Monetary Policy and the Uncovered Interest Rate Parity Puzzle*

David K. Backus†, Federico Gavazzoni‡, Chris Telmer§ and Stanley E. Zin¶

December 2011
(First Draft: May 2010)

Abstract

High interest rate currencies tend to appreciate. This is the *uncovered interest rate parity (UIP) puzzle*. It is primarily a statement about *short-term* interest rates and how they are related to exchange rates. Short-term interest rates are strongly affected by monetary policy. The UIP puzzle, therefore, can be restated in terms of monetary policy. When one country has a high interest rate *policy* relative to another, why does its currency tend to appreciate? We represent monetary policy as foreign and domestic Taylor rules. Foreign and domestic pricing kernels determine the relationship between these Taylor rules and exchange rates. We examine different specifications for the Taylor rule and ask which can resolve the UIP puzzle. We find evidence in favor of *asymmetries*. If the domestic Taylor rule responds more aggressively to inflation than does the foreign Taylor rule, the excess expected return on foreign currency increases. A related effect applies to Taylor rules that respond to exchange rates and/or lagged interest rates. A calibrated version of our model is consistent with many empirical observations on real and nominal exchange rates, including the negative correlation between interest rate differentials and currency depreciation rates.

*Thanks to Jordi Galí, Isaac Kleshchelski, Kristoffer Nimark, Jaume Ventura, Adrien Verdelhan and, in particular, Burton Hollifield for many helpful comments and discussions. Telmer thanks CREI, Universitat Pompeu Fabra, for generous hospitality while some of this work was completed.

†Stern School of Business, New York University; dbackus@stern.nyu.edu

‡Tepper School of Business, Carnegie Mellon University; federicogavazzoni@cmu.edu

§Tepper School of Business, Carnegie Mellon University; chris.telmer@cmu.edu

¶Stern School of Business, New York University; stan.zin@nyu.edu

1 Introduction

Uncovered interest rate parity (UIP) predicts that high interest rate currencies will depreciate relative to low interest rate currencies. Yet for many currency pairs and time periods we seem to see the opposite. The inability of asset-pricing models to reproduce this fact is what we refer to as the *UIP puzzle*.

The UIP evidence is primarily about *short-term* interest rates and currency depreciation rates. Monetary policy exerts substantial influence over short-term interest rates. Therefore, the UIP puzzle can be restated in terms of monetary policy: Why do countries with high interest rate *policies* have currencies that tend to appreciate relative to those with low interest rate *policies*?

The risk-premium interpretation of the UIP puzzle asserts that high interest rate currencies pay positive risk premiums. The question, therefore, can also be phrased in terms of currency risk: When a country pursues a high-interest rate monetary policy, why does this make its currency risky? For example, when the Fed sharply lowered rates in 2001 and the ECB did not, why did the euro become relatively risky? When the Fed sharply reversed course in 2005, why did the dollar become the relatively risky currency? This paper formulates a model of interest rate policy and exchange rates that can potentially answer these questions.

To understand what we do it's useful to understand previous work on monetary policy and the UIP puzzle.[1] Most models are built upon the basic Lucas (1982) model of international asset pricing. The key equation in Lucas' model is

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1} e^{-\pi^*_{t+1}}}{n_{t+1} e^{-\pi_{t+1}}} \ , \tag{1}$$

where S_t denotes the nominal exchange rate (price of foreign currency in units of domestic), n_t denotes the intertemporal marginal rate of substitution of the domestic representative agent, π_t is the domestic inflation rate and asterisks denote foreign-country variables. Equation (1) holds by virtue of complete financial markets. It characterizes the basic relationship between interest rates, nominal exchange rates, real exchange rates, preferences and consumption.

Previous work has typically incorporated monetary policy into Equation (1) via an explicit model of *money*. Lucas (1982), for example, uses cash-in-advance constraints to map Markov processes for money supplies into the inflation term, $\exp(\pi_t - \pi^*_t)$, and thus into exchange rates. His model, and many that follow it, performs poorly in accounting for data. This is primarily a reflection of the weak empirical link between measures of money and exchange rates.

[1] Examples are Alvarez, Atkeson, and Kehoe (2007), Backus, Gregory, and Telmer (1993), Bekaert (1994), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006), Canova and Marrinan (1993), Dutton (1993), Grilli and Roubini (1992), Macklem (1991), Marshall (1992), McCallum (1994) and Schlagenhauf and Wrase (1995).

Our approach is also built upon Equation (1). But — like much of the modern theory and practice of monetary policy — we abandon explicit models of money in favor of interest rate rules. Following the New Keynesian macroeconomics literature (*e.g.*, Clarida, Galí, and Gertler (1999)), the policy of the monetary authority is represented by a Taylor (1993) rule. Basically, where Lucas (1982) uses money to restrict the inflation terms in Equation (1), we use Taylor rules. Unlike his model, however, our allows for dependence between the inflation terms and the real terms, n_t and n_t^*. This is helpful for addressing the evidence on how real and nominal exchange rates co-move.

A sketch of what we do is as follows. The simplest Taylor rule we consider is

$$i_t = \tau + \tau_\pi \pi_t + \tau_x x_t \quad , \tag{2}$$

where i_t is the nominal short-term interest rate, π_t is the inflation rate, x_t is consumption growth (analogous to the output-gap in a model with nominal frictions)," and τ, τ_π and τ_x are policy parameters. We also assume that the private sector can trade bonds. Therefore the nominal interest rate must also satisfy the standard (nominal) Euler equation,

$$i_t = -\log E_t \, n_{t+1} e^{-\pi_{t+1}} \quad , \tag{3}$$

where (as above) n_{t+1} is the real marginal rate of substitution. An equilibrium inflation rate process must satisfy both of these equations at each point in time, which requires inflation to solve the nonlinear stochastic difference equation:

$$\pi_t = -\frac{1}{\tau_\pi}\left(\tau + \tau_x x_t + \log E_t \, n_{t+1} \, e^{-\pi_{t+1}}\right) \quad . \tag{4}$$

A solution to Equation (4) is an endogenous inflation process, π_t, that is jointly determined by the response of monetary authority and the private sector to the same underlying shocks. By substituting such a solution back into the Euler equation (3), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) (GHPZ) refer to as a 'monetary policy consistent pricing kernel:' a (nominal) pricing kernel that depends on the Taylor-rule parameters τ and τ_π. Doing the same for the foreign country, and then using Equation (1), we arrive at a nominal exchange rate process that also depends on the policy parameters τ, τ_π and τ_x. Equations (1)–(4) (along with specifications for the shocks) fully characterize the joint distribution of interest rates and exchange rates and, therefore, any departures from UIP.

Given a Taylor rule such as (2), and its foreign counterpart, we can ask whether the implied exchange rate process in (1) tends to appreciate when the implied interest rate in (3) is relatively high. If so, then the source of UIP deviations can be associated with this Taylor rule. Moreover, we can generalize the specification of the Taylor rule in Equation (2) and analyze the consequences of alternative monetary policies for exchange rates. In addition, we can ask whether the Taylor rule parameters are identified by the UIP facts. Cochrane (2007) provides examples in which policy parameters and the dynamics of the shocks are not separately identified by the relationship between interest rates and inflation. Our framework has the potential for identifying monetary policy parameters from the properties of exchange rates.

A more specific description of our question goes as follows. A number of papers (*e.g.*, Backus, Foresi, and Telmer (2001), Lustig, Roussanov, and Verdelhan (2009)) have demonstrated the importance of *asymmetries* between foreign and domestic pricing kernels for explaining the UIP puzzle. Inspection of Equation (1) shows why. Exchange rates are all about *differences* in nominal pricing kernels, $n_{t+1}\exp(-\pi_{t+1})$ and $n^*_{t+1}\exp(-\pi^*_{t+1})$. If there are no differences, then the exchange rate is a constant. Many previous papers have come up with *statistical* models of such differences, but far fewer have come up with *economic* models. Our basic question asks whether differences in monetary policies are a plausible source of the asymmetries that we know we need. We ask if asymmetric domestic and foreign Taylor rule coefficients can help explain the UIP puzzle. We find that they can.

Our main result is as follows. If the domestic country has a relatively tight monetary policy — as measured by a relatively large value for the inflation-stabilization coefficient, τ_π, from Equation (2) — then the unconditional means of domestic inflation and interest rates are relatively *low*, and the unconditional mean of the *foreign* currency risk premium is positive. Moreover, the *conditional* risk premium is more highly variable, relative to the case of symmetric monetary policies. The reason, explained in more detail in Section 4, is that a tighter monetary policy, *ceteris paribus*, *increases* the variability of the nominal pricing kernel. Our main result, then, is consistent with the broad set of facts that characterize tight-policy countries such as Germany, Japan and Switzerland as having low interest rates (and inflation), and having 'funding currencies' for the foreign currency carry trade.

Some supplementary results are as follows. Section 5 attempts to take a closer look at *exactly* how the Taylor rule affects nominal exchange rates by ignoring variation in real exchange rates. This means that $n_t = n^*_t$ and, according to Equation (1), relative PPP holds: $\log(S_t/S_{t-1}) = \pi_t - \pi^*_t$. We also go one step further and set $n_t = n^*_t = e^r$, thus abstracting from real interest rate variation (this doesn't really matter for nominal exchange rates and it makes the analysis easier). The resulting Euler equation for the nominal interest rate (with lognormality) is as follows.[2]

$$i_t = r + E_t\,\pi_{t+1} - \frac{1}{2}\,Var_t(\pi_{t+1}) \ \ . \tag{5}$$

The Taylor rule (2) becomes $i_t = \tau + \tau_\pi \pi_t + x_t$, where z_t is a 'policy shock.' The model therefore boils down to two equations for each country — two Euler equations and two Taylor rules — along with specifications for the policy shocks. As is shown below, the latter must necessarily feature stochastic volatility. Otherwise the conditional variance in Equation (5) would be a constant and UIP would hold (up to a constant). The solution for inflation is of the form $\pi(z_t, v_t)$, where v_t is the volatility of z_t. Most of our analysis focuses on variation arising from v_t because only it affects currency risk.

What we find here is negative in nature. We find that simple Taylor rules of the form (2) can generate deviations from UIP, but not as large as those typically focused upon in the literature.[3] The basic reason is straightforward and, we think, informative. The Euler equation (5) imposes restrictions between the current interest rate and *moments* of future inflation. The Taylor rule imposes an additional, *contemporaneous* restriction between the current interest rate and current inflation. It says that a volatility shock that increases inflation by 1% must increase the interest rate by *more than* 1%. This is because $\tau_\pi > 1$, the so-called "Taylor principle" required for the inflation solution to be non-explosive. However, if inflation is a stationary, positively autocorrelated process, then its conditional mean in Equation (5) must increase by *less than* 1%. The only way that both can be satisfied is if the conditional variance in Equation (5) *decreases*. But this means that the mean and variance of the (log) pricing kernel are positively correlated, something which contradicts Fama's (1984) necessary conditions for resolving the anomaly. There are two ways around this. The first is that volatility is negatively autocorrelated. This is empirically implausible. The second is that the volatility shock that affects inflation also affects the real interest rate (and the real exchange rate). This is the subject of Section 6.

[2] Equation (5) also shows how our paper — at least the initial part — relates to the benchmark New-Keynesian setup. All that really distinguishes the two is the conditional variance term. But, for us, this is where all the action is. That is, if inflation were homoskedastic then the nominal interest rate would satisfy the Fisher equation (up to a constant), the difference equation (4) would be linear, and the solution for inflation would be in the same class as, say, Clarida, Galí, and Gertler (1999). What would also be true, however, is that UIP would be satisfied (up to a constant) and well-known regression (Bilson (1981), Fama (1984)) of the depreciation rate on the interest rate differential would yield a (population) slope coefficient of 1.0. Our paper would be finished before it even began. Stochastic volatility, therefore, is not a choice, it is a *requirement*. The only issue is where it comes from.

[3] Specifically, our model (without real rate variation) can generate slope coefficients from the regression of depreciation rates on interest rate differentials that are less than unity, but not less than zero.

This reasoning — spelled out in detail in Section 5.4 — is admittedly complex. But the basic point is not. Taylor rules of the form (2) imply restrictions on the co-movement of the mean and variance of the pricing kernel. Getting this co-movement right is critical for resolving the UIP puzzle, so these restrictions can be binding. Models of the inflation term in Equation (1) that are driven by exogenous money supplies do not impose such restrictions. Neither do models in which an exogenous inflation process is used to transform real exchange rates into nominal exchange rates. The sense in which we're learning something about how the conduct of modern monetary policy relates to exchange rates is the sense in which these restrictions identify the policy parameters, τ and τ_π, and the parameters of the shock process z_t.

Our next results are more positive. While continuing to abstract from real exchange rate and interest rate variability, we examine two alternative Taylor rules relative to that in Equation (2). In both cases there are parameterizations of the model that admit UIP deviations similar to those observed in data. The first alternative introduces an additional variable and an asymmetry to the Taylor rule (2). The variable is the contemporaneous currency depreciation rate, $\log(S_t/S_{t-1})$. The asymmetry is that the foreign central bank reacts more to the exchange rate than does the domestic central bank. Or, in concrete terms, the Bank of England reacts to variation in the pound/dollar exchange rate, but the Fed does not. Such an asymmetry seems plausible. The international role of the U.S. dollar versus the pound is certainly not symmetric. A small country like New Zealand might pay closer attention to the kiwi/yen exchange rate than a large country like Japan. There is also some empirical and theoretical support for such an asymmetry (*c.f.* Benigno (2004), Benigno and Benigno (2008), Clarida, Gal, and Gertler (1998), Eichenbaum and Evans (1995), Engel and West (2006)).

The second alternative Taylor rule we consider is based on McCallum (1994) and is emphasized in Woodford (2003). We include the lagged interest rate into Equation (2). Like McCallum, we find parameterizations of the model that work. Our approach extends his work by endogenizing the currency risk premium which, in his paper, is exogenous.[4] This is an important step since it constrains the sense in which the UIP anomaly is driven by endogenous equilibrium *inflation risk*. That is, in our model, a shock is realized, the Taylor rule responds to that shock, and as a result so does inflation. Whether or not this shock commands a risk premium depends on the parameters of the model. We can then ask if the way in which monetary policy reacts to shocks is consistent with risk premiums that are capable

[4]Engel and West (2006) also study a model of how Taylor rules affect exchange rates. Their analysis, while focusing on a different set of questions, is related to McCallum's in that they interpret their 'policy shock' as an amalgamation of an actual policy shock and an exogenous risk premium. Our paper relates to theirs in that both derive an exchange rate process as the solution to a forward-looking difference equation. The main difference is that our deviations from UIP are endogenous.

of creating sizable deviations from UIP.

The final sections of our paper re-introduce variability in real exchange rates and take the model to the data. We use Epstein and Zin (1989) preferences and we model foreign and domestic consumption as following long-run risk processes as in Bansal and Yaron (2004) and the exchange rate applications in Bansal and Shaliastovich (2008) and Colacito and Croce (2008). Our model does not feature nominal frictions, so inflation reacts to consumption shocks (since they appear in the Taylor rule) but not the other way around.[5] We solve for endogenous inflation in the same manner as described above, but inclusive of the n_t term in the difference Equation (4).

Our last set of results are both qualitative, as above, and quantitative. We characterize conditions under which real and nominal exchange rates will resolve the UIP puzzle and show that the latter depend on the Taylor rule parameters. We show that, as above, Taylor rule-implied inflation tends to hinder the model's performance, reducing the deviations from UIP. The logic is basically the same as our simplest, nominal-variability-only model described above. Nevertheless, we are able to find a calibration that satisfies the following criteria: (i) the Bilson-Fama UIP regression coefficient is negative, (ii) UIP holds unconditionally, so that the mean of the risk premium is is zero, (iii) changes in real and nominal exchange rates are highly correlated (Mussa (1986)), (iv) exchange rate volatility is high relative to inflation differentials, (v) exchange rates exhibit near random-walk behavior but interest rate differentials are highly autocorrelated, (vi) international pricing kernels are highly correlated but international aggregate consumption growth rates are not (Brandt, Cochrane, and Santa-Clara (2006)), (vii) domestic real and nominal interest rates are highly autocorrelated with means and volatilities that match data. Our calibrated values for the Taylor rule parameters satisfy conditions required for a solution to exist and, interestingly, are also in the ballpark of typical reduced-form estimates. We find $\tau_\pi = \#$ and $\tau_l = \#$, where the latter is the coefficient on consumption growth, the analog of the output gap in our setting.

The remainder of the paper is organized as follows. In Section 2 we provide a terse overview of existing results on currency risk and pricing kernels that are necessary for our analysis. Section 3 develops our baseline model of real and nominal exchange rates. Section 4 presents our main result on how asymmetric policies can help account for the UIP puzzle. Section 5 examines the special case of zero variability in real exchange rates. Section 6 enhances the model of Section 3

[5] Similar to Bansal and Shaliastovich (2008), Colacito and Croce (2008), and Verdelhan (2010) we treat foreign and domestic consumption *exogenously*, remaining silent on the goods-market equilibrium that gives rise to the consumption allocations. We simply exploit the fact that, with complete financial markets, Equation (1) will hold in any such equilibrium. We then calibrate the joint distribution of foreign and domestic consumption to match the data and ask if the implied real and nominal exchange rates and interest rates fit the facts. A more formal treatment and justification is provided in Section 3.

to be more amenable to the data and Section 7 conducts a calibration and presents quantitative results. Section 8 concludes.

2 Pricing Kernels and Currency Risk Premiums

We begin with a terse treatment of existing results in order to fix notation. The level of the spot and one-period forward exchange rates, in units of U.S. dollars (USD) per unit of foreign currency (say, British pounds, GBP), are denoted S_t and F_t. Logarithms are s_t and f_t. USD and GBP one-period interest rates (continuously compounded) are denoted i_t and i_t^*. Covered interest parity implies that $f_t - s_t = i_t - i_t^*$. Fama's (1984) decomposition of the interest rate differential (forward premium) is

$$\begin{aligned} i_t - i_t^* = f_t - s_t &= \big(f_t - E_t s_{t+1}\big) + \big(E_t s_{t+1} - s_t\big) \\ &\equiv p_t + q_t \end{aligned}$$

This decomposition expresses the forward premium as the sum of q_t, the expected USD depreciation rate, and p_t, the expected payoff on a forward contract to receive USD and deliver GBP. We define the latter as the *foreign currency risk premium.* We define *uncovered interest parity* (UIP) as $p_t = 0$. The well-known rejections of UIP are manifest in negative estimates of the parameter b from the regression

$$s_{t+1} - s_t = c + b\big(i_t - i_t^*\big) + \text{residuals} \quad . \tag{6}$$

The population regression coefficient — we'll call it the "Bilson-Fama coefficient" — can be written

$$b = \frac{Cov(q_t, p_t + q_t)}{Var(p_t + q_t)} \quad . \tag{7}$$

Fama (1984) noted that necessary conditions for $b < 0$ are

$$Cov(p_t, q_t) < 0 \tag{8}$$
$$Var(p_t) > Var(q_t) \tag{9}$$

Our approach revolves around the standard (nominal) pricing-kernel equation,

$$b_t^{n+1} = E_t \, m_{t+1} b_{t+1}^n \quad , \tag{10}$$

where b_t^n is the USD price of a nominal n-period zero-coupon bond at date t and m_t is the pricing kernel for USD-denominated assets. The one-period interest rate is $i_t \equiv -\log b_t^1$. An equation analogous to (10) defines the GBP-denominated pricing kernel, m_t^*, in terms of GBP-denominated bond prices, b_t^*.

