j.q_j} = -\frac{L^d_{j1}}{\sqrt{L^d_{11}L^d_{jj}}}. \tag{11}$$

From the complex partial wavelet coherency the partial phase delay, i.e. the phase difference between x_1 and x_2 given all other series can be computed as[4]

$$\phi_{1j,q_j} = \arctan\left(\frac{\Im\left\{\rho_{1j.q_j}\right\}}{\Re\left\{\rho_{1j.q_j}\right\}}\right). \tag{12}$$

3 Empirical results

We use quarterly data for the monetary aggregates M3 and M1, and for the Harmonized Index of Consumer Prices (HICP) in the euro area over the sample period 1970Q1 to 2012Q4.[5] For later adjustments of the monetary aggregates (see below) we use euro area

[3] However, the economic interpretation of the lead-lag pattern is not as clear cut. See Section 3, footnote 15.

[4] For more details on the multivariate extension of wavelet analysis, see Aguiar-Conraria and Soares (2014), pp. 357.

[5] The series are standardised before the wavelet analysis is applied.

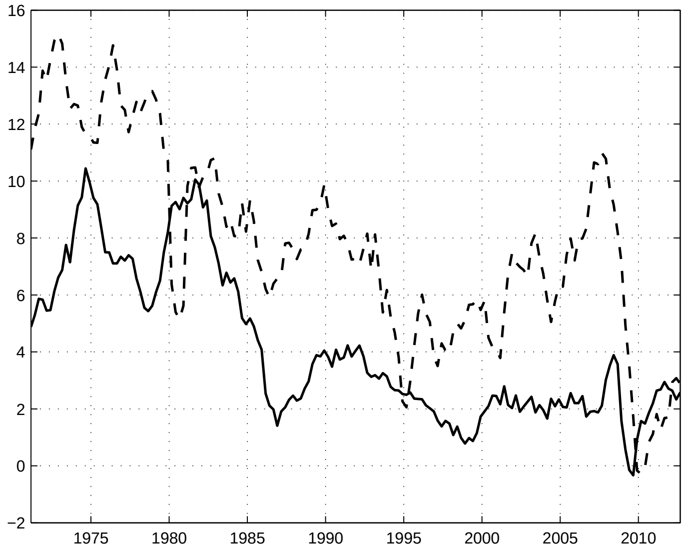

Figure 1: Annual growth rates (percent) of M3 (dashed line) and HICP (solid line) in the euro area.

real GDP and a euro area short-term interest rate. The latter series is taken from the area wide model (AWM) database up to 2010Q4 and extended with the three-months euribor rate from 2011Q1 onwards. All other series were downloaded from the ECB's Statistical Data Warehouse.[6] We extended the real GDP series from the SDW which begins in 1980Q1 backwards to 1970Q1 using growth rates for real GDP from the AWM database. For the monetary aggregates we used series of notional stocks which are constructed from transactions-based flow data and are adjusted for non-transaction related changes such as statistical reclassifications, revaluations etc.[7]

We start by estimating the wavelet coherency between the annual growth rate of euro area M3 and the annual inflation rate of the euro area HICP.[8] Figure 1 displays the annual growth rate of euro area M3 and annual HICP inflation while Figure 2 shows the wavelet coherency of both series across time (horizontal axis) and frequencies/periods (in years) (vertical axis). The coherency is increasing from the blue to the red colored areas as indicated by the scale on the right hand side. The grey (black) lines indicate coherency significantly different from zero at the 10%- (5%-) level.[9] Note that only estimates between the curved red bands should be interpreted, however, since the limited length of the data set leads to a deterioration in the information content beyond these bands.[10] According

[6] http://sdw.ecb.europa.eu/

[7] For details, see the technical notes to the European Central Bank's Monthly Bulletin.

[8] All estimations were performed using the AST-toolbox for MATLAB by Aguiar-Conraria and Soares. https://sites.google.com/site/aguiarconraria/joanasoares-wavelets/

[9] Significance is established by a simulation procedure: Uncorrelated AR processes are fitted to both series from which, by Monte Carlo simulations or bootstrapping, artifical time series can be simulated. From estimating wavelet coherency for each simulated pair of series a distribution of wavelet coherencies is obtained and critical values can be derived. For details, see Aguiar-Conraria and Soares (2014).

[10] If there is only an insufficient number of past or future observations available to estimate the wavelet

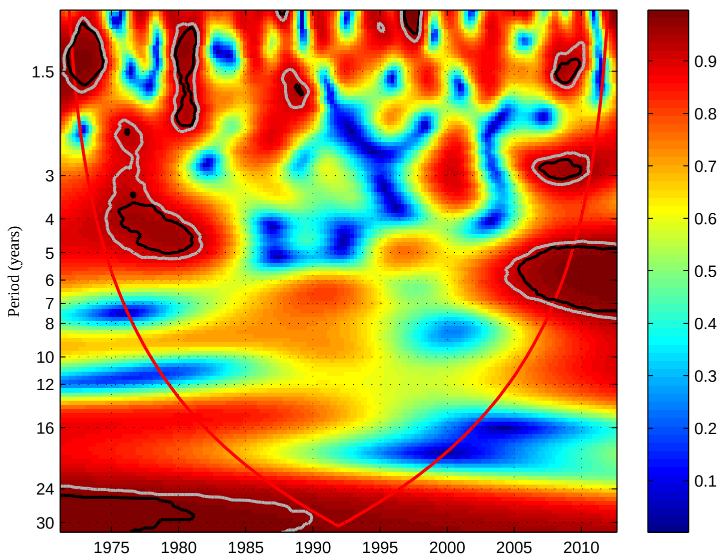

Figure 2: Wavelet coherency of annual growth rate of M3 and HICP inflation.

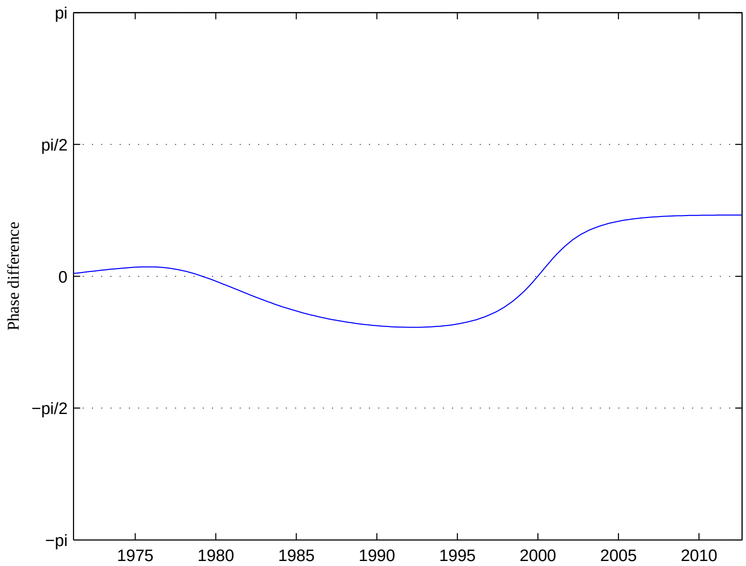

Figure 3: Phase difference of HICP inflation and annual growth rate of M3 - periods of 8-12 years.

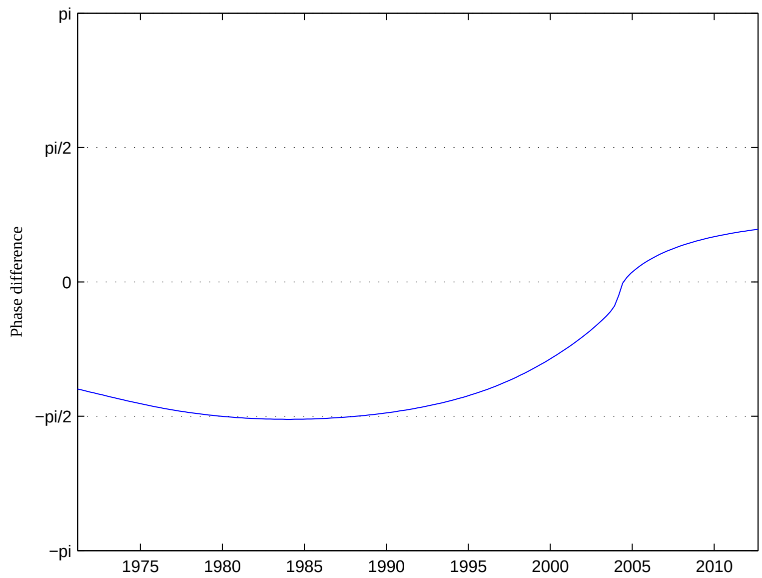

Figure 4: Phase difference of HICP inflation and annual growth rate of M3 - periods of 12-16 years.

