Liquidity–Saving Mechanisms: Quantifying the Benefits in TARGET2

Martin Diehl – Uwe Schollmeyer[1]

Abstract
This paper quantifies the benefits of the liquidity-saving mechanisms (LSM) in TARGET2. It builds on two different models which were developed for the quantification of the benefits of LSM in an environment of fee-based liquidity provision, such as Fedwire, and for a collateral-based payment system. Calibrating with data from 2010 we conclude that considerable positive welfare effects of the implemented LSM in TARGET2 do indeed exist. Depending on the theoretical approach these welfare effects can reach the order of 170.000 to 300.000 Euro per day. However, the institutional setup for the liquidity provision for any specific RTGS has to be taken into account in any case.

JEL classification: E42, E58, G21

1 Introduction

The Eurosystem (the ECB and the national central banks of the Eurozone) operates the Trans-European Automated Real-Time Gross Express Transfer System (TARGET2) to ensure the efficient and sound clearing of large-value payments in 23 countries of the European Union. Indeed a smooth functioning of the payment system is a necessary precondition for the functioning of the transmission of the monetary policy and it constitutes the backbone of a resilient market infrastructure which is essential for financial stability. It is therefore obvious that the design of such a large-value payment system is thoroughly and constantly evaluated with regard to both reliability and efficiency.

TARGET2 builds on the user requirements and on the experiences of its predecessor-RTGS of the three providing central banks (Banca d'Italia, Banque de France and Bundesbank). It is equipped with various optimization algorithms as well some different tools for the liquidity management provided to the banks. Since the start of operations in November 2007, TARGET2 is a continuously updated high-end payment system.

The financial benefits of TARGET2 in comparison to a plain-vanilla RTGS have so far never been quantified. A first approach that was conducted by Renault/Pecceu (2007) restricted itself to the increase of the settlement efficiency of a non-FIFO offsetting algorithm compared to a FIFO (first-in first-out) algorithm.

The increased efficiency of a real-time gross settlement system (RTGS) with liquidity-saving mechanisms (LSM) given unaltered behaviour of banks is easy to proof. However, all LSM do also provide an opportunity for strategic withholding of liquidity by single actors. These opportunities may counteract the efficiency gains of the LSM as such. Therefore, models have to be employed which allow for both, liquidity-saving and liquidity-withholding.

The theoretical analysis of liquidity-saving mechanisms (LSM) in large-value payment systems which focused on both aspects starting with Martin and McAndrews (2008) has

[1] Martin Diehl and Uwe Schollmeyer are payment system analysts at the Deutsche Bundesbank. This chapter represents their judgements and views and does not necessarily reflect the opinion of the Deutsche Bundesbank.

shown that the introduction of an LSM would normally increase welfare, but under certain conditions welfare might also be reduced. A numerical solution based on simulations presented by Galbiati and Soramäki (2010) gives broadly the same results. In a simulation study based on synthetically created data Schulz (2011) differences between small, medium and large participants and notes that a LSM may have an unequal effect on differently sized banks. When the system is collateral-based instead of fee-based, the introduction of an LSM will arguably always increase the welfare (Jurgilas, Martin, 2010a).

Martin and McAndrews (2008) stated: *„Future research in this area can usefully focus on the question of the empirical magnitudes of the parameters of interest. The important parameters in the model are the cost of delay, the cost of borrowing intraday funds from the CB, the relative size of the payments made to the settlement system versus other payments, and the proportion of time-critical payments. […] [and] the probability that queued payments offset.“* Hitherto, the magnitudes of the welfare gains and of the important parameters in the model are still largely unexplored. One sole exception so far is presented in Atalay et alii (2010) with regard to the Fedwire System in the USA. This latter paper, however, cannot explore the benefits of a collateral-based payment system as the liquidity provision in Fedwire is fee-based.

Our paper tries to fill these gaps with respect to TARGET2. We show that even a relatively simply modeled LSM saves at about 45.000 to 58.000 Euro per day compared to a plain-vanilla RTGS depending on the equilibrium reached. In a more advanced setup, the savings calculate as about 170.000 to 292.000 Euro per day. This compares to the results of Atalay et alii (2010) for Fedwire where the welfare effects are calculated as between $500,000 and $2 million. The different level of savings may in part be explained by the higher transaction value of Fedwire taken into account by the respective authors. In addition, one has to keep in mind that the results are not only depending on such factors as the cost of delay, the cost of the collateral or borrowing intraday funds from the central bank, the relative size of the payments made to the system versus other payments and the proportion of time-critical payments. TARGET2, moreover, features more than a simple LSM. Besides some highly developed queuing arrangements which are the focus of our paper. It also has some other liquidity management tools such as reservations, liquidity pooling and bilateral and multilateral limits. These reservations and limits, however, could potentially also lead to a socially less efficient use of available liquidity, which has to be kept in mind when interpreting the results.

