

Public Debt and Changing Inflation Targets

Michael U. Krause and Stéphane Moyen
Deutsche Bundesbank*

April 29, 2011

Abstract

We develop a monetary DSGE model with a maturity structure of long-term government debt and analyze to what extent and under which conditions a government can reduce its real debt burden by increasing inflation. The success of such a policy depends on the maturity structure of public debt and on the extent to which a central bank can use the credibility of its inflation target to exploit expectations of low inflation. We model the maturity structure by means of a callable perpetuity with stochastic call date, and credibility as a private-sector inference problem concerning the central bank's inflation target. Slow updating of a formerly established target reflects high credibility. Calibrating the model to the currently observed average maturity of U.S. government debt, and simulating the projected increase in debt after the financial crisis, we find that, only if the change in the inflation target rate is expected to be highly persistent, or if the average maturity of public debt were much higher, can the government significantly reduce its real debt burden.

Keywords: Government debt, learning, inflation target, debt maturity

JEL classification: E4, E5.

*Address: Deutsche Bundesbank, Economic Research Center, Wilhelm-Epstein-Straße 14, 60431 Frankfurt, Germany. Email: michael.u.krause@bundesbank.de, stephane.moyen@bundesbank.de. The authors thank Dale Henderson, Thomas Laubach, Eric Leeper, Wolfgang Lemke, Thomas Lubik, Lars Svensson, Leopold von Thadden and Alexander Wolman, and participants of the 2nd JLS-Workshop at Goethe University, Frankfurt, the SED 2010 Conference in Montreal, the EEA 2011 Annual Congress in Glasgow, and seminar participants at the universities of Bonn, Nürnberg, and at Humboldt University, Berlin, for comments and discussions. The views expressed in this paper are those of the authors, and do not necessarily reflect a position of the Deutsche Bundesbank or the European System of Central Banks.

1 Introduction

Large increases in government deficits during the economic crisis of 2008/2009 initiated a debate on whether the real value of public debt should be reduced by raising inflation, at least temporarily.¹ The effectiveness of such a strategy depends on two factors: the response of long-term inflation expectations and the maturity structure of public debt. Inflation expectations affect current inflation due to forward-looking price-setting, and affect long-term nominal rates on newly-issued debt, because lenders want to be compensated in real terms as reflected in the Fisher equation. The maturity structure of debt determines the fraction of outstanding real debt that can be inflated away over time as it was priced under the previously prevailing, lower long-term inflation expectations.²

How inflation expectations evolve after a rise in inflation depends crucially on how the price setters perceive the central bank's inflation target. This in turn is influenced by the central bank's communication strategy and its credibility. If the intention of a higher inflation rate were announced and believed, inflation and inflation expectations may jump instantaneously, possibly without much effect on the real economy. In contrast, if the central bank does not announce an inflationary strategy, and enjoys a high credibility of its established target, inflation expectations may stay low in spite of rising inflation. While this keeps nominal interest rates on newly-issued debt low, it may require a central bank to induce a large change in the output gap in order to bring up current inflation relative to a maintained expectation of future low inflation.

In this paper we quantitatively analyze the role of debt maturity and inflation expectations after a surge in U.S. government debt as observed during the crisis followed by a hypothetical increase in the central bank's inflation target. To this end, we present a New Keynesian monetary business cycle model with two non-standard features. First, we assume that agents may need to infer from observed inflation the monetary authority's

¹Most notably, Kenneth Rogoff in Project Syndicate has at the end of 2008 and 2010 suggested to allow some 6 to 7 percent inflation. Similar statements have been made by Paul Krugman and others. See Cochrane (2011) and Miller (2009). Some interpret the proposal by Blanchard, Dell'Ariccia and Mauro (2010) of a higher inflation target as an indirect attempt to prepare the public for inflation that erodes government debt.

²For example, if all debt outstanding were of 10 year maturity, so that only a tenth of this debt becomes due each year, then the remaining 90 percent will have nominal interest rates that are set based on the lower inflation expectations of the past.

true inflation target. Variations in the speed at which agents revise their perception of the target then can be seen to reflect alternative scenarios concerning the degree of credibility of monetary policy and the evolution of long-term inflation expectations. Technically, agents face a signal extraction problem as to whether changes in monetary policy are transitory or due to persistent changes in the inflation target. As in Erceg and Levin (2003), agents solve this problem by means of a Kalman Filter.³

Second, we introduce a maturity structure of debt by means of callable perpetuities of which a given fraction matures each period. Introducing this new type of bond allows us to calibrate the model to the observed average maturity of public debt. Thus, a realistic fraction of the real value of debt is susceptible to inflation even when inflation expectations and thus long-term nominal interest rates have adjusted. Furthermore, we can track a long-run nominal interest rate and the average interest rate on outstanding debt, and thus reveal countervailing forces not at work in standard models. In all other respects, though, the model is a standard New Keynesian dynamic stochastic general equilibrium model with monopolistic competition and sticky prices. We calibrate the size of the debt shock and the average maturity of debt held by the public for the United States as a benchmark, and later generalize our setting to other countries.

We find that of the additional real U.S. public debt accrued after the economic crisis of 2008/2009 about a third is cumulatively inflated away after ten years if the inflation target is permanently raised by four percentage points. In contrast, a temporary change in the inflation target of the same size, as suggested by Rogoff (2008, 2010), has only negligible effects after ten years. The reason is while a small amount of previously-issued debt is in fact inflated away, the rise in nominal rates on newly issued debt raises debt servicing costs even after inflation has returned to the old target. A change in the inflation target from two to only four percent, suggested by Blanchard et al. (2010) as an insurance against hitting the zero-lower bound on interest rates, is also too low to have a dramatic effect on real U.S. public debt.

A higher average maturity of outstanding public debt – or, conversely, a smaller fraction of debt maturing each period – always increases the amount of real debt reduced

³The authors consider the case of a downward change in the inflation target, albeit without consideration of the maturity of public debt.

within a given time period, taking as given the way expectations are formed. Thus a country with a higher average maturity can be seen to face a larger temptation to increase inflation. The other important factor is how a change in the inflation target is interpreted by the public. On the one hand, a central bank may announce a higher inflation target. If the public believes this, then both actual inflation and the long-run nominal interest rate will adjust swiftly as both are forward-looking variables. On the other hand, if a higher inflation target is not announced, the public will only slowly update its perception of the inflation target, depending on how firmly anchored a previously established inflation target was. Then both actual inflation and the long-run nominal interest rate on debt will only slowly adjust upward.

Of course, at very low average maturities, and thus a high fraction of debt being rolled over each period, a large drop in real debt can only be achieved if the public falsely believes in a low inflation target, so that newly-issued debt is priced at too low interest rates. But note that a perceived low inflation target also anchors actual inflation at a low level. An increase in actual inflation may then only obtain through a strong decrease in the policy interest rate that sufficiently raises the output gap. This of course is infeasible in situations where interest rates are close to the zero-lower bound, as seen since the crisis of 2008/2009 or in Japan. With a maturity of roughly 13 years as in the U.K. in 2010, rather than the four and a half years as in the U.S., the amount of additional real debt reduced after ten years is about 30 percent rather than 24 percent for a four percentage point increase in the inflation target.

There are two studies that analyze the possibility of the U.S. government to inflate away its real debt. Aizenman and Marion (2009) carefully document the evolution of debt, inflation and the maturity of debt since World War II. In a simple partial equilibrium model with a fixed interest rate they show that the incentives to inflate to reduce debt are large. In contrast, with a model that includes the endogenous interest rates and forward-looking expectations, we show that the consequences of raising inflation may be much lower. In a paper about the measurement of interest paid on government debt, Hall and Sargent (2010) also show that under their improved measures, the fraction of U.S. real debt inflated away was lower than previously estimated. Furthermore, they emphasize

that instead high real GDP growth made the largest contribution to real debt reduction, and not inflation.⁴

Davig, Leeper and Walker (2011) explain how increases in public debt may endogenously lead to a monetary policy regime switch when debt reaches a ‘fiscal limit’. In other words, high debt may trigger the central bank to be passive while fiscal policy actively determines the price level, in the terminology of Leeper (1991). Once a fiscal limit is reached, monetary policy switches stochastically to a passive stance, and inflation serves to bring the public debt back to a sustainable level. In our paper, we explore the conditions under which an inflationary episode can reduce real government debt sufficiently, and focus on the role of debt maturity and the public’s beliefs about the monetary stance. In Davig et al. (2011), there is full information of an inflationary policy switch, whereas we make the switch imperfectly observed.

The paper now proceeds to the following section where we give a brief overview of the current fiscal situation of the U.S. government, including details on the maturity structure of debt.⁵ Section 3 develops the model, and introduces the long-term stochastic bond designed to approximate a realistic maturity structure of government debt, and specifies the signal extraction problem regarding the long-term inflation target of the central bank. In section 4, the analysis is presented followed by a deeper discussion focusing on debt maturity and the persistence of the inflation target. In Section 5 we will try to answer the following question, How sensitive is real public debt to changing inflation targets? Finally, section 6 concludes.

2 The U.S. fiscal situation

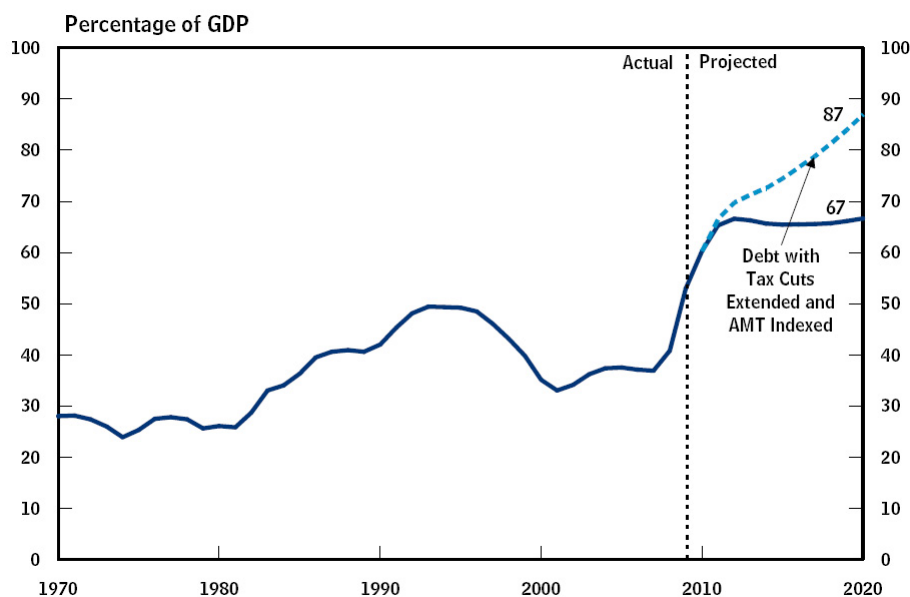
In early 2010, debt of the U.S. government is projected to increase substantially. The baseline forecast as of 2009 by the congressional budget office sees debt rising to 67 percent of GDP by 2012, and then remaining constant until 2020 (see Figure 1). In 2008, government debt held by the public was at 40.8 percent. The government’s annual spending on net interest will more than triple between 2010 to 2020 in nominal terms,

⁴See Persson, Persson and Svensson (1996) for an early attempt to address this issue for the Swedish case.

