

Fiscal Policy, Default Risk and Euro Area Sovereign Bond Spreads

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Abstract

This paper develops an arbitrage-free affine term structure model of potentially defaultable sovereign bonds to model a cross-section of six euro area government bond yield curves. We make use of the coexistence of a common monetary policy under European Monetary Union, which determines the short end of the yield curve that is common to all countries, and decentralized debt policies which drive expected default probabilities and thereby spreads at the long end. The factors of our term structure model are observable macroeconomic variables, including measures of government solvency. When applying this model to yield curves of six EMU member countries over the period January 1999 to March 2010, we find strong evidence for a break in the relationship between the fiscal variable and the default intensities in early 2008. Despite using no latent factors, our model produces an excellent fit to both yield levels and spreads. For highly indebted countries, following the break the sensitivity of spreads to the fiscal variable rises sharply.

JEL classification: E6, H6.

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1 Introduction

Recent years have not been lacking in drama in financial markets. At the time of this writing, government bond markets in the euro area are at the center of attention amidst intense speculation about the possibility of default by one or more member countries of the euro area. Indeed, yield spreads between government bonds of euro area countries and German government bonds of comparable maturity have experienced a striking regime shift sometime during early 2008. For example, during the period January 1999 to February 2008 the yield spread (in our data set, which we will describe in detail below) of 10-year government bonds for France, Italy and Spain over German yields of the same maturity averaged 6.8, 24.3 and 12.2 basis points respectively, with standard deviations of 5.9, 8.2 and 12.1 basis points. During the period from March 2008 to March 2010, these averages rose to 29.2, 82.1 and 57.6 basis points respectively, and their standard deviations doubled or tripled. (Developments in some euro area government bond markets since then make even these spreads appear modest.) It is therefore crucial to understand what determined the spreads before and after this evident break, and what might have caused the break.

In this paper, we use recent advances in term structure modeling to explain the evolution of euro area sovereign yield spreads, with the goal of understanding the role of macroeconomic variables and especially of fiscal policies in determining yield spreads at all maturities, both before and since the onset of the crisis.¹ Specifically, we jointly model the zero-coupon yield curves of government bonds of six euro area countries within an affine term structure model of potentially defaultable bonds, using only macroeconomic variables as factors. To estimate this model, we make use of a rich data set of government bond yields at many maturities of those six countries that covers the period from the beginning of stage three of European Monetary Union (EMU) in January 1999 to the spring of 2010. The probabilities of default perceived by investors are linked to the same macroeconomic fundamentals that drive yields. Our key findings are that a small set of macroeconomic variables can explain the term structures of all six countries remarkably well, and that the governments' debt

¹From the perspective of academic research, the events discussed above play out against the backdrop of intense efforts over the past decade to arrive at a better understanding of the macroeconomic determinants of asset prices in general, and of the linkages between the term structure of interest rates and the macroeconomy in particular. Gürkaynak and Wright (2010) provide an up-to-date survey of research on the term structure from a macroeconomic perspective.

service commitments have a significant influence on yields, which was minuscule before the spring of 2008 and has since had a major effect on perceived default probabilities.

Spreads among euro area government bonds since the beginning of stage three of European Monetary Union have attracted substantial attention in the literature. We discuss the related literature in the following subsection. Euro area yield spreads are particularly interesting because they allow us to study the macroeconomic determinants of term premia and default risk premia. Under the assumption (maintained throughout our study) that the probability of a country leaving the euro is considered nil, expectations of future short-term interest rates are identical across countries and exchange-rate risk is not priced. Hence our data set allows us to focus on term and default risk premia and their relation to the common monetary policy on one hand and country-specific fiscal policies on the other.

The literature in this area has mostly focused on regressions of yield spreads of other euro area members vis-à-vis Germany at certain maturities on country-specific variables such as fiscal variables, proxies for liquidity (such as size of the outstanding debt), proxies for time-varying risk aversion (as captured by private credit spreads) etc. We share with this literature the focus on fiscal variables (in addition to macroeconomic determinants of the common short-term interest rate) as explanatory variables. We depart from these earlier studies by estimating a multicountry affine term structure model, thereby using the entire cross-sectional information in the term structure by imposing the restrictions implied by ruling out arbitrage across maturities and borrowers, and by allowing for the interaction between macroeconomic variables and prices of risk.

Within the finance literature on the term structure, our paper is the first application of an affine term structure model of defaultable bonds to euro area yield curves that uses macroeconomic variables as factors. Linking the term structures to macroeconomic variables is challenging for several reasons. Because of the still limited sample since the introduction of the euro, we rely on monthly data, which is a compromise between the higher-frequency yield data and the lower-frequency macro data. Even then, since we are particularly interested in fiscal developments, using national accounts data interpolated to monthly frequency is problematic because of smoothing. We therefore rely in our estimation on the Kalman filter to infer missing observations of fiscal variables rather than a more mechanical interpolation routine.

Another major challenge is the clear evidence for a structural break in the relation-

ship between macroeconomic variables and spreads sometime during the first half of 2008. Because this break dwarfs any evidence for instability in this relationship that may have existed prior to 2008, and because euro area government bond spreads have yet to return to a more normal pattern, using a model with regime switching would necessarily lead to the conclusion that the crisis state is absorbing. We therefore have at this point no choice but to model the break as exogenous.

Our sample for yields starts with the beginning of Stage Three of EMU in January 1999 (with Greek yields starting shortly after EMU accession in 2001) and ends in March 2010. We are reluctant to extend the sample period beyond this point for two reasons. First, our term structure model of multiple, potentially defaultable issuers (with Germany assumed to be free of default risk so as to normalize spreads relative to German benchmark yields) assumes that each issuer is not bearing risk of other issuers defaulting. From the spring of 2010 on, this assumption becomes questionable in light of the support extended through the European Financial Stability Facility. Second, from May 2010 on the ECB intervened in government bond markets under its Securities Markets Programme, which reportedly had large effects on yields for certain issuers that are unrelated to the macroeconomic determinants we are interested in.

We first conduct some preliminary analysis to decide on both the most relevant fiscal variable to include and the most likely break point between macroeconomic variables and spreads. Our term structure model of multiple defaultable issuers that we then estimate conditional on our choices of fiscal variable and break date fits yields for all countries and a wide range of maturities impressively well, despite the fact that we do not use latent factors in our term structure model and restrict ourselves to two euro area-wide and for each country one fiscal factor. Measurement error standard deviations are on average across countries and maturities only about 20 basis points. We model the break as occurring in the linear relationship between the factors and the default intensities that drive spreads. There is strong evidence for such a break, with the sensitivity of spreads to especially the fiscal factors increasing sharply during the post-break sample.

In the following subsection we review related literature. In section 2 we present some exploratory results using OLS regressions, which help us to determine the states to include in the term structure model as well as the break date. Section 3 describes the affine term structure model. Section 4 presents our results. Section 5 offers conclusions. Details on the

data and the model specification are in appendices.

1.1 Relation to the literature

As mentioned earlier, a large empirical literature has studied government bond spreads in the euro area since the beginning of the common monetary policy in 1999 with the goal of identifying the determinants of term and risk premia in the absence of exchange-rate risk. Many of these studies rely on regressions of yield spreads at certain maturities on candidate explanatory variables. A common finding in this literature, beginning with Codogno et al. (2003) and Bernoth et al. (2006) and including more recent studies such as Manganelli and Wolswijk (2009), Haugh et al. (2009) and Schuknecht et al. (2010), is that euro area sovereign yield spreads seem to strongly comove. Principal component analysis regularly reveals that the first principal component accounts for more than 80% in the total variation of yield spreads. This finding suggests that a common factor, frequently interpreted as time-varying risk aversion of international investors that affects all yield spreads through the repricing of given country-specific risk characteristics, is the dominant force, making it difficult *a priori* to identify the role of country-specific variables such as fiscal policies in the determination of spreads. Laubach (2010), however, presents evidence that the strength of comovement among yield spreads varies substantially over time and has weakened since 2009.

If we assume that investors assign zero probability to the event of a member country leaving the euro, yield spreads can be explained as compensation for either liquidity risk or default risk. How to distinguish between these two interpretations of spreads has been a source of disagreement in the literature. Although since the eruption of the Greek fiscal crisis in November 2009 it seems plausible that default risk has been the dominant market concern, the relative importance of liquidity versus default risk is less clear during the first ten years of EMU. In their early study based on four years of monthly data, Codogno et al. (2003) concluded that “the risk of default is a small but important component of yield differentials” while liquidity factors seemed to be of lesser importance. But it seems difficult to explain persistent positive spreads of government bonds of AAA-rated countries over Bunds as being driven by factors other than liquidity risk.

Several recent studies conclude that the importance of liquidity risk seems to vary over time with proxies of international investor risk aversion. Beber et al. (2009), using intraday

European bond quotes from the period April 2003 to December 2004, find that differences in credit quality among countries play a major role, but that “in times of market stress, investors chase liquidity, not credit quality.” By contrast, Favero et al. (2010) conclude that the interaction between liquidity demand and risk is *negative*. They attribute the difference between their results and those of Beber et al. to the fact that Beber et al. “control for country-specific risk but do not consider aggregate risk factors.” In pooled regressions of quarterly spread data for ten euro area countries including an interaction term between a proxy for risk aversion and the volume of bonds outstanding (as proxy for liquidity) as well various fiscal variables to account for credit risk, Haugh et al. (2009) find a significant role for liquidity in line with the sign of Beber et al., with liquidity (or lack thereof) making a large contribution to the spreads of Irish and Finnish government bonds in late 2008 and early 2009.² While we do not deny that liquidity risk may in some instances and for some countries (those with small size of debt outstanding relative to euro area sovereign debt overall) have a sizeable role to play, we interpret the results from this literature as pointing more consistently to an important role of credit risk factors emanating from public finances, and therefore concentrate on those. This view is furthermore vindicated by the fact that we mainly focus on countries whose debt market is generally considered large and liquid, as it is the case for the biggest four euro-area countries or for a largely indebted but nevertheless "core" EMU-country like Belgium. Admittedly, this claim is weaker concerning the last country in our panel, Greece [figures here for debt to GDP ? share of euro area wide debt ?], but we think that it would be difficult to argue that concerns about fiscal sustainability in Greece were not the key driver of the surge in Greek bond yields over the past two years.

We depart from the literature discussed so far by using a no-arbitrage term structure model so as to exploit the information contained in the entire maturity spectrum of yield spreads. Not only can we multiply manifold the number of observations used in the analysis, we can also sharpen the conclusions regarding the determinants of yield spreads by estimating their effects on bonds of different maturities. The “essentially affine” class of term structure models that we use was first proposed by Duffee (2002) as a special case of affine term structure models. Beginning with the work of Ang and Piazzesi (2003), a growing literature has explored the role of macroeconomic variables as factors.³ Lemke

²Abmann and Boysen-Hogrefe (2010) model time-varying risk aversion as a latent variable and conclude, similar to Beber et al. (2009) and Haugh et al. (2009), that liquidity matters in times of stress.

³This use of macroeconomic variables as factors is not uncontroversial. Duffee (2009) and Joslin et al.

(2008) estimates a model with only observable macroeconomic factors for German bond yields during the euro area. For the U.S., Dai and Philippon (2006) and Laubach (2010) include fiscal variables as factors among the variables

In order to study the role of default risk in determining yield *spreads*, we employ the extension of affine term structure models to defaultable bonds proposed by Duffie and Singleton (1999). Geyer et al. (2004) provide an early application of such a model to euro area spreads, without, however, including macroeconomic variables as factors.⁴ More recently, Monfort and Renne (2010) generalize this model to account for regime switching and both default risk and liquidity factors and apply this model to jointly model the sovereign yield curves of Germany, France and Italy during EMU, using latent factors. For the reasons discussed above, we abstract here from regime switching.

Lastly, our results have implications for the long empirical literature on the effects of fiscal policy on interest rates (e.g. Ardagna et al. 2007). For the euro area, Faini (2006) provides evidence that fiscal policy (as measured by the deficit/GDP and debt/GDP ratios) significantly affects the level of average euro area long-term yields, with significant spillover effects from one country's fiscal policy stance to the euro-area wide level and very small effects on spreads. Our results confirm his findings during the period prior to the break in early 2008, but also document the increase in spread sensitivities since then.

