Short-term GDP forecasting with a mixed frequency dynamic factor model with stochastic volatility

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Plan of the talk

- Motivation
- The model
- Estimation strategy
- An empirical application: forecasting euro area GDP
- Full sample results
- Daily business: some bayesian tools for nowcasting
- Out of sample: point and density forecast evaluation
- Some concluding remarks
Motivation 1

- Interest in policy making and forecasting in probability distributions around a central forecast
- Density forecasts are sensitive to shifts in the parameters of the model: Jore, Mitchell, and Vahey (2010), Clark (2011)
- Discrete breaks far away in the past: use sample split
- More recently: increasing interest in modeling small continuous breaks (time varying parameter models, Cogley and Sargent, 2003, Primiceri, 2005, large literature following)
- The Great Recession: spotlight on volatility breaks (end to the Great Moderation?)
Most of the work on density forecast/time varying models falls in the medium/long term forecasting literature.

Nowcasting/Short term forecasting is a world of its own:
- mixed frequency data
- ragged edge data
- different timeliness (soft/hard data)

There are existing tools that deal with the above issues but:
- No applications on density forecasts
- Time constant parameters (some allow for discrete random breaks in the mean: MS models)

Our contribution

- Extend the mixed frequency factor model by Mariano and Murasawa (2003) to account for continuous shifts in volatility
- Derive some interesting tools:
  - Density forecasts / fan charts for GDP short term forecasts
  - Probability distributions of the news content of indicator releases
Empirical application: forecasting euro area GDP

- We document a dramatic increase in both common and idiosyncratic business cycle volatility in the euro area in the past few years.
- Evaluate the contribution to forecast accuracy of stochastic volatility in terms of:
  - Point forecast accuracy (RMSE)
    - S-vol lowers uniformly but marginally RMSE
  - Ability to produce normalized forecast errors (computed via PITS) which are close to normal
    - The model produces good pits with and without S-vol
  - Interval forecast accuracy (coverage rates)
    - S-vol improves significantly the coverage rates
\[
\begin{pmatrix}
y_1^* \cr y_2^*
\end{pmatrix} =
\begin{pmatrix}
\mu_1^* \\
\mu_2
\end{pmatrix} + \beta f_t +
\begin{pmatrix}
u_{1,t} \\
u_{2,t}
\end{pmatrix}
\]

\[
y_{1t} = \frac{1}{3} y_{1,t}^* + \frac{2}{3} y_{1,t-1}^* + y_{1,t-2}^* + \frac{2}{3} y_{1,t-3}^* + \frac{1}{3} y_{1,t-4}^*
\]

\[
\begin{pmatrix}
y_{1t} \\
y_{2t}
\end{pmatrix} =
\begin{pmatrix}
\mu_1 \\
\mu_2
\end{pmatrix} + \left(\begin{array}{c}
\beta_1 \left( \frac{1}{3} f_t + \frac{2}{3} f_{t-1} + f_{t-2} + \frac{2}{3} f_{t-3} + \frac{1}{3} f_{t-4} \right) \\
\beta_2 f_t
\end{array}\right) +
\begin{pmatrix}
\frac{1}{3} u_{1,t} + \frac{2}{3} u_{1,t-1} + u_{1,t-2} + \frac{2}{3} u_{1,t-3} + \frac{1}{3} u_{1,t-4} \\
u_{2,t}
\end{pmatrix}
\]
Idiosyncratic shocks: the baseline model

\[ f_t = \sum_{j=1}^{p_f} \phi_j^f f_{t-j} + \epsilon_t^f \quad \epsilon_t^f \sim N(0, \sigma_f) \]

\[ u_{1,t} = \sum_{j=1}^{p_1} \phi_j^1 u_{1,t-j} + \epsilon_t^1 \quad \epsilon_t^1 \sim N(0, \sigma_1) \]

\[ u_{2,t} = \sum_{j=1}^{p_2} \phi_j^2 u_{2,t-j} + \epsilon_t^2 \quad \epsilon_t^2 \sim N(0, \sigma_2) \]
Stochastic volatility

\[ f_t = \sum_{j=1}^{p_f} \phi_j^f f_{t-j} + \epsilon_t^f (\lambda_f, t)^{0.5} \quad \epsilon_t^f \sim N(0, 1) \]

\[ u_{1,t} = \sum_{j=1}^{p_1} \phi_j^1 u_{1,t-j} + \epsilon_t^1 (\lambda_1, t)^{0.5} \quad \epsilon_t^1 \sim N(0, \sigma_1) \]

\[ u_{2,t} = \sum_{j=1}^{p_2} \phi_j^2 u_{2,t-j} + \epsilon_t^2 (\lambda_2, t)^{0.5} \quad \epsilon_t^2 \sim N(0, \sigma_2) \]