⁵We do not discuss in depth the determinants of the debt structure, or its optimality.

and double as a share of GDP. This baseline assumes that previous, but temporary, tax cuts, and an alternative minimum tax would expire. An alternative scenario sees debt held by the public rise to 100 percent of GDP, almost double the ratio of 53 percent in 2009.⁶ Note that considering an increase in inflation in response to rising public debt is not an outlandish possibility in the case of the U.S. As Reinhart and Rogoff (2010) show for data from 1790 and 2009 (their Figure 3), inflation was significantly higher in the U.S. whenever debt was at levels above 90% of debt to GDP. The lack of an explicit inflation target of the Fed even today may make an inflation easier to achieve.

Fig. 1 - DEBT BURDEN



Source: CBO Deputy Director Robert Sunshine’s presentation to the Association of Government Accountants.

In terms of maturity, the currently outstanding debt of 7.1 Trillion Dollars held by the public is divided up as follows at the end of June, 2009: 30% is in Treasury Bills (maturities less than one year), 50% is in Treasury Notes (maturities between one and ten years), and 9.7% is in Treasury Bonds (with maturities of more than ten years). 8% is in TIPS (inflation-indexed bonds).⁷ Remarkably, the fraction of Treasury Bonds

⁶CBO Testimony before the Committee on the Budget, United States Senate, January 28, 2010.

⁷Taken from the Monthly Statement of the Public Debt, June 30, 2009. U.S. Treasury, Bureau of

outstanding has decreased from a level of about 20% in 2001, as the government moved towards shorter maturities which have higher liquidity. Thus, if the government were willing to inflate away part of the debt, it would have to do so in an amount of time short enough to affect a substantial portion of its debt. Any newly issued debt would price in inflation expectations. The average maturity of U.S. debt is currently at 49 months, so roughly 4 years.⁸ This is down from 70 months in 2000.

Finally, note that foreign owners of U.S. debt would also be affected. Almost 50% of U.S. publicly held federal debt is held by foreigners, which would also have to sustain losses due to inflation in the U.S. Central banks of other countries hold two thirds of that debt, in particular the central banks of Japan and China. Overall, lenders from Japan and China hold 47% of foreign-owned U.S. debt. These countries are not likely to let their currencies appreciate substantially should the U.S. allow inflation to rise. Also, many developed countries face similar or larger sovereign debt problems. There is most likely a strategic complementarity: inflation rising in a large economy such as the U.S. will put pressure on others to follow. Therefore, the model we develop is relevant for either a closed economy, or one where debt is only held domestically,⁹ or an open economy, where all countries act symmetrically, for the strategic reasons just stated.

3 A model of long-term debt and inflation

The model is a standard New Keynesian framework, with the addition of long-term bonds with stochastic maturity and of learning about the inflation target. The central bank follows a Taylor rule which sets the short-term nominal interest rate as a function of the inflation rate gap and the output gap; there is a one-period government bond priced at that interest rate, in addition to the newly introduced stochastic bond. Firms are monopolistic competitors selling differentiated products at prices that are allowed to

the Public Debt.

⁸As of April 2009. Taken from US Department of the Treasury (2009) *Report to the Secretary of the Treasury from the Treasury Borrowing Advisory Committee of the Securities Industry and Financial Markets Association*, April 29, 2009. See also the Report from August 5, 2009 which reports that "... the average maturity of issuance now exceeds the average maturity of marketable debt outstanding. This suggests that the decline in the average maturity of debt outstanding that that we have witnessed over the past seven years – from a high of approximately 70 months in 2000 to a low of approximately 50 months earlier this year should be arrested and begin to slowly lengthen going forward."

⁹A case in point is Japan.

adjust in a stochastic fashion as in Calvo (1983).¹⁰ Consumers maximize lifetime utility from consumption, labor input, and real money holdings. Government budget dynamics are determined by a fiscal rule.

3.1 The maturity structure of public debt

The central element of the model is an approximation of the maturity structure of public debt in terms of a stochastic, long-term, bond. Each period, an individual bond of this type pays the interest determined when the bond was issued or matures with a given probability, in which case it pays back the face value plus interest. Technically, the bond is a callable perpetuity with stochastic call date, which is independent across bonds. Since the government issues a large number of these bonds, the fraction of bonds maturing each period is identical to the call probability. Private Agents are assumed to hold the same, representative, portfolio of the bonds. The stochastic bond allows to calibrate the average maturity of outstanding debt to that observed in the data, or to the fraction of total debt that matures. Every period, the government is assumed to issue new debt, to replenish the depleted debt and reduce or increase total public debt.¹¹

With probability α the stochastic bond matures, and with probability $1 - \alpha$ it survives into the next period. Denote the stock of long-term bond with B_t^L , while the more familiar risk-free one-period bond is denoted simply with B_t . The total stock of long-term bonds then evolves as

$$B_t^L = (1 - \alpha)B_{t-1}^L + B_t^{new}, \quad (1)$$

where B_t^{new} denotes the amount of newly issued bonds, while $(1 - \alpha)B_{t-1}^L$ is the value of bonds not maturing. Because of the large number of bonds, the fraction of bonds maturing is equal to the probability of maturing. There is always a stock of bonds that was not redeemed, and all ages of bonds are present in the market.

Let the interest rate of bonds newly issued in period t be given by i_t^{new} , and the

¹⁰Our results are robust to the inclusion of sticky wages à la Erceg, Henderson and Levin (2000), thus we stick to the simplest model. Details on this conclusion are available upon request from the authors.

¹¹We exclude the possibility of explicit government default, other than implicitly by inflation. See Hatchondo and Martinez (2009) or Arellano and Ramanarayanan (2008) for recent examples. Also, we do not explore inflation risk premia and term structure implications of our model. On this, see for example Rudebusch and Swanson (2008). These authors use an assumption on declining payment streams on consols, which in the aggregate shows some similarities with the bond structure developed here.

average interest rate of all current and previously issued stochastic bonds by i_t^L . Then the latter is given by

$$i_t^L = \frac{B_t^{new}}{B_t^L} i_t^{new} + (1 - \alpha) \frac{B_{t-1}^{new}}{B_t^L} i_{t-1}^{new} + (1 - \alpha)^2 \frac{B_{t-2}^{new}}{B_t^L} i_{t-2}^{new} + \dots$$

The weights on the interest rates of previously issued bonds depend on the fraction of those bonds that has survived until date t and the value of these bonds relative to current long-term debt. Thus the average interest rate on outstanding long-term debt can be tracked in recursive form

$$i_t^L B_t^L = (1 - \alpha) i_{t-1}^L B_{t-1}^L + i_t^{new} B_t^{new}. \quad (2)$$

The interest rate i_t^{new} is priced according to an appropriate arbitrage condition between the one-period and the stochastic bonds, derived below from the households first-order conditions.

3.2 Households

The representative household is assumed to maximize the present value of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{(M_t/P_t)^{1-\sigma_m}}{1-\sigma_m} - \varphi \frac{N_t^{1+\phi}}{1+\phi} \right),$$

with C_t consumption, M_t/P_t real money balances, and β the discount factor, σ the inter-temporal elasticity of substitution (and the inverse of risk aversion), σ_m governs the interest elasticity of money demand, and χ a utility weight. Labor services provided enter negatively, with ϕ the inverse of the Frisch elasticity of labor supply, and φ scales labor disutility. The consumption good is an aggregate of a continuum of differentiated products $C_t(z)$, and given by the function $C_t = \left(\int_0^{\infty} C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}$, with $\epsilon > 1$ a constant elasticity of substitution. The individual goods are supplied by a continuum of monopolistically competitive firms at price $P_t(z)$ for each firm z .

Maximization takes place subject to the evolution of the interest rate on the portfolio of bonds (2), and the budget constraint

$$\begin{aligned} \frac{B_t}{P_t} + \frac{B_t^{new}}{P_t} + \frac{M_t}{P_t} + C_t = & (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + (\alpha + i_{t-1}^L) \frac{B_{t-1}^L}{P_t} + \frac{M_{t-1}}{P_t} \\ & + (1 - \tau_t) \frac{W_t}{P_t} N_t + \int_0^1 \frac{\Pi_t(z)}{P_t} dz, \end{aligned}$$

where W_t/P_t is the real wage and τ_t is a proportional tax rate on labor income. $\Pi_t(z)$ is nominal income from dividends of monopolistically competitive intermediate firms – indexed z – owned by households. As mentioned, the one period bond issued in period t is denoted by B_t and pays interest i_t in the following period. In contrast, only a fraction α of long-term bonds B_{t-1}^L is redeemed each period, and a quantity B_t^{new} of bonds are newly issued.

Combining equation (1) with equation (2) and the aforementioned budget constraint, the representative household maximizes its intertemporal utility with respect to C_t , B_t , B_t^L , i_t^L , M_t , N_t , and $C_t(z)$. Note at this point that, because i_t^L changes with the amount of new debt a household chooses to purchase, it is part of the optimization problem.

Consumption smoothing and the holdings of the two types of bonds are guided by the familiar Euler equation for short-term bonds,

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + i_t], \quad (3)$$

and a similar Euler equation for long-term bonds

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + i_t^{new} - \mu_{t+1}(1 - \alpha)\Delta i_{t+1}^{new}], \quad (4)$$

where λ_t is the marginal utility of wealth, which must be equal to the marginal utility of consumption

$$\lambda_t = C_t^{-\sigma}. \quad (5)$$

The second Euler condition deserves further comment. It relates the nominal stochastic discount factor $\beta(\lambda_{t+1}/\lambda_t)P_t/P_{t+1}$ to the interest rate on newly-issued long-term debt, i_t^{new} , corrected for its expected change, $\Delta i_{t+1}^{new} = i_{t+1}^{new} - i_t^{new}$. An expected increase in the long-term interest rate on long-term bonds, reduces the incentives to invest in such bonds today, which requires a higher long-term interest rate today to ensure agents are indifferent to investing in short-term bonds.

