This Is What’s in Your Wallet...and How You Use It∗

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Abstract

Data from the 2012 Diary of Consumer Payment Choice (DCPC) is used to estimate payment instrument choice for U.S. consumers. The data shows substantial changes compared to a similar study by Klee (2008) (which used data from 2001): Checks have virtually disappeared from purchase transactions, while still play a role in bill payments. Cash, on the other hand, still plays a large role for low-value transactions. As opposed to analyzing payment instrument choices in isolation the data allows to look at sequences of payment instrument choices and how they relate to cash withdrawals. Preliminary results indicate that such forward-looking behavior of consumers might be important.

Keywords: payment instrument choice, money demand, cash withdrawals, payment cards, Diary of Consumer Payment Choice

JEL Classification: E41, E42

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Non-technical summary

The 2012 Diary of Consumer Payment Choice offers fresh insights into the transformation of the money and payment system from paper to electronics in the United States, where consumers choose to adopt, carry, and use one of nearly a dozen means of payment to buy goods and services. According to this new data, checks became nearly extinct from point-of-sale transactions over the last decade, which is a remarkable change considering that earlier studies, based on data from 2001, found that they commanded an almost 40 percent market share of high-value grocery store transactions. Looking at a broader spectrum of possible payment contexts shows that non-automatic bill payments are the only segment where checks are still used to a considerable extent. Their popularity is on par with that of online banking bill payment and bank account number payments for those types of payments.

The same data can also be used to jointly study consumers’ payment instrument choice and cash management practices. This is a novel approach in the literature, which in the past has focused on one or the other, but data limitations precluded a more comprehensive approach. In the model we set up and estimate, consumers optimally choose payment instruments and time cash withdrawals taking into account that using cash for a transaction might limit their options for paying with cash later or force them to make costly cash withdrawals.

The results indicate that accounting for such forward-looking behavior is important: Consumers planning to make several transactions on a day are much less likely to use cash for the first transaction than similar consumers who make only one purchase on a day. The main reason for that is that cash withdrawals are rather costly, for example, for consumers with both credit and debit cards it takes over seven transactions before the withdrawal costs are recouped. Therefore consumers tend to hang on to their cash holdings during their early transactions.
A popular commercial campaign by the U.S. bank Capital One asks listeners, “What’s in your wallet?” This paper attempts to answer this question using a panel of micro data from the new 2012 Diary of Consumer Payment Choice (DCPC). Aside from prurient interests in other peoples’ wallets, the question and answer offers fresh insights into the transformation of the money and payment system from paper to electronics in the United States, where consumers choose to adopt, carry, and use one of nearly a dozen means of payment to buy goods and services.

There have been a number of recent contributions on this topic in various countries using micro-level transactions data, see, for example, Fung, Huynh, and Sabetti (2012) for Canada, Bounie and Bouhdaouï (2012) for France, von Kalckreuth, Schmidt, and Stix (2009) for Germany, Klee (2008) and Cohen and Rysman (2012) for the U.S.. First, we replicate the analysis of Klee (2008) the DCPC data. The result shows that over the last decade payment instrument choice has undergone a remarkable transformation: checks have virtually disappeared from point-of-sale transactions.

Second, a similar finding across these studies is that transaction values are very important in explaining the payment instrument choice decision: low value payments are mostly paid for with cash, debit is used for higher value transactions and credit for the largest ones. Since the DCPC records how much cash respondents carry in their wallet during the day and transaction values we can test whether a cash-in-advance constraint can account for this general finding. The advantage of building such a model is that it links explicitly payment instrument choice and the demand for various liquid assets. Klee (2008) was trying to make this connection, but data limitations only allowed her to link transaction values (not cash holdings) to payment instrument choice. Eschelbach and Schmidt (2013) finds in a reduced form estimation on German data that “the probability of a transaction being settled in cash declines significantly as the amount of cash available at one’s disposal decreases”.

The inclusion of cash inventory management decisions into the analysis establishes a link between successive transactions. This means that when consumers make payment choices they will not only consider the current benefits of using a
particular instrument, but also the effect of this choice on the potential utilities derived from future transactions. To illustrate this, take, for example, a consumer who has $10 in her purse along with a credit card and is planning to make two low-value transactions worth $8 and $3, respectively. Clearly, a choice to use cash for the $8 transaction will force her to either use the credit card for the $3 transaction or to withdraw cash, which could be costly in utility terms. The random utility maximizing framework used in the aforementioned studies lends itself naturally to such extension. In particular, we follow the methodology of Rust (1987) (see Chapter 7.7 in Train (2009) for a concise description), which (at the cost of restrictive assumptions) yields closed form solutions for the payment instrument choice probabilities.