2 Fiscal sustainability and euro-area sovereign bond yields: preliminary evidence

Affine term structure models rely on the assumption that linear relations hold between bond prices or yields and the observable macro factors that drive the yield curve. Hence, as a first pass, simple OLS regressions can provide us with useful insights about the set of variables that are likely to span the curve of each country, as shown in Dai and Philippon (2006). Following these authors, we present in this section the results of regressions of bond yields

(2010) have pointed to the importance of unspanned macro risks, i.e. that current macroeconomic variables cannot be recovered from current yields, but that macroeconomic variables nonetheless can affect future yield curves through their impact on expected future short-term interest rates. See Gürkaynak and Wright (2010) for further discussion.

⁴Amato and Luisi (2006) is to our knowledge the first use of an affine term structure model of defaultable bonds with macroeconomic variables as factors, but applied to U.S. corporate bond spreads.

of several maturities on measures of the fiscal stance and usual macroeconomic controls for each of the six countries in our sample. The idea is that if, for any given country, a given fiscal variable fails to explain significantly bond yields of different maturities in simple reduced-form regressions, then there is no point including this variable as a factor in our more sophisticated (and heavily constrained) no-arbitrage multicountry term structure model.

For each of the six countries, we use monthly observations of government bond yields at 2, 5 and 10 years maturities, that we regress on the 1-month risk free short term rate, a monthly indicator for the position in the domestic business cycle, domestic HICP inflation and four alternative measures of national fiscal imbalances. Appendix A details the sources and methodology for these series, and notably for the zero-coupon yields used throughout. The short term rate is measured using prices of 1 month OIS swaps rather than euro area money market rates, which have been obviously comprising a certain amount of premia for credit and liquidity risks since the start of the financial crisis in August 2007. For each country, the monthly national business cycle indicator is constructed as the first principal component of sectoral activity indices taken from Eurostat's business surveys. Figure 1 shows the short term interest rate for the euro area, the German 10-year government bond yield and our coincident indicator of the euro area wide business cycle, computed here as a weighted average of the six national activity indicators.

The appropriate choice of the most relevant measure of fiscal imbalances at the national level is less clear. Previous studies frequently consider the deficit to GDP or the debt to GDP ratios, or forecasts thereof (see e.g. Codogno et al., 2003, for the euro area and Laubach, 2009, Dai and Philippon, 2006, for the US). Bernoth et al. (2006) argue that debt service (defined as the ratio of gross interest payments to current government revenue) is more appropriate when trying to assess the impact of fiscal balances on euro area bond yields, if only because governments have less incentive to manipulate it than other measures that are used officially to monitor whether national fiscal positions meet the obligations set out by the Stability and Growth Pact. Furthermore, Haugh et al. (2009) find that both fiscal deficit and debt service help to explain a substantial part of cross-sectional variations in euro area bond yields during the recent crisis. It can be also noted that debt service is routinely monitored by bond markets participants in order to gauge the sustainability of issuing countries' fiscal imbalances.

Either one of these three measures suffers potentially from an endogeneity problem. In practice, as long as the average maturity of countries' debt is not too short, so that the share of total debt that needs to be refinanced each period is small, the contemporaneous effect of changes in interest rates on either the deficit/GDP ratio or the debt service ratio is rather modest.⁵ The primary deficit/GDP ratio obviously does not suffer from this potential problem of reverse causality. As a consequence, we consider here four alternative measures of fiscal imbalances: total fiscal and primary deficit to GDP, the debt to GDP ratio as well as the debt service to income ratio. We take the corresponding series from the OECD Economic Outlook database. OECD data are provided on a semi-annual basis with quarterly frequency. For the needs of the preliminary regressions conducted in this section, we first interpolated these quarterly series using simple cubic splines. Note however that, in the subsequent estimation of the complete affine term structure model, missing observations of the fiscal variable are dealt with in a more satisfying manner using the Kalman filter, as detailed below in section 4.1. For illustration, Figure 3 shows the debt service ratios as interpolated at monthly frequency with the Kalman filter.

As evidenced by Figure 2, the level and dynamics of euro area spreads seem to have undergone a structural break at some point during the year 2008, a move that has been generally commented as reflecting a repricing of country-specific risk, notably credit risk that would have been neglected prior to the 2007- crisis. We thus added a break in the constant and in the regression coefficients of the fiscal variable in our regressions. In order to limit the arbitrariness inherent to the selection of any exogenous break, we had our choice guided by a simple statistic: the average R^2 over regressions for all countries and maturities, conditional on a given break date. Figure 4 shows the resulting statistic for each of the candidate four fiscal variables when the break date is allowed to vary between the end of 2006 and the end of 2009. Two facts emerge from this exercise. First, the explanatory power of debt to GDP and debt service is higher than that of the deficit to GDP ratios. Second, we reach at least local maxima of the average R^2 for both debt to GDP and debt service regressions at the end of the the first and fourth quarters of 2008. On this basis,

⁵Gross debt issuance in 2010 ranged from between 8 and 10 percent of GDP for Germany, France and Spain, to nearly 17 percent for Italy and Greece. An increase in the spread of 100 bps for a country that needs to refinance debt in the amount of 10 percent of GDP would add in the same year at most 0.1 percent of GDP to the deficit. Only for Greece, which at the end of our sample was facing spreads around 300 bps that have since risen to 900 bps, would the endogeneity problem be serious.

we finally chose March 2008 as our preferred break date, which leaves enough observations available after the break for estimation purposes.

Tables 1 to 4 present the results of the regressions. Each table stands for one of the candidate measures of fiscal imbalances. Standard errors are corrected for heteroscedasticity using the Newey-West procedure. We first note that, whatever the fiscal variable, the sensitivity of longer term yields to the short term rate and the share of variance explained by macroeconomic factors decreases with maturity, consistently with common results in the yield curve literature (see e.g. Ang and Piazzesi, 2003). Overall, the crisis dummy, which takes the value one from March 2008 onward, as well as domestic activity, the fiscal variable and the interacted term standing for non-linear effects of fiscal imbalances in crisis times all turn out to be significant. By contrast, domestic inflation is generally not significant. In the analysis that follows, we are therefore omitting inflation from the state vector.

Due to sign conventions in the construction of fiscal variables, we should expect a negative sign for coefficients of deficit measures (i.e. more negative fiscal balances should imply higher yields accounting for larger risk premia), and a positive sign for the debt ratio and debt service measures. Table 1 reveals that the deficit variable enters significantly but with the wrong sign for four out of six countries, although the sign is reversed in crisis times as expected. Conversely, debt to GDP turns out to be negatively correlated with French and German yields, even in crisis times, which may be interpreted as signalling these two “core” countries as relative safe havens in times of heightened risk perception. Last but not least, the debt service to income variable is positively correlated with yields in all countries in quiet times, while pushing yields of “core” versus more “peripheral” countries into opposite directions in crisis times (respectively downward and upward). Overall, based on these preliminary results, we decided to use the ratio of debt service to government income as our best measure of fiscal imbalances in the following.

3 An affine term structure model of defaultable bonds

3.1 Dynamics of the pricing factors under the historical measure

Let us denote respectively by r_t and x_t the one-period rate –or short-term rate– and the Euro area business cycle indicator. These two variables, together with the country-specific fiscal variables $f_{i,t}$, make up the set of pricing factors, which are stacked in a vector $X_t =$

$[x_t \ r_t \ f_{1,t} \ \dots \ f_{N,t}]'$.⁶ We assume that the factors depend on their lagged values and are affected by a vector ε_t of idiosyncratic shocks, where the ε_t 's are i.i.d. $N(0, I)$. Accordingly, the factors are modelled as following a VAR(p), with p the number of lags:

$$X_t = \mu_X + \Phi_1 X_{t-1} + \dots + \Phi_p X_{t-p} + \Sigma_X \varepsilon_t \quad (1)$$

With $N = 6$ countries and thus eight variables overall in X_t , the number of parameters to be estimated in the VAR grows very rapidly with p , which requires that we impose some *a priori* restrictions on the model. For parsimony, we first assume that the covariance matrix Σ_X is diagonal, which implies that the innovations to the factors are orthogonal. Moreover, some constraints have also to be imposed on the auto-regressive matrices Φ_i . First, consistently with the prohibition of any monetary financing of public debt as enacted by the Maastricht Treaty as a guarantee for the independence of the European Central Bank (ECB), we assume that the short term rate r_t is not affected by past values of the national fiscal variables $f_{i,t}$. Therefore, the short term rate equation –akin to a reaction function of the ECB– reads:

$$r_t = \mu_r + (\theta_1 x_{t-1} + \rho_1 r_{t-1}) + \dots + (\theta_p x_{t-p} + \rho_p r_{t-p}) + \sigma_r \varepsilon_{r,t} \quad (2)$$

Such a specification clearly departs from the usual Taylor-type rule, where the short term rate is routinely depicted as reacting both to the business cycle and inflation. We think however that it is not a major problem, since, first, the ECB's objective of maintaining price stability over the medium term does not imply that they have to respond systematically to current inflation or even lagged inflation, and second, available empirical evidence suggests that in fact current inflation and even expected inflation do not often come out as a significant driver of the ECB's policy rate decisions.⁷ Note also that the six countries in our sample account for roughly 85% of total euro area GDP (in 2010), which makes our aggregate business cycle indicator a reasonable proxy for the true euro area activity measures to which the ECB is likely to react.

Second, we assume that the European business cycle x_t depends on a euro-area wide measure of fiscal imbalances, that is equal to the GDP-weighted sum of the national fiscal variables. The business cycle equation thus reads:

⁶Note that we do not include any additional latent factor in the state space, contrary to Ang and Piazzesi (2003), Dai and Philippon (2006) and many others.

⁷See e.g. Gerlach, 2007, and Jensen and Aastrup, 2010, for two recent assessments.

$$x_t = \mu_x + (\alpha_1 x_{t-1} + \beta_1 r_{t-1} + \omega_1 \sum_{j=1}^N W_j f_{j,t-1}) + \dots + (\alpha_p x_{t-p} + \beta_p r_{t-p} + \omega_p \sum_{j=1}^N W_j f_{j,t-p}) + \sigma_x \varepsilon_{x,t} \quad (3)$$

Finally, we also assume that the parameters that govern the dynamics of the N fiscal variables are not country-specific, except for the standard deviations of the innovations. Concretely, we have for any country j :

$$f_{j,t} = \mu_f + (\kappa_1 x_{t-1} + \zeta_1 r_{t-1} + \psi_1 f_{j,t-1}) + \dots + (\kappa_p x_{t-p} + \zeta_p r_{t-p} + \psi_p f_{j,t-p}) + \sigma_{f,j} \varepsilon_{f,t} \quad (4)$$

Formally, these constraints imply the following form for the matrices Φ_i and Σ :

$$\Phi_i = \begin{bmatrix} \begin{bmatrix} \alpha_i & \beta_i \\ \theta_i & \rho_i \\ \kappa_i & \zeta_i \\ \vdots & \vdots \\ \kappa_i & \zeta_i \end{bmatrix} & \begin{bmatrix} \omega_i W_1 & \dots & \omega_i W_N \\ 0 & \dots & 0 \\ & & \psi_i \mathbf{Id} \end{bmatrix} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_x & 0 & \dots & & 0 \\ 0 & \sigma_r & & & \\ \vdots & \ddots & \sigma_{f,1} & \ddots & \vdots \\ & & & \ddots & 0 \\ 0 & & \dots & 0 & \sigma_{f,N} \end{bmatrix}.$$

It will prove convenient in the pricing framework that follows to turn the model into its companion VAR(1) representation. To that end, let us define a new state vector F_t in which the vectors from X_t to X_{t-p+1} are stacked. The dynamics of F_t is:

$$F_t = \mu + \Phi F_{t-1} + \Sigma \varepsilon_t, \quad (5)$$

with

$$\mu = \begin{bmatrix} \mu_X \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_p \\ \mathbf{Id} & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & \mathbf{Id} & 0 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_X & 0 \\ 0 & 0 \end{bmatrix}. \quad (6)$$

3.2 Dynamics of the factors under the risk-neutral measure

It is well known that the absence of arbitrage opportunities is a necessary and sufficient condition for the existence of a positive stochastic discount factor (see, e.g., Hansen and Richard, 1987). Following, amongst many others, Ang and Piazzesi (2003), we postulate the following form for this stochastic discount factor $m_{t,t+1}$:

$$m_{t,t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t} \quad (7)$$

where the Radon-Nikodym derivative ξ_t follows a log-normal process defined by:

$$\xi_t = \xi_{t-1} \exp\left(-\frac{1}{2}\lambda'_{t-1}\lambda_{t-1} - \lambda'_{t-1}\varepsilon_t\right)$$

where λ_t is the vector of the time-varying market prices of risk associated with the innovations ε_t to the pricing factors in F_t . We parameterize the λ_t as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 F_t \quad (8)$$

