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Lethal lapses – how a positive interest rate shock might stress German life insurers

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Non-technical summary

Research Question

Interest rate risk is the single most important type of risk for life insurance companies. Particularly the risks arising from the prevailing low interest rates are currently a topic of debate. The main concern is that, in a prolonged period of low interest rates, the investment income of life insurers may no longer be sufficient to finance the rate of interest guaranteed to the customers.

However, a substantial hike in interest rates also harbors risks. In its aftermath, policyholders would profit directly from the higher capital market rates in the event of policy lapses. If they maintained their policies, by comparison, they would continue to participate in the life insurers' portfolios, the return on which would be encumbered by legacy holdings of low-yielding securities.

Contribution

This study contributes to analyse these risks. It presents a solvency balance sheet to identify a life insurer's capital buffers which prevent a "policyholder run". As a simple stress test, it can be used to derive individual critical interest rates which would cause the buffers to be depleted. Using a unique dataset with supervisory data, these critical interest rates are estimated for the around 60 bigger German life insurers from 2005 to 2013.

Results

At the end of 2013 German life insurers in aggregate would have been at risk of a policyholder run if interest rates had risen abruptly by 2.1 percentage points. Critical interest rate levels generally declined in the course of the financial and sovereign debt crisis from around 6.3% (end-2007) to around 3.8% (end-2011) for the larger German life insurance companies as a whole. Despite the challenges presented by the low-interest-rate environment, and in spite of investments in longer-dated bonds which boosted the interest rate sensitivity of life insurers' assets, the situation has not deteriorated since then. The additional interest provision ("Zinszusatzreserve") which German life insurers have been required to set aside since 2011 contributed to sending the critical interest rate level back to around 4.1% by the end of 2013. Life insurers may find it difficult to continue this positive trend in light of the quantitative easing (QE) of monetary policy in the euro area. According to anecdotal evidence, the companies are increasingly investing in long-term bonds to achieve a yield pickup. This would boost the duration of assets and increase the vulnerability to a positive interest rate shock.

Nichttechnische Zusammenfassung

Fragestellung

Das Zinsänderungsrisiko ist für Lebensversicherer die bedeutendste Risikoart. Derzeit wird vor allem das Risiko anhaltend niedriger Zinsen diskutiert. Denn langfristig fällt es Lebensversicherern bei niedrigen Marktzinsen schwerer, die Mindestverzinsung zu erwirtschaften, die sie den Versicherungsnehmern garantiert haben.
Jedoch birgt auch ein erheblicher Zinsanstieg Risiken. Denn anschließend würden die Kunden bei einer Kündigung unmittelbar von den gestiegenen Kapitalmarktzinsen profitieren. Bei einer Fortführung ihrer Verträge partizipierten sie dagegen weiter am Portfolio der Lebensversicherer, dessen Rendite durch den Altbestand gering verzinster Wertpapiere belastet würde.

Beitrag

Die Studie soll dazu beitragen, die genannten Risiken zu untersuchen. Sie stellt eine Solvenzbilanz vor, aus der unmittelbar ersichtlich ist, welche Kapitalpuffer einen Lebensversicherer vor einem Run schützen. Als einfacher Stresstest lassen sich daraus versicherungsspezifische kritische Zinsniveaus ermitteln, bei denen diese Puffer aufgebraucht wären. Mit Hilfe aufsichtlicher Einzeldaten werden diese kritischen Zinsniveaus für die größten rund 60 deutschen Lebensversicherer von 2005-2013 geschätzt.

Ergebnisse

Das Aggregat größerer deutscher Lebensversicherer wäre Ende 2013 bei einem Zinsanstieg von mehr als 2,1 Prozentpunkten von einem Run bedroht gewesen. Die kritischen Zinsniveaus sind im Laufe der Finanz- und Staatsschuldenkrise generell gesunken, im Aggregat größerer deutscher Lebensversicherer von rund 6,3% Ende 2007 auf rund 3,8% Ende 2011. Trotz der Herausforderungen aus dem Niedrigzinsumfeld und trotz einer gestiegenen Laufzeit der festverzinslichen Wertpapiere im Bestand hat sich die Situation seither etwas entschärft. Auch aufgrund der Zinszusatzreserve, die deutsche Lebensversicherer seit dem Jahr 2011 bilden müssen, erhöhte sich das kritische Zinsniveau bis Ende 2013 wieder auf rund 4,1%. Angesichts der quantitativen Lockerung der Geldpolitik im Euro-Währungsgebiet (Quantitative Easing) dürfte es den Lebensversicherern schwerfallen, diesen positiven Trend fortzusetzen. Laut anekdotischer Evidenz investieren die Unternehmen in zunehmend länger laufende Anleihen, um weiterhin auskömmliche Renditen zu erwirtschaften. Das erhöht die Zinssensitivität der Aktiva und somit die Anfälligkeit der Versicherer gegenüber einem positiven Zinsschock.

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Lethal lapses - how a positive interest rate shock might stress German life insurers *

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Abstract

Life insurers typically grant policyholders a surrender option. We demonstrate that the resulting lapse risk could materialise in the form of a "policyholder run" if interest rates were to increase sharply. An inverse stress test based on a unique set of regulatory panel data suggests that German life insurers have become less resistant to an upward interest rate shock in the course of the financial and sovereign debt crisis from 2007 to 2011. Despite the challenges presented by the low-interest-rate environment, the situation has not deteriorated since then. In light of the quantitative easing (QE) of monetary policy in the euro area, life insurers may find it difficult to continue this positive trend.

Keywords: Life insurance, interest rate risk, lapse risk, rational policyholder run, inverse stress test.

JEL classification: G22, G33, C72, C13.

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1 Introduction

Life insurers rank among Germany's most important financial intermediaries. At the end of 2013, they held assets worth nearly €800 billion at book values.[1] Owing to their traditional business models, the bulk of life insurers are exposed to similar interest rate risks. On the one hand, they grant their customers long-term guarantees. These can become difficult to honor in a low-interest-rate environment. On the other hand, they grant their customers a surrender option which allows the latter to terminate their policies at predetermined surrender values. This exposes German life insurers to risks from positive interest rate shocks.

In extreme cases, lapse risk may materialize in the form of a "policyholder run". According to Harrington (1992) three of six overindebted US life insurers experienced a mass lapse before they filed for bankruptcy in 1991. These insurers had previously suffered large losses on their junk bond investments. Although German life insurers invest almost exclusively in high quality assets, the basic risk channel appears to be transferable. If interest rates were to rise, the value of the life insurers' fixed-income assets would decrease as new investments became more attractive relative to their legacy assets. For policyholders, the difference between their surrender value and the value of participating further in the life insurer's assets would narrow.[2] When interest rates exceed a certain threshold, it would become rational for policyholders to terminate their contracts.[3]

If policyholders were to act as rational investors, the individual point in time at which they exercise the lapse option would be the solution to an optimal stopping problem.[4] This problem is difficult to solve due to several features of traditional participating life insurance contracts. For example, the profit participation and the interest rate guarantees of cliquet-style contracts create path-dependencies.[5] To derive one common threshold for the termination of heterogenous contracts with one insurer, one additionally has to consider the interaction between lapses of different policies.

The problem is greatly simplified if an instantaneous shock for an existing life insurer is analyzed. Assuming the shareholders have a limited liability, one single threshold can be derived from the life insurer's financial position, beyond which a lapse is rational for all policyholders.[6] This threshold is independent of future capital market movements. On the basis of the model of Albizzati and Geman (1994), we show that a policyholder run

[1] These data and all of that to follow disregard business for the account and at the risk of policyholders (unit-linked policies).