The paper is organized as follows: Section 2 draws a quick comparison between the DCPC data and Klee (2008) and estimates simple multinomial logit models for several types of transactions. Section 3 describes the dynamic extension of the payment instrument choice model and discusses how it can be solved. Section 4 extends that model to allow for withdrawals, linking payment instrument choice and cash demand. Section 5 describes the results of the estimation, while Section 6 concludes.

2 Payment instrument choice

2.1 Payments transformation 2001-2012

This subsection replicates the econometric analysis in Klee (2008) on the DCPC data. First, we need to restrict our data to make sure that the results are comparable. The transactions used in her estimation all came from a grocery store chain that accepted cash, check, debit and credit cards (signature debit was recorded as credit card payment), moreover she restricted her sample to transaction values between $5 and $150 (2001 dollar prices). The DCPC has a much broader scope, it tries to cover all consumer transactions, not just purchases at grocery stores. In fact, it also has information on not-in-person payments (such as on-line purchases), bill payments and automatic bill payments. For the results in this sub-

\footnote{Note that her data is not meant to be representative of the U.S. payment system.}
section we only used transactions carried out at “grocery, pharmacy, liquor stores, convenience stores (without gas stations)”, where cash, check, debit or credit card was used\(^2\), and kept the range of transaction values unchanged in 2001 dollars.

As in Klee (2008) we estimate a multinomial logit model of the payment instrument choice. The choice of respondent \(n\) from using payment instrument \(p\) in transaction \(t\) depends on the indirect utilities \(u_{ntp}\):

\[
p^* = \arg\max_p u_{ntp}
\]

\[
u_{ntp} = x_t \beta_{1p} + z_n \beta_{2p} + \epsilon_{ntp},
\]

where vector \(x_t\) collects transaction specific explanatory variables (transaction value, indicator variable for weekend) while vector \(z_n\) denotes respondent specific variables (household income, age, education, gender, marital status) and \(\epsilon_{ntp}\) is assumed to be an i.i.d. Type I Extreme Value distributed error term. Note that since the variables in \(x_t\) and \(z_t\) do not vary by payment instrument, the coefficients \(\beta\) are assumed to be different for each payment instrument. The assumption about the error terms guarantees a closed form solution for the choice probabilities:

\[
\Pr(p|x_t, z_n) = \frac{\exp(u_{ntp})}{\sum_p \exp(u_{ntp})}.
\]

The variables were chosen so as to match the specification in Klee (2008) as close as we could\(^3\).

Figure 1 compares the estimated payment choice probabilities at different transaction values in 2001 and 2012. The left panel is taken from Klee (2008), while the right panel is obtained from carrying out the estimation on the DCPC data. The most striking difference between the two panels is that checks have virtually disappeared from grocery stores over the past decade. Second, the probability of choosing cash has roughly halved at all transaction values and it is used overwhelm-

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\(^2\)The DCPC also has data on prepaid card, bank account number payment, money order, travelers’ checks, text message and other payments. For grocery stores, however, their share is negligible.

\(^3\)We have no information on the number of items bought and if the respondent used a manufacturer coupon to get some discount, nor do we have information on whether she resides in urban or rural area an if she is a home-owner or not.
Table 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Marginal effects (1)</th>
<th>Marginal effects (2)</th>
<th>Marginal effects (3)</th>
<th>Marginal effects (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items bought</td>
<td>0.024* (0.000)</td>
<td>0.018* (0.000)</td>
<td>0.003* (0.000)</td>
<td>0.002* (0.000)</td>
</tr>
<tr>
<td>Items bought^2</td>
<td>0.001* (0.000)</td>
<td>0.000* (0.000)</td>
<td>0.000* (0.000)</td>
<td>0.000* (0.000)</td>
</tr>
<tr>
<td>Value of sale</td>
<td>0.008* (0.000)</td>
<td>0.003* (0.000)</td>
<td>0.003* (0.000)</td>
<td>0.003* (0.000)</td>
</tr>
<tr>
<td>Manufacturer coupons</td>
<td>0.014* (0.000)</td>
<td>0.006* (0.000)</td>
<td>0.006* (0.000)</td>
<td>0.002* (0.000)</td>
</tr>
<tr>
<td>Day of week</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monday</td>
<td>0.016* (0.001)</td>
<td>0.024* (0.001)</td>
<td>0.005* (0.000)</td>
<td>0.013* (0.001)</td>
</tr>
<tr>
<td>Tuesday</td>
<td>0.029* (0.001)</td>
<td>0.037* (0.001)</td>
<td>0.006* (0.000)</td>
<td>0.014* (0.001)</td>
</tr>
<tr>
<td>Wednesday</td>
<td>0.030* (0.001)</td>
<td>0.041* (0.001)</td>
<td>0.004* (0.000)</td>
<td>0.015* (0.001)</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.017* (0.001)</td>
<td>0.038* (0.000)</td>
<td>0.000* (0.000)</td>
<td>0.021* (0.001)</td>
</tr>
<tr>
<td>Friday</td>
<td>0.022* (0.001)</td>
<td>0.011* (0.001)</td>
<td>0.006* (0.000)</td>
<td>0.027* (0.001)</td>
</tr>
<tr>
<td>Saturday</td>
<td>0.012* (0.001)</td>
<td>0.012* (0.001)</td>
<td>0.006* (0.000)</td>
<td>0.017* (0.000)</td>
</tr>
<tr>
<td>Median household income</td>
<td>0.004* (0.000)</td>
<td>0.009* (0.000)</td>
<td>0.005* (0.000)</td>
<td>0.018* (0.000)</td>
</tr>
</tbody>
</table>