By employing the quoted models of Martin/McAndrews and Jurgilas/Martin we endeavour to support an established line of modeling payment system and behaviour of banks in payment systems. Rather than developing our own model we try to make use of available ideas. Thereby, we hope to contribute to an ongoing process of joint development of payment economics. In addition, in calibrating the models we try to stick as close as possible to the published quantification by Atalay et alii. This contributes to making the results comparable among various large-value payment systems.

The remainder of the paper is structured as follows: In section 2 some basic facts about TARGET2 as well as its technical features are presented. Section 3 outlines the basic lines of the models developed by Martin and McAndrews (2008) and Jurgilas and Martin (2010a). In Section 4 we present our calibrations of the relevant parameters with respect to the conditions in the Eurozone and discuss our findings and compare to the results of other research in this field. Finally in section 5 we offer a brief conclusion and propose some field for future research.

2 Overview of TARGET2

TARGET2 is an integrated market infrastructure provided by the Eurosystem for the processing of primarily high value and urgent payments in euro. TARGET2 is run by the Eurosystem and is the responsibility of the Governing Council of the ECB. Compared to its forerunner TARGET, which was an association of 17 differential components, TARGET2 is operated on one highly resilient single technical platform (so called Single Shared Platform, SSP). Three Eurosystem central banks – the Banca d'Italia, the Banque de France and the Deutsche Bundesbank (3CBs) – jointly provide this technical infrastructure and operate it on behalf of the Eurosystem. Nevertheless, from a legal point of view, each participating and connected central bank is responsible for the operation of its system component and maintains the business relationships with their local participants.

TARGET2 is Europe's most important payment system for urgent payments and processes a daily average of around 340,000 payments with a total value of almost 2.3 trillion Euro.

In its modular architecture, TARGET2 offers a high degree of flexibility to both central banks and participants. The actual settlement process takes place in the payments module (PM) where each of the 866 direct participants and 69 ancillary systems maintain an account. Intraday liquidity is provided free of interest in the PM either via credit lines on RTGS or central bank accounts (based on a pool of pre-deposited collateral) or via intraday repo transactions with the respective national central bank which is responsible for the business relation with the banks of its country.

Payments can be classified as "normal", "urgent", or in exceptional cases as "highly urgent". Payments can further be warehoused, i.e. the submission times can be predetermined. This together with the liquidity management tools determines the payment processing in the entry disposition where a first bilateral optimization mechanism (offsetting of payments) is employed. Normally a basic FIFO mechanism will resolve the payments in the entry disposition. However, in cases where a liquidity increase for (highly) urgent payments would result, normal payments can be processed by a FIFO by-passing principle which is an additional mechanism for saving liquidity.

If the entry disposition fails to settle a payment, this is queued according to its priority status. When in a queue, the settlement manager of a bank can intervene e.g. by reordering transactions within the queue, revoking it or by changing the priority or the set execution time. For the queued payments three different optimization procedures (algorithms) are then available to resolve the queue.

With these optimization procedures and liquidity-saving mechanisms, TARGET2 settles 50% of all transactions within 29 seconds and 90% within 42 seconds. Furthermore, only 0,21% of the volume and 1,8% of the value of all sent payments were not settled on account of a lack of funds or for breaching the sender's limit at the time the system closed.

TARGET2 offers several distinct liquidity management tools for the banks. A direct participant in the payment module has the option to control the use of available liquidity by means of a reservation and a limit system, which may be combined as required. In TARGET2, it is possible for participants to reserve liquidity for urgent and highly urgent payments and to dedicate liquidity to the settlement of ancillary systems. Participants can also define bilateral and multilateral sender limits. Furthermore, banks can use a liquidity pooling functionality within a group to view and use their liquidity, irrespective of the account on which it is held. Increased visibility within the system is also indirectly contributing to more efficient liquidity management. TARGET2 offers online information tools that allow access to all information needed in relation to the payment and liquidity situation of RTGS participants.

Overall, use of liquidity-saving features may depend on several factors. First, it is expected to vary depending on the liquidity situation. Overall use of such features can be expected to be high in tight liquidity situations and low in an environment where liquidity is abundant. Consequently, a relatively low level of recourse to the optimization procedures need not indicate that the liquidity-saving features are inefficient, but that the participants had a sufficient level of liquidity.