[2] Russell et al. (2013) put it another way: "the rates credited to the cash value inside a life insurance policy generally mimic a moving average of intermediate term interest rates or reflect the performance of the insurer's general account. Thus, crediting rates would tend to lag behind market rates in an increasing interest rate environment and make life insurance policies less attractive than alternative investments."

[3] The assumption of non-linear or S-shaped lapse rates is common in the insurance industry, cf. Loisel and Milhaud (2011).

[4] Where policyholders judge the biometric risks (eg mortality, longevity, occupational disability) covered by their life insurance to be high, their decisions may differ. This aspect is disregarded in the rest of the analysis, as are liquidity needs of the policyholders.

[5] The path-dependency implies that it is impossible to value a contract some time after inception exclusively on the basis of the initial contract data, the current interest rate and the future interest rate process. The value of the contract also depends on the historical path of interest rates.

[6] While it might even be rational for some policyholders to terminate their contracts outside this situation, all policyholders share the incentive to lapse their policies whenever such a situation arises.

is rational when the life insurer's assets no longer cover the aggregate surrender values of policyholder's claims. We refer to this situation as "economic overindebtedness".

Economic overindebtedness would not be reported as such in the solvency balance sheets under Solvency I and II.[7] However, a simple modification eliminates this problem. All necessary inputs are contained in the financial statements of German life insurers prepared according to the German Commercial Code (HGB). A policyholder run is rational when the reserves in the modified balance sheet are depleted.

The modified balance sheet is used to derive insurer-specific critical interest rates, above which a policyholder run is rational. We show that German life insurance companies became less resilient to a positive interest rate shock during the financial and sovereign debt crisis. For larger German life insurers as a whole, the critical interest rate level decreased from 6.3% at the end of 2007 to 3.8% at year-end 2011. Despite the challenges presented by the low-interest-rate environment and despite the increasing maturities of investments, the situation has not deteriorated since then. The critical interest rate level was higher, at 4.1%, at the end of 2013.

Owing to the quantitative easing (QE) of monetary policy in the euro area, we expect that life insurers will find it difficult to continue this positive trend. Anecdotal evidence indicates that the companies are increasingly investing in long-term bonds to achieve a yield pickup. This strategy is also fostered by the future regulation according to Solvency II which promotes an asset-liability match under a going-concern assumption.[8] However, this investment strategy increases the vulnerability to a positive interest rate shock. As a counter measure, German life insurers could strengthen their resilience by further reducing dividend payments and participation of the policyholders in their surplus. Derivative hedging might provide further protection, but could prove costly.

To the best of the authors' knowledge, this is the first study to address insurance runs apart from the rather descriptive article of Harrington (1992). However, this study can build on the broad literature on bank runs which sets out basic mechanisms being transferable to life insurance companies. For instance, the seminal model of Diamond and Dybvig (1983) demonstrates that a rational bank run may arise when customers expect other customers to withdraw their deposits. As all customers try to be ahead of the curve, they ultimately create a self-fulfilling prophecy.

Additionally, there is a large theoretical and empirical literature on lapse which has been reviewed by Eling and Kochanski (2013). For traditional participating (or with-profit) policies, Albizzati and Geman (1994) consider a simple case in which an insurer shares a fixed portion of the initial yield of a zero-coupon bond with the holders of a policy with an identical maturity. In the absence of (re)investment risk, the life insurers' profits are deterministic, and the profit participation equals a constant guaranteed interest rate. The only source of uncertainty in the model stems from stochastic interest rates which

[7]Under Solvency II, the solvency balance sheet aims to show best estimates of capitalized long-term cash flows under a going-concern assumption. Lapse risk is addressed in the solvency capital requirements rather than in the solvency balance sheet. However, we believe that our approach is a more straightforward means of assessing the likelihood of the extreme case of a policyholder run.

[8]Due to the surrender option of the policyholders, the duration of liabilities is uncertain. It typically exceeds the duration of assets, if all policyholders keep their long-lasting contracts. However, it is close to zero, if all policyholders lapse their contracts. If the duration of assets matches the *expected* duration of liabilities, this would turn out to be a substantial asset-liability mismatch in case of a policyholder run.

drive the market value of the zero-coupon bond in combination with the lapse option. Termination of the contract is rational if the terminal value of an alternative contract after transaction costs and taxes exceeds the terminal value of the initial contract. A closed-form pricing-formula for a European-type surrender option is derived under the model of Heath et al. (1992). The result is generalized to cover other possible exercise dates and multiple contracts on the basis of assumed risk-pooling effects that are not explicitly modeled. Grosen and Jørgensen (2000) separate a participating policy into a risk-free bond, a European-type bonus option and an American-type surrender option. This study may be considered the first analysis of the American-type lapse option for a true participating contract, as asset returns are modeled stochastically according to Black and Scholes (1973). As in Bacinello (2003a) and Bacinello (2003b) the binomial method of Cox et al. (1979) is used to price the American-type surrender option. Closed-form solutions on the basis of partial differential equations are derived by Jensen et al. (2001), Tanskanen and Lukkarinen (2003) and Bauer et al. (2006). Among others, Nordahl (2008) uses the least-squares Monte Carlo simulation method proposed by Longstaff and Schwartz (2001) for more complex simulations.

There is a wide gap between the literature on the rational lapse of life insurance contracts and the lapse rate models applied in the insurance industry (e.g. models that assume a logarithmic dependence of lapse rates). Studies as those of De Giovanni (2010) and Le Courtois and Nakagawa (2013) try to bridge this gap, assuming a partial (ir)rationality on the part of policyholders. Loisel and Milhaud (2011) question current practitioners' deterministic and stochastic models. They discuss whether typical assumptions are suitable in the extreme event of a massive interest rate shock. As such an event has rarely occurred hitherto, their analysis builds on thought experiments rather than on statistical data.

The empirical evidence on determinants of lapse is mixed. Early studies by Dar and Dodds (1989) and Outreville (1990) mainly support the emergency fund hypothesis (EFH) that lapses are driven by emergency cash needs. Later studies such as those of Kuo et al. (2003), Kim (2012) and Russell et al. (2013) also confirm the interest rate hypothesis (IRH) that interest rates are positively related to lapse rates.[9] By contrast, the results of Kiesenbauer (2012) for the German market contradict both the EFH and the IRH except in the case of unit-linked policies.

The remainder of this paper is structured as follows. Section 2 provides a short overview of the German life insurance market (2.1) and the implicit options typically embedded in life insurance contracts (2.2). Section 3 derives a theoretical model which shows that a policyholder run would be a plausible outcome if interest rates exceeded a critical level. Section 4 explains why solvency balance sheets according to Solvency I and II fail to present a fair view of this risk (4.1). A modified solvency balance is derived (4.2) which may be used for a simple stress test (4.3). Section 5 conducts this inverse stress test for the biggest German life insurers, which number roughly 60. Following a short presentation of the data (5.1), we discuss the life insurers' individual buffers (5.2), modified asset durations (5.3) and critical interest rates (5.4). Section 6 concludes the paper.

[9] Studies of Pesando (1974), Schott (2013) and Carson and Hoyt (1992) on policy loans support the IRH. Policy loans are a withdrawal of funds from a life insurance policy while keeping the contract in force.

2 The German market for life insurance

This section illustrates the importance of life insurers for the German financial system. Additionally, it describes the characteristics of traditional German life insurance contracts as a source of interest rate risk.

2.1 Life insurance companies as important financial intermediaries in Germany

German life insurers held assets with a market value of €918 billion at the end of 2013,[10] which means they rank among Germany's most important financial intermediaries. At book values, these assets were worth €846 billion, of which 94%, or €794 billion, was investments and 6%, or €52 billion, was other assets.[11] The capital assets had net hidden reserves of €72 billion. This difference between the market value and the book value of assets is recorded outside the balance sheet. It is, however, disclosed in the notes to the financial statements prepared according to the German Commercial Code (HGB).