Age

- 35–44: 0.281* (0.009)
- 45–54: 0.300* (0.010)
- 55–64: 0.269* (0.011)
- 65–74: 1.080* (0.011)

Education

- High school: 0.309* (0.009)
- Some college: 0.514* (0.004)
- College: 0.474* (0.005)

Married

- 0.466* (0.006)

Female-headed household

- 0.341* (0.011)

Urban

- 0.068* (0.001)

Owner-occupied

- 0.055* (0.003)

Pseudo R-squared: 0.117

Observations: 6,204,845

Robust standard errors in parentheses.

*Significant at 1%.

Figure 1: Payment instrument choice at grocery stores in 2001 (left, from Klee (2008)) and 2012 (right)

...ingly for low-value transactions. Credit and debit cards have stepped into the void left by the decline of cash at low transaction values and checks at larger values of sale. In particular, while the choice probability for PIN debit (orange dash-dotted line) exhibits a hump-shaped pattern, credit (including signature debit) increases monotonically over this range of purchase values.

2.2 Payment instrument use in different contexts

In this subsection we drop the data restrictions imposed by the need for comparability in the previous subsection and re-do the same estimations on the broad range of payment contexts covered in the DCPC. The qualitative results from Figure 1 carry over to more general settings. Checks, for example, only play a significant role in (non-automatic) bill payments and are largely absent elsewhere. Cash transactions play a significant role for low-value in-person transactions in general and are also used to pay for some bills. For obvious reasons they are not present in not-in-person transactions and automatic bill payments.

2.2.1 In-person vs. not-in-person purchases

Figure 2 shows payment instrument choice probabilities by transaction values. For in-person transaction (left panel) the graph tells a similar story to the one for grocery stores only (note that the scale of both axes has changed). Checks are rather unimportant, the change in cash use probability between low and high transaction values is by far the biggest among all payment instruments, though
credit card use increases fairly quickly and does not level off even at transaction values as high as $1,000.

Unlike the studies that rely on scanner data, we are able to separate out signature debit transactions from credit cards. This is interesting, because according to Figure 2 signature debit transactions are very similar to PIN debit transactions, but quite different from credit transactions. There is not much of a difference between the two types of debit cards, though PIN debit use seems to level off at somewhat higher transaction values. This suggests that when making a payment, consumers are primarily concerned about the funds that debit and credit cards tap into and not about the network through which these transactions are routed through. The increase in the “Other” category with the transaction value is largely the result of a few purchases made using money order, which are of fairly high value. Since there aren’t many large value transactions (the 99th percentile is at $341), these are a non-trivial portion of all large transactions.

Not-in-person purchases are dominated by credit and signature debit card payments, while bank account number payments (subsumed in the other category) represent about 10 percent of all not-in-person transactions.

2.3 Bill payments

Lastly, we look at bill payments, where two more lines have been added to the graphs to represent online banking bill payments and bank account number payments. Together they make up about half of the bill payments and about three-quarters of automated bill payments. More interestingly, checks are just as frequently used for bill payments as either of its electronic counterparts. For automatic bill payments, checks, obviously, disappear and their role is largely taken over by online banking bill payments. There is still a non-trivial share of cash payments, predominantly for lower value bills. Interestingly, credit and debit cards are not often used to make bill payments (automatic or not).
3 Dynamic model of consumer payment choice

As noted in the Introduction, the strong interdependence between transaction values and payment instrument choice exhibited in Figures 1-3 has also been found in a number of previous studies. One possible explanation for this phenomenon is that the availability of various payment instruments differs. Consumer have to have enough currency on them to make a cash payment, while they “only” need to keep a high enough balance on their checking account to make a debit transaction or have enough available credit to use a credit card. Ideally, we would like to have a model that captures the availability of all payment instruments. Unfortunately, we are only able to infer the amount of cash in respondents wallet from the DCPC data, but do not have the neccessary information on account balances to take more constraints into account. Cash in the wallet, however, is likely to be the constraint that limits the payment instrument choice most often.

The goal of this Section is to build a dynamic model of consumer payment choice that incorporates a cash-in-advance (CIA) constraint for cash payments.
Throughout this Section we assume that cash in the wallet at the beginning of the day is given exogenously and consumers have no way of withdrawing cash. Withdrawals will be incorporated in Section 4.