Under these assumptions, it can be shown that the dynamics of the pricing factors under the risk-neutral measure \mathbb{Q} is defined by:

$$F_t = \mu^* + \Phi^* F_{t-1} + \Sigma \varepsilon_t^* \quad (9)$$

where the ε_t^* 's are i.i.d. $N^{\mathbb{Q}}(0, I)$ and with:

$$\begin{aligned} \mu^* &= \mu - \lambda_0 \Sigma \\ \Phi^* &= \Phi - \lambda_1 \Sigma. \end{aligned} \quad (10)$$

3.3 Bond pricing with default risk

Let us denote by $P(t, h)$ the price at time t of a (credit) risk-free zero-coupon bond of residual maturity h . This price is given by:

$$P(t, h) = E(m_{t,t+1} \dots m_{t,t+h}) \quad \text{or} \quad P(t, h) = E^{\mathbb{Q}}(\exp(-r_t - r_{t+1} \dots - r_{t+h-1})).$$

Remember, that in the following, we assume that one country, Germany, is viewed as credit risk-free by investors. To price the other five countries' bonds, that are perceived as subject to credit risk, we define a default intensity –or hazard rate– for each country. The default intensity of country $j, j = 2..6$, is denoted by $s_{j,t}$ and reflects the credit risk embedded in the bonds issued by this country. If recovery rates were nil, the default intensity at time t would be the default probability of the considered debtor at that period. However, recovery rates are strictly positive processes. Therefore, the hazard rates $s_{j,t}$ should be more rigorously termed as "recovery-adjusted default intensities" (see, e.g. Monfort and Renne, 2010).⁸ Duffie and Singleton (1999) show that, under the assumption that the loss is proportional

⁸Intuitively, with a constant recovery rate of R , the recovery-adjusted default intensity $s_{j,t}$ would be approximately equal to $(1 - R)\tilde{s}_{j,t}$ where $\tilde{s}_{j,t}$ is the default probability of country j at time t .

to the market value, defaultable bonds can be priced using the same machinery than for risk-free bonds by simply replacing the short-term risk-free rate r_t by the default-adjusted short-term rate $r_t + s_{j,t+1}$. Formally, denoting by $P_j(t, h)$ the price at time t of a bond of residual maturity h issued by country j , we have:

$$P_j(t, h) = E^{\mathbb{Q}}(\exp[-(r_t + s_{j,t+1}) - \dots - (r_{t+h-1} + s_{j,t+h})]).$$

Appendix B shows that bond prices are exponential affine in the factors F_t when the hazard rates are themselves affine in the same factors, that is:

$$P_j(t, h) = \exp(A_{j,h} + B_{j,h}F_t) \quad (11)$$

where the vectors $A_{j,h}$ and $B_{j,h}$ are obtained by applying recursive formulas. The continuously compounded yield, denoted by $y_{j,h,t}$ and defined by $y_{j,h,t} = -\log(P_j(t, h))/h$, are then given by:

$$y_{j,h,t} = \bar{A}_{j,h} + \bar{B}_{j,h}F_t \quad (12)$$

with $\bar{A}_{j,h} = -A_{j,h}/h$ and $\bar{B}_{j,h} = -B_{j,h}/h$.

3.4 Introducing a break in the hazard rates

The modeling approach is completed by the introduction of an exogenous break at time τ . The period posterior to τ corresponds to the crisis period. We assume that the break only applies to the parameters of the hazard rates $s_{j,t}$, while the stochastic discount factor as well as the historical dynamics of the pricing factors F_t are not affected by that break. Underlying is the assumption that there is more inertia in the specifications of the dynamics of the factors than in the specifications of the default intensities. Then, the main changes implied by the crisis in terms of bond pricing would regard the way investors form expectations about the default probabilities of the countries.

In that context, the hazard rates are given by:

$$s_{j,t} = \mathbb{I}(t < \tau) \times [\gamma_{j,0}^{bb} + \gamma_{j,1}^{bb}F_t] + \mathbb{I}(t \geq \tau) \times [\gamma_{j,0}^{pb} + \gamma_{j,1}^{pb}F_t] \quad (13)$$

where the *bb* and *pb* subscripts respectively stand for "before break" and "post break".

Further, we assume that the likelihood of such a break in the future is not even taken into consideration before the break date: whereas agents' expectations regarding future values of the hazard rates $s_{j,t+h}$ –for any horizon h – are based on the vectors of parameters

$\gamma_{j,0}^{bb}$ and $\gamma_{j,0}^{pb}$ until time $\tau - 1$, these expectations get based on the $\gamma_{j,0}^{pb}$'s and on the $\gamma_{j,1}^{pb}$'s from time τ onwards. To put it differently, the break we model was unforeseeable and, once it occurred, is considered as permanent by investors. In terms of bond pricing, the presence of the break implies the existence of two sets of vectors $\{\bar{A}_{j,h}^{bb}, \bar{B}_{j,h}^{bb}\}_{j,h}$ and $\{\bar{A}_{j,h}^{pb}, \bar{B}_{j,h}^{pb}\}_{j,h}$. Whereas the former are used to price bonds before the break, the latter are used to price the bonds after the break.

4 Estimation and results

4.1 Estimation

The estimation is based on two steps. In the first one, we estimate the parameters that enter the historical dynamics of the factors, namely $\Theta_1 = [\alpha', \beta', \theta', \rho', \kappa', \zeta', \omega', \sigma']'$, where α , for instance, is equal to $[\alpha_1, \alpha_2, \dots, \alpha_p]'$ and σ is the vector $[\sigma_1, \dots, \sigma_N]'$. In the second step, the parameters defining the risk-neutral dynamics of the factors, namely $\Theta_2 = [\alpha^{*'}, \beta^{*'}, \theta^{*'}, \rho^{*'}, \kappa^{*'}, \zeta^{*'}, \omega^{*'}]'$, as well as the γ 's are estimated.

The first step of the estimation deals with the historical dynamics of F_t , as given by Equation (5). For parcimony, we limit to 2 the number of lags p in the historical VAR. As said, the model does not include any unobservable, or latent, factor. However, given that the frequency of the fiscal variable is lower than the monthly one retained for the estimation, we have to deal with missing data. This problem is solved using the Kalman filter, which provides us also with the log-likelihood of the model defined by (5). More precisely, the vector of (unobserved) monthly fiscal variables is defined as a latent vector whose dynamics follows (4). Measurement equations then state that the fiscal variables are observed without error at quarterly frequency whenever the OECD data are available.

In a second step, we estimate the parameters that govern the risk neutral dynamics of the factors and the parameters of the default intensities, conditionally on the results obtained with the first step. This estimation is based on a non-linear least square procedure (see, e.g. Moench, 2008). The procedure consists in minimizing the sum of pricing errors across time, countries and maturities. Specifically, let us denote by $y_{j,h,t}^o$ the observed continuously-compounded zero-coupon yield of maturity h of country j . The model-based counterpart of $y_{j,h,t}^o$ is a function of Θ_2 and F_t .⁹ The function of Θ_2 that we want to minimize is the

⁹Actually, the model-based yield also depends on the σ 's. However, in that second step of the estimation,

following expression:¹⁰

$$\sum_{h,t} \underbrace{(y_{1,h,t}(\Theta_2, F_t) - y_{1,h,t}^o)^2}_{\text{Error on German yields}} + \sum_{j>1,h,t} \underbrace{(y_{j,h,t}(\Theta_2, F_t) - y_{1,h,t}(\Theta_2, F_t) - (y_{j,h,t}^o - y_{1,h,t}^o))^2}_{\text{Error on spreads vs. Germany}}.$$

Whereas we optimize the fit of the German yields, we look for the best fit of the spreads vs. Germany for other countries (*credit spreads* hereinafter). This is done in order to favour the fit of the credit spreads by compelling the hazard rates to reflect differences in credit risk amongst countries.

The computation of the first-step covariance matrix of the estimates is based on the Hessian. Since we only use observable factors to model the yields, the pricing errors are subject to heterogeneity and auto-correlation. To cope with that, the computation of the second-step covariance matrix uses Newey-West (1987) heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimators (see Appendix C).

4.2 Discussion

Figure 5 plots the observed yields of German bonds with maturities of 1 to 10 years against the yields simulated with our baseline model (in the following called model I; alternative model versions will be introduced below). Similarly, Figure 6 shows the observed and estimated bond spreads towards Germany for the remaining five euro area countries and two maturities (5 and 10 years). Overall, the model does a reasonably good job in tracking both the absolute level of yields of German government bonds and the level of spreads of other countries vis-à-vis Germany. As appears on the last row of Table 6, the standard deviation of the measurement errors over all countries and maturities amounts to a modest 20 basis points, which compares favourably with the usual fit of affine term structure macrofinance models. This is all the more remarkable because we restrict the state space of factors spanning the six yield curves to observable factors only, while most studies in this literature also incorporate latent factors in the model (see e.g. Ang and Piazzesi, 2003, Dai and Philippon, 2005, Rudebusch and Wu, 2008). Considering all countries and maturities, the model principally fails to capture a spike in yields in mid-2002 (which is not obviously related to any monetary policy decision neither to a particular fiscal event at that time),

we suppose that these are fixed to their first-step estimated value.

¹⁰The minimization is carried out on Matlab, by using iteratively a Newton algorithm and a Nelder-Mead simplex algorithm until reaching convergence.

as well as a hump in the levels of long term spreads over 2001-2002. Note however that the fit to observed spreads is particularly good from 2003 on, which means that the model provides quite an appropriate tool to analyze the changes in bond risk pricing that were brought along by the 2007- financial turmoil.

The first two columns of Tables 5 and 6 show the estimates of the parameters of the default intensities $s_{j,t}$ for our baseline model, while contrasting the two sub-periods before and after the assumed break date of March 2008. Focusing first of the sensitivities of default intensities to the fiscal variable, γ_j^f , we first find that the domestic debt service variable stands out as a significant determinant of spreads for most countries, and with the expected positive sign. Our findings also confirm the widespread intuition that the crisis brought about a rise in the magnitude of these sensitivities to fiscal sustainability measures, which we view as a major contribution of this study. Indeed, while they come out as negligible on the pre-crisis period, they consistently rise with the crisis, magnifying afterwards the impact of aggravated national fiscal imbalances on intra-EMU spreads. For instance, whereas before the crisis a one point increase in the Italian debt service ratio would have induced only a tiny change in the compensation required for the increased default risk (by some 2 basis points), the required compensation after the crisis would have been more than ten times larger, by some 27 basis points.¹¹

Regarding other parameters, we also find that the crisis was associated with a significant increase in the average level γ_0 of default intensities for all countries except France, witnessing an upward shift in the required compensation for default risk in both highly indebted and peripheral countries. The decline in γ_0 for France after the break could indicate safe-haven flows into French bonds. By contrast, the sensitivities of default intensities to either the euro area business conditions or the short term interest rate (γ_j^x and γ_j^r respectively) come out as weak and generally non significant.

Considering these first results, we also checked for the possibility of imposing additional constraints on the default intensities parameters in order to reduce the number of estimated parameters and thereby hopefully improve the reliability of our parameter estimates. We based our assessment on a series of Wald tests, looking first at parameter estimates obtained

¹¹The lack of precision of some of the post-break estimates is due to the short post-break sample of 25 months. Note that, strictly speaking, the consequences on spreads (in bp) of changes to the γ 's can be read directly from the estimates only for the one-period-ahead spread. At longer horizons, the impact is proportional to a complex convolution of virtually all the parameters.

with the baseline model (denoted model I hereafter), then at alternative, more constrained specifications, but still nested in the baseline model. Table 7 presents the results of tests of restrictions imposed to the baseline model. The first column gives the p values of a test of the null of no break in the parameters, considering them all together or by successive blocks for each variable. The successive columns provide block-by-block tests of the null that the default price coefficients are common across all five countries or equally null, considering in turn the pre- and post-break periods. Results of the tests confirm that a break in each of the γ 's is required by the data, and that both γ_0 and γ_j^f are non-zero and different across countries in each sub-period. In contrast, the tests validate the null of zero values for the sensitivities to the cycle and the monetary policy rate on the pre-crisis period. Finally, columns 3 to 8 of Tables 5 and 6 present the results that we obtain for alternative specifications based on the tests just described. Besides, Tables 8 to 10 show the results of the corresponding Wald tests. Whatever the restrictions imposed following the outcome of the first series of tests as in Table 7, note that our previous findings regarding the significant and changing role of fiscal sustainability in default intensities remain qualitatively unaffected.