Backus, Foresi, and Telmer (2001) translate Fama's (1984) decomposition into pricing kernel language. First, assume complete markets so that the currency depreciation rate is

$$s_{t+1} - s_t \quad = \quad \log\left(m^*_{t+1}/m_{t+1}\right)$$

Fama's (1984) decomposition becomes

$$\begin{aligned} i_t - i^*_t &= \log E_t\, m^*_{t+1} - \log E_t\, m_{t+1} && (11)\\ q_t &= E_t \log m^*_{t+1} - E_t \log m_{t+1} && (12)\\ p_t &= \left(\log E_t\, m^*_{t+1} - E_t \log m^*_{t+1}\right) - \left(\log E_t\, m_{t+1} - E_t \log m_{t+1}\right) && (13)\\ &= Var_t(\log m^*_{t+1})/2 - Var_t(\log m_{t+1})/2 \ , && (14) \end{aligned}$$

where Equation (14) is only valid for the case of conditional lognormality. Basically, Fama's (1984) conditions state that the means and the variances must move in opposite directions and that the variation in the variances must exceed that of the means.

Our objective is to write down a model in which $b < 0$. Inspection of Equations (8) and (14) indicate that a necessary condition is that p_t vary over time and that, for the lognormal case, the log kernels *must* exhibit stochastic volatility.

3 Model

Consider two countries, home and foreign. The home-country representative agent's consumption is denoted c_t and preferences are of the Epstein and Zin (1989) (EZ) class:

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}$$

where β and ρ characterize patience and intertemporal substitution, respectively, and the certainty equivalent of random future utility is

$$\mu_t(U_{t+1}) \equiv E_t[U^\alpha_{t+1}]^{1/\alpha} \ ,$$

so that α characterizes (static) relative risk aversion (RRA). The relative magnitude of α and ρ determines whether agents prefer early or late resolution of uncertainty ($\alpha < \rho$, and $\alpha > \rho$, respectively). Standard CRRA preferences correspond to $\alpha = \rho$. The marginal rate of intertemporal substitution, defined as n_{t+1}, is

$$n_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho} . \qquad (15)$$

We also refer to n_{t+1} as the *real* pricing kernel. The *nominal* marginal rate of substitution — the pricing kernel for claims denominated in USD units — is then

$$m_{t+1} \quad = \quad n_{t+1}e^{-\pi_{t+1}} \ , \qquad (16)$$

where π_{t+1} is the (continuously-compounded) rate of inflation between dates t and $t+1$. The foreign-country representative agent's consumption, c_t^*, and preferences are defined analogously. Asterisks' are used to denote foreign variables. Foreign inflation is π_{t+1}^*.

The domestic pricing kernel satisfies $E_t(m_{t+1}R_{t+1}) = 1$ for all USD-denominated asset returns, R_{t+1}. Similarly, $E_t(m_{t+1}^* R_{t+1}^*) = 1$ for all GBP-denominated returns. The domestic pricing kernel must also price USD-denominated returns on GBP-denominated assets:

$$E_t\left(m_{t+1}\frac{S_{t+1}}{S_t}R_{t+1}^*\right) = 1 \ . \tag{17}$$

We assume that international financial markets are complete for securities denominated in goods units, USD units and GBP units. This implies the uniqueness of the nominal and real pricing kernels and therefore, according to Equation (17),

$$\frac{S_{t+1}}{S_t} = \frac{m_{t+1}^*}{m_{t+1}} = \frac{n_{t+1}^* e^{-\pi_{t+1}^*}}{n_{t+1} e^{-\pi_{t+1}}} \ . \tag{18}$$

Equation (18) must hold in any equilibrium with complete financial markets. This is true irrespective of the particular goods-market equilibrium that gives rise to the consumption allocations c_t and c_t^* that are inherent in n_t and n_t^*. Our approach is to specify c_t and c_t^* *exogenously* and calibrate them to match the joint behavior of data on domestic and foreign consumption. We are silent on the model of international trade that gives rise to such consumption allocations. Bansal and Shaliastovich (2008), Colacito and Croce (2008), Gavazzoni (2008), Verdelhan (2010) and others follow a similar approach. Hollifield and Uppal (1997), Sercu, Uppal, and Hulle (1995) and the appendix in Verdelhan (2010) — all building upon Dumas (1992) — are examples of more fully-articulated complete markets models in which imperfectly-correlated cross-country consumption is generated by transport costs. Basically, our approach is to these models what Hansen and Singleton's (1983) first-order-condition-based approach was to Mehra and Prescott's (1985) general equilibrium model.

Domestic consumption growth, $x_{t+1} \equiv \log(c_{t+1}/c_t)$, follows an AR(1) process with stochastic volatility u_t.

$$\begin{aligned} x_{t+1} &= (1-\varphi_x)\theta_x + \varphi_x x_t + \sqrt{u_t}\epsilon_{t+1}^x \\ u_{t+1} &= (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \epsilon_{t+1}^u \end{aligned}$$

The innovations ϵ_t^x and ϵ_t^u are standard normal and independent of each other. The analogous foreign consumption process is denoted with asterisks: $x_{t+1}^* \equiv \log(c_{t+1}^*/c_t^*)$, with volatility u_t^*. Foreign parameter values are assumed to be identical ('symmetric') to their domestic counterparts and the innovation-pairs, $(\epsilon_t^x, \epsilon_t^{x^*})$

and $(\epsilon_t^u, \epsilon_t^{u^*})$ are assumed to be correlated so that (i) cross-country consumption correlations are low, and (ii) cross-country volatility correlations are high (numerical values are calibrated below). The former is a well-documented empirical fact whereas the latter is what we mean by the 'common factor' which, as argued above, plays a critical role in our story.[6]

The final ingredients are domestic and foreign Taylor rules:

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t \qquad (19)$$

$$i_t^* = \bar{\tau}^* + \tau_\pi^* \pi_t^* + \tau_x x_t^* \qquad (20)$$

Here, we make the foreign specification explicit so as to emphasize the asymmetry that is a focal point of our paper, $\tau_\pi \neq \tau_\pi^*$. The Taylor rules (19) and (20) are fairly typical in the literature, the main exception being that we use consumption growth instead of the 'output gap.' In our model, which abstracts from any frictions that can give rise to a 'gap,' the distinction is meaningless. In Appendix # we extend this basic specification to include a (possibly asymmetric) exchange rate term and a lagged interest rate term.

3.1 Solution

What *is* a 'solution?' Since we take foreign and domestic consumption to be exogenous it is just a stochastic process for domestic inflation and one for foreign inflation such that the nominal interest rates implied by the nominal pricing kernels, (16), are the same as those implied by the Taylor rules, (19) and (20).

We proceed as follows. Starting with the domestic country, we derive an expression for the *real* pricing kernel in terms of the model's state variables, x_t and u_t. Next, we solve for domestic inflation and, therefore, obtain an endogenous expression for the domestic *nominal* pricing kernel. We can do this independently of the foreign country because (i) consumptions are exogenous, and (ii) there is no cross-country interaction in the Taylor rules (condition (ii) is relaxed in Appendix #). Next, we do the same things for the foreign country. Finally, we compute the nominal exchange rate as a ratio of the foreign and domestic nominal pricing kernels.

Following Hansen, Heaton, and Li (2005), we linearize the logarithm of the real pricing kernel, Equation (15), around zero. The result is

$$-\log n_{t+1} = \delta^r + \gamma_x^r x_t + \gamma_u^r u_t + \lambda_x^r \sqrt{u_t} \epsilon_{t+1}^x + \lambda_u^r \sigma_u \epsilon_{t+1}^u \quad , \qquad (21)$$

[6] We represent a common factor as highly-correlated volatility processes instead of the cleaner case of $u_t = u_t^*$. We do this because we seek to isolate the effect of asymmetric monetary policy by restricting all parameter values to be symmetric *except* Taylor rule coefficients. Given this, the case of $u_t = u_t^*$ implies a *real* exchange rate risk premium of zero and a higher-order factor structure for nominal exchange rates than for real exchange rates (two versus one). Both are strongly at odds with the data.

where

$$\gamma_x^r = (1-\rho)\varphi_x \quad , \quad \gamma_u^r = \frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 \quad ,$$

$$\lambda_x^r = (1-\alpha) - (\alpha-\rho)\omega_x \quad , \quad \lambda_u^r = -(\alpha-\rho)\omega_u \quad ,$$

where $\omega_x > 0$ and $\omega_u > 0$ are positive linearization coefficients. Expressions for these, and the constant δ^r, are given along with derivations in Appendix D. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_x^r \ \gamma_u^r]^\top$ as real factor loadings and to $\lambda^r = [\lambda_x^r \ \lambda_u^r]^\top$ as real prices of risk. The one-period real interest rate is $r_t = -\log E_t n_{t+1} = \overline{r} + \gamma_x^r x_t + (\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2)u_t$, where $\overline{r} = \delta^r - \frac{1}{2}(\lambda_u^r \sigma_u)^2$.

The Euler equation for the nominal one-period interest rate is

$$i_t = -\log E_t\, n_{t+1} e^{-\pi_{t+1}} \quad . \tag{22}$$

This, combined with the domestic Taylor rule (19), implies that a solution for endogenous inflation, $\pi(x_t, u_t)$, must solve the difference equation.

$$\pi_t = -\frac{1}{\tau_\pi}\left(\bar{r} + \tau_x x_t + \log E_t\, n_{t+1}\, e^{-\pi_{t+1}}\right) \quad . \tag{23}$$

We guess that the solution is of the form

$$\pi_t = a + a_x x_t + a_u u_t \tag{24}$$

for coefficients a, a_x and a_u to be determined. In Appendix # we prove that the unique "minimum state variable" solution is

$$\begin{aligned} a_x &= \frac{(1-\rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \\ a_u &= \frac{\frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 - \frac{1}{2}\Big((1-\alpha) - (\alpha-\rho)\omega_x + a_x\Big)^2}{\tau_\pi - \varphi_u} \quad , \end{aligned}$$

with the (relatively inconsequential) solution for a relegated to Appendix #. Putting together Equations (21), (22) and (24), we arrive at the nominal pricing kernel

$$-\log m_{t+1} \quad = \quad \delta + \gamma_x x_t + \gamma_u u_t + \lambda_x \sqrt{u_t}\epsilon_{t+1}^x + \lambda_u \sigma_u \epsilon_{t+1}^u \quad , \tag{25}$$

where

$$\delta = \delta^r + a + a_x(1-\varphi_x)\theta_x + a_u(1-\varphi_u) \quad ;$$

$$\gamma_x = \gamma_x^r + a_x\varphi_x; \quad \gamma_u = \gamma_u^r + a_u\varphi_u;$$

$$\lambda_x = \lambda_x^r + a_x; \quad \lambda_u = \lambda_u^r + a_u \quad .$$

The nominal one-period interest rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{\iota} + \gamma_x x_t + (\gamma_u - \frac{1}{2}\lambda_x^2)u_t \quad , \end{aligned} \tag{26}$$

where $\bar{\iota} = \delta - \frac{1}{2}(\lambda_u \sigma_u)^2$.

Analogous calculations for the foreign country yield solutions for n_{t+1}^*, π_{t+1}^*, m_{t+1}^*, r_t^* and i_t^*. We omit these calculations since they are almost identical, the only differences being that (i) the state variables, x_t^* and u_t^* are imperfectly correlated with x_t and u_t, and (ii) the foreign Taylor-rule coefficient τ_π^* is distinct from its domestic counterpart τ_π.

3.2 Exchange Rates

Using Equation (18), the nominal deprecation rate is $d_{t+1} \equiv \log(S_{t+1}/S_t) = m_{t+1}^*/m_{t+1}$. Following Section 2, the forward premium, expected depreciation rate and risk premium can be written

$$\begin{aligned} f_t - s_t &= i_t - i_t^* = (\iota - \iota^*) + (\gamma_x x_t - \gamma_x^* x_t^*) + (\gamma_u - \frac{1}{2}\lambda_x^2)u_t - (\gamma_u^* - \frac{1}{2}(\lambda_x^*)^2)u_t^* \\ q_t &= (\delta - \delta^*) + (\gamma_x x_t - \gamma_x^* x_t^*) + (\gamma_u u_t - \gamma_u^* v_t^*) \\ p_t &= (\bar{\iota} - \bar{\iota^*}) - (\delta - \delta^*) - \frac{1}{2}\left(\lambda_x^2 u_t - (\lambda_x^*)^2 u_t^*\right) \end{aligned}$$

The Bilson-Fama regression coefficient is $b = Cov(f_t - s_t, q_t)/Var(f_t - s_t)$. A useful reference point is the special case of $\varphi_x = 0$. If the Taylor rule parameters are symmetric (*i.e.*, $\tau = \tau^*$), then

$$b = \frac{\gamma_u}{\gamma_u - \frac{1}{2}\lambda_x^2} \quad .$$

If volatility is positively autocorrelated ($\varphi_u > 0$), then *sufficient* conditions for $b < 0$ are (i) $\rho > \alpha$, so that the representative agent has preference for the early resolution of uncertainty, and (ii) ## FILL IN. For the more general case of $\varphi_x \neq 0$ and $\tau \neq \tau^*$, the same is true for the empirically relevant range of the parameter space.

This last point highlights an important feature of our setup thus far. The extent to which the Bilson-Fama coefficient is negative hinges on *preferences*. The sign of the Bilson-Fama coefficient is driven by real exchange rate behavior, not by the properties of endogenous inflation. Indeed, in Appendix # we show that (i) the *real* Bilson-Fama coefficient — the slope coefficient of a regression of the real depreciation rate on the real interest rate differential — is unambiguously negative if $\rho > \alpha$, and (ii) the nominal Bilson-Fama coefficient is typically greater than its real counterpart. Endogenous inflation, in other words, pushes us *toward* UIP. Exogenous inflation, in contrast, imposes no such restrictions. Papers such as Bansal and Shaliastovich (2008) and Lustig, Roussanov, and Verdelhan (2009) take the latter route and show that ##. Comments ##.

Two related points are as follows. First, this is not a general feature of Taylor-rule implied inflation. In Appendix # we show that more elaborate Taylor rules — *e.g.*, incorporating exchange rates and/or lagged nominal interest rates — *can* generate a nominal Bilson-Fama coefficient that is less than its real counterpart. We choose, however, the simpler setting because it articulates our main point most clearly. Second, suppose that our model *did* generate nominal deviations from UIP that were substantially larger than real deviations. While this would support our cause — our view that monetary policy is important for exchange rate behavior — it would also rise to an important empirical tension. We know that real and nominal exchange rates behave quite similarly (Mussa (1986)), and that real and nominal Bilson-Fama regressions also look quite similar (Engel (2011) is a recent example). The most obvious way around this tension — if one wants to continue down the road in which monetary policy plays an important role — is to consider environments in which there is some feedback between nominal variables/frictions and real exchange rates. This is beyond the scope of this paper.

4 Asymmetric Monetary Policy

What is the effect of $\tau_\pi > \tau_\pi^*$?

To answer this, we work with the excess expected return on a forward contract that is long GBP and short USD. The definition of p_t above is the opposite. Therefore we work with $-p_t$.[7] In addition, we work with the currency risk premium in *levels*, not in logs. This is because, as has become commonplace in the literature, we'd like to evaluate our model in terms of the Sharpe ratio on a traded portfolio. With lognormality, (logs of) expected returns in levels are expected returns in logs plus half the conditional variance. Therefore, the main object of interest is

$$-p_t + \frac{1}{2} Var_t d_{t+1} = -Cov_t(\log n_{t+1}, d_{t+1}) + Cov_t(\pi_{t+1}, d_{t+1}) \quad ,$$

where, recall, $d_{t+1} \equiv \log(S_{t+1}/S_t)$, the depreciation rate of USD. For our model,

[7] We do so because we find it more intuitive to say "this is the risk premium on holding *foreign* currency," as opposed to the premium on holding domestic currency. Our notational convention for p_t, from Section 2, follows that which is common in the literature.

the covariance terms can be written

$$
\begin{aligned}
-p_t + \frac{1}{2} Var_t d_{t+1} &= -\lambda_u \sigma_u^2 (\lambda_u^* \eta_{u,u^*} - \lambda_u) + \lambda_x \sqrt{u_t} (\lambda_x \sqrt{u_t} - \lambda_x^* \sqrt{u_t^*} \eta_{x,x^*}) \\
&\approx \lambda_u \sigma_u^2 (\lambda_u^* - \lambda_u) - \lambda_x^2 u_t \quad , \qquad (27)
\end{aligned}
$$

where η_{x,x^*} and η_{u,u^*} are the cross-country correlations of consumption growth and volatility, respectively. The approximation that gives rise to the last line is valid if (i) η_{x,x^*} is close to zero, and (ii) η_{u,u^*} is close to one. The former, while obviously extreme, captures the empirical feature of "low cross-country consumption correlations." The latter (in combination with the former) captures one of our motivational facts, the Lustig, Roussanov, and Verdelhan (2009) result that carry trade profits are dominated by a global risk factor.

How do the Taylor rule parameters affect the risk premium, (27)? First, in Appendix # we show that note that the first term, $\lambda_u \sigma_u^2 (\lambda_u^* - \lambda_u)$, is negligible. Asymmetries in the price of risk of stochastic volatility ($\lambda_u \neq \lambda_u^*$) do not play a significant role in the FX risk premium.[8] This leaves us with λ_x. Recalling that $\lambda_x = \lambda_x^r + a_x$, and that λ_x^r (the real price of consumption growth risk) is independent of monetary policy, we are left with

$$
a_x = \frac{(1-\rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \quad . \qquad (28)
$$

If $0 < \rho < 1$ (so that the EIS is greater than 1), and the autocorrelation in consumption growth is small enough (which is empirically plausible), then a strong enough reaction of the monetary authority to consumption growth, τ_x, implies that $a_x < 0$. Inspection of (28) then indicates that an increase in τ_π must make a_x *less negative*, the nominal price of consumption risk, λ_x, larger, and, therefore, the risk premium on GBP *larger*. This establishes our main comparative static result; *ceteris paribus* if the domestic Taylor rule responds more aggressively to inflation, then the risk premium on foreign currency will increase.[9]

4.1 Economic Intuition

We define a country with a *weak* price-stabilization policy as one with a relatively *low* value for the inflation coefficient, τ_π, from the Taylor rule (19). Such a country (in our model) will have unconditionally high inflation and nominal interest rates,

[8] This result is standard in consumption models with stochastic volatility. It basically says that the conditional variance of consumption growth, u_t, is much larger than the conditional variance of stochastic volatility, σ_u^2.

[9] Note that if we worked with the log risk premium instead of the level, then the pivotal term in Equation (27) would involve the coefficient $(\lambda_x - \lambda_x^*)$, not just λ_x. Thus, the nature of our comparative static result would be unaffected, since it would hold τ_π^* fixed.

and is characterized by a relatively weak response to an inflation surprise.[10] Similarly, we define a country with a *strong* employment-stabilization policy as one with a relatively *high* value for the 'output gap' coefficient, τ_x.