3 Model setup

3.1 Fee-based liquidity provision

Martin and McAndrews (2008) provide for a model of LSM in an environment of fee-based liquidity provision, such as Fedwire. This model is further analyzed for welfare effects by Atalay et alii (2008). The model set-up is as follows:

- the day in the payment system is divided into two periods, morning and afternoon,
- participants form a unit mass of banks of equal size but heterogeneous in their payments,
- each bank must make and receive one payment a day,
- a fraction of θ of the banks must make a time-critical payment, for delaying a time-critical payment the banks face delay costs of γ,
- banks may face a liquidity shock in the morning which comes as the net payment to settlement systems;
    - a fraction σ receives a positive liquidity shock of size $1-\mu$,
    - a fraction σ receives a negative liquidity shock of size $1-\mu$,
    - a fraction $1-2\sigma$ receives no liquidity shock,
- banks having a negative balance at the end of the morning must pay an overdraft fee R.

The costs that the banks might face are either R, the overdraft fee from the payment system provider or γ, the delay cost. As it is assumed that there is no market for intraday-liquidity, no interest can be earned by borrowing positive balances through the day.

The provision of a LSM would offer the banks a third option besides borrowing at cost R or delaying at cost γ: queuing. A queued payment will be released when the account of the bank is at least balanced by an incoming payment in the morning period. Implicitly the authors assume the special case of a Balance Reactive Gross Settlement System (BRGS).[2] Finally, all payments are assumed to settle at least in the afternoon period.

[2] See Norman (2010) for an overview on different liquidity-saving mechanisms.

The fraction of banks that delay may decrease with ratio γ/R (cost of delay / cost of overdraft). However, the optimal strategy is not so simple: The banks form a belief about the probability of receiving a payment in the morning. And the equilibrium depends on the probability of the liquidity-shock and of the time-critical payments. Martin and McAndrews show, that for some parameter constellations multiple equilibria for both cases (i.e. with or without LSM) coexist. As the strategy formation involves ex-ante beliefs about the probabilities of receiving a payment in the morning period (π) and the probabilities of being part of the groups of banks that receive a liquidity shock (σ) or a time-critical payment (θ), the socially best solution (the planner's solution) might deviate from the actual solution in the market. In fact, there are fractions λ_i^j of banks with j denoting the banks which delay, queue or pay early and i denoting the membership in the six possible groups defined by the liquidity shock and the time-critical payments.[3]

3.2 Collateral-based liquidity provision

Jurgilas and Martin (2010a) extend Martin and McAndrews (2008) to a model for a collateral-based payment system. With the liquidity shock σ of size 1-μ and probability π, the probability for time-critical payment θ and the delay cost γ being equal to the latter model, the cost R is now defined differently. The loss of reputation is assumed to be costly at this rate R>0. Furthermore Jurgilas and Martin have to introduce new variables with regard to the initial level of collateral L_0 posted at the central bank at cost κ per unit. Additional collateral has to be added during the day at cost Ψ per unit. Note that $\Psi>\kappa$. If collateral is added at the end of the day, the cost would be Γ.

The cost function that is to be minimized by the banks is now becoming more complex, as the banks would also have to form a belief ω about the sufficiency of its initial level of collateral (L_0) taking into account the payment activities with other banks denoted Φ. The latter refers to receiving an offsetting payment in the morning period from another bank. The collateral choice is thus crucial for the equilibrium in an environment with and without LSM.

The authors additionally introduce two different cases of payment cycles. Either two payments constitute a short cycle, i.e. bilateral offsetting, or all payments form a unique long cycle, the latter being deemed more representative for existent payment systems. In short cycles only one equilibrium exists, whereas in a long cycle two equilibria may occur in the case of the absence of a LSM. If a LSM is introduced, there is always exactly one equilibrium and welfare is also always improved compared to a plain-vanilla collateral-based RTGS.

This notion contrasts sharply to the ambiguous result of Martin and McAndrews (2008) regarding the introduction of an LSM into a fee-based payment system. Also the inherent preferences of the banks to delay payments vanish when the liquidity provision is conducted via collateralization. This is so, because the marginal cost of borrowing is zero in the latter case.[4]

[3] See for further details Annex 1 and the original paper.
[4] See for further details Annex 2 and the original paper.

4 Quantifying the variables for TARGET2

For the quantification of all variables and calibration of the model with regard to TARGET2 we used information from the national German component of TARGET2 (TARGET2-BBK) for the year 2010. For some variables we counter tested our results with information from interviews with bank managers of larger German banks.

As the deliberations of Atalay et alii (2010) had shown, the quantification entails a considerable amount of plausibility considerations since clear construct in the theoretic model do in many cases not match one to one with observable data. Since we try to stick as close to the calibration in Atalay et alii (2010) we followed as far as possible their proceedings.