Based on the Wald tests above, we finally use the restrictions corresponding to Model II to simulate counterfactuals of the intra-EMU spreads under the hypothesis of an absence of structural break in the parameters of default risk prices. Figure 7 shows the outcome, plotted against both the realized and fitted spreads at the 5 and 10 year maturities. A comparison of the fitted values of the 10 year spreads to the simulated counterfactuals at the end of our sample in March 2010 confirms that the repricing of default risk brought about by the crisis implied only minor increases in spreads of core-countries like France and Belgium (between 10 and 20 bp). In contrast, the repricing move is estimated to have induced much larger increases in the spreads of the other three countries (from 60-70 bp for Italy and Spain up to some 180 bp for Greece).

5 Conclusion

In this paper we take a first step towards exploring the information contained in yields across sovereign issuers, maturities and time to obtain a better understanding of the macroeconomic determinants of euro area sovereign yield spreads through the lens of an arbitrage-free

term structure model. Our main results at this preliminary stage are that a small number of macroeconomic variables can explain the evolution of yields at various maturities and across time and issuers impressively well, that there is a significant relationship between debt service as a fraction of tax revenues and yield spreads, and that this relationship has undergone a major break in early 2008.

In future work we plan to expand the analysis in several directions. One important direction is to model the information set of investors yet more carefully, by relying on real-time data (which we have from the OECD for a subset of vintages during our sample) and on survey expectations of fiscal and other variables as noisy measurements of model-implied investors' expectations. Doing so would at least partially address the problem of "fiscal foresight," whereby frequently investors have more information about future fiscal policies than a simple VAR model that an econometrician would use to proxy for investors' expectations (Leeper et al., 2008). Adding the information contained in real-time fiscal forecasts should also contribute to refine the the estimates of the historical dynamics of the pricing factors. It would be then possible to extract reliable measures of expected bond default probabilities from the modeled spreads, as well as time-varying estimates of credit risk premia embedded in bond yields at various maturities. Finally, another interesting direction goes towards a more detailed analysis of the contributions of the various factors to yield spreads over time.

References

- [1] Amato, Jeffery and Maurizio Luisi, 2006. "Macro factors in the term structure of credit spreads." BIS working paper no. 203.
- [2] Ang, Andrew and Monika Piazzesi, 2003. "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables." *Journal of Monetary Economics* 50 (4), 745-787.
- [3] Ardagna, Silvia, Francesco Caselli and Timothy Lane, 2007. "Fiscal discipline and the cost of public debt service: Some estimates for OECD countries." *The B.E. Journal of Macroeconomics* 7.
- [4] Aßmann, Christian, and Jens Boysen-Hogrefe, 2009. "Determinants of government bond spreads in the euro area – in good times as in bad." Kiel Institute for the World Economy, working paper 1548.
- [5] Attinasi, Maria-Grazia, Cristina Checherita and Christiane Nickel, 2009. "What explains the surge in euro area sovereign spreads during the financial crisis of 2007-09?" European Central Bank, working paper no. 1131, December.
- [6] Beber, Alessandro, Michael Brandt, and Kenneth Kavajecz, 2009. "Flight-to-quality or flight-to-liquidity? Evidence from the euro-area bond market." *Review of Financial Studies* 22 (3), 925-957.
- [7] Bernoth, Kerstin and Burcu Erdogan, 2010. "Sovereign bond yield spreads: A time-varying coefficients approach." DIW Berlin discussion paper 1078.
- [8] Bernoth, Kerstin, Jürgen von Hagen and Ludger Schuknecht, 2006. "Sovereign risk premiums in the European government bond market." Available at <http://www.sfbtr15.de/dipa/151.pdf>.
- [9] BIS, 2005. "Zero-coupon yield curves: technical documentation." BIS Working Papers 25.
- [10] Codogno, Lorenzo, Carlo Favero and Alessandro Missale, 2003. "Yield spreads on EMU government bonds." *Economic Policy* Vol. 18, Issue 37, 503-532.

- [11] Dai, Qiang, and Thomas Philippon, 2006. "Fiscal policy and the term structure of interest rates." Working paper, November.
- [12] Duffee, Gregory, 2002. "Term premia and interest rate forecasts in affine models." *Journal of Finance* 57, 405-443.
- [13] Duffee, Gregory, 2009. "Information in (and not in) the term structure." Manuscript, Johns Hopkins University, August.
- [14] Duffie, Darrell and Kenneth Singleton, 1999. "Modeling term structures of defaultable bonds." *Review of Financial Studies* 12 (4), 687-720.
- [15] Ejsing, Jacob and Wolfgang Lemke, 2009. "The Janus-headed salvation: Sovereign and bank credit risk premia during 2008-09." European Central Bank, working paper no. 1127, December.
- [16] Faini, Riccardo, 2006. "Fiscal policy and interest rates in Europe." *Economic Policy* Vol. 21, Issue 47, 443-489.
- [17] Favero, Carlo, Marco Pagano, and Ernst-Ludwig von Thadden, 2010. "How does liquidity affect government bond yields?" *Journal of Financial and Quantitative Analysis* 45 (1), 107-134.
- [18] Gerlach, Stefan, 2007. "Interest rate setting by the ECB, 1999-2006: Words and Deeds." *International Journal of Central Banking* March, 1-45.
- [19] Geyer, Alois, Stephan Kossmeier, and Stefan Pichler, 2004. "Measuring systematic risk in EMU government yield spreads." *Review of Finance* 8 (2), 171-197.
- [20] Gurkaynak, Refet, Brian Sack and Jonathan Wright, 2006. "The U.S. treasury yield curve: 1961 to the present." Feds working papers, Federal Reserve Bank.
- [21] Gürkaynak, Refet and Jonathan Wright, 2010. "Macroeconomics and the term structure." Manuscript, Johns Hopkins University, September.
- [22] Haugh, David, Patrice Ollivaud and David Turner, 2009. "What drives sovereign risk premiums? An analysis of recent evidence from the euro area." OECD Economics Department, working paper no. 718, July.

- [23] Jensen, Henrik and Morten Aastrup, 2010. “What Drives the European Central Bank’s Interest-Rate Changes?” CEPR Discussion Paper No. 8160, December 2010.
- [24] Joslin, Scott, Marcel Pribsch and Kenneth Singleton, 2010. “Risk premiums in dynamic term structure models with unspanned macro risks.” Manuscript, Stanford University, May.
- [25] Kim, Don H. and Athanasios Orphanides, 2005. “Term structure estimation with survey data on interest rate forecasts.” Federal Reserve Board, Finance and Economics Discussion Series 2005-48.
- [26] Laubach, Thomas, 2009. “New Evidence on the Interest Rate Effects of Budget Deficits and Debt,” *Journal of the European Economic Association* 7, 858-885
- [27] Laubach, Thomas, 2010. “Fiscal policy and interest rates: The role of sovereign default risk.” mimeo.
- [28] Leeper, Eric, Todd Walker, and Shu-Chun Yang, 2008. “Fiscal foresight: Analytics and econometrics.” NBER Working Paper No. 14028, May.
- [29] Lemke, Wolfgang, 2008. “An affine macro-finance term structure model for the euro area.” *North American Journal of Economics and Finance* 19, 41-69.
- [30] Manganelli, Simone, and Guido Wolswijk, 2009. “What Drives Spreads in the Euro Area Government Bond Market?” *Economic Policy* Vol. 24, Issue 58, 191-240.
- [31] Mönch, Emanuel, 2008. “Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach.” *Journal of Econometrics*
- [32] Monfort, Alain, and Jean-Paul Renne, 2010. “Default, liquidity and crises: An econometric framework.” Working paper, May.
- [33] Nelson, Charles and Andrew Siegel, 1987. “Parsimonious modeling of yield curves.” *Journal of Business*, 60(4):473-89.
- [34] Schuknecht, Ludger, Jürgen von Hagen and Guido Wolswijk, 2010. “Government Bond Risk Premiums in the EU Revisited: The Impact of the Financial Crisis.” European Central Bank, working paper no. 1152, February.

- [35] Sgherri, Silvia, and Edda Zoli, 2009. “Euro area sovereign risk during the crisis.” IMF working paper WP/09/222.
- [36] Svensson, Lars, 1994. “Estimating and interpreting forward interest rates: Sweden 1992–1994.” NBER Working Papers 4871, National Bureau of Economic Research.

	Germany			France			Italy		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	0,64 (1,54)	0,68 (1,6)	0,61 (1,61)	0,67 (1,37)	1,2** (2,4)	1,6*** (3,29)	-0,57 (-1,44)	0,36 (0,63)	1,3 (1,59)
π_i	-0,07 (-0,64)	-0,17 (-1,49)	-0,25** (-2,31)	0,0067 (0,09)	0,036 (0,43)	0,061 (0,63)	-0,076 (-0,86)	0,017 (0,15)	0 (0,74)
$x_{i,t}$	0,081 (1,15)	0,0087 (0,12)	-0,056 (-0,75)	0,11*** (2,66)	0,021 (0,41)	-0,068 (-1,16)	0,22*** (5,09)	0,12* (1,8)	-0,0062 (-0,07)
r_t	0,58*** (5,96)	0,48*** (3,67)	0,44*** (3,02)	0,5*** (5,84)	0,34** (2,51)	0,27 (1,42)	0,43*** (6,74)	0,25* (1,74)	0,16 (0,78)
$f_{i,t}$	0,083 (0,82)	0,063 (0,54)	0,032 (0,28)	0,27*** (3,91)	0,31** (2,56)	0,29* (1,71)	0,28*** (4,16)	0,34*** (2,71)	0,34** (2)
$f_{i,t}.\delta_{cris,t}$	-0,036 (-0,28)	-0,073 (-0,57)	-0,11 (-0,94)	-0,21** (-2,2)	-0,31*** (-2,89)	-0,36*** (-3,18)	-0,029 (-0,3)	-0,26** (-1,97)	-0,44** (-2,5)
R^2	0,85	0,69	0,52	0,9	0,72	0,48	0,92	0,75	0,53
	Spain			Belgium			Greece		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	-0,66* (-1,65)	-0,32 (-0,63)	0,082 (0,16)	-0,17 (-0,61)	-0,016 (-0,04)	0,24 (0,45)	2,9*** (2,86)	2,4** (2,19)	1,8 (1,52)
π_i	0,0027 (0,05)	-0,058 (-0,98)	-0,094 (-1,54)	-0,062 (-1,07)	-0,11 (-1,16)	-0,12 (-0,92)	0,16 (1,07)	0,082 (0,54)	-0,005 (-0,04)
$x_{i,t}$	0,24*** (3,36)	0,19** (2,16)	0,11 (1,38)	0,17*** (3,55)	0,07 (0,91)	0,0031 (0,03)	0,15** (2,55)	0,00018 (0)	-0,12** (-2,21)
r_t	0,73*** (12,17)	0,55*** (8,84)	0,4*** (6,54)	0,62*** (8,05)	0,47*** (5,61)	0,36*** (3,07)	0,52*** (3,78)	0,45*** (3,06)	0,43*** (2,66)
$f_{i,t}$	0,068* (1,76)	-0* (-1,89)	-0,27*** (-4,46)	0,084* (1,79)	0,18** (2,35)	0,23** (2,12)	0,13* (1,89)	0,043 (0,59)	-0,063 (-0,81)
$f_{i,t}.\delta_{cris,t}$	-0** (-2,18)	0,048 (0,75)	0,2*** (2,89)	-0,054 (-0,77)	-0,17 (-1,57)	-0,25* (-1,71)	-0,44*** (-3,88)	-0,34*** (-2,97)	-0,21* (-1,72)
R^2	0,86	0,72	0,68	0,89	0,69	0,49	0,61	0,49	0,5