Our basic result is that a weak price-stabilization policy implies a riskier currency. The logic is a simple implication of Equation (14), an expression for the currency risk premium that applies to *any* lognormal model. We reproduce it here for clarity:

$$- \ p_t = Var_t(\log m_{t+1})/2 - Var_t(\log m^*_{t+1})/2 \ \ . \tag{29}$$

Recalling that *minus* p_t is the risk premium on *foreign* currency, this expression says something that, at first blush, might seem counterintuitive. It says that the country with the *low* pricing-kernel variability is the country with the *risky* currency. Low variance ... high risk! At second blush, however, it makes perfect sense. It is a general characteristic of the "change-of-units risk" that distinguishes currency risk from other forms of risk. Change-of-units risk is a relative thing. It measures how I perceive the risk in unit-changing *relative* to how you perceive it.

To understand this, recall that the (log) depreciation rate is the difference between the two (log) pricing kernels. If they are driven by the same shocks (and if loadings are symmetric), then the conditional variances in Equation (29) are the same and currency risk is zero for both foreign and domestic investors. If, instead, domestic shocks are much more volatile than foreign shocks, then variation in the exchange rate and variation in the *domestic* pricing kernel are more-or-less the same thing. The domestic investor then views foreign currency as risky because its value is being dominated by the same shocks as is his marginal utility. The foreign investor, in contrast, feels *relatively* sanguine about exchange rate variation. It is *relatively* unrelated to whatever it is that is affecting his marginal utility. Hence, the risk premium on the foreign currency must be positive and the premium on the domestic currency (which, of course is the "foreign" currency for the foreign investor) must be negative. While the latter might seem counterintuitive — the foreign investor views currency as a *hedge* — it is inescapable. If GBP pays a positive premium then USD *must* pay a negative premium.[11] Remember it like

[10] The language, 'inflation surprise,' while commonplace in the literature, is obviously loose. Inflation is an endogenous variable. Better language — language that is precise in our specific setting — is that a relatively large value for τ_π translates into a stronger impulse response in the nominal interest rate to any shock that generates a positive impulse response in inflation.

[11] One should take care not to confuse this with "Siegel's Paradox," the statement that — because of the ubiquitous Jensen's inequality term — the forward rate cannot equal the conditional mean of the future spot rate, irrespective of the choice of the currency numeraire. This *is not* what is going on here. The Jensen's term is (one half of) the variance in the difference of the (log) kernels, not the difference in the variances from Equation (29). To make this crystal-clear, consider the case in which this entire discussion *would be* a futile exercise in Siegel's Paradox. Suppose that the log kernels are independent of one-another with constant and identical conditional variances.

this; "high variability in marginal utility means low tolerance for exchange-rate risk and, therefore, a high risk premium on foreign currency."

Now that we've established intuition for Equation (29) we can turn to monetary policy. Why does a weak price stabilization policy result in a *highly* variable pricing kernel? The key, interestingly, is the other policy parameter, τ_x. Recall the basic definition of the nominal pricing kernel:

$$\begin{aligned} \log m_{t+1} &= \log n_{t+1} - \pi_{t+1} \\ \Longrightarrow Var\big(\log m_{t+1}\big) &= Var\big(\log n_{t+1}\big) + Var\big(\pi_{t+1}\big) - 2\,Cov\big(\log n_{t+1}\,,\,\pi_{t+1}\big)\ . \end{aligned}$$

In Appendix # we show something quite intuitive. If τ_x is large enough, then the covariance term is typically positive enough so that $Var(m_{t+1}) < Var(n_{t+1})$; the nominal pricing kernel is less variable than the marginal rate of substitution. This is not the case for any set of parameter values, but it is for the economically-interesting ones that we characterize in our calibration (Section 7). It is a fairly typical characteristic of most New Keynesian models (# references) and was first pointed out in our specific class of models by Gallmeyer, Hollifield, Palomino, and Zin (2007). It is also empirically-plausible in the sense that a large value for τ_x implies an endogenous inflation process that is negatively correlated with consumption growth — thus implying a positive correlation between inflation and marginal utility in Equation (30) — something we typically see in the data.

We find this implication of monetary policy to be interesting, irrespective of how things work out for exchange rates. What's intriguing is that a policy which seeks to fulfill the "dual mandate" by reacting to real economic activity will typically generate *inflation risk*; a negative (positive) correlation between consumption growth (marginal utility) and inflation. This means that securities denominated in nominal units will have expected returns that incorporate an inflation risk premium. What it also means, however, is that *nominal risk is less than real risk*; the nominal pricing kernel is less variable than its real counterpart. Sharpe ratios on nominal risky assets will therefore tend to be *less* than those on real risky assets (Hansen and Jagannathan (1991)). Of course, we don't observe data on the latter, but the implication seem interesting nevertheless.

Turning now to the policy parameter of direct interest, τ_π, we get to the heart of the story. Why does a relatively large value for τ_π translate into a larger *foreign* currency premium? Because it undoes the effect of τ_x. That is, as τ_π gets large, the variance of endogenous inflation decreases (*i.e.*, the as the central bank 'cares more' about inflation it drives the variability of inflation to zero). Thus, the extent to which $Var(m_{t+1}) < Var(n_{t+1})$ is mitigated which, because $Var(n_{t+1})$ is exogenous

Then the log risk premium, p_t is zero, and the level risk premium is $-Var_t(d_{t+1})/2$, the familiar Siegel-Jensen term. The above discussion, and our model, presume no such independence nor homoscedasticity. The name-of-the-game is the covariance term, $Cov_t(d_{t+1}, m_{t+1})$ (or its foreign-kernel equivalent).

in our setting, must mean that $Var(m_{t+1})$ increases. Thus the foreign-currency risk premium from Equation (29) increases.

Summarizing, then, the economic intuition goes as follows. A procyclical interest rate rule makes the nominal economy "less risky" than the real economy. A stronger interest rate reaction to inflation *undoes* this. It makes domestic state prices more variable so that domestic residents view currency as being more risky *relative to* foreign residents. In a nutshell, 'weak' interest rate rules make for riskier currencies. This seems to accord with the data. The countries with (supposedly) strong anti-inflation stances — *e.g.*, Germany, Japan, Switzerland — have, on average, had low interest rates, low inflation and negative risk premiums.

5 Abstracting from Real Exchange Rates

The crux of our question asks "how does Taylor-rule-implied inflation affect exchange rates?" In this section we try to clarify things further by abstracting from real exchange rate variation. We set $n_t = n_t^*$, implying that $\log(S_t/S_{t-1}) = \pi_t - \pi_t^*$, so that relative PPP holds exactly. We don't take this specification seriously for empirical analysis. We use it to try to understand exactly how the Taylor rule restricts inflation dynamics and, therefore, nominal exchange rate dynamics. As we'll see in Section 6, the lessons we learn carry over to more empirically-relevant models with both nominal and real variability.

We use simplest possible variant of the Taylor rule:

$$i_t = \tau + \tau_\pi \pi_t + z_t \quad , \tag{30}$$

where z_t is a 'policy shock' that follows the process

$$\begin{aligned} z_{t+1} &= (1-\varphi_z)\theta_z + \varphi_z z_t + \sqrt{v_t}\epsilon^z_{t+1} & (31) \\ v_{t+1} &= (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon^v_{t+1} \quad . & (32) \end{aligned}$$

There are, of course, many alternative specifications. A good discussion related to asset pricing is Ang, Dong, and Piazzesi (2007). Cochrane (2007) uses a similar specification to address issues related to price-level determinacy and the identification of the parameters in Equation (30). We begin with it for reasons of tractability and clarity. We then go on to include the nominal depreciation rate and the lagged interest rate.

In addition to $n_t = n_t^*$, we abstract from real interest rate variation by setting $n_t = n_t^* = 1$. For exchange rates, conditional on having $n_t = n_t^*$, this is without loss of generality. The (nominal) short interest rate, $i_t = -\log E_t\, m_{t+1}$, is therefore

$$\begin{aligned} i_t &= -\log E_t\, e^{-\pi_{t+1}} \\ &= E_t\, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1}) \quad . & (33) \end{aligned}$$

The Taylor rule (30) and the Euler equation (33) imply that inflation must satisfy the following difference equation:

$$\pi_t = -\frac{1}{\tau_\pi}\left(\tau + z_t + E_t\,\pi_{t+1} - \frac{1}{2}\,Var_t(\pi_{t+1})\right) \quad . \tag{34}$$

Given the log-linear structure of the model, guess that the solution has the form,

$$\pi_t = a + a_1 z_t + a_2 v_t \quad . \tag{35}$$

Instead of solving Equation (34) forward, just substitute Equation (35) into the Euler equation (33), compute the moments, and then solve for the a_i coefficients by matching up the result with the Taylor rule (30). This gives,

$$\begin{aligned} a &= \frac{C-\tau}{\tau_\pi} \\ a_1 &= \frac{1}{\varphi_z - \tau_\pi} \\ a_2 &= \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} \end{aligned}$$

where

$$C \equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2 \quad .$$

More explicit derivations are given in Appendix A. Inflation and the short rate can now be written as:

$$\begin{aligned} \pi_t &= \frac{C-\tau}{\tau_\pi} + \frac{1}{\varphi_z-\tau_\pi} z_t + \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} v_t \\ i_t &= C + \frac{\varphi_z}{\varphi_z-\tau_\pi}\, z_t + \frac{\tau_\pi}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}\, v_t \\ &= C + \varphi_z a_1 z_t + \tau_\pi a_2 v_t \quad , \end{aligned}$$

and the pricing kernel as

$$\begin{aligned} -\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\epsilon^z_{t+1} + \sigma_v a_2 \epsilon^v_{t+1} \\ &= D + \frac{1}{\varphi_z-\tau_\pi}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} v_t \\ &\quad + \frac{1}{\varphi_z-\tau_\pi} v_t^{1/2}\epsilon^z_{t+1} + \frac{\sigma_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}\epsilon^v_{t+1} \end{aligned} \tag{36}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2 \quad .$$

Now consider a foreign country, say the UK. Denote all foreign variables with an asterisk. The foreign Taylor rule is

$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + z_t^* \quad .$$

with z_t^* and its volatility following processes analogous to Equations (31–32). For now, z_t and z_t^* can have any correlation structure. Repeating the above calculations for the UK and then substituting the results into Equations (11–14) we get

$$\begin{aligned} i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_\pi a_2 v_t - \tau_\pi^* a_2^* v_t^* \\ q_t &= D - D^* + a_1 \varphi_z z_t - a_1^* \varphi_z^* z_t^* + a_2 \varphi_v v_t - a_2^* \varphi_v^* v_t^* \\ p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right) \end{aligned}$$

where $D \equiv C + (\sigma_v a_2)^2/2$. It is easily verified that $p_t + q_t = i_t - i_t^*$.

Result 1: *Symmetry and* $\varphi_z = 0$

If all foreign and domestic parameter values are the same and $\varphi_z = \varphi_z^* = 0$, then the UIP regression parameter (7) is:

$$b = \frac{Cov(i_t - i_t^* \, , \, q_t)}{Var(i_t - i_t^*)} \quad = \quad \frac{Cov(p_t + q_t, q_t)}{Var(p_t + q_t)} \tag{37}$$

$$= \quad \frac{\varphi_v}{\tau_\pi} \tag{38}$$

Calculations are provided in Appendix A.

5.1 Discussion

The sign of $Cov(p_t, q_t)$ does not depend on φ_z. That is, $Cov(p_t, q_t)$ is essentially the covariance between the kernel's mean and its variance and, while v_t appears in both, z_t appears only in the mean. The assumption $\varphi_z = 0$ is therefore relatively innocuous in the sense that it has no effect on one of the two necessary conditions (8) and (9).

We require $\tau_\pi > 1$ for the solution to make sense. Therefore, according to Equation (38), $0 < b < 1$ unless $\varphi_v < 0$. The latter is implausible. Nevertheless, the UIP regression coefficient can be significantly less than unity and the joint distribution of exchange rates and interest rates will admit positive expected excess returns on a suitably-defined trading strategy.

We cannot, at this point, account for $b < 0$. But the model does deliver some insights into our basic question of how Taylor rules restrict inflation dynamics and, consequently, exchange rate dynamics. We summarize with several remarks.

Remark 1: *This is not just a relabeled affine model*

Inspection of the pricing kernel, Equation (36), indicates that it is basically a log-linear function of two unobservable factors. Is what we are doing just a relabeling of the class of latent-factor affine models described in Backus, Foresi, and Telmer (2001)? The answer is no and the reason is that the Taylor rule imposes economically-meaningful restrictions on the model's coefficients.

To see this consider a pricing kernel of the form

$$-\log m_{t+1} \quad = \quad \alpha + \chi v_t + \gamma v_t^{1/2} \varepsilon_{t+1} \tag{39}$$

where v_t is an arbitrary, positive stochastic process, and an analogous expression describes m^*_{t+1}. Backus, Foresi, and Telmer (2001) show that such a structure generates a UIP coefficient $b < 0$ if $\chi > 0$ and $\chi < \gamma^2/2$. The former condition implies that the mean and variance of negative the log kernel move in the same direction — this gives $Cov(p_t, q_t) < 0$ — and the latter implies that the variance is more volatile so that $Var(p_t) > Var(q_t)$.

Now compare Equations (39) and (36). The Taylor rule imposes the restrictions that χ can only be positive if φ_v is negative (because $a_2 < 0$ since $\tau_\pi > 1$) and that $\chi/\gamma = \rho_v/(2\tau_\pi(\tau_\pi - \varphi_v))$. Both χ and γ are restricted by value of the policy parameter τ_π, and the dynamics of the volatility shocks. In words, the UIP evidence requires the mean and the variance of the pricing kernel to move in particular ways relative to each other. The Taylor rule and its implied inflation dynamics place binding restrictions on how this can happen. The unrestricted pricing kernel in Equation (39) can account for $b < 0$ irrespective of the dynamics of v_t. Imposing the Taylor rule says that v_t must be negatively autocorrelated.

Remark 2: *Reason that negatively-correlated volatility is necessary for $b < 0$?*

First, note that $a_2 < 0$, so that an increase in volatility v_t decreases inflation π_t. Why? Suppose not. Suppose that v_t increases. Then, since $\tau_\pi > 1$, the Taylor rule implies that the interest rate i_t must increase *by more* than inflation π_t. However this contradicts the stationarity of inflation which implies that the conditional mean must increase *by less* than the contemporaneous value. Hence $a_2 < 0$. A similar argument implies that $a_1 < 0$ from Equation (35). The point is that the dynamics of Taylor-rule implied inflation, at least as long as the real exchange rate is constant, are driven by the *muted response of the interest rate* to a shock, relative to that of the inflation rate.

Next, to understand why $\varphi_v < 0$ is necessary for $b < 0$, consider again an increase in volatility v_t. Since $a_2 < 0$, the U.S. interest rate i_t and the contemporaneous inflation rate π_t must decline. But for $b < 0$ USD must be expected to *depreciate*. This means that, although π_t decreases, $E_t\pi_{t+1}$ must increase. This means that volatility must be negatively autocorrelated.

Finally, consider the more plausible case of positively autocorrelated volatility, $0 < \varphi_v < 1$. Then $b < 1$ which is, at least, going in the right direction (*e.g.*, Backus, Foresi, and Telmer (2001) show that the vanilla Cox-Ingersoll-Ross model generates $b > 1$). The reasoning, again, derives from the 'muted response of the interest rate' behavior required by the Taylor rule. This implies that $Cov(p_t, q_t) > 0$ — thus violating Fama's condition (8) — which says that if inflation and expected inflation move in the same direction as the interest rate (because $\varphi_v > 0$), then so must the USD currency risk premium. The Bilson-Fama regression (6) can be written

$$q_t \quad = \quad c + b(p_t + q_t) - \text{forecast error} \ \ ,$$

where 'forecast error' is defined as $s_{t+1} - s_t - q_t$. Since $Cov(p_t, q_t) > 0$, then $Var(p_t + q_t) > Var(q_t)$ and, therefore, $0 < b < 1$.

Even more starkly, consider the case of $\varphi_v = 0$ so that $b = 0$. Then the exchange rate is a random walk — *i.e.*, $q_t = 0$ so that $s_t = E_t s_{t+1}$ — and all variation in the interest rate differential is variation in the risk premium, p_t. Taylor rule inflation dynamics, therefore, say that for UIP to be a good approximation, changes in volatility must show up strongly in the conditional mean of inflation and that this can only happen if volatility is highly autocorrelated.

Remark 3: *Identification of policy parameters*

Cochrane (2007) provides examples where policy parameters like τ_π are impossible to distinguish from the parameters of the unobservable shocks. Result 1 bears similarity to Cochrane's simplest example. We can estimate b from data but, if we can't estimate φ_v directly then there are many combinations of φ_v and τ_π that are consistent with any estimate of b.

Identification in our special case, however, is possible because of the conditional variance term in the interest rate equation: $i_t = E_t\pi_{t+1} - Var_t\pi_{t+1}$. To see this note that, with $\varphi_z = 0$, the autocorrelation of the interest rate is φ_v and, therefore, φ_v is identified by observables. Moreover,

$$\frac{i_t}{E_t\pi_{t+1}} = \frac{\tau_\pi}{\varphi_v} \ \ ,$$

which identifies τ_π because the variables on the left side are observable.

The more general case of $\varphi_z \neq 0$ doesn't work out as cleanly, but it appears that the autocorrelation of inflation and the interest rate jointly identify φ_z and φ_v and the above ratio again identifies the policy parameter τ_π. These results are all special cases of those described in Backus and Zin (2008).

5.2 Asymmetric Taylor Rules

The series of affine models outlined in Backus, Foresi, and Telmer (2001) suggest that asymmetries between the foreign and domestic pricing kernels are likely to play a critical role in achieving $b < 0$. Their approach is purely statistical in nature. There are many parameters and few sources of guidance for which asymmetries are plausible and which are not. This section asks if foreign and domestic Taylor rule asymmetries are plausible candidates.

Suppose that foreign and domestic Taylor rules depend on the exchange rate in addition to domestic inflation and a policy shock:

$$i_t = \tau + \tau_\pi \pi_t + z_t + \tau_3 \log(S_t/S_{t-1}) \tag{40}$$
$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + z_t^* + \tau_3^* \log(S_t/S_{t-1}) \tag{41}$$

The asymmetry that we'll impose is that $\tau_3 = 0$ so that the Fed does not react to the depreciation rate whereas the Bank of England does. Foreign central banks reacting more to USD exchange rates seems plausible. It's also consistent with some empirical evidence in, for example, Clarida, Galí, and Gertler (1999), Engel and West (2006), and Eichenbaum and Evans (1995).