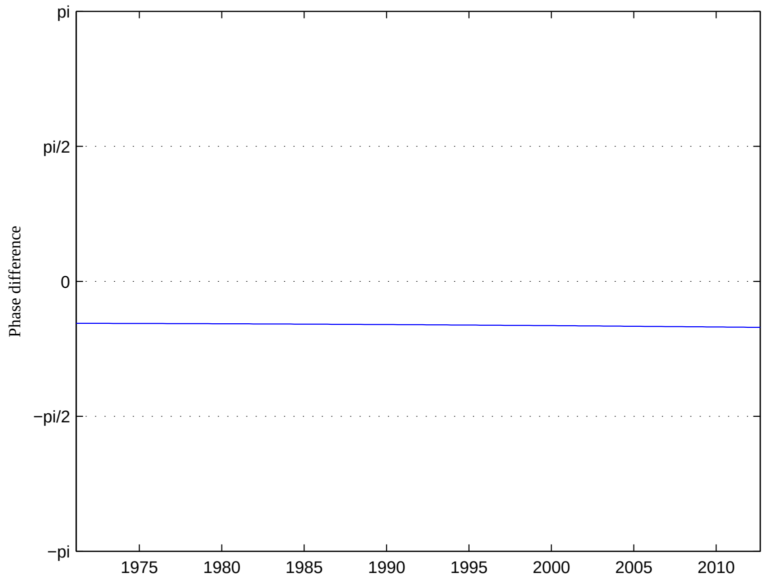

Figure 5: Phase difference of HICP inflation and annual growth rate of M3 - periods of 24-32 years.

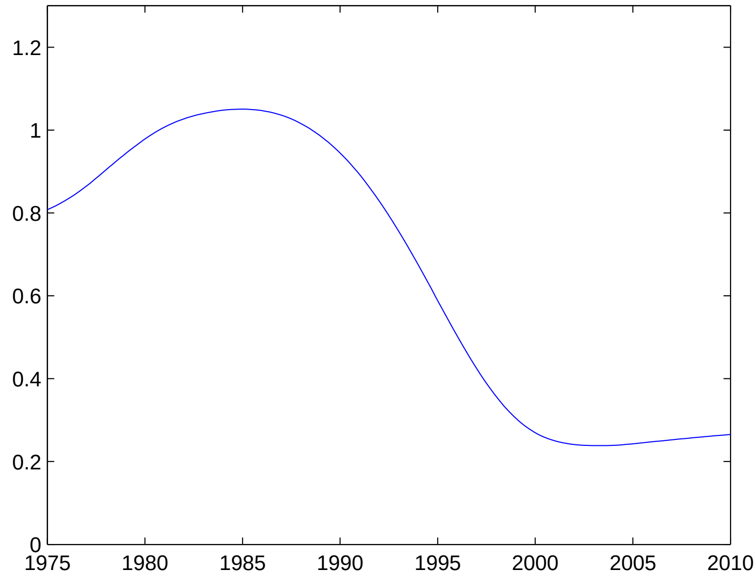

Figure 6: Cross-wavelet gain of of annual growth rate of M3 and HICP inflation, periods of 8-16 years.

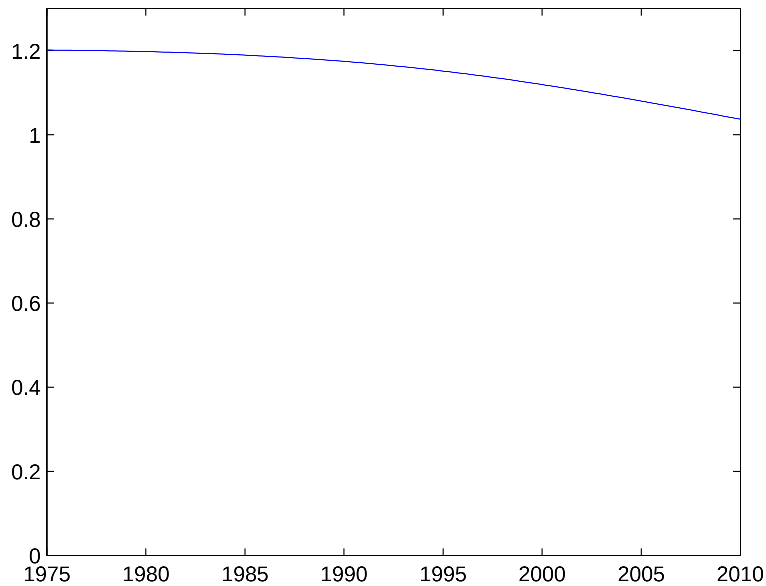

Figure 7: Cross-wavelet gain of of annual growth rate of M3 and HICP inflation, periods of 24-32 years.

to Figure 2 there are only few significant estimated coherencies between money growth and inflation.[11] Significant comovements between these two variables occured for periods about three to seven years in the early 2000s and at frequencies with periods less than five years between the late 1970s and the early 1980s. Other significant coherencies are estimated for only very brief time periods at high frequencies or are not contained within the interpretable region. For periods of about eight up to 20 years coherencies are relatively high (up to 0.7 or 0.8) until the mid 1990s when the relationship between money growth and inflation becomes much weaker. Strong and consistent comovements with coherency higher than 0.9 are only found at very low frequencies (periods of 28 years and longer). Basically, the relationship shown at the bottom of Figure 2 represents an approximation to the comovement in the long-run trends of money growth and inflation. Benati (2009) using windowed Fourier analysis finds a similarly strong relationship between euro area money growth and inflation with a coherency of close to one at frequency zero. Because of our different methodology we cannot investigate the relationship at arbitrarily low frequencies but are limited by the length of the sample period. This implies that as we move to lower and lower frequencies the time period our results can be interpreted for finally collapses to just one point in time in the early 1990s.[12] Furthermore, for these very low frequencies most or all of the data points available are used to estimate the wavelet coherencies which implies a strong overlap between neighboring estimates and, therefore, only modest time-variation, i.e. the power of the wavelet analysis to pinpoint changes in time becomes much lower at these low frequencies.[13] As one focus of our analysis is on using the ability of wavelet analysis to investigate time variation in the money-growth-inflation relationship we therefore will also consider somewhat higher frequencies in the following analysis.

Our results at frequencies between eight and 16 years are in strong contrast to those in Rua (2012a) who estimates significant coherencies at these frequencies between euro area M3 growth and inflation throughout the whole sample period. This difference to our results is due to different inflation data, since Rua uses the officially published HICP only after 1991 and a HICP series constructed by himself from weighted national CPI data for the time before. By contrast, we use the officially published series throughout. This results in the annual inflation rate used by Rua being higher and more volatile than the official series, especially in the 1970s and early 1980s, which causes him to estimate significant wavelet coherencies for a much longer time span.[14]

power spectra and the wavelet cross spectrum at a given point in time the algorithm extends the sample backwards or forward by "reflecting" the first/last observations. The red lines indicate the time periods in which the results are affected by this procedure. The region of usable estimates becomes smaller for lower frequencies as the flexible determination of the observation window length that enters the wavelet transform implies broader windows and, hence, the use of more observations for extracting lower frequency components.

[11] The simulation procedure described in footnote 9 generally leads to very wide intervals for which zero coherency cannot be rejected and, thus, makes it hard to obtain significant results.

[12] This is indicated by the convergence of the two red lines for fluctuations with periods of 32 years. Results for lower frequencies are not interpretable any more because fitting wavelets is not possible without reflecting.

[13] Nevertheless, as explained in Section 2 the results from wavelet analysis should be more efficient with respect to the time-frequency resolution than those obtained by windowed Fourier analysis.

