Sovereign risk and the effects of fiscal retrenchment in deep recessions

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Abstract

Government debt levels are rising rapidly. On the one hand, delaying fiscal consolidation may thus lead to a rising sovereign risk premium and affect private sector funding conditions adversely. On the other hand, with monetary policy remaining severely constrained by the zero lower bound on policy rates, early retrenchment may have a strong recessionary impact. We analyze these countervailing consideration within a new Keynesian model featuring a risk-premium channel and show how the short-run effects of fiscal retrenchment depend on the state of the economy and therefore on the timing of its implementation. We find that for most parameterizations an immediate retrenchment tends to reduce economic activity—although spending cuts may raise economic activity if the economy is under severe fiscal and, simultaneously, the zero lower bound is likely to remain a constraint on monetary policy for an extended period.

Keywords: Fiscal consolidation, Monetary policy, Zero lower bound, Risk premium

JEL-Codes: E62, E52, E32

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"Those countries with serious fiscal challenges need to accelerate the pace of consolidation. We welcome the recent announcements by some countries to reduce their deficits in 2010..." (G20 Communiqué, June 2010)

"The right thing, overwhelmingly, is to do things that will reduce spending and/or raise revenue after the economy has recovered ... (Paul Krugman, June 2010)

1 Introduction

The global financial crisis has sent public debt in the industrialized world on a sharply higher trajectory.[1] As a result, the sustainability of public finances has moved into the focus of market concerns, and fiscal consolidation has become a policy priority. Yet the appropriate timing of consolidation remains subject to intense controversy. On the one hand, consolidation efforts may prematurely withdraw support from a still-fragile economic recovery, at a time when monetary policy remains constrained by the zero lower bound (ZLB) on nominal interest rates. On the other hand, delaying consolidation measures for too long may lead to sharply rising risk premia, which in turn weigh down on private demand.

In this paper, we take up the question of when to start fiscal retrenchment in the wake of deep recessions. Specifically, we analyze how the short-run effects of government spending cuts depend on (a) the severity of deep recessions, that is, the expected duration for which the ZLB is binding, and (b) the extent of fiscal strain. Regarding the former, Christiano et al. (2010) and Woodford (2011) among others, have shown that government spending raises output more strongly if monetary policy is constrained by the ZLB.[2] In normal times, the expansionary effect of government spending triggers a rise in inflation and a monetary tightening, which crowds out private demand. Hence, output multipliers are typically moderate. In contrast, when the ZLB on policy rates is binding the central bank will not respond to the positive fiscal impulse; any increase in inflation due to higher government spending translates into lower real interest rates, raising private demand. For the same reason, fiscal retrenchment at the ZLB can be particularly harmful to economic activity, as monetary policy is unable to lower rates in response to falling inflation.

During deep recessions, nonetheless, credible plans for cutting spending in the *future* can

[1] For an analysis of fiscal policy during the global recession see e.g. Benetrix and Lane (2010).

[2] Other important contributions include Eggertsson (2001), Erceg and Lindé (2010), and Devereux (2010).

enhance economic activity, provided that the fiscal retrenchment does not start too early into the economic recovery (see Corsetti et al. 2010). The mechanism relies on the effect of fiscal policy decisions on the path of real interest rates: firms facing nominal rigidities have an incentive, all else equal, to set lower prices ahead of the period in which the government is expected to reduce spending. This, in turn, creates disinflationary pressures which allows the central bank to lower policy rates. If cuts are projected to occur sufficiently far in the future, when the economy is already on the recovery path and the ZLB no longer binding, the anticipated fiscal retrenchment immediately lowers long-term interest rates and thus stimulates activity during the recessionary period.

Yet concerns about fiscal strain and increasing sovereign risk lead to a countervailing consideration. As government revenue declines during the recession and the public debt stock rises, steps to reduce the deficit may be necessary to prevent or at least contain a rise in the risk premium that investors demand for holding the country's sovereign debt. To the extent that fiscal consolidation succeeds in lowering this risk premium, it may also have a stimulating effect on economic activity. A key reason is the close link between public and private financing costs. As emphasized in both International Monetary Fund (2010*a*) and European Central Bank (2010), government bond yields typically set a floor for, or at least have a strong influence on, domestic corporate bond yields. Consequently, a reduction in sovereign risk premia can lower funding costs in the wider economy and stimulate private consumption and investment.

Against this background, we investigate how the short-run effects of spending cuts depend on the state of economy, namely, on the severity of the recession and the extent of fiscal strain. Our analysis is based on a new Keynesian model, which we specify so as to capture key features that have shaped the fiscal policy debate in major industrialized countries since the beginning of the global financial crisis. Specifically, we posit an interest rate spread which may emerge from heterogeneity in the private sector and costly financial intermediation as in Cúrdia and Woodford (2009). This interest rate spread in turn is tightly linked to the sovereign risk premium. Sovereign default risk is assumed to increase in the level of fiscal strain, possibly in a non-linear manner (as in the risk premium specification by García-Cicco et al. (2010)). We are thus in a position to investigate the quantitative importance of a "risk-premium channel" in fiscal policy transmission, that is, the effect of fiscal strain on private sector interest rate spreads.

Overall, we find that the presence of such a channel raises the stakes for the timing of fiscal consolidation. Importantly, however, while the strength of the recession changes the impact for spending cuts irrespectively of whether a risk-premium channel is present, the latter plays out strongly only in case the recession is severe. Key to this result is that sovereign risk becomes relevant for economic activity only to the extent that its effect on interest rate spreads cannot be cushioned by monetary policy, that is, as long the ZLB is an effective constraint. In fact, unless circumstances are particularly extreme, we find that delaying credible fiscal retrenchment until after the economy has exited from the recession is beneficial in terms of stimulating economic activity during the recession.[3]

The text is structured as follows. Section 2 presents empirical evidence with respect to the presence of a risk-premium channel whose role for fiscal policy transmission is analyzed in this paper. Section 3 describes the model economy. Analytical results as well as results from model simulations are reported in section 4 and 5, respectively. Section 6 concludes.

2 Evidence on the sovereign-risk channel

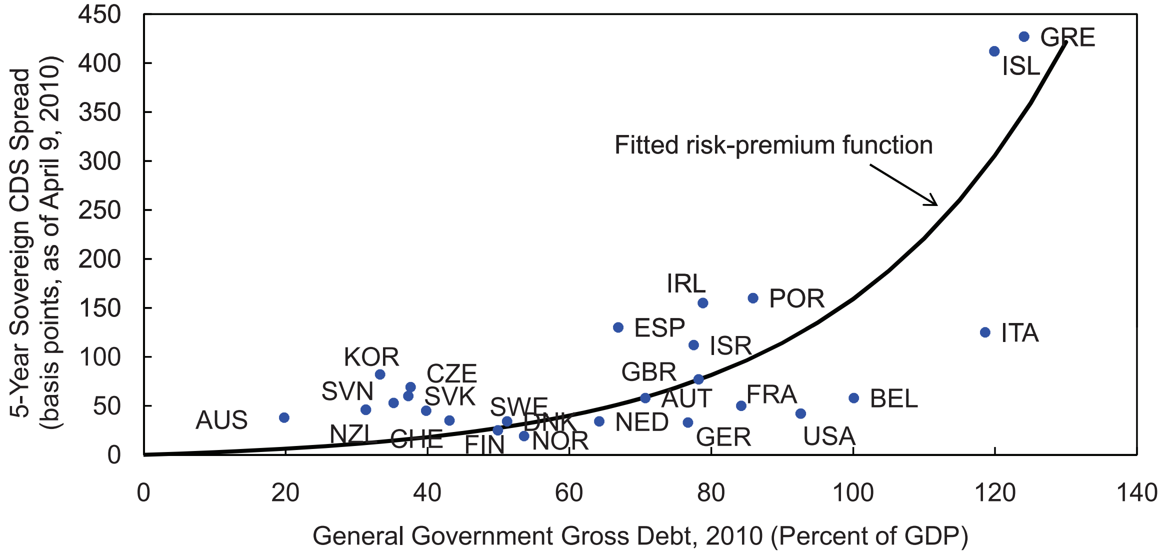
Our model analysis below accounts for the possibility that higher public debt leads to an increased sovereign-risk premium which eventually affects private sector financing conditions. In the following we briefly discuss some evidence on the presence of such a sovereign-risk channel.

Recent developments in sovereign bond markets confirm the notion that a large current and/or projected future stock of public debt is a key indicator of sovereign risk. Figure 1 bears out this relationship. It plots credit default swap spreads of industrialized economies against the level of projected end-2010 gross public debt.[4] For the countries shown in the figure, CDS spreads are systematically higher, the higher the level of projected gross public debt. In fact, the risk premia appear to rise disproportionately as the debt level goes up.

[3] Under these extreme circumstances, we find indeed that government spending cuts may raise output, that is we find non-Keynesian effects of government spending. This possibility has been analyzed in earlier work, among others, by Giavazzi and Pagano (1990), Bertola and Drazen (1993), Sutherland (1997), Perotti (1999), Alesina and Ardagna (2010), Alesina and Perotti (1995), and, more recently, by the IMF 2010 World Economic Outlook (especially chapter 3).

[4] Credit default swaps (CDS) are insurance contracts that cover the repayment risk on an underlying bond. The CDS spread indicates the annual insurance premium to be paid by the buyer. Accordingly, a higher perceived default probability on the underlying bond implies, ceteris paribus, a higher CDS spread.

Figure 1: Sovereign Risk Premia vs. General Government Gross Debt 2010

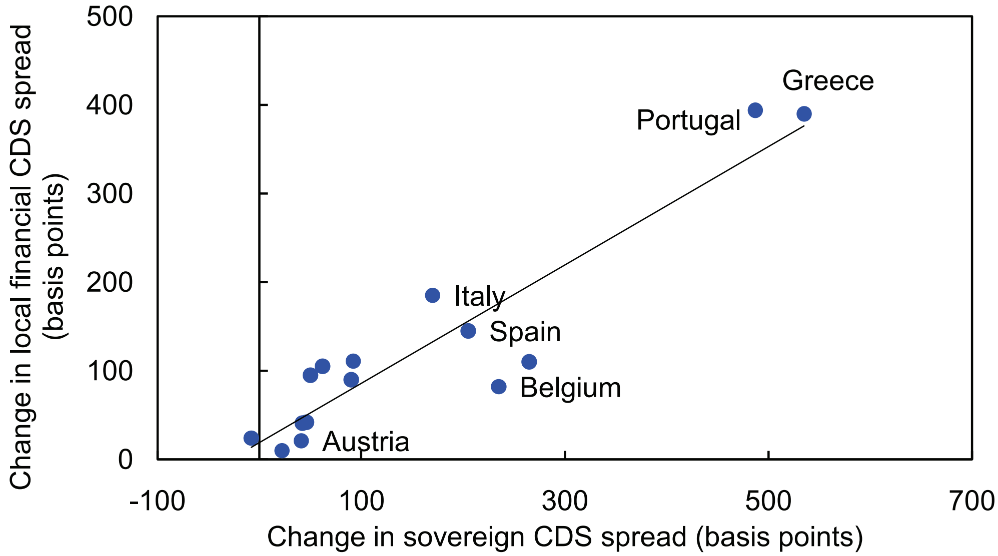


Notes: Source: IMF, Global Financial Stability Report, April 2010. Note: Excludes Japan.

The finding of a systematic relationship between fiscal variables and yields on government bonds is by no means new or specific to the recent turmoil in sovereign bond markets. Instead, similar relationships have been documented in a number of papers, including Reinhart and Sack (2000), Ardagna et al. (2007), Baldacci et al. (2008), Haugh et al. (2009), Baldacci and Kumar (2010), or Laubach (2009). These studies generally use regression techniques to determine the marginal impact of particular fiscal variables, notably the levels of debt and deficits, on long-term government bond rates. A common finding is that countries with higher debt and deficits face higher financing costs. This is consistent with the notion of increased default risk. Ardagna et al. (2007) explicitly focus on possible nonlinearities in the relationship and find that bond rates rise disproportionately for very high levels of debt, as suggested by the previous figures. Needless to say, actual financing costs appear to be affected by a range of additional factors as well, from which we abstract here. Nonetheless, high current and/or projected debt is consistently found to be a key determinant of government financing costs.[5] Long-term government bond yields, in turn, are an important benchmark for broader financing

[5] Borgy et al. (2011) estimate an arbitrage-free affine term structure model of potentially defaultable sovereign bonds on a panel of euro area countries. Allowing for a regime shift in the dynamics of yield spreads during early 2008, they find a significant relationship between debt service (relative to tax revenues) and yield spreads.