Assuming the same processes for the state variables as Equations (31) and (32) (and their foreign counterparts), guess that the inflation solutions look like:

$$\begin{aligned}
\pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\
\pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t
\end{aligned}$$

and collect the state variables into the vector

$$X_t^\top \equiv \left[z_t \ z_t^* \ v_t \ v_t^* \right]^\top .$$

Interest rates, from Euler equations with real interest rate = 0, must satisfy:

$$\begin{aligned}
i_t &= C + B^\top X_t \\
i_t^* &= C^* + B^{*\top} X_t
\end{aligned}$$

where,

$$\begin{aligned}
B^\top &\equiv \left[\begin{array}{cccc} a_1 \varphi_z & a_2 \varphi_z^* & (a_3 \varphi_v - \frac{a_1^2}{2}) & (a_4 \varphi_v^* - \frac{a_2^2}{2}) \end{array} \right] \\
C &\equiv a + a_1 \theta_z (1 - \varphi_z) + a_2 \theta_z^* (1 - \varphi_z^*) + a_3 \theta_v (1 - \varphi_v) + a_4 \theta_v^* (1 - \varphi_v^*) - \frac{1}{2}\left(a_3^2 \sigma_v^2 + a_4^2 \sigma_v^{*2}\right) \\
B^{*\top} &\equiv \left[\begin{array}{cccc} a_1^* \varphi_z & a_2^* \varphi_z^* & (a_3^* \varphi_v - \frac{a_1^{*2}}{2}) & (a_4^* \varphi_v^* - \frac{a_2^{*2}}{2}) \end{array} \right] \\
C^* &\equiv a^* + a_1^* \theta_z (1 - \varphi_z) + a_2^* \theta_z^* (1 - \varphi_z^*) + a_3^* \theta_v (1 - \varphi_v) + a_4^* \theta_v^* (1 - \varphi_v^*) - \frac{1}{2}\left(a_3^{*2} \sigma_v^2 + a_4^{*2} \sigma_v^{*2}\right)
\end{aligned}$$

The solution for the a coefficients and the following result are provided in Appendix B.

Result 2: *Asymmetric reaction to exchange rates*

> If foreign and domestic Taylor rules are Equations (40) and (41), with $\tau_3 = 0$ and all remaining foreign and domestic parameter values the same, then $b < 0$ if $\tau_3^* > \tau_\pi$.

Remark 4: *Pathological policy behavior?*

Interpreted literally, $\tau_3^* > 0$ means that the Bank of England reacts to an *appreciation* in GBP by increasing the British interest rate. However, at the same time, there exist sensible calibrations of the model in which $Cov(i_t^*, \log(S_t/S_{t-1})) > 0$. This makes the obvious point that the Taylor rule coefficients must be interpreted with caution since all the endogenous variables in the rule are responding to the same shocks.

5.3 McCallum's Model

McCallum (1994), Equation (17), posits a policy rule of the form

$$i_t - i_t^* \quad = \quad \lambda\big(s_t - s_{t-1}\big) + \sigma\big(i_{t-1} - i_{t-1}^*\big) + \zeta_t \ \ ,$$

where ζ_t is a policy shock. He also defines UIP to include an exogenous shock, ξ_t, so that

$$i_t - i_t^* = E_t\big(s_{t+1} - s_t\big) + \xi_t \ \ .$$

McCallum solves the implicit difference equation for $s_t - s_{t-1}$ and finds that it takes the form

$$s_t - s_{t-1} \quad = \quad -\sigma/\lambda\big(i_t - i_{t-1}\big) - \lambda^{-1}\zeta_t + \big(\lambda + \sigma\big)^{-1}\xi_t$$

He specifies values $\sigma = 0.8$ and $\lambda = 0.2$ — justified by the policy-makers desire to smooth interest rates and 'lean-into-the-wind' regarding exchange rates — which resolve the UIP puzzle by implying a regression coefficient from our Equation (6) of $b = -4$. McCallum's insight was, recognizing the empirical evidence of a risk premium in the interest rate differential, to understand that the policy rule and the equilibrium exchange rate must respond to the same shock that drives the risk premium.

In this section we show that McCallum's result can be recast in terms of our pricing kernel model and a policy rule that targets the interest rate itself, not the interest rate differential. The key ingredient is a lagged interest rate in the policy rule:

$$i_t = \tau + \tau_\pi \pi_t + \tau_4 i_{t-1} + z_t \quad , \tag{42}$$

where the processes for z_t and its volatility v_t are the same as above. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t + a_3 i_{t-1} \quad , \tag{43}$$

Substitute Equation (43) into the pricing kernel and compute the expectation:

$$i_t = \frac{1}{1-a_3}\Big(C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t\Big) \quad ,$$

where

$$C \equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2$$

Match-up the coefficients with the Taylor rule and solve for the a_j parameters:

$$\begin{aligned}
a &= \frac{C}{\tau_\pi + \tau_4} - \frac{\tau}{\tau_\pi} \\
a_1 &= \frac{\tau_\pi + \tau_4}{\tau_\pi(\varphi_z - \tau_\pi - \tau_4)} \\
a_2 &= \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi^2(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} \\
a_3 &= -\frac{\tau_4}{\tau_\pi}
\end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_\pi - \tau_4)}$$

and that matching coefficients imply

$$\frac{a_1\varphi_z}{1-a_3} = 1 + \tau_\pi a_1 \quad ; \quad \frac{a_2\varphi_v - a_1^2/2}{1-a_3} = \tau_\pi a_2.$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C}{\tau_\pi + \tau_4} - \frac{\tau}{\tau_\pi} + \frac{\tau_\pi + \tau_4}{\tau_\pi(\varphi_z - \tau_\pi - \tau_4)} z_t + \\
&+ \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi^2(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} v_t - \frac{\tau_4}{\tau_\pi} i_{t-1} \\
i_t &= \frac{\tau_\pi}{\tau_\pi + \tau_4} C + \frac{\varphi_z}{\varphi_z - \tau_\pi - \tau_4} z_t + \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} v_t \\
&= \frac{1}{1-a_3}\Big(C + \varphi_z a_1 z_t + (\tau_\pi + \tau_4) a_2 v_t\Big)
\end{aligned}$$

The pricing kernel is

$$-\log m_{t+1} = D + \frac{a_1\varphi_z}{1-a_3}z_t + \frac{a_2\varphi_v - a_3a_1^2/2}{1-a_3}v_t + a_1v_t^{1/2}\epsilon_{t+1}^z + \sigma_v a_2\epsilon_{t+1}^v$$

where

$$D \equiv \frac{C}{1-a_3} + (\sigma_v a_2)^2/2$$

The GBP-denominated kernel and variables are denoted with asterisks. If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$), the interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$\begin{aligned}
i_t - i_t^* &= \frac{a_1\varphi_z}{1-a_3}(z_t - z_t^*) + \frac{a_2\varphi_v - a_1^2/2}{1-a_3}(v_t - v_t^*) \\
q_t &= \frac{a_1\varphi_z}{1-a_3}(z_t - z_t^*) + \frac{a_2\varphi_v - a_3a_1^2/2}{1-a_3}(v_t - v_t^*) \\
p_t &= -\frac{1}{2}a_1^2(v_t - v_t^*)
\end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

The nominal interest rate and the interest rate differential have the same autocorrelation:

$$\begin{aligned}
Corr(i_{t+1}, i_t) &= Corr(i_{t+1} - i_{t+1}^*, i_t - i_t^*) \\
&= 1 - (1-\varphi_z)(1+\tau_\pi a_1)^2\frac{Var(z_t)}{Var(i_t)} - (1-\varphi_v)(\tau_\pi a_2)^2\frac{Var(v_t)}{Var(i_t)}.
\end{aligned}$$

If we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned}
b &= \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} \\
&= \frac{\varphi_v - \tau_4}{\tau_\pi}
\end{aligned}$$

To see the similarity to McCallum's model define $\zeta \equiv z_t - z_t^*$, and subtract the UK Taylor rule from its U.S. counterpart in (42). Assuming symmetry, we get

$$\begin{aligned}
i_t - i_t^* &= \tau_\pi(\pi_t - \pi_t^*) + \tau_4\big(i_t - i_t^*\big) + \zeta_t \\
&= \tau_\pi(s_t - s_{t-1}) + \tau_4\big(i_t - i_t^*\big) + \zeta_t\,,
\end{aligned}$$

where the second equality follows from market completeness and our simple pricing kernel model. This is the same as McCallum's policy rule with $\tau_\pi = \lambda$ and $\tau_4 = \sigma$.

His UIP "shock" is the same as our $p_t = -a_1^2(v_t - v_t^*)/2$, with $\varphi_z = \varphi_v = 0$. With $\varphi_v = 0$ we get the same UIP regression coefficient, $-\tau_4/\tau_\pi$. McCallum's model is basically a two-country Taylor rule model with a lagged interest rate in the policy rule and no dynamics in the shocks. Allowing for autocorrelated volatility diminishes the model's ability to account for a substantially negative UIP coefficient, a feature that McCallum's approach does not recognize. A value of $b < 0$ can only be achieved if volatility is less autocorrelated that the value of the interest rate smoothing policy parameter.

5.4 Summary

The goal of this section has been to ascertain how the imposition of a Taylor rule restricts inflation dynamics and how these restrictions are manifest in the exchange rate. What have we learned?

A good context for understanding the answer is the Alvarez, Atkeson, and Kehoe (2008) (AAK) paper. The nuts and bolts of their argument goes as follows. With lognormality, the nominal interest is

$$i_t \quad = \quad -E_t\big(\log m_{t+1}\big) - Var_t\big(\log m_{t+1}\big)/2$$

AAK argue that if exchange rates follow a random walk then variation in the conditional mean term must be small.[12] Therefore (according to them), "almost everything we say about monetary policy is wrong." The idea is that, in many existing models, the monetary policy transmission mechanism works through its affect on the conditional mean of the nominal marginal rate of substitution, m_t. But if exchange rates imply that the conditional mean is essentially a constant — so that 'everything we say is wrong' — then the mechanism must instead be working through the conditional variance.

If one takes the UIP evidence seriously, this isn't quite right. The UIP puzzle *requires* variation in the conditional means (*i.e.*, it says that exchange rates *are not* a random walk).[13] Moreover, it also requires that this variation be negatively correlated with variation in the conditional variances, and that the latter be larger than the former. In terms of monetary policy the message is that the standard

[12] *i.e.*, random walk exchange rates mean that $E_t \log(S_{t+1}/S_t) = 0$, and, from Equation (12), $E_t \log(S_{t+1}/S_t) = -E_t(\log m_{t+1} - \log m^*_{t+1})$. Random walk exchange rates, therefore, imply that the *difference* between the mean of the log kernels does not vary, not the mean of the log kernels themselves. More on this below.

[13] Of course, the variation in the forecast error for exchange rates dwarfs the variation in the conditional mean (*i.e.*, the R^2 from the Bilson-Fama-regressions is very small). Monthly changes in exchange rates certainly exhibit 'near random walk' behavior, and for policy questions the distinction may be a second-order effect. This argument, however, does not affect our main point regarding the AAK paper: that exchange rates are all about *differences* between pricing kernels and its hard to draw definitive conclusions about their *levels*.

story — that a shock that increases the mean (of the marginal rate of substitution) *decreases* the interest rate — is wrong. The UIP evidence says that we need to get used to thinking about a shock that increases the mean as *increasing* the interest rate, the reason being that the same shock must *decrease* the variance, and by more than it increases the mean.

Now, to what we've learned. We've learned that symmetric monetary policies as represented by Taylor rules of the form (30) can't deliver inflation dynamics that, by themselves, satisfy these requirements. The reason is basically what we label the 'muted response of the short rate'. The evidence requires that the conditional mean of inflation move by more than its contemporaneous value. But the one clear restriction imposed by the Taylor rule — that the interest rate must move *less* than contemporaneous inflation because the interest rate must also be equal to the conditional mean future inflation — says that this can't happen (unless volatility is negatively autocorrelated).

This all depends heavily on the real interest rate being a constant, something we relax in the next section. What's going on is as follows. In general, the Euler equation and the simplest Taylor rule can be written as

$$i_t = r_t + E_t\,\pi_{t+1} - \frac{Var_t(\pi_{t+1})}{2} + Cov_t(n_{t+1}, \pi_{t+1}) \tag{44}$$

$$i_t = \tau + \tau_\pi \pi_t + z_t \ . \tag{45}$$

The Euler equation (44) imposes restrictions between the current short rate and *moments* of future inflation. The Taylor rule (45) imposes an additional *contemporaneous* restriction between the current interest rate and current inflation. To see what this does, first ignore the real parts of Equation (44), r_t and the covariance term. Recalling that endogenous inflation will be a function $\pi(z_t, v_t)$, consider a shock to volatility that increases inflation by 1%.[14] The Taylor rule says that i_t must increase by *more than* 1%, say 1.2%. But, if inflation is a positively autocorrelated stationary process, then its conditional mean, $E_t\pi_{t+1}$, must increase by *less than* 1%, say 0.9%. Equation (44) says that the only way this can happen is if the conditional variance *decreases* by 0.2%; a volatility shock that increases π_t must decrease $Var_t\pi_{t+1}$. Therefore the mean and variance of the pricing kernel must move in the *same* direction, thus contradicting what Fama (1984) taught us is necessary for $b < 0$.

Phrased in terms of the exchange rate, the logic is equally intuitive. The increase in the conditional mean of inflation implies an expected devaluation in USD — recall that relative PPP holds if we ignore real rates — which, given the increasing interest rate implied by the Taylor rule, moves us in the UIP direction: high interest rates associated with a devaluing currency. Note that, if volatility

[14] A shock to z_t isn't particularly interesting in this context because it doesn't affect both the mean and variance of the pricing kernel.

were negatively autocorrelated, $E_t\pi_{t+1}$ would *fall* and the reverse would be true; we'd have $b < 0$.[15]

So, the contemporaneous restriction implied by the Taylor rule is very much a binding one for our question. This points us in two directions. First, it suggests that an interaction with the real interest rate is likely to be important. None of the above logic follows if r_t and $Cov_t(n_{t+1}, \pi_{t+1})$ also respond to a volatility shock. We follow this path in the next section. Second it points to something else that the AAK story doesn't get quite right. Exchange rate behavior tells us something about the *difference* between the domestic and foreign pricing kernels, not necessarily something about their *levels*. The above logic, and AAK's logic, is about levels, not differences. Symmetry makes the distinction irrelevant, but with asymmetry it's important. What our asymmetric example delivers is (i) inflation dynamics that, in each currency, satisfies 'muted response of the short rate' behavior, and (ii) a *difference* in inflation dynamics that gets the *difference* in the mean and the variance of the kernels moving in the right direction.

To see this, recall that $X_t^\top \equiv \begin{bmatrix} z_t & z_t^* & v_t & v_t^* \end{bmatrix}^\top$ and consider the foreign and domestic pricing kernels in the asymmetric model:

$$\begin{aligned}
-\log m_{t+1} &= \text{constants} + a_1\varphi_z z_t + a_3\varphi_v v_t + a_1 v_t^{1/2}\epsilon_{t+1}^z + a_3\sigma_v\epsilon_{t+1}^v \\
-\log m_{t+1}^* &= \text{constants} + A^\top \Lambda X_t + V(X_t)^{1/2}\begin{bmatrix}\epsilon_{t+1}^z \; \epsilon_{t+1}^{z^*} \; \epsilon_{t+1}^v \; w_{t+1}^*\end{bmatrix}^\top
\end{aligned}$$

where Λ is a diagonal matrix of autoregressive coefficients, and $V(X_t)$ is a diagonal matrix of conditional standard deviations. The asymmetric restriction that $\tau_3 = 0$ and $\tau_3^* \neq 0$ effectively makes this a 'common factor model' with asymmetric loadings on the common factors. A number of recent papers, Lustig, Roussanov, and Verdelhan (2009) for example, have argued persuasively for such a specification. What we've developed is one economic interpretation of their statistical exercise.[16]

More explicitly, consider the *difference* in the mean and variance of the log kernels from the symmetric and asymmetric examples of Sections 5 and 5.2. For the symmetric case we have

$$\begin{aligned}
p_t &= -\frac{1}{2}a_1^2(v_t - v_t^*) \\
q_t &= a_2\varphi_v(v_t - v_t^*)
\end{aligned}$$

[15]This intuition is also useful for understanding why we get $0 < b < 1$ with positively autocorrelated volatility. The RHS of the regression, the interest rate spread, contains *both* the mean and the variance of inflation. The LHS contains only the mean. If (negative) the mean and the variance move in the same direction, then the RHS is moving more than the LHS and the population value of b is less than unity.

[16]Note that if the conditional mean coefficients on z_t and v_t were the same across m and m^* then, contrary to AAK's assertion, monetary policy *could* affect the mean of the pricing kernel while still allowing for a random walk exchange rate. This is simply because z_t and v_t *would not appear* in the difference between the means of the two log kernels.

whereas for the asymmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2}\big(a_1^2 - a_1^{*2}\big)v_t + \frac{1}{2}a_4^* v_t^* \\ q_t &= \varphi_v\big(a_3 - a_3^*\big)v_t - a_4^* v_t^* \end{aligned}$$

where the a coefficients are functions of the model's parameters, outlined above and in more detail in the appendix. What's going on in the symmetric case is transparent. p_t and q_t can only be negatively correlated if $\varphi_v < 0$ (since $a_2 < 0$). The asymmetric case is more complex, but it turns out that what's critical is that $(a_3 - a_3^*) < 0$. This in turn depends on the difference $(\tau_\pi - \tau_3^*)$ being negative. Overall, what the asymmetric Taylor rule does is that it introduces an asymmetry in how a common factor between m and m^* affect their conditional means. This asymmetry causes the common factor to show up in exchange rates, and it can also flip the sign and deliver $b < 0$ with the right combination of parameter values.

6 Enhanced Model

We now turn to a calibration and quantitative assessment of our model. The specification in Section 3 is empirically inadequate. We know, for example, that it cannot account for volatility observed in interest rates, exchange rates and the nominal pricing kernel, at the same time as having realistic implications for the cross-country correlation of consumption growth (Brandt, Cochrane, and Santa-Clara (2006)). As a result, we follow Bansal and Yaron (2004), and the application to exchange rates of Bansal and Shaliastovich (2008), by modeling domestic consumption growth, x_{t+1}, as containing a small and persistent component (its 'long-run risk') with stochastic volatility:

$$\log(c_{t+1}/c_t) \equiv x_{t+1} = \mu + l_t + \sqrt{u_t}\,\epsilon^x_{t+1} \tag{46}$$

$$l_{t+1} = \varphi_l l_t + \sqrt{w_t}\,\epsilon^l_{t+1} \tag{47}$$

where

$$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \epsilon^u_{t+1} \tag{48}$$

$$w_{t+1} = (1-\varphi_w)\theta_w + \varphi_w w_t + \sigma_w \epsilon^w_{t+1} \tag{49}$$

Foreign consumption growth, x^*_{t+1} is defined analogously. The innovations are assumed to be multivariate normal and independent *within*-country: $(\epsilon^x, \epsilon^l, \epsilon^u, \epsilon^w)' \sim \text{NID}(0, I)$, but we allow for correlation *across* countries: $\eta_{\epsilon^j} \equiv Corr(\epsilon^j, \epsilon^{j^*})$, for $j = (x, l, u, w)$.