[14] Since the wavelet coherency is based on observations from (endogenously determined) windows a stronger correlation during the 1970s and 1980s will to some extent also affect estimated coherency into

The lead-lag structure between the two series can be analyzed for selected frequency bands by estimating the phase difference (8).[15] Figures 3 to 5 display the estimated phase differences for movements with periods of eight to 12 and 12-16 years, which covers the relatively high coherencies in the upper part of the lower half of Figure 2 and for fluctuations with periods between 24 and 32 years which corresponds to the very low frequencies with coherencies close to one at the bottom of Figure 2. Phase differences for periods outside of the red bands in 2 should again not be interpreted. The phase difference for the first frequency range indicates a maximum lead of money growth before inflation of about one year between the mid 1980s and mid 1990s which, however, begins to disappear over the following years and turns into a lag of money growth behind inflation in the early 2000s.[16] The phase difference for the 12-16 years frequency range implies a lead of money growth before inflation of about 3.5 years until the mid 1990s with the lead narrowing over the following years. For the very long-term fluctuations with periods between 24 and 32 years the phase difference is about $-\frac{\pi}{6}$ which implies a lead of money growth before inflation of approximately 2.5 years, but which can only be interpreted between the late 1980s and the mid 1990s.

While cross-spectral coherency is a measure of the extent of covariability in the fluctuations of both time series, comparable to an (absolute) correlation coefficient. The relative size of the comovements, comparable to a regression coefficient is measured by the cross-spectral gain. The gain can be interpreted as a regression coefficient in a regression of a frequency component of inflation on the respective frequency component of money growth. At low frequencies the gain captures the long-run level relationship between money growth and inflation as in Lucas (1980). An estimated gain of one would imply a quantity-theoretic relationship, i.e. that inflation and money growth move in a one-to-one fashion. Figures 6 and 7 show the gain (10) for two selected frequency ranges.[17] While the gain for the very low frequencies is close to one (about 1.1) over the interpretable time period (late 1980s to mid 1990s) and thus indicates a quantity-theory type relationship the gain for fluctuations with periods of 8-16 years exhibits strong time-variation.[18] Focusing on the

more recent years.

[15]The phase difference describes how far series x leads series y along the circumference of the unit circle. A problem in the interpretation of the phase difference arises because along the unit circle any given lead of variable x before variable y can also be interpreted as a lead of variable y (money growth) for variable x (inflation). For example, assume a phase difference of $\frac{\pi}{2}$, i.e. series x is at its high point on the unit circle while series y is at zero. This relationship could either be viewed as x leading y by $\frac{\pi}{2}$ or as series x leading y by $\frac{3\pi}{2}$.

[16]For the eight to 12 years frequency band with an average period of 10 years an average phase difference of about $\frac{\pi}{6}$ around 1990 implies approximately a lead of money growth before inflation of $\frac{10}{2\pi}\frac{\pi}{6} = \frac{5}{6}$.

[17]Estimating the gain at frequency zero as in Benati (2009) using wavelet methods would require an infinitely long sample period. Parametric methods can be used to estimate cross spectra at low frequency even on finite data sets as they allow to "extrapolate" the relationships between the variables to frequencies not actually contained in the data. See, for example, Sargent and Surico (2011) who estimate a time-varying VAR and derive the cross-spectral gain at frequency zero from the estimated VAR coefficients.

[18]Using a simulation approach similar to the one designed to test for the significance of the coherencies we simulated the distribution of the cross wavelet gain under the hypothesis of uncorrelated AR processes for money growth and inflation, i.e. under the null hypothesis of a zero gain. The simulated distribution turned out to be very wide with critical values for rejecting the null hypothesis larger than one, i.e. the region of nonrejection covered both zero and one. This is similar to the very broad probability bands around the estimated gain at frequency zero in Sargent and Surico (2011) which over long episodes also

interpretable region between the early 1980s and about 2005 the gain starts out close to one but declines strongly beginning in the late 1980s and declines to values around 0.2 in 2005. This indicates that at these moderately long periods the relationship between the levels of money growth and inflation was quite strong until the end of the Great Inflation and became much less pronounced afterwards. Using a time-varying VAR for the U.S. Sargent and Surico (2011) estimate cross spectral gains between money growth (M2) and inflation (measured by the growth in the GDP deflator). Their results indicate that the cross-spectral gain at frequency zero was not significantly different from one in the 1970s but declined significantly below one after 1980 with point estimates around 0.25.[19] Benati (2009) using windowed Fourier analysis shows that in the Euro area the cross spectral gain at frequency zero between M3 money growth and inflation was not significantly different from one throughout the 1980s but declined afterwards and fell significantly below one in the mid 1990s. Sargent and Surico attribute the strong decline in the cross spectral gain at frequency zero to a more aggressive monetary policy reaction function which implies a shift in the cross spectral gain towards zero. Based on a DSGE model they show that a cross spectral gain at low frequencies around one results from the central bank allowing persistent innovations in money by reacting too weakly to inflationary pressures. In contrast, a monetary policy reaction function in which the central bank responds aggressively to inflationary pressures leads to a gain close to zero. Alternatively, Benati (2009) explains the estimated decline in the cross-spectral gain with velocity shocks. As explained above, our estimates for the very long-run gain are not very informative about possible time variation. However our results for the frequency band with periods between 8 and 16 years are broadly consistent with this evidence. To sum up, our results so far indicate that at least at very low frequencies there is a strong relationship between money growth and inflation with money growth leading inflation. At somewhat higher but still relatively low frequencies the comovement of both series becomes weaker and deteriorates in time with the lead of money growth becoming shorter or turning into a lag.

As a first robustness check Figure 8 presents wavelet coherencies for M3 growth and HICP inflation where the growth rate of M3 has been adjusted for real GDP growth.[20] If money growth moved strongly with real GDP growth this effect could obscure the money growth-inflation relationship. Compared to Figure 2 the results become somewhat stronger for fluctuations with periods of six to ten years showing significant coherencies from the mid 1970s to the mid 1990s and, generally, resulting in higher values for the coherencies. The results for fluctuations at the low end of the frequency spectrum remain essentially unchanged. Figure 9 shows the phase difference for fluctuations with periods between eight and 12 years, i.e. for the frequency band for which we estimate the significant coherencies. The results indicate, that for this frequency range adjusted M3 growth is actually lagging inflation by about one year.[21] In contrast, the phase difference

cover both one and zero.

[19]See Figure 5 in Sargent and Surico (2011).

[20]See, for example, Assenmacher-Wesche and Gerlach (2007), Teles and Uhlig (2010) and Amisano and Fagan (2010). To construct the adjusted money growth rate, we subtract the annual growth rate of real GDP from the annual growth rate of M3.

[21]Given that the phase difference refers to the relative position of the two time series cycle on the unit circle the distinction between lag and lead is not entirely clear, as a short lead could also be interpreted as a long lag and vice versa. Hence, given the average period of ten years the estimated lag of about one year could also be interpreted as a lead of about nine years.

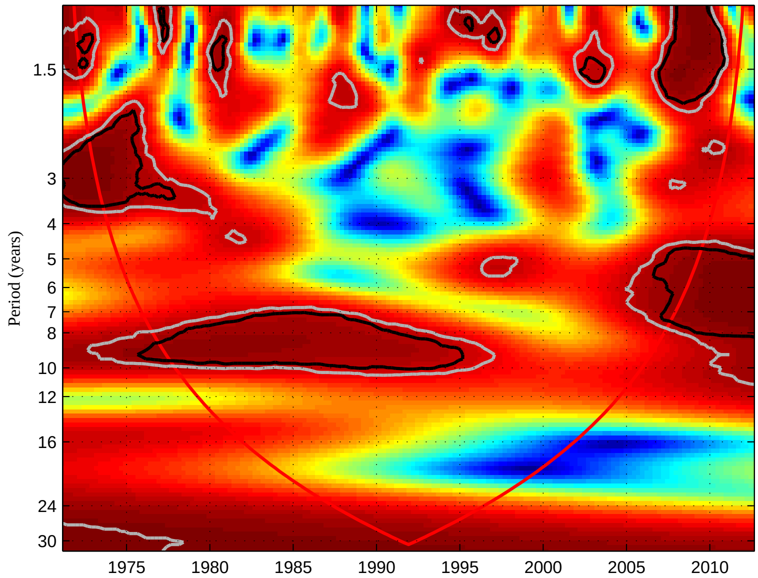

Figure 8: Wavelet chereny of annual growth rate of M3 (adjusted for real GDP growth) and HICP inflation.