The change in the long-run interest rate is valued by a stochastic discount factor for the long-term bond

$$\mu_t = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + \mu_{t+1}(1 - \alpha)], \quad (6)$$

where μ_t is the Lagrangean multiplier on (2). This condition can be seen as defining the appropriate discount factor for long-term bonds as a recursively weighted average of the

current and future short-term stochastic discount factors. For example, relatively low expected inflation in future periods will tend to raise future expected discount factors, and thus raise current and future μ_t . By virtue of (4), a higher μ_{t+1} will also lead agents to postpone investment in the long-term bond, in order to avoid capital losses.

The remaining optimality conditions are the familiar conditions for money demand,

$$\frac{M_t}{P_t} = \left[\chi C_t^\sigma \frac{1+i_t}{i_t} \right]^{1/\sigma_m}, \quad (7)$$

labor supply,

$$\varphi N_t^\phi = C_t^{-\sigma} (1 - \tau_t) \frac{W_t}{P_t}, \quad (8)$$

and the demand for differentiated products, $C_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} C_t$, where the price level is defined as the cost of the minimum-expenditure combination of the $C_t(z)$ to obtain a given value of C_t , $P_t \equiv \left(\int_0^\infty P_t(z)^{1-\epsilon} dz \right)^{1/(1-\epsilon)}$.

3.3 Firms

Firms are monopolistic competitors each facing iso-elastic demand for their differentiated products derived above, and demand labor to produce. Production is linear in labor, $Y_t(z) = AN_t(z)$, where A is the aggregate productivity level. Prices are sticky in that each period, following Calvo (1983), only a fraction $(1 - \theta)$ of firms is able to optimally adjust prices. If a firm cannot re-optimize its price, the nominal price evolves according to the indexation rule $P_t(z) = \pi_t^* P_{t-1}(z)$, where π_t^* is the actual (or, with imperfect information, perceived) inflation target. Thus $\pi_t = P_t/P_{t-1}$ is the gross aggregate inflation rate. The inclusion of the actual or perceived inflation target in the indexing rule is crucial for the issue at hand, because we deal with potentially permanent changes in inflation, and want to ensure that long-run monetary neutrality holds.¹²

Taking into account that it might not be able to set its price optimally in a near future, a firm z chooses the optimal price, $P_t^*(z)$, by maximizing intertemporal profits subject to the demand it faces and taking into account the indexing rule. The first-order

¹²We also experimented the more general indexation scheme $P_t(z) = \tilde{\pi}_t P_{t-1}(z)$, where we allowed $\tilde{\pi}_t = \pi_{t-1}^\xi \pi_t^{*(1-\xi)}$ to depend on both lagged actual inflation and the actual (or perceived) inflation target π_t^* . We found that our main results are barely affected by this assumption.

condition for this program is

$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}}$$

where

$$\mathcal{Z}_{1,t} = \lambda_t m c_t C_t + \theta \beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^*} \right)^{-\epsilon} \mathcal{Z}_{1,t+1} \right] \quad (9)$$

and

$$\mathcal{Z}_{2,t} = \lambda_t C_t + \theta \beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^*} \right)^{1-\epsilon} \mathcal{Z}_{2,t+1} \right], \quad (10)$$

which is the same for all firms that can adjust their price in period t . Real marginal costs are given by $m c_t = (W_t/P_t)/A$ and λ_t is the marginal utility of consumption, which appears by the assumption of perfect capital markets. The aggregate price index can be shown to evolve according to

$$1 = \theta \pi_t^{*(1-\epsilon)} \pi_t^{-(1-\epsilon)} + (1 - \theta) \left(\frac{\epsilon}{\epsilon - 1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} \right)^{1-\epsilon}. \quad (11)$$

3.4 The fiscal and monetary authorities

The fiscal authority follows a fiscal rule that adjusts the tax rate depending on the deviation of real debt from a long-run level of real debt, assumed as given. The tax rule is given by

$$\tau_t - \tau = \rho_\tau (\tau_{t-1} - \tau) + \phi_\tau \widehat{d}_t, \quad (12)$$

where τ is the steady-state tax rate and \widehat{d}_t is the percent deviation of total real short- and long-term debt, $d_t = (B_t + B_t^L)/P_t = b_t + b_t^L$, with $b_t = B_t/P_t$ and $b_t^L = B_t^L/P_t$, from its long-run, steady-state, level, which is exogenously given. The tax smoothing parameter ρ_τ prevents excessive jumps in the tax rate, and ϕ_τ determines the responsiveness of the tax rate to variations in real debt. The latter parameter we assume to be low.

Aggregate public debt evolves then according to the consolidated budget constraint of the public sector, written here in real terms as

$$\tau_t w_t N_t + m_t - \frac{m_{t-1}}{\pi_t} + b_t + b_t^{new} = g + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} + (\alpha + i_{t-1}^L) \frac{b_{t-1}^L}{\pi_t}. \quad (13)$$

Government revenue consists of tax revenue $\tau_t w_t N_t$, seignorage revenue, where $m_t = M_t/P_t$, and newly issued debt, $b_t + b_t^{new}$, while expenditure consists of (exogenous) real

government spending g , redeemed bonds $b_{t-1} + \alpha b_{t-1}^L$ and real interest paid on bonds $(i_{t-1}b_{t-1} + i_{t-1}^L b_{t-1}^L)/\pi_t$. Rewriting the evolution of long-term debt (1) in real terms

$$b_t^L = (1 - \alpha) \frac{b_{t-1}^L}{\pi_t} + b_t^{new},$$

shows how the real value of debt that has not matured, $(1 - \alpha)b_{t-1}^L$, is susceptible to inflation, π_t . In the following, we assume that the one-period bond is issued at an infinitesimal quantity $b_t = b$, for it to be merely relevant for the pricing of assets via the short-term policy interest rate i_t . The monetary authority follows an interest rate rule given by

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[i + \widehat{\pi}_t^* + \phi_\pi (\widehat{\pi}_t - \widehat{\pi}_t^*) + \phi_y (\widehat{Y}_t - \widehat{Y}_t^n) \right] + \eta_t, \quad (14)$$

with i the steady-state value of the nominal interest rate, $\widehat{\pi}_t^*$ the time-varying inflation target, \widehat{Y}_t is actual output and \widehat{Y}_t^n is the natural rate of output being defined as the level of output that would prevail under fully flexible prices, all expressed as deviations from steady state. Hence, $i + \widehat{\pi}_t^*$ is the variation of the nominal rate that is governed by changes in the inflation target. The policy interest rate adjusts with inertia, as given by ρ_i . The interest rate rule is additionally subject to a monetary policy white noise disturbance η_t with variance σ^2 .

The percentage deviation of the inflation target from steady state is assumed to evolve according to follow the AR(1) process

$$\widehat{\pi}_t^* = \rho_\pi \widehat{\pi}_{t-1}^* + \eta_t^\pi$$

with η_t^π a white noise process with variance σ_π^2 . The persistence parameter ρ_π is between zero and one so that variations of the target are potentially very persistent. The shocks are i.i.d. normal, with the variance of η_t^π substantially lower than that of η_t .

3.5 Market clearing and equilibrium

Aggregate demand is given by total private and government consumption:

$$Y_t = C_t + g, \quad (15)$$

and the market clearing condition on goods market is given by:

$$\Delta_{p,t} Y_t = AN_t, \quad (16)$$

where $N_t = \int_0^1 N_t(z)dz$ is aggregate labor input and the term $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} dz$ measures the price dispersion arising from staggered price setting. Similar to the aggregate price index, the price distortion has a law of motion that can be shown to be:

$$\Delta_{p,t} = \theta \Delta_{p,t-1} \left(\frac{\pi_t}{\pi_t^*}\right)^\epsilon + (1 - \theta) \left(\frac{\epsilon}{\epsilon - 1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}}\right)^{-\epsilon}. \quad (17)$$

The competitive equilibrium of our model is a set of stationary processes $B_t, B_t^L, B_t^{new}, C_t, \Delta_{p,t}, i_t, i_t^L, i_t^{new}, \lambda_t, M_t, \mu_t, N_t, \pi_t, \tau_t, W_t, Y_t, \mathcal{Z}_{1,t}, \mathcal{Z}_{2,t}$, satisfying the relations (1) to (17) and $b_t = b$, given the exogenous stochastic processes η_t, η_t^π and Y_t^n , and the initial conditions $B_{-1}, B_{-1}^L, i_{-1}, i_{-1}^L, \Delta_{p,-1}$ and π_{-1} .

3.6 Imperfect information and credibility

We allow for the possibility that private agents do not have perfect knowledge of the central bank's objectives, in particular the inflation target. Agents only receive a signal on an aggregate monetary policy shock, defined as

$$\varepsilon_t^\pi \equiv (1 - \rho_i)(1 - \phi_\pi)\widehat{\pi}_t^* + \eta_t. \quad (18)$$

The signal extraction problem entails backing out the two components $\widehat{\pi}_t^*$ and η_t in the Taylor rule (14). Formally, given their knowledge about the driving process of the shocks and of the standard deviation of the inflation target and policy shock, agents use a simple Kalman filter to extract the optimal estimates of the two unobserved components of ε_t^π .¹³ The latter evolve according to:

$$\widetilde{E}_t \widehat{\pi}_t^* = \widetilde{E}_{t-1} \widehat{\pi}_t^* + \frac{k}{\rho_\pi} (\varepsilon_t^\pi - \widetilde{E}_{t-1} \varepsilon_t^\pi); \quad (19)$$

with $k = \frac{\rho_\pi(1-\rho_i)(1-\phi_\pi)\mathcal{P}}{((1-\rho_i)(1-\phi_\pi))^2\mathcal{P} + \sigma^2}$ the Kalman gain parameter of the steady-state Kalman filter, where \mathcal{P} solves $\mathcal{P}^2 + [(1 - \rho_\pi^2)\sigma^2 / [(1 - \rho_i)(1 - \phi_\pi)]^2 - \sigma_\pi^2] \mathcal{P} - (\sigma_\pi\sigma / [(1 - \rho_i)(1 - \phi_\pi)])^2 = 0$, and,

$$\widetilde{E}_t \eta_t = \varepsilon_t^\pi - (1 - \rho_i)(1 - \phi_\pi)\widetilde{E}_{t-1} \widehat{\pi}_t^*. \quad (20)$$

¹³Examples of such kind of imperfect information mechanism can be found in Erceg and Levin (2003), Darracq-Pariès and Moyen (2009), Melecky, Palenzuela and Söderström (2009) or Fève, Matheron and Sahuc (2010).