Table 1: Simplified accumulated balance sheet of German life insurers 2013

Assets			**Liabilities**		
Item	Book value (€bn)	Share	Item	Book value (€bn)	Share
Investments	794	93.8%	Premium reserve (provision)	710	83.9%
Other assets	52	6.2%	*thereof: additional interest provision**	*13*	*1.6%*
			Bonus and rebate provisions	51	6.0%
			thereof: BRP eligible for own funds	*41*	*4.9%*
			Other liabilities	70	8.3%
			Equity**	15	1.8%
Total	846	100.0%	Total	846	100.0%
Net hidden reserves	72	8.5%			

* The additional interest provision (Zinszusatzreserve) is a provision for future interest rate payments. It is the additional premium reserve that results from applying a market-oriented discount rate rather than the higher guaranteed interest rate.
** Equity including profit-participation capital, subordinated liabilities and special items with reserve elements.

On the liabilities side of the balance sheet, the premium reserve accounted for €710 billion, or 84%, of total liabilities. This item reflects the discounted surplus of the prospective cash outflows over the remaining premium payments by policyholders. It is common practice

[10] In order to disregard business conducted for the account and at the risk of the customer (unit-linked policies), the corresponding additional assets and liabilities in the amount of €77 billion were deducted from both sides of the balance sheet.

[11] Deposit receivables from insurance business assumed as reinsurance cover are classified as other assets.

among German life insurers to use the contract-specific guaranteed return as the technical interest rate (or discount rate). Consequently, the premium reserve mostly matches the surrender value of the policies,[12] with the exception of the additional interest provision (Zinszusatzreserve) which life insurers have been setting aside since 2011.[13] A further €51 billion, or 6%, of total liabilities are bonus and rebate provisions (Rückstellung für Beitragsrückerstattung). These provisions are used to smooth the policyholders' profit participation. Policyholders do not have any actual entitlements to €41 billion of these provisions, which makes them eligible as own funds.[14] Life insurers' equity amounts to €15 billion, or 2%, of total assets.

At the end of 2013 the lion's share of 88.7% of investments was in fixed-income securities. Equity securities accounted for another 5.7%, real estate investments for 3.9% and other assets for 1.8%.[15] Owing to the predominance of fixed-income securities, the market value of life insurers' assets is highly sensitive to interest rate changes.

2.2 Features of life insurance contracts as a source of interest rate risk

German life insurers offer three main types of contracts. Endowment policies, which pay a lump sum either after a predefined time horizon or in case of the death of the policyholder, account for 41.7% of contracts and for 41.2% of the total premium amount at the end of 2013. Annuity policies have grown in popularity recently owing to changes in German tax laws, accounting for 45.1% of all contracts. Term life insurance policies, which only make payments in case of the death of the policyholder, are less important, accounting for 13.2% of all contracts.[16]

Endowment policies and annuity policies have several important features in common. They are *participating* (or *with profits*), ie they grant the policyholders (i) a predetermined minimum return[17] and (ii) participation in the surplus. Additionally, they contain (iii) a surrender option, which allows the policyholder to terminate the contract and collect a predetermined surrender value, and (iv) protection against mortality. Both types of participating contracts can thus be decomposed into a fixed-term deposit (basic contract), a bonus option, a surrender option, and a mortality protection. Annuity policies additionally offer protection against longevity.[18]

[12] According to Section 25 (2) of the German Regulation on the Principles Underlying the Calculation of the Premium Reserve (Deckungsrückstellungsverordnung), the premium reserve must not fall below the surrender values. Until 1994 the premium reserve always equalled the sum of the surrender values. Save for the additional interest provision, that is still common practice.

[13] Life insurers are obliged by law to set aside an additional interest provision (Zinszusatzreserve) in a low-interest-rate environment. Technically, a dynamic, market-oriented cap on the discount rate is applied to calculate the premium reserve.

[14] Subject to the approval of the German Federal Financial Supervisory Authority (BaFin), life insurers may use these provisions, which are eligible as own funds, to avert financial difficulties.

[15] cf. GDV (2014b), p. 44.

[16] cf. GDV (2014a), pp. 14 ff.

[17] This minimum return does not relate to the policyholder's overall paid-in capital, but only to the savings portion, ie the accumulated premiums less costs eg the insurance broker's commission.

[18] Longevity risk refers to the biometric risk that individuals live longer than expected. If individuals are spending down their savings, longevity may force them to reduce their living standards in the old age.

The guaranteed return and the surrender option are the main sources of interest rate risk for German life insurers. A contract containing only these two features and no profit participation can be modeled as a fixed-term deposit and an American put option. The put option makes it impossible for insurance companies to exactly match the duration of their assets and liabilities without making use of derivatives. Instead, life insurers typically invest in fixed-income securities with a duration that

1. is lower than the duration of the liabilities if lapse rates stay low,
2. is higher than the duration of the liabilities in the extreme case of a mass lapse.

This duration mismatch can be disadvantageous in different interest rate environments. When interest rates are low, customers tend to keep their contracts to earn the guaranteed return (case 1 above where contracts have a high duration). Insurers' fixed-income securities thus mature before the contracts expire. Consequently, the companies have to reinvest in securities with a lower yield. The longer the low interest rates persist, the more difficult it becomes for insurers to earn the return they have guaranteed their customers ("death by a thousand cuts").

However, a sudden increase in interest rates can be disadvantageous for life insurers as well. The market value of their fixed-income securities declines, possibly creating hidden liabilities if their prices fall below book values. These hidden liabilities indicate that an insurer's return on its fixed-income securities is lower than the market rate. Policyholders are thus incentivized to terminate their contracts in order to profit from higher market rates rather than to continue participating in the insurer's portfolio (case 2 above where contracts have a low duration).

The higher interest rates rise, the more sharply the prices of fixed-income securities on the insurer's books decline, and the more difficult it becomes for the life insurer to pay the guaranteed surrender values in the case of policyholder termination.

3 A model of policyholder runs

This section presents the literature on lapses of life insurance policies and their relation to interest rate risk. On this basis, it models the potential impact of an extreme increase in interest rates on the termination of life insurance contracts. It builds upon the work of Albizzati and Geman (1994) while additionally considering the limited liability of the shareholders. As this model disregards credit risk and reinvestment risk, it allows us to carve out very clearly how an interest rate shock – as the single remaining source of risk – might cause a policyholder run.

3.1 Model with homogenous policyholders

Albizzati and Geman (1994) model a life insurer with a single contract. At time $t_1 = 0$, the policyholder makes a single payment K_0, of which a portion β is passed on to insurance agents and brokers. The remaining premium $K_0^* = (1-\beta)K_0$ is invested in a risk-free zero-coupon bond with a continuous yield of $R_{0,T}$. Let the maturity of both the bond and the policy be T, when the bond pays $V_T^M = K_0^* e^{TR_{0,T}}$.

