A Search Model of Bank Default

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Abstract

This paper develops a model in which bank default is endogenously determined, and depends notably on bank size and bankers' behavior. By accounting for heterogeneity in entrepreneurs' productivity and information asymmetry at the expense of financial investors, moral hazard arises following a sectoral productivity shock: bankers tend to choose investments that are more profitable in the short-run but whose risk is borne by the financiers. This 'risk-shifting' mechanism magnifies credit rationing in the economy, particularly for safe borrowers, and contributes to bank default since financial investors may prefer not to (re-)capitalize intermediaries as long as they cannot control for bankers' choices. The search theory helps to depict a financial market freeze, i.e a slow down in fund-raising for even sound borrowers.

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1 Introduction

Recent years have seen renewed interest for the study of financial markets and the banking sector. The 2007-2009 crisis has emphasized the need for a better understanding of the role of financial intermediaries, from banks' behavior to general equilibrium effects of credit crunch and regulation issues. The scope of investigation is still important as far as the interactions between three types of well-known but theoretically isolated aspects are concerned, namely, the structure of the banking sector *per se* (size, leverage, incentives), real macroeconomic outcomes of banking activities (credit rationing), and financial interconnections (systemic risk, financial regulation). Some of these aspects have been extensively studied, however bank default is still hardly explained in the literature.

This paper analyzes how a downturn in a specific real sector — the housing market for instance — may lead to mistrust about banks' investment choices, and consequently fuels bank default and credit rationing. First, heterogeneity in entrepreneurs' productivity is introduced, both at the sectoral and at the individual levels. Second, information asymmetry at the expense of capital holders about banks' investment opportunities — or equivalently about bankers' choices between alternative investments — is accounted for. A sectoral productivity shock creates a 'risk-shifting' incentive through which bankers over-invest in the entrepreneurial market hit by the downturn at the expense of the financiers. As the latter cannot control for bankers' choices but bear the investment costs, they would have preferred that the intermediaries invest in long-duration credit relationships whereas the bankers tend to favor short-run profitability.

This 'risk-shifting' mechanism has two major macroeconomic consequences. As sound financial intermediaries find it more difficult to raise funds, credit rationing is in turn magnified, even for viable entrepreneurs, since there are fewer banks able to finance investment projects. The introduction of financial and credit market frictions allows to stress out the potential inefficiency due to the fact that bankers' behavior thus contributes to excessive credit rationing in distressed times. Second, the bank default rate goes up as fewer financiers are willing to (re-)capitalize the banking sector following the shock. More particularly, I find that the combination of uncertainty and information asymmetry is a necessary condition for bank default react to a sectoral productivity shock.