Table 1: Preliminary regressions – Endogenous variable: deficit to GDP

	Germany			France			Italy		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	1,1 (0,36)	3,5 (1,28)	5,7*** (3,16)	1,2 (0,72)	4,4** (2,44)	7,4*** (4,49)	-5,8** (-2,04)	-2,1 (-0,52)	2,5 (0,56)
π_i	0,069 (0,59)	0,11 (0,95)	0,14 (1,56)	-0,054 (-0,88)	-0,0062 (-0,11)	0,052 (1,04)	-0,14 (-1,51)	-0,034 (-0,29)	0,11 (0,85)
$x_{i,t}$	0,19*** (4,35)	0,15*** (3,51)	0,096*** (2,83)	0,24*** (8,57)	0,21*** (5,01)	0,15*** (3,46)	0,25*** (4,46)	0,14* (1,7)	-0,0055 (-0,06)
r_t	0,37*** (4,2)	0,051 (0,73)	-0,15** (-2,16)	0,3*** (4,7)	-0,051 (-0,63)	-0,27*** (-2,8)	0,63*** (10,72)	0,5*** (6,29)	0,43*** (3,89)
$f_{i,t}$	-0,08*** (-3,41)	-0,14*** (-6,34)	-0,18*** (-9,79)	-0,11*** (-4,58)	-0,17*** (-8,07)	-0,22*** (-11,48)	0,03* (1,84)	0,051** (2,06)	0,069*** (2,7)
$f_{i,t}.\delta_{cris,t}$	0,015 (0,34)	0,053 (1,35)	0,088*** (3,38)	0,029 (1,22)	0,077*** (3,26)	0,12*** (5,96)	-0,044* (-1,83)	-0,014 (-0,4)	0,023 (0,58)
R^2	0,88	0,82	0,83	0,92	0,87	0,87	0,89	0,71	0,54
	Spain			Belgium			Greece		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	0,14 (0,15)	0,0037 (0)	-0,034 (-0,03)	-3 (-1,07)	-1,5 (-0,45)	-0,31 (-0,1)	17*** (3,18)	12*** (3,33)	8,3*** (2,91)
π_i	-0,0066 (-0,11)	-0,03 (-0,52)	-0,033 (-0,68)	-0,057 (-1,17)	-0,079 (-1,44)	-0,072 (-1,29)	0,15 (1,1)	0,043 (0,35)	-0,047 (-0,44)
$x_{i,t}$	0,21** (2,41)	0,064 (0,66)	-0,075 (-0,94)	0,18*** (5,05)	0,097** (2,3)	0,042 (1,25)	0,12** (1,99)	-0,0044 (-0,08)	-0** (-2,37)
r_t	0,73*** (11,61)	0,54*** (8,5)	0,37*** (5,36)	0,62*** (11,18)	0,48*** (8,29)	0,36*** (5,03)	0,73*** (8)	0,58*** (5,39)	0,49*** (3,86)
$f_{i,t}$	-0,0027 (-0,28)	0,029*** (2,69)	0,055*** (5,13)	0,014** (2,37)	0,03*** (4,01)	0,042*** (5,68)	0,032 (1,22)	0,039 (1,53)	0,048** (2,45)
$f_{i,t}.\delta_{cris,t}$	0,017 (1,14)	0,006 (0,32)	-0,003 (-0,15)	-0,029 (-0,98)	-0,011 (-0,32)	0,0011 (0,04)	0,17*** (3,44)	0,13*** (3,67)	0,084*** (3,22)
R^2	0,86	0,74	0,7	0,9	0,78	0,75	0,71	0,61	0,62

Table 2: Preliminary regressions – Endogenous variable: debt to GDP

	Germany			France			Italy		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	-7,4 (-0,67)	-11 (-1,12)	-17* (-1,96)	-1,2 (-0,57)	-4,8** (-2,08)	-8,6*** (-4,06)	4 (1,62)	0,46 (0,14)	-3,8 (-1,08)
π_i	-0,018 (-0,2)	-0,048 (-0,47)	-0,037 (-0,34)	-0 (-1,37)	-0,078 (-0,75)	-0,029 (-0,27)	-0,2** (-2,36)	-0,13 (-1,14)	-0,019 (-0,17)
$x_{i,t}$	0,14*** (2,73)	0,068 (1,23)	0,0017 (0,03)	0,18*** (3,12)	0,15** (2,14)	0,12* (1,77)	0,29*** (4,57)	0,22*** (2,59)	0,094 (1,27)
r_t	0,61*** (7,1)	0,42*** (5,08)	0,28*** (3,36)	0,61*** (7,77)	0,38*** (4,34)	0,2** (2,32)	0,54*** (7,64)	0,33*** (4,01)	0,18** (2,29)
$f_{i,t}$	0,49 (1)	1** (1,97)	1,6*** (3,3)	0,17 (0,88)	0,69*** (2,79)	1,3*** (5,1)	0,089** (2,19)	0,18*** (3,12)	0,26*** (4,54)
$f_{i,t}.\delta_{cris,t}$	-1,2 (-0,69)	-1,8 (-1,12)	-2,6* (-1,94)	-0,21 (-0,53)	-0,84** (-1,99)	-1,5*** (-3,95)	0,45* (1,9)	0,13 (0,41)	-0,28 (-0,86)
R^2	0,85	0,72	0,61	0,88	0,73	0,66	0,89	0,75	0,67
	Spain			Belgium			Greece		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	0,53 (0,45)	1,2 (0,83)	1,7 (1,18)	21*** (3,5)	15** (2,09)	3,5 (0,56)	15*** (2,85)	10*** (3,12)	4,7 (1,53)
π_i	-0,0039 (-0,06)	-0,0076 (-0,13)	0,0031 (0,06)	-0,028 (-0,81)	-0,074* (-1,86)	-0,087* (-1,92)	0,19 (1,24)	0,082 (0,63)	-0,022 (-0,21)
$x_{i,t}$	0,21** (2,5)	0,075 (0,83)	-0,054 (-0,77)	0,2*** (5,5)	0,11*** (2,68)	0,049 (1,45)	0,19*** (3,16)	0,086 (1,57)	0,0094 (0,19)
r_t	0,73*** (11,19)	0,5*** (8,11)	0,3*** (5,01)	0,56*** (9,02)	0,39*** (6,29)	0,26*** (3,77)	0,6*** (6,22)	0,42*** (5,92)	0,28*** (3,74)
$f_{i,t}$	-0,012 (-0,27)	0,13*** (2,71)	0,25*** (5,48)	0,074*** (2,98)	0,16*** (4,68)	0,21*** (6,4)	0,15* (1,88)	0,23*** (3,25)	0,31*** (4,8)
$f_{i,t}.\delta_{cris,t}$	0,27 (1,23)	0,34 (1,29)	0,38 (1,38)	2,7*** (3,53)	1,9** (2,14)	0,53 (0,66)	1,2*** (3,08)	0,86*** (3,49)	0,43* (1,88)
R^2	0,86	0,75	0,72	0,91	0,8	0,77	0,69	0,63	0,71

Table 3: Preliminary regressions – Endogenous variable: debt service to income

	Germany			France			Italy		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	0,54*** (3,64)	0,51** (2,41)	0,38 (1,49)	0 (0,38)	0,39 (1,29)	0,69** (2,2)	-0,76*** (-4,12)	-0,93*** (-3,68)	-0,87*** (-3,63)
π_i	-0,058 (-0,5)	-0,16 (-1,3)	-0,23** (-2,08)	0 (0,12)	0,06 (0,69)	0,11 (1,28)	-0,069 (-0,65)	0,057 (0,47)	0,18 (1,55)
$x_{i,t}$	0,077 (1,11)	-0,0012 (-0,02)	-0,072 (-0,92)	0,12*** (2,9)	0,026 (0,52)	-0,071 (-1,27)	0,25*** (5,15)	0,15** (2,4)	0,023 (0,37)
r_t	0,57*** (5,68)	0,46*** (3,42)	0,42*** (2,75)	0,5*** (6,04)	0,31** (2,57)	0,21 (1,29)	0,41*** (7,92)	0,17** (2,24)	0,02 (0,15)
$f_{i,t}$	0,087 (0,87)	0,079 (0,67)	0,059 (0,5)	0,25*** (3,74)	0,33*** (3,23)	0,36*** (2,66)	0,19*** (4,69)	0,28*** (6,05)	0,33*** (6,77)
$f_{i,t}.\delta_{cris,t}$	-0,042 (-0,32)	-0,083 (-0,64)	-0,12 (-1)	-0,21** (-2,19)	-0,33*** (-3,11)	-0,41*** (-3,98)	0,027 (0,29)	-0,16 (-1,53)	-0,34*** (-2,96)
R^2	0,85	0,7	0,53	0,9	0,74	0,53	0,92	0,81	0,69
	Spain			Belgium			Greece		
	3-year	5-year	10-year	3-year	5-year	10-year	3-year	5-year	10-year
$\delta_{cris,t}$	-0,91** (-2,37)	-0,52 (-1,02)	0,036 (0,06)	-0,59*** (-2,9)	-1,1*** (-4,63)	-1,4*** (-5,67)	0,55* (1,69)	0,58 (1,45)	0,68 (1,35)
π_i	0,021 (0,4)	-0,075 (-1,2)	-0,15** (-2,26)	-0,023 (-0,44)	-0,019 (-0,36)	0,011 (0,26)	0,14 (1,05)	0,067 (0,48)	-0,013 (-0,1)
$x_{i,t}$	0,22*** (3,14)	0,24*** (2,82)	0,23*** (2,87)	0,18*** (4,65)	0,089* (1,79)	0,028 (0,57)	0,2*** (3,01)	0,038 (0,64)	-0* (-1,81)
r_t	0,71*** (11,72)	0,56*** (8,21)	0,43*** (5,88)	0,55*** (7,3)	0,32*** (5,07)	0,15*** (2,59)	0,41*** (2,58)	0,35** (2,31)	0,33** (2,13)
$f_{i,t}$	0,16*** (3,49)	-0,076 (-1,05)	-0,33*** (-3,69)	0,11*** (3,04)	0,25*** (5,8)	0,34*** (7,96)	0,2** (2,4)	0,14 (1,58)	0,067 (0,65)
$f_{i,t}.\delta_{cris,t}$	-0,19*** (-3,68)	0,029 (0,35)	0,28*** (2,97)	-0,075 (-1,27)	-0,23*** (-3,82)	-0,34*** (-8,32)	-0,46*** (-4,1)	-0,4*** (-3,26)	-0,29** (-2,13)
R^2	0,87	0,71	0,62	0,9	0,8	0,77	0,62	0,5	0,49

Table 4: Preliminary regressions – Endogenous variable: primary deficit to GDP

		Model I		Model II		Model III		Model IV	
		before bk	after bk	before bk	after bk	before bk	after bk	before bk	after bk
γ_0	FR	0,0098 (0,83)	0,005 (0,08)	0,0062 (0,68)	-0,024 (-0,34)	-0,00088 (-0,04)	0,021 (0,18)	-0,00046 (-0,03)	0,068 (0,93)
	IT	0,067** (2,34)	0,48*** (4,28)	0,074*** (3,97)	0,46*** (4,29)	0,064*** (3,49)	0,52*** (3,27)	0,062*** (3,28)	0,54*** (2,72)
	ES	0,0088 (0,48)	0,43*** (3,3)	0,036** (2,29)	0,35*** (4,13)	0,02 (1,04)	0,27* (1,73)	0,022 (0,78)	0,31*** (2,85)
	BE	0,017 (0,75)	0,79*** (2,86)	0,047** (2,44)	0,84*** (3,68)	0,052*** (2,94)	0,93 (0,94)	0,052*** (3,48)	0,95 (1,27)
	GR	0,065** (2,42)	1,6** (2,3)	0*** (3,12)	1,7** (2,36)	0,17*** (4,84)	1,5 (1,6)	0,16*** (10,14)	1,5* (1,78)
	γ_x	FR	-0,024 (-1,02)	0,011 (0,23)	0	-0,003 (-0,05)	0	-0,0053 (-0,03)	0
	IT	-0,014 (-0,38)	0,0019 (0,01)	id.	-0,012 (-0,11)	id.	id.	id.	id.
	ES	0,011 (0,38)	-0,084 (-0,7)	id.	-0,085 (-0,94)	id.	id.	id.	id.
	BE	-0,005 (-0,14)	-0,0059 (-0,06)	id.	0,0049 (0,05)	id.	id.	id.	id.
	GR	0,044 (0,86)	0,67 (0,98)	id.	0,7 (1,12)	id.	id.	id.	id.

Table 5: Estimates of default intensities parameters (first part), for alternative model specifications. Student-t are reported below the parameter estimates (in parenthesis).

