# The dynamics of sovereign debt crises in a monetary union. Bundesbank Conference Hamburg 2011

Francisco Roch<sup>1</sup> and Harald Uhlig<sup>1</sup>

<sup>1</sup>University of Chicago Department of Economics huhlig@uchicago.edu

May 19, 2011

# Outline

#### Introduction

- A single country: the model
   State space representation
  - Debt pricing

## 3 No bailouts



- Bailouts
- 5 Contagion and amplification

# Conclusions

## Outline

#### Introduction

- A single country: the model
   State space representation
   Debt pricing
- 3 No bailouts
- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

### **Motivation**

- 2010: fears of a Greek default.
- Contagion? Euro crisis? European banking crisis?
- May 2010: EMU-IMF rescue package.
- European Financial Stability Facility (EFSF)
- Nonetheless, spreads remain high and fears persist.















Introduction

### Greek-German Yield Spreads: Feb 2008 vs Apr 2011



Introduction

# Portugal-Ger. Yield Spreads: Feb 2008 vs Apr 2011



Introduction

## Spain-German Yield Spreads: Feb 2008 vs Apr 2011



## Greece GDP vs 2011:Q2



### Greece GDP vs Argentine GDP



### Goal

- Modelling financial crises ...
  - self-fulfilling default possibility, Cole-Kehoe (2000)
  - random income shocks, Arellano (2008)
  - short-sighted politicians, Beetsma-Uhlig (1999)
- ... and the role of bailouts.
- Contagion and amplification in a monetary union.

# Fiscal policy in a monetary union

- Cooper-Kempf (2004) / Cooper-Kempf-Peled (2009, 2010): fiscal policy in a monetary union is crucial. Spillovers will be there. Incentives for excessive deficits.
- Beetsma-Uhlig (1999): a growth-and-stability pact is needed.
- Uhlig (2003): "what if" a sovereign defaults in EMU?

# Outline

#### Introduction



#### 3 No bailouts

- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

# The government

Arellano (2008).

• Government objective:

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - \chi_t \delta_t)$$

- ► *c*<sub>t</sub>: gov. spending.
- y<sub>t</sub>: tax receipts.
- $\delta_t = 1$ : default in *t*.  $\chi_t$ : loss in "face".
- Budget constraint:

$$c_t + (1-\theta)B_t = y_t + q_t(B_{t+1})(B_{t+1} - \theta B_t)$$

- $0 < \theta \le 1$ : maturity.
- Once defaulted, forever excluded. Then,  $c_t = y_t$ .

# Debt pricing

- Risk neutral traders. Discount future with R.
- Debt pricing schedule:  $q_{m,t}(B_{t+1})$ .
- International assistance ("bailout"):  $q_{a,t}(B_{t+1}) \ge 0$ .
- (Future version: conditionality...)

$$q_t(B_{t+1}) = \max\{q_{m,t}(B_{t+1}), q_{a,t}(B_{t+1})\}$$

# Timing

- Shocks are realized.
- Pricing schedule established.
- Government decides:
  - Default or
  - pay and issue new debt.
- government consumes.

### Outline

#### Introduction

A single country: the model
State space representation
Debt pricing

#### 3 No bailouts

- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

### State space representation

Cole-Kehoe (2000)

State is

$$\mathbf{s} = (\mathbf{B}, \mathbf{d}, \mathbf{z})$$

where B: debt, d: default status, z: exogenous random variables,

$$\boldsymbol{z} = (\boldsymbol{y}, \zeta, \psi)$$

- y ∈ [y<sub>L</sub>, y<sub>H</sub>], 0 < y<sub>L</sub> ≤ y<sub>H</sub>, with strictly positive and continuous density iid.
- $\zeta \in [0, 1]$  uniform, iid: "crisis" sunspot.
- $\psi \in [0, 1]$  uniform, iid: "bailout" sunspot.