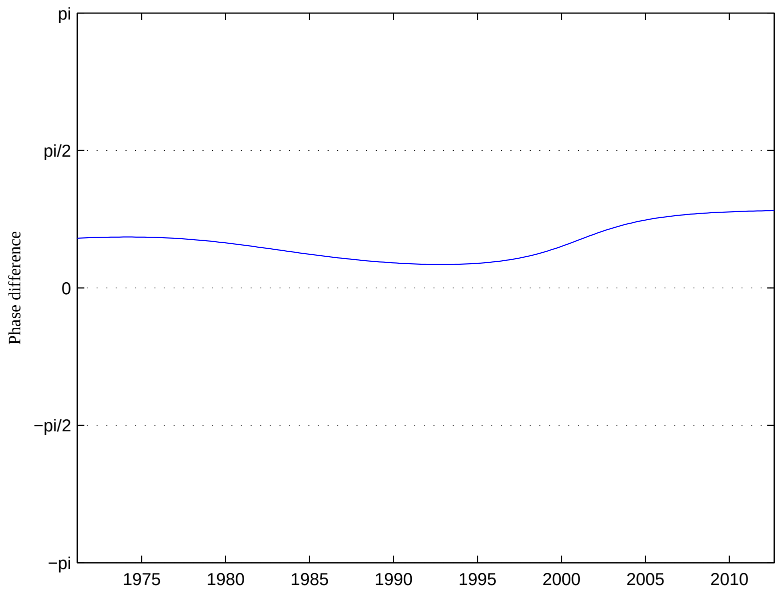

Figure 9: Phase difference of HICP inflation and annual growth rate of M3 (adjusted for real GDP growth) - periods of 8-12 years.

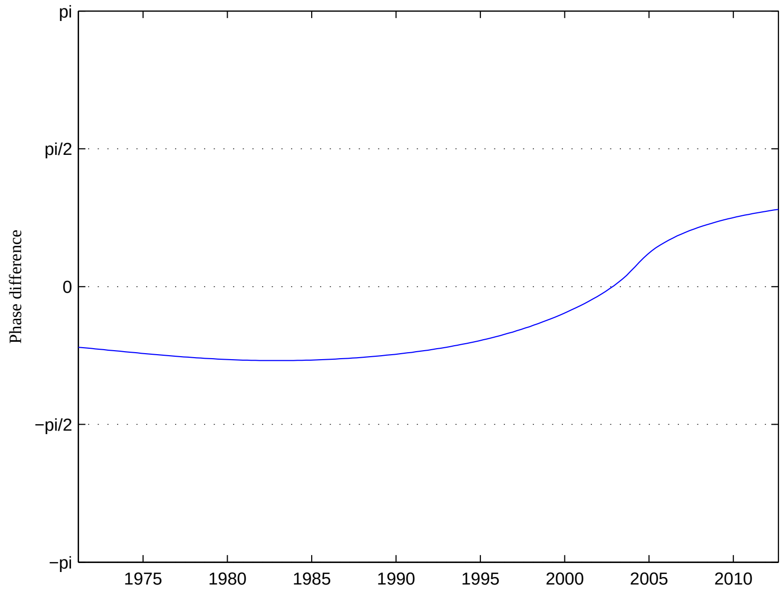

Figure 10: Phase difference of HICP inflation and annual growth rate of M3 (adjusted for real GDP growth) - periods of 12-16 years.

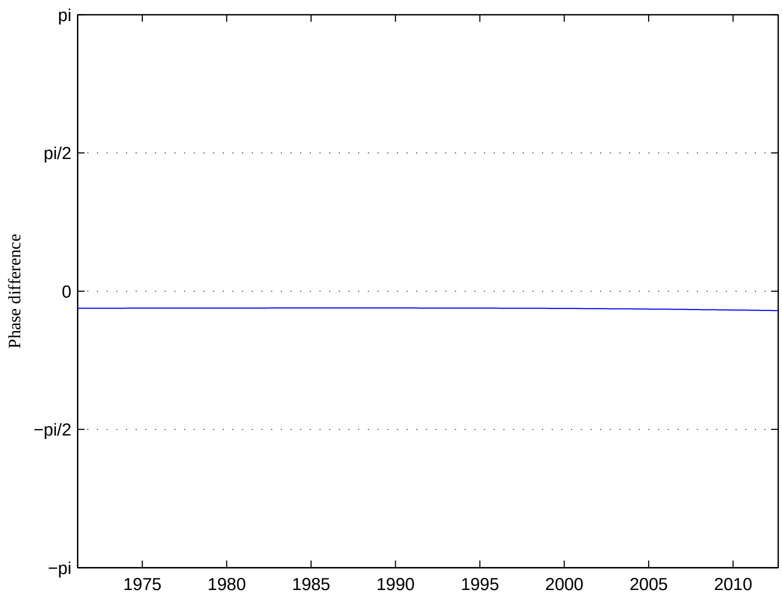

Figure 11: Phase difference of HICP inflation and annual growth rate of M3 (adjusted for real GDP growth) - periods of 24-32 years.

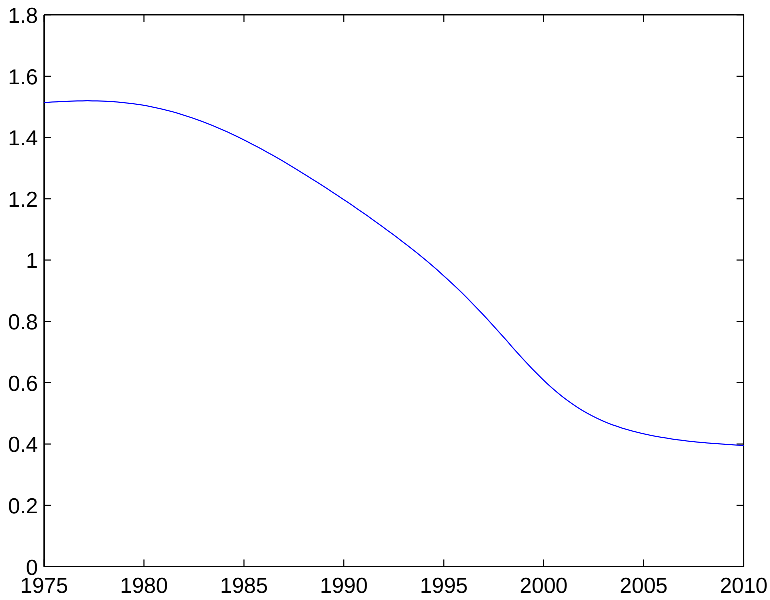

Figure 12: Cross-spectral gain of of annual growth rate of M3 (adjusted for real GDP growth) and HICP inflation, periods of 8-12 years.

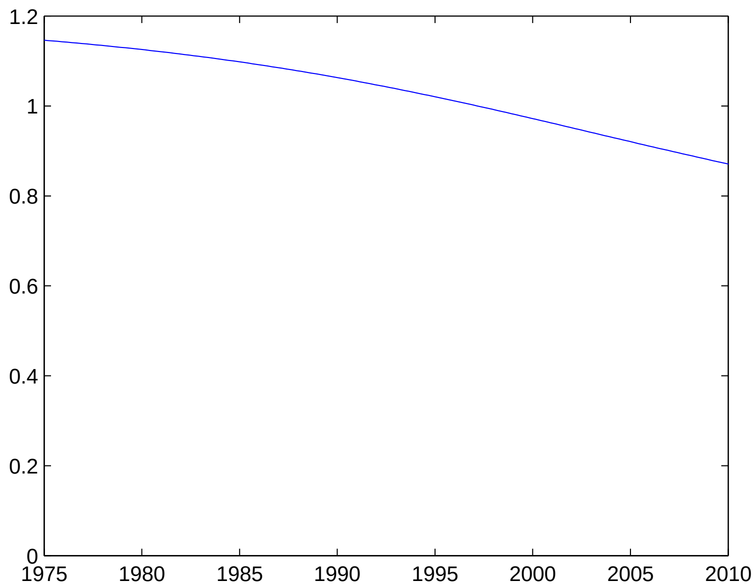

Figure 13: Cross-spectral gain of of annual growth rate of M3 (adjusted for real GDP growth) and HICP inflation, periods of 24-32 years.

for cycles with periods of 12-16 years in Figure 10 represents a lead of money growth of about 1.4 to two years but with a declining coherency which indicates a looser relationship from the 1990s on. The phase differences for fluctuations between 24-32 years (Figure 11) change little compared to Figure 5. For these frequencies, the gain for the adjusted money growth series is also similar to the one for the unadjusted series (Figure 7) while for the 8-12 years range our estimates in Figure 12 again indicate a strong decline in the gain which starts from surprisingly high levels.