3.1 The dynamic problem

Given that the availability of one of the payment instruments, cash, changes if it is used in a transaction, a link exists between current and future transactions: Deciding to use cash now, may reduce the number of available instruments in future transactions, leading to a drop in the expected utility derived from that transaction. If cash balances are insufficient to settle a transaction, the consumer will no longer be able to take advantage of a high realization of random utility associated with cash transactions. A forward-looking consumer will take this potential loss of utility into account when making the payment instrument choice in the current

Figure 3: Payment instrument choice for bill payments (left) and automatic bill payments (right)
transaction. That is, she would maximize

\[
V(m_t, t) = \max_{i_t \in \{h, c, d\}} u^i_{ndt} + E[V(m_{t+1}, t+1)]
\]

\[
u^i_{ndt} = \beta_i x_{ndt} + \gamma x_{ni} + \epsilon_{nti} = \delta_{nti} + \epsilon_{nti},
\]

where \(V(m_t, t)\) denotes the value of having \(m_t\) amount of cash before making the \(t\)th transaction, and \(E[.]\) is the mathematical expectation operator taken over the realizations of the shocks for future transactions. The instantaneous utility from using a payment instrument has three parts. Some variables \(x_{ndt}\) only differ across individuals (\(n\)) or days (\(d\)) or transactions (\(t\)), but not across payment instruments (\(i\)). Demographic variables and transaction values would be the obvious examples. For these variables separate coefficients (\(\beta_i\)) will have to be estimated for each payment instrument. Other explanatory variables are specific to a payment instrument (for example, whether a credit card gives rewards) and are only included in the indirect utility function for that instrument. For these variables only a single parameter is estimated and these are collected in \(\gamma\). The deterministic part of the indirect utility \(\beta_i x_{ndt} + \gamma x_{ni}\) will be denoted by \(\delta_{nti}\). Finally, there is a random component of the utility distributed independently and identically Type I generalized extreme value. The \(n\) and \(d\) subscripts will be dropped in what follows.

The consumer chooses between credit, debit and cash if she has enough of it to pay for the \(t\)th transaction (\(m_t \geq p_t\)). The evolution of \(m\) is given by

\[
m_{t+1} = m_t - p_t \cdot I(i_t = h),
\]

where \(I\) is an indicator variable taking the value of 1 if cash is chosen (\(i = h\)) and 0 otherwise. The program has a finite number of “periods” (transactions) \(T\), which is known to the consumer, and can be solved by evaluating the expectation on the right-hand side from the last period backwards.

Note that we assume throughout the model, that the consumer knows with certainty, at the beginning of the day, not only the number of transactions that she will make, but also the deterministic part \(\delta_{nti}\) of the indirect utility for each of these transactions. Though there are some variables in \(\delta_{nti}\), such as are demographic characteristics, for which this information structure seems reasonable,
assuming that she knows exactly the dollar value of each transaction is clearly an extreme assumption.

To start the backward iteration, we need to fix the value of having an amount \( m \) of cash left after the last transaction. For simplicity, for now, assume that there is no value to carrying cash over from one day to the next, resulting in

\[
V(m_T; T) = \begin{cases} 
\max_{i \in \{h,c,d\}} u_i^T & \text{if } m_T \geq p_T \\
\max_{i \in \{c,d\}} u_i^T & \text{if } m_T < p_T
\end{cases},
\]

i.e. the continuation value after transaction \( T \) is 0, regardless of the amount of cash on hand after the final transaction of the day.

### 3.2 Period \( T - 1 \)

Note that, given the simplifying assumption about the value of end-of-day cash holdings, the last period collapses to the multinomial logit choice problem, with expected utilities given by

\[
E[V(m_T; T)] = \begin{cases} 
\ln \left( \sum_{i \in \{h,c,d\}} \exp(\delta_{Ti}) \right) + \gamma & \text{if } m_T \geq p_T \\
\ln \left( \sum_{i \in \{c,d\}} \exp(\delta_{Ti}) \right) + \gamma & \text{if } m_T < p_T
\end{cases}, \tag{1}
\]

just like in the static case of the previous section. This means that, iterating backwards, the choice problem for \( T - 1 \) is

\[
V(m_{T-1}; T - 1) = \begin{cases} 
\max_{i \in \{h,c,d\}} u_i^{T-1} + E[V(m_{T-1} - p_{T-1} \cdot I(i_{T-1} = h); T)] & \text{if } m_{T-1} \geq p_{T-1} \\
\max_{i \in \{c,d\}} u_i^{T-1} + E[V(m_{T-1}; T)] & \text{if } m_{T-1} < p_{T-1}
\end{cases}. \tag{2}
\]

While this function looks complicated, it is not hard to evaluate it. Given \( m_{T-1} \) we know which one of the two branches in equation (2) is relevant.
3.2.1 Insufficient cash for the current transaction, $m_{T-1} < p_{T-1}$

Starting with the simpler case, assume that $m_{T-1} < p_{T-1}$, meaning that: (i) in the current period only debit or credit can be chosen and therefore (ii) $m_T = m_{T-1}$.