		Model I		Model II		Model III		Model IV	
		before bk	after bk	before bk	after bk	before bk	after bk	before bk	after bk
γ_r	FR	0,055 (1,24)	-0,06 (-1,06)	0	-0,06 (-0,98)	0	-0,023 (-0,2)	0	0
	IT	0,077 (1,38)	-0,15** (-2,2)	id.	-0,13 (-1,6)	id.	id.	id.	id.
	ES	0,043 (1)	0,19 (1,1)	id.	0,14 (1,31)	id.	id.	id.	id.
	BE	0,071* (1,77)	0,044 (0,7)	id.	0,02 (0,35)	id.	id.	id.	id.
	GR	0,061 (0,96)	-0,9 (-1,11)	id.	-0,85 (-1,36)	id.	id.	id.	id.
	γ_f	FR	0,024* (1,68)	0,16 (1,45)	0,026* (1,89)	0,17 (1,36)	0,034 (1,15)	0,15* (1,76)	0,037* (1,85)
	IT	0,023** (2,34)	0,27* (1,95)	0,036*** (3,73)	0,24* (1,93)	0,042*** (4,66)	0,17*** (2,69)	0,04*** (3,33)	0,14** (1,99)
	ES	0,011** (2,44)	0,2 (1,26)	0,022*** (3,17)	0,13* (1,75)	0,023*** (3,93)	0,094*** (3,23)	0,024*** (3,66)	0,084** (2,11)
	BE	0,0053 (0,97)	0,31*** (2,58)	0,014** (2,57)	0,33*** (3,4)	0,012** (2,49)	0,42 (0,88)	0,012** (2,51)	0,4 (1,01)
	GR	0,0026 (0,47)	0,31 (1,05)	-0,021** (-2,18)	0,29 (1,31)	0,019*** (3,5)	0,62 (1,47)	0,0086* (1,93)	0,57 (1,32)
	stdv	0,199		0,202		0,206		0,206	

Table 6: Estimates of default intensities parameters (continued), for alternative model specifications. Student-t are reported below the parameter estimates (in parenthesis). The bottom line presents the average standard deviation of the measurement errors (in percentage points), across countries and maturities.

H0: no break		H0: country-specific γ 's			H0: null γ 's		
		Before bk	After bk		Before bk	After bk	
$\gamma_i = \gamma_i^b$	0,00						
$\gamma_{0,i} = \gamma_{0,i}^b$	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00
$\gamma_{x,i} = \gamma_{x,i}^b$	0,00	$\gamma_{x,i} = \gamma_x$	0,35	0,03	$\gamma_{x,i} = \gamma_x$	0,26	0,00
$\gamma_{r,i} = \gamma_{r,i}^b$	0,00	$\gamma_{r,i} = \gamma_r$	0,79	0,00	$\gamma_{r,i} = \gamma_r$	0,42	0,01
$\gamma_{f,i} = \gamma_{f,i}^b$	0,00	$\gamma_{f,i} = \gamma_f$	0,02	0,03	$\gamma_{f,i} = \gamma_f$	0,02	0,00

Table 7: Wald tests – this table presents p-values of Wald tests performed using Model I's parameter estimates and the covariance matrix of these parameters.

H0: no break		H0: country-specific γ 's			H0: null γ 's		
		Before bk	After bk		Before bk	After bk	
$\gamma_i = \gamma_i^b$	-						
$\gamma_{0,i} = \gamma_{0,i}^b$	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00
$\gamma_{x,i} = \gamma_{x,i}^b$	-	$\gamma_{x,i} = \gamma_x$	-	0,00	$\gamma_{x,i} = \gamma_x$	-	0,00
$\gamma_{r,i} = \gamma_{r,i}^b$	-	$\gamma_{r,i} = \gamma_r$	-	0,00	$\gamma_{r,i} = \gamma_r$	-	0,00
$\gamma_{f,i} = \gamma_{f,i}^b$	0,00	$\gamma_{f,i} = \gamma_f$	0,03	0,00	$\gamma_{f,i} = \gamma_f$	0,00	0,00

Table 8: Wald tests – this table presents p-values of Wald tests performed using Model II's parameter estimates and the covariance matrix of these parameters.

H0: no break		H0: country-specific γ 's			H0: null γ 's		
		Before bk	After bk		Before bk	After bk	
$\gamma_i = \gamma_i^b$	-						
$\gamma_{0,i} = \gamma_{0,i}^b$	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00
$\gamma_{x,i} = \gamma_{x,i}^b$	-	$\gamma_{x,i} = \gamma_x$	-	-	$\gamma_{x,i} = \gamma_x$	-	0,98
$\gamma_{r,i} = \gamma_{r,i}^b$	-	$\gamma_{r,i} = \gamma_r$	-	-	$\gamma_{r,i} = \gamma_r$	-	0,84
$\gamma_{f,i} = \gamma_{f,i}^b$	0,00	$\gamma_{f,i} = \gamma_f$	0,01	0,52	$\gamma_{f,i} = \gamma_f$	0,00	0,00

Table 9: Wald tests – this table presents p-values of Wald tests performed using Model III's parameter estimates and the covariance matrix of these parameters.

H0: no break		H0: country-specific γ 's			H0: null γ 's		
		Before bk	After bk		Before bk	After bk	
$\gamma_i = \gamma_i^b$	0,00						
$\gamma_{0,i} = \gamma_{0,i}^b$	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00	$\gamma_{0,i} = \gamma_0$	0,00	0,00
$\gamma_{x,i} = \gamma_{x,i}^b$	-	$\gamma_{x,i} = \gamma_x$	-	-	$\gamma_{x,i} = \gamma_x$	-	-
$\gamma_{r,i} = \gamma_{r,i}^b$	-	$\gamma_{r,i} = \gamma_r$	-	-	$\gamma_{r,i} = \gamma_r$	-	-
$\gamma_{f,i} = \gamma_{f,i}^b$	0,00	$\gamma_{f,i} = \gamma_f$	0,00	0,41	$\gamma_{f,i} = \gamma_f$	0,00	0,00

Table 10: Wald tests – this table presents p-values of Wald tests performed using Model IV's parameter estimates and the covariance matrix of these parameters.

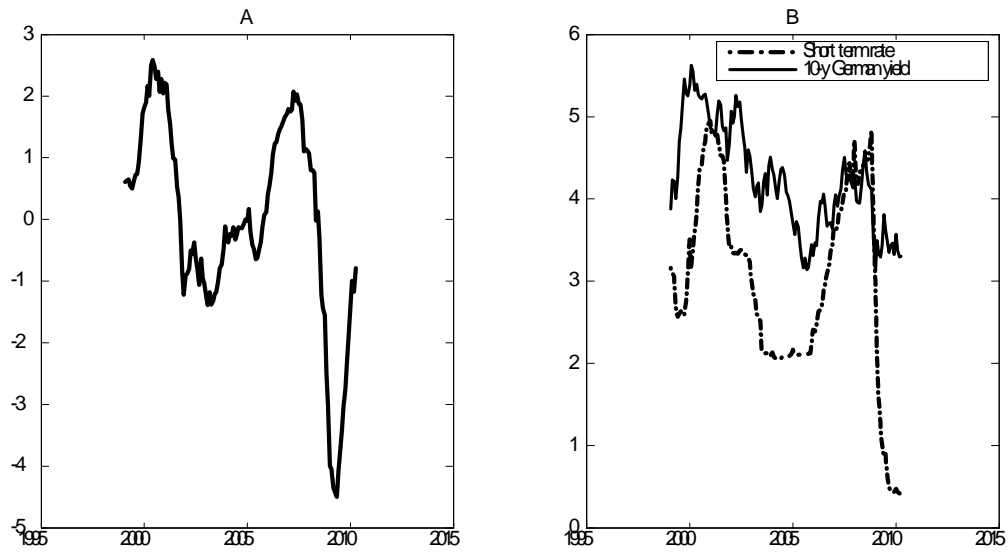


Figure 1: Euro area business cycle and interest rates. Panel A: Euro area business cycle indicator. Panel: 1-month Euro area interest rate and German Bund 10-year yield.

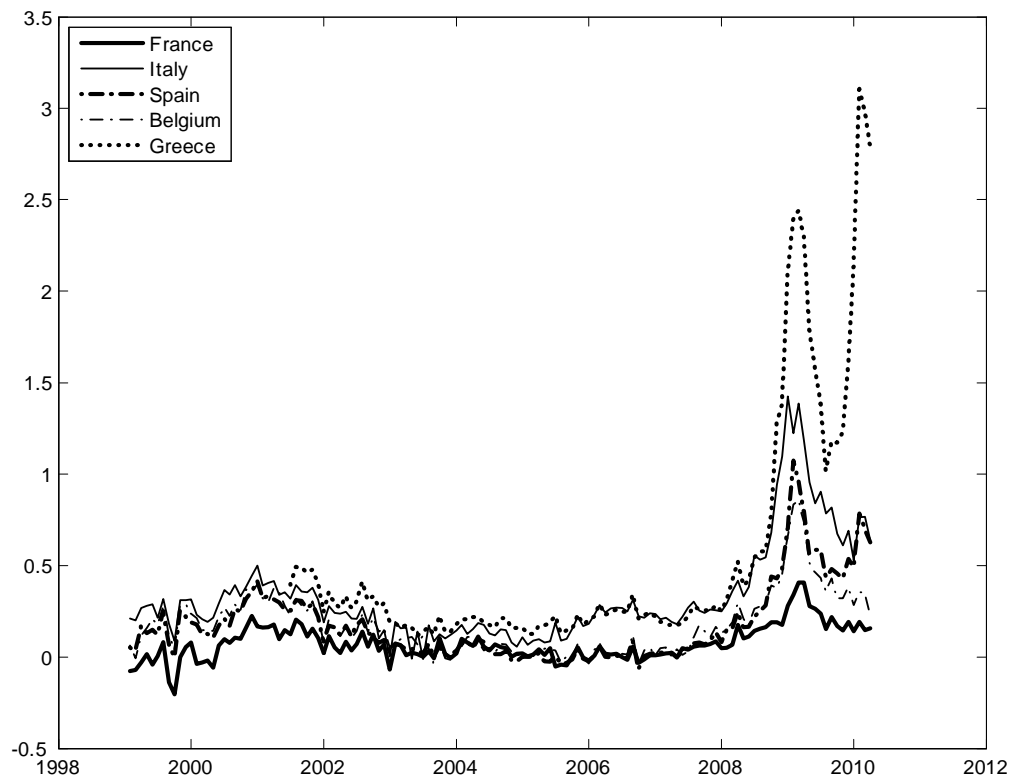


Figure 2: 10-year sovereign spreads against Germany, selected countries.

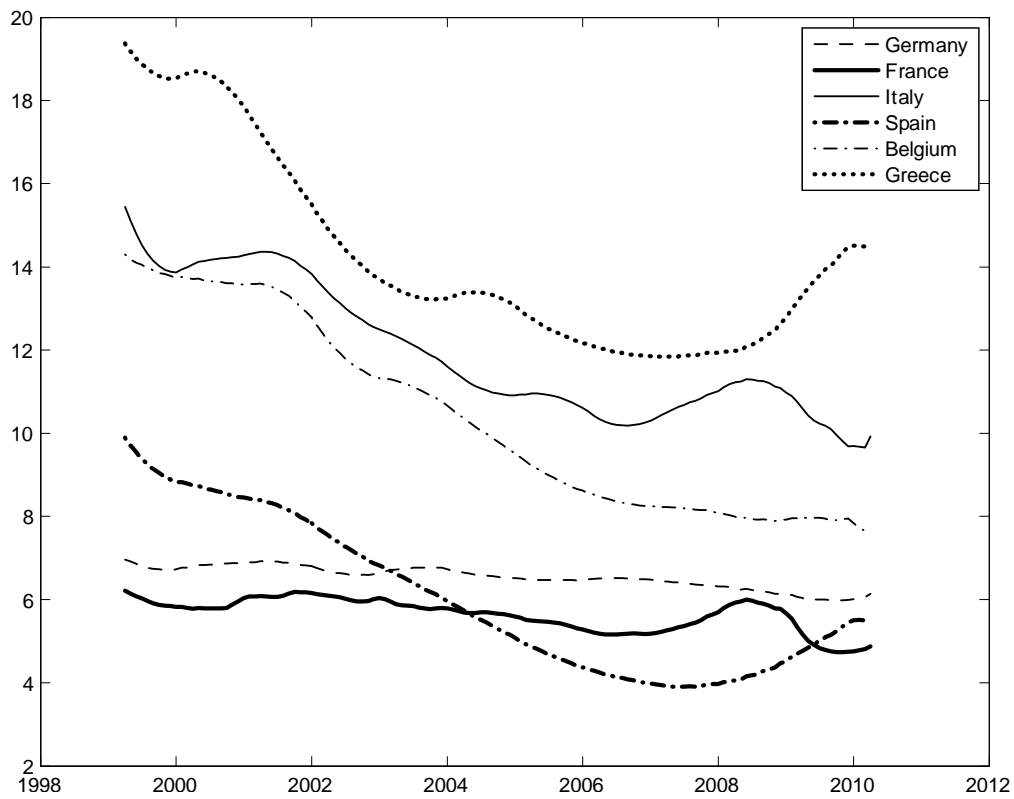


Figure 3: Debt service to income ratio, selected Euro-area countries. Monthly series interpolated from quarterly OECD data using the Kalman filter.