The process (46)–(49) looks complicated, but each of the ingredients are necessary. Stochastic volatility is necessary because without it the currency risk premium would be constant and the UIP regression parameter, b, would be 1.0. Long-run risk — by which we mean time variation in the conditional mean of consumption growth, l_t — isn't critical for exchange rates, but it is for achieving a realistic calibration of interest rates. It decouples the conditional mean of consumption growth from other moments of consumption growth, thereby permitting persistent and volatile interest rates to co-exist with relatively smooth and close-to-*i.i.d.* consumption growth. Finally, cross-country correlation in the innovations is critical for achieving realistic cross-country consumption correlations. The latter imposes substantial discipline on our calibration (*c.f.*, Brandt, Cochrane, and Santa-Clara (2006)).

We also allow for more flexibility in terms of the Taylor rules.[17]

$$i_t = \tau + \tau_\pi \pi_t + \tau_l l_t + z_t \ , \tag{50}$$

where z_t is a policy shock governed by

$$z_{t+1} = (1-\varphi_z)\theta_z + \varphi_z z_t + \sqrt{v_t}\epsilon^z_{t+1} \tag{51}$$

$$v_{t+1} = (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon^v_{t+1} \ . \tag{52}$$

Analogous equations, denoted with asterisks, characterize the foreign-country Taylor rules. We include the exogenous policy shocks, z_t, in order to allow for some flexibility in the distinction between real and nominal variables. Without policy shocks endogenous inflation will depend only on consumption shocks. The same will therefore be true of nominal exchange rates. We find it implausible that monthly variation in nominal exchange rates is 100% attributable to real shocks. Note that stochastic volatiliy in the policy shocks is a necessary condition for them to have any affect on currency risk premiums.

As before, we use the Hansen, Heaton, and Li (2005) linearization of the real pricing kernel around zero. The result is

$$-\log(n_{t+1}) = \delta^r + \gamma^r_l l_t + \gamma^r_u u_t + \gamma^r_w w_t \tag{53}$$

$$+ \lambda^r_x \sqrt{u_t}\epsilon^x_{t+1} + \lambda^r_l \sqrt{w_t}\epsilon^l_{t+1} + \lambda^r_u \sigma_u \epsilon^u_{t+1} + \lambda^r_w \sigma_w \epsilon^w_{t+1} \tag{54}$$

where

$$\gamma^r_l = (1-\rho); \quad \gamma^r_u = \frac{\alpha}{2}(\alpha-\rho); \quad \gamma^r_w = \frac{\alpha}{2}(\alpha-\rho)\omega^2_l$$

$$\lambda^r_x = (1-\alpha); \quad \lambda^r_l = -(\alpha-\rho)\omega_l; \quad \lambda^r_v = -(\alpha-\rho)\omega_u; \quad \lambda^r_w = -(\alpha-\rho)\omega_w$$

[17] For parsimony, we use expected consumption growth, l_t, and not its current level, x_t, as is instead standard in the literature. Doing so reduces our state space by one variable. The model can readily be extended to allow for a specification that includes x_t instead of l_t.

Details for the derivation, together with the expressions for the constant δ^r and the linearization coefficients ω_l, ω_u, and ω_w, can be found in Appendix D. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_l^r\ \gamma_u^r\ \gamma_w^r]'$ as real factor loadings and to $\lambda^r = [\lambda_x^r\ \lambda_l^r\ \lambda_u^r\ \lambda_w^r]'$ as real prices of risk.

The conditional mean of the real pricing kernel is equal to

$$E_t \log n_{t+1} = -(\delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t)$$

and its conditional variance is

$$Var_t \log n_{t+1} = \left(\lambda_x^r\right)^2 u_t + \left(\lambda_l^r\right)^2 w_t + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2$$

The conditional mean depends both on expected consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility processes only. Notice that, in the standard time and state separable utility case, volatility is not priced as a separate source of risk and the real pricing kernel collapses to the familiar:

$$-\log n_{t+1} = \delta^r + (1-\alpha) l_t + (1-\alpha)\sqrt{u_t}\epsilon_{t+1}^x$$

Next, the real short rate is

$$\begin{aligned} r_t &\equiv -\log E_t(n_{t+1}) \\ &= \bar{r} + \gamma_l^r l_t + r_u^r u_t + r_w^r w_t \end{aligned}$$

where

$$\bar{r} = \delta^r - \frac{1}{2}[(\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2] \ ,$$

and

$$r_u^r = \gamma_u^r - \frac{1}{2}(\lambda_x^r)^2; \qquad r_w^r = \gamma_w^r - \frac{1}{2}(\lambda_l^r)^2 \ .$$

Assuming symmetry, the expression for the expected real depreciation, q_t^r, the real forward premium, $f_t^r - s_t^r$, and the real risk premium, p_t^r, are:[18]

$$q_t^r = \gamma_l^r (l_t - l_t^*) + \gamma_u^r (u_t - u_t^*) + \gamma_w^r (w_t - w_t^*) \ ,$$

$$f_t^r - s_t^r = \gamma_l^r (l_t - l_t^*) + r_u^r (u_t - u_t^*) + r_w^r (w_t - w_t^*) \ ,$$

$$p_t^r = -\frac{1}{2}\left((\lambda_x^r)^2 (u_t - u_t^*) + (\lambda_l^r)^2 (w_t - w_t^*)\right) \ .$$

[18]Symmetry means that both the parameters governing the motion of the state variables and the preference parameters are the same across countries.

Result 3: *The real UIP slope coefficient*

If all foreign and domestic parameter values are the same, then the real UIP regression parameter, obtained by the regressing the real interest rate differential on the real depreciation rate is:

$$\begin{aligned} b^r &= \frac{Cov(f_t^r - s_t^r, q_t^r)}{Var(f_t^r - s_t^r)} \\ &= \frac{(\gamma_l^r)^2\, Var(l_t - l_t^*) + \gamma_u^r r_u^r\, Var(u_t - u_t^*) + +\gamma_w^r r_w^r\, Var(w_t - w_t^*)}{(\gamma_l^r)^2\, Var(l_t - l_t^*) + (r_u^r)^2\, Var(u_t - u_t^*) + (r_w^r)^2\, Var(w_t - w_t^*)} \end{aligned}$$

Without the presence of both stochastic volatility and EZ preferences, b^r is equal to one and, in real terms, UIP holds identically. Also, when the long-run state variables, l_t and w_t, are perfectly correlated across countries, the slope coefficient reduces to $b^r = \gamma_u^r / r_u^r$. This is the case considered by Bansal and Shaliastovich (2008).

For b^r to be negative, we require $Cov(f_t^r - s_t^r, q_t^r) < 0$. The expression above makes it evident that only stochastic volatility terms can contribute negatively to this covariance. In particular, a necessary condition for a negative real slope coefficient is that the γ^r and $r^r = (r_u^r, r_w^r)'$ coefficients have opposite sign, for at least one of the stochastic volatility processes. A preference for the early resolution of risk ($\alpha < \rho$) and an EIS larger than one ($\rho < 0$) deliver the required covariations.

6.1 Inflation and the Nominal Pricing Kernel

Following the technique developed above, we guess that the solution for endogenous inflation has the form

$$\pi_t = a + a_1 l_t + a_2 u_t + a_3 w_t + a_4 z_t + a_5 v_t \ ,$$

substitute it into the Euler equation (3), compute the moments, and then solve for the a_j coefficients by matching up the result with the Taylor rule (50). This gives,

$$a_1 = \frac{\gamma_l - \tau_l}{\tau_\pi - \varphi_l}; \quad a_2 = \frac{\gamma_u - \frac{1}{2}\lambda_x^2}{\tau_\pi - \varphi_u}; \quad a_3 = \frac{\gamma_w - \frac{1}{2}\lambda_l^2}{\tau_\pi - \varphi_w};$$

$$a_4 = \frac{-1}{\tau_\pi - \varphi_z}; \quad a_5 = \frac{-\frac{1}{2}a_4^2}{\tau_\pi - \varphi_v}$$

$$\begin{aligned} a &= \frac{1}{\tau_\pi - 1}[\delta - \tau + a_2(1-\varphi_u)\theta_u + a_3(1-\varphi_w)\theta_w + a_5(1-\varphi_v)\theta_v \\ &\quad - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2 + (\lambda_v\sigma_v)^2] \end{aligned}$$

where the constant term, the factor loadings and the pricing of risk of the nominal pricing kernel are

$$\delta = \delta^r + a + a_2(1-\varphi_u)\theta_u + a_3(1-\varphi_w)\theta_w + a_5(1-\varphi_v)\theta_v$$

$$\gamma_l = \gamma_l^r + a_1\varphi_l; \quad \gamma_u = \gamma_u^r + a_2\varphi_u; \quad \gamma_w = \gamma_w^r + a_3\varphi_w; \quad \gamma_z = a_4\varphi_z; \quad \gamma_v = a_5\varphi_v$$

$$\lambda_x = \lambda_x^r; \quad \lambda_l = \lambda_l^r + a_1; \quad \lambda_u = \lambda_u^r + a_2; \quad \lambda_w = \lambda_w^r + a_3; \quad \lambda_z = a_4; \quad \lambda_v = a_5$$

The linearized nominal pricing kernel is

$$\begin{aligned} -\log m_{t+1} &= -\log n_{t+1} + \pi_{t+1} \\ &= \delta + \gamma_l l_t + \gamma_u u_t + \gamma_w w_t + \gamma_z z_t + \gamma_v v_t \\ &+ \lambda_x \sqrt{u_t}\epsilon_{t+1}^x + \lambda_l \sqrt{w_t}\epsilon_{t+1}^l + \lambda_u \sigma_u \epsilon_{t+1}^u \\ &+ \lambda_w \sigma_w \epsilon_{t+1}^w + \lambda_z \sqrt{v_t}\epsilon_{t+1}^z + \lambda_v \sigma_v \epsilon_{t+1}^v \ . \end{aligned}$$

The Taylor rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices of risk. This would not be the case if the inflation process was exogenously specified. On the other hand, the factor loadings and the prices of risk of the nominal state variables, z_t and v_t, depend exclusively on the choice of the Taylor rule parameters.

The nominal short rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{\iota} + \gamma_l l_t + \gamma_z z_t + r_u v_t + r_w w_t + r_v v_t \ , \end{aligned}$$

where

$$\bar{\iota} = \delta - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2 + (\lambda_v\sigma_v)^2] \ ;$$

$$r_u = \gamma_u - \frac{1}{2}\lambda_x^2; \qquad r_w = \gamma_w - \frac{1}{2}\lambda_l^2; \qquad r_v = \gamma_v - \frac{1}{2}\lambda_z^2 \ .$$

The nominal interest rate differential, the expected depreciation rate and the risk premium can be derived from Equations (11–14). Assuming symmetry across countries, we have

$$q_t = \gamma_l(l_t - l_t^*) + \gamma_z(z_t - z_t^*) + \gamma_u(u_t - u_t^*) + \gamma_w(w_t - w_t^*) + \gamma_v(v_t - v_t^*) \ ,$$

$$f_t - s_t = \gamma_l(l_t - l_t^*) + \gamma_z(z_t - z_t^*) + r_u(u_t - u_t^*) + r_w(w_t - w_t^*) + r_v(v_t - v_t^*) \ ,$$

$$p_t = -\frac{1}{2}\left(\lambda_x^2(u_t - u_t^*) + \lambda_l^2(w_t - w_t^*) + \lambda_z^2(v_t - v_t^*)\right) \ .$$

Result 4: *The nominal UIP slope coefficient*

If all foreign and domestic parameter values are the same, the nominal UIP slope coefficient is

$$\begin{aligned} b &= \frac{Cov(f_t - s_t, q_t)}{Var(f_t - s_t)} \\ &= \frac{\gamma_l^2\, Var(l_t - l_t^*) + \gamma_z^2\, Var(z_t - z_t^*) + \gamma_u r_u\, Var(u_t - u_t^*) + \gamma_w r_w\, Var(w_t - w_t^*) + \gamma_v r_v\, Var(v_t - v_t^*)}{\gamma_l^2\, Var(l_t - l_t^*) + \gamma_z^2\, Var(z_t - z_t^*) + r_u^2\, Var(u_t - u_t^*) + r_w^2\, Var(w_t - w_t^*) + (r_v)^2\, Var(v_t - v_t^*)}\,. \end{aligned}$$

As was the case for the real UIP slope coefficient, without EZ preferences and stochastic volatility in consumption growth, long run risk and policy shock, $b = 1$.

6.1.1 Discussion

The results obtained in this section rely crucially on three ingredients: EZ preferences, stochastic volatility and the choice of the Taylor rule parameters. We now analyze their impact on the UIP slope coefficient and risk premium.

Remark 5: *With EZ preferences, volatility is priced as a separate source of risk*

From the previous section, we learned that if we want to explain the UIP puzzle we need stochastic volatility. In the model with real exchange rate variability, the necessary variation for the real UIP slope, b^r, comes from consumption growth, in the form of short-run volatility, u_t, and long-run volatility, w_t.

With standard expected utility ($\alpha = \rho$), both the volatility real factor loadings γ_u^r and γ_w^r, and the real prices of risk, λ_u^r and λ_w^r, collapse to zero. Consequently, the real UIP slope coefficient is identically equal to one. EZ preferences allow agents to receive a compensation for taking volatility risk, to which they would not be entitled with standard time-additive expected utility preferences. The contemporaneous presence of both stochastic volatility and EZ preferences is needed to explain the anomaly in real terms. Without stochastic volatility in the real pricing kernel, the real currency risk premium is constant and both of Fama's condition are violated. Without EZ preferences, stochastic volatility in consumption growth is not priced at all.

Remark 6: *The role of the Taylor parameters in the UIP slope coefficient*

Work in progress.

Remark 7: *The role of persistence in stochastic volatility*

Similarly to the purely nominal symmetric example of section 5, the persistence of country specific volatility φ_u plays a crucial role in the determination of the sign of the UIP slope. Too see this, consider again for simplicity the case in which the long-run factor l_t and its volatility w_t are perfectly correlated across countries. Also, assume the policy shock z_t is not autocorrelated ($\varphi_z = 0$). The nominal slope coefficient simplifies to

$$b \;=\; \frac{Cov(f_t - s_t, q_t)}{Var(f_t - s_t)} = \frac{\gamma_u r_u\, Var(u_t - u_t^*) + \gamma_v r_v\, Var(v_t - v_t^*)}{r_u^2\, Var(u_t - u_t^*) + (r_v)^2\, Var(v_t - v_t^*)} \ .$$

For the necessary condition of $Cov(f_t - s_t, p_t) < 0$ to be satisfied, we investigate the coefficients on short-run consumption volatility and policy shock volatility. First, γ_v and r_v cannot have opposite sign. The reason is the same as in the symmetric purely nominal example of the previous section: a shock to a *nominal* state variable of inflation, z_t or v_t, together with $\tau_\pi > 1$, imply the muted response of interest rate to a *nominal* shock, relative to that of the inflation rate. Therefore, unless the policy shock volatility is negatively autocorrelated, the contribution of the nominal state variables to $Cov(f_t - s_t, p_t)$ is necessarily of the wrong sign. As was the case for the purely nominal example, introducing asymmetries across countries, or allowing for interest rate smoothing in the Taylor rules can overcome this problem.

A different mechanism is at work for short-run consumption volatility. In this case, similarly to what we have seen above for the real slope coefficient, γ_u and r_u can have different signs, provided that the agents in the economy have preference for the early resolution of risk. However, a positive autocorrelation in stochastic volatility necessarily works against it. This is a direct consequence of endogenizing inflation and deriving the GHPZ monetary policy consistent pricing kernel. To see this, recall that

$$\gamma_u = \gamma_u^r + a_2 \varphi_u \ , \qquad a_2 = \frac{\gamma_u - \frac{1}{2}\lambda_x^2}{\tau_\pi - \varphi_u} = \frac{r_u}{\tau_\pi - \varphi_u} \ .$$

Since we require γ_u and r_u to have opposite signs for the resolution of the puzzle, we *must* have $a_2 < 0$. Therefore, $\gamma_u < \gamma_u^r$, and, all other things being equal, we require a stronger preference for the early resolution ($\alpha << \rho$) of risk, relative to the one we needed for the real case.

Consequently, it is in general harder to get a negative *nominal* UIP slope rather than a negative *real* UIP slope. As we have seen, the first reason is that the contribution of the nominal state variables necessarily goes in the wrong direction, at least in our simple symmetric case with Taylor rules reacting to (expected) consumption growth and current inflation. The second reason is that, with endogenous inflation, positive autocorrelated consumption volatility makes it harder to get the required magnitudes of the factor loadings and prices of risk of short-and long-run volatility. Nonetheless, a careful choice of Taylor parameters can deliver the required Fama conditions.

7 Quantitative Results

We'd like our model to be able to account for the following exchange rate facts. Foremost, of course, is the negative nominal UIP slope coefficient. But other important features are (i) UIP should hold unconditionally, so that the mean of the risk premium, p_t is zero, (ii) changes in real and nominal exchange rates are highly correlated (Mussa (1986)), (iii) exchange rate volatility is high relative to inflation differentials, (iv) exchange rates exhibit near random-walk behavior but interest rate differentials are highly autocorrelated, (v) international pricing kernels are highly correlated but international aggregate consumption growth rates are not (Brandt, Cochrane, and Santa-Clara (2006)). In addition, domestic real and nominal interest rates should be highly autocorrelated with means and volatilities that match data.

We calibrate our model using a monthly frequency. We begin by tying-down as much as we can using consumption data. The parameters for domestic and foreign aggregate consumption growth are chosen symmetrically so that (i) the mean and standard deviation match U.S. data, (ii) the autocorrelation is close to zero, (iii) the cross-country correlation is 0.30, and (iv) the autocorrelation of the conditional mean, l_t, is 0.993 and its cross-country correlation is 0.90 (following, roughly, Bansal and Shaliastovich (2008), Bansal and Yaron (2004) and Colacito and Croce (2008)). The autocorrelations of the short and long-run volatilities are chosen, primarily, to match the autocorrelation in interest rates and inflation rates. The parameters of the policy shock processes, z_t and z_t^*, are set so that the shocks are independent across countries and uncorrelated across time (*i.e.*, $\varphi_z = \varphi_z^* = 0$). Finally, the level and persistence of the volatility of the policy shocks are chosen — alongside risk aversion, intertemporal substitution, and the Taylor rule parameters — to match (i) the variance of the nominal exchange rate, (ii) the mean and variance of inflation and the nominal interest rate, (iii) the autocorrelation of the interest rate differential (forward premium), and (iv) the UIP regression parameter, b. The resulting parameter values are reported in Table 1.

Our model's population moments, evaluated at the parameter values of Table 1, are reported in Table 2. By and large, the model performs pretty well. Endogenous inflation — the focal point of our paper — matches the the sample mean, variance and autocorrelation of the U.S. data. The same applies for interest rates and the interest rate differential (the forward premium). Simulations of these variables are reported in Figures 1 and 2. Real and nominal exchange rates fit the Mussa (1986) evidence. See Figure 3. Nominal exchange rate variability is higher than in the data, but only slightly, at 17.41% versus 15.0%. This is good news in light of the point made by Brandt, Cochrane, and Santa-Clara (2006); the high pricing kernel variability required to explain asset prices (Hansen and Jagannathan (1991)) requires either highly correlated foreign and domestic pricing kernels, highly variable exchange rates, or some combination of the two. With standard preferences, low cross-country consumption correlations rule out the former, thus implying that observed exchange rate variability is *too small* relative to theory. Our model resolves this tension with the combination of recursive preferences and high correlation in cross-country long-run risk processes. This point has been made previously by Colacito and Croce (2008). Its empirical validity is an open question.