4.1 Calibration for a fee-based set-up

Although TARGET2 works with a collateral-based liquidity provision we tried firstly, to assess the welfare effects of LSM in TARGET2 following the fee-based set-up. In doing so we enhanced the comparability of our results towards other calibrations since the first calibration overall was done by Atalay et alii for this sort of set-up. Obviously, some changing assumptions were necessary to get a plausible calibration for TARGET2 and its specific features. They turn out to be quite significant in some cases and are explained for each parameter in detail below.

Calibration of μ and σ

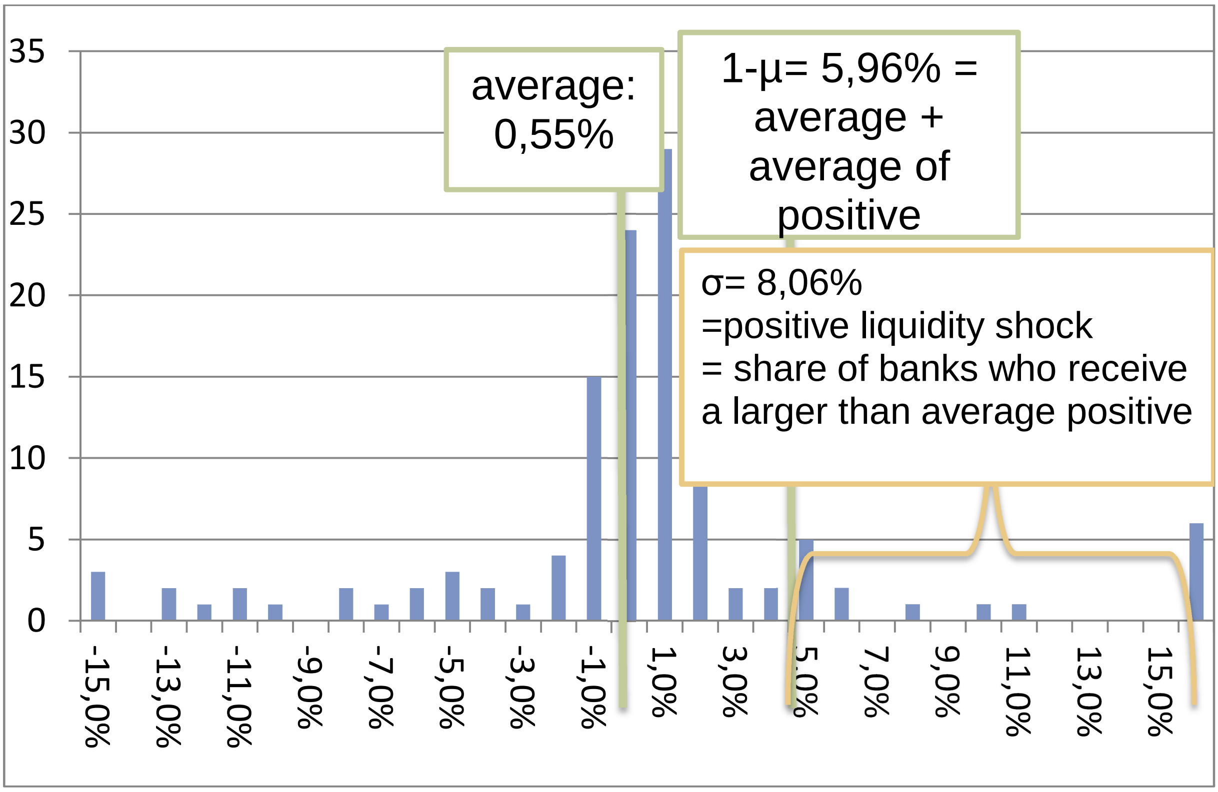
Following Atalay et alii (2010) we firstly calibrate the liquidity shock μ and derive a value for the size of the share of banks σ which is subject to this liquidity shock. This can be calculated by the following formula:

$$1 - \mu = \frac{net\ postion\ with\ ancillary\ systems}{time - critical\ payments}$$

The average value of net transfers to ancillary systems divided by the time-critical payments is +0.55% in our data.[5] This is the mean value for distribution of banks in TARGET2-BBK. As we concentrate on one side of the distribution, we took the average of the positive values 5.41%. Adding both, gives us the cut-off value μ=5.96% and consequently 1-μ=0.94. As 8.06% of all banks are above this value, we calibrated σ=0.08.

[5] See for explanation Figure 1.

Figure 1: Distribution of 1-µ in TARGET2-BBK, 2010



Source: Own calculations.