Finally, optimal forecasts of the future inflation targets and monetary policy shock can be obtained:

$$\begin{bmatrix} \widetilde{E}_t \widehat{\pi}_{t+i}^* \\ \widetilde{E}_t \eta_{t+i} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ 0 & 0 \end{bmatrix}^i \begin{bmatrix} \widetilde{E}_t \widehat{\pi}_t^* \\ \widetilde{E}_t \eta_t \end{bmatrix}$$

It is important to keep in mind that, if the intention of a higher inflation rate is announced and believed, expectations of future inflation are correct in the sense that agents would not be making systematic errors in predicting inflation. We will refer to this as the full information case. The signal extraction problem is used here to capture different degrees of credibility of the central bank's established inflation target. Under imperfect information, i.e., when the central bank does not announce its new inflationary strategy, agents repeatedly make forecast errors, since over many periods their perception of the target differs from the actual realization of the target. Slow learning about the true increasing target reflects a high credibility of the a previously prevailing – low – inflation target. Changes in ε_t^π will be ascribed mainly to the transitory shocks, and inflation expectations for a longer time will remain anchored at a low level. Our analysis is thus the opposite to the case considered in Erceg and Levin (2003), who analyze slowly revised perceptions about a reduction in the Federal Reserve's inflation target, as it took place during the Volcker disinflation in the 1980s. Slow revisions could then be taken to represent a low credibility of announcement to reduce inflation.

3.7 Calibration and solution procedure

The model is calibrated at the quarterly frequency, with a discount factor of $\beta = 0.99$, which implies a steady state annual real interest rate of about 4%. The intertemporal elasticity of substitution is $\sigma = 1.5$ following the estimates in Smets and Wouters (2007), and the disutility of labor is $\varphi = 2$ in line with Domeij and Floden (2006). We set the money demand elasticity σ_m to 2.56 in line with Chari, Kehoe and McGrattan (2000), while the scale factor χ is set to match the long-term ratio of the monetary base to output in the U.S. The monopolistic markup factor is set to 20 %, resulting from a demand elasticity for the differentiated products of $\epsilon = 6$. The average level of hours worked is calibrated to one third.

The probability of the stochastic bond maturing is $\alpha = 0.055$, which corresponds to an

average maturity of about 4.5 years, or 55 months. This value corresponds to the actual average maturity of the U.S. (see Section 2) as recorded in the Report of the Treasury Borrowing Advisory Committee of August 5, 2009. The steady state debt to GDP ratio is assumed to be 45 percent, a value consistent with the pre-crisis level of U.S. government debt. The shock will drive up this debt to slightly above 67 percent. The steady state government spendings to GDP ratio is set to 20 percent. Since the short-term bond is only used to determine the stochastic discount factor, its actual quantity is assumed constant and close to zero. So the average maturity essentially depends on the properties of the long-term bond.

Tab. 1 - BASELINE CALIBRATION

Parameter	Value	Description
<u>Preferences</u>		
β	0.99	Time discount factor
σ	1.5	Intertemporal elasticity of substitution
σ_m	2.56	Inverse of the interest elasticity of money demand
χ	5.2×10^{-6}	Scale factor to utility of money balances, targets $m/Y = 0.07$
ϕ	2.00	Inverse of the Frish of labor supply
φ	35.94	Scale factor to disutility of work, targets $h = 1/3$
<u>Bonds market</u>		
α	0.055	Quarterly probability of maturing debt
<u>Firms</u>		
ϵ	6	Price markup of 20%
θ	0.75	One year price contracts
<u>Monetary policy</u>		
ρ_i	0.75	Interest rate smoothing parameter
ϕ_π	1.5	Response of interest rate to inflation
ϕ_y	0.5	Response of interest rate to output gap
<u>Fiscal policy</u>		
ρ_τ	0.5	Tax rate smoothing parameter
ϕ_τ	0.02	Tax feedback to deviations of debt from steady-state

The parameters of the monetary policy rule assume the fairly standard values of $\phi_\pi = 1.5$ and $\phi_y = 0.5$ and a persistence parameter of $\rho_i = 0.75$. The values guarantee determinacy of the rational expectations equilibrium in models with balanced budget rules, as well as in models with sufficiently aggressive fiscal policy rules. In the baseline

case, the inflation target has a persistence of $\rho_\pi = 0.99$.

For the fiscal rule, we assume $\rho_\tau = 0.5$ and $\phi_\tau = 0.02$, which yields determinate and non-explosive equilibria in all our simulations. The tax response to deviations of debt from the steady-state sustainable level is very mild and thus gives the potentially strongest role for inflation to contribute to debt consolidation. Of course, a high tax responsiveness is possible, and could easily take care of the higher debt. We show its effects below, but this is the very scenario that political constraints will most likely make difficult to follow, and may be avoided by raising inflation instead.¹⁴ We describe the calibration of the shocks' standard deviations in the next sections along with the presentation of our different scenarios.

The calibrated models' rational expectations equilibrium dynamics are derived numerically using Dynare, which utilizes the ideas of Sims (2002). To determine the solutions under imperfect information, the resulting state space solution is then augmented by the evolution of the inferences on the permanent and transitory shocks to the monetary policy rule. That is, rather than simulating the model response to the shocks following the AR(1) processes specified above, the perceived shocks derived from the Kalman filter are fed into the model, and the corresponding impulse responses displayed.

4 Analysis

In this section, we analyse the dynamic adjustment of public debt and other key variables with and without a change in the inflation target. The starting point of the simulations is an increase of public debt of the magnitude observed in the U.S. since the onset of the economic crisis of 2008 and 2009. To clarify, we consider first such a debt shock, i.e., an helicopter drop of government bonds¹⁵, absent changes in the target, and compare two possible tax policies.

¹⁴Throughout, we assume that the model's approximation is far enough from a fiscal limit, where further tax increases would lead to falling tax revenue due to Laffer curve effects.

¹⁵See also Leith and Wren-Lewis (2011).

4.1 A debt shock

In all the scenarios, debt is assumed to increase by about 65 percent from the current debt-to-GDP ratio. For the U.S., this corresponds to the increase of debt from about 45 percent of GDP in the middle of 2009 to the projected 67 in 2012. Figure 2 shows the subsequent evolution of real government debt. The solid line depicts the dynamic response for a fiscal rule where the tax rate adjusts to higher debt just sufficiently to keep debt from exploding. Then real debt barely falls over the following 20 years. The associated tax rate increase is about two percentage points. Because of the higher stock of debt, of which each period a fraction $\alpha = 5.5\%$ becomes due, a correspondingly higher amount of new debt is issued each period.

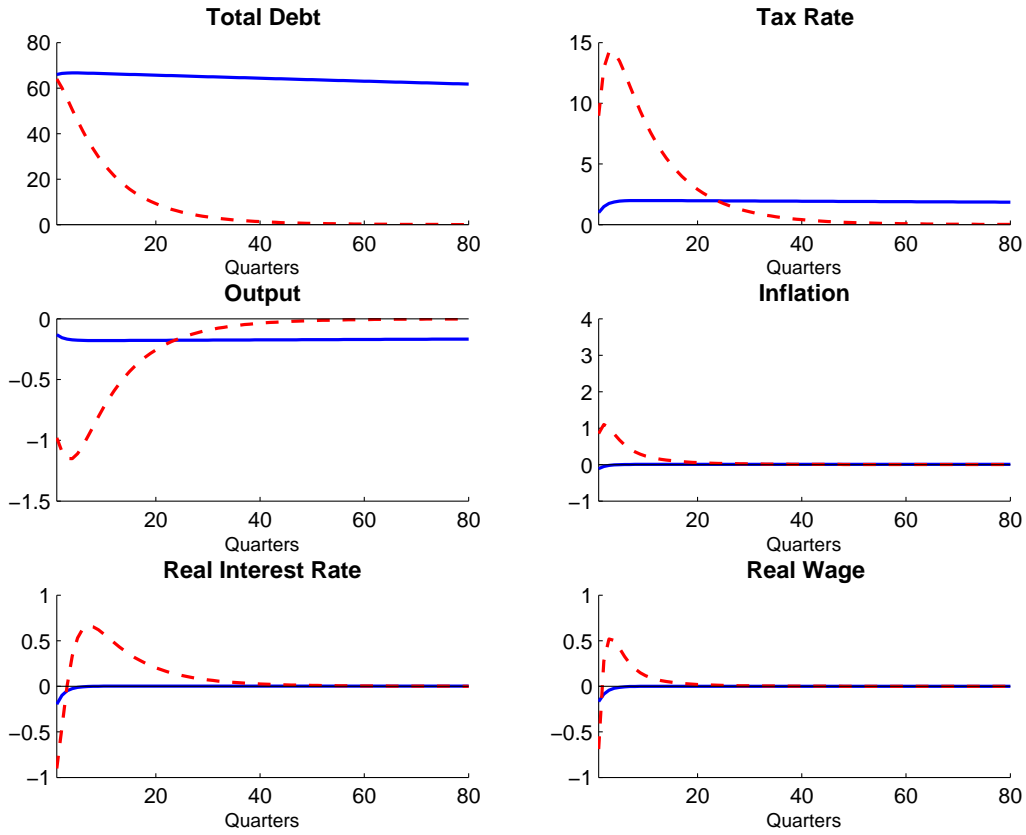
The higher tax rate leads to drop in after-tax *net* real wages for workers, reducing their labor supply and consequently output by about 0.2 percent below steady state. This distortionary effect lasts until debt returns to its long-run level. The inflation rate only initially slightly falls below the long-run target, following a short-lived contractionary effect of the tax rate on labor supply and thus the real wage, which reduces real marginal costs. However, after a few quarters, inflation, real wages, and the real interest rate return back to, or close to, the steady state.

Contrast this with a tax policy that reduces the additional debt within a short amount of time, depicted by the dashed line in Figure 2. This tax policy is represented in the fiscal authority's tax rule by a high coefficient on debt. The resulting tax increase is up to 15 percentage *points* above the steady state tax rate, and leads to a much larger drop in output over several periods, almost 1.2 percent in the second quarter. At the same time, inflation rises by over one percentage point, which induces the policy interest rate, and also the real interest rate, to increase after a short drop. Finally, real gross wages rise because of the reduced labor supply after the tax increase.