Budget, if not defaulted:

$$c + (1 - \theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s))$$

## Value functions

Default:

$$v_D(z) = u(y(z)) + \beta E \left[ v_D(z') \mid z \right]$$

No default:

$$\begin{split} v_{\mathsf{ND}}(s) &= \max_{c,B'} \{ u(c) + \beta E\left[ v(s') \mid z \right] \mid \\ c + (1-\theta)B(s) &= y(s) + q(B';s)(B'-\theta B(s)) \\ s' &= (B',d(s),z') \} \end{split}$$

Overall:

$$\mathbf{v}(\mathbf{s}) = \max_{\delta \in \{0,1\}} (1-\delta) \mathbf{v}_{ND}(\mathbf{s}) + \delta(\mathbf{v}_D(\mathbf{z}(\mathbf{s})) - \chi(\mathbf{s}))$$

## Equilibrium

- Given q(B'; s), the government maximizes its utility with the choices c(s), δ(s) and B'(s), s.t. the budget constraint.
- The market pricing function  $q_m(B'; s)$  is consistent with risk-neutral pricing of government debt.
- The pricing function satisfies

$$q(B';s) = \max\{q_m(B';s), q_a(B';s)\}$$

# Outline

#### Introduction



Debt pricing

#### 3 No bailouts

- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

# Continuation probabilities

• Given *B*, let *A*(*B*) be the set of no-default *z*:

$$A(B) = \{z \mid \delta(s) = 0 \text{ for } s = (B, 0, z)\}$$

• Continuation probability:

$$P(B'; s) = \operatorname{Prob}(z' \in A(B') \mid s) = E\left[1_{\delta(s')=0} \mid s
ight]$$

•  $q(B'; s) = \bar{q}(B'; s)$ : if no default today.

• If  $\theta = 0$ , i.e one-period debt: market price is

$$ar{q}_m(B';s) = rac{1}{R}P(B';s)$$

# Sunspots

Suppose, q(B'; s) = q
 q(B'; s) is consistent with debt repayment now. Let

$$ar{v}_{\mathcal{ND}}(s) = \max_{c,B'} \{u(c) + eta E \left[v(s') \mid z
ight] \mid \ c + (1- heta)B(s) = y(s) + ar{q}(B';s)(B' - heta B(s))\}$$

Maximal debt level:

$$\bar{B}(z) = \inf\{B \mid \bar{v}_{ND}(s = (B, 0, z)) \leq v_D(z(s)) - \chi(s = (B, 0, z))\}$$

• Suppose,  $q(B'; s) = q_a(B'; s)$  is consistent with default now. Let

$$\underline{v}_{ND}(s) = \max_{c,B'} \{ u(c) + \beta E \left[ v(s') \mid z \right] \mid \\ c + (1 - \theta)B(s) = y(s) + q_a(B'; s)(B' - \theta B(s)) \}$$

Maximal debt level:

$$\underline{B}(z) = inf\{B \mid \underline{v}_{ND}(s = (B, 0, z)) \le v_D(z(s)) - \chi(s = (B, 0, z))\}$$

#### Sunspot assumption

#### Assumption A.

For some parameter  $\pi \in [0, 1]$ , and all s with  $\underline{B}(z) \leq B(s) \leq \overline{B}(z)$ , we have  $q_m(B'; s) = \overline{q}_m(B'; s)$ , if  $\zeta(s) \geq \pi$  and  $q_m(B'; s) = 0$ , if  $\zeta < \pi$ .

# Sunspot equilibrium

Cole-Kehoe (2000): assume constant probability  $\pi$  of sunspot default.

- If  $B > \overline{B}(z)$ , default now. Market price for new debt is zero.
- 2 If  $\underline{B}(z) \leq B \leq \overline{B}(z)$ ,
  - default with probability π (i.e. ζ(z) < π).</li>
     Market price for new debt is zero.
  - ② continue with probability  $1 \pi$  (i.e.  $\zeta(z) \ge \pi$ ). Market price for new debt is  $\bar{q}_m(B'; s)$ .
- If  $B < \underline{B}(z)$ , no default. Market price for new debt is  $\bar{q}_m(B'; s)$ .

### Crisis zone

# $B' \in \mathcal{B} = \left[\min \underline{B}(z), \max \overline{B}(z)\right]$

## Outline

#### Introduction

- A single country: the model
  State space representation
  Dobt pricing
  - Debt pricing

### No bailouts

- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

# Default decision



### Sunspot



Roch-Uhlig (University of Chicago)

# Debt pricing



## Debt pricing with and without sunspots



# A first order condition

#### Proposition

If optimum = first-order condition, then

$$q(B';s) + q_1(B';s)B' = \beta E\left[\frac{u'(c(s'))}{u'(c(s))}(1 - \theta + \theta q(B''(s');s'))1_{\delta(s')=0}\right]$$