From (ii) we know which branch of $E[V(m_T, T)]$ in equation (1) is the relevant one, so all the terms in equation (2) are known and the choice probability of, for example, credit will given by

$$\Pr(i_{T-1} = c|m_{T-1} < p_{T-1}) = \frac{\exp(\delta^c_{T-1} + E[V(m_{T-1}, T)])}{\exp(\delta^c_{T-1} + E[V(m_{T-1}, T)]) + \exp(\delta^d_{T-1} + E[V(m_{T-1}, T)])},$$

which collapses to the logit choice probability, since the expected utility terms for period T added to $\delta^c_{T-1}$ are the same and they all appear additively in the argument of the exp(.) operator, that is

$$\Pr(i_{T-1} = c|m_{T-1} < p_{T-1}) = \frac{\exp(\delta^c_{T-1}) \cdot \exp(E[V(m_{T-1}, T)])}{\exp(\delta^c_{T-1}) \cdot \exp(E[V(m_{T-1}, T)]) + \exp(\delta^d_{T-1}) \cdot \exp(E[V(m_{T-1}, T)])} = \frac{\exp(\delta^c_{T-1})}{\exp(\delta^c_{T-1}) + \exp(\delta^d_{T-1})}.$$

It is worth to keep this simple and intuitive principle in mind: Dynamic considerations only affect payment instrument choice if the current choice reduces the expected utility when entering into the next transaction. In this model, card use cannot do that\(^4\). The probability for debit card use will be analogous.

3.2.2 Cash is an option in $T-1$, $m_{T-1} \geq p_{T-1}$

Going back to equation 2, if $m_{T-1} \geq p_{T-1}$, then next period’s expected utility will be a bit more complicated to compute. Looking again at the choice probability

\(^4\)In reality, it could be the case that checking account balances drop to levels where they cannot be used, or that consumers max out their credit card(s). Unfortunately we do not have data on that.
for credit cards for this case will highlight the difference:

\[ \Pr(i_{T-1} = c|m_{T-1} \geq p_{T-1}) = \frac{\exp(\delta_{T-1}^c + E[V(m_{T-1}, T)])}{\exp(\delta_{T-1}^h + E[V(m_{T-1} - p_{T-1}, T)]) + \sum_{j=c,d} \exp(\delta_{T-1}^j + E[V(m_{T-1}, T)])} \]

Note the new first term in the denominator (the terms referring to credit and debit have been collapsed into a summation). Since cash can now be chosen in period \( T - 1 \) debit and credit probabilities will decrease somewhat, hence the appearance of the new term.

Importantly, however, the formula reveals that the continuation utility after choosing cash may be different than the continuation utility after choosing cards. In particular, the first argument of \( E[V(\cdot, T)] \) is now \( m_{T-1} - p_{T-1} \) if cash is chosen in \( T - 1 \), whereas it is \( m_{T-1} \) if cards are used in period \( T - 1 \). This is the way consumers account for the fact that cash use now may limit their choices in the following transaction. Note, however, that the principle stated above still applies: If (i) \( m_{T-1} - p_{T-1} \geq p_T \) or (ii) \( m_{T-1} < p_T \) then there is no “real” effect of the payment instrument choice in \( T - 1 \) on the value function in \( T \); since in (i) the consumer has enough cash to make both the \( (T-1) \)th and the \( T \)th transaction with cash and in (ii) she would not have enough cash to pay for the \( T \)th transaction even if she did not use cash for transaction \( T - 1 \). This argument extends to more transactions: If (i) \( m_t - p_t \geq \sum_{s=t+1}^{T} p_s \) or (ii) \( m_t < \min\{p_s\}_{s=t+1}^{T} \) then the expected utilities in the formulas will be the same and the choice probabilities will collapse to the logit probabilities\(^5\).