However, the distribution of banks according to the share of net-transfers to ancillary systems divided by the time-critical debits in the morning is for TARGET2 not symmetric as was assumed by Atalay et alii (2010). If the alternative route via the negative values of the distribution is taken, the values for 1-µ and σ would change to 0.96 and 0.14 respectively.

Calibration of θ, time-critical payments

Next we try to find a plausible value for the size of the fragment of banks with time-critical payments. First of all, the natural approach for the calculation of time-sensitive payments would have been the share of the submitted urgent and highly urgent payments in TARGET2. However, this proved to be inconclusive as it seemed that regarding the present liquidity conditions in the Eurozone as well as the technical efficiency of TARGET2 itself, liquidity managers can be sure that a payment settles in a reasonable time. As was mentioned earlier, more than 90% of all payments settled within one minute, and the share of the entry disposition together with its offsetting mechanism settled around 95% of all submitted payments. In addition, checking with liquidity managers we found out that the use of the “urgent”- and the “highly-urgent”-category is extremely heterogenous and does not warrant any conclusions for the true size of time-critical payments. The interviewees gave quite disparate estimations for their time-sensitive payments which in average (7%) were also astonishingly small and they included rather heterogenous sorts of payments different from each other (e.g. Cash-in-Transit-company payments, securities settlement, money leg of other trades, exceptional third-party transfers). Thus, we discarded this approach early.

We decided rather to follow the approach of Atalay et alii (2010) and classified only customer payments as non-time critical (and delete all technical payments) and received $\theta=0{,}5$, i.e. a share of 50% time-critical payments in TARGET2-BBK (in terms of value).

Calibration of the delay costs

Calibrating the delay costs (γ) is the next step. Delay costs are not systematically monitored, neither in the US nor in the Eurosystem. Atalay et alii derived them from the model using a technical requirement to hold and calibrating the ratio of γ/R. They could do so, because the overdraft costs are known to the system. TARGET2 does not work with overdraft costs and therefore, we could not apply this model-based calibration. We asked the market participants. They referred to market usance and some institution specific delay costs.

In the case of Clearstream, the delay costs are defined in absolute Euro terms by time brackets and are further differentiated by the cumulative occurrences of delays in a certain time. This is inconclusive for a calculation within the model. A more general approach is the definition of delay costs in the European Interbank Compensation Guidelines of the European Banking Association (Revision 2010). They recommend a value of EONIA added by 0.25 base points plus 100 Euro administration costs. Not regarding the fix part this would in 2010 have amounted to an average of $\gamma=0.9377\%$.

Calibration of the overdraft fee

As overdraft fees are not applicable to the Eurosystem, and an estimation according to the relation γ/R as in Atalay et alii (2010) gave results way off any true collateral cost, we substituted R by κ, the cost of collateral. Not every interviewed bank applied opportunity costs of collateral holding. In addition, the transaction costs of pledging collateral would normally hold a high share of fixed costs. The real costs of pledging collateral differ according to the various sorts of collateral and according to the way of transaction. Thus we calculated for 2010 an average over all classes of collateral of approximately one basis point and applied therefore, $\kappa=0{,}0001$.

Putting all calibrated data together we calculated the welfare costs for the system without and with LSM using the formula in Annex 1. We concluded that the minimum savings of a simple LSM in TARGET2-BBK that would function on a fee-basis would be 45.000 € per day. If we calculate the second case for the calibration of μ and σ ($1-\mu=0.96$ and $\sigma=0.14$), the minimum savings would amount to 58.000 € per day. These numbers are high in comparison to any reasonable calculation of the costs for implementing LSM.

However, these numbers compare to the more impressing 500.000 USD per day from Atalay et alii (2010). A significant difference comes from the fact that in Fedwire overdraft costs of six basis points are applied whereas the real collateral costs for TARGET2-BBK are much less (and we substituted the former by the latter for the sake of applicability). Another difference is the total turnover, which was for Fedwire assumed as being much higher than for TARGET2. In addition, the calibration is quite sensitive to the parameters. As has been shown, the model does not entirely fit into the institutional frame of the Eurozone and any additional assumptions would naturally influence the results. We did for some of the reasonable cases also observe, that the absolute values in Euro for the total costs are calculated at negative values, which is an indication for either wrong assumptions or deficiencies of the model employed.