The policyholder receives a share λ between zero and one of the continuous yield of the zero-coupon bond by way of profit participation. He or she can terminate the contract at any time without incurring any additional fees.[19]
The surrender value for the policyholder in $t \leq T$ thus gives $V_t^S = K_0^* e^{\lambda t R_{0,T}}$.[20] The life insurer is thus hedged against interest rate risk if the customer holds the contract until maturity T. In this case, the life insurer makes a risk-free profit of

$$V_T^M - V_T^S = K_0^* \left(e^{TR_{0,T}} - e^{\lambda TR_{0,T}} \right).$$

If the policyholder terminates the contract in $t < T$ before the contract matures, the life insurer has to sell the bond. At this point of time, the insurer is not hedged and might suffer a loss if the market value of the bond $V_t^M = K_0^* e^{TR_{0,T}-(T-t)R_{t,T}}$ at the new interest rate $R_{t,T}$ is below the policyholder's surrender value. The profit or loss for the life insurer is

$$V_t^M - V_t^S = K_0^* \left(e^{TR_{0,T}-(T-t)R_{t,T}} - e^{\lambda t R_{0,T}} \right).$$

Unlike Albizzati and Geman (1994) we assume that the insurer has limited liability. The insurer's shareholders will only bear losses until its own funds have been depleted. Upon inception of the life insurance contract, the insurer receives a capital contribution of E_0 from its shareholders. These own funds are invested in risk-free zero-coupon bonds which mature in T. Thus, the insurer holds total investments of

$$A_t = (E_0 + K_0^*)\, e^{TR_{0,T}-(T-t)R_{t,T}}.$$

Shareholders in the life insurer receive the residual of the assets after all policyholder claims have been satisfied. Shareholders' liability is limited to their capital contribution. Consequently, the value of their equity investment cannot fall below its floor of zero. On the flip side, policyholders will never receive more than the market value of the insurers' total assets A_t, even if their surrender value (V_t^S) is higher. As we assume that there are no additional costs in case of an insolvency, the life insurer pays its policyholders V_t^P and shareholders V_t^E in the event of termination, with

$$V_t^P = min\left[V_t^S; A_t\right],$$
$$V_t^E = max\left[A_t - V_t^S; 0\right].$$

The surrender value might exceed the market value of the life insurer's total assets $(V_t^S > A_t)$.[21] In that case, a termination of the contract would force the life insurer into insolvency. The policyholder receives the market value of assets A_t, with an equivalent final value of

$$A_T = (E_0 + K_0^*)\, e^{TR_{0,T}}.$$

[19] Albizzati and Geman (1994) cite the fierce competition in the life insurance market to justify this assumption. In Germany this assumption seems equally plausible as the *Bundesgerichtshof*, the highest German court for civil and criminal proceedings, has declared common clauses on cancellation fees invalid.

[20] If the policyholder holds the contract until maturity, this equals a termination of the contract at time T.

[21] In the model, this can only occur before the contract matures. Otherwise, the life insurer is hedged against interest rate risks.

Staying with the same example, if the policyholder decides, by contrast, to hold the contract until maturity, he or she only receives the fixed final value of

$$V_T^S = K_0^* e^{\lambda T R_{0,T}}.$$

Consequently, it is rational for the policyholder to terminate the contract. In addition to the fixed final value, he or she receives the compounded capital contribution of shareholders $E_0 e^{\lambda T R_{0,T}}$ as well as their profit participation $K_0^* \left(e^{T R_{0,T}} - e^{\lambda T R_{0,T}}\right)$. If the policyholder does not terminate the contract, he or she would lose part of these payments in the event of a decline in interest rates. Additionally, he or she cannot profit from a further increase in interest rates by continuing to hold the contract.[22]

3.2 Model with heterogenous policyholders

For a life insurer with two ongoing contracts, we consider an identical scenario in which at time t the sum of both surrender values exceeds the market value of assets A_t and the single values stay below A_t:

$$V_{1,t}^S + V_{2,t}^S > A_t, \quad \text{but} \quad V_{j,t}^S \leq A_t. \tag{1}$$

The first contract is again opened at $t_1 = 0$ with a savings portion of the initial single premium $K_{1,0}^*$ and a profit participation λ_1. The second contract is opened at $t_2 > t_1$ with a savings portion K_{2,t_2}^* and a profit participation λ_2. Thus, the surrender values of the contracts at time $t \geq t_2$ equal

$$V_{1,t}^S = K_{1,0}^* e^{\lambda_1 t R_{0,T}} \quad \text{and} \quad V_{2,t}^S = K_{2,t_2}^* e^{\lambda_2 (t-t_2) R_{t_2,T}}.$$

The market value of assets at time $t \geq t_2$ equals

$$A_t = \left[(E_0 + K_{1,0}^*) e^{T R_{0,T}} + K_{2,t_2}^* e^{(T-t_2) R_{t_2,T}}\right] e^{-(T-t) R_{t,T}}.$$

From (1) we see that for both contracts the inequality

$$V_{j,t}^S > \frac{V_{j,t}^S}{V_{1,t}^S + V_{2,t}^S} A_t$$

has to be satisfied. It implies that for each policyholder j the surrender value $V_{j,t}^S$ is greater than the proportional claim on the life insurer's assets in case of an insolvency. Without loss of generality we may assume that the following inequality between the interest rates and the profit participations holds:[23]

$$(T - t_2) R_{t_2,T} - \lambda_2 (t - t_2) R_{t_2,T} \geq T R_{0,T} - \lambda_1 t R_{0,T}.$$

From this inequality we obtain a lower bound for the interest rate at time t (see Appendix):

$$(T - t) R_{t,T} > (T - \lambda_1 t) R_{0,T}. \tag{2}$$

Both policyholders decide simultaneously whether or not to terminate their contracts at time t. Regardless of the second policyholder's decision, it is rational for the first policyholder to lapse his or her contract.

[22] Our model differs strongly from that of Albizzati and Geman (1994) in this respect.

[23] If this inequality is not fulfilled, one can show $(T-t)R_{t,T} > [T - t_2 - \lambda_2(t - t_2)]R_{t_2,T}$ instead of inequality (2). For the remaining arguments one has to switch the roles of the policyholders.

i) If only the first policyholder lapses his or her contract, he or she receives the surrender value $V^S_{1,t}$ corresponding to a final value of $K^*_{1,0}e^{\lambda_1 t R_{0,T}}e^{(T-t)R_{t,T}}$. Using inequality (2), we find that this final value exceeds the final value without lapsing the contract:

$$K^*_{1,0}e^{\lambda_1 t R_{0,T}}e^{(T-t)R_{t,T}} > K^*_{1,0}e^{TR_{0,T}} > V^S_{1,T}.$$

ii) If both policyholders lapse their contract at time t, each of them can only receive a fraction of the assets' market value. Assuming the first policyholder receives his or her proportional share of the life insurer's assets $\frac{V^S_{1,t}}{V^S_{1,t}+V^S_{2,t}}A_t$, this corresponds to a final value of

$$A_T - \frac{V^S_{2,t}}{V^S_{1,t}+V^S_{2,t}}A_T.$$

From (1) it follows that this final value exceeds the final value without lapsing the contract:

$$A_T - \frac{V^S_{2,t}}{V^S_{1,t}+V^S_{2,t}}A_t e^{(T-t)R_{t,T}} > A_T - V^S_{2,t}e^{(T-t)R_{t,T}} = \left(A_t - V^S_{2,t}\right)e^{(T-t)R_{t,T}}.$$

As the first policyholder has an incentive to terminate his or her contract, it is rational for the second policyholder to anticipate this decision and to terminate his or her contract as well. In that case the second policyholder receives the final payoff $\frac{V^S_{2,t}}{V^S_{1,t}+V^S_{2,t}}A_t e^{(T-t)}$, which is larger than the final payoff in case of continuation of the contract $\min\{V^S_{2,T}, (A_t - V^S_{1,t})e^{(T-t)R_{t,T}}\}$. If the market value of a life insurer's assets falls below the sum of the surrender values of the policyholders, it is always rational for at least one policyholder to lapse his or her contract. Anticipating this decision, it becomes rational for at least one other policy- holder to terminate his or her contract as well. Consequently, in the Nash equilibrium, all policyholders lapse their contracts simultaneously.
A relaxation of the assumptions of our model would leave the main findings unaffected. However, the introduction of taxes, biometric risks and cancellation fees could increase the critical threshold, above which a policyholder run occurs. For cancellation fees, the prohibitive impact seems rather low from a legal perspective. Most German life insurers charge negligible fees as the *Bundesgerichtshof*, the highest German court for civil and criminal proceedings, has declared common clauses on cancellation fees invalid. However, the high cancellation fees as provided for in old contracts may still have a psychological impact and thus constrain lapses.[24] Regulatory measures might work in two directions. While an occasional reduction of surrender values could prevent a policyholder run, the anticipation of such a measure might have the opposite effect.[25] It could not only lower the hurdle for policy lapses, but even create spill-over effects within the industry. Another property of the model is that it tends to estimate the critical threshold very conservatively. It derives one single threshold beyond which a termination of all policies becomes equally

[24]The empirical analysis of Kiesenbauer (2012) provides evidence that it has long been common sense in Germany to terminate life insurance policies only in case of urgent liquidity needs.