4.2 Real debt and changes of the inflation target

We now turn to the question how much a change in the inflation target rather than raising taxes would contribute to a reduction in government debt. That is, we take the baseline change in debt of 65 percent, the fiscal rule that implies a minimal reaction of the tax

Fig. 2 - DEBT SHOCK



Notes: The impulse responses portray selected variables responses to the debt shock described in the text for two different scenarios. The solid line depicts the response of the economy under the baseline calibration while the dashed line illustrates the dynamics under a debt reducing tax policy.

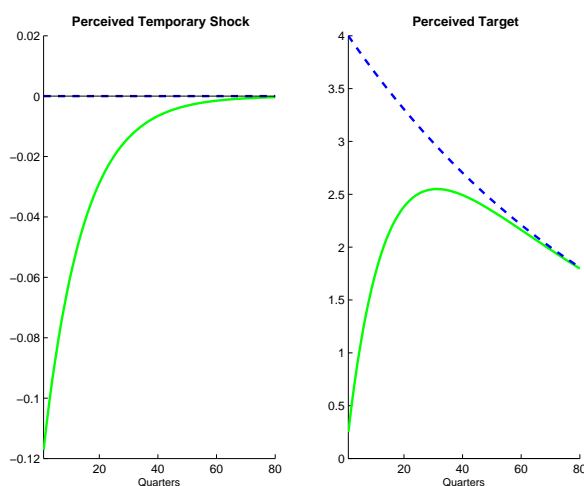
rate, and simulate a persistent change in the annualized inflation target by 4 percentage points from 2 to 6 percent, which we denote $\hat{\pi}_{a,0}^* = 4 \times \hat{\pi}_0^* = 4$. The persistence parameter of the target process is set to a high value of $\rho_\pi = 0.99$. Later, we consider less persistent changes.¹⁶

Throughout, we compare here the evolution of the economy after changes in the inflation target both when it is perfectly observed and when the public cannot distinguish a change of the target from the transitory monetary policy shock. Recall that we take

¹⁶Furthermore, for clarity, we assume that all the new debt is priced at the steady-state nominal interest rates, so that the shock to the inflation target occurs after the increase in debt. This avoids confounding effects of the target change with effects from the debt change as such.

imperfect information to reflect a high credibility of a previously prevailing low inflation target, when the change in the target has not been communicated to the public. The degree of misperception depends crucially on how volatile the public perceives the inflation target to be, which in turn determines the speed of learning according to the Kalman filter. In this calibration, the perceived volatility of the target is set such that perceived and actual inflation target coincide after 20 years.

Fig. 3 - PERSISTENT TARGET SHOCK

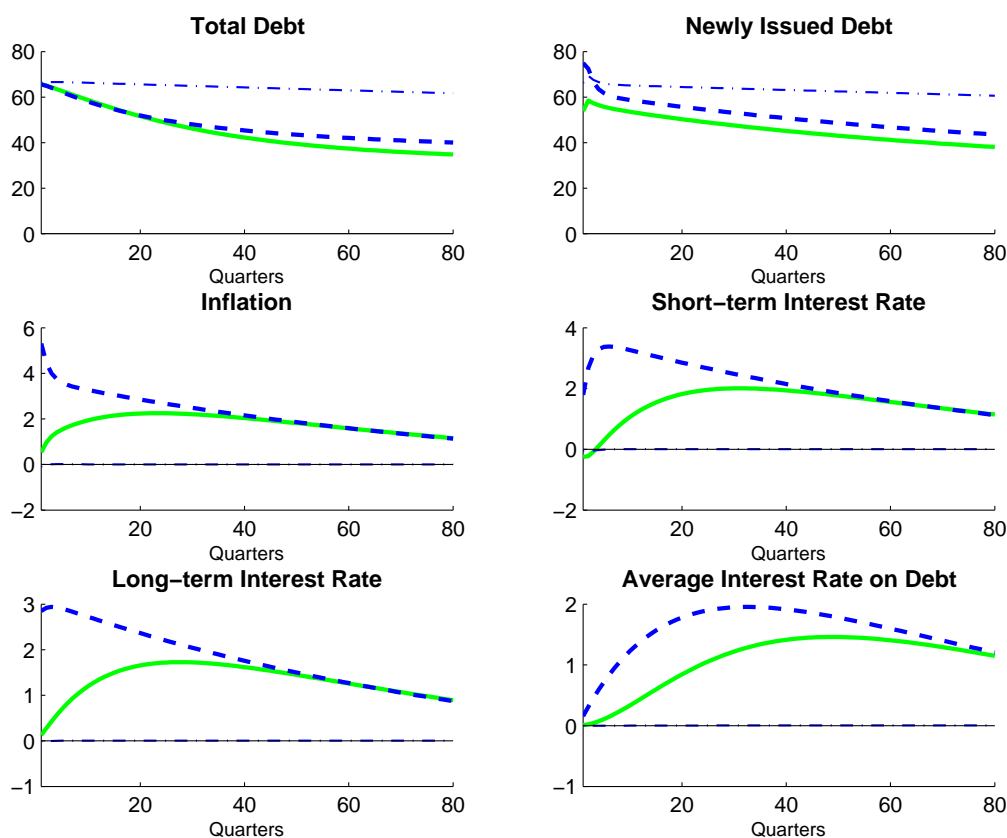


Notes: The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning.

Figure 3 shows the evolution of the actual and perceived inflation target after the one time increase of four percentage points. The target follows the process specified above, as depicted by the dashed line in the right-hand-side panel. In contrast, when agents only slowly learn about the changed inflation trend, it takes long for the difference between actual and perceived target to vanish. This is because, initially, agents assign a large fraction of the change in the nominal interest to the transitory shock, as can be seen clearly in the left panel.

The corresponding evolution of the economic variables of interest are shown in Figures 4 and 5. The former focuses on real government debt held by the public, the realized

Fig. 4 - PERSISTENT TARGET SHOCK



Notes: The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning. Finally, the dashed dotted line represents the benchmark constant target case.

inflation rate, and the three measures of interest generated by the model. The latter figure shows the evolution of output, real interest and government spending on interest, the real wage, the marginal tax rate, and tax revenue.

Consider first the responses under full information about the inflation target in Figure 4, again depicted by the dashed lines. The addition to total debt that followed the debt shock falls over time, but at a decelerating rate. After ten years, about 23 percent of this increase in total real debt has been inflated away. Recall that we show here only the additional debt above steady-state debt. Along with total debt, newly-issued debt follows a similar path in percentage deviations from its steady state, which is of course

only a fraction α of total debt. The initial jump in new debt is due to a substitution from money holdings to bonds, induced by a higher nominal interest rate that induces to a drop in money demand. The government budget constraint mandates a commensurate increase in bonds.¹⁷

Actual inflation follows the same time path as the target, except for a short-lived initial cost push effect, which follows from the assumed inertial behavior of the short-term, policy, interest rate. Thus the policy interest rate in the fourth panel does not immediately adjust to the higher target. Of course, after a few periods, the Fisher relationship has to hold, and the deviations of the short-term rate from steady state track that of inflation closely. This relationship is exploited in more detail below.

The behavior of the long-term interest rates on newly-issued debt, i_t^{new} , shows the implications of the long-term bond introduced in this paper. To see this most clearly, linearize about the steady state the two Euler equations (3) and (4) for the short- and long-term bonds, to find that, up to first order: $i_t^{new} \approx \alpha^n i_t + (1 - \alpha^n) E_t i_{t+1}^{new}$, or

$$i_t^{new} \approx \alpha^n \sum_{s=0}^{\infty} (1 - \alpha^n)^s E_t i_{t+s}, \quad (21)$$

where $\alpha^n \equiv (\alpha + i)/(1 + i)$. The long-term interest rate on newly-issued debt is the weighted sum of all future expected short-term nominal interest rates, with declining weights $(1 - \alpha^n)^s$ as the time horizon s increases. This is borne out in the bottom left graph.

While the long-term interest rate is an average of future short rates, the average interest rate of outstanding debt is an average of all long-term rates set in the past. Thus the average interest rate paid on debt can only sluggishly follow the evolution of the long-term interest rates on newly-issued debt. This can be made explicit by linearizing equation (2), resulting in the recursion $i_t^L \approx \alpha^L i_t^{new} + (1 - \alpha^L) i_{t-1}^L$ or

$$i_t^L \approx \alpha^L \sum_{s=0}^{\infty} (1 - \alpha^L)^s i_{t-s}^{new}, \quad (22)$$

where $\alpha^L \equiv (1 - (1 - \alpha)/\pi)$, which is close to α for the gross inflation rate π close to one. The average interest rate on outstanding debt is thus approximately a weighted average

¹⁷In our baseline calibration, this increase in bonds almost exactly offsets the initial reduction in outstanding debt arising from the jump in inflation. Alternative calibrations of the money demand parameters imply slightly changed initial dynamics, but do not affect our main results.

of all past long-term interest rates, with declining weights $(1 - \alpha^L)^s$, as the time s since issuance increases. This explains the slowly rising (dashed) line for the average interest rate on debt. Only after about 30 quarters it begins to fall again. Before turning to other variables of interest, we now discuss the adjustments under imperfect information.