The choice probability for cash will be, conditional on \( m_{T-1} \geq p_{T-1} \), simply,

\[ \Pr(i_{T-1} = h|m_{T-1} \geq p_{T-1}) = \frac{\exp(\delta_{T-1}^h + E[V(m_{T-1} - p_{T-1}, T)])}{\exp(\delta_{T-1}^h + E[V(m_{T-1} - p_{T-1}, T)]) + \sum_{j=c,d} \exp(\delta_{T-1}^j + E[V(m_{T-1}, T)])}. \]

\(^5\)Checking whether either of these special cases does in fact hold, speeds up the evaluation of the expected utility tremendously for consumers who make many transactions a day.
3.3 Period $T - 2$

With these probabilities in mind we can move back one more period in the iteration to complete the description of the backward induction process. The value function still looks similar to equation (2),

$$V(m_{T-2}, T - 2) =$$

$$\begin{cases} 
\max_{i \in \{h, c, d\}} u^i_{T-2} + E[V(m_{T-2} - p_{T-2} \cdot I(i_{T-2} = h), T - 1)] & \text{if } m_{T-2} \geq p_{T-2} \\
\max_{i \in \{c, d\}} u^i_{T-2} + E[V(m_{T-2}, T - 1)] & \text{if } m_{T-2} < p_{T-2} 
\end{cases}$$

but the expected utility calculations are a bit more involved (see Rust (1987) for details):

$$E[V(m_{T-1}, T - 1)] =$$

$$\begin{cases} 
\ln \left( \sum_{i \in \{h, c, d\}} \exp \left( \delta_{T-1} + E[V(m_{T-1} - p_{T-1} \cdot I(i_{T-1} = h), T)] \right) \right) + \gamma & \text{if } m_{T-1} \geq p_{T-1} \\
\ln \left( \sum_{i \in \{c, d\}} \exp \left( \delta_{T-1} + E[V(m_{T-1}, T)] \right) \right) + \gamma & \text{if } m_{T-1} < p_{T-1} 
\end{cases}$$

The only remaining piece of the puzzle is to spell out $E[V(m_{T-1} - p_{T-1} \cdot I(i_{T-1} = h), T)]$, and $E[V(m_{T-1}, T)]$, given $m_{T-2}$.

3.3.1 Case of $m_{T-2} < p_{T-2}$

Again, starting with the simpler case of $m_{T-2} < p_{T-2}$, when only cards can be used in $T - 2$ and $m_{T-1} = m_{T-2}$. If it is also true that $m_{T-2}$ is not enough to pay for transaction $T - 1$, either (i.e. $m_{T-2} < p_{T-1}$), then $m_T = m_{T-2}$ and checking if $m_{T-2} \geq p_T$ or $m_{T-2} < p_T$ determines which branch of equation (1) is relevant, but in any case $E[V(m_{T-2}, T)]$ can easily be evaluated using that now $m_T = m_{T-2}$.

For $m_{T-2} \geq p_{T-1}$, there are two possibilities: cash may or may not be used in transaction $T - 1$. The probability of cash being used was derived in equation (3) so simple substitution gives

$$E[V(m_{T-1} - p_{T-1} \cdot I(i_{T-1} = h), T)] =$$

$$Pr(i_{T-1} = h|m_{T-2} >= p_{T-1}) \cdot E[V(m_{T-2} - p_{T-1}, T)] \cdot (1 - Pr(i_{T-1} = h|m_{T-2} >= p_{T-1})) \cdot E[V(m_{T-2}, T)], \quad (4)$$

13
where the expectations on the RHS are given by equation (1).

3.3.2 Case of $m_{T-2} \geq p_{T-2}$

These cases are dealt with similarly, the point is again to go back all the way to evaluating the known $E[V(., T)]$ function and figuring out the probabilities of all possible branches. There are two possibilities in period $T-2$: (i) cash is used or (ii) cash is not used. (ii) implies $m_{T-1} = m_{T-2}$ and leads to similar calculations to the ones in the previous subsection.

(i) Implies $m_{T-1} = m_{T-2} - p_{T-2}$ and now this value of cash-on-hand will have to be checked against $p_{T-1}$ and $p_T$ to figure out the expected values, but these calculations are again parallel to the ones in the previous section.

Thus we have demonstrated, that the terms $E[V(m_{T-2} - p_{T-2} : I(i_{T-2} = h), T - 1)]$ and $E[V(m_{T-2}, T - 1)]$ can be computed from functions that are readily known, hence we are again left with the task of computing the choice probabilities in transaction $T - 2$ given $m_{T-2}$ using equation (3), and can continue the recursion all the way up to the first transaction.

4 Incorporating withdrawals

The dynamic model of Section 3 can be used to calculate the benefits of having a certain amount of cash on hand. The goal of this section is to use that information and data on withdrawals to estimate the costs associated with obtaining cash in order to characterize cash demand. Theoretical models of cash demand show that the assumptions made about the cash spending behavior of consumers affects parameters of the cash demand function in an important way. For example, the $-\frac{1}{2}$ interest elasticity of the Baumol-Tobin model drops to $-\frac{1}{3}$ in the slightly different setting of Miller and Orr (1968), whereas the interest elasticity is not constant in Alvarez and Lippi (2009). This is where the payments diary data becomes very useful since it has observations on both actual cash spending and cash withdrawal behavior so the econometrician does not have to rely on an assumptions about consumers’ cash use, when estimating cash demand, but can estimate cash use and cash inventory management simultaneously.
4.1 Simple model of withdrawals