[25]The German Federal Financial Supervisory Authority (BaFin) is permitted to reduce the surrender values of financially troubled life insurers.

rational. For some contracts, a termination may be rational even earlier.[26] In sum, the modeled critical threshold seems to be a fair point estimate.

4 Measuring the solvency of German life insurers with respect to lapse risk

This section illustrates the regulation under Solvency I and II with regard to lapse risk. It argues that the current balance sheet according to Solvency I might, in a modified version, still be useful for detecting short-run risks. Instead of one single solvency balance sheet as provided for under Solvency II, two separate solvency balance sheets might enhance clarity and robustness.

4.1 Solvency balance sheets under Solvency I and II

The current Solvency I regulatory framework takes on a rather *short-term gone-concern perspective*. It mainly shows an insurer's short-term capacity to fulfill the claims of policyholders and other debtors.[27] However, two properties block this view on short-term risks. First, financial assets are mostly carried at (amortized) cost. Changes to their market value are recorded off balance sheet. For example, rising interest rates typically reduce the market value of fixed-income assets but not their book value.[28] Second, the liabilities blend current claims with an additional interest provision. The technical provisions for each contract must at least equal the surrender value floor. What is more, German life insurers have been required to set aside an additional interest provision (Zinszusatzreserve) since 2011.

Solvency II was designed purely to take on a *long-term going-concern perspective*. Both assets and liabilities are reported on a market-consistent basis. For most financial assets, market values are readily available. For the bulk of an insurer's liabilities, a transfer value between knowledegable and willing parties in an arm's length transaction has to be estimated. The resulting technical provision of a contract may be lower than its surrender value or even negative.[29] The difference between the amount payable on surrender and the best estimate of a policy's value is denoted as surrender strain. In sum, the solvency balance sheet represents best estimates of future cash flows. The long-term perspective implies that the short-term lapse risk cannot be shown in the solvency balance sheet. It is addressed by the lapse risk module of the solvency capital requirements (SCR). These are designed to ensure that the insurer holds enough capital to cope with deviations from

[26] The empirical analysis even amplifies this conservatism. The estimated surrender values disregard any participation in the terminal bonus in case of a lapse.

[27] Solvency I builds on local GAAP which in Germany is prescribed by the Commercial Code (HGB).

[28] Insurance companies are allowed to classify most investments as fixed assets in their financial statement prepared according to the German Commercial Code (section 341b HGB) which is also used for regulatory purposes under Solvency I. The book value of fixed assets is only written down if the market value is expected to stay below the book value on the long run. That is typically not the case for fixed-income assets which lose value if interest rates increase. Such changes in market value are regarded as temporary.

[29] See European Commission (2014), p. 16 (11)+(12).

this best estimate in 99.5% of all years. However, we believe that an additional short-term-oriented balance sheet is the most straightforward and transparent way to analyse this risk.

4.2 A modified solvency balance sheet

We present a modified balance sheet to fulfill this task. A policyholder run is rational when the market value of the insurer's assets falls below the surrender values of the policyholders' claims. *Vice versa*, an insurer is safe as long as it is capable of fulfilling the claims of policyholders and other debtors. Exactly this condition would be scrutinized by a modified solvency balance sheet according to Solvency I if financial assets were to be reported at fair value and technical provisions were to be reported at the surrender values of the policies.

All necessary inputs are contained in the financial statements of German life insurers prepared according to the German Commercial Code (HGB) as presented in section 1. The market value of total assets $A_{i,t}$ can be computed by adding the book value of total assets $A_{i,t}^{BV}$ and the net hidden reserves $NSR_{i,t}$ which are provided in the notes. The surrender values of policyholders $V_{i,t}^{S}$ can be estimated as the premium reserve $PR_{i,t}$ less the additional interest provision $AIP_{i,t}$.

Table 2: Alternative solvency balance sheet of German life insurers 2013

Assets			Liabilities		
Item	Market value (€ bn)	Share	Item	Book value (€ bn)	Share
Investments	865	94.3%	Surrender values	696	75.9%
Other assets	52	5.7%	Other liabilities*	81	8.8%
			Capital buffer	140	15.3%
Total	918	100.0%	Total	918	100.0%

* Other liabilities include components of equity and the bonus and rebate provisions that do not qualify as regulatory own funds under Solvency I.

Table 2 shows the modified solvency balance sheet for the aggregate of German life insurance companies at the end of 2013. The market value of their investments amounted to around € 865 billion, while the market value of other assets stood at about € 52 billion. These figures were set against policyholders' cumulated surrender values of around € 696 billion, estimated as premium reserves less additional interest provisions. In addition, there were liabilities and provisions that do not qualify as own funds to the tune of around € 81 billion. In the event of policy lapses, the difference between these assets and liabilities would leave a buffer of about € 140 billion.[30] This capital buffer, which protects German life insurers against a policyholder run, is relatively large. Viewed in relation to

[30]This buffer is calculated as the market value of assets of € 917.6 billion, less the premium reserve without the additional interest provision of € 696.3 billion, and less other liabilities and provisions that do not qualify as own funds of € 81.0 billion. The figure of € 140.3 billion also corresponds to the sum of additional interest provisions, own funds and valuation reserves.

the market value of the assets, it is equivalent to around 15%.

4.3 A simple stress test

A policyholder run is rational when the capital buffer in the modified solvency balance sheet has been depleted. In the short run, this situation might arise if the market value of asset declines by the size of the capital buffer, denoted as $\Delta A_{i,t}^{crit}$, due to a positive interest rate shock. Using the modified duration of life insurers' assets $DurA_{i,t}$, it is possible to estimate the critical change in interest rates $\Delta r_{i,t}^{crit}$ which would cause the buffer to be depleted:

$$\Delta A_{i,t}^{crit} \approx DurA_{i,t} \Delta r_{i,t}^{crit} A_{i,t}.$$

The modified duration is equivalent to the percentage change in the market value of assets in the event of a parallel upward shift in the yield curve by one percentage point. On aggregate for the larger German life insurers which each had more than €1 billion in premium reserves, the estimated modified duration is 7.2 at the end of 2013. An interest rate rise of 1 percentage point would approximately reduce the market value of all assets of those companies by around 7.2%.
This duration is then used to estimate the critical increase in interest rates

$$\Delta r_{i,t}^{crit} \approx \frac{\frac{\Delta A_{i,t}^{crit}}{A_{i,t}}}{DurA_{i,t}}. \tag{3}$$

Equation (3) is straightforward if it is decomposed into two elements. In the nominator, the capital buffer is related to the market value of all assets. The denominator contains the interest rate sensitivity of the market value of assets. The whole term reveals which positive interest rate shock would diminish the market value of assets by more than the size of the buffer.
At the end of 2013 the buffer of all German life insurers accounted for €140 billion of the assets' market value of €918 billion, ie about 15.3%. Given the asset duration of 7.2, the market value of assets falls by this amount if interest rates rise by $\frac{15.3\%}{7.2\%}$ or 2.1 percentage points. If the interest rate level were to rise abruptly by 2.1 percentage points, the aggregate buffer of all German life insurers in aggregate would be fully depleted.