4.2 Welfare effects for an inclusion of a LSM within a fee-based liquidity provision

For most of the values of the calibration of the model by Jurgilas and Martin (2008), we could use the above mentioned data. The cost of additional collateral during the day Ψ was assumed to be only slightly higher than the cost for the provision of the level of initial collateral κ because of the general characteristic of the costs (mainly transaction costs) as fixed costs. The same holds true for collateral that is added at the end of the day (Γ). This makes the choice of the initial level of collateral L_0 less crucial. As a consequence we could disregard the belief ω about the sufficiency of its initial level of collateral. Additionally, some interesting features such as the possibility of auto-collateralization developed by Bundesbank and Clearstream lead to a high amount of collateral available for the purposes of conducting monetary policy with German banks. As Baglioni and Montecini (2008) note, it "is difficult to provide a reliable estimate of [cost of intraday borrowing from the central bank] because it is not always clear whether a bank is actually constrained to hold those securities or holds them as part of its optimal portfolio management."

As the model of Jurgilas and Martin (2008) differentiates three possible cases, one of them is not applicable to our setup, we can derive two values for daily savings attributable to the imaginary introduction of a LSM into TARGET2 by using the formula in Annex 2. These welfare effects are calculated as ca. 170.000 € per day and ca. 292.000 € per day, respectively. In comparison to the values for the welfare effects in a fee-based environment, these numbers are higher by the factor four to five and are much closer to the figures for Fedwire by Atalay et alii. Arguably, a set-up within a collateral-based liquidity provision applies better to TARGET2. A minor drawback is just that up to now, we are not aware of any comparable calculations for other RTGS with collateral-based liquidity provision.

4.3 Calibration of the necessary collateral

Jurgilas and Martin (2010b) calculate the potential savings in terms of collateral for CHAPS. They conclude that introducing an LSM to CHAPS could reduce the necessary collateral to 50 per cent of the actual level for 2010. Following their reasoning we calculated the level of necessary collateral implied by the model for a collateral-based system and calibrated it with the values as given above. Interestingly, we found that the actual level of collateral used is less than 90 per cent of the minimum suggested by the model in its most favourite case (for an RTGS with LSM). Two explanations occurred to us: Firstly, TARGET2 uses already (for a long time) a sophisticated set of LSM including many other features for managing liquidity as described above. Secondly, the costs for additional collateral are that low that banks do not fear an unsurmountable intraday lack of collateral.

5 Conclusion

We applied existing models for measuring the effects of a liquidity saving mechanism (LSM) onto the specific institutional conditions in the Eurozone, namely TARGET2 and its liquidity provisioning mechanisms. We found that even the hardly applicable comparison to a fee-based system such as Fedwire can show that a LSM leads to an increased social welfare in the dimension of about 45.000 to 58.000 € per day. If a better-fitting model with a collateral-based liquidity provision is chosen, the welfare effects are even more pronounced at about 170.000 to 292.000 € per day. The only comparable value from Atalay et alii (2010) gives a magnitude of 500.000 USD per day. Both values are comparable since the latter was calibrated according to a much higher turnover.

Both calibrations are not free of reasonable doubts. To apply real numbers to some model values requires in some cases a considerable level of simplification. The adaption of the model to the specific conditions of the Eurozone makes it necessary to estimate some crucial parameters, so that our results can so far only be indicative. Specifically, the share of time-critical payments had to be chosen somehow arbitrary. Further research could follow a number of directions:

- A deeper investigation of the cost of collateral taking into account the heterogeneity of banks.
- An enlargement of the database to all countries of the Eurozone, for the investigation of national structures that have – with regard to payment behaviour – so far not fully integrated despite the multi-national character of TARGET2 and the ongoing financial integration.
- Taking into account the usage of bilateral and multilateral limits by certain banks or national banking communities.
- A refinement of the methodology for investigation of the (marginal) welfare effects of multiple LSM within the same system.
- An improved method for the measurement of some crucial variables such as the time-sensitivity of payments or the (marginal) cost of collateral.

References:

Atalay, E – Martin A – McAndrews J (2008) The Welfare Effects of a Liquidity-Saving mechanism. Federal Reserve Bank of New York Staff Report no. 331. (Originally published 2008, revised 2010)

Atalay, E – Martin A – McAndrews J (2010) Quantifying the Benefits of a Liquidity-Saving mechanism. Federal Reserve Bank of New York Staff Report no. 447.

Baglioni A – Montecini A (2008) The Intraday Price of Money: Evidence from the e-MID Interbank Market. Journal of Money, Credit and Banking, vol. 40(7), 1533-1540

European Banking Federation (2010) European Interbank Compensation Guidelines. Available via https://www.ebaportal.eu/_Download/Legislation/2010/14-06-European_Interbank_Compensation_Guidelines-Revision_2010-D1659K-2009.pdf

European Central Bank (2011) TARGET Annual Report 2010. Available via: http://www.bundesbank.de/download/target2/t2_jahresbericht_2010.en.pdf

Galbiati, M – Soramäki K (2010) Liquidity-saving mechanisms and bank behavior. Bank of England Working Paper No. 400.