The dynamics under imperfect information are depicted by the solid lines in Figure 4. The perceived inflation target differs from the actual target because agents assign a large fraction of observed interest rate changes to the monetary policy shock, rather than the target. Recall the definition (18) of the signal, which is now filtered according to the Kalman filter. Consequently, actual inflation moves only slowly upwards, since the perceived target $\tilde{E}_t \hat{\pi}_t^*$ enters firms' price setting, as given by the linearized Phillips curve for full indexation to the perceived target rate:

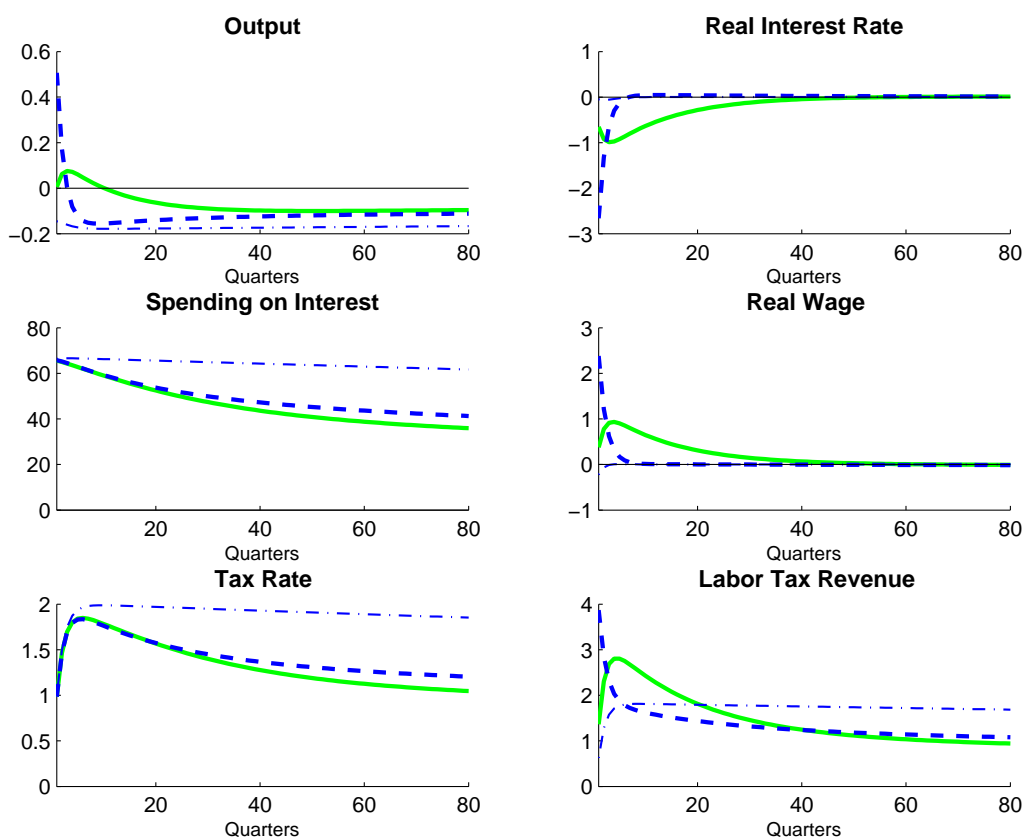
$$\hat{\pi}_t = \tilde{E}_t \hat{\pi}_t^* + \beta(E_t \hat{\pi}_{t+1} - \tilde{E}_t \hat{\pi}_{t+1}^*) + \kappa \hat{m}c_t, \quad (23)$$

with κ a nonlinear function of some models' structural parameters. Thus a high credibility of a previously established inflation target lowers the responsiveness of actual inflation to an inflationary change in the central bank's target.

The initial impact on real debt of the change on the inflation target differs only slightly from that under full information. The surprise effect of inflation on outstanding debt is smaller than before, but this time, newly issued debt increases by less, due to a smaller drop of seignorage revenue. The main difference to the full information case bears out over time, however, as agents underprice newly-issued debt, because their inflation expectations are persistently lower than the actual inflation rates. This shows up in the correspondingly slow movements of the short and long-term interest rates.

Figure 5 shows the movements of the remaining variables of interest. Output initially rises under both information scenarios, because the inertial interest rate rule allows real interest rates to drop after the increase in inflation. Over time, output falls below steady state because of the distortionary effect of the higher tax rate. In proportion to the higher debt level, the public sector's spending on interest increases and follows the same dynamics. The behavior of labor tax revenue mirrors the dynamics of output and the tax rate.

Fig. 5 - PERSISTENT TARGET SHOCK



Notes: The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning. Finally, the dashed dotted line represents the benchmark constant target case.

4.3 Inspecting the mechanism

We now turn to a qualitative analysis of how debt maturity and persistence of the inflation target determine the observed evolution of public debt. A few modest simplifications of the model allow us to give transparent analytical representations of the key mechanisms. First, we keep labor supply constant, which removes any feedback effects of taxes on output. Secondly, we ignore seignorage, for simplicity, and because seignorage revenue is known to empirically play only a small role in government revenue dynamics for the ranges of inflation considered here. Finally, we assume perfectly flexible prices.

Under these assumptions, the equation for the evolution of debt combined with the

real government budget constraint can be reduced to $\tau_t wN + b_t^L = g + (1 + i_{t-1}^L) b_{t-1}^L / \pi_t$. Then, with a slightly simplified fiscal rule $\tau_t = \tau + \phi_\tau (b_t^L / b^L - 1)$, we obtain

$$b_t^L = \frac{1}{1 + (\phi_\tau / b^L) wN} \left[g - (\tau - \phi_\tau) wN + (1 + i_{t-1}^L) \frac{b_{t-1}^L}{\pi_t} \right]. \quad (24)$$

The evolution of debt essentially depends on the coefficient $(1 + (\phi_\tau / b^L) wN)^{-1}$ and the relative dynamics of i_{t-1}^L and π_t . The smaller ϕ_τ , the slower the adjustment of b_t^L will be to any variations of the right-hand side variables. However, as long as i_{t-1}^L and π_t do not act systematically to stabilize debt, ϕ_τ must be strictly positive and high enough to guarantee a non-explosive path of debt.¹⁸

The future dynamics of i_t^L and π_t depend crucially on the expected path of inflation, which in turn depends on the expected path of the inflation target, π_t^* . This dependence can now be easily made explicit. The assumption of flexible prices implies that the monetary authority directly determines the inflation rate through its control over the short-term nominal interest rate. Setting $\rho_i = \phi_y = 0$ in the Taylor rule (14), and using the consumption-Euler equation (3), inflation can be easily found to follow:¹⁹

$$\widehat{\pi}_t = \frac{\phi_\pi - 1}{\phi_\pi - \rho_\pi} \widehat{\pi}_t^* + \frac{1}{\phi_\pi} \eta_t.$$

The stated equation holds under full information, and shows that, after a change in the inflation target, future inflation can be expected to evolve directly proportional to the expected inflation target, since $E_t \widehat{\pi}_{t+s}^* = \rho_\pi^s \widehat{\pi}_t^*$ for $s \geq 0$, or

$$E_t \widehat{\pi}_{t+1+s} = \frac{\phi_\pi - 1}{\phi_\pi - \rho_\pi} \rho_\pi^{1+s} \widehat{\pi}_t^*. \quad (25)$$

Under imperfect information, we have to substitute the target and monetary policy shocks by their respective perceptions at time t , i.e., $\widetilde{E}_t \widehat{\pi}_t^*$ and $\widetilde{E}_t \eta_t$, which will be further used below. Then of course, actual and target inflation will not necessarily move closely.

¹⁸In fact, for $\phi_\tau = 0$, the tax rate would be constant. Then, since in steady state, $g - \tau wN = (1 - (1 + i)/\pi) b^L$, we can linearize equation (24) to get

$$\widehat{b}_t^L = \frac{1 + i^L}{\pi} \widehat{b}_{t-1}^L + \frac{b^L}{\pi} \widehat{i}_{t-1}^L - (1 + i^L) b^L \widehat{\pi}_t$$

Since the real interest rate rate $(1 + i)/\pi$ is larger than one in steady state, debt would be explosive up to first order.

¹⁹To obtain this relationship, combine the simplified interest rate rule and the Euler equation and solve forward. Ruling out explosive paths gives the stated (unique) solution to the inflation rate.

To determine the expected evolution of different nominal interest rates, we can make use of the two relationships (21) and (22) in combination with the consumption Euler equation (3) and the expected evolution of the inflation target, $E_t \widehat{\pi}_{t+s}^* = \rho_\pi^s \widehat{\pi}_t^*$. Since there are no movements in the simplified model's natural real rate of interest, $1/\beta - 1$, the evolution of the short-term nominal interest rate is solely determined by the expected inflation rate. Then the Euler equation implies $E_t i_{t+s} - i = E_t \widehat{\pi}_{t+1+s}$. Inserting (25) yields

$$E_t i_{t+s} - i = \omega \rho_\pi^{s+1} \widehat{\pi}_t^*,$$

where we have defined a scale factor $\omega \equiv (\phi_\pi - 1)/(\phi_\pi - \rho_\pi)$. Using the approximation (21) for i_t^{new} , the interest rate on newly-issued long-term debt can now be written as

$$i_t^{new} - i \approx \frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \omega \widehat{\pi}_t^*. \quad (26)$$

Furthermore, inserting this into (22) delivers the average interest rate on outstanding debt as a function of the process for the inflation target

$$i_t^L - i \approx \frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \alpha^L \sum_{s=0}^{\infty} (1 - \alpha^L)^s \omega \widehat{\pi}_{t-s}^*, \quad (27)$$

which is the second expression needed to characterize the evolution of government debt.²⁰

To gain some further intuition, consider first the factor in front of $\omega \widehat{\pi}_t^*$ in equation (26). It reflects the relevant aspects of the forward-looking nature of long-term nominal interest rates: maturity of debt and expected evolution of inflation. When $\rho_\pi = 0$, that interest rate will be equal to the steady-state rate in all periods, $i_t^{new} = i$, since target inflation is i.i.d. about its steady state. In contrast, for an inflation target close to a random walk, $\rho_\pi \approx 1$, the interest rate follows the same process as the target. In fact, then also $\omega \approx 1$. Only for intermediate values of ρ_π does the maturity structure exert its influence on the long-term interest rate, which then on average compensates for future inflation rates. In contrast, for an economy with only short-term bonds, $\alpha = \alpha^n = 1$, as in the standard New Keynesian model, the nominal interest rate only needs to compensate for one period-ahead inflation. $i_t^{new} - i \approx \rho_\pi \omega \widehat{\pi}_t^*$, which is of course the short-term nominal interest rate. For (27), the shorter the average maturity, i.e., α and thus α^L close to 1, the smaller the weights on past inflation rates will be. Then, again, the average rate

²⁰Recall that $\alpha^n = \frac{\alpha+i}{1+i}$ and $\alpha^L = 1 - \frac{1-\alpha}{\pi}$, which are close to α for low steady-state values of i and π .

tends to equal the long-term rate and the short-term rate as well. It is clear that under full information, unless α and ρ_π are at extreme values, the average long-term interest rate will be unable to compensate even for fully and correctly anticipated inflation that follows a change in the inflation target.