- We let the log-stochastic volatility components follow a random walk without drift

\[ \log(\lambda_{i,t}) = \log(\lambda_{i,t-1}) + \eta_{i,t} \quad \eta_{i,t} \sim N(0, \sigma_{\eta,i}) \]
State space representation

- The model has a time varying state space representation

\[ y_t = F \mu_t \]
\[ \mu_t = H\mu_{t-1} + \eta_t \quad \eta_t \sim N(0, Q_t) \]
\[ \Lambda_t = \Lambda_{t-1} + \zeta_t \quad \zeta_t \sim N(0, \Xi) \]

- \( y_t \) collects both quarterly and monthly variables,
- \( \mu_t \) includes the unobserved factor and idiosyncratic shocks
- \( Q_t \) collects the drifting volatilities \( \sigma_i \lambda_{i,t} \)
6 blocks of parameters: 6 steps Metropolis Hastings within Gibbs algorithm

\[ y_t = F \mu_t \]
\[ \mu_t = H \mu_{t-1} + \eta_t \quad \eta_t \sim N(0, Q_t) \]
\[ \Lambda_t = \Lambda_{t-1} + \zeta_t \quad \zeta_t \sim N(0, \Xi) \]

1. Elements of F (\(\beta\))
2. Elements of H (\(\phi\))
3. Time constant elements of \(Q_t\) (\(\sigma_i\))
4. Time varying elements of \(Q_t\) (\(\lambda_{i,t}\))
5. Variances of s-vol (\(\sigma_{\eta,i}\))
6. The unobserved state vector (\(\mu_t\))

Uncorrelated disturbances: estimation can be performed equation by equation
Steps 1-2: $\beta_i$ and $\sigma_i$

- Take a measurement equation

\[ y_{i,t} = \beta_i f_t + u_{i,t} \]

- Autocorrelated $1 - \phi(L)$ and heteroscedastic $\lambda_{i,t}^{0.5}$ residuals
- Filter with $1 - \phi(L)$ and divide by $\lambda_{i,t}^{0.5}$
- $f_t$ and all other parameters can be treated as known
- This is a standard regression
- Normal-gamma conjugate prior $\rightarrow$ Normal-gamma posterior
Step 3: $\phi_i$

- Take a transition equation

$$\mu_{i,t} = \sum_{j=1}^{p_i} \phi_i \mu_{i,t-j} + \eta_{i,t}$$

- heteroscedastic $\lambda_{i,t}^{0.5}$ residuals
- divide by $\lambda_{i,t}^{0.5}$
- This is a standard regression
- Normal conjugate prior $\rightarrow$ Normal posterior
- Discard explosive roots
Step 4-5: $\lambda_{i,t}, \sigma_{\eta,i}$

- Use block-by-block Jacquier-Polson-Rossi algorithm
- Involves drawing from a candidate density (log-normal)
- Metropolis acceptance step
Step 6: $\mu_t$

- Conditional on $F(\beta)$, $H(\phi)$, $Q_t(\sigma_i, \lambda_{i,t})$ use state space representation
- Durbin and Koopman disturbance smoother gives draws of $\mu_t$
- Missing values in GDP equation are treated as in Mariano/Murasawa (skip the filtering step)
Estimate with OLS on a training sample of $\tau$ initial observations.