Figure 2: Comovement Sovereign and Local Financial CDS Spreads, 10/2009-06/2010



Notes: Source: IMF, Global Financial Stability Report, April 2010, Figure 4.

conditions in a given country. This is prominently embedded in the notion of a "sovereign ceiling." In a strict interpretation, this notion posits that no debtor in a given country can have a better credit quality than the government, a primary reason being the latter's capacity to extract private sector resources through taxation. In reality, some authors, including Durbin and Ng (2005), have documented exceptions to this rule, notably for firms with substantial export earnings or close links to a foreign firm. Even then, however, sovereign and corporate bond yields comove significantly (see, for instance, the literature review in Cavallo and Valenzuela 2007).

This observation is in line with the recent experience. As figure 2 shows, the risk premium on European banks rose nearly one-for-one with the risk premia of the corresponding sovereigns. As such, weak fiscal positions can affect borrowing conditions in the economy as a whole—notably in economies were households and corporations rely heavily on financial intermediaries for access to credit. However, in our analysis below we abstract default risk of financial intermediaries. Instead, we assume that interest rate spreads paid by private borrowers increase directly with the perceived sovereign risk. The IMF 2011 Global Financial Stability Report provides evidence for a positive relationship between the sovereign risk premium and the risk premium faced by non-financial corporations.

3 The model

We analyze the effects of fiscal retrenchment within a variant of the basic new Keynesian model. The model that we use is a special case of Cúrdia and Woodford (2009), augmented by a sovereign risk channel. To simplify the exposition, we defer (a discussion of) the micro-foundations to the appendix and merely summarize the equilibrium conditions in terms of deviations from the steady state (a tilde denotes the deviation of a variable from steady state, a hat denotes a log deviation). The linearized format will allow us to analytically characterize how the effects of spending cuts depends a) on the severity of the recession (that is, how long the economy is expected to remain at the ZLB) and b) on the presence of sovereign risk (which, in turn, is increasing in the fiscal strain experienced by the economy).

Two equations characterize the behavior of the private sector. First, there is the new Keynesian Phillips curve, which relates inflation, $\widehat{\Pi}_t$, to expected inflation and output, $\tilde{y}_t$, and government purchases, $\tilde{g}_t$:

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \kappa_y \tilde{y}_t - \kappa_g \tilde{g}_t, \tag{1}$$

where E_t is the expectations operator. $\beta \in (0,1)$, and κ_g, κ_y are positive constants. Second, there is a dynamic IS-relationship

$$\tilde{y}_t - \tilde{g}_t = E_t \tilde{y}_{t+1} - E_t \tilde{g}_{t+1} - \varrho \left[\widehat{R}_t + \widehat{\Delta}_t + \widetilde{\psi}_t - E_t \widehat{\Pi}_{t+1} \right], \tag{2}$$

where $\widehat{R}_t$ is the short-term interest rate controlled by the central bank (policy rate), $\widetilde{\psi}_t$ is an interest rate spread and $\widehat{\Delta}_t$ is an exogenously determined consumption preference shock. ϱ is a positive constant. Except for the interest rate spread, equation (1) and (2) represent the canonical form of the basic New Keynesian model, see Woodford (2003) and Galí (2008) for a textbook treatment. The interest rate spread emerges if the population is heterogenous so that borrowing and lending takes place in equilibrium and financial intermediation is costly.[6]

In what follows we make two assumptions, such that the interest rate spread is endogenously related to the state of public finances. First, we allow for the possibility that the government defaults on a certain fraction of its debt obligations. In equilibrium, there is thus a risk premium on public debt. Second, we assume that in a state of sovereign default a fraction of

[6] Cúrdia and Woodford (2009) provide an explicit model and Appendix A of the current paper discusses conditions under which equations (1) and (2) emerge as a special case of their model.

private-sector loans will not perform, since the government diverts repayment (see Mendoza and Yue 2010 for a similar assumption). Furthermore, the risk that a loan will not perform in a state of default is orthogonal to the characteristics of borrowers. Financial intermediaries are thus unable to discriminate among borrowers and charge a spread that is directly linked to the probability of sovereign default, as reflected by the sovereign risk premium.

Sovereign default is the more likely the more fiscally strained the economy, that is, the higher the higher the stock of public debt. As in Schabert and van Wijnbergen (2008), we abstract from the distributional effects of sovereign default and assume that there are "appropriate" lump-sum transfers which ensure that ensure actual default does not have redistribution effects in equilibrium. However, because these transfers are not proportional to bond holdings or loan exposure, creditors will nevertheless ask for a higher interest rate whenever a sovereign default is perceived to be more likely.[7] For our baseline scenario we assume that the probability of default increases linearly in the primary budget deficit (net of transfer payments). While this assumption is restrictive, it allows us to characterize the equilibrium allocation analytically.[8] Finally, we assume that taxes are lump-sum, but that tax revenues also depend on the level of economic activity. Under these assumptions, we have that the spread is given by

$$\widetilde{\psi}_t = \xi E_t(\tilde{g}_{t+1} - \chi \tilde{y}_{t+1}). \tag{3}$$

where $0 \leq \chi < 1$ measures the elasticity of taxes with respect to output.[9] The parameter $\xi \geq 0$ summarizes a) how strongly the perceived default probability increases in the deficit and b) the extent to which the government diverts the repayment of loans in case of sovereign default. Completing the assumptions on fiscal policy, in our baseline scenario we assume that government spending is determined exogenously.

Finally, regarding monetary policy, we postulate a simple feedback rule which prescribes an

[7] We provide a more detailed discussion of these assumptions in appendix B. Given that actual default is neutral, we do not have to take a stand as to what decision leads to default. In this regard the literature has pursued two distinct approaches. First, Arellano (2008) and others, following Eaton and Gersovitz (1981) have modeled default as a strategic decision of the sovereign which balances the gains form foregone repayment against the costs of exclusion from international credit markets. Second, and more recently, Bi (2010) and Juessen et al. (2011), consider default as the consequence of the government's inability to raise the funds which are necessary to honor its debt obligations.

[8] In Section 5 below we report results from model simulations which illustrate the robustness of our results with respect to an alternative assumption whereby the probability of default, and therefore the interest rate spread, is related to the level of public debt.

[9] In addition, we assume that there is a lump-sum tax component that responds mildly to the level of government debt but that does not affect the interest rate spread, see appendix B.

adjustment of the policy rate in response to inflation and the interest rate spread, provided that the zero lower bound (ZLB) does not prevent a further reduction:

$$\widehat{R_t} = \max\{\phi_\pi \widehat{\Pi}_t - \alpha\tilde{\psi}_t, -\log(R)\}. \tag{4}$$

Here $\alpha \in (0, 1]$ stands for an attempt by the central bank to sterilize the effect of the sovereign risk premium on economic outcomes. Also, $\phi_\Pi > 1$, so that determinacy is ensured when the economy is not at the ZLB; and R marks the steady-state value of the central bank's policy rate.

We consider a deep recession scenario triggered by $\widehat{\Delta}_t$ which induces a sizeable decline in private expenditure and pushes the policy rate to the ZLB. For this shock we assume a Markov-structure, as in Christiano et al. (2010) and Woodford (2011): it persists in the next period period with probability $\mu \in [0, 1)$. Given that there are no endogenous state variables, the expected duration of the recession, that is, the expected length of the ZLB episode, is given by $1/(1 - \mu)$. Once the shock ceases to persist, the economy immediately reverts back to the steady state.

4 The effects of fiscal retrenchment

We now analyze in some detail how the effects government spending cuts on current economic activity depend on the state of the economy. In a first step, we analyze the equilibrium properties of the model in the presence of a sovereign-risk channel and—to highlight an important transmission mechanism in the model—how they may depend on an endogenous fiscal response to the state of the economy.

4.1 Equilibrium determinacy and endogenous spending cuts

In our baseline scenario the level of government spending is determined exogenously. For this case, we find that the presence of a sovereign-risk channel alters the determinacy properties of the model while the ZLB is binding. In the following, we establish restrictions on parameters which ensure that the equilibrium is (locally) determinate.[10]

[10] Here we focus on local determinacy once the economy has reached the lower bound. Another strand of the literature examines global determinacy in the New Keynesian model and is concerned with preventing the economy to fall into a liquidity-trap in the first place. Benhabib et al. (2002) for example propose switching

Proposition 1 *In the economy summarized by equations* (1) – (4), *let* $\widehat{\Delta}_t$ *take on a positive value* $\widehat{\Delta} > 0$ *in period zero, and remain such with probability* μ *in each subsequent period, until it reverts to* $\widehat{\Delta}_t = 0$ *forever. Furthermore, let the value of* $\widehat{\Delta}$ *be large enough that the lower bound is binding initially. There is a unique bounded equilibrium if and only if*

$$a) \quad a < 1/(\beta\mu), \quad \text{and} \quad b) \quad (1 - \beta\mu)(1 - a) > \mu\varrho\kappa_y,$$

where $a := \mu + \mu\xi\chi\varrho$ *and* $\kappa_y := \kappa[\omega + 1/\varrho]$.

Proof. See Appendix C. ■

In the absence of an endogenous risk premium, $\xi = 0$, as in Christiano et al. (2010) and Woodford (2011), condition a) is always satisfied. So there will be a unique bounded equilibrium if and only if condition b) holds, that is if $(1 - \beta\mu)(1 - \mu) > \mu\varrho\kappa_y$. The previous literature has shown that the set of "fundamental" parameters for which this condition holds is the larger (i) the less persistent the lower bound situation (in our parameterization, the smaller μ), (ii) the lower the interest-sensitivity of demand (the smaller ϱ) and (iii) the flatter the Phillips curve (the smaller κ_y). Relative to these findings, our analysis shows that the range of parameters for which the equilibrium is determinate actually shrinks in the presence of a sovereign-risk channel. Namely, with $\xi > 0$, condition a) is violated if either the interest rate spread is sufficiently responsive to the deficit or if the tax revenue is sufficiently responsive to output (χ is large enough). Note that the same parameters are also key determinants for whether condition b) is satisfied.

It is instructive to contrast this result for the baseline scenario with a situation where government spending adjusts endogenously, possibly to contain sovereign risk, while the economy is at the ZLB. The following proposition summarizes the conditions for the existence of a unique bounded equilibrium.

Proposition 2 *In the economy specified in Proposition 1, let government spending* $\tilde{g}_t$ *take on a value of* $\tilde{g}_t = \varphi\tilde{y}_t$, *when the economy is at the ZLB,* $\tilde{g}_t = 0$. *otherwise. Suppose further*

to non-Ricardian fiscal policy. These mechanisms will rule out liquidity traps by making the low-inflation steady state fiscally unsustainable. Mertens and Ravn (2010) study the efficacy of fiscal policy in belief-driven equilibria.

that $\varphi < 1$. *Define* $a^* := \mu + \mu\xi\chi^*\varrho^*$; $\kappa_y^* = \kappa_y - \varphi\kappa_g$; $\chi^* := \chi - \varphi$, *and* $\varrho^* = \varrho/(1-\varphi)$. *There exists a unique bounded equilibrium if and only if:*

1. *with* $a^* > 0$

$$a)\quad a^* < 1/(\beta\mu), \quad \textit{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\varrho^*\kappa_y^*,$$

$$[\ \textit{and if}\ \varphi > 1], \quad c)\quad (1+\beta\mu)(1+a^*) > -\mu\varrho^*\kappa_y^*$$

2. *with* $a^* < 0$:

$$a)\quad (1+\beta\mu)(1+a^*) > -\mu\varrho^*\kappa_y^* \quad \textit{and} \quad b)\quad (1-\beta\mu)(1-a^*) > \mu\varrho^*\kappa_y^*.$$

Proof. See Appendix C. ■

To appreciate the implications, consider first the possibility that there is no sovereign-risk channel ($\xi = 0$). In this case the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ($\varphi < 0$). Interestingly, however, with an endogenous risk premium, the opposite may hold. More precisely, if $\xi > 0$ and if in addition the conditions of Item 1. of Proposition 2 hold, then subject to some limits on the elasticity of taxes with respect to output, namely $\chi < 1 - \frac{\kappa\omega}{(1-\beta\mu)\xi}$, the range of fundamental parameters for which the equilibrium is determinate is at least as large with a pro-cyclical spending response, $\varphi \in (0,1)$, as without any response, and can be larger. Note that the latter case is the more likely the less elastic the tax revenue to the state of the economy (the smaller χ), and the more responsive the interest rate spread to the deficit (the larger ξ). Put differently, a pro-cyclical fiscal stance may reduce the risk of equilibrium indeterminacy (see Appendix C, corollary 5 for details).
This deserves some discussion. While previous work has focused on the case in which sovereign debt and a rising risk premium imply explosive debt, we consider a situation in which debt ultimately will always be stabilized, either through default or through one-off tax measures. In such an environment, we find that an economy with an endogenous risk premium can be prone to belief-driven equilibria. In this environment, spending cuts during recessions may actually help to anchor expectations on a stable equilibrium. To see why, assume that during the ZLB period agents expect some drop in output. A drop in output means less tax

revenue and, in the absence of a fiscal response, higher deficits, and thus, ultimately, a higher interest rate spread. As this rise in the interest rate spread cannot be offset by monetary action at the ZLB, it immediately raises the real interest rate. As a result, expectations of negative output developments can become self-fulfilling in high-debt economies, with a high and rising interest rate spread weighing heavily on output, thus confirming agents' beliefs in equilibrium. In contrast, a pro-cyclical fiscal stance may be sufficient to prevent an adverse expectational shock from being self-fulfilling, because expected spending cuts may offset the expected decline in tax revenues triggered by the output decline.