Since, solving the dynamic model of Section 3 is already computationally involved we propose a simple model for withdrawals. Consumers start the day with an exogenously amount of cash. Before every purchase transaction they can decide if they want to withdraw cash first. If they choose to do so, we assume that they withdraw enough cash to possibly settle all of their remaining transactions with cash. That is, we assume, for now, that there is no limit on how much cash they can withdraw (clearly, a simplifying assumption for cashbacks) and that there is no variable cost of carrying cash within the day. The main reason for this assumption is to keep the model as simple and easy to solve as possible. The fixed cost of making a withdrawal and the lack of carrying/holding cost implies that consumers will make at most one withdrawal during the day, moreover, there is no reason to make a withdrawal after the last point of sale transaction.

Formally, if a consumer decides to make a withdrawal before transaction \( t \), her new cash balances will be \( m_t = \bar{m}_t \equiv \sum_{s=t}^{T} p_s \). The costs to making a withdrawal will be modeled as

\[
c_t = \alpha z_{nd} + \epsilon_t,
\]

where \( z_{nd} \) is a vector of consumer and day specific explanatory variables and \( \epsilon_t \) follows a logistic distribution.

The choice of the consumer before each transaction is now:

\[
E[W(m_t, t)] = \begin{cases} 
E[V(\bar{m}_t, t, W = 1)] - c_t & \text{if } I^w_t = 1 \\
E[V(m_t, t, W = 0)] & \text{if } I^w_t = 0
\end{cases}
\]

(5)

where \( I^w_t \) is an indicator variable for withdrawals (1 if there is a withdrawal, 0 otherwise). Note that due to the one withdrawal a day limit, \( W \) is a state variable: If a withdrawal was made before on the day consumers will not have the option (nor the need) to make additional ones since they will be able to make all payments using cash. On the other hand, if they have not used up their withdrawal opportunity, than in the current or in any one of the future transactions they may do so.
Formally,

\[ E[V(\bar{m}_t, t, W = 1)] = \max_{i \in \{h, c, d\}} u^i_t + E[V(m_t - p_t \cdot I(i_t = h), t + 1, W = 1)], \]

with \( m_t - p_t = \sum_{s=t+1}^{T} p_s \), meaning that the choice probabilities will not be affected by the cash-in-advance constraint, since it will not bind in the remaining transactions.

The more computationally involved part will be the evaluation of \( E[V(m_t, t, W = 0)] \), where the possibility of a future withdrawal will have to be included at each future transaction. However, the backward iteration described in Section 3 will still work in principle, with the appropriate modifications. In particular, the random component of the withdrawal cost was chosen to still yield closed form solutions, as the withdrawal choice is now essentially a simple logit choice model, with the latent utilities described by equation (5).

5 Results

The model is estimated by choosing parameters \((\alpha, \beta, \gamma)\) to maximize the likelihood of observing the sequence of payment instrument and withdrawal choices.

5.1 Marginal effects

The marginal effects are computed for the final transaction in Table 1, so they coincide with what a multinominal model (with the same estimated parameters) would give. The main difference compared to Klee (2008) is that the effect of transaction values on cash use drop to about a quarter of what see found. Part of the explanation is obviously the inclusion of a dummy variable for small value transactions, which was motivated by the fact that some merchants only take cash for small transactions. The other reason is that our dynamic framework controls explicitly for one of the main reasons transaction values might matter: the cash in advance constraint.
Marginal effects

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Debit</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>TransVal</td>
<td>-0.0013</td>
<td>0.0009</td>
<td>0.0004</td>
</tr>
<tr>
<td>Under $10r</td>
<td>0.2012</td>
<td>-0.1205</td>
<td>-0.0807</td>
</tr>
<tr>
<td>HHIIncome</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Age</td>
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<td>-0.0017</td>
<td>-0.0000</td>
</tr>
<tr>
<td>Female</td>
<td>0.0122</td>
<td>0.0085</td>
<td>-0.0207</td>
</tr>
<tr>
<td>RewardDC</td>
<td>0.0152</td>
<td>-0.0168</td>
<td>0.0016</td>
</tr>
<tr>
<td>Revolver</td>
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<td>0.0309</td>
<td>-0.1180</td>
</tr>
<tr>
<td>RewardCC</td>
<td>-0.0367</td>
<td>-0.0131</td>
<td>0.0498</td>
</tr>
</tbody>
</table>

*For dummy variables, marginal effect is a change from 0 to 1. TransVal=$12.53, income, age at sample average.

Table 1: Marginal effects for the final transaction on a day

5.2 Are consumers forward-looking?

Our model and the rest of the literature on payment choice can be thought of as two extremes: We endow consumers with a lot of information about their future transactions while the rest of the literature thinks about them as completely myopic. Does this difference make a difference empirically? The simplest answer to this question is to compare the choice probabilities from the two models. As noted before, the choice probabilities coincide with what a multinomial logit model would give for the final transaction, but may differ if consumer have some transactions left.