Where our model falls somewhat short is in the magnitude of the *nominal* UIP slope coefficient. While Fama's conditions are satisfied — see Figure 4 and the "carry trade" graph, Figure 5 — we nevertheless get $b = -1.07$ whereas a rough average from the data is around $b = -2.00$. Herein lies our overall message, which echos that of Section 5. The restrictions on inflation imposed by the Taylor rule are binding in the sense that, although the slope coefficient for real variables may be strongly negative, its nominal counterpart is less so. Put differently, if the mapping between real and nominal variables is an *exogenous* inflation process, then, given our real model, a realistic nominal slope coefficient would be easy to obtain. *Endogenous* inflation, on the other hand, ties one's hands in an important manner.

8 Conclusions

How is monetary policy related to the UIP puzzle? Ever since we've known about the apparent profitability of the currency carry trade people have speculated about a lurking role played by monetary policy. The story is that, for some reason, central banks find themselves on the short side of the trade, borrowing high yielding currencies to fund investments in low yielding currencies. In certain cases this has seemed almost obvious. It's well known, for instance, that in recent years the Reserve Bank of India has been accumulating USD reserves and, at the same time, sterilizing the impact on the domestic money supply through contractionary open-market operations. Since Indian interest rates have been relatively high, this policy basically *defines* what it means to be on the short side of the carry trade. This leads one to ask if carry trade losses are in some sense a *cost* of implementing Indian monetary policy? If so, is this a good policy? Is there some sense in which it is *causing* the exchange rate behavior associated with the carry trade?

Our paper's questions, while related, are less ambitious than these speculations about India. What we've shown goes as follows. It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency *units*:

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1} \exp(-\pi^*_{t+1})}{n_{t+1} \exp(-\pi_{t+1})} \quad .$$

It is less a tautology that we can write down sensible stochastic processes for these four variables that are consistent with the carry trade evidence.[19] Previous work has shown that such processes have many parameters that are difficult to identify with sample moments of data. Our paper shows two things. First, that by incorporating a Taylor rule for interest rate behavior we reduce the number of parameters. Doing so is sure to deteriorate the model's fit. But the benefit is lower dimensionality and parameters that are economically interpretable. Second, we've shown that some specifications of Taylor rules work and others don't. This seems helpful in and of itself. It also shows that there exist policy rules which, when combined with sensible pricing kernels, are *consistent* with the carry trade evidence. This is a far cry from saying that policy is *causing* carry trade behavior in interest rates and exchange rates, but it does suggest a connection that we find intriguing. In our models, for instance, there exist changes in the policy parameters, τ_π and τ_x, under which the carry trade profits go away.

Finally, it's worth noting that India, of course, is much more the exception than the rule. Most central banks — especially if we limit ourselves to those from OECD countries — don't have such explicit, foreign-currency related policies. However, many countries *do* use nominal interest rate targeting to implement domestic policy and, therefore, we can think about central banks and the carry trade in a *consolidated* sense. For example, in early 2004 the UK less U.S. interest rate differential was around 3%. Supposing that this was, to some extent, a policy *choice*, consider the open-market operations required to implement such policies. The Bank of England would be contracting its balance sheet — selling UK government bonds — while (at least in a relative sense) the Fed would be expanding its balance sheet by buying U.S. government bonds. If the infamous carry-trader is in between, going long GBP and short USD, then we can think of the Fed funding the USD side of the carry trade and the Bank of England providing the funds for the GBP side. In other words, the consolidated balance sheets of the Fed and Bank of England are short the carry trade and the carry-trader is, of course, long. In this sense, central banks and their interest-rate policies may be playing a more important role than is apparent by just looking at their foreign exchange reserves.

[19] See, for example, Backus, Foresi, and Telmer (2001), Bakshi and Chen (1997), Bansal (1997), Brenna and Xia (2006), Frachot (1996), Lustig, Roussanov, and Verdelhan (2009), and Saá-Requejo (1994).

References

Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe, 2007, Time-varying risk, interest rates, and exchange rates in general equilibrium, Working paper 371, Federal Reserve Bank of Minneapolis.

——— , 2008, If exchange rates are random walks, then almost everything we say about monetary policy is wrong, *Federal Reserve Bank of Minneapolis Quarterly Review* 32, 2–9.

Ang, Andrew, Sen Dong, and Monika Piazzesi, 2007, No-arbitrage taylor rules, Unpublished manuscript, Columbia University.

Backus, David K., Silverio Foresi, and Christopher I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* 56, 279–304.

Backus, David K., Allan W. Gregory, and Christopher I. Telmer, 1993, Accounting for forward rates in markets for foreign currency, *Journal of Finance* 48, 1887–1908.

Backus, David K., and Stanley E. Zin, 2008, Notes on identification of taylor rules, Unpublished notes.

Bakshi, Gurdip, and Zhiwu Chen, 1997, Equilibrium valuation of foreign exchange claims, *Journal of Finance* 52, 799–826.

Bansal, Ravi, 1997, An exploration of the forward premium puzzle in currency markets, *Review of Financial Studies* 10, 369–403.

——— , and Ivan Shaliastovich, 2008, A long-run risks explanation of predictability puzzles in bond and currency markets, Working Paper.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run, a potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Bekaert, Geert, 1994, Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model, *Journal of International Economics* 36, 29–52.

Benigno, Gianluca, 2004, Real exchange rate persistence and monetary policy rules, *Journal of Monetary Economics* 51, 473 – 502.

——— , and Pierpaolo Benigno, 2008, Exchange rate determination under interest rate rules, *Journal of International Money and Finance* 27, 971 – 993.

Bilson, John, 1981, The speculative efficiency hypothesis, *Journal of Business* 54, 435–452.

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671–698.

Brenna, Michael J., and Yihong Xia, 2006, International capital markets and foreign exchange risk, *Review of Financial Studies* 19, 753–795.

Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, 2006, The returns to currency speculation, Working Paper, Northwestern University.

Canova, Fabioi, and Jane Marrinan, 1993, Profits, risk and uncertainty in foreign exchange markets, *Journal of Monetary Economics* 32, 259–286.

Clarida, Richard, Jordi Gal, and Mark Gertler, 1998, Monetary policy rules in practice some international evidence, *European Economic Review* 42, 1033 – 1067.

Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, The science of monetary policy: A new keynesian perspective, *Journal of Economic Literature* XXXVII, 1661–1707.

Cochrane, John H., 2007, Inflation determination with taylor rules: A critical review, Working paper, University of Chicago.

Colacito, Ricardo, and Mariano Massimiliano Croce, 2008, Risks for the long run and the real exchange rate, Unpublished manuscript, University of North Carolina.

Dumas, Bernard, 1992, Dynamic equilibrium and the real exchange rate in a spatially separated world, *Review of Financial Studies* 5, 153–180.

Dutton, John, 1993, Real and monetary shocks and risk premia in forward markets for foreign exchange, *Journal of Money, Credit and Banking* 25, 731–754.

Eichenbaum, Martin S., and Charles L. Evans, 1995, Some empirical evidence on the effects of monetary policy shocks on exchange rates, *Quarterly Journal of Economics* 110, 975–1009.

Engel, Charles, 2011, The real exchange rate, real interest rates, and the risk premium, Unpublished manuscript, University of Wisconsin.

——— , and Kenneth D. West, 2006, Taylor rules and the deutschmark-dollar real exchange rate, *Journal of Money, Credit and Banking* 38, 1175–1194.

Epstein, L. G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Fama, Eugene F, 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–38.

Frachot, Antoine, 1996, A reexamination of the uncovered interest rate parity hypothesis, *Journal of International Money and Finance* 15, 419–437.

Gallmeyer, Michael F., Burton Hollifield, Francisco Palomino, and Stanley E. Zin, 2007, Arbitrage-free bond pricing with dynamic macroeconomic models, Working paper, Carnegie Mellon University.

Gavazzoni, Federico, 2008, Uncovered interest rate parity puzzle: An explanation based on recursive utility and stochastic volatility, Working Paper.

Grilli, Vittorio, and Nouriel Roubini, 1992, Liquidity and exchange rates, *Journal of International Economics* 32, 339–352.

Hansen, Lars P., John C. Heaton, and Nan Li, 2005, Consumption strikes back?: Measuring long-run risk, NBER Working Paper No. 11476.

Hansen, Lars P., and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.

Hansen, Lars Peter, and Kenneth Singleton, 1983, Stochastic consumption, risk aversion and the temporal behavior of stock returns, *Journal of Political Economy* 91, 249–265.

Hollifield, Burton, and Raman Uppal, 1997, An examination of uncovered interest rate parity in segmented international commodity markets, *Journal of Finance* 52, 2145–2170.

Lucas, Robert E, 1982, Interest rates and currency prices in a two-country world, *Journal of Monetary Economics* 10, 335–60.

Lustig, Hanno N., Nikolai L. Roussanov, and Adrien Verdelhan, 2009, Common risk factors in currency markets, Unpublished manuscript, UCLA.

Macklem, Tiff, 1991, Forward exchange rates and risk premiums in artificial economies, *Journal of International Money and Finance* 10, 365–391.

Marshall, David, 1992, Phd thesis,, Carnegie Mellon University.

McCallum, Bennett T., 1994, A reconsideration of the uncovered interest rate parity relationship, *Journal of Monetary Economics* 33, 105–132.

Mehra, R, and E Prescott, 1985, The equity premium puzzle, *Journal of Monetary Economics* 15, 145–61.

Mussa, Michael, 1986, Nominal exchange rate regimes and the behavior of real exchage rates: Evidence and implications, *Carnegie-Rochester Series on Public Policy* 25 (Autumn): 117-214.

Saá-Requejo, Jesús, 1994, The dynamics and the term structure of risk premium in foreign exchange markets, Unpublished manuscript, INSEAD.

Schlagenhauf, Don E., and Jeffrey M. Wrase, 1995, Liquidity and real activity in a simple open economy model, *Journal of Monetary Economics* 35, 431–461.

Sercu, Piet, Raman Uppal, and C. Van Hulle, 1995, The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity, *Journal of Finance* 50, 1309–1319.

Taylor, John B., 1993, Discretion versus policy rules in practice, *Carnegie Rochester Conferance Series on Public Policy* 39, 195–214.

Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, *Journal of Finance* 65, 123–145.

Woodford, Michael, 2003, *Interest and Prices* (Princeton University Press: Princeton NJ 08540 USA).

Appendix A
Symmetric Model

The short rate must satisfy both the Euler equation and the Taylor rule:

$$i_t = -\log E_t m_{t+1} \tag{A1}$$
$$i_t = \tau + \tau_\pi \pi_t + z_t \ , \tag{A2}$$

where the processes for z_t and its volatility v_t are

$$z_t = \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z$$
$$v_t = \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v$$

where ϵ_t^z and ϵ_t^v are *i.i.d.* standard normal. Given that $m_{t+1} = n_{t+1}P_t/P_{t+1}$ and $\pi_{t+1} = \log(P_{t+1}/P_t)$, set the real pricing kernel to a constant so that $m_{t+1} = \exp(-\pi_{t+1})$. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t \ , \tag{A3}$$

Substitute Equation (A3) into the Euler equation (A1) and compute the expectation. The result is

$$i_t = C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t \ , \tag{A4}$$

where

$$C \equiv -n + a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2$$

Substiute the postulated solution (A3) into the Taylor rule, match-up the resulting coefficients with those in Equation (A4), and solve for the a_i coefficients:

$$a = \frac{C-\tau}{\tau_\pi}$$
$$a_1 = \frac{1}{\varphi_z - \tau_\pi}$$
$$a_2 = \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_\pi)} \ .$$

Note that this is the same as saying that

$$\frac{\partial i_t}{\partial v_t} = \tau_\pi \frac{\partial \pi_t}{\partial v_t} = \frac{\partial E_t \pi_{t+1}}{\partial v_t} - \frac{1}{2}\frac{\partial Var_t \pi_{t+1}}{\partial v_t}$$

Similarly, $a_1 = 1/(\varphi_z - \tau_\pi)$ is the same as saying that

$$\frac{\partial i_t}{\partial z_t} = \tau_\pi \frac{\partial \pi_t}{\partial z_t} + 1 = \frac{\partial E_t \pi_{t+1}}{\partial z_t} - \frac{1}{2}\frac{\partial \, Var_t \pi_{t+1}}{\partial z_t} \quad .$$

Both of these things are kind of trivial. They just say that the effect of a shock on the Taylor rule equation must be consistent with the effect on the Euler equation.

Note also that

$$C = \frac{\tau_\pi}{\tau_\pi - 1}\Big(- n - \frac{\tau}{\tau_\pi} + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2\Big)$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C-\tau}{\tau_\pi} + \frac{1}{\varphi_z - \tau_\pi} z_t + \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v - \tau_\pi)} v_t \\
i_t &= C + \frac{\varphi_z}{\varphi_z - \tau_\pi}\, z_t + \frac{\tau_\pi}{2(\varphi_z-\tau_\pi)^2(\varphi_v - \tau_\pi)}\, v_t \\
&= C + \varphi_z a_1 z_t + \tau_\pi a_2 v_t
\end{aligned}$$

The pricing kernel is

$$\begin{aligned}
-\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\epsilon^z_{t+1} + \sigma_v a_2 \epsilon^v_{t+1} \\
&= D + \frac{1}{\varphi_z - \tau_\pi}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} v_t \\
&\quad + \frac{1}{\varphi_z - \tau_\pi} v_t^{1/2}\epsilon^z_{t+1} + \frac{\sigma_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}\epsilon^v_{t+1}
\end{aligned}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2$$

The GBP-denominated kernel and variables are denoted with asterisks. The interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_\pi a_2 v_t - \tau_\pi^* a_2^* v_t^* \\
q_t &= D - D^* + a_1\varphi_z z_t - a_1^*\varphi_z^* z_t^* + a_2\varphi_v v_t - a_2^*\varphi_v^* v_t^* \\
p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right)
\end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$) and if we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned}
b &= \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} \\
&= \frac{\varphi_v}{\tau_\pi}
\end{aligned}$$

Appendix B
Asymmetric Taylor Rule

Taylor rules

$$\begin{aligned} i_t &= \tau + \tau_\pi \pi_t + z_t + \tau_3 d_t \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + z_t^* + \tau_3^* d_t \\ d_t &\equiv \log(S_t/S_{t-1}) = \pi_t - \pi_t^* \end{aligned}$$

State variables,

$$\begin{aligned} z_t &= \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v \end{aligned}$$

and the associated foreign-country processes with asterisks and with all shocks *i.i.d.*. Collect them in the state vector, X_t:

$$X_t \equiv \left[z_t \; z_t^* \; v_t \; v_t^* \right]^\top$$

Inflation solutions:

$$\begin{aligned} \pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

Interest rates, from Euler equations with real interest rate = 0:

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where,

$$\begin{aligned} B^\top &\equiv \left[\; a_1\varphi_z \quad a_2\varphi_z^* \quad (a_3\varphi_v - \frac{a_1^2}{2}) \quad (a_4\varphi_v^* - \frac{a_2^2}{2}) \; \right] \\ C &\equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_z^*(1-\varphi_z^*) + a_3\theta_v(1-\varphi_v) + a_4\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\big(a_3^2\sigma_v^2 + a_4^2\sigma_v^{*2}\big) \\ B^{*\top} &\equiv \left[\; a_1^*\varphi_z \quad a_2^*\varphi_z^* \quad (a_3^*\varphi_v - \frac{a_1^{*2}}{2}) \quad (a_4^*\varphi_v^* - \frac{a_2^{*2}}{2}) \; \right] \\ C^* &\equiv a^* + a_1^*\theta_z(1-\varphi_z) + a_2^*\theta_z^*(1-\varphi_z^*) + a_3^*\theta_v(1-\varphi_v) + a_4^*\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\big(a_3^{*2}\sigma_v^2 + a_4^{*2}\sigma_v^{*2}\big) \end{aligned}$$

Taylor rules become:

$$\begin{aligned} i_t &= \tau + \tau_\pi(a + A^\top X_t) + z_t + \tau_3\big(a + A^\top X_t - a^* - A^{*\top} X_t\big) \\ &= \tau + \tau_\pi a + \tau_3(a - a^*) + \big(\tau_\pi A^\top + \iota_z^\top + \tau_3[A^\top - A^{*\top}]\big) X_t \\ i_t^* &= \tau^* + \tau_\pi^*(a^* + A^{*\top} X_t) + z_t^* + \tau_3^*\big(a + A^\top X_t - a^* - A^{*\top} X_t\big) \\ &= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) + \big(\tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*[A^\top - A^{*\top}]\big) X_t \end{aligned}$$

where $\iota_z^\top \equiv [\,1\;\;0\;\;0\;\;0\,]$ and $\iota_z^{*\top} \equiv [\,0\;\;1\;\;0\;\;0\,]$. Matching-up the coefficients means

$$\begin{aligned} C &= \tau + \tau_\pi a + \tau_3(a - a^*) \\ C^* &= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) \\ B &= \tau_\pi A^\top + \iota_z^\top + \tau_3(A^\top - A^{*\top}) \\ B^* &= \tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*(A^\top - A^{*\top}) \end{aligned}$$

To solve for the constants (the first two equations):

$$\begin{bmatrix} 1 - \tau_\pi - \tau_3 & \tau_3 \\ -\tau_3^* & 1 - \tau_\pi^* + \tau_3^* \end{bmatrix} \begin{bmatrix} a \\ a^* \end{bmatrix} = \begin{bmatrix} \tau - stuff \\ \tau^* - stuff^* \end{bmatrix}$$

where $stuff$ and $stuff^*$ are everything on the LHS of the solutions for C and C^*, except the first terms, a and a^*.

The B equations are eight equations in eight unknowns, A and A^*. Conditional on these, the C equations are two-in-two, a and a^*. The B equations can be broken into 4 blocks of 2. It's useful to write them out because you can see where the singularity lies.

$$\begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_z) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_z) \end{bmatrix} \begin{bmatrix} a_1 \\ a_1^* \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_z^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_z^*) \end{bmatrix} \begin{bmatrix} a_2 \\ a_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_v) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_v) \end{bmatrix} \begin{bmatrix} a_3 \\ a_3^* \end{bmatrix} = \begin{bmatrix} -a_1^2/2 \\ -a_1^{*2}/2 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_v^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_v^*) \end{bmatrix} \begin{bmatrix} a_4 \\ a_4^* \end{bmatrix} = \begin{bmatrix} -a_2^2/2 \\ -a_2^{*2}/2 \end{bmatrix}$$

Two singularities exist:

- *UIP holds exactly.* If $\tau_3 = 0$ (so that the Fed ignores the FX rate), $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ (complete symmetry in parameters, save τ_3 and τ_3^*) then a singularity is $\tau_3^* = \tau_\pi - \varphi_v$. As τ_3^* approaches this from below or above, the UIP coefficient goes to 1.0.