4.2 Output effects of spending cuts

Keeping the determinacy conditions discussed in the previous section in mind, in the following we focus on the effects of exogenous spending cuts and concentrate on parametrizations that imply a stable and unique equilibrium. As we are eventually interested in understanding how the effect of spending cuts depends on the state of the economy, we proceed in two steps. First, we analyze the effects of spending cuts during the ZLB episode. In doing so, we follow Woodford (2011) and Christiano et al. (2010) and assume that government spending takes on a value that differs from its steady-state level only while the economy is at the ZLB, namely a level of $\tilde{g}_t = g_l$. Otherwise government spending is set to its steady-state level. In a second step, we analyze how a spending cut after the ZLB episode impacts on economic activity during the ZLB episode. In the following we establish our result for the first step.

Proposition 3 *Under the conditions spelled out by Proposition 1 for a unique bounded equilibrium to exist, let government spending take on a value of g_l whenever the lower bound is binding, and zero otherwise. As before, define $a = \mu + \mu\xi\chi\varrho$, and $b = \mu + \mu\varrho\xi$. Then, while the economy is at the ZLB, output is given by*

$$y_l = \vartheta_r(\log(R) - \Delta) + \vartheta_g \; g_l,$$

where

$$\vartheta_r = \frac{\varrho(1-\beta\mu)}{(1-\beta\mu)(1-a) - \mu\varrho\kappa_y} > 0 \tag{5}$$

and

$$\vartheta_g = \frac{(1-\beta\mu)(1-b) - \mu\varrho\kappa_g}{(1-\beta\mu)(1-a) - \mu\varrho\kappa_y}. \tag{6}$$

Proof. See Appendix C. ■

Note that ϑ_g provides a measure for the government spending multiplier on output at the ZLB. It is characterized in more detail by corollary 6 in Appendix C. Specifically, under the determinacy conditions established above, (6) implies that the multiplier is positive if and only if

$$(1-\mu) - \frac{\mu\varrho\kappa_g}{1-\beta\mu} > \mu\xi\varrho. \tag{7}$$

If this condition is satisfied, a spending cut at the ZLB will reduce output. If $\xi = 0$, this will always be the case; moreover, the government spending multiplier will be strictly larger than one, see Christiano et al. (2010) and Woodford (2011). In contrast, if $\xi > 0$, the government spending multiplier at the ZLB may actually be either strongly positive, or negative, such that spending cuts raise output. In our quantitative exploration below, we try to assess how likely such a scenario is in terms of the parameter restrictions which need to be satisfied.

The above results pertain to fiscal retrenchment while the economy is still at the ZLB. We now consider a retrenchment that is designed to take effect only once the economy has left the ZLB. As discussed in Corsetti et al. (2010), fiscal consolidation some time in the future does reduce demand and inflation contemporaneously, but may have positive output effects well before it is implemented. In fact, if spending consolidation is implemented when the central bank is no longer constrained by the ZLB, in reaction to its effect on inflation the policy rate (R_t) will fall in both nominal and real terms (recall, $\phi_\pi > 1$). In order to assess this formally, we state the following proposition.

Proposition 4 *In the economy specified in Proposition 1, let $\tilde{g}_t$ take on a value of $\tilde{g}_t = 0$ whenever the ZLB is binding. Once the ZLB ceases to bind, $\tilde{g}_t = g_a < 0$, in the first period, and subsequently with probability $\nu \in [0,1)$. Otherwise $\tilde{g}_t = 0$ forever. Assuming that the conditions for determinacy are satisfied, output while at the ZLB is given by*

$$\begin{aligned} y_l \;=\; \tfrac{1}{d}\big[\; & \varrho(1-\beta\mu)[\log(R)-\Delta] + (1-\mu)(1-\beta\mu)(1+\varrho\xi\chi)(y_a - g_a) + \varrho(1-\mu)\pi_a \\ & - (1-\mu)(1-\beta\mu)\varrho\xi(1-\chi)g_a \; \big], \end{aligned} \tag{8}$$

where $d = (1-\beta\mu)[1-a] - \varrho\mu\kappa_y$, $a := \mu + \mu\xi\chi\varrho$ *as in Proposition 1, and* y_a *and* π_a *denote, respectively, output and inflation in the austerity period, equal to*

$$y_a = \frac{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_g}{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_y} g_a, \tag{9}$$

and

$$\pi_a = \frac{(1-\nu)(\kappa_y - \kappa_g)}{(1-\nu)(1-\beta\nu) + \varrho(\phi_\pi - \nu)\kappa_y} g_a. \tag{10}$$

Proof. See Appendix C. ■

Here we are primarily interested in the output effects during the ZLB period, corollary 7 in Appendix C provides a detailed characterization. In the absence of a risk-premium channel, the corollary shows that future spending cuts raise output today unless too much of the retrenchment is expected to occur too close after the exit from the ZLB, i.e., future retrenchment efforts need to be sufficiently persistent; compare Corsetti et al. (2010) and Woodford (2011).[11] Moreover, given that future spending cuts have an expansionary effect on current activity in the absence of a risk-premium channel, the corollary shows that the expansionary effect will be unambiguously magnified in case $\xi > 0$. In other words, a delayed implementation of spending cuts can have a strong positive effect on output while the economy is at the ZLB, and particularly so if the initial fiscal situation is bleak.

4.3 Quantitative illustration

In the following we fix a number of parameters at conventional values and illustrate the quantitative dimension of our theoretical results. In order to do so, we consider a range of values for the parameters that capture those features of the economic environment of particular importance for the fiscal transmission mechanism. First, regarding μ we consider

[11] The scenario that is described in Proposition 4 strictly speaking mixes two different elements of a retrenchment, namely the timing of it and its persistence. We have computed an alternative scenario which disentangles these two elements. The scenario assumed that some time after the economy had left the ZLB there is a period of retrenchment with persistence ν. The timing of the start of the retrenchment was random, however, and retrenchment would start with probability $1-\gamma$, $\gamma \in [0,1)$ in any given period after the ZLB had ceased to bind. The results were, qualitatively similar to those displayed here: future retrenchment had a positive effect on output while at the ZLB if retrenchment was expected not to start too early after the end of the ZLB phase (nor too late). The formulae in that scenario were unwieldy, however, so we rather report the simpler case given by Proposition 4.

values that imply an expected duration of the ZLB episode between 4 and 8 quarters (the expected duration is given by $1/(1-\mu)$).

Second, we obtain a range of plausible values for ξ on the basis of the evidence discussed in section 2, notably the evidence that the sovereign risk premium increases disproportionately in the level of public debt. To formalize this empirical relationship in a stylized manner, we postulate the nonlinear functional form[12]

$$rp_t = 1/100 \left(\exp\left[\left(\frac{E_t b_{t+1}}{4y} - \psi_1 \right) / \psi_2 \right] + \psi_3 \right), \tag{11}$$

where the parameters values $\psi_1 = 1.28$, $\psi_2 = 0.32$, and $\psi_3 = -0.02$ minimize the sum of squared deviations between this specification and the data shown in figure 1.

However, in the baseline scenario of the model we assume that the sovereign risk premium depends on the expected deficit rather than on the level of debt. To quantify ξ, we now assume that the sovereign risk premium and the interest rate spread move one-for-one. Let $\xi^{\text{one period}}(\cdot)$ be the slope of the risk premium with respect to the deficit at a specific debt level, evaluated in steady state. So $\xi^{\text{one period}} = \frac{1}{400\psi_2 y} \exp[(b/(4y) - \psi_1)/\psi_2]$. The first column of Table 1 reports the resulting values for $\xi^{\text{one period}}$ for alternative debt levels, which do appear fairly small. However, the values for $\xi^{\text{one period}}$ are likely to understate the size of the response of the interest rate spread to the fiscal situation. In particular, while the computations so far were based on the assumption that the interest rate spread depends on the current deficit only, an appropriate mapping from the slope into the simplified model environment needs to take into account the horizon over which deficits accumulate. The following expression is meant to capture this effect: $\xi = \xi^{oneperiod} \frac{2-\mu}{1-\mu}$.[13] The right panel of Table 1 reports the corresponding values of ξ for different initial debt levels if the ZLB has an expected duration of six, seven or eight quarters. These calculations suggest that a value of ξ of about 0.1 cannot

[12] See García-Cicco et al. (2010) for a similar specification in an open economy context.

[13] The derivation of the formula is as follows. Focus on the interest rate spread in the IS curve, neglecting other terms. This gives a relationship of $\tilde{y}_t = \mu E_t \tilde{y}_{t+1} - \mu \varrho \xi^{\text{one period}} E_t \left\{ \tilde{b}_{t+1} \right\}$. Iterating this forward gives $\tilde{y}_t = -\varrho \xi^{\text{one period}} E_t(\mu \tilde{b}_{t+1} + \mu^2 \tilde{b}_{t+2} + ...)$. Abstracting from effects on the valuation of debt, $\tilde{b}_t$ is the sum of an initial debt level and accumulated deficits. Abstract further from the initial debt level. The impact of additional deficits is $\tilde{y}_t = -\varrho \xi^{\text{one period}} E_t(\mu(\widetilde{deficit}_t + \widetilde{deficit}_{t+1}) + \mu^2(\widetilde{deficit}_t + \widetilde{deficit}_{t+1} + \widetilde{deficit}_{t+2}) + ...)$. Our calculations above result in the same deficit in every period at the lower bound, labeled $deficit_l$. Then, $y_l = -\varrho \xi^{\text{one period}} \mu \frac{2-\mu}{(1-\mu)^2} \widetilde{deficit}$. The current setup in the analytical model gives $y_l = \mu y_l - \varrho \xi \mu deficit_l$, or, equivalently, since deficits are the same in each period that the lower bound is binding, $y_l = -\varrho \xi \mu \frac{1}{1-\mu} deficit_l$. Reconciling the two different expressions for y_l gives $\xi = \xi^{oneperiod} \frac{2-\mu}{1-\mu}$.