5 Empirical analysis

In this section, critical interest rate *shocks* are calculated for individual life insurers. These are added to the yields on listed Bunds with a residual maturity of ten years to give enterprise-specific critical interest rate *levels*. The subsections follow the decomposition of criticial interest rate shocks into a firm-specific capital buffer and a firm-specific asset duration. Both elements are analysed separately before the resulting critical interest rates are assessed.

5.1 Data

The modified solvency balance sheets for German life insurers between 2005 and 2013 are prepared using data provided by the German Federal Financial Supervisory Authority

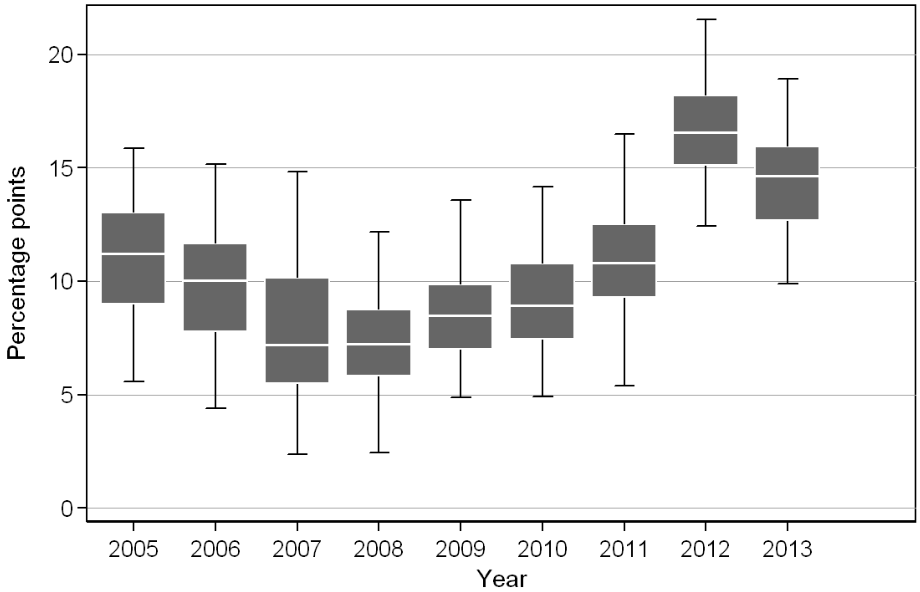
(BaFin) on the life insurers' investments (book and market values) and their balance sheets.[31] These data are also contained in the insurers' financial statements prepared according to the German Commercial Code (HGB), which are publicly available. The modified asset durations of the individual life insurers are estimated using non-public BaFin projections. These projections are conducted at least once a year. BaFin asks about 90 German insurance companies to predict selected positions of their financial statements at year-end under different scenarios. The life insurers mostly have to simulate the consequences of a decline in a stock market index and an increase in interest rates.
Numerous checks have shown virtually no data errors for the larger German life insurers. Outliers in the estimated modified durations and other variables (e.g. asset growth and relative buffers) are restricted to smaller companies and mostly result from special business models. To avoid a potential bias, we disregarded German life insurers which each had less than €1 billion in premium reserves.

5.2 Relative buffers

At the end of 2013, the capital buffer accounted for 14.6% of the market value of aggregated assets for the median of larger German life insurance companies which each had less than €1 billion in premium reserves. The minimum figure at the end of 2013 was 9.3%, and the maximum figure was 20.9% of the assets' market value.
Figure 1 shows that the capital buffer shrank from the end of 2005 until the onset of the financial crisis in 2007. After that, it increased almost steadily until the end of 2012.

[31] These data are taken from the BaFin reporting templates Formblatt 100 and Nachweis 101.

Figure 1: Relative capital buffers of large German life insurers.



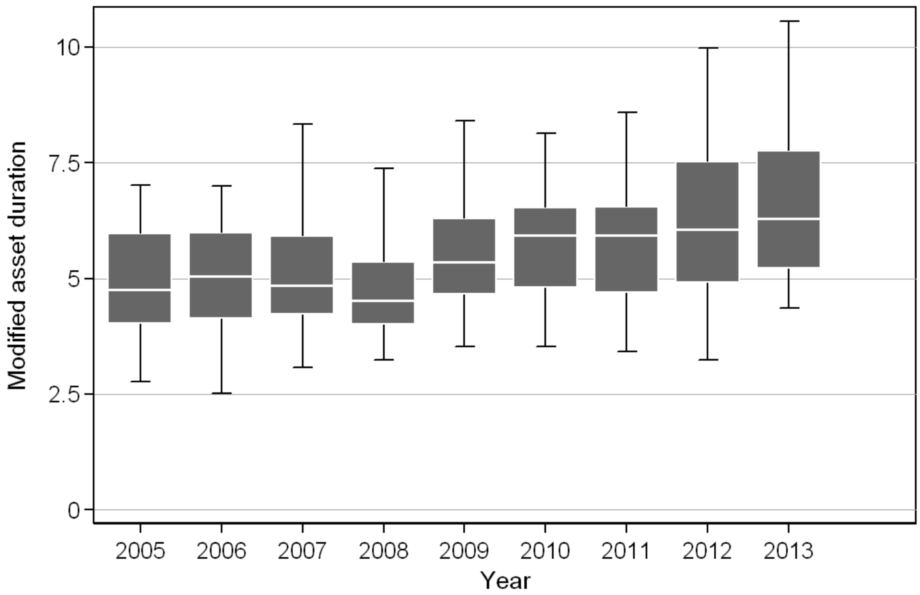
Notes. The boxplots show the capital buffer of the around 60 largest German life insurers which each had more than €1 billion in premium reserves from the end of 2005 to year-end 2013. The capital buffer includes own funds, the additional interest provision and net hidden reserves. It is related to the market value of assets, excluding those held for the account and at the risk of the customer.
The outer lines (whiskers) start at the 5% and 95% percentiles. The inner boxes mark the median, the 25% and the 75% percentiles.

This reflects the low-interest-rate environment following the financial and sovereign debt crisis. The fall in interest rates increased the market value of life insurers' legacy fixed-income securities with their relatively high yield. This more than compensated insurers for the effect of higher counterparty risks. However, this exogenous shock has a limited effective period. Even when interest rates remain low, the high yielding legacy assets of the insurers draw nearer maturity. That reduces the net silent reserves which make up a large part of the insurers' capital buffers. This effect will become observable at the latest when long-run interest rates have approached their lower bound.

5.3 Duration of assets

Figure 2 shows that the average modified duration of assets has increased since 2005. This partly reflects the efforts of German life insurers to improve their (expected) asset-liability match. Additionally, the companies have been looking to earn a yield pick-up since the beginning of the low interest environment. Stepping up their investment in longer-dated bonds, they boosted the interest rate sensitivity of their assets.

Figure 2: Modified duration of German life insurers' assets



Source: Own calculations based on BaFin projections.
Notes. The boxplots show the modified duration of assets of the around 60 largest German life insurers which each had more than €1 billion in premium reserves from the end of 2005 to year-end 2013. The modified duration estimates the percentage by which a parallel upward shift in the entire yield curve by one percentage point would affect the market value of the life insurers' assets.
The outer lines (whiskers) start at the 5% and 95% percentiles. The inner boxes denote the median, the 25% and the 75% percentiles.