As another robustness exercise, we also control for the impact of short-term interest rates on both money growth and inflation by computing the partial wavelet coherency of money growth (adjusted for GDP growth) and inflation conditional on the change in the short-term interest rate. Euro area interest rates have been following a falling trend since the early 1980s (Figure 14) and may have led to low frequency increases in money demand due to lower opportunity costs. As this increase in money demand need not be inflationary it might obscure the estimated relationship between money growth and inflation.[22] Comparing Figures 2 and 15 this conditioning does not lead to a much higher coherency of money growth and inflation at the 8-12 years frequency band but the "gap" in the red areas tends to move somewhat to the left. For the very low frequencies with periods of 24-32 years the phase difference in the mid 1990s declines to about -$\frac{\pi}{4}$ in Figure 17 compared to about -$\frac{\pi}{6}$ for the case of unadjusted money growth (Figure 5) which implies a lead for money growth of approximately 3.5 years. For the shorter periods of 8-12 years for which Figure 15 indicates relatively high coherencies except for the mid to late 1980s the phase difference shown in Figure 16 shows a marked increase in the early 1980s close to $\frac{\pi}{2}$ which corresponds to a lag of money growth somewhat less than 2.5 years with a following decline to a lag of about 1.25 years ($\frac{\pi}{4}$).[23] For the 8-12 years fluctations the estimated gain fluctuates between slightly below 1.5 and 0.9 and, thus, is pushed up to higher values by the adjustment of money growth. However, the opposite is the case for the gain for the long-run movements which turns out to be well below the values found for unadjusted M3 and M3 only adjusted for real GDP growth. Accounting for changes in interest rates seems to soak up a substantial part of the level effect of money growth on inflation.

In a further step, we replace the broad monetary aggregate M3 with the narrow aggregate M1. According to Figure 20 we estimate high but slightly lower coherencies than for M3 for fluctuations with periods of more than 24 years. For the frequency band of 8-12 years coherencies appear slightly stronger than for M3. Significant coherencies are shown for some time periods at business cycle frequencies (2-6 years). The signficant coherencies in the late 2000s at these frequencies are likely to be driven by the financial crisis when both inflation and money growth declined strongly. The phase differences for

[22]The partial wavelet coherency controls for the effect of interest rate changes on both inflation and money growth. For this reason this conditioning would not allow us to detect comovements between these two variables which are correlated with changes in the short-term interest rate. For example, if a reduction in interest rate leads to inflation as well as to an increase in money growth this comovement between money growth and inflation would not show up in our estimates. Adjusting only money growth for inflation would require an estimate of the interest rate elasticity of money demand at the relevant frequencies.

[23]Expressed as a lead of money growth the lead would be about 7.5 and 8.75 years, respectively, which appears relatively large. The phase difference for the 12-16 years band turned out to be very unstable and is not shown here.

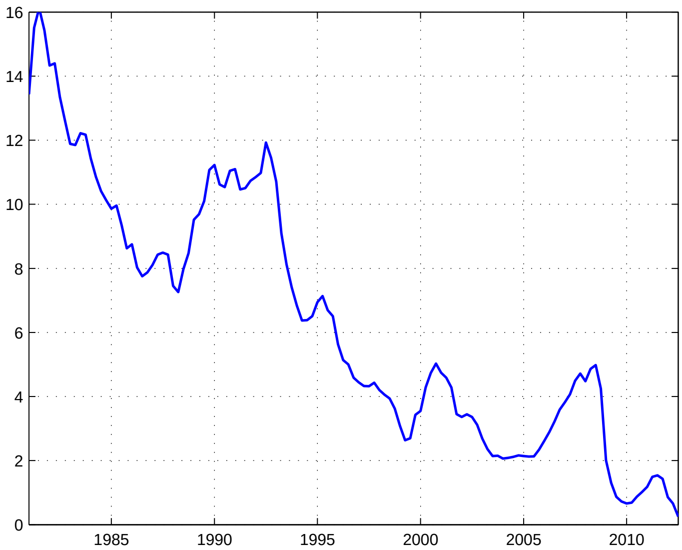

Figure 14: Euro area short-term interest rate (percent)

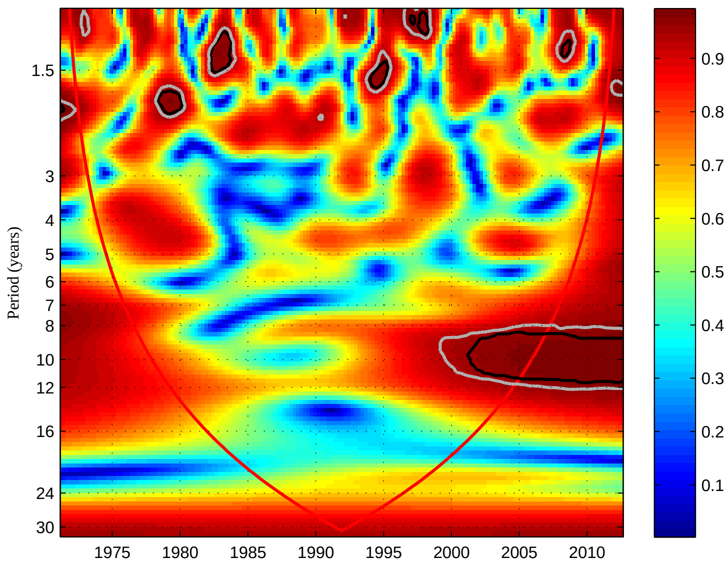

Figure 15: Wavelet chereny of annual growth rate of M3 (adjusted for real GDP growth) and HICP inflation, controlling for changes in short-term interest rates.

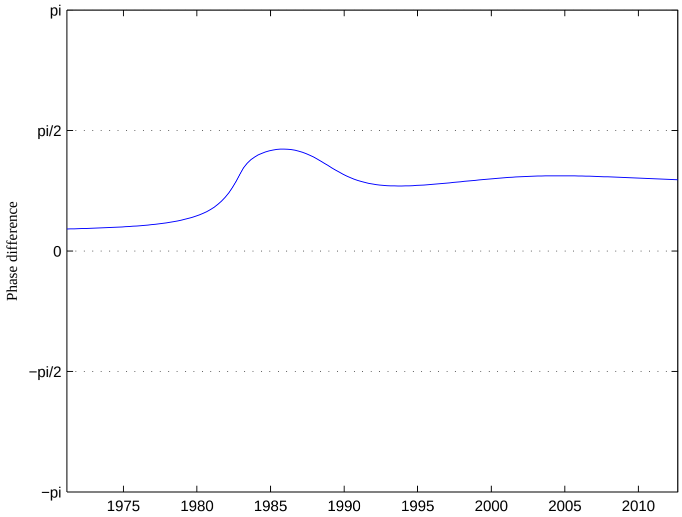

Figure 16: Phase difference of HICP inflation and annual growth rate of M3 (adjusted for real GDP growth), controlling for the change in short-term interest rates - periods of 8-12 years.

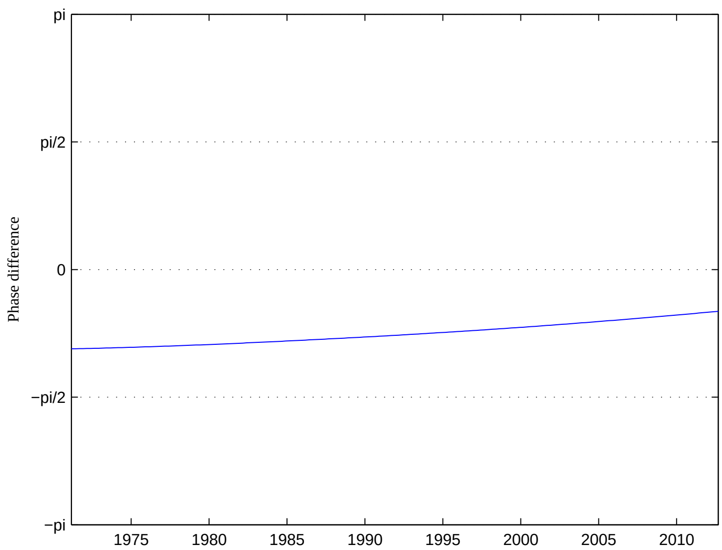

Figure 17: Phase difference of HICP inflation and annual growth rate of M3 (adjusted for real GDP growth), controlling for the change in short-term interest rates - periods of 24-32 years.