Table 1: Quantifying parameter ξ

		ξ depending on average length of lower bound (qtrs)		
debt/GDP	$\xi^{\text{one period}}$	6	7	8
60 percent	.001	.007	.007	.008
90 percent	.002	.017	.019	.022
110 percent	.004	.031	.036	.040
140 percent	.012	.081	.093	.104

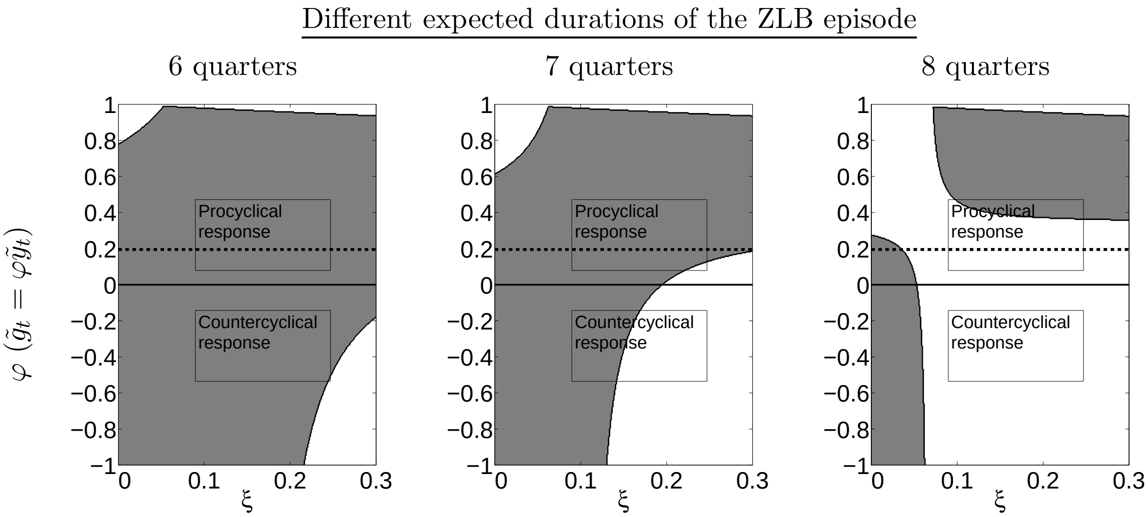
Notes: The table presents estimates for the slope of the risk premium with respect to the deficit, ξ, for different average lengths of the lower bound situation and for different debt/GDP ratios. The entries in the columns on the right are based on the formula $\xi = \xi^t extoneperiod\frac{2-\mu}{1-\mu}$ that is explained in detail in the main text.

be ruled out if the initial level of debt is high and the recessionary shock is persistent. Before turning to a quantitative analysis of the role of these parameters, we briefly discuss the values assumed for the remaining parameters on the basis of observations for U.S. data. A time period in the model is one quarter. The steady-state level of government spending (consumption and investment) relative to the size of GDP is $\overline{g} = 0.19$. The inflation target, $\overline{\Pi}$, is set to an annualized rate of 2 percent. The steady-state level for the central bank's target interest rate, R, is 4.6 percent annualized. These values are broadly in line with U.S. averages over the last 20 years and imply a value for the time-discount factor of $\beta = 0.9937$. The coefficient on inflation in the Taylor rule is $\phi_\Pi = 1.5$. We set $\alpha = 1$, such that the central bank normally is able to neutralize the effect of sovereign debt on economic activity. The parameters κ_y and κ_g are given by $\kappa\,[\omega + 1/\varrho]$ and κ/ϱ, respectively, where $\kappa = (1-\beta\theta)(1-\theta)/\theta$ and $\varrho = (1-g)/\sigma$. For our baseline scenario, we assume $\sigma = 1$, corresponding to log-utility. We set $\omega = 1/1.9$, in line with the arguments provided by Hall (2009) regarding plausible values for the Frisch elasticity of labor supply. θ measures the degree of price stickiness: we set it to 0.9, a value which implies a plausible slope of the Phillips curve.[14] We normalize steady-state output to unity (by appropriately choosing z) and set the value of κ_n so as to target steady-state employment of $n = 1/3$. Finally, estimates provided by the OECD for the output responsiveness of the government budget suggest the parametrization $\chi = 0.34$.[15]

[14]Specifically, our parametrization implies a slope coefficient of $\kappa = 0.012$. Galí and Gertler (1999) report estimates for the slope of the Philips curve, given by $(1-\beta\theta)(1-\theta)/\theta$, in the range between 0.007 and 0.047. More recently, Altig et al. (2010) report an estimate of 0.014.

[15]See Girouard and André (2005); to obtain these estimates, the OECD follows a disaggregated approach, distinguishing four sources of tax revenues: personal income tax, social security contributions, corporate income tax and indirect taxes; in addition the estimates take into account unemployment-related transfers. For all five

Figure 3: Determinacy regions with endogenous response of government spending



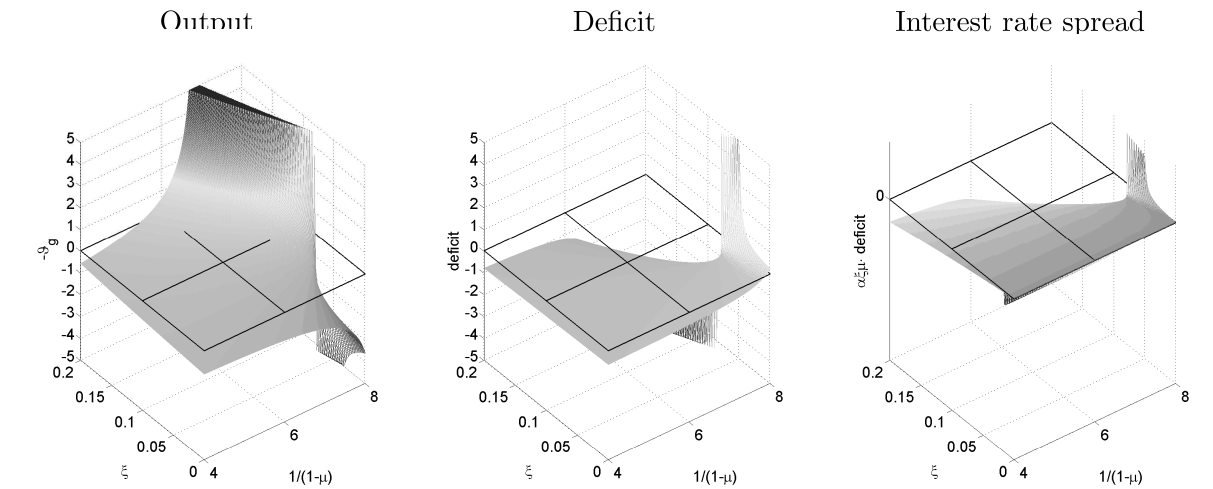
Notes: Determinacy regions with endogenous response of government spending to economic activity during a deep recession. Grey areas mark parameterizations that imply determinacy. y-axis: response of government spending to output, φ ($\tilde{g}_t = \varphi\tilde{y}_t$). x-axis: response of the interest rate spread to the deficit, ξ. From left to right: ZLB is expected to bind for 6, 7, or 8 quarters (or, $\mu = 5/6, 6/7, 7/8$).

For the baseline parameterization, figure 3 illustrates the results of propositions 1 and 2. Each panel of the figure contains results for a different value of μ implying, from left to right, an expected duration of the ZLB episode of 6, 7 and 8 quarters, respectively. For different values of ξ, measured on the horizontal axis, and φ, measured on the vertical axis, we evaluate whether a unique equilibrium exists. Grey areas indicate determinacy regions, while white areas indicate equilibrium indeterminacy. In case the expected duration at the ZLB is long (right panel), we find that there may be no countercyclical spending policy that ensures determinacy.

We now turn to a quantitative assessment of the output effects of spending cuts, illustrating the result stated in proposition 3. Recall that we focus on determinate equilibria in this case. Figure 4 displays the effect of a government spending cut during the ZLB episode for different levels of fiscal strain, measured by alternative values for ξ, and for different assumptions regarding the expected length of the recession, as measured by alternative values of $1/(1-\mu)$. The other parameters underlying these computations remain as laid out earlier. The left

categories, the output elasticity is decomposed into i) the tax-base elasticity of a particular revenue/expenditure type and ii) the output elasticity of the tax/expenditure base in question. These components are quantified on the basis of different estimation strategies and combined to compute the output semi-elasticity of the budget.

Figure 4: Effects of early retrenchment



Notes: The figure shows the effects of a unit cut in government spending for the length of the ZLB episode. Effect on output (left panel), on the deficit (center: negative means deficit falls) and on the interest rate spread (right). On the axes: responsiveness of risk-premium to deficit, ξ, and expected duration of ZLB episode: $1/(1-\mu)$. Only parameterizations that imply determinacy are shown. For better readability, multipliers and deficits were capped at the maximum level indicated in the charts.

panel shows the result for output, the middle panel shows the result for the deficit and the right panel shows the result for the interest rate spread.

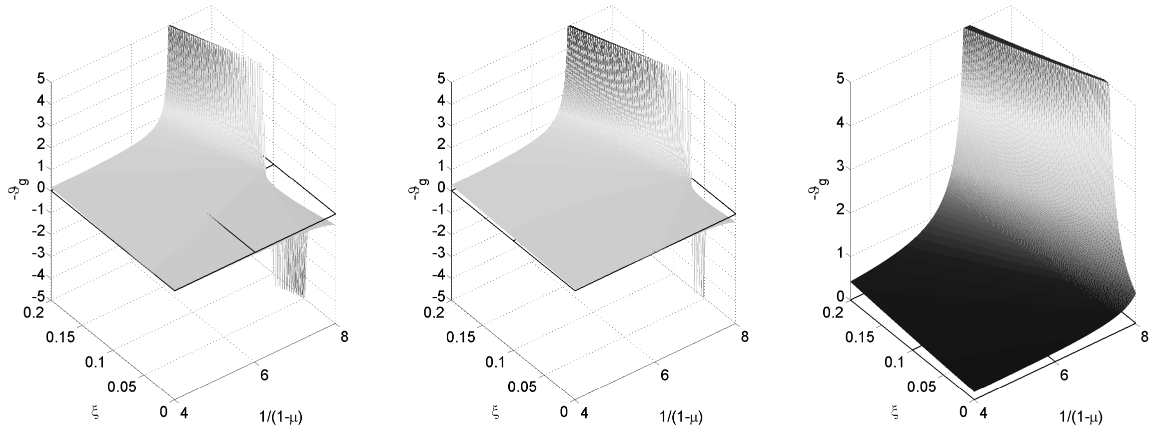
The output response to a spending cut depends on both dimensions under consideration. Consider first the case where $\xi = 0$, that is, a situation when there is no sovereign-risk channel. In this case, a spending cut induces a sizeable decline of output. In fact, for an expected duration of the ZLB episode of eight quarters, the government spending multiplier on output reaches a value of about 3, a result recently stressed in Christiano et al. (2010). Turning to the sovereign risk channel, it is important to emphasize that, as the interest rate spread becomes more responsive to the deficit, that is as ξ takes on bigger values, the multiplier tends to decline, that is, output tends to fall by less. Indeed, for very high values of ξ we find that government spending cuts may actually raise output if the ZLB episode is expected to be relatively long-lasting.

To understand this finding, it is useful to consider the response of the deficit and of the risk premium. If fiscal strain is pervasive, a cut in government spending may reduce the risk premium considerably. This, in turn, lowers the interest rate spread, stimulates private demand and tax revenues—setting in motion a virtuous cycle of a further decline in interest rate spreads, increased economic activity and a further improvement of the fiscal outlook.

Figure 5: Effects of future retrenchment

Expected duration of retrenchment

2 quarters ($\nu = 0.5$) | 10 quarters ($\nu = 0.9$) | 20 quarters ($\nu = 0.95$)



Notes: The figure shows the effect of a unit cut in government spending after the end of the ZLB episode on output at the lower bound. From left to right: different expected persistence of the future retrenchment. From left to right: persistence $\nu = 0.5, 0.9, 0.95$. On the axes: responsiveness of risk-premium to deficit, ξ, and expected duration of ZLB episode: $1/(1-\mu)$. Only parameterizations that imply determinacy are shown. For better readability, multipliers and deficits were capped at the maximum level indicated in the charts.

Note, however, that this outcome requires a fairly high value for the slope of the risk premium, ξ. Also, for this effect to be strong the ZLB episode needs to be expected to be long-lasting, because once the economy has exited the ZLB monetary policy is assumed to sterilize the interest rate spread rendering the risk premium irrelevant at that stage.

Figure 5 provides a graphical illustration of the results stated in proposition 4. We focus on the effects of future spending cuts on current output, that is, on output during the ZLB episode, entertaining alternative values for the persistence of future austerity measures, ν. Again, we display the output effect for different scenarios of fiscal strain, measured by alternative values for ξ, and for different assumptions regarding the expected length of the recession, measured by alternative values of μ. We find that future retrenchment stimulates current activity in almost all scenarios, and particularly so if the retrenchment is sufficiently persistent.