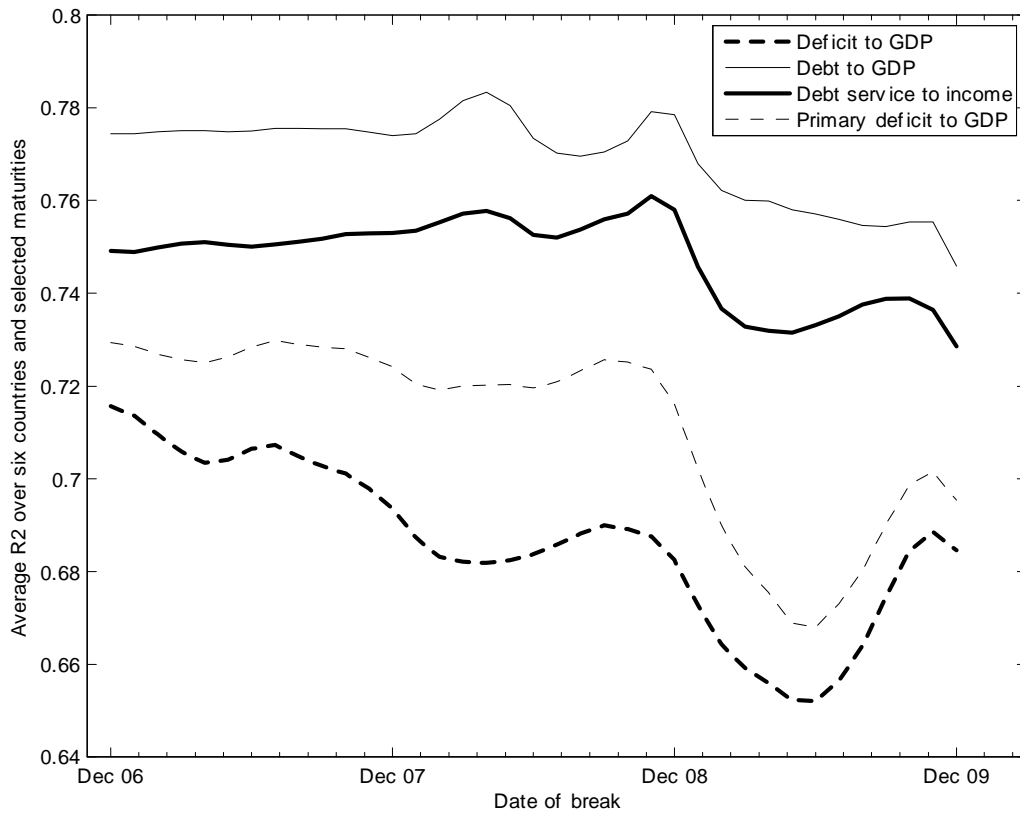


Figure 4: Choosing the break date. The Figure shows average R^2 of regressions of bond yields on fiscal and other macroeconomic determinants over six countries and 2,5,10 year maturities (see section 2 for details). Each line links the average R^2 values for a given fiscal variable when the imposed break date varies between end 2006 and end 2009.

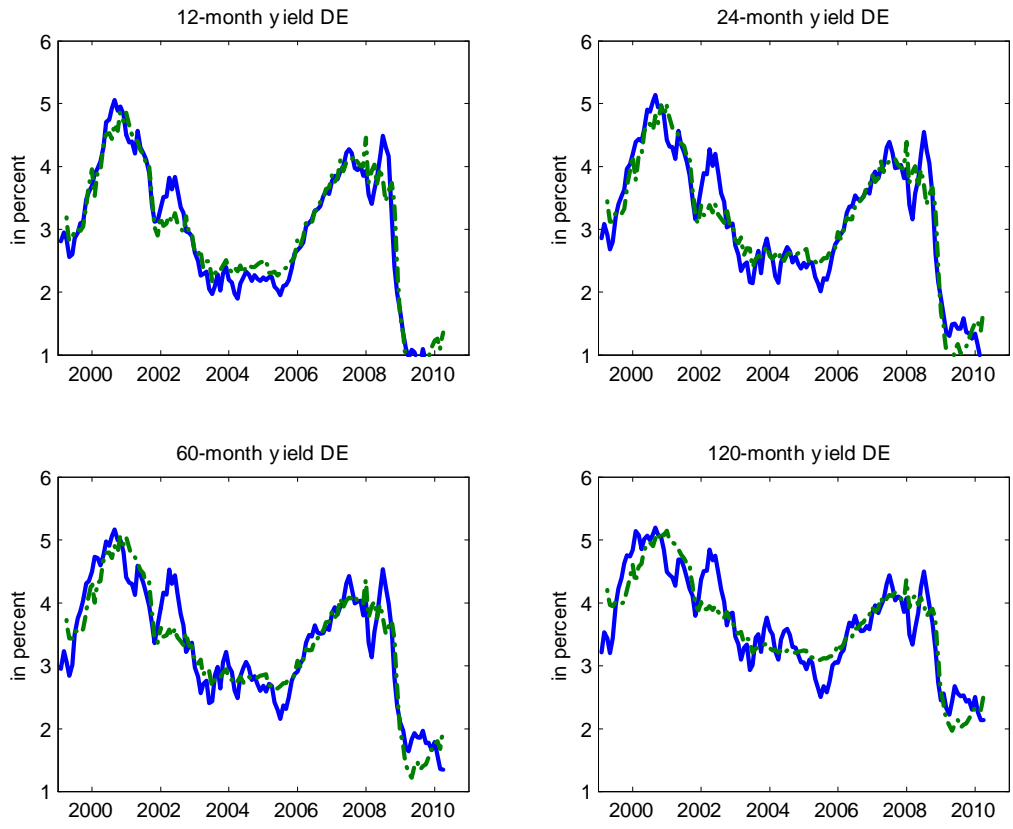


Figure 5: Actual vs. model-implied German yields (Model I)

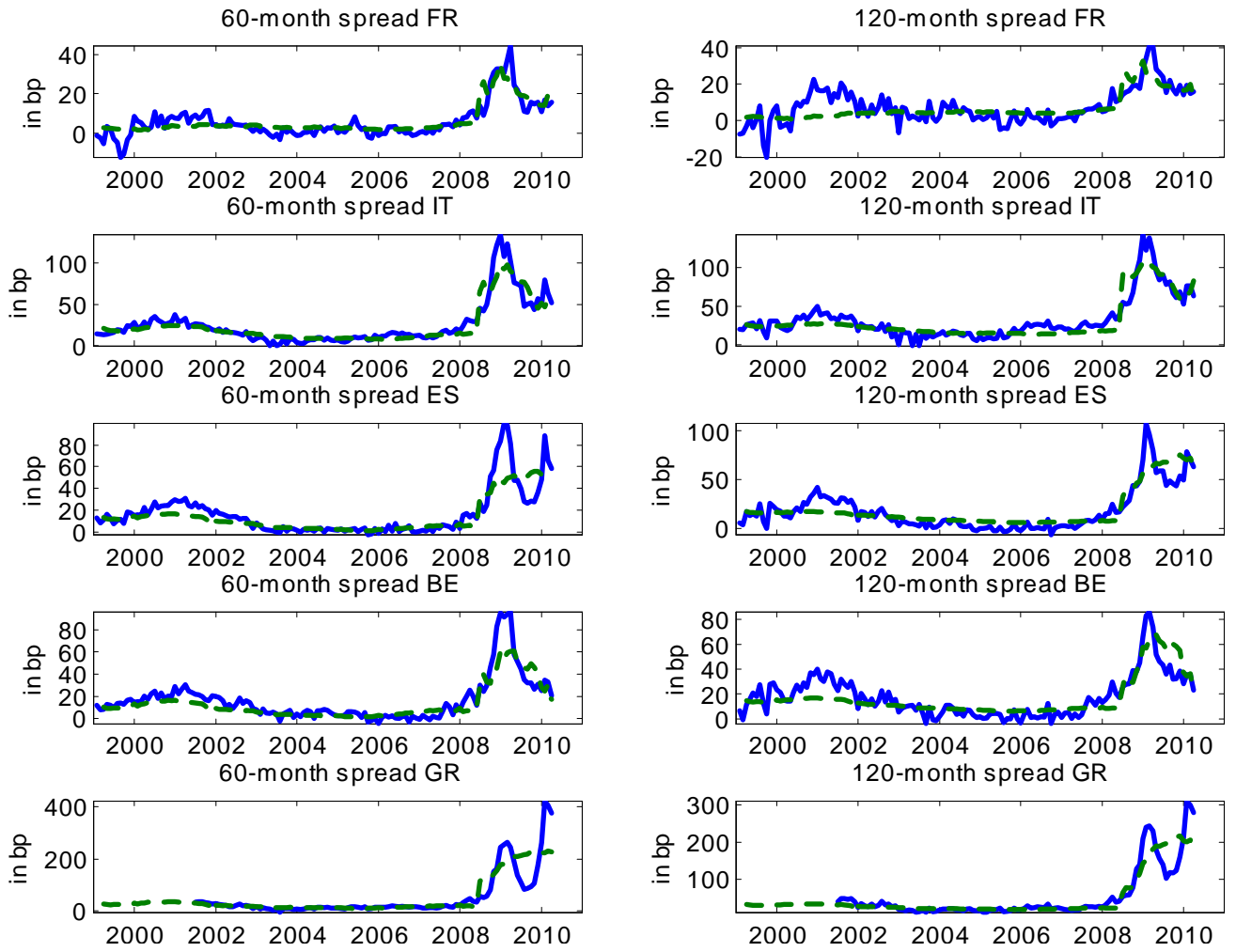


Figure 6: Actual vs. model-implied spreads (Model I)

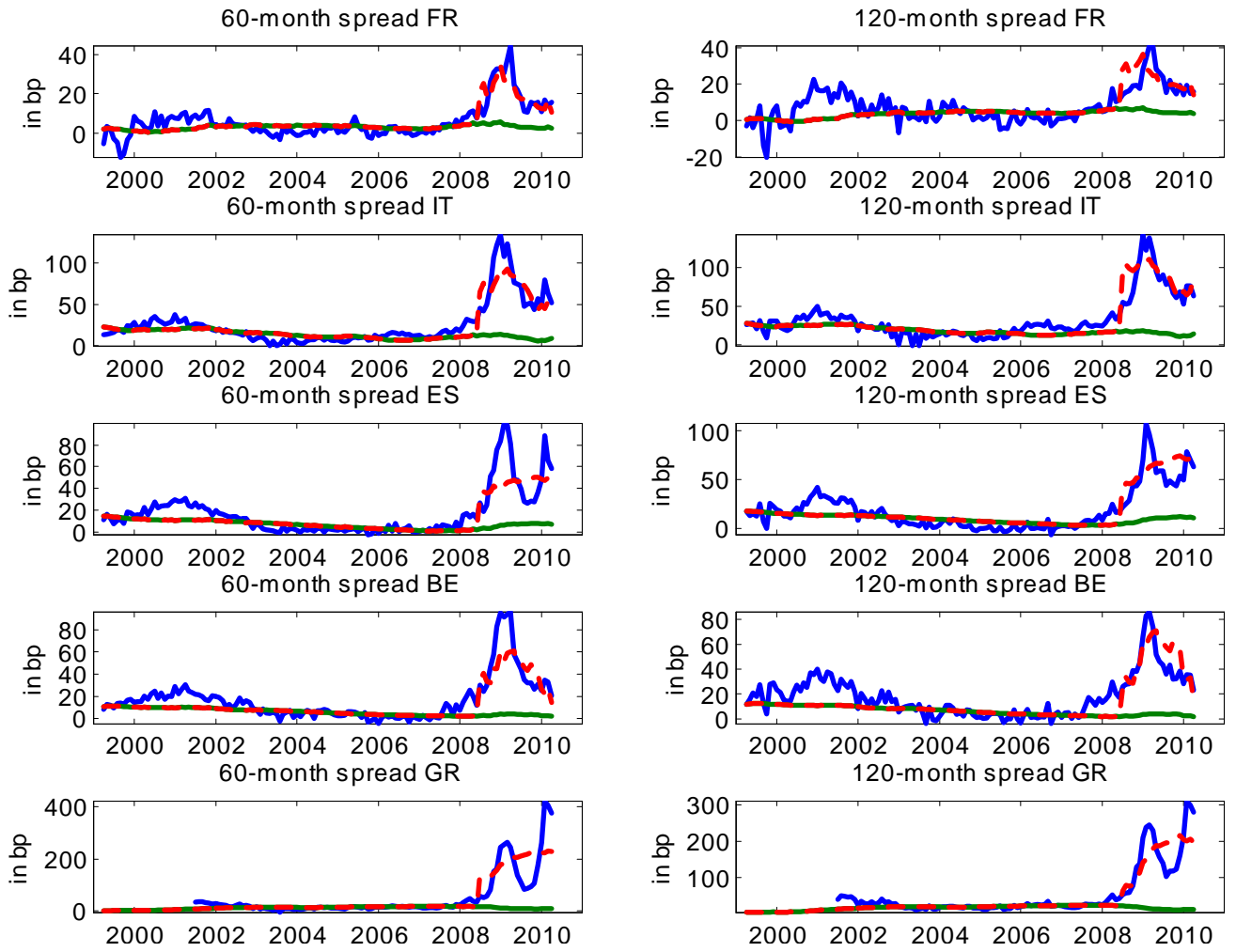


Figure 7: Realized, fitted and counterfactual spreads vs Germany, assuming no break in the default intensity parameters (model II).