- *Anomaly resolved.* Similarly, if $\tau_3 = 0$, $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ then a singularity is $\tau_3^* = \tau_\pi$. As τ_3^* approaches from *below*, the UIP coefficient goes to infinity. As τ_3^* approaches from *above*, it goes to *negative* infinity.

The latter condition is where the UIP regression coefficient changes sign. This says that we need $\tau_3^* > \tau_3$. This may seem pathological. It says that — if we interpret these coefficients as policy responses (which we shouldn't) — the ECB responds to an appreciation in EUR by increasing interest rates *more* than 1:1 (and more than the 'Taylor principle' magnitude of $\tau_\pi > 1$).

Appendix C
Derivations for McCallum Model

Write the interest rate coefficients as follows:

$$\begin{aligned} i_t &= \frac{1}{1-a_3}\Big(C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t\Big) \\ &= c_i + c_{iz}z_t + c_{iv}v_t \end{aligned}$$

and, for reasons that will become clear, define

$$\tilde{i}_t \equiv i_t - \theta_z c_{iz} - \theta_v c_{iv}$$

The exogenous state variables obey

$$\begin{aligned} z_t &= \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v \end{aligned}$$

where a mean is now incorporated for z. I'm not sure if this thing is identified or not. Denote the state vector as $X_t^\top = [z_t \; v_t \; \tilde{i}_{t-1}]^\top$ so that we can write

$$X_t = (I-\Phi)\theta + \Phi X_{t-1} + V(X_{t-1})^{1/2}s_{t-1}$$

where

$$\begin{aligned} \theta^\top &= [\theta_z \; \theta_v \; c_i]^\top \\ \Phi &= \begin{bmatrix} \varphi_z & 0 & 0 \\ 0 & \varphi_v & 0 \\ c_{iz} & c_{iv} & 0 \end{bmatrix} \\ V(X_{t-1}) &= \begin{bmatrix} v_{t-1} & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ s_t^\top &= [\epsilon_t^z \; \epsilon_t^v \; 0]^\top \end{aligned}$$

The mean, variance and autocovariance of X are

$$\mu_X^\top = [\theta_z \quad \theta_v \quad C/(1-a_3)]$$

$$\begin{aligned} \Gamma_0 &= \begin{bmatrix} \frac{\theta_v}{1-\varphi_z^2} & 0 & \frac{c_{iz}\varphi_z\theta_v}{1-\varphi_z^2} \\ & \frac{\sigma_v^2}{1-\varphi_v^2} & \frac{c_{iv}\varphi_v\sigma_v^2}{1-\varphi_v^2} \\ & & \frac{c_{iz}^2\theta_v}{1-\varphi_z^2} + \frac{c_{iv}^2\sigma_v^2}{1-\varphi_v^2} \end{bmatrix} \\ \Gamma_1 &= \Phi\Gamma_0 \end{aligned}$$

Moments

- *Inflation.* Let $\pi_t = a_\pi + A_\pi^\top X_t$ where $A_\pi^\top = [a_1\ a_2\ a_3]$. Since

$$\pi_t = a + a_1 z_t + a_2 v_t + a_3 i_{t-1} \quad ,$$

we must have

$$a_\pi = a + a_3 \left(c_{iz}\theta_z + c_{iv}\theta_v \right) \quad .$$

The unconditional moments are:

$$\begin{aligned} \mu_\pi &= a_\pi + A_\pi^\top \mu_X \\ \sigma_\pi^2 &= A_\pi^\top \Gamma_0 A_\pi \\ Corr(\pi_t, \pi_{t-1}) &= A_\pi^\top \Gamma_1 A_\pi / \sigma_\pi^2 \end{aligned}$$

I worked one out by hand as a check:

$$\sigma_\pi^2 = (a_1\varphi_z + a_3 c_{iz})^2 \frac{\theta_v}{1-\varphi_z^2} + (a_2\varphi_v + a_3 c_{iv})^2 \frac{\sigma_v^2}{1-\varphi_v^2} + a_1^2\theta_v + (a_2\sigma_v)^2$$

The conditional moments are:

$$\begin{aligned} E_t\pi_{t+1} &= a_\pi + A_\pi^\top \big((I-\Phi)\theta + \Phi X_{t-1} \big) \\ Var_t\pi_{t+1} &= A_\pi^\top \begin{bmatrix} v_t & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_\pi \end{aligned}$$

- *Interest rate.* Let $i_t = c_i + C_i^\top X_t$, where $C_i^\top = [c_{iz}\ c_{iv}\ 0]$ and

$$\begin{aligned} C &= a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2 \\ c_i &= C/(1-a_3) \\ c_{iz} &= \varphi_z a_1/(1-a_3) \\ c_{iv} &= (\tau_\pi + \tau_4) a_2/(1-a_3) \end{aligned}$$

The moments are:

$$\begin{aligned} \mu_i &= c_i + C_i^\top \mu_X \\ \sigma_i^2 &= C_i^\top \Gamma_0 C_i \\ Corr(i_t, i_{t-1}) &= C_i^\top \Gamma_1 C_i / \sigma_i^2 \end{aligned}$$

- *Depreciation rate*: $d_t = \pi_t - \pi_t^*$. With independence across countries we have

$$\begin{aligned} \mu_\pi &= a_\pi - a_{\pi^*} + A^\top \mu_X - A^\top \mu_{X^*} \\ \sigma_d^2 &= \sigma_\pi^2 + \sigma_{\pi^*}^2 \\ Corr(d_t, d_{t-1}) &= \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma_{\pi^*}^2} Corr(\pi_t, \pi_{t-1}) + \frac{\sigma_{\pi^*}^2}{\sigma_\pi^2 + \sigma_{\pi^*}^2} Corr(\pi_t^*, \pi_{t-1}^*) \end{aligned}$$

So — obviously, in this model where relative PPP holds exactly — we have a strong counterfactual. The autocorrelation of the depreciation rate and the inflation rate are the same. Relaxing these things may work, to some extent. Here's a start:

$$\begin{aligned}
\mu_\pi &= a_\pi - a_\pi^* + A_\pi^\top \mu_X - (A_\pi^*)^\top \mu_{X^*} \\
\sigma_d^2 &= A_\pi^\top \Gamma_0 A_\pi + (A_\pi^*)^\top \Gamma_0^* A_\pi^* + Cov() \\
Corr(d_t, d_{t-1}) &= Cov(\pi_t, \pi_{t-1}) + Cov(\pi_t^*, \pi_{t-1}^*) + Cov(\pi_t, \pi_{t-1}^*) + Cov(\pi_{t-1}^*, \pi_t)
\end{aligned}$$

- *Interest rate differential*: $i_t - i_t^*$.

$$i_t - i_t^* = c_i - c_{i^*} + C_i^\top X_t - C_{i^*}^\top X_t^*$$

With independence, the moments are

$$\begin{aligned}
\mu_\pi &= c_i - c_{i^*} + C_i^\top \mu_X - C_{i^*}^\top \mu_{X^*} \\
Var(i_t - i_t^*) &= \sigma_i^2 + \sigma_{i^*}^2 \\
Corr(i_t - i_t^*\ ,\ i_{t-1} - i_{t-1}^*) &= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_{i^*}^2}\rho_i + \frac{\sigma_{i^*}^2}{\sigma_i^2 + \sigma_{i^*}^2}\rho_{i^*}
\end{aligned}$$

- *UIP Coefficient.* First the expected depreciation rate, with symmetry, is

$$\begin{aligned}
q_t = E_t d_{t+1} &= E_t(\pi_{t+1} - \pi_{t+1}^*) \\
&= a_\pi - a_{\pi^*} + A_\pi^\top \Phi X_t - A_{\pi^*}^\top \Phi^* X_t^* \ .
\end{aligned}$$

So the covariance (with independence) is

$$\begin{aligned}
Cov(i_t - i_t^*, q_t) &= Cov\left(C_i^\top X_t - C_{i^*}^\top X_t^*\ ,\ A_\pi^\top \Phi X_t - A_{\pi^*}^\top \Phi^* X_t^*\right) \\
&= C_i^\top \Gamma_0 \Phi^\top A_\pi + C_{i^*}^\top \Gamma_0^* \Phi^{*\top} A_{\pi^*}
\end{aligned}$$

and the regression coefficient is

$$b = \frac{C_i^\top \Gamma_0 \Phi^\top A_\pi + C_{i^*}^\top \Gamma_0^* \Phi^{*\top} A_{\pi^*}}{Var(i_t - i_t^*)}$$

- *p and q*

$$\begin{aligned}
q_t &= E_t \pi_{t+1} - E_t \pi_{t+1}^* \\
p_t &= -\frac{1}{2}\left(Var_t \pi_{t+1} - Var_t \pi_{t+1}^*\right) \\
&= -\frac{1}{2}\left(A_\pi^\top \begin{bmatrix} v_t & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_\pi - A_{\pi^*}^\top \begin{bmatrix} v_t^* & 0 & 0 \\ 0 & \sigma_v^{*2} & 0 \\ 0 & 0 & 0 \end{bmatrix} A_{\pi^*}\right)
\end{aligned}$$

where the formulae for the conditional means is above (under the italicized heading *Inflation*).

Appendix D
Linearization for the Pricing Kernel

The log of the equilibrium domestic marginal rate of substitution in Equation (15) is given by

$$\log(n_{t+1}) = \log\beta + (\rho-1)x_{t+1} + (\alpha-\rho)[\log W_{t+1} - \log\mu_t(W_{t+1})],$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is the log of the ratio of domestic observed consumption in $t+1$ relative to t and W_t is the value function. The first two terms are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from EZ preferences.

We work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton, and Li (2005). In particular, I focus on the the value function of each representative agent, scaled by the observed equilibrium consumption level

$$\begin{aligned} W_t/c_t &= [(1-\beta) + \beta(\mu_t(W_{t+1})/c_t)^{\rho}]^{1/\rho} \\ &= \left[(1-\beta) + \beta\mu_t\left(\frac{W_{t+1}}{c_{t+1}} \times \frac{c_{t+1}}{c_t}\right)^{\rho}\right]^{1/\rho}, \end{aligned}$$

where I use the linear homogeneity of μ_t. In logs,

$$wc_t = \rho^{-1}\log[(1-\beta) + \beta\exp(\rho g_t)],$$

where $wc_t = \log(W_t/c_t)$ and $g_t \equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1})))$. Taking a linear approximation of the right-hand side as a function of g_t around the point $\bar{m}$, I get

$$\begin{aligned} wc_t &\approx \rho^{-1}\log[(1-\beta) + \beta\exp(\rho\bar{m})] + \left[\frac{\beta\exp(\rho\bar{m})}{1-\beta+\beta\exp(\rho\bar{m})}\right](g_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa g_t \end{aligned}$$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the "log aggregator", the linear approximation is exact with $\bar{\kappa} = 1-\beta$ and $\kappa = \beta$.

Given the state variables of the economy, l, u and w, and the log-linear structure of the model, we conjecture a solution for the value function of the form,

$$wc_t = \bar{\omega} + \omega_l l_t + \omega_u u_t + \omega_w w_t,$$

where $\bar{\omega}$, ω_l, ω_u and ω_w are constants to be determined. Therefore

$$wc_{t+1} + x_{t+1} = \bar{\omega} + \omega_l l_{t+1} + \omega_u u_{t+1} + \omega_w w_{t+1} + x_{t+1}$$

and, using the properties of lognormal random variables, g_t can be expressed as

$$\begin{aligned} g_t &\equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1}))) \\ &= \log(E_t[\exp(wc_{t+1} + x_{t+1})^{\alpha}]^{\frac{1}{\alpha}})] \\ &= E_t[wc_{t+1} + x_{t+1}] + \frac{\alpha}{2}\mathrm{Var}_t[wc_{t+1} + x_{t+1}]. \end{aligned}$$

Using the above expression, we solve for the value-function parameters by matching coefficients

$$\begin{aligned} \omega_l &= \kappa(\omega_l \varphi_l + 1) \\ \Rightarrow \omega_l &= \left(\frac{\kappa}{1-\kappa\varphi_l}\right) \\ \omega_u &= \kappa(\omega_u \varphi_u + \frac{\alpha}{2}) \\ \Rightarrow \omega_u &= \frac{\alpha}{2}\frac{\kappa}{1-\kappa\varphi_u} \\ \omega_w &= \kappa(\omega_w \varphi_w + \frac{\alpha}{2}\omega_l^2) \\ \Rightarrow \omega_w &= \frac{\alpha}{2}\omega_l^2 \frac{\kappa}{1-\kappa\varphi_u} \ . \end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1} - \log \mu_t(W_{t+1})]$ in the pricing kernel in Equation (8):

$$\begin{aligned} \log W_{t+1} - \log \mu_t(W_{t+1}) &= wc_{t+1} + x_{t+1} - \log \mu_t(\exp(wc_{t+1} + x_{t+1})) \\ &= \omega_l \sqrt{w_t}\epsilon^l_{t+1} + \omega_u \sigma_u \epsilon^u_{t+1} + \omega_w \sigma_w \epsilon^w_{t+1} + \sqrt{u_t}\epsilon^x_{t+1} \\ &- \frac{\alpha}{2}(\omega_l^2 w_t + \omega_u^2\sigma_u^2 + \omega_w^2 \sigma_w^2 + u_t) \ . \end{aligned}$$

Equation (53) follows by collecting terms. In particular,

$$\delta^r = -\log\beta + (1-\rho)\mu + \frac{\alpha}{2}(\alpha-\rho)[(\omega_u\sigma_u)^2 + (\omega_w\sigma_w)^2]$$

$$\gamma_l^r = (1-\rho); \quad \gamma_u^r = \frac{\alpha}{2}(\alpha-\rho); \quad \gamma_w^r = \frac{\alpha}{2}(\alpha-\rho)\omega_l^2$$

$$\lambda_x^r = (1-\alpha); \quad \lambda_l^r = -(\alpha-\rho)\omega_l; \quad \lambda_v^r = -(\alpha-\rho)\omega_u; \quad \lambda_w^r = -(\alpha-\rho)\omega_w$$

$$\omega_l = \left(\frac{\kappa}{1-\kappa\varphi_l}\right); \quad \omega_u = \frac{\alpha}{2}\left(\frac{\kappa}{1-\kappa\varphi_u}\right); \quad \omega_w = \frac{\alpha}{2}\left(\frac{\kappa}{1-\kappa\varphi_w}\right)\omega_l^2$$

Appendix E
Moment Conditions

- Consumption growth:

$$E_t(x_{t+1}) = \mu + l_t \ , \qquad Var_t(x_{t+1}) = u_t \ ,$$
$$E(x_{t+1}) = \mu \ , \quad Var(x_{t+1}) = \theta_u + Varl_t \ ,$$
$$Cov(x_{t+1}, x_t) = \varphi_l \, Var_l \ , \quad Corr(x_{t+1}, x_t) = \frac{\varphi_l \, Varl_t}{\theta_u + Varl_t}$$

- Long run risk:

$$E_t(l_{t+1}) = \varphi_l l_t \ , \quad Var_t(l_{t+1}) = w_t \ ,$$
$$E(l_{t+1}) = 0 \ , \quad Var(l_{t+1}) = \frac{\theta_w}{1-\varphi_l^2} \ ,$$
$$Cov(l_{t+1}, l_t) = \varphi_l \, Varl_t \ , , \quad Corr(l_{t+1}, l_t) = \varphi_l$$

- Short-run volatility:

$$E_t(u_{t+1}) = (1-\varphi_u)\theta_u \ , \quad Var_t(u_{t+1}) = \sigma_u^2 \ ,$$
$$E(u_{t+1}) = \theta_u \ , \quad Var(u_{t+1}) = \frac{\sigma_u^2}{1-\varphi_u^2} \ ,$$
$$Cov(u_{t+1}, u_t) = \varphi_u \, Varu_t \ , \quad Corr(u_{t+1}, u_t) = \varphi_u$$

- Long-run volatility:

$$E_t(w_{t+1}) = (1-\varphi_w)\theta_w \ , \quad Var_t(w_{t+1}) = \sigma_w^2 \ ,$$
$$E(w_{t+1}) = \theta_w \ , \quad Var(w_{t+1}) = \frac{\sigma_w^2}{1-\varphi_w^2} \ ,$$
$$Cov(w_{t+1}, w_t) = \varphi_w \, Varw_t \ , \quad Corr(w_{t+1}, w_t) = \varphi_w$$

- Real pricing kernel:

$$\begin{aligned}
E_t \log n_{t+1} &= -(\delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t) \\
Var_t \log n_{t+1} &= \left(\lambda_x^r\right)^2 u_t + \left(\lambda_l^r\right)^2 w_t + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2 \\
E \log n_{t+1} &= -(\delta^r + \gamma_u^r \theta_u + \gamma_w^r \theta_w) \\
Var \log n_{t+1} &= \left(\lambda_x^r\right)^2 \theta_u + \left(\lambda_l^r\right)^2 \omega_u + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2 \\
&+ \left(\gamma_l^r\right)^2 Var(l_t) + \left(\gamma_u^r\right)^2 Var(u_t) + \left(\gamma_w^r\right)^2 Var(w_t)
\end{aligned}$$

- Real risk free interest rate:

$$\begin{aligned}
E(r_t) &= \bar{r} + r_u^r \theta_u + r_w^r \theta_w \\
Var(r_t) &= (\gamma_l^r)^2 \, Var(l_t) + \left(r_u^r\right)^2 Var(v_t) + \left(r_w^r\right)^2 Var(w_t)
\end{aligned}$$

$$Corr(r_{t+1}, r_t) = 1 - (1-\varphi_l)\big(\gamma_l^r\big)^2 \frac{Var(l_t)}{Var(r_t)} - (1-\varphi_u)\big(r_u^r\big)^2 \frac{Var(u_t)}{Var(r_t)} - (1-\varphi_w)\big(r_w^r\big)^2 \frac{Var(w_t)}{Var(r_t)}$$

- Cross-country moments (symmetric coefficient):[20]

$$\begin{aligned} Cov(x_t, x_t^*) &= Cov(l_t, l_t^*) + \eta_{\epsilon^x} E(\sqrt{u_t}\sqrt{u_t^*}) \\ Cov(l_t, l_t^*) &= \frac{\eta_{\epsilon^l} E(\sqrt{w_t}\sqrt{w_t^*})}{1-\varphi_l^2} \\ Cov(v_t, v_t^*) &= \frac{\eta_{\epsilon^u}\sigma_u^2}{1-\varphi_u^2} \\ Cov(w_t, w_t^*) &= \frac{\eta_{\epsilon^w}\sigma_w^2}{1-\varphi_w^2} \end{aligned}$$

- Real depreciation rate:

$$E_t(d_{t+1}^r) = q_t^r \ , \quad E(d_t^r) = 0 \ ,$$

$$Var(d_{t+1}^r) = 2[Var(\log n_{t+1}) - Cov(\log n_{t+1}, \log n_{t+1}^*)]$$

- Inflation:

$$E(\pi_t) = a + a_2\theta_u + a_3\theta_w + a_5\theta_v$$
$$Var(\pi_t) = a_1^2 Var(l_t) + a_2^2 Var(u_t) + a_3^2 Var(w_t) + a_4^2 Var(z_t) + a_5^2 Var(v_t)$$

$$\begin{aligned} Corr(\pi_{t+1}, \pi_t) &= 1 - (1-\varphi_l)a_1^2 \frac{Var(l_t)}{Var(\pi_t)} - (1-\varphi_u)a_2^2 \frac{Var(u_t)}{Var(\pi_t)} - (1-\varphi_w)a_3^2 \frac{Var(w_t)}{Var(\pi_t)} \\ &- (1-\varphi_z)a_4^2 \frac{Var(z_t)}{Var(\pi_t)} - (1-\varphi_v)a_5^2 \frac{Var(v_t)}{Var(\pi_t)} \end{aligned}$$

$$corr(x_{t+1}, \pi_t) = a_1 \frac{Var(l_t)}{Stdev(x_t)Stdev(\pi_t)}, \quad corr(x_t, \pi_t) = corr(x_{t+1}, \pi_t)\varphi_l$$

- Nominal interest rate:

$$E(i_t) = \bar{\imath} + r_u\theta_u + r_w\theta_w + r_v\theta_v$$

$$Var(i_t) = \gamma_l^2 Var(l_t) + \gamma_z^2 Var(z_t) + r_u^2 Var(u_t) + r_w^2 Var(w_t) + \big(r_v\big)^2 Var(v_t)$$

$$\begin{aligned} Corr(i_{t+1}, i_t) &= 1 - (1-\varphi_l)\gamma_l^2 \frac{Var(l_t)}{Var(i_t)} - (1-\varphi_z)\gamma_z^2 \frac{Var(z_t)}{Var(i_t)} \\ &- (1-\varphi_u)r_u^2 \frac{Var(u_t)}{Var(i_t)} - (1-\varphi_w)r_w^2 \frac{Var(w_t)}{Var(i_t)} - (1-\varphi_v)\big(r_v\big)^2 \frac{Var(v_t)}{Var(i_t)} \end{aligned}$$

[20]The expressions for cross-country moments greatly simplify if we assume either independence or perfect correlation in the stochastic volatility processes, u_t and w_t.