Jurgilas M – Martin A (2010a) Liquidity-saving mechanisms in collateral-based RTGS payment systems. Bank of England Working Paper No. 389.

Jurgilas M – Martin A (2010b) Liquidity-saving mechanisms in collateral-based RTGS payment systems. Presentation. Mimeo.

Norman B (2010) Liquidity-saving in real-time gross settlement systems – an overview. Bank of England Financial Stability Paper No. 7.

Martin A – McAndrews J (2008) Liquidity-saving mechanisms. Journal of Monetary Economics, Vol. 55(3), 554-567.

Renault, F – Pecceu, J-B (2007) From PNS to TARGET2: the cost of FIFO in RTGS payment systems. In Leinonen, H (ed.) Simulation studies of liquidity needs, risks and efficiency in payment networks. Bank of Finland Studies E:39.

Schulz C (2011) Liquidity Requirements and Payment Delays Participant Type Dependent Preferences. European Central Bank Working Paper No. 1291

TARGET2-SSP General Functional Specifications (Version 2.1). Available via: http://www.bundesbank.de/download/target2/target_2_user_v2_1.pdf.

Annex 1 : The model of fee-based liquidity provision

Martin and McAndrews differentiate six different type of banks according to two features:

- banks with or without time-sensivite payments (s or r)
- banks with positive, negative or no liquidity shock (s_+, s_-, s_0, r_+, r_-, r_0)

They derive four different equilibria in a world of spontaneous action and three for a social planner. The various types of banks react in each of the equilibria according to the following table, where E stands for “sending a payment early” and D stands for “delay”. In the case with LSM the third option “Q” (meaning: queue the payment) occurs.

Equilibria without LSM

Type	s+	s0	s-	r+	r0	r-
1-Equilibrium	E	E	E	D	D	D
2-Equilibrium	E	E	D	D	D	D
3-Equilibrium	E	D	D	D	D	D
4-Equilibrium	D	D	D	D	D	D
1-Planner	E	E	E	E	E	E
2-Planner	E	E	E	E	E	D
3-Planner	E	E	D	E	E	D

Equilibria with LSM

Type	s+	s0	s-	r+	r0	r-
1-Equilibrium	E	E	E	E	E	E
2-Equilibrium	E	E	E	Q	Q	D
3-Equilibrium	E	Q	Q	Q	Q	D
4-Equilibrium	E	Q	D	Q	Q	D
1-Planner	E	E	E	E	E	E
2-Planner	E	E	E	E	Q	D
3-Planner	E	Q	Q	E	Q	D
4-Planner	E	Q	D	E	Q	D

To calculate the welfare effects of an LSM Atalay et alii use a calculation of the welfare costs as follows:

W =	Welfare costs
$-\sigma[(\theta\lambda^{e}_{s+}+(1-\theta)\lambda^{e}_{r+})(1-\pi)(2\mu-1)R]$	overdraft costs of banks with positive liquidity shock and who pay early, but did not receive a payment in the morning
$-\sigma\theta\lambda^{q}_{s+}(1-\pi)\gamma$	costs of delaying a time-critical payment of banks who queued, received a positive liquidity shock and did not receive a payment in the morning
$-\sigma\theta\lambda^{d}_{s+}\gamma$	costs of delaying a time-critical payment of banks who delayed and received positive liquidity shock
$-(1-2\sigma)[(\theta\lambda^{e}_{s0}+(1-\theta)\lambda^{e}_{r0})(1-\pi)\mu R]$	overdraft costs of banks without liquidity shock who payed early
$-(1-2\sigma)\theta\lambda^{q}_{s0}(1-\pi)\gamma$	delay costs of banks without liquidity shock who queued
$-(1-2\sigma)\theta\lambda^{d}_{s0}\gamma$	delay costs of banks without liquidity shock who delayed
$-\sigma[(\theta\lambda^{e}_{s-}+(1-\theta)\lambda^{e}_{r-})(1-\mu\pi)R]$	overdraft costs of banks with negative liquidity shock who payed early
$-\sigma[\theta\lambda^{q}_{s-}(1-\pi)\gamma+(\theta\lambda^{q}_{s-}+(1-\theta)\lambda^{q}_{r-}(1-\mu)R]$	overdraft costs of banks with negative liquidity shock who queued
$-\sigma[\theta\lambda^{d}_{s-}\gamma+(\theta\lambda^{d}_{s-}+(1-\theta)\lambda^{d}_{r-})(1-\pi)(1-\mu)R]$	overdraft costs of banks with negative liquidity shock who delayed

Where:

Variable	meaning
σ	fraction of banks with negative liquidity shock = fraction of banks with positive liquidity shock
μ	size of payments between banks
μ	size of liquidity shock
θ	fraction of banks with time-critical payment
γ	cost of delaying time-critical payment
R	overdraft fee
λ_i^j	fraction of banks that pay early/delay/queue
π	probability of receiving payment in the morning
π^0	probability of receiving payment in the morning conditionally on not putting the payment in the queue
π^q	probability of receiving a payment in the morning conditionally on putting the payment in the queue

Finally, they multiply the calculated value for W with the turnover of Fedwire and the result is the welfare cost of the respective system.