If  $\theta = 0$  (only short-term debt), then **FOC** 

$$q(B';s) + q_1(B';s)B' = eta E\left[rac{u'(c(s'))}{u'(c(s))} \mathbf{1}_{\delta(s')=0}
ight]$$

or

$$1 - h(B'; s)B' = \beta RE \left[\frac{u'(c(s'))}{u'(c(s))} \mid \delta(s') = 0\right]$$

where

$$h(B';s) = -\frac{\partial E[\delta(s') = 0] / \partial B}{E[\delta(s') = 0]}$$

Roch-Uhlig (University of Chicago)

Sovereign Debt Crises

# The market price q(B')



# The market price q(B') vs the lhs of **FOC**



# The market price q(B') vs the lhs of **FOC**



### The two sides of **FOC**



### The two sides of **FOC**



## Current income, future debt



Roch-Uhlig (University of Chicago)

### FOC, when income fluct. are small.



**FOC**, when B = 0 and income fluct. are small.



Debt dynamics,  $\beta R = 1$ , small income fluct.



Debt dynamics,  $\beta R < 1$ , small income fluct.



Debt dynamics,  $\beta R \ll 1$ , small income fluct.



## Stationary debt dynamics, $\beta R \ll 1$



### Outline

#### 1 Introduction

- A single country: the model
   State space representation
   Debt pricing
  - Debt pricing

#### 3 No bailouts



## Contagion and amplificatio

#### Conclusions

#### **Bailouts**

• Country can sell debt at some "assisted" price  $0 < q_a < 1/R$  to some outside facility, provided the total amount *B*' of debt does not exceed some upper limit  $\overline{B}_a$ .

• 
$$q_a(B';s) \equiv q_a$$
 for  $B' \leq \bar{B}_a$ .

• Probabilistic bailout: with exog. probability  $\omega$ .

$$q_a(B';s) \equiv 0 \text{ for } B' \leq \overline{B}_a, \text{ if } \psi(s) < \omega.$$

2) 
$$q_a(B'; s) \equiv q_a$$
 for  $B' \leq B_a$ , if  $\psi(s) > \omega$ .

Conditionality ...

### FOC, no assistance



# FOC, with one-time assistance



Roch-Uhlig (University of Chicago)

# FOC, with permanent assistance



Stationary debt dynamics, one-time assistance



Stationary debt dynamics, permanent assistance



Debt pricing: no vs probabilistic assistance



**Bailouts** 

Stationary debt dynamics, probabilistic assistance



# Conditionality

- So far, two alternatives: default, don't default. Assisted pricing.
- Better: three alternatives:
  - Default.
  - Don't default.
  - 3 Accept outsider plan: assisted pricing, but reduced consumption.
- Incentive compatibility.
- Future versions.

# Outline

#### Introduction

A single country: the model
 State space representation
 Debt pricing

#### 3 No bailouts

#### Bailouts



#### 6 Conclusions

# Contagion

Several countries.

State

$$\mathbf{s} = (\mathbf{s}_1, \ldots, \mathbf{s}_J)$$

where

$$s_j = (B_j, d_j, z_j)$$

and where

$$\mathbf{z}_{j} = (\mathbf{y}_{j}, \chi_{j}, \zeta_{j}, \psi_{j}, \pi_{j}, \omega_{j})$$

- Suppose  $\chi_j \equiv \chi$  iid. Or suppose  $\pi_j \equiv \pi$  iid.
- Suppose  $\omega_i \equiv \omega$  iid? Or should it be negatively correlated?

# Amplification

- Debt pricing so far: risk neutral traders.
- Instead: debt is held by outside banks.
- When some banks need to sell: cash-in-market by other banks determines debt.
- Contagion:
  - Falling debt price in country 1 ...
  - … means closure of fragile banks …
    - ... debt gets dumped, all debt prices fall ...
    - ... increasing default probability in country 2.
- Why not also in Germany? Answer: outside crisis zone.
- Cash-in-market? Why not private purchasers of debt?

## Outline

#### Introduction

A single country: the model
 State space representation
 Debt pricing

#### 3 No bailouts

- 4 Bailouts
- 5 Contagion and amplification

#### 6 Conclusions

### Conclusions

- Dynamics of debt default:
  - Arellano (2008) [income fluct.] meets
    - 2 Cole-Kehoe (2000) [sunspot defaults] meets
    - Beetsma-Uhlig (1999) [short-sighted politicians]
- Stationary debt dynamics: precarious.
- Assisted pricing: debt level increases, still precarious or assured default. Postpones the "day of reckoning".
- Conditionality, contagion, amplification, banking system: future versions / future work.