- Nominal depreciation rate:

$$E_t(d_{t+1}) = q_t \ , \quad E(d_t) = 0$$

$$Var(d_{t+1}) \quad = \quad 2[Var(\log m_{t+1}) - Cov(\log m_{t+1}, \log m^*_{t+1})]$$

Table 1
Calibrated Parameter Values

Parameter		Value
Subjective discount factor	β	0.999
Mean of consumption growth	μ	0.0016
Long run risk persistence	φ_l	0.99
Short run volatility level	θ_u	1.75E-05
Short run volatility persistence	φ_u	0.98
Short run volatility of volatility	σ_u	2.10E-06
Long run volatility mean	θ_w	2.80E-08
Long run volatility persistence	φ_w	0.98
Long run volatility of volatility	σ_w	3.40E-09
Policy shock persistence	φ_z	0
Policy shock volatility level	θ_v	8.33E-06
Policy shock volatility persistence	φ_v	0.98
Volatility of policy shock volatility	σ_v	1.05E-06
Cross-correlation short run growh shocks ϵ^x	η_x	0.292
Cross-correlation long run growth shocks ϵ^l	η_l	1
Cross-correlation short run volatility shocks ϵ^u	η_u	0
Cross-correlation long run volatility shocks ϵ^w	η_w	1
Cross-correlation policy shocks ϵ^z	η_z	0
Cross-correlation volatility policy shocks ϵ^v	η_v	0
Risk aversion	$1-\alpha$	10
Elasticity of intertemporal substitution	$1/(1-\rho)$	2
Taylor-rule parameter, constant	$\bar{\tau}$	-0.00035
Taylor-rule parameter, inflation	τ_1	1.4
Taylor-rule parameter, expected consumtpion growth	τ_2	1.5

Table 2
Sample and Population Moments

Moment	Sample	Population (Model)
$E(x_t) \times 12$	1.92	1.92
$\sigma(x_t) \times \sqrt{12}$		1.50
$Corr(x_t, x_t^*)$	≈ 0.3	0.32
$E(r_t) \times 12$		1.24
$\sigma(r_t) \times 12$		0.82
$\sigma(r_t - r_t^*) \times 12$		0.13
$E(i_t) \times 12$	6.42	5.60
$\sigma(i_t) \times 12$	3.72	2.99
$\sigma(i_t - i_t^*) \times 12$		0.43
$E(\pi_t) \times 12$	4.34	4.30
$\sigma(\pi_t) \times \sqrt{12}$	1.32	1.26
$Corr(r_t, r_{t-1})$		0.99
$Corr(i_t, i_{t-1})$	0.98	0.99
$Corr(i_t - i_{t-1}, i_t^* - i_{t-1}^*)$		0.98
$Corr(\pi_t, \pi_{t-1})$	0.71	0.67
b^r		-5.89
b	≈ -2	-1.07
$\sigma(s_{t+1} - s_t) \times \sqrt{12}$	≈ 15.0	17.41

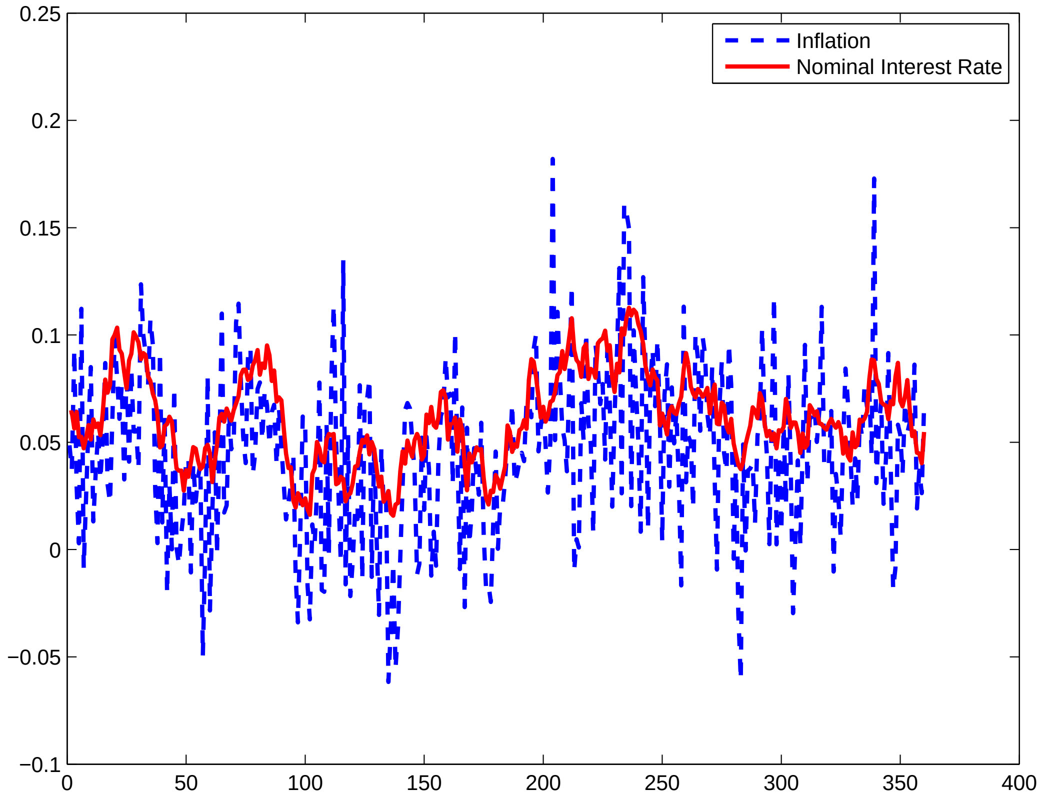

Figure 1: Annualized Inflation and the Nominal Interest Rate, 30-Year Simulation. Discussion in Section 7.

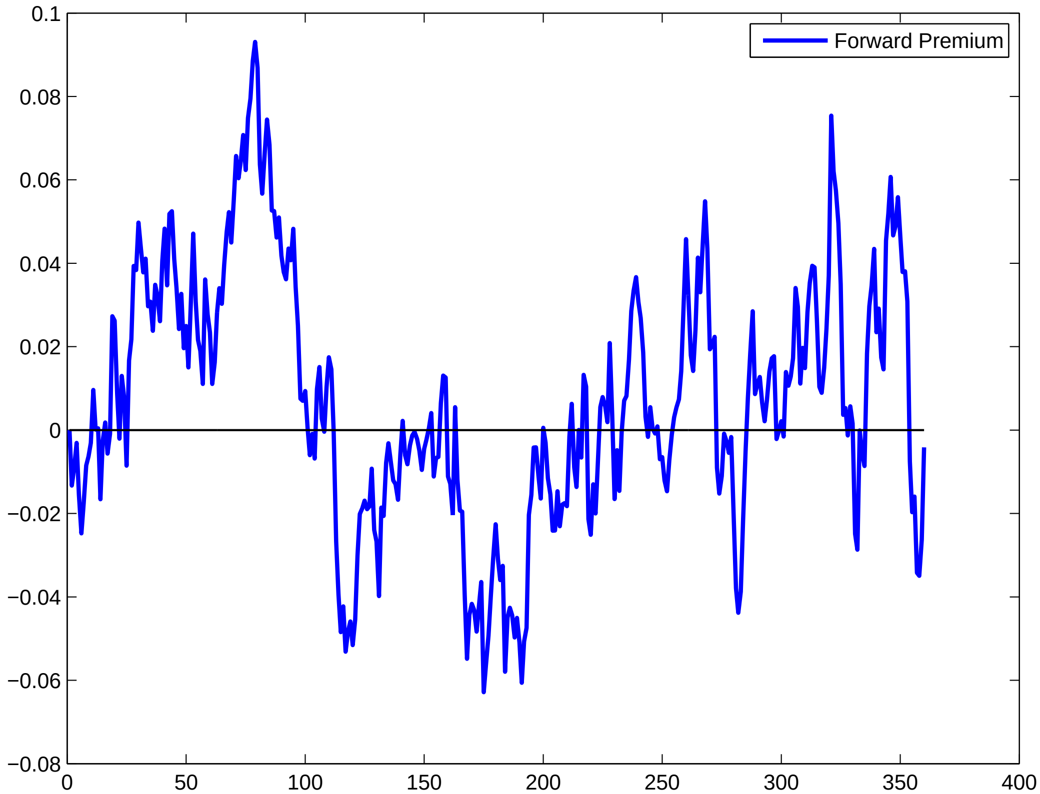

Figure 2: Annualized Interest Rate Differential (Forward Premium), 30-Year Simulation. Discussion in Section 7.

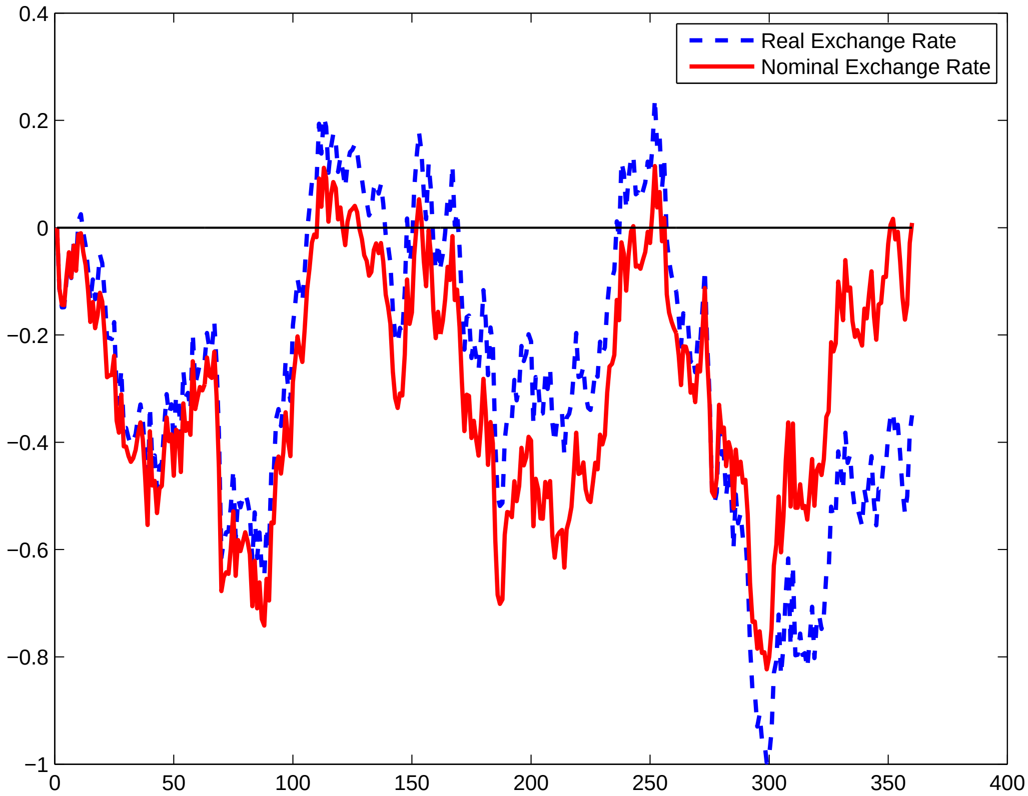

Figure 3: Log Real and Nominal Exchange Rate, 30-Year Simulation. Discussion in Section 7.

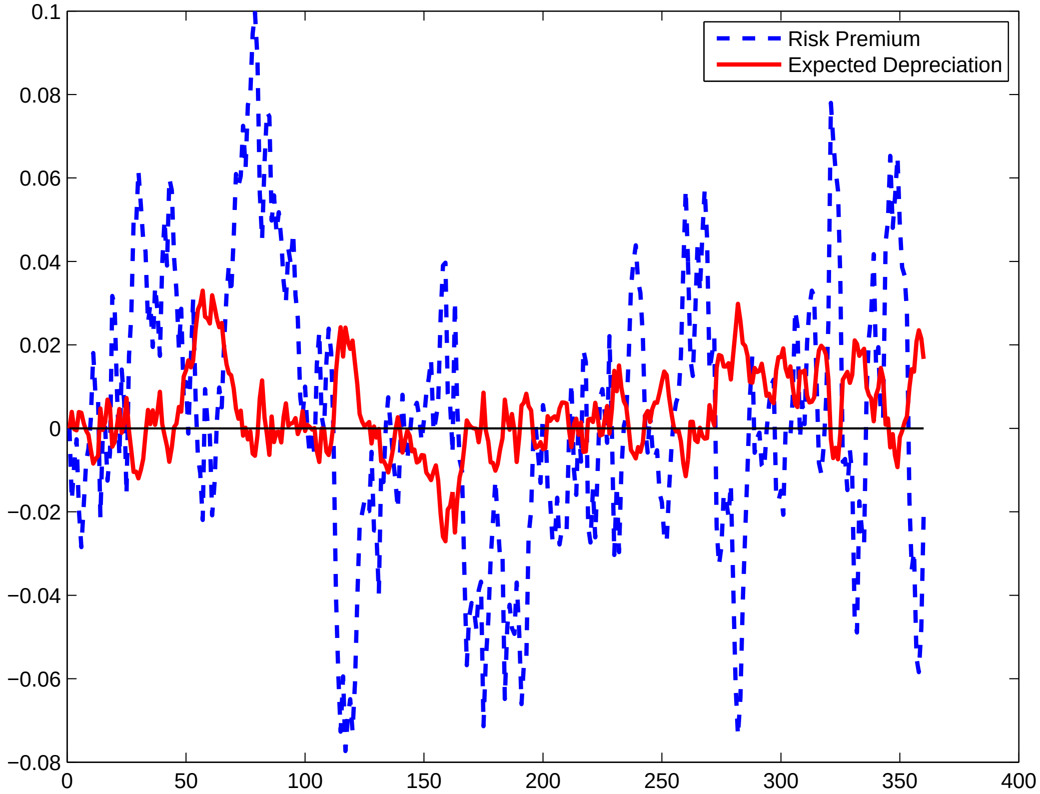

Figure 4: Currency Risk Premium and Expected Depreciation, 30-Year Simulation. Discussion in Section 7.

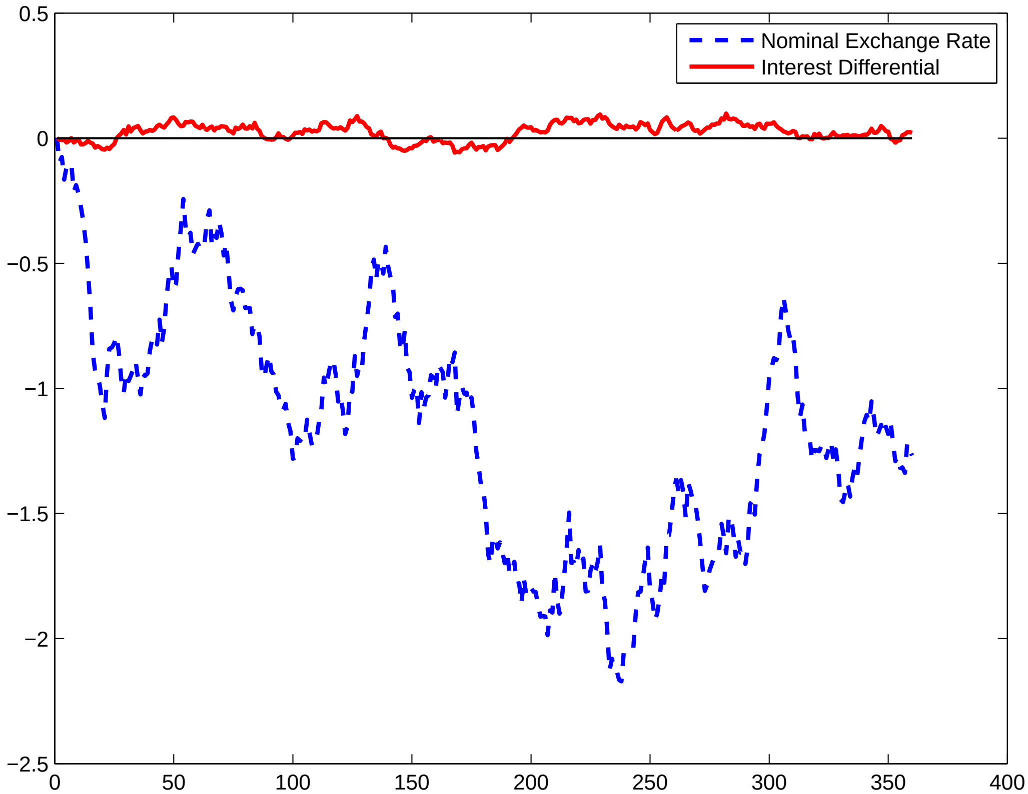

Figure 5: Log Nominal Exchange Rate and Interest Rate Differential, 30-Year Simulation. The interest rate differential is USD less GBP. The exchange rate is "price of GBP." So, UIP predicts that when the red line is above zero, the blue line will *increase*. The profitability of the carry trade is premised upon the opposite. While it's obviously not clear from the graph (as in an analogous graph of data), the latter tends to happen slightly more than the former. The graph also highlights the riskiness of the carry trade. Variation in nominal exchange rates is large relative to the interest differential and its components, p and q. This graph is discussed in Section 7.