To sum up, our model economy accounts for two main channels through which fiscal consolidation affects the economy. On the one hand, when the policy rate is at the ZLB, any reduction in the sovereign risk premium directly translates into a reduction of the real interest rate faced by private agents. This channel is quantitatively important if a) the ZLB episode is expected to be long-lasting (such that elevated interest rate spreads cannot be accommodated by monetary policy) and b) if fiscal strain is high (such that spreads respond strongly to changes in the fiscal outlook). On the other hand, a reduction in government spending lowers demand and thus exerts a disinflationary effect. If this cannot be accommodated by monetary policy, because the economy is at the ZLB, disinflation translates directly into an increase of real interest rates, with adverse effects on economic activity and, through the fall in tax revenue, on the government budget and the risk premium. An early retrenchment likely reduces the risk premium. Overall, our quantitative assessment suggests that under most circumstances delaying fiscal retrenchment until the economy has exited from the ZLB is conducive to stimulating economic activity during the ZLB episode. That said, if the fiscal strain is exceptionally pervasive *and* the ZLB episode expected to be long-lasting, economic activity can rise in response to an early retrenchment as well.

5 Dynamic analysis

We now revisit the above results in an environment that accounts more accurately for the dynamics of the sovereign-risk channel. For once, we now allow for the sovereign risk premium to depend on the level of debt according to equation (11), rather than on the expected deficit. We also maintain the assumption that the interest rate spread moves one-for-one with changes in the sovereign risk premium. This constitutes a limiting case of particular interest in which fiscal retrenchment may have a particular strong effect on interest rate spreads. This should be borne in mind when interpreting the results below.

In addition, we depart from the simplifying assumption that the expected duration of the ZLB episode is constant. Instead, we envisage a scenario in which the initial debt level is instrumental for the depth of the recession and in which fiscal retrenchment can affect the length of the lower bound episode. Our setup thus allows us to account for a possible state dependence of government spending cuts along two dimensions: the ZLB and the extent of fiscal strain.[16]

[16]At time same time we abstract from any non-linearities inherent in the new Keynesian model itself. Consequently, for the simulation we use the linearized model along with the lower bound constraint, equation (4), and the risk-premium specification in equation (11). That is, we assume that there is a one-to-one relationship between sovereign risk and the risk premium: $\psi_t - \psi = rp_t - rp$ and replace equation (3) with (11). As a result, the slope of the risk premium differs for different actual debt levels while the steady steady will be the same. If we had instead linearized the risk-premium as well, we would have lost the steepness of

5.1 A deep recession and the timing of retrenchment

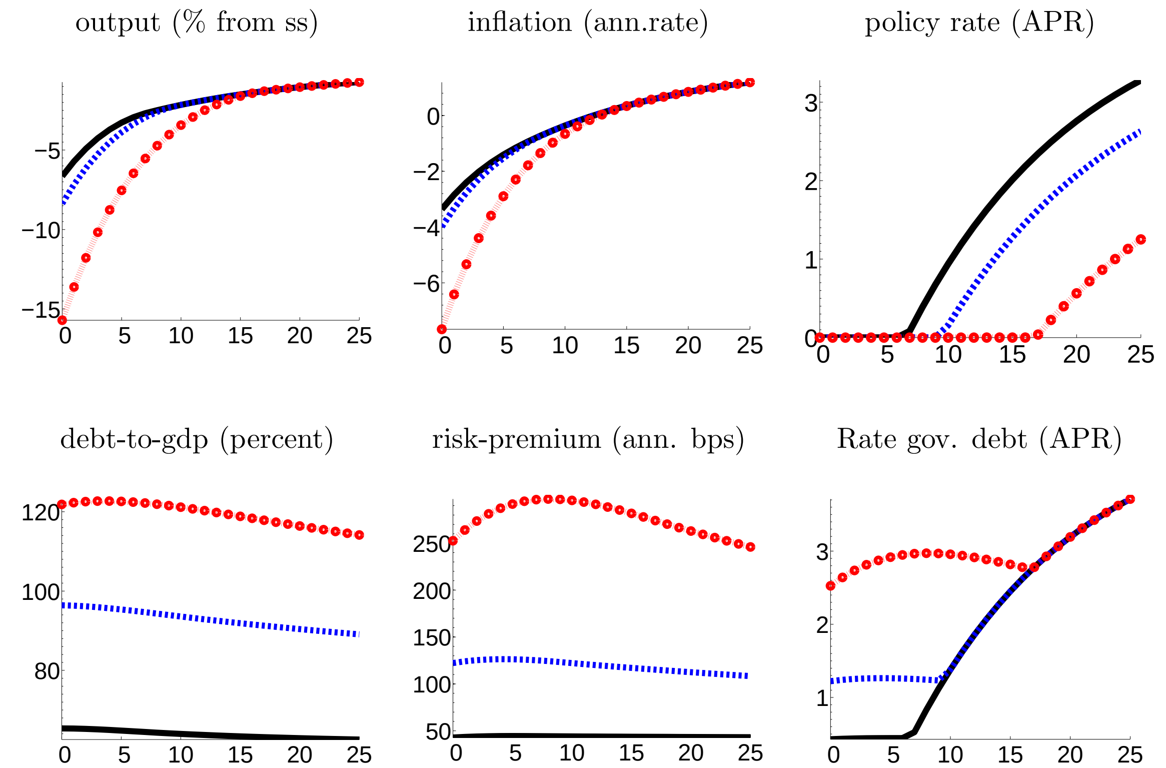
Our goal is to contrast the macroeconomic transmission of early fiscal retrenchment measures with measures that will take effect later in time. To set the stage for our analysis, we begin by studying the impact of a large recessionary shock in the absence of any spending cuts. We postulate that Δ_t follows a first-order autoregressive process. The size of the initial shock and the shock's persistence are calibrated to match the output losses in the U.S. in the last recession. In particular, the Congressional Budget Office (2011) estimates that the output gap reached 6.7 percent in 2009 and that it will still be at a 1.7 percent level in 2014. Taken together, in our baseline economy with a level of debt to GDP of 60 percent (which was the U.S.'s debt level when it entered the recession), we need an initial impact of $\Delta_0 = 0.01075$ and a persistence of $\rho_\Delta = 0.93$ to roughly replicate those values. For the simulations, we also assume that taxes respond to the debt level in a debt-stabilizing way. In particular, we set $\phi_{tax,b} = 0.015$. This response is twice as large as would be required to ensure stable debt dynamics in the absence of adverse movements in the risk premium.

For three different initial levels of government indebtedness, 60 percent of GDP, 90 and 110 percent, Figure 6 shows the evolution of the economy absent fiscal retrenchment measures. In our simulations, not only is the recession the deeper the higher the initial level of debt, but a persistently high risk premium also implies that the time span over which the lower bound remains binding is longer. In the simulations shown here, the ZLB episode is extended by as much as ten quarters; see Figure 6. In addition, the depth of the recession rises disproportionately with the level of debt. The large additional output losses that ensue at higher debt levels clearly underscore the importance of *ex ante* precautionary fiscal prudence in good times. A more timely question, however, is whether *ex post* a high level of indebtedness provides a rationale for early retrenchment.

Starting from the baseline laid out in Figure 6, we thus explore the effect of fiscal retrenchment measures that differ in their timing. So, by way of example, Figure 7 presents the evolution of the economy for an intermediate level of indebtedness (90 percent of GDP). The baseline response for that level of indebtedness is shown as a black solid line. We also plot the effects of different packages of spending cuts to 2 percent of (steady-state) GDP per period. First,

the premium for high debt levels. As is customary, we solve the model economy under perfect foresight using standard techniques.

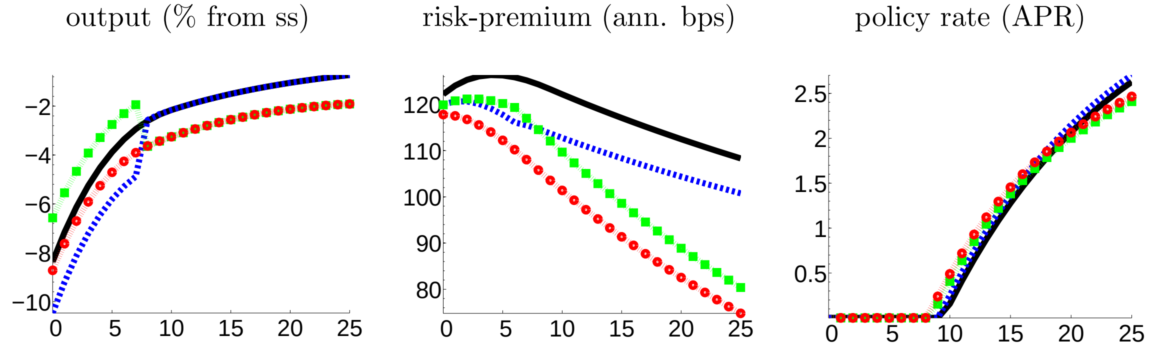
Figure 6: The deep recession without retrenchment



Notes: The figure shows responses to the recessionary shock. The underlying economies are identical but for the initial debt level. Black solid line: 60 percent debt-to-GDP, blue dashed: 90 percent, red dots: 110 percent. Output is expressed in terms of percentage deviations from steady state. Inflation and interest rates are in annualized percentage rates. The debt-to-gdp ratio is reported in percent, the risk premium in basis points annualized.

a package of immediate cuts for two years ("Immediate retrenchment", blue dashed line). Second, a package of spending cuts that starts two years after the initial recessionary impact and that lasts for 10 years ("Delayed retrenchment" green squares). Third, a combination of the two packages ("Persistent retrenchment," so spending cuts start immediately and last for 12 years in total, red circles). Observe, first, that all three austerity packages are effective in reducing the deficit (in line also with Figure 4). As a result, retrenchment reduces the level of sovereign debt and thereby the risk premium. The effect of the spending cuts on output differs quite considerably, however, across the different scenarios. With immediate retrenchment, all of the spending cuts materialize while monetary policy remains severely constrained by the zero lower bound. In the simulations shown, the stimulating effect from a persistent reduction in the risk-premium is dominated: inflation falls in the face of weaker

Figure 7: Retrenchment scenarios at a 90 percent debt level



Notes: The figure shows the response of the economy to a deep recessionary shock and, in addition, to austerity packages. All lines refer to an economy with a 90 percent debt-GDP ratio at the onset of the recession. The black solid shows the baseline without spending cuts. The other lines show austerity packages that cut government spending by 2 percent of GDP in each period. The blue dashed line reports austerity that lasts for two years and is implemented immediately. The green line marked by squares shows an austerity package that is implemented two years in the future and lasts for 40 quarters. The red line marked by circles merges the former two austerity packages. For the scaling of variables, refer to Figure 6.

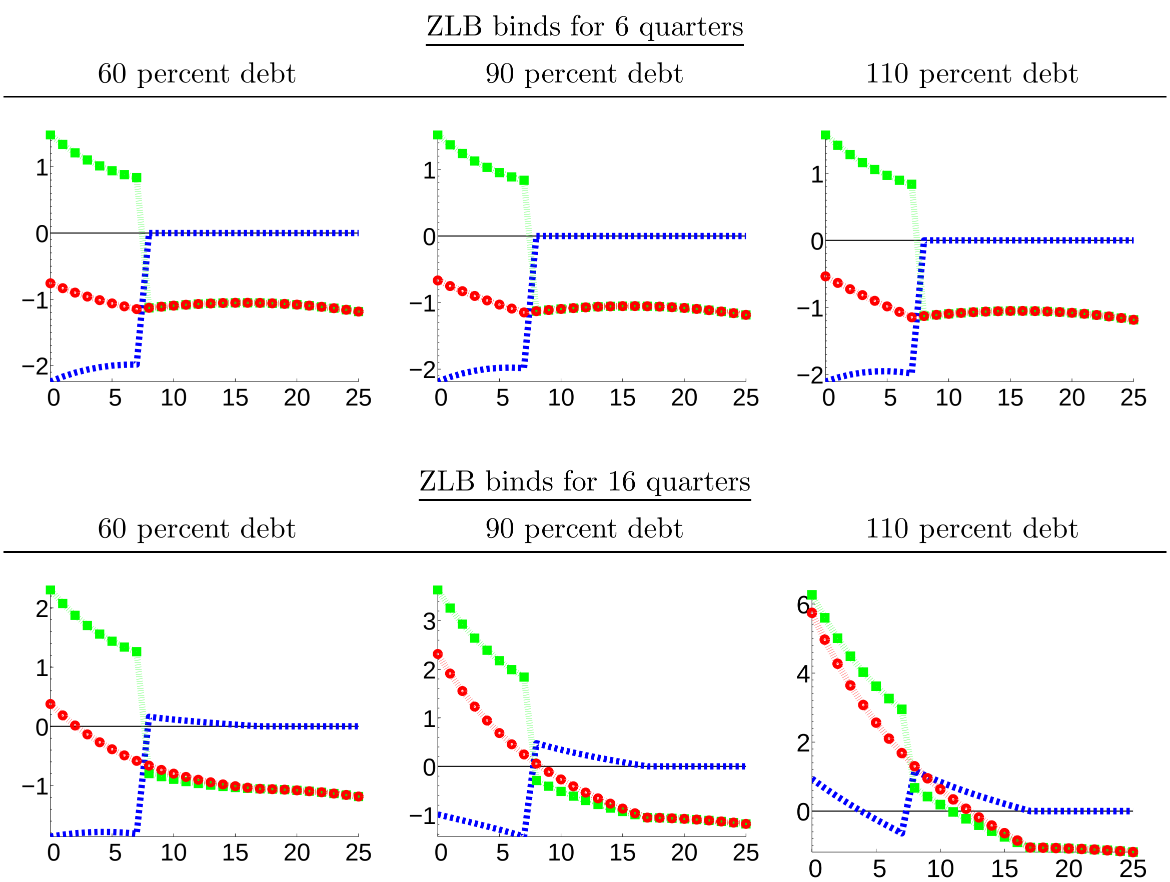
demand so that the long real rate increases. As a result, consumption is crowded out. For a 90 percent debt to GDP ratio, the resulting drop in output therefore is reminiscent of the lower bound discussions in the literature (see, for example, Woodford 2011).