The equation for the evolution of real debt, (24), can now be rewritten in deviations from its steady-state level:

$$\widehat{b}_t^L = \Phi \left[\frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \alpha^L \sum_{s=1}^{\infty} (1 - \alpha^L)^{s-1} \omega \widehat{\pi}_{t-s}^* - \widehat{\pi}_t + \widehat{b}_{t-1}^L \right]$$

with $\Phi = 1/(1 + (\phi_\tau/b^L)\omega N)$. This equation shows most directly the role of inflation persistence and average debt maturity α , and gives a simple characterizations for the evolution of debt for different values for the parameters α and ρ_π . To further understand the role of the determinants of real debt dynamics, assume again that there is no long-term debt, i.e., $\alpha = 1$. Then $\alpha^n = \alpha^L = 1$, and debt follows

$$\widehat{b}_t^L = \Phi \left[E_{t-1} \widehat{\pi}_t - \widehat{\pi}_t + \widehat{b}_{t-1}^L \right] \quad (28)$$

since $E_{t-1} \widehat{\pi}_t = \rho_\pi \widehat{\pi}_{t-1}$. Under full information, a one-time increase in the inflation target would have an effect on real debt only in the period of the change, since future nominal rates adjust to compensate for the predictable path of inflation. That is, if $E_{t-1} \widehat{\pi}_t = 0$, then a rise in inflation deflates real debt by $-\widehat{\pi}_t$. But absent further shocks, there will be no expectational errors, since $E_t \widehat{\pi}_{t+1} - \widehat{\pi}_{t+1} = \omega(E_t \widehat{\pi}_{t+1}^* - \widehat{\pi}_{t+1}^*) = \omega(\rho_\pi \widehat{\pi}_t^* - \rho_\pi \widehat{\pi}_t^*) = 0$. In other words, without long-term debt, and under full information, only inflation surprises can affect the real value of the stock of outstanding debt. This is different under imperfect information.

Recall that the inflation rates $\widehat{\pi}_t$ themselves are the outcome of realizations of the inflation target $\widehat{\pi}_t^*$ or the beliefs $\widetilde{E}_t \widehat{\pi}_t^*$ under imperfect information. In the latter case, when agents slowly learn the true inflation target, they will make repeated expectational errors, even if all debt matures after one period. Thus real debt will be affected by inflation even when no further surprise shock to the target rate occurs. This explains the differences between the impulse responses under full information and under imperfect information in the previous section. To be explicit, we can write the evolution of agents'

optimal estimate of the target as

$$\tilde{E}_t \hat{\pi}_{t+1}^* = \rho_\pi \tilde{E}_{t-1} \hat{\pi}_t^* + k' (\hat{\pi}_t^* - \tilde{E}_{t-1} \hat{\pi}_t^*) \quad (29)$$

with $k' = \frac{\rho_\pi(1-\phi_\pi)\mathcal{P}'}{(1-\phi_\pi)^2\mathcal{P}'+\sigma^2}$ again the Kalman gain parameter, with \mathcal{P}' solving the equation $\mathcal{P}'^2 + [(1-\rho_\pi^2)\sigma^2/(1-\phi_\pi)^2 - \sigma_\pi^2]\mathcal{P}' - (\sigma_\pi\sigma/(1-\phi_\pi))^2 = 0$.

The object of interest here are the future expectational errors for the inflation target $\tilde{E}_{t+s-1} \hat{\pi}_{t+s}^* - \hat{\pi}_{t+s}^*$ which, up to the factor ω , determine future expectational errors for inflation $E_{t+s-1} \hat{\pi}_{t+s} - \hat{\pi}_{t+s}$ in the equation for debt (28). It is easy to show that the expectational error for inflation must evolve according to

$$\tilde{E}_{t+s-1} \hat{\pi}_{t+s}^* - \hat{\pi}_{t+s}^* = (\rho_\pi - k')^s \left[\tilde{E}_{t-1} \hat{\pi}_t^* - \hat{\pi}_t^* \right]$$

after a one-time surprise increase in the target.²¹ Under imperfect information, this expectational error only slowly declines as agents update their perception of the inflation target.

5 How sensitive is real public debt to changing inflation targets?

After having displayed the dynamic behavior of the baseline calibration of the model, along with a specific scenario concerning the inflation target and a qualitative analysis, we now turn to a deeper quantitative analysis. We first show the importance of debt maturity for the observed evolution of public debt. Then we explore how the characteristics of the process of the inflation target influence the degree to which a change in the target affects public debt. It turns out that the persistence of the change is essential for an inflationary policy to have a noticeable impact on the real value of government debt.

To proceed, we need to decide on a metric that summarizes the effects of changing inflation targets on public debt. Recall Figure 4, where a large part of the debt reduction was achieved after 40 quarters, and the difference between full and imperfect information

²¹The future evolution of the perception after a one-time target shock is

$$\tilde{E}_{t+s-1} \hat{\pi}_{t+s}^* = \rho_\pi^s \hat{\pi}_t^* + (\rho_\pi - k')^s \left[\tilde{E}_{t-1} \hat{\pi}_t^* - \hat{\pi}_t^* \right]$$

while the actual evolution of the target after the shock at time t is $\hat{\pi}_{t+s}^* = \rho_\pi^s \hat{\pi}_t^*$. For persistent target shocks and noisy signals, ρ_π close to one and k' small, $(\rho_\pi - k')^s$ will decline monotonically in s .

was noticeable. Therefore, we compute the relative percentage-point difference between the real debt level under the changed inflation target and its level in the absence of that change, at a ten-year horizon. Formally, this is expressed by the following debt multiplier (DM hereafter) measure,

$$DM(h) = (\widehat{d}_{t+h}^{RS} / \widehat{d}_{t+h} - 1) \times 100,$$

with $h = 40$ quarters, and where \widehat{d}_{t+h}^{RS} and \widehat{d}_{t+h} are, respectively, the percent-deviations from steady state of the levels of real debt after the target shock and the level of debt under no target shock. Thus the measure basically compares the difference between the dashed-dotted line and the solid or dashed lines in Figure 3.

5.1 Debt maturity and Credibility

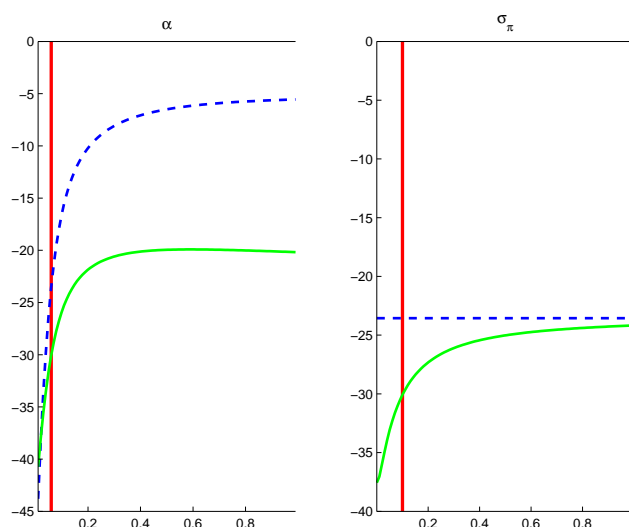
The left panel of Figure 6 highlights the importance of the maturity structure of public debt for the susceptibility of real debt to a higher inflation target. The change in the target is the four-percentage-point increase considered before, and the vertical line indicates the baseline value $\alpha = 0.055$ for the fraction of debt that matures each period. Recall that $1/\alpha$ is then the average expected maturity of debt, in this case four and a half years. The intersections of the vertical line with the dashed and solid curves show the debt reductions under full and imperfect information, respectively. To be precise, the intersections shows the debt reduction as shown in Figure 4, relative to the debt reduction in Figure 2, after ten years. While under full information, the debt reduction is a little under 24 percent, under learning it is 30 percent. Since debt barely changed even after 10 years when the inflation target was held constant, these values almost fully correspond to the reduction of debt due to the changed inflation target.

Consider first higher values of α , which imply lower average maturities of debt. As α increases, the gains from the baseline increase in the inflation target shrink, but do not vanish. In the extreme case with one-quarter bonds only, when $\alpha = 1$, the reduction in real debt after a fully perceived increase in the target falls to about 5 percent.²² In contrast, if a change in inflation is only slowly perceived to be due to a change in the

²²In fact, when $\alpha = 1$, and the target change is fully-perceived, the drop in real debt is solely achieved by the initial surprise jump in inflation that deflates the existing debt before it is rolled over at a higher nominal interest rate.

target, then inflation expectations are too low. Then also interest rates on rolled-over debt are repeatedly set too low, and real debt can be deflated by more than 20 percent after then years. Thus the shorter the average maturity of public debt, the higher is the role of more firmly-anchored past inflation expectations on the sensitivity of real debt to higher actual inflation.

Fig. 6 - AVERAGE MATURITY AND CREDIBILITY



Notes: Difference in debt reduction $DM(40)$ as a function of the parameter on top of each panel. Keeping the other parameters fixed, that parameter is varied in the reported range. The vertical solid bar indicates its base-line value. The dashed line and the solid line respectively depict the full information and the learning case.

In contrast, a higher maturity implied by values of α below 0.055 further facilitates inflating away debt. For lower values of α , that is, higher average maturity of debt, the possible real debt reduction from the changed inflation target increases up to a maximum beyond 40 percent. Consider a ‘British’ scenario, with an average maturity of over 13 years ($\alpha \approx 0.019$), much higher than the close to four and a half years for the U.S. In that case, after 10 years, inflation would reduce the additional debt by more than 35 percent, irrespective on whether there is full information or not. The difference between full information and imperfect information remains relatively small, as in the baseline. Thus for realistic maturity structures, the credibility of a previously established inflation

target does not strongly affect the degree to which pushing up inflation can reduce real government debt.