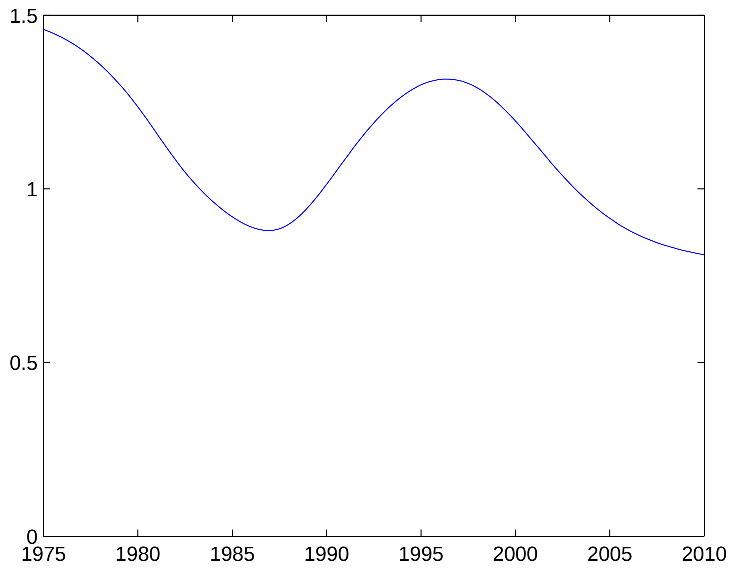

Figure 18: Cross-spectral gain of of annual growth rate of M3 and HICP inflation, , controlling for the change in short-term interest rates - periods of 8-12 years.

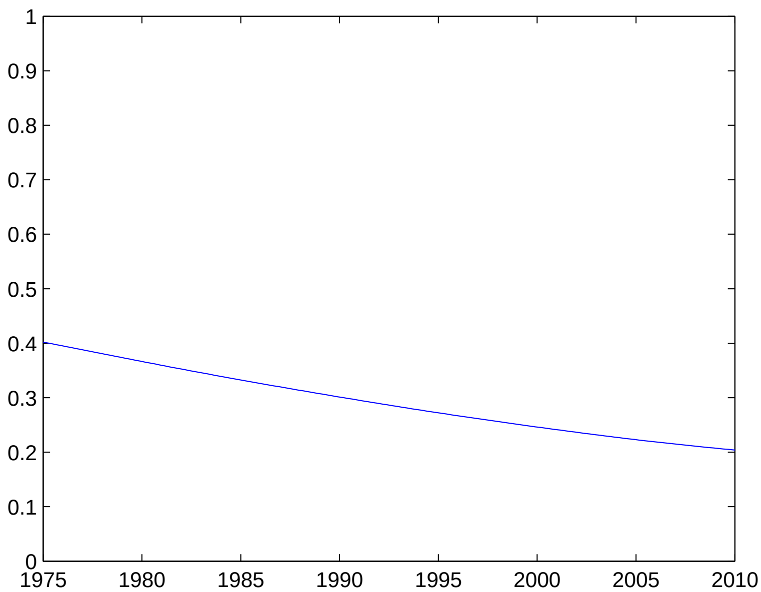

Figure 19: Cross-spectral gain of of annual growth rate of M3 and HICP inflation, periods of 24-32 years.

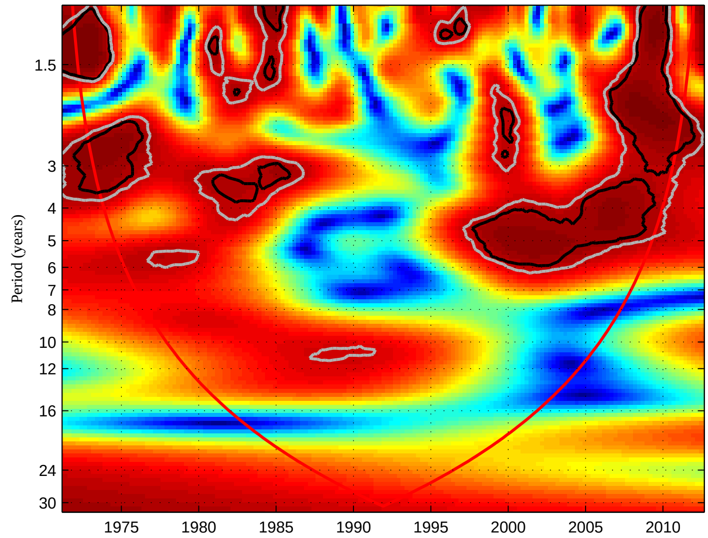

Figure 20: Wavelet chereny of annual growth rate of M1 and HICP inflation.

fluctuations with periods between 8 and 12 years turn out to be very unstable and are not shown here. For the low frequency fluctuations at the bottom of Figure 20 the estimated phase difference in Figure 21 indicates a lead of money growth before inflation of about 3.5 years, similar to the estimates for M3. The gains for both frequency ranges displayed in Figures 6 and 22 show results similar to those for M3.

Adjusting M1 growth for real GDP growth and computing the partial wavelet coherency conditional on the change in the short-term rate leads to coherencies close to one at almost all frequencies with fluctuations of eight years and longer (Figure 26) but with money growth lagging behind inflation when considering all of this frequency range (Figure 25) and a gain between 0.8 and 1.0 (Figure 24).

4 Conclusions

The empirical literature surveyed in Section 1 has found evidence for a stable relationship between money growth and inflation in the euro area, in particular between the low frequency components of these time series. However, some more recent studies indicate that this relationship might have weakened through time. Our results provide evidence for strong comovements in money growth and inflation at low frequencies with money growth as the leading variable and an estimated gain that indicates approximately a one-to-one relationship. The particular strength of the wavelet analysis is its ability to investigate time-variation in the money growth-inflation relationship. Given that the length of the time series available with about 40 years limits the frequency spectrum we can analyse using this methodology, we focus in our analysis of time variation on medium-to-long run frequencies with fluctuations of eight years up to 16 years. For these we find evidence for strong comovements between both time series up to the mid 1990s and signs of a weakening of the relationship afterwards which also reflects in a deterioration of the leading property

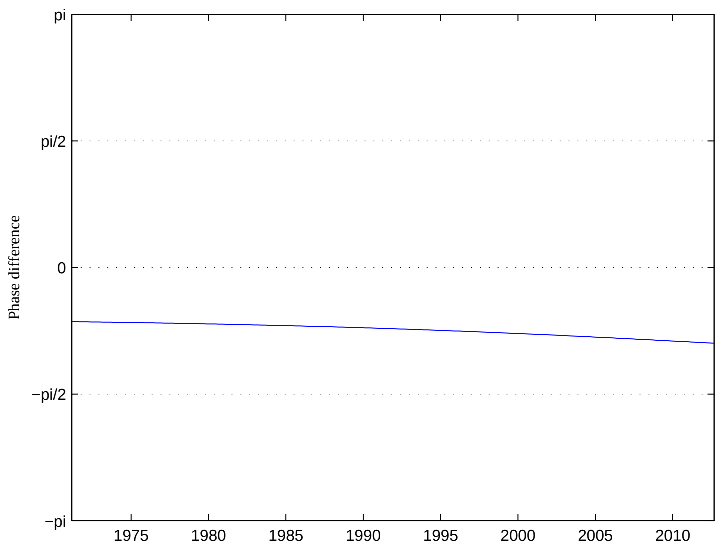

Figure 21: Phase difference of HICP inflation and annual growth rate of M1 - periods of 24-32 years.

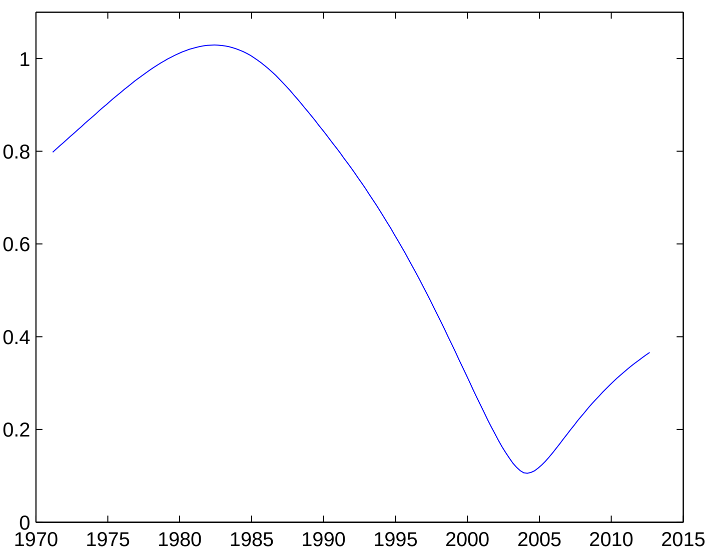

Figure 22: Cross-wavelet gain of of annual growth rate of M1 and HICP inflation, periods of 8-16 years.