In what follows we will compare the choice probabilities for the first transaction of the day and vary the total number of daily transactions. The consumer is assumed to start the day with $20, has average household income, average age and all daily transactions are assumed to be $12.53 (median transaction value).

Table 2 shows that the model predicts widely different choice probabilities in the five scenarios. In particular, the probability of using cash drops from 40 percent in the case of a single transaction, to just below 30 percent even if she makes only one more transaction. The drop in the probability of using cash is monotonic, in the case of a third transaction it is only roughly half of what it would otherwise be. Since our choice model (like other multinomial logit model) posses
Table 2: Choice probabilities of the first daily transaction for different total number of transactions

the independence of irrelevant alternatives property the relative probabilities of debit and credit do not change.

5.3 Withdrawal costs

Given the estimates of $\alpha, \beta, \gamma$ the model can be used to conduct a cost-benefit analysis of cash withdrawals. In particular, given $\hat{\alpha}$, we compute the average withdrawal cost for our sample:

$$\bar{c} = \frac{\sum n \sum d \hat{\alpha} z_{nd}}{N \cdot D},$$

where the denominator is the number of respondent-days used in the estimation. To get a sense of how big withdrawal costs are, we relate it to the expected benefit of having cash defined as:

$$\Delta EV = E \left[ V(pT, 0, T) \right] - E \left[ V(0, 0, T) \right],$$

that is the difference in the expected utilities from making a payment of $12.53 for the average consumer (see previous subsection). In fact we compute this difference for debit and credit card holder, debit card holders who do not have a credit card

<table>
<thead>
<tr>
<th>Daily transactions</th>
<th>Cash</th>
<th>Debit</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4070</td>
<td>0.2397</td>
<td>0.3533</td>
</tr>
<tr>
<td>2</td>
<td>0.2947</td>
<td>0.2851</td>
<td>0.4202</td>
</tr>
<tr>
<td>3</td>
<td>0.2289</td>
<td>0.3117</td>
<td>0.4595</td>
</tr>
<tr>
<td>4</td>
<td>0.1827</td>
<td>0.3303</td>
<td>0.4870</td>
</tr>
<tr>
<td>5</td>
<td>0.1484</td>
<td>0.3442</td>
<td>0.5074</td>
</tr>
</tbody>
</table>

*Dummy variables set to 1, except for “Under$10". TransVal=$12.53, income, age at sample average.
and credit card holders who do not own a debit card.

\[
\frac{\bar{c}}{\Delta EV^{DC}} \sim 6.15 \\
\frac{\bar{c}}{\Delta EV^{D}} \sim 3.24 \\
\frac{\bar{c}}{\Delta EV^{C}} \sim 4.19
\]

The calculations show that average withdrawal costs are not recouped until the seventh transaction for debit and credit card holders. For consumers, with a smaller set of available payment instruments, having cash is more valuable and the fourth (debit) or fifth (credit) transactions can tip the balance in favor of a withdrawal.

5.4 Shadow value of cash

There is another way to measure the usefulness of cash, in line with the monetary economics literature, by computing the shadow value of cash, denoted by \( \lambda \). Originally, that measures the change in the utility from relaxing the cash-in-advance constraint by an infinitesimal amount. We measure it by adding \( \Delta \$ = $1, $5, $12.53 \) to the beginning of day cash holdings of each individual on each day and compute the average of the resulting changes in expected utilities

\[
\lambda_{\Delta \$} = E[V(m_{nd} + \Delta \$, t = 1)] - E[V(m_{nd}, t = 1)].
\]

Again, the same concept of \( \Delta EV \) is used to normalize \( \lambda \):

\[
\frac{\lambda_{\$1}}{\Delta EV^{DC}} \sim 0.0164 \\
\frac{\lambda_{\$5}}{\Delta EV^{DC}} \sim 0.1117 \\
\frac{\lambda_{\$12.53}}{\Delta EV^{DC}} \sim 0.2892
\]

The costless relaxation of everybody’s budget constraint by the median transaction amount yields on average about a quarter of the expected utility of increasing the payment instrument choice set from debit and credit to cash, debit and credit of
the hypothetical consumer of the previous subsection. This suggests a number of people in our sample are already able to use cash for all or their transactions; for them the shadow value is zero. Of course, doing away with the restriction of zero continuation value at the end of the day would change this result.

6 Conclusion

Payment instrument choice is ultimately a dynamic decision: using an instrument for a transaction may limit its availability in future transactions. The Diary of Consumer Payment Choice allows us to study this decision for the case of cash. The (preliminary) results of the simple model in this paper show that these effects may be substantial and can help to better understand how consumers make payments.

Bibliography