The right panel of Figure 6 shows the role of the perceived volatility σ_π of the inflation target, which enters the Kalman gain calculation, on the reduction of debt after a given target change of four percentage points. That is, we illustrate here for the case of imperfect information how much the effect of inflation on debt is changing when agents interpret a change in the signal ε_t^π more or less strongly as a change in the inflation target. The perceived volatility of the inflation target relative to the perceived volatility of the monetary policy shock determines the gain in the Kalman gain, and thus the speed of learning. Again, the vertical line in the graph depicts the baseline case, which corresponds to the dynamics of learning as shown in Figure 3. The more of a change in the signal ε_t^π is assigned to a change in the target, that is, the higher σ_π , the lower is the gain from not fully communicating the target changes. Conversely, the lower σ_π , the less agents believe that a change in the target is possible, and stick to their previous inflation expectations. Then the effect on real debt is much larger, since nominal interest rates compensate to little for subsequently higher inflation.²³

5.2 Inflation target process

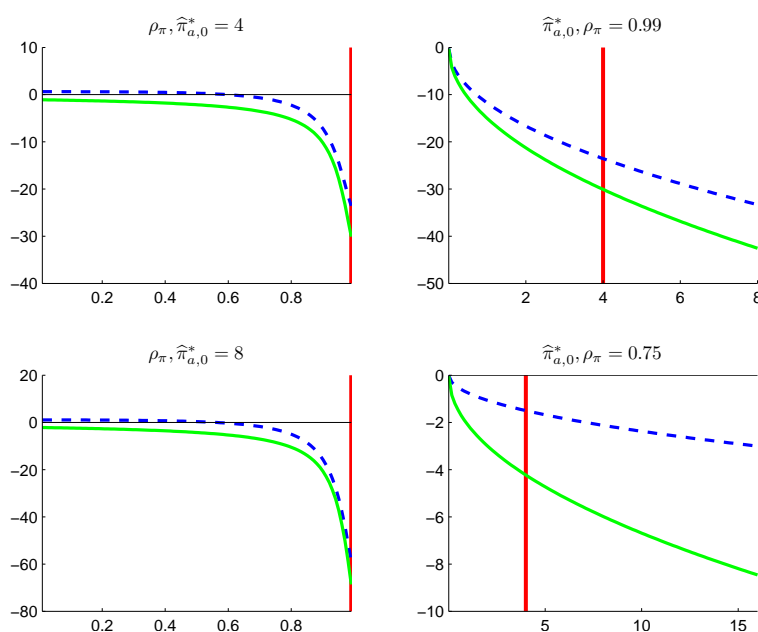
In the baseline scenario, the inflation target is increased by four percentage points and then only slowly reverts to the previous, low-inflation steady state. An observer concerned with high and possibly unsustainable debt may be interested in a scenario where inflation increases only temporarily, but possibly by a larger amount. This would correspond to the proposals made by Rogoff (2008, 2010). Such a setting may be particularly relevant when the average maturity of debt is comparatively low. Then most of the effect on real debt would come from an initial spike in inflation, before interest rates on rolled-over debt can have increased. Thus we analyse here the role of the persistence and the size of the inflation target change. Note that we keep the perceived volatility, σ_π , and thus the speed of learning, constant.

Figure 7 depicts the role of target persistence for given size of the target change, and,

²³Melecky et al. (2009) estimate the volatilities of the inflation target and the policy shock, and interpret a higher perceived volatility as an overestimate by households.

conversely, the role of the target change for different degrees of persistence. The top row starts from the baseline scenario, and varies shock persistence for given baseline shock size in the left panel, and varies shock size for given persistence in the right panel. Varying the persistence ρ_π of the inflation target shows that only for high values is the amount of debt reduction high, be it under full or under imperfect information. A temporary increase in the target, say one with a half-life of 2 years, which implies a value of $\rho_\pi = 0.917$, would lead to a debt reduction of ten percent. A half-life of 1 year implies $\rho_\pi = 0.84$. Recall that this comes along with an already strong increase of inflation from a steady-state target of 2 percent to 6 percent annualized.

Fig. 7 - INFLATION TARGET PROCESS



Notes: Difference in debt reduction $DM(40)$ as a function of the parameter on top of each panel. Keeping the other parameters fixed, that parameter is varied in the reported range. The vertical solid bar indicates its baseline value. The dashed line and the solid line respectively depict the full information and the learning case.

For a persistent target change, increasing the size of the shock obviously increases the impact on real government debt. For an increase from 2 percent to 10 percent, the reduction of debt under full information goes beyond 30 percent of the additional debt

accrued the debt shock, and under imperfect information is even higher than 40 percent. Thus if a government or monetary authority were to give up the low inflationary stance of the last decades, higher inflation may lead to a reduction in real debt after ten years.

The two bottom panels show the corresponding effects for a higher shock size, and for a lower shock persistence. Raising the shock size to 8 percentage points does increase the negative effect on debt substantially if taken to be persistent. Thus the maximum effect is about 70 percent of the additional debt. For lower persistencies, the effect is much weaker. Finally, for a relatively low persistence of $\rho_\pi = 0.75$, even a very strong increase in inflation by 16 percentage points would reduce debt by less than 3 percent under full information, and by a little over 8 percent with learning. Thus it appears that short-lived changes of the inflation target, have comparatively moderate effects on the real value of government debt unless the magnitudes of the change in inflation are very large.

6 Conclusion

This paper investigated to what extent and under which conditions higher inflation reduces the real value of public debt. We take the proposals made by Rogoff (2008, 2010) as a starting point, who suggested a two to three year increase in U.S. inflation of about four percentage points, with the aim of alleviating private and public sector balance sheets. To reiterate our main finding obtained in a New Keynesian model with long-term debt and changing inflation targets: to achieve a reduction of about 25 percent of the additional real government debt accrued after the crisis, requires a permanent increase in the targeted inflation rate. In contrast, the proposed temporary changes in inflation have substantially smaller effects.

We conducted our analysis in the context of the U.S. budgetary situation which appears dire by many accounts.²⁴ But our theoretical framework equally applies to any other country. Many developed countries have experiences large increases in government debt, and face various degrees of temptation to increase inflation. This can be easily modeled in our framework by appropriate choice of the average maturity of public debt and the level of of debt. As our analysis suggests, countries with higher average maturi-

²⁴See Congressional Budget Office (2010).

ties stand to gain more strongly, in particular the U.K. As of now, no country appears to have yielded to the temptation.

We leave it to future research and the reader to decide whether the effect of a 25 percent reduction is to be judged large or small relative to the cost paid in terms of persistently higher inflation. To answer this normative question would require embedding a mechanism that assigns costs to particular levels of debt and inflation. Because of our theoretical requirement that monetary policy be neutral in the long-run, implemented through an indexation assumption, there is no explicit cost to any inflation target change other than the distortion in monetary holdings induced by a nominal interest rate above the Friedman rule of zero. Only if we allowed for price dispersion or recurrent price adjustment or information costs, would a cost of higher inflation targets be incorporated. Similarly, debt as such has no particular cost, other than the distortion brought about by the proportional labor tax. Future research ought to shed light on the trade-offs faced by policy-makers. This notwithstanding, we expect the magnitudes and mechanics of debt dynamics presented here to prevail.

In our scenario with imperfect information and learning, the slow learning of the inflation target by the public can be interpreted as a high credibility of the previously established low inflation target. A government would have to weigh the benefits of an increased budget surplus through lower debt servicing with the cost in terms of reputation. Once inflation expectations have adjusted to a higher level, and the public has doubts about the independence of the central bank, future interest rates will incorporate a risk premium for inflation risk. This in turn would affect the government's incentive later on to reduce inflation back to a lower level, as the real value of outstanding debt would be higher than expected. The modeling of the dynamics of reputation, the risk of losing credibility and other game-theoretic aspects is beyond the scope of the paper.

References

- Aizenman, J., Marion, N., 2009. Using Inflation to Erode the U.S. Public Debt. NBER Working Paper 15562.
- Arellano, C., Ramanarayanan, A., 2008. Default and the Maturity Structure in Sovereign Bonds. Federal Reserve Bank of Minneapolis Staff Report 410.
- Blanchard, O., Dell’Ariccia, G., Mauro, P., 2010. Rethinking Macroeconomic Policy. IMF Staff Position Note 10/03.
- Calvo, G., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 383–398.
- Chari, V., Kehoe, P., McGrattan, E., 2000. Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? *Econometrica* 68, 1151–1180.
- Cochrane, J., 2011. Understanding policy in the great recession: Some unpleasant fiscal arithmetic. *European Economic Review* 55, 2–30.
- Darracq-Pariès, M., Moyen, S., 2009. Monetary Policy and Inflationary Shocks Under Imperfect Credibility. Working Paper Series. European Central Bank.
- Davig, T., Leeper, E.M., Walker, T.B., 2011. Inflation and the fiscal limit. *European Economic Review* 55, 31–47.
- Domeij, D., Floden, M., 2006. The labor-supply elasticity and borrowing constraints: Why estimates are biased. *Review of Economic Dynamics* 9, 242–262.
- Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46, 281–313.
- Erceg, C.J., Levin, A.T., 2003. Imperfect credibility and inflation persistence. *Journal of Monetary Economics* 50, 915–944.
- Fève, P., Matheron, J., Sahuc, J.G., 2010. Disinflation shocks in the Eurozone: A DSGE perspective. *Journal of Money, Credit and Banking* 42, 289–323.

- Hall, G.J., Sargent, T.J., 2010. Interest Rate Risk and Other Determinants of Post-WWII U.S. Government Debt/GDP Dynamics. NBER Working Paper 15702.
- Hatchondo, J.C., Martinez, L., 2009. Long-duration bonds and sovereign defaults. *Journal of International Economics* 79, 117–125.
- Leeper, E.M., 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics* 27, 129–147.
- Leith, C., Wren-Lewis, S., 2011. Discretionary policy in a monetary union with sovereign debt. *European Economic Review* 55, 93–117.
- Melecky, M., Palenzuela, D.R., Söderström, U., 2009. Inflation target transparency and the macroeconomy, in: Schmidt-Hebbel, K., Walsh, C.E., Loayza, N. (Eds.), *Monetary Policy under Uncertainty and Learning*. Central Bank of Chile. chapter 10, pp. 371–411.
- Miller, R., 2009. US needs more inflation to speed recovery, say Mankiw, Rogoff, may 19, 2009. bloomberg.com.
- Persson, M., Persson, T., Svensson, L.E.O., 1996. Debt, Cash Flow and Inflation Incentives: A Swedish Example. NBER Working Paper 5772.
- Reinhart, C.M., Rogoff, K.S., 2010. Growth in a Time of Debt. NBER Working Paper 15639.
- Rogoff, K.S., 2008. Inflation is now the lesser evil. Project Syndicate, December. <http://www.project-syndicate.org/commentary/rogoff51/English>.
- Rogoff, K.S., 2010. Why America isn't working. Project Syndicate, December. <http://www.project-syndicate.org/commentary/rogoff72/English>.
- Rudebusch, G.D., Swanson, E.T., 2008. Examining the bond premium puzzle with a dsge model. *Journal of Monetary Economics* 55, 111–126.
- Sims, C.A., 2002. Solving linear rational expectations models. *Computational Economics* 20, 1–20.

Smets, F., Wouters, R., 2007. Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review* 97, 586–606.

US Department of the Treasury, 2009. Treasury borrowing advisory committee report to the secretary of the treasury. <http://www.treas.gov/press/releases/tg254.htm>.