Annex 2: The model for collateral-based liquidity provision

The sequence of the banks‘ actions is:

- choose amount of initial collateral: L_0,
- observe liquidity shock λ and liquidity in the morning: $L_1=L_0+\lambda(1-\mu)$,
- observe type of payment to be made (time critical or non-time critical),
  - share of time critical payments: θ,
  - delay costs for time critical payments: γ,
- submit a payment (P=1) or delay (P=0) until afternoon,
- with LSM decide if to queue (Q=1) or not (Q=0),
- observe incoming payments,
- post additional collateral at the end of day if needed at costs ψ.

The strategy of the banks is:

- minimize sum of delay and collateral costs
- dependend on liquidity shock, time criticality of payments and on belief about probality to receive a(nother) payment in the morning (ω)

A: The derived solution for an RTGS without LSM is:

$$\min_{L_0} \mathop{\mathrm{E}}_{\lambda,\gamma}\left[\min_{P} \mathop{\mathrm{E}}_{\phi(\omega)}(C_1 + C_2)\right]$$

where:

$$C_1 = \kappa L_0 + PI(L_1 < \mu)\left(1-\omega^i\right)(R+\gamma) + (1-P)\gamma$$
$$C_2 = \left[(1-P)\left(1-\omega^i\right) + PI(L_1 < \mu)\left(1-\omega^i\right)\right]\max\{\mu - L_1, 0\}\,\Gamma$$
$$\Gamma = \frac{(1-\tau_s)^{n-1}}{n}\psi < \psi$$
$$\tau_s: P = 1 \; and \; L_1 \geq \mu$$
$$L_1 = L_0 + \lambda(1-\mu)$$

There exist multiple equilibria for the choice of the initial level of collateral L_0:

- (i) if $(1-\mu)\kappa<\gamma\theta(1-\pi)$ and $(2\mu-1)\kappa<\gamma\theta(1-\pi)$: **$L_0=\mu$**, $\omega^i=1-\pi$, P*=1 für λ=0,1 und 0 für λ=-1

- (ii) if $(1-\mu)\kappa>\gamma\theta(1-\pi)$ and $(3\mu-2)\kappa<\gamma\theta\pi$: **$L_0=2\mu-1$**, $\omega^i=1-\pi$, P*=1 für λ=1 und 0 für λ=-1,0

- (iii) if $(3\mu-2)\kappa>\gamma\theta\pi$ and $(2\mu-1)\kappa>\gamma\theta(1-\pi)$: **$L_0=1-\mu$**, $\omega^i=0$, P*=0

For the case of a social planner two solutions exist:

- if $(3\mu-2)\kappa > \gamma\theta$: $L_0 = 1-\mu$, whereas P(λ,v,L_0)=0 und $P(\mu,\gamma,L_0) = 0, \omega^i = 0\forall\lambda,\gamma$
- otherwise: $L_0 = 2\mu - 1, P(\lambda,\gamma,L_0) = 1, \omega^i = 1\forall\lambda,\gamma$.

B: The derived solution for an RTGS with LSM is:

$$\min_{L_0} \underset{\lambda,0}{\mathrm{E}} \left[\min_{P,Q} \underset{\phi(\omega)}{\mathrm{E}} (C_1 + C_2) \right]$$

where

$$C_1 = (1-Q)\left[PI(L_1 < \mu)\left(1-\omega^i\right)(R+\gamma) + (1-P)\gamma\right] + Q(1-P)(1-\omega^q)\gamma + \kappa L_0$$

$$C_2 = \left\{(1-Q)\left(1-\omega^i\right)[(1-P) + PI(L_1 < \mu)] + Q(1-P)(1-\omega^q)\right\}x \max \ (\mu - L_1, 0)\Gamma.$$

The optimal collateral choice is the same for the market equilibrium and the social planner:

$$\boldsymbol{L_0 = 1 - \mu}, \qquad P(\lambda, \gamma, L_0) = 0, \qquad \omega^i = 0 \ \forall \lambda, \gamma \ if \ (3\mu - 2)\kappa > \gamma\theta$$

***