Under delayed retrenchment gains from fiscal forward guidance are possible. In this case spending cuts are implemented when monetary policy can again add to stabilizing the economy by means of lowering the policy rate. The prospect of a lower long-term real rate crowds in consumption, and thus output, in the early periods of the recession, and quite strongly so. This leads to higher tax revenues and implies an immediate reduction in the risk premium, which further stimulates demand. Last, under a persistent retrenchment scenario we have combination of the two previous scenarios. In the simulations shown here, it is most successful in reducing the risk premium (red dotted line). However, consumption is crowded in by less than with medium term austerity alone, which leads to lower economic activity.

5.2 The effects of retrenchment and the state of the economy

The different impact of austerity on demand and the risk premium opens the possibility that the initial debt level may have a strong bearing on the effects of retrenchment, in particular when accounting for the endogenous length of the ZLB episode. For the sake of exposition,

Figure 8: Output effect of spending cuts and the state of the economy



Notes: The effect of the three austerity packages (described in the notes to Figure 7) when, for each initial debt level, the depth of the recession without austerity is calibrated such that it implies that the ZLB will bind until quarter 6 (upper row) and 16 (lower row). Regardless of the austerity package implemented, for this figure, monetary policy is assumed to keep the nominal rate at zero for that time period and not to tighten any earlier.

we disentangle these two dimensions. Figure 8 shows the effect of the austerity packages on output for different debt levels, but fixing the length of the ZLB episode at six (upper panels) and sixteen (lower panels) quarters.[17]

If the lower bound is expected to be binding only for a short period of time (upper panels), the effect of spending cuts hardly varies with the debt level. In addition, for this case, it is hard to make a case for early austerity measures, in line with our findings in the analytical part of the paper. These findings change dramatically, however, if the lower bound is expected to be binding for longer and if, therefore, monetary policy cannot sterilize the effect of the risk

[17]That length replicates the duration of the lower bound with a 60 and 110 percent of debt to GDP ratio in figure 6, respectively. In the underlying computations we re-scale the recessionary shock appropriately.

premium on economic activity for a substantial period of time (lower panels). Overall, we find our earlier results (see figure 4 above) confirmed: immediate retrenchment may have a beneficial effect on economic activity if both fiscal strain is severe and the ZLB is expected to bind for an extended period.

5.3 Further sensitivity analysis

This section assesses the sensitivity of our results with respect to two modifications that could, in principle, shift the incentive for austerity forward in time. We start by assessing what happens if distortionary taxes—rather than lump-sum taxes—are used to finance government debt. In the absence of the lower bound, higher distortionary taxes reduce economic activity. To the extent that an early retrenchment reduces the need for distortionary taxation in the future, one would expect such an early retrenchment to show more positive effects on output than we have reported so far.

Figure 9 assesses this for the 110 percent debt to GDP level.[18] The left panel shows the effect of the early, medium term, and immediate consolidation packages in the baseline model with lump-sum taxation.[19] The second panel shows how distortionary taxation alters the effect of the austerity packages. It does not shift the emphasis toward early austerity. While distortionary taxation does reduce future economic activity, and thus weighs on current demand, in our simulations a second, countervailing, effect is present. Without a retrenchment taxation rises more strongly (with a view toward stabilizing the rising debt burden). As stressed by Eggertsson (2009), to the extent that higher labor taxes raise inflation, at the lower bound this can help stabilize output. Since labor taxes are lower with early austerity, this reduces the stimulating impact of higher taxes while at the ZLB which more than outweighs the positive effects coming from lowering the debt burden. One may argue, however, about the extent to which taxes would actually be increased in a lower bound situation. Since the timing of

[18] For this exercise, we introduce distortionary labor taxes into the model. The tax rate is calibrated to be 35 percent in steady state, in line with the 2006 average U.S. marginal income tax rate reported by Barro and Redlick (2010). Apart from this, the labor tax process replicates the assumptions used for lump-sum taxes above:

$$\tilde{\tau}_t = \widetilde{\tau_{w,t} w_t h_t} = \chi \tilde{y}_t + \phi_b \tilde{b}_{t-1},$$

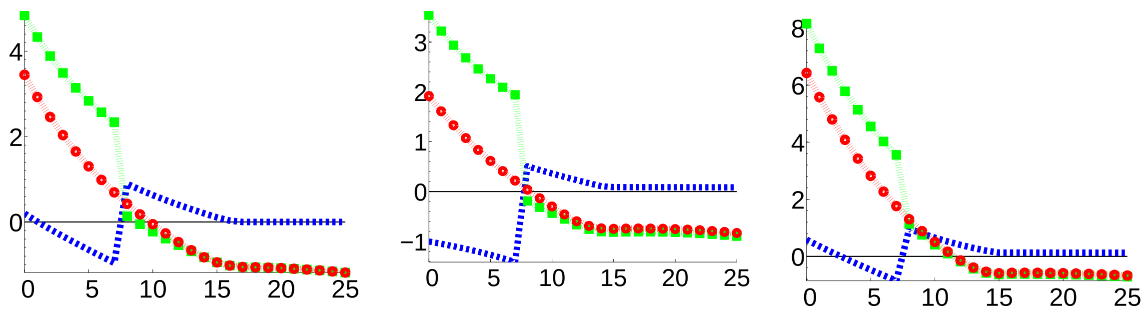
where a tilde marks deviations from steady state.

[19] Figure 9 allows for an endogenous timing of the exit from the lower bound whereas the right-most panel in **??** fixed the ZLB episode at 16 quarters. As a result, under any of the austerity packages shown, in the left-most panel of Figure 9 the economy exits the ZLB earlier than in **??**. The earlier exit from the ZLB renders the retrenchment somewhat lets stimulative, as can be seen in the smaller output effects.

Figure 9: Sensitivity to assumption about taxation

Effects of austerity on output (110 percent debt to GDP)

lump-sum taxes | only labor taxes | labor tax after ZLB



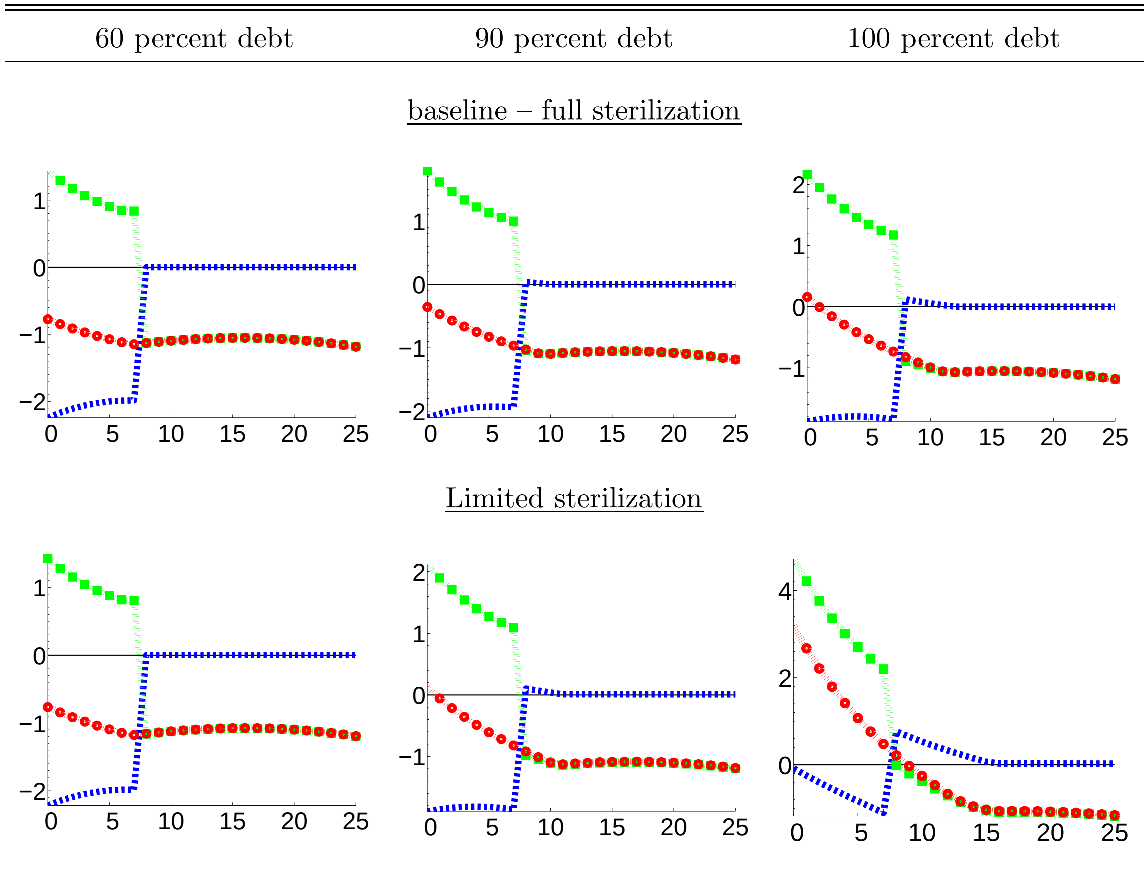
Notes: The figure shows the effect on output of the three austerity packages described in Figure 7 for a 110 percent initial debt-to-GDP level. The left panel reports the effects in the baseline with lump-sum taxes. The center panel reports the effect of austerity when, instead of lump-sum taxes, only labor taxes are used to stabilize the debt. The right panel shows the case where beyond quarter 16, only labor taxes are used to stabilize the debt. The labor tax rate is assumed to remain at its steady-state level for the first 16 quarters.

taxation is important for the results, the right-most panel of the figure purges the effects shown from the effects of labor taxes at the lower bound. If labor taxes rise only after the lower bound has ceased to bind, the case for austerity becomes stronger with distortionary taxation than without. Most of the shift is witnessed for delayed retrenchment, but also early austerity becomes less detrimental.

As a final sensitivity check, we consider the sensitivity of our results with regard to the extent to which the central bank will, in fact, be capable of sterilizing sovereign risk. In particular, our baseline simulations have assumed that once out of the lower bound the central bank can set interest rates in order to prevent the sovereign spread from affecting economic activity. This may be too benign an assumption. Figure 10 therefore assesses the effect of spending cuts if the central bank does not, or cannot, fully neutralize the sovereign spread.

Comparing the simulation results in figure 10 with those in the earlier figures suggests that, quite naturally, this is immaterial for the responses at low, say 60 percent of GDP, debt levels, and thus a low sensitivity of the risk premium with respect to the fiscal balance. However, for higher debt levels the outcomes of retrenchment are much more favorable if the central bank cannot perfectly sterilize the sovereign risk premium in the future. Note that, for this case, retrenchment bears more favorable effects on output across the board, regardless of the

Figure 10: Effects of austerity on output when CB cannot fully neutralize risk premium



Notes: The effect of the three austerity packages (described in Figure 7), when the central bank does not fully absorb the risk premium even once the economy has left the lower bound. Shown is the case in which the central bank absorbs only 80 percent of the risk premium ($\alpha = 0.8$).

specifics of the timing.