A Data

In this appendix, we describe the data that we use throughout the paper. In particular, we detail the procedure that we used to get zero-coupon government bond yields at different maturities.

A.1 Macroeconomic data

For the activity data, we rely on monthly sectoral business survey data published by the DG ECFIN of the European Commission. For each country, our national business cycle indicator is the first principal component that we extract from confidence indicator surveys. We consider the following sectoral indicators: construction, consumers, industrial, retail and services components of the national business cycle indicator. The Euro area business cycle indicator is then computed on the basis of the GDP-weighted average of this national business cycle indicator (normalized by the total weight of GDP for the six European countries included in our sample). The short term rate is measured using prices of 1 month OIS. Inflation data are the domestic harmonized consumer price index (HCPI).

For the fiscal data, we rely on OECD data from Economic Outlook published in May 2010 (n°87). We use the deficit to GDP, primary deficit to GDP, debt to GDP and debt service to income receipt ratio. These quarterly fiscal data are revised data. In the preliminary regressions presented in section 2, we interpolated these quarterly series using simple cubic splines in order to deal with monthly frequency. In the affine term structure model developed in sections 3 and 4 we used the Kalman filter procedure in order to deal with monthly frequency for the fiscal data in a more sophisticated way as described in section 4.1.

A.2 Zero-coupon yields

The estimation of the model requires zero-coupon yields. However, governments usually issue coupon-bearing bonds. In order to have the most comparable data across countries, we estimate the zero-coupon yield curves using the same methodology for five countries: Germany France, Italy, Spain and Greece. For some maturities and some dates, Belgian yields obtained with this methodology present some unsatisfactory level in the early 2000s (with Belgian long-term yields slightly lower than the German ones). Hence, for Belgium,

we use zero-coupon yields computed by the National Bank of Belgium. The series of Greek zero-coupon yields start in mid-2001, a few months after Greece joined the euro-area (leaving aside convergence effects in early 2001).

As Gurkaynak, Sack and Wright (2005) , we resort to a parametric approach (see BIS, 2005, for an overview of zero-coupon estimation methods). We choose the parametric form originally proposed by Nelson and Siegel (1987). Specifically, the yield of a zero-coupon bond with a time to maturity m for a point in time t is given by:¹²

$$y_t^m(\theta) = \beta_0 + \beta_1 \left(-\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_2 \left[\left(\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \exp\left(-\frac{m}{\tau_1}\right)\right]$$

where θ is the vector of parameters $[\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]'$. Assume that, for a given country and a given date t , we dispose of observed prices of N coupon-bearing bonds (with fixed coupon), denoted by $P_{1,t}, P_{2,t}, \dots, P_{N,t}$. Let us denote by $CF_{k,i,t}$ the i^{th} (on n_k) cash flows that will be paid by the k^{th} bond at the date $\tau_{k,i}$. We can use the zero-coupon yields $\{y_t^m(\theta)\}_{m \geq 0}$ to compute a modeled (dirty) price $\hat{P}_{k,t}$ for this k^{th} bond:

$$\hat{P}_{k,t}(\theta) = \sum_{i=1}^{n_k} CF_{k,i,t} \exp\left(-\tau_{k,i} y_t^{\tau_{k,i}-t}(\theta)\right).$$

The approach then consists in looking for the vector θ that minimizes the distance between the N observed prices and modeled bond prices. Specifically, the vector θ_t is given by:

$$\theta_t = \arg \min_{\theta} \sum_{k=1}^N \omega_k (P_{k,t} - \hat{P}_{k,t}(\theta))^2$$

where the ω_k 's are some weights that are chosen with respect to the preferences that one may have regarding the fit of different parts of the yield curve. Intuitively, taking the same value for all the ω_k 's would lead to large yield errors for financial instruments with relatively short remaining time to maturity. This is linked to the concept of duration (i.e. the elasticity of the price with respect to one plus the yield): a given change in the yield corresponds to a small/large change in the price of a bond with a short/long term to maturity or duration.

¹²We use the Nelson-Siegel form rather than the extended version of Svensson (1994) because the latter requires more data to be estimated properly (and for some countries and some dates, we have too small a number of coupon-bond prices).

Since we do not want to favour a particular segment of the yield-curve fit, we weight the price error of each bond by the inverse of the remaining time to maturity.¹³

Coupon-bond prices come from Datastream.¹⁴ In the same spirit as Gurkaynak et al. (2005), different filters are applied in order to remove those prices that would obviously bias the obtained yields. In particular, the prices of bonds that were issued before 1990 or that have atypical coupons (below 1% or above 10%) are excluded. In addition, the prices of bonds that have a time to maturity lower than 1 month are excluded.¹⁵

B Derivation of the recursive formulas

Let $P_j(t, h)$ denote the price, at time t , of a zero-coupon bond issued by country j with a residual maturity of h periods. Assume that, for a given $h \geq 1$, there exist some matrices $A_{j,1}, \dots, A_{j,h-1}$ and $B_{j,1}, \dots, B_{j,h-1}$ that are such that, for any period t and any maturity $n \in \{1, \dots, h-1\}$, $B_j(t, n) = A_{j,n} + B_{j,n}F_t$. Naturally, the latter formula is valid only if country j has not defaulted before t (otherwise, we would have the trivial prices $P_j(t, n) = 0$ for any maturity n).

Let us consider the price of a h -period bond at time t . This price is given by:

$$\begin{aligned} P_j(t, h) &= \exp(-r_t)E^{\mathbb{Q}}(P_j(t+1, h-1)) \\ &= \exp(-A_1 - B_1F_t)E^{\mathbb{Q}}(P_j(t+1, h-1)) \\ &= \exp(-A_1 - B_1F_t)E^{\mathbb{Q}}[\mathbb{I}(d_{j,t+1} = 0) \times \{A_{j,h-1} + B_{j,h-1}F_{t+1}\}] \end{aligned}$$

where $d_{j,t}$ is a default indicator which is equal to 1 if country j has defaulted at or before t and is equal to 0 otherwise. We have:

¹³Using remaining time to maturity instead of duration has not a large effect on estimated yields as long as we are not concerned with the very long end of the yield curve.

¹⁴Naturally, the number of bonds used differ among the countries (from 19 bonds for the Netherlands to 175 bonds for Germany).

¹⁵The trading volume of a bond usually decreases considerably when it approaches its maturity date.

$$\begin{aligned}
P_j(t, h) &= \exp(-A_1 - B_1 F_t) E^{\mathbb{Q}} \left[E^{\mathbb{Q}}(\mathbb{I}(d_{j,t+1} = 0) \times \exp\{A_{j,h-1} + B_{j,h-1} F_{t+1}\} | F_{t+1}) \right] \\
&= \exp(-A_1 - B_1 F_t) E^{\mathbb{Q}} \left[\exp\{A_{j,h-1} + B_{j,h-1} F_{t+1}\} E^{\mathbb{Q}}(\mathbb{I}(d_{j,t+1} = 0) | F_{t+1}) \right] \\
&= \exp(-A_1 - B_1 F_t) E^{\mathbb{Q}} [\exp\{A_{j,h-1} + B_{j,h-1} F_{t+1}\} \exp(-s_{j,t+1})] \\
&= \exp(-A_1 - B_1 F_t) E^{\mathbb{Q}} [\exp\{A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) F_{t+1}\}] \\
&= \exp(-A_1 - B_1 F_t) E^{\mathbb{Q}} [\exp\{A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) (\mu^* + \Phi^* F_t + \varepsilon_t^*)\}] \\
&= \exp(-A_1 - B_1 F_t + A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) (\mu^* + \Phi^* F_t) + \\
&\quad \frac{1}{2} (B_{j,h-1} - \gamma_1) \Sigma \Sigma' (B_{j,h-1} - \gamma_1)').
\end{aligned}$$

Therefore, with

$$\begin{cases} A_{j,h} &= -A_1 + A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) \mu^* + \frac{1}{2} (B_{j,h-1} - \gamma_1) \Sigma \Sigma' (B_{j,h-1} - \gamma_1)' \\ B_{j,h} &= -B_1 + (B_{j,h-1} - \gamma_1) \Phi^*, \end{cases}$$

we have $P_j(t, h) = A_{j,h} + B_{j,h} F_t$. Hence, we have shown how to compute recursively the $A_{j,h}$'s and $B_{j,h}$'s.

C Computation of the covariance matrix of the estimators

In this appendix, we present the methodology used to compute the covariance matrix of the parameter estimates obtained by minimization of the sum of squared pricing errors. This refers to the second step of estimation presented in 4.1. The parameters are the entries of vector Θ_2 , that parameterizes the risk-neutral dynamics as well as the hazard rates of the different countries. As explained in 4.1:

$$\Theta_2 = \arg \min_{\theta} \sum_{j,h,t} (y_{j,h,t}^o - y_{j,h,t}(\theta, F_t))^2.$$

This estimator must satisfy the first-order conditons:

$$\sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\Theta_2, F_t)) = 0,$$

where the left-hand side of the previous equation is of dimension $k \times 1$ (the dimension of vector Θ_2). Deriving the Taylor expansion of the previous equation in a neighborhood of the

limit value $\underline{\Theta}_2$, and multiplying by \sqrt{T} leads to:

$$0 \simeq \sqrt{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t)) + \\ \sqrt{T} (\Theta_2 - \underline{\Theta}_2) \left[\sum_{j,h,t} \frac{\partial^2 y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta \partial \theta'} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t)) \right. \\ \left. - \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \left(\frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \right)' \right].$$

Since $E(y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t)) = 0$, we have

$$\frac{1}{T} \sum_{j,h,t} \frac{\partial^2 y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta \partial \theta'} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t)) \xrightarrow{a.s.} 0.$$

Therefore:

$$\sqrt{T} (\Theta_2 - \underline{\Theta}_2) \simeq \left[\frac{1}{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \left(\frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \right)' \right]^{-1} \times \\ \frac{1}{\sqrt{T}} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t)).$$

Hence, the asymptotic distribution of $\sqrt{T} (\Theta_2 - \underline{\Theta}_2)$ is given by $\hat{\mathcal{J}}^{-1} \hat{\mathcal{I}} \hat{\mathcal{J}}^{-1}$ where:

$$\hat{\mathcal{J}}^{-1} = \left[\frac{1}{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \left(\frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} \right)' \right]^{-1}.$$

As regards $\hat{\mathcal{I}}$ —that is the covariance matrix of $\frac{1}{\sqrt{T}} \sum_t \gamma_t$ with $\gamma_t = \sum_{j,h} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t))$ —, we use the Newey-West (1987) HAC estimator. This estimate is given by:

$$\hat{\mathcal{I}} = \sum_{i=-(T-m+1)}^{i=T-m-1} \kappa \left(\frac{i}{m} \right) \hat{c} \hat{v}(\hat{\gamma}_t, \hat{\gamma}_{t+i})$$

where $\hat{\gamma}_t = \sum_{j,h} \frac{\partial y_{j,h,t}(\underline{\Theta}_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\underline{\Theta}_2, F_t))$ and where $\hat{c} \hat{v}$ denoting the sample covariance matrix. We use the Bartlett kernel $\kappa(x) = 1 - |x|$.¹⁶

¹⁶The kernel bandwidth m is taken equal to 6 (one semester) in our analysis, which is of the order of magnitude of $T^{1/3}$ (that is usually seen as a “rule-of-thumb” guide for this value).