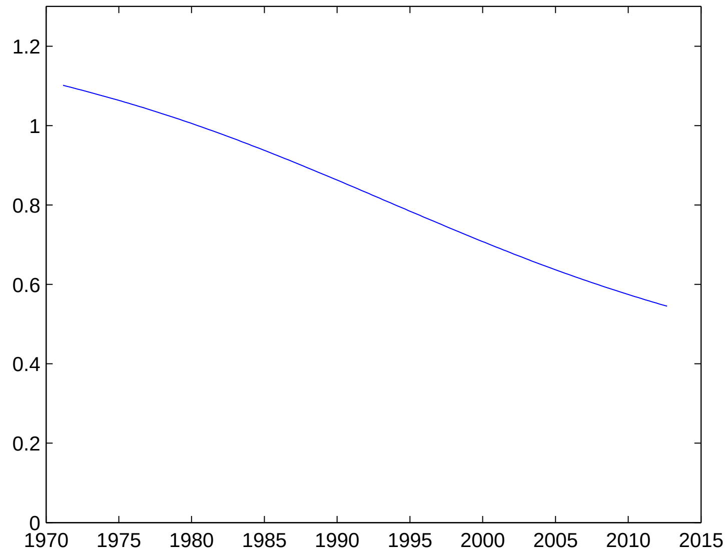

Figure 23: Cross-wavelet gain of of annual growth rate of M1 and HICP inflation, periods of 24-32 years.

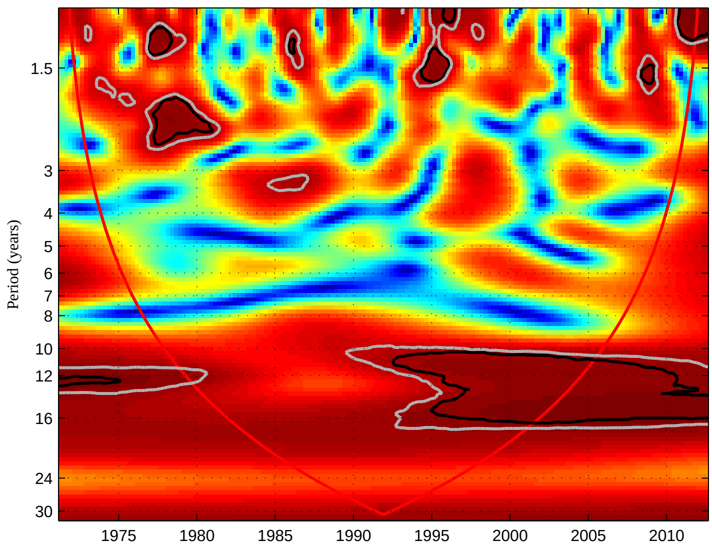

Figure 24: Wavelet cohereny of annual growth rate of M1 (adjusted for real GDP growth) and HICP inflation, controlling for changes in short-term interest rates.

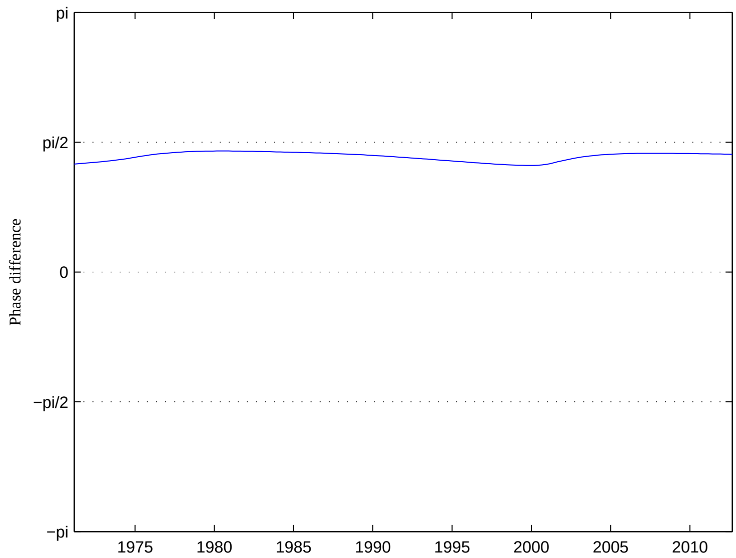

Figure 25: Phase difference of HICP inflation and annual growth rate of M1 (adjusted for real GDP growth), controlling for the change in short-term interest rates - periods of 8-32 years.

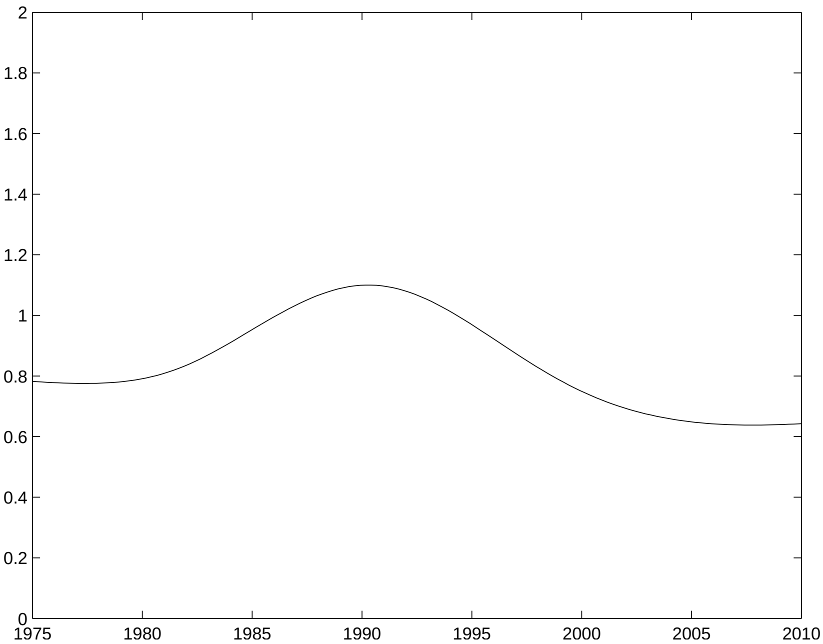

Figure 26: Cross-wavelet gain of of annual growth rate of M1 (adjusted for real GDP growth) and HICP inflation, controlling for changes in short-term interest rates - periods of 8-32 years.

of money growth before inflation as well as in a decline in the cross wavelet gain in the later part of the sample. Adjusting money growth for real GDP growth leads to broadly similar results. If we control for the correlation of changes in short-term interest rates with both money growth and inflation the strength of the relationship seems to weaken even further. The use of the monetary aggregate M1 in place of M3 also leaves the results broadly intact.

One possible interpretation of our results is that the relationship between money growth and inflation at the frequencies investigated in this paper might be regime dependent and that strong comovements are not a general but an episodic feature, such as in the 1970s and early 1980s. For example, De Grauwe and Polan (2005) show that money growth has only a significant effect on inflation in high inflation countries (countries with inflation rates of 10% or higher). Their interpretation is that inflation rates in low inflation countries are dominated by shocks unrelated to money growth with the velocity of circulation adjusting endogenously to inflation and output shocks. For high inflation countries, however, accelerating money growth leads to increasing inflation and increasing velocity which reinforces the inflationary process. Similarly, the results in Benati (2009) suggest that the relationship between money growth and inflation might be tighter in periods of high inflation and high money growth than in those where inflation and money growth are low. During low inflation episodes the relationship between money growth and inflation might be obscured by non-monetary shocks to inflation and shocks to the velocity of circulation. Sargent and Surico (2011) argue that monetary policy is central to understanding the shifting relationship between money growth and inflation: if monetary policy responds aggressively to inflation the relationship between money growth and inflation will break down. Structural change is another candidate explanation for the weakened money growth-inflation relationship. For example, changes in financial markets might have caused monetary overhangs no longer to lead quickly to an increase in consumer prices but to unload into asset price increases first.[24]

[24] See, e.g. Adalid and Detken (2007) or Bruggemann (2007).