- Normal prior means are set at OLS estimates, variances at $10^3$ the OLS variances.
- Gamma degrees of freedom set to $\tau + 1$ for time constant variances.
- Gamma degrees of freedom for the variance of $\lambda_{i,t}$ set to 1 and scale parameter to 0.025 (in line with Clark, 2011).
- We set $\tau$ to 36 (first three years of data).
Empirical application: Euro area GDP forecasting - indicators

**Table: Variable selection summary**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly series</strong></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>Euro Area</td>
</tr>
<tr>
<td><strong>Monthly series</strong></td>
<td></td>
</tr>
<tr>
<td>Industrial Production</td>
<td>Euro Area</td>
</tr>
<tr>
<td>Industrial Production - Pulp/paper</td>
<td>Euro Area</td>
</tr>
<tr>
<td>Business Climate - IFO</td>
<td>Germany</td>
</tr>
<tr>
<td>Economic Sentiment Indicator</td>
<td>Euro Area</td>
</tr>
<tr>
<td>PMI composite</td>
<td>Euro Area</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>US-Euro</td>
</tr>
<tr>
<td>10y spread</td>
<td>US-Euro</td>
</tr>
<tr>
<td>Michigan Consumer Sentiment</td>
<td>US</td>
</tr>
</tbody>
</table>
FULL SAMPLE RESULTS
## Factor Loadings - posterior estimates

**Table:** Factor Loadings - posterior estimates

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.27</td>
<td>0.38</td>
<td>0.54</td>
</tr>
<tr>
<td>IP</td>
<td>0.40</td>
<td>0.49</td>
<td>0.60</td>
</tr>
<tr>
<td>IP-PULP</td>
<td>0.23</td>
<td>0.29</td>
<td>0.36</td>
</tr>
<tr>
<td>IFO</td>
<td>0.10</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>ESI</td>
<td>0.10</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>PMI</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>US $ TO EURO</td>
<td>-0.08</td>
<td>-0.05</td>
<td>-0.02</td>
</tr>
<tr>
<td>US-spread</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>Michigan Consumer</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Stochastic volatilities - factor and selected indicators

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# Stylized data release calendar

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Timing</th>
<th>Publication lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP</td>
<td>11\textsuperscript{th} – 15\textsuperscript{th} of month</td>
<td>2</td>
</tr>
<tr>
<td>IP-PULP</td>
<td>11\textsuperscript{th} – 15\textsuperscript{th} of month</td>
<td>2</td>
</tr>
<tr>
<td>GDP</td>
<td>1 day after IP</td>
<td>2</td>
</tr>
<tr>
<td>IFO</td>
<td>20\textsuperscript{th} – 30\textsuperscript{th} of month</td>
<td>0</td>
</tr>
<tr>
<td>PMI</td>
<td>20\textsuperscript{th} – 30\textsuperscript{th} of month</td>
<td>0</td>
</tr>
<tr>
<td>ESI</td>
<td>20\textsuperscript{th} – 30\textsuperscript{th} of month</td>
<td>0</td>
</tr>
<tr>
<td>Michigan Consumer</td>
<td>Last Friday of the month</td>
<td>0</td>
</tr>
<tr>
<td>dollar-euro</td>
<td>Last day of month (Monthly ave.)</td>
<td>0</td>
</tr>
<tr>
<td>US-spread</td>
<td>Last day of month (Monthly ave.)</td>
<td>0</td>
</tr>
</tbody>
</table>
Note: ratio of the RMSE of the factor model with stochastic volatility to that of a naive constant growth model.
Forecast dispersion at different releases

Note: Standardized interquartile range (difference between the 75th and the 25th percentiles standardized by the median)
Log-predictive score at different releases
Kalman smoother allows you to “dissect" the news content of each data release, taking into account the ragged-edge nature of monthly releases.

Various definitions of “news" in the literature: they all have flaws.

Recent contribution by Banbura-Modugno settles the issue. They show how to map monthly variables forecast errors into projection revisions.