These long-term investments are widely regarded as improving of the life insurers' asset-liability match. That is true if lapse rates remain low. However, life insurers become more vulnerable to a positive interest rate shock. This is illustrated by the firm-specific critical interest rate, which contracts if companies invest in longer-dated bonds. It has to be added that not all life insurers follow this investment strategy. Heterogeneity among companies has increased in recent years.

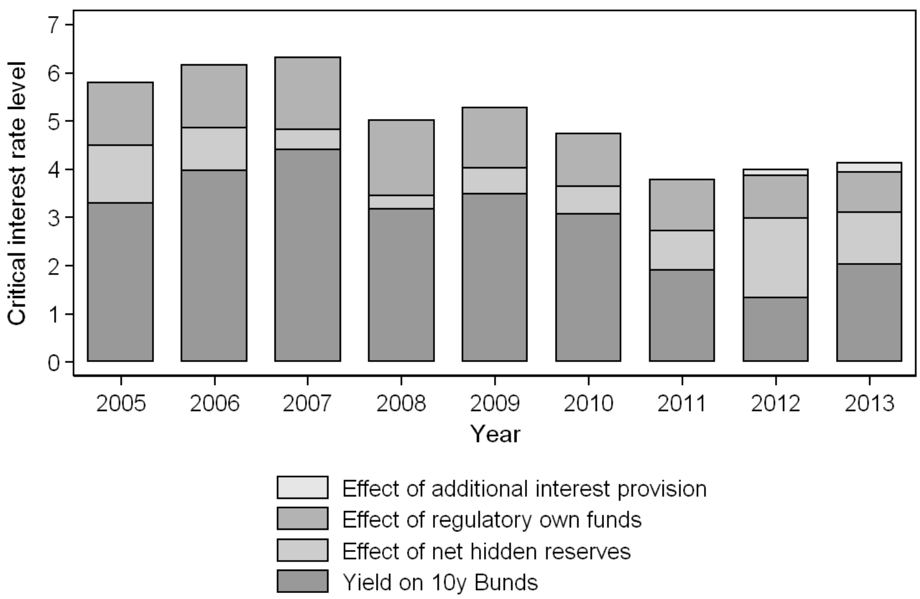
5.4 Critical interest rate

The firm-specific critical interest rate *level* depends on two factors. The first factor is the firm-specific critical interest rate *shock* which results from the interaction of each company's buffer and the duration of its assets. The second factor is the interest rate level.

Figure 3 decomposes the critical interest level of the around 60 largest German life insurers which each had more than €1 billion in premium reserves. It illustrates that the critical interest rate level is not affected by mere changes in interest rates. In the short run, such changes in interest rates are nullified by the effect of the associated changes in life insurers' buffers. For example, the short-term fluctuations in interest rates from 2011 to 2013 are offset by the effect of the corresponding change in the net silent reserves. The critical interest rate level thus reflects (i) any change in buffers except those that result directly from short-term fluctuations of interest rates, and (ii) any change in the duration of assets.

The figure shows that larger German life insurers as a whole built up reserves from 2005 to 2007. The critical interest rate level has generally declined during the financial and sovereign debt crisis. For larger German life insurers as a whole, it decreased from 6.3% at the end of 2007 to 3.8% at year-end 2011. The onset of the financial crisis in 2008 and the emergence of the European sovereign debt crisis in 2010 both had a negative impact.

Figure 3: Critical interest rate levels (in per cent)



Notes. The bar charts show the critical interest rate level for the aggregate of the around 60 largest German life insurers which each had more than €1 billion in premium reserves from the end of 2005 to year-end 2013. Above the critical interest rate levels, the market value of the life insurers' total assets would no longer be sufficient, on an aggregate level, to cover policyholders' surrender values and other liabilities. The critical interest rate level is decomposed into the interest rate *level* of German government bonds with a residual maturity of 10 years and the critical interest rate *shock*. The critical interest rate *shock* results from three buffers: the net hidden reserves, the regulatory own funds and the additional interest provision.

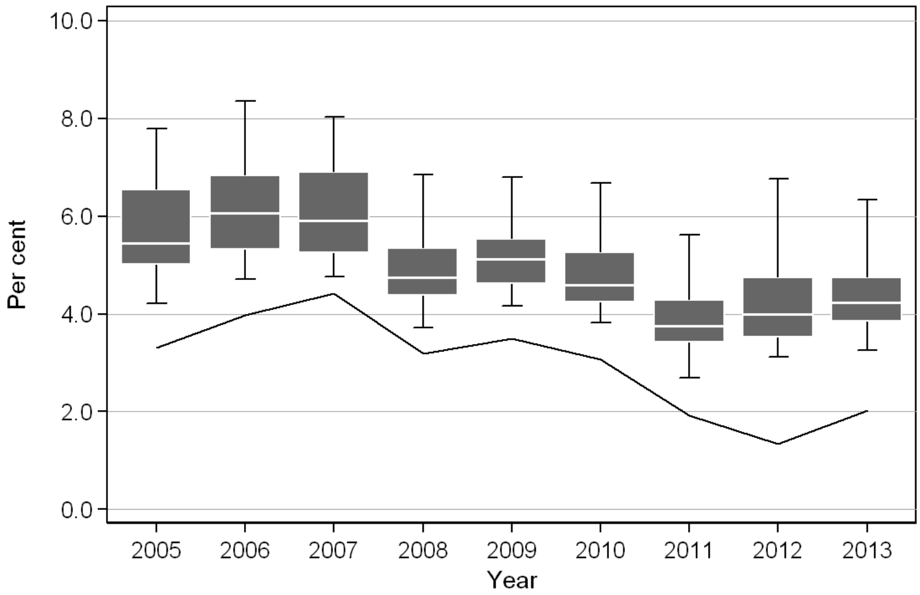
The critical interest rate level increased after that to 4.1% at the end of 2013, which is remarkable given the challenges presented by the low-interest-rate environment and the increasing duration of the life insurers' assets. On the one hand, German life insurers reduced their profit distributions to shareholders and customers to preserve their regulatory own funds.[32] On the other hand, they are required by law to set aside an additional inter-

[32] For life insurers, it is up to the companies to decide on the level of profits they want to report. Currently, this profit can be set almost arbitrarily by realizing hidden reserves. Of this profit, a high share has to be distributed to policyholders. Life insurers are thus not free to retain earnings once these have been reported. They are, however, allowed to state low profits to retain funds in the company and to build up buffers outside the balance sheet.

est provision (Zinszusatzreserve) in a low-interest-rate environment. This buffer seems to play a small, but nonetheless essential role in safeguarding the internal capital adequacy of German life insurers.

Figure 4 presents the distribution of critical interest rates among German life insurers. It shows that the dispersion of critical interest rates among life insurers narrowed with the onset of the financial crisis in 2008. It appears to have been rather stable since then.[33]

Figure 4: Critical interest rate levels and actual interest rate level (in per cent)



Notes. The boxplots show the critical interest rate level of the around 60 largest German life insurers which each had more than €1 billion in premium reserves from the end of 2005 to year-end 2013. Above the critical interest rate levels, the market value of the life insurers' total assets would no longer be sufficient to cover policyholders' surrender values and other liabilities. The critical interest rate levels relate to Bunds with a residual maturity of 10 years. They assume a parallel upward shift in the entire yield curve.
The outer lines (whiskers) start at the 5% and 95% percentiles. The inner boxes denote the median, the 25% and the 75% percentiles.
The line below shows the historical yield of Bunds with a residual maturity of 10 years.