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Appendix: Wavelet power spectra

Our empirical analysis of comovements in money growth and inflation across times and frequencies rests on the assumption that the frequency components highlighted in the main text above actually are important to the behaviour of the overall time series. For this reason the following figures show the wavelet power spectra (4) for the time series with the power spectrum increasing from dark blue to dark red.[25] For the inflation rate the power spectrum indicates important flucations at frequencies of length of 20 years and above and at frequencies with fluctations around 8 to 10 years. However, the importance of these flucations has been declining through time. The most important flucations of M3 have periods of about 12-16 years and of 24 years and longer. The results for M3 adjusted for real GDP growth are similar with additional contributions of medium and higher frequencies. Thus, overall the frequencies analysed in the main text make marked contributions to the overall variations in M3 money growth and HICP inflation. Adjusting M1 growth with real GDP growth leads to more widely distributed contributions to the overall movements.

[25]Since the power spectrum is not normalized to the region between zero and one as the coherency, the colour scale in these graphs differs from the one used for the cross wavelet coherencies. The highest values for the power spectra are coded dark red and the lowest values dark blue with the other colours used relative to this scale.

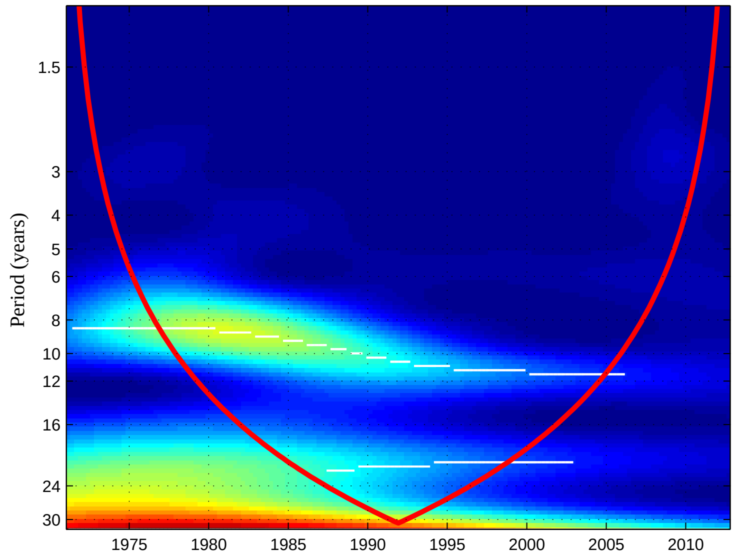

Figure 27: Wavelet power spectrum - annual HICP inflation.

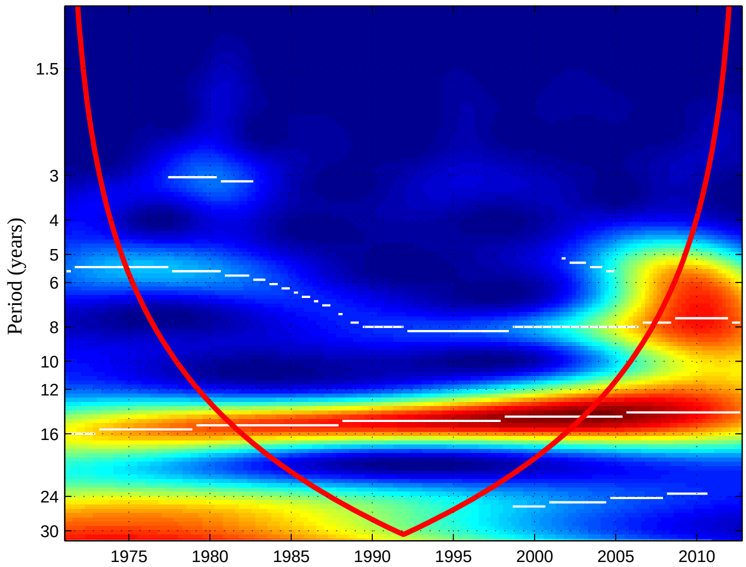

Figure 28: Wavelet power spectrum - annual growth rate of M3.

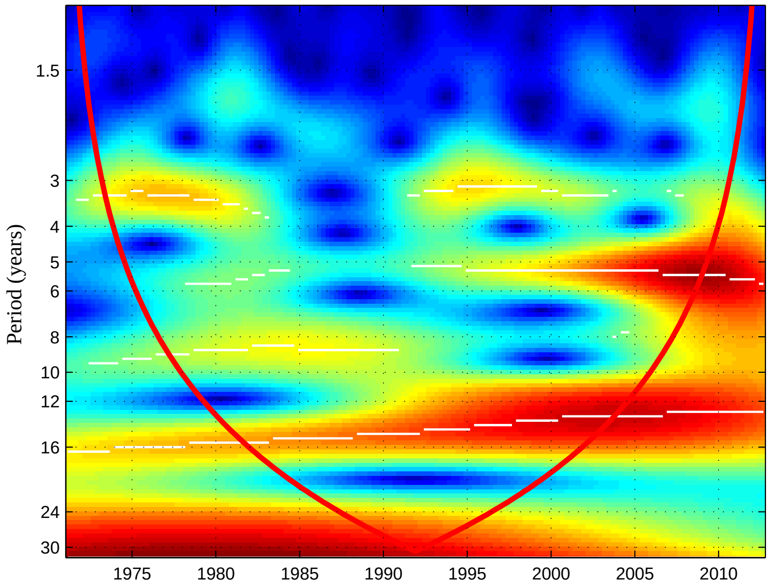

Figure 29: Wavelet power spectrum - annual growth rate of M3, adjusted for real GDP growth.

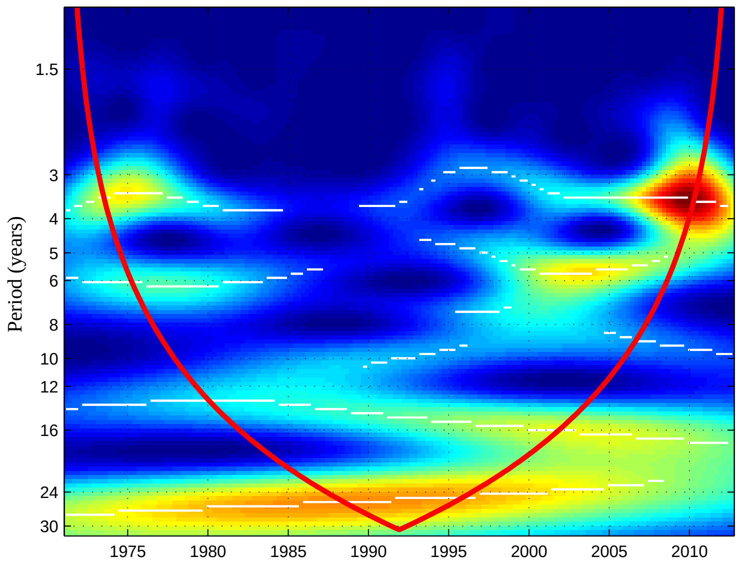

Figure 30: Wavelet power spectrum - annual growth rate of M1.

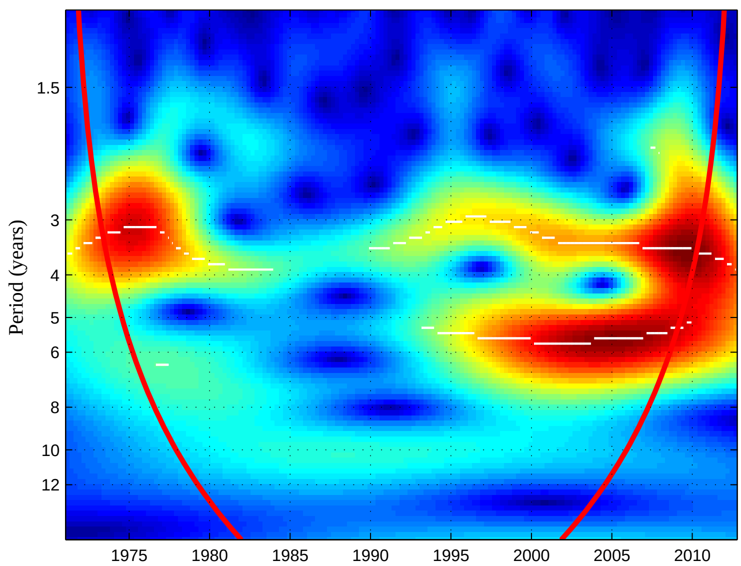

Figure 31: Wavelet power spectrum - annual growth rate of M1, adjusted for real GDP growth.