Understanding the Equity Premium Puzzle and the Correlation Puzzle

R. Albuquerque, M. Eichenbaum, S. Rebelo

Discussion by K. Adam

Mannheim University & CEPR

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Summary

- Lucas AP model + EZ preferences + persistent time pref shocks

  => sizable equity premium (2.7%) with low risk aversion (≈ 1)

  => disconnect between AP returns & cons./div. growth

  + range of other moments of APs & fundamentals

- Simplicity & elegance of the model:

  => sizable quantitative improvement relative to Lucas case

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- Observable fundamentals (D & C): iid growth rates

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- Significant advance over more complicated alternatives:

  Long-run risk models with EZ preferences (Bansal et al)

  - growth rates: persistent + stochastic volatility
  - persistent comp. of growth is AR(1) + stoch vola shocks
  - eqn describing evolution of volatility

  $=>$ less successful in replicating the disconnect puzzle

  $=>$ need much higher risk aversion ($\sim$10)

Outline of Remaining Discussion

1. How does the model achieve the resolution of the

   - disconnect puzzle?

   - the equity premium puzzle?

2. What 'side effects' are generated?

3. General comment on pref-based explanations of AP fluctuations

The Disconnect Puzzle from a Slightly Different Angle

- Fundamental AP Equation

$$P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})]$$

Forward iteration + TVC =>

$$\frac{P_t}{D_t} = E_t[M_{t+1}\frac{D_{t+1}}{D_t} + M_{t+1}M_{t+2}\frac{D_{t+2}}{D_{t+1}} + ...]$$

M : some function of C and its lags

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- P/D on the left largely unrelated to future C and D: 'disconnect puzzle'

How to Resolve The Disconnect Puzzle?

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- Adam, Marcet & Nicolini (2011): learning-induced low frequency swings in optimism about future returns

How to Resolve The Disconnect Puzzle?

- New proposal in this paper: persistent & time-varying discount factor

$$\log(\lambda_{t+1}/\lambda_t) = 0.9992 \log(\lambda_t/\lambda_{t-1}) + \varepsilon_{t+1}$$

- 'Almost closed-form' expression for log SDF:

$$\log M_{t+1} = a + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) \underbrace{r_{c,t+1}}_{\text{Return of C claim}}$$

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- When & how does it give rise to a disconnect & EP?

Resolving The Disconnect Puzzle

- Instead of Campbell-Shiller approximation use

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\begin{aligned}
r_{c,t+1} &= \log \frac{P^c_{t+1} + C_{t+1}}{P^C_t} \\
&= \log\left(P^c_{t+1} / C_{t+1} + 1\right) - \log P^c_t / C_t + \log C_{t+1} / C_t \\
&\approx \log(P^c_{t+1} / C_{t+1}) - \log(P^c_{t+1} / C_{t+1}) + \Delta c_{t+1}
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- With

$$\log P^c_t / C^c_t = A_{c0} + A_{c1} \log \frac{\lambda_{t+1}}{\lambda_t} \text{ and } \rho \approx 1$$

we get

$$r_{c,t+1} \approx A_{c1}\varepsilon_{t+2} + \Delta c_{t+1}$$

Resolving The Disconnect Puzzle

- **Approximate closed form expression for SDF:**

$$\log M_{t+1} \approx a + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\left(A_{c1}\varepsilon_{t+2} + \Delta c_{t+1}\right)$$

- All terms iid except for $\log(\lambda_{t+1}/\lambda_t)$, which is highly persistent

  $=>$ **disconnect if** $\theta \neq 0$!

- Disconnect even if $\theta = 1$: CRRA case

Matching the EP Puzzle

- Consider the AP equation

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- Risk premium emerges: discount effect is proportional to

$$(\theta - 1)A_{c1}A_{d1}\sigma_\varepsilon^2$$

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  $=>$ emerges in all estimations $\theta \approx -0.5$
- IES largely irrelevant for EP: only $\theta = (1-\gamma)/(1-1/\psi)$ matters!

Which Side Effects from Discount Factor Shocks?

- SDF: $\log M_{t+1} \propto \theta \log(\lambda_{t+1}/\lambda_t)$
- Low frequency effect of $\theta \log(\lambda_{t+1}/\lambda_t)$ shows up also in safe assets $=>$ safe assets are also 'disconnecting'

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- Vola of ex-ante risk vs. ex-post risk free rate:

$$E_t\left[\frac{1+i_t}{1+\pi_{t+1}}\right] \quad \text{versus} \quad \frac{1+i_t}{1+\pi_{t+1}}$$

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  Price of long bonds declines as PD falls: in the data?

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- Too much persistence?

  Monthly persistence of $\log \lambda_{t+1}/\lambda_t$ is 0.9992 => 0.9904 annually

  PD ratio persistence in the model 0.99?

  => Annual persistence of PD ratio in the data: ∼0.72

A General Remark on RE AP Models

- EM approach to AP: Boom and bust cycles in APs

  are fundamentally justified

- Adam & Marcet (2011): RE-component of EM AP models

  => return expectations low when PD ratio high (and vice versa)

- Follows from low frequency & mean-reverting disconnect of PD ratio

  + unpredictability of D growth

Investors' Return Expectations in the US

Figure: Average 1 year ahead stock market return exp. (UBS/Gallup Survey Data).

Summary

- Great paper! (to be written)
- Clean and elegant AP model:

  2 simple departures from Lucas AP model

  $=>$ impressive quantitative improvement
- Some open issues for further research