An empirical investigation reveals no systematic relationship between the key characteristics of German life insurers such as size, legal form and ownership and their resilience against a positive interest rate shock. Only the current average return on the life insurers' investment is significantly different from zero on the 5%-level at the end of 2013 (see Appendix B).

[33]The distribution of the critical *change* in interest rates is almost identical to the distribution of the corresponding levels shown in Figure 4 and is therefore not discussed here.

6 Conclusion

The risks arising for German life insurers from the prevailing low-interest-rate environment, which is intensified by the quantitative easing (QE) of monetary policy in the euro area, have been widely discussed. However, an exit from low interest rates also harbors risks. A substantial hike in interest rates could incentivize policyholders to lapse their contracts. Policyholders doing so would profit directly from the higher capital market rates instead of further participating in the life insurers' portfolios, the return on which would be encumbered by legacy holdings of low-yielding securities.

In extreme cases, lapse risk can materialize in the form of a policyholder run. On a theoretical foundation, this study shows which buffers protect a life insurer against this risk. These buffers can be scrutinized using a solvency balance sheet according to Solvency I. The only modifications are that financial assets have to be reported at fair value and technical provisions at the surrender values of the policies. The solvency balance sheet as provided for under Solvency II takes on a long-term perspective, while turning a blind eye on short-term lapse risk. It might thus prove useful to amend the Solvency II regulation by the presented modified balance sheet.

This modified balance sheet is used for a simple (inverse) stress test. We derive insurer specific critical interest rates, above which a policyholder run is rational. We demonstrate that German life insurance companies became less resilient to a positive interest rate shock during the financial and sovereign debt crisis. On aggregate, the critical interest rate level of bigger German life insurers decreased from 6.3% at the end of 2007 to 3.8% at year-end 2011. Despite the challenges presented by the low-interest-rate environment, and in spite of investments in longer-dated bonds which boosted the interest rate sensitivity of assets, the situation has not deteriorated since then. As German life insurers reduced their profit distributions to shareholders and customers, and set aside an additional interest provision (Zinszusatzreserve), the critical interest rate level was higher, at 4.1%, at the end of 2013. Owing to the quantitative easing (QE) of monetary policy in the euro area, we expect that life insurers will find it difficult to continue this positive trend. Anecdotal evidence indicates that the companies are increasingly investing in long-term bonds to achieve a yield pickup. This strategy boosts the duration of assets and increases the vulnerability to a positive interest rate shock. Solvency II with its *long-term going-concern perspective* let alone seems insufficient to address this risk appropriately.

Future research could further analyse the expectations channel with regard to contagion effects and regulatory measures. Our model considers the policyholders' expectations only insofar as these anticipate the lapse behavior of their peers within the same insurance company. Possible contagion effects between different insurance companies remain unconsidered. Likewise, regulatory measures are disregarded in our analysis. For example, a reduction in surrender values may be anticipated by policyholders and thus lower the hurdle for policy lapses. The implementation of this measure could also create spillover effects from financially troubled life insurers to otherwise sound companies. These potentially undesired effects of regulation seem to warrant further attention.

Appendix A: Derivation of inequality (2)

Inequality (2) is derived assuming the following inequality between the interest rates and the profit participation:

$$(T-t_2)R_{t_2,T} - \lambda_2(t-t_2)R_{t_2,T} \geq TR_{0,T} - \lambda_1 tR_{0,T}.$$

This leads to

$$\begin{aligned}
&K^*_{1,0}e^{\lambda_1 tR_{0,T}} = V^S_{1,t}\\
&> \frac{V^S_{1,t}}{V^S_{1,t}+V^S_{2,t}}A_t\\
&= \frac{K^*_{1,0}e^{\lambda_1 tR_{0,T}}}{K^*_{1,0}e^{\lambda_1 tR_{0,T}} + K^*_{2,0}e^{\lambda_2(t-t_2)R_{t_2,T}}}\left((E_0+K^*_{1,0})e^{TR_{0,T}} + K^*_{2,0}e^{(T-t_2)R_{t_2,T}}\right)e^{-(T-t)R_{t,T}}\\
&= K^*_{1,0}e^{-(T-t)R_{t,T}}e^{TR_{0,T}}\frac{E_0+K^*_{1,0}+K^*_{2,t_2}e^{(T-t_2)R_{t_2,T}-TR_{0,T}}}{K^*_{1,0}+K^*_{2,t_2}\underbrace{e^{\lambda_2(t-t_2)R_{t_2,T}-\lambda_1 tR_{0,T}}}_{\leq e^{(T-t_2)R_{t_2,T}-TR_{0,T}}}}\\
&\geq K^*_{1,0}e^{TR_{0,T}-(T-t)R_{t,T}}.
\end{aligned}$$

It follows that

$$(T-t)R_{t,T} \geq (T-\lambda_1 t)R_{0,T}.$$

Appendix B: Key characteristics of German life insurers and their resilience against a policyholder run.

We also performed cross-sectional regressions to explore whether the resilience of German life insurers against a policyholder run is systematically related to the insurers' key characteristics.

As dependent variable we use the insurer-specific critical interest rate shock at the end of the years 2005, 2007, 2009, 2011 and 2013. We exclude variables from the regression which explain this dependent variable *by definition* such as the share and the duration of the life insurers' fixed-income investments and the buffers that protect the life insurers against a policyholder run.[34]

Table 5 shows that the key characteristics of bigger German life insurers such as their size, legal form and ownership structure are not significantly related to their resilience against a policyholder run. Only at the end of 2013, the current average return on a life insurers investment in per cent is significantly different from zero on the 5%-level.

[34] The critical interest rate shock as the dependent variable can be decomposed into a firm-specific capital buffer and a firm-specific asset duration. Consequently, both variables are mechanically related to the critical interest rate shock.

Figure 5: Summary statistics from pooled OLS regressions of the critical positive interest rate shock.

	Critical positive interest rate shock				
	2005	2007	2009	2011	2013
Current average ROI	0.376	0.0144	0.405	0.345	0.788**
	(1.34)	(0.06)	(1.01)	(0.75)	(2.28)
Size	0.00652	0.0656	0.0450	-0.124	-0.133
	(0.04)	(0.55)	(0.52)	(-1.11)	(-1.48)
Share of regulated contracts	-1.910	-0.498	-0.499	0.546	0.434
	(-1.51)	(-0.56)	(-0.72)	(0.69)	(0.66)
Public limited company	-0.702	-0.606*	-0.456	-0.342	-0.325
	(-1.57)	(-1.80)	(-1.67)	(-1.13)	(-1.22)
Public sector	0.111	0.312	-0.340	-0.223	-0.259
	(0.17)	(0.59)	(-0.93)	(-0.48)	(-0.67)
Constant	2.077	1.703	0.152	1.783	0.706
	(0.97)	(1.20)	(0.07)	(0.86)	(0.46)
N	56	57	59	60	60
R2	0.139	0.0678	0.133	0.0905	0.217

Notes. The table reports summary statistics from a pooled OLS regression of the insurer-specific critical interest rate shock at the end of the years 2005, 2007, 2009, 2011 and 2013. We analysed the around 60 largest German life insurers which each had more than €1 billion in premium reserves.
Variables: *Current average ROI* denotes the current average return on a life insurers investment in per cent, *Size* is the natural logarithm of the premium reserve, *Share of regulated contracts* refers to life insurance policies that have been taken out before the liberalization of the European insurance market in 1994, *Public limited company* is a dummy variable that equals one if life insurers have this legal form (as opposed to mutual insurance associations), *Public sector* is a dummy variable that equals one if life insurers belong to the public-sector.
Asterisks *, ** and *** indicate significance at the 0.1, 0.05 and 0.01 level, respectively; two-tailed test; t statistics in parentheses

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