We use their methodology to decompose the forecast revisions for 2010Q2 as new information accumulates.
News and forecast evolution 2010Q2: median posterior estimate

Our model assigns a probability to the overall revision...
... and to the contributions!
FORECAST EVALUATION
Relative RMSE at different horizons

### Coverage baseline

**Table: Coverage Rates - Baseline Model**

<table>
<thead>
<tr>
<th>Nom Cov</th>
<th>Backcast</th>
<th></th>
<th>Nowcast</th>
<th></th>
<th>1 step ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coverage</td>
<td>P-value</td>
<td>Coverage</td>
<td>P-value</td>
<td>Coverage</td>
</tr>
<tr>
<td>0.1</td>
<td>0.14</td>
<td>0.63</td>
<td>0.15</td>
<td>0.25</td>
<td>0.17</td>
</tr>
<tr>
<td>0.2</td>
<td>0.32</td>
<td>0.26</td>
<td>0.23</td>
<td>0.60</td>
<td>0.26</td>
</tr>
<tr>
<td>0.3</td>
<td>0.50</td>
<td><strong>0.08</strong></td>
<td>0.41</td>
<td><strong>0.08</strong></td>
<td>0.42</td>
</tr>
<tr>
<td>0.4</td>
<td>0.59</td>
<td><strong>0.09</strong></td>
<td>0.50</td>
<td>0.11</td>
<td>0.55</td>
</tr>
<tr>
<td>0.5</td>
<td>0.59</td>
<td>0.41</td>
<td>0.56</td>
<td>0.33</td>
<td>0.58</td>
</tr>
<tr>
<td>0.6</td>
<td>0.64</td>
<td>0.73</td>
<td>0.67</td>
<td>0.26</td>
<td>0.59</td>
</tr>
<tr>
<td>0.7</td>
<td>0.77</td>
<td>0.44</td>
<td>0.73</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>0.8</td>
<td>0.86</td>
<td>0.41</td>
<td>0.79</td>
<td>0.81</td>
<td>0.65</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td><strong>0.09</strong></td>
<td>0.88</td>
<td>0.60</td>
<td>0.74</td>
</tr>
</tbody>
</table>

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## Coverage Stochastic Volatility

### Table: Coverage Rates - Model with Stochastic Volatility

<table>
<thead>
<tr>
<th>Nom Cov</th>
<th>Backcast Coverage</th>
<th>Backcast P-value</th>
<th>Nowcast Coverage</th>
<th>Nowcast P-value</th>
<th>1 step ahead Coverage</th>
<th>1 step ahead P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.09</td>
<td>0.89</td>
<td>0.14</td>
<td>0.40</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>0.2</td>
<td>0.18</td>
<td>0.83</td>
<td>0.26</td>
<td>0.29</td>
<td>0.23</td>
<td>0.60</td>
</tr>
<tr>
<td>0.3</td>
<td>0.32</td>
<td>0.86</td>
<td>0.32</td>
<td>0.75</td>
<td>0.30</td>
<td>0.96</td>
</tr>
<tr>
<td>0.4</td>
<td>0.45</td>
<td>0.62</td>
<td>0.41</td>
<td>0.88</td>
<td>0.44</td>
<td>0.52</td>
</tr>
<tr>
<td>0.5</td>
<td>0.59</td>
<td>0.41</td>
<td>0.47</td>
<td>0.63</td>
<td>0.48</td>
<td>0.81</td>
</tr>
<tr>
<td>0.6</td>
<td>0.68</td>
<td>0.43</td>
<td>0.61</td>
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<td>0.69</td>
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<tr>
<td>0.7</td>
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<td>0.62</td>
<td>0.71</td>
<td>0.83</td>
</tr>
<tr>
<td>0.8</td>
<td>0.91</td>
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<td>0.82</td>
<td>0.71</td>
<td>0.73</td>
<td>0.19</td>
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<tr>
<td>0.9</td>
<td>0.95</td>
<td>0.24</td>
<td>0.89</td>
<td>0.87</td>
<td>0.77</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Some concluding remarks

- We introduce a mixed frequency factor model with stochastic volatility, and develop a Bayesian procedure for its estimation.
- We use it to model quarterly euro area GDP growth and a set of monthly indicators.
- In sample results show the relevance of changes in volatility. In addition, the estimated monthly GDP tracks very well the much more complex Eurocoin.
- We also show how, in a given quarter, the factor model can be used to assess the uncertainty around the news content of monthly releases of hard, soft and financial indicators.
- Finally, we evaluate out of sample point and density forecasts accuracy of the model, finding that SV improves substantially density forecasts.
THANK YOU FOR THE ATTENTION