6 Conclusion

Given the sharp deterioration of public finances in the wake of the global financial crisis, most industrialized countries are facing a period of significant fiscal retrenchment, including sizeable spending cuts. Starting from this observation we have analyzed how the economic impact of such spending cuts depends on the state of the economy and thus on the timing of their implementation. The question has particular relevance in the wake of deep recessions, when monetary policy is constrained by the zero lower bound (ZLB) on nominal interest rates. Under these circumstances, delaying fiscal retrenchment until monetary policy is no longer constrained may be beneficial for economic activity, since the disinflationary effect of the fiscal retrenchment could then be offset by accommodative monetary policy. In anticipation of such fiscal-monetary interaction, real long-term interest rates would fall prior to the actual spending cuts, supporting demand during the initial recession period. This mechanism is discussed in detail by Corsetti et al. (2010).

In the present paper, we enrich the analysis by accounting for a sovereign-risk channel. Consistent with the empirical evidence, we assume that the sovereign risk premium rises as a function of the stock of public debt and that private sector funding costs increase simultaneously. All else equal, early fiscal consolidation may thus have a positive effect on aggregate demand and output to the extent that it reduces the real interest rate faced by private borrowers. The contribution of this paper is to evaluate quantitatively, in a new Keynesian model, the importance of this channel.

A key finding of our analysis is that sovereign risk as such has little bearing on the fiscal transmission mechanism. This holds even if fiscal strain is pervasive, because unless monetary policy is severely constrained, it may cushion the effect of sovereign risk on interest rate spreads. We thus find quite generally that delaying fiscal retrenchment is beneficial in term of economic activity while the economy is still in a deep recession. If, however, both fiscal strain is pervasive and the constraint on monetary policy is expected to be relatively long-lived, the effect of government spending cuts on the economy have a distinct non-Keynesian flavor: they may actually boost economic activity while the economy is stuck at the ZLB.

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A Limiting case of Curdia-Woodford model

In the following we discuss conditions under which the new Keynesian Phillips curve and the dynamics IS relationship derived by Cúrdia and Woodford (2009) simplifies to (1) and (2).

Regarding the dynamic IS relation, substituting for i_t^{av} using their equation and for Ω_t (using eq. 2.10, eq. 2.4 and eq. 2.8 in CW), we obtain the interest rate term

$$i_t^d + (\pi_b + s_\Omega)\omega_t + (s_\Omega(\hat{\delta} - 1) - \psi_\Omega)\Omega_{t+1}.$$

Note that

$$\begin{aligned} \hat{\delta} &= \chi_b + \chi_s - 1, \\ \psi_\Omega &= \pi_b(1 - \chi_b) - (1 - \pi_b)(1 - \chi_s), \\ \lim_{\delta \to 1} \chi_\tau &= \lim_{\delta \to 1} \beta(1 + \bar{r}^\tau)(\delta + (1 - \delta)\pi_\tau) = \beta(1 + \bar{r}^\tau) \approx 1. \end{aligned}$$

Hence, if $\delta \to 1$, expression (12) can be approximated by

$$i_t^d + (\pi_b + s_\Omega)\omega_t, \tag{12}$$

that is, the interest term in the dynamic IS relationship is the policy rate plus the current spread.

Moreover, if $s_b\sigma_b >> s_s\sigma_s$, s_Ω (given by expression (2.7) in CW) simplifies to

$$s_\Omega = \pi_b\pi_s \frac{s_b\sigma_b - s_s\sigma_s}{\pi_b s_b\sigma_b + \pi_s s_s\sigma_s} = \frac{\pi_b\pi_s s_b\sigma_b}{\pi_b s_b\sigma_b + \pi_s s_s\sigma_s} - \frac{\pi_b\pi_s s_s\sigma_s}{\pi_b s_b\sigma_b + \pi_s s_s\sigma_s} \approx \pi_s.$$

Such that, in this case, (12) further simplifies to $i_t^d + \omega_t$.

In CW, variable Ω_t enters the new Keynesian Phillips curve with coefficient $\xi(s_\Omega + \pi_b - \gamma_b)$, with

$$\gamma_b = \pi_b \left(\frac{\psi\bar{\lambda}^b}{\psi_b\bar{\tilde{\lambda}}} \right)^{1/\nu}.$$

Expression (1.22) of CW evaluated in steady state gives

$$\tilde{\lambda} = \psi \left[\pi_b \left(\frac{\lambda^b}{\psi_b} \right)^{1/\nu} + (1 - \pi_b) \left(\frac{\lambda^s}{\psi_s} \right)^{1/\nu} \right]^\nu.$$

Using that $\lambda^s = \lambda^b/\Omega$ we have

$$\tilde{\lambda} = \psi \left[\pi_b \left(\frac{\lambda^b}{\psi_b} \right)^{1/\nu} + (1-\pi_b) \left(\frac{\lambda^b}{\Omega \psi_s} \right)^{1/\nu} \right]^{\nu} = \lambda^b \psi \left[\pi_b \left(\frac{1}{\psi_b} \right)^{1/\nu} + (1-\pi_b) \left(\frac{1}{\Omega \psi_s} \right)^{1/\nu} \right]^{\nu}$$
$$= \frac{\lambda^b \psi}{\psi_b} \left[\pi_b + (1-\pi_b) \left(\frac{\psi_{bs}}{\Omega} \right)^{1/\nu} \right]^{\nu},$$

where $\psi_{bs} = \psi_b/\psi_s$ is the relative weight of disutility of labor across the two types (it determines hours worked in steady state). This implies

$$\gamma_p = \pi_b \left[\pi_b + (1-\pi_b) \left(\frac{\psi_{bs}}{\Omega} \right)^{1/\nu} \right]^{-1}. \tag{13}$$

So for a given Ω, setting ψ_{bs} determines γ_p.

B Sovereign risk premium and interest rate spreads

We assume that, as in Cúrdia and Woodford (2009), savers hold government debt. However, we allow for the possibility of sovereign default. Moreover, in case of a default the government diverts some of the loans issued by financial intermediaries. As a result, the interest rate spread (between private sector borrowing and lending rates) is tightly linked to the sovereign risk premium. At the same time, we abstract from modeling the redistribution that this default scenario entails. Rather, as in Schabert and van Wijnbergen (2008), we assume that while the government "taxes" owners of debt securities in the case of a sovereign default, it does so in a budget-neutral way. More precisely, there are lump-sum transfers that (in the aggregate) neutralize the budgetary consequences of default for all agents in the economy.[20] In the following, we provide a formal exposition of our setup.

Let b_t be real end-of-period government debt and let $R_{g,t}$ be the interest on that debt. The government budget constraint is given by

$$b_t = (1-d_t)\frac{b_{t-1}R_{g,t-1}}{\Pi_t} + g_t - rev_t, \tag{14}$$

where, $d_t = 0$ in states in which default does not occur, and $d_t \in (0,1)$ in states of default.

[20] Note that there measures are not proportional to an individual's bond holdings, however. As a result individual creditors ask for a premium in equilibrium.

Regarding real tax revenues, rev_t, we distinguish four components:

$$rev_t = const + \chi y_t + d_t \frac{b^p_{t-1} R_{p,t-1}}{\Pi_t} - d_t \left(\frac{b_{t-1} R_{g,t-1}}{\Pi_t} + \frac{b^p_{t-1} R_{p,t-1}}{\Pi_t} \right) + \phi_{tax,b} b_{t-1}.$$

The first component is a constant. The second component captures the fact that tax revenues fluctuate with the cycle. The third component marks the funds diverted from private sector intermediation in the case of a sovereign default. The fourth component are the lump-sum transfers to creditors and financial intermediaries in the case of a default. These payments neutralize the distributional consequences of default in equilibrium. The last component ensures that taxes adjust in order to stabilize debt. Substituting in (14) gives

$$b_t = \left(\frac{R_{g,t-1}}{\Pi_t} - \phi_{tax,b} \right) b_{t-1} + (g_t - const - \chi y_t), \tag{15}$$

Note that due to "appropriate" lump-sum transfers there is no direct impact of default on the government's indebtedness. Nevertheless, the probability of default has a bearing on government indebtedness since it is crucial for the risk premium charged by creditors. In particular, the first-order condition for sovereign debt and central bank instruments is given by: $1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_{g,t}}{\pi_{t+1}} (1 - d_{t+1}) \right\}$ and $1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \right\}$. As a result, there is a spread between the two nominal rates.

Given that default is neutral on the budget in the above sence, there is no strategic default decision as in Arellano (2008) and others. As lump-sum taxes are available, there is no fiscal limit as in Bi (2010) and Bi and Leeper (2010). Nevertheless, as suggested by these analyses, we assume that the probability of default, and hence, the interest rate spread is positively related to the level of government debt. To pin down this relationship, we consider the empirical relationships discussed in section 2.[21]

[21] Bi and Leeper (2010) specify a distribution for a fiscal debt limit as a stand-in for the underlying political process of sovereign default. In their setup, each period, the government sector draws a limit from a known distribution. A default (with a fixed haircut) occurs whenever the level of sovereign debt exceeds the fiscal limit. As a result, the risk of sovereign default rises sharply with the debt level, and so does the sovereign risk premium, very similar to our environment (see also Bi (2010) and Juessen et al. (2011)). Schabert and van Wijnbergen (2008) instead assume an upper bound on taxation: creditors expect default to occur if the level of taxation required to stabilize the debt reaches levels that are impossible to implement politically. They assume that there is some uncertainty around that level, but that the distribution of beliefs is known. Taxes rise in the debt level. As a result, the perceived probability of expected default increases strictly in the real value of debt as well, as in our formulation, and savers will ask for a risk premium in order to hold government debt.

C Proofs

C.1 Proof of Proposition 1

The economy, stripped from exogenous variables, is given by

$$E_t z_{t+1} = A z_t,$$

where $z_t = [\tilde{y}_t; \widehat{\pi}_t]$ and

$$A = \frac{1}{a\mu\beta}\begin{bmatrix} \mu\beta + \varrho\mu\kappa_y & -\varrho\mu \\ -a\kappa_y & a \end{bmatrix},$$

where $a = (\mu + \mu\varrho\alpha\xi\chi)$. The Blanchard-Kahn conditions for determinacy require that matrix A have two roots outside the unit circle. Woodford (2003) gives the following necessary and sufficient conditions for determinacy:
either (Case I): (i) $det(A) > 1$, (ii) $det(A) - tr(A) > -1$, and (iii) $det(A) + tr(A) > -1$,
or (Case II): (i) $det(A) - tr(A) < -1$ and (ii) $det(A) + tr(A) < -1$.

In the current case, $det(A) = \frac{1}{a\mu\beta}$ and $tr(A) = \frac{1}{a\mu\beta}[\mu\beta + \varrho\mu\kappa_y + a]$. Since both $det(A) > 0$ and $tr(A) > 0$ Case II cannot be satisfied. Checking Case I, condition (iii) holds since both terms are positive. Condition (i) is equivalent to condition a) in the proposition. Condition (ii) of Case I is equivalent to condition b) in the proposition. □

C.2 Proof of Proposition 2

In this case

$$A = \frac{1}{a^*\mu\beta}\begin{bmatrix} \mu\beta + \varrho^*\mu\kappa_y^* & -\varrho^*\mu \\ -a^*\kappa_y^* & a^* \end{bmatrix},$$

where a^*, ϱ^* and κ_y^* are defined in the proposition.

1. Note that under the restriction that $a^* > 0$, $det(A) > 0$. Therefore it cannot be the case that $det(A) - tr(A) < -1$ and $det(A) + tr(A) < -1$. This means that determinacy can only obtain under the conditions of Case I. In addition, if $\varphi < 1$, then $tr(A) > 0$, so $det(A) + tr(A) > -1$. Condition c) is therefore obsolete if $\varphi < 1$.

2. For $a^* < 0$, $det(A) < 0$, so Case I cannot hold. The conditions given in the proposition are those pertaining to Case II. □

C.3 Corollary to proposition 2

Corollary 5 *Under the conditions of Proposition 2, the following special cases obtain:*

1. *With no endogenous risk premium ($\xi = 0$), the range of parameters for which the equilibrium is determinate is larger if spending is countercyclical ($\varphi < 0$), rather than acyclical. In addition, the range of fundamental parameters implying determinacy of the equilibrium is smaller, the larger φ.*

2. *With an endogenous risk premium $\xi > 0$, instead, the range of parameters for which the equilibrium is determinate is often larger if spending is pro-cyclical, i.e., if spending is cut during a deep recession. More precisely:*

   (a) If the conditions of Part 1) of Proposition 2 hold as well as $\varphi \in (0,1)$ and $\chi < 1 - \frac{\kappa\omega}{(1-\beta\mu)\xi}$, then the range of fundamental parameters for which the equilibrium is determinate is at least as large as in the absence of an endogenous response in spending, and can be larger. Note that this case is more likely, the less elastic the tax revenue to the state of the economy (the smaller χ), and the more responsive the country's risk premium to the deficit (the larger ξ).

   (b) It may occur that the equilibrium is indeterminate if government spending does not respond to output, but becomes determinate with a mild procyclical response that satisfies $\frac{1+\xi\varrho\chi}{1+\xi\varrho} < \varphi < 1$ (this is is only case under which the conditions of Case 2) of Proposition 2 can hold). Note that this inequality is more likely satisfied the steeper the risk premium and the less elastic the response of taxes.

Proof.

1. If $\alpha\xi = 0$, $a^* = \mu > 0$. As a result condition Case 1 of Proposition 2 gives the relevant condition. First note that condition a) will be satisfied always. Condition c) holds for $\varphi < 1$. What remains to be checked therefore is whether condition b) holds for $\varphi < 0$ whenever it holds for $\varphi = 0$, and holds for some fundamental parameters for which it would not hold otherwise. That is true if

$$(1-\beta\mu)(1-a^*) - \mu\varrho^*\kappa_y^* > (1-\beta\mu)(1-a) - \mu\varrho\kappa_y,$$

   or equivalently (for $\alpha\xi = 0$),

$$\mu\varrho^*\kappa_y^* - \mu\varrho\kappa_y < 0.$$

   Substituting, the condition reads

$$\mu\kappa\left[\frac{\varrho\omega}{1-\varphi} + 1\right] - \mu\kappa\left[\varrho\omega + 1\right] < 0.$$

This reduces to $\varphi/(1-\varphi) < 0$ which is true for $\varphi < 0$. So the range of fundamental parameters for which determinacy obtains is bigger with a countercyclical government spending response in this case than in the absence of any response. What remains to be shown is that a stronger response further increases the range of fundamental parameters for which determinacy obtains. To see this, observe that the left-hand side of condition b) in Case 1 of Proposition 2 is independent of φ. The right-hand side is given by

$$\mu\varrho^*\kappa_y^* = \mu\kappa[1 + \omega\varrho/(1-\varphi)].$$

The right-hand side is strictly increasing in φ. As a result, the set of parameters for which the condition will bind will be the larger the more negative φ is.

2. (a) The range of fundamental parameters for which determinacy holds is bigger if $a^* < a$, and if

$$(1-\beta\mu)(a-a^*) > \mu\varrho^*\kappa_y^* - \mu\varrho\kappa_y.$$

   $a^* < a$ boils down to $\frac{\chi-\varphi}{1-\varphi} < \chi$, which is true for $\varphi < 1$. The second condition reduces to

$$(1-\beta\mu)\alpha\xi(1-\chi)\frac{\varphi}{1-\varphi} > \kappa\omega\frac{\varphi}{1-\varphi}.$$

   For $\varphi \in (0,1)$ this yields $\chi < 1 - \frac{\kappa\omega}{(1-\beta\mu)\alpha\xi}$, the condition in the corollary.

   (b) $a^* < 0$ means $\frac{1+\alpha\xi\varrho\chi}{1+\alpha\xi\varrho} < \varphi < 1$, so this is the only case in which Part 2 of Proposition 2 can be satisfied.

■

C.4 Proof of Proposition 3

The assumed Markov structure means that output, inflation and government spending (in deviation from steady state) will take on the same values while the lower bound binds, y_l, π_l and g_l, respectively, and values of zero thereafter. The IS curve thus implies

$$y_l - g_l = \mu(y_l - g_l) - \varrho[-\log(R) + \Delta + \mu\alpha\xi(g_l - \chi y_l) - \mu\pi_l].$$

And the Phillips curve implies

$$\pi_l = \mu\beta\pi_l + \kappa_y y_l - \kappa_g g_l.$$

Solving these equations for y_l and π_l gives for y_l:

$$y_l = \vartheta_r[\log(R) - \Delta] \ + \vartheta_g g_l,$$

where ϑ_r and ϑ_g take on the values given in the proposition. In addition, $\vartheta_r > 0$: the numerator is positive, and the denominator is positive, too, by the condition b) for determinacy in Proposition 1. □

C.5 Corollary to proposition 3

Corollary 6 *Under the parameter restrictions of Proposition 1:*

1. *The government spending multiplier, ϑ_g, is positive if and only if*

$$(1-\mu) - \frac{\mu\varrho\kappa_g}{1-\beta\mu} > \mu\xi\varrho. \tag{16}$$

   Note that, conversely, the spending multiplier will be negative if the risk premium sufficiently affects the economy, i.e., if ξ is large enough.

2. *If $\xi = 0$, provided that the conditions for determinacy in Proposition 1 are satisfied, the government spending multiplier is strictly larger than one. This case corresponds to the analysis by Christiano et al. (2010) and Woodford (2011).*

3. *If $\xi > 0$, the government spending multiplier is unambiguously larger than one if $\chi > 1 - \frac{\kappa\omega}{\xi(1-\beta\mu)}$, that is, if the tax revenue rises sufficiently fast with output. In addition, government spending at the lower bound is self-financing if $\vartheta_g > 1/\chi$.*

Proof.

1. Under the restrictions for determinacy of Proposition 1, the denominator of ϑ_g is unambiguously positive. $\vartheta_g > 0$ thus requires $(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > 0$ which solves to the expression in equation (16).

2. The conditions for determinacy require that $(1-\beta\mu)(1-a) - \mu\varrho\kappa_y > 0$, so the denominator of ϑ_g is positive. The same condition can be also used to prove that the numerator of ϑ_g is positive. Extending the above inequality yields:

$$(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > -(1-\beta\mu)(b-a) - \mu\varrho(\kappa_g - \kappa_y).$$

Note that $\kappa_g < \kappa_y$. In addition, note that $b = a$ if $\alpha\xi = 0$. This proves that $(1-\beta\mu)(1-b) - \mu\varrho\kappa_g > 0$ if $\alpha\xi = 0$ and under the conditions of Proposition 1.

3. For $\alpha\xi > 0$, $\vartheta_g > 1$ is equivalent, after substituting for κ_g and κ_y, to $\chi > 1 - \frac{\kappa\phi}{\alpha\xi(1-\beta\mu)}$. The deficit is given by $g - \chi y_l$. Spending will thus be self-financing if $1 - \chi\vartheta_g < 0$.

■

C.6 Proof of Proposition 4

For the austerity phase the IS equation is given by

$$(y_a - g_a)(1-\nu) = -\varrho\left[\phi\pi_a - \nu\pi_a\right].$$

The Phillips curve is given by

$$\pi_a = \beta\nu\pi_a + \kappa_y y_a - \kappa_g g_a.$$

These two equations solve to expressions (9) and (10). While at the lower bound, the IS equation is given by

$$y_l(1-\mu) = (1-\mu)(y_a - g_a) - \varrho\left[-\log(R) + \Delta + \alpha\xi[(1-\mu)g_a - \chi(1-\mu)y_a - \chi\mu y_l] - \mu\pi_l - (1-\mu)\pi_a\right].$$

The Phillips curve is given by

$$\pi_l = \beta\mu\pi_l + \beta(1-\mu)\pi_a + \kappa y_l.$$

Solving the latter two equations leads to the equation for output, y_l, (8).

C.7 Corollary to Proposition 4

Corollary 7 *Under the conditions in Proposition 4,*

1. *if $\xi = 0$, future austerity enhances current economic activity unless too much of it is expected to occur too close to the exit from the ZLB. More precisely, there exists a value of $\nu \in [0, 1)$ such that $y_l > 0$ if $g_a < 0$. This is the case for any $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.*

2. *if $\xi > 0$ and, furthermore, $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$, the effect of future austerity on current economic activity (y_l) is more positive than in the absence of the risk premium, regardless of the size of the tax elasticity.*

3. *Provided that the effect of future austerity on y_l is positive, the magnitude of such effect will be larger, the more sensitive the economy is to the risk premium (the larger is ξ)(as long as determinacy obtains).*

4. *In addition, there are parameterizations for which the effect of future austerity is positive if $\xi > 0$, while it is negative if $\xi = 0$. At the same time, ill-timed austerity for which $y_l < 0$ may be particularly detrimental if $\xi > 0$ and $\chi > 0$.*

Proof.

1. For the case $\xi\alpha = 0$, we have that $y_l = 1/d[(1 - mu)(1 - \beta\mu)(y_a - g_a) + \varrho(1 - \mu)\pi_a]$.

   Note that $d = (1 - \mu)(1 - \beta\mu) - \varrho\mu\kappa_y > 0$ since Proposition 4 assumes determinacy of the equilibrium (cp. condition b) in Proposition 1).

   $y_l > 0$ thus requires

$$(1 - \mu)(1 - \beta\mu)(y_a - g_a) + \varrho(1 - \mu)\pi_a > 0. \tag{17}$$

   Substitute for y_a and π_a using equations (9) and (10). Further note that the denominator in the expressions for π_a and y_a is positive (we have assumed determinacy in Proposition 4, so $\phi_\pi > 1$, and therefore especially $\phi_\pi - \nu > 0$). Furthermore, observe that $\kappa_y - \kappa_g > 0$, and that $g_a < 0$. Using these observations, inequality (17) resolves to $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

2. Let y_l^w denote the size of output at the lower bound with a response of the risk-premium ($\alpha\xi > 0$). Denote with a superscript ^{o} the terms in the absence of a response of the risk-premium. For example, let y_l^o denote the size of output in the absence of a response of the risk premium ($\alpha\xi = 0$).

   Note, first, that y_a, g_a, π_a are independent of the risk-premium.

   Note, second, that $d^w = (1 - \beta\mu)(1 - \mu - \mu\alpha\xi\chi\varrho) - \varrho\mu\kappa_y < d^o$.

   Note, third, that $d^w > 0$ by the assumption of determinacy. Thus $[\frac{d^o}{d^w} - 1] > 0$.

   Condition $y_l^w > y_l^o$, after substituting for y_a and π_a, and after dividing by $g_a < 0$, is equivalent to

$$\begin{aligned}
&\left[\tfrac{d^o}{d^w} - 1\right] \varrho(1 - \mu)(\kappa_g - \kappa_y)\left[(1 - \beta\mu)(\phi_\pi - \nu) - 1 + \nu\right] \\
&+ \tfrac{d^o}{d^w}\varrho(\phi_\pi - \nu)(\kappa_g - \kappa_y)\varrho\alpha\xi\chi(1 - \mu)(1 - \beta\mu) \\
&+(1 - \mu)(1 - \beta\mu)\left[(1 - \nu)(1 - \beta\nu) + \varrho(\phi_\pi - \nu)\kappa_y\right]\varrho\alpha\xi(\chi - 1) < 0.
\end{aligned}$$

   The second row is non-positive. The third row is strictly negative (since $\chi \in [0, 1)$).

$\kappa_g - \kappa_y < 0$, so the first row will be strictly negative if $(1-\beta\mu)(\phi_\pi - \nu) - 1 + \nu > 0$, which is equivalent to $\nu > \frac{1+\phi_\pi(\beta\mu-1)}{\beta\mu}$.

3. $\frac{\partial y_l}{\partial(\alpha\xi)} = -\frac{1}{d^2} n \frac{\partial d}{\partial(\alpha\xi)} + \frac{1}{d}\frac{\partial n}{\partial(\alpha\xi)}$, where n denotes the denominator in the expression for y_l. d is positive in the case of determinacy. n is positive for those cases in which y_l reacts positively to future austerity.

   $\frac{\partial d}{\partial(\alpha\xi)} = -(1-\beta\mu)\mu\varrho\chi < 0$.

   $\frac{\partial n}{\partial(\alpha\xi)} = -(1-\mu)(1-\beta\mu)\varrho(1-\chi)g_a + (1-\mu)(1-\beta\mu)\varrho\chi(y_a - g_a) > 0$, since $g_a < 0$ and $(y_a - g_a) > 0$. Putting these elements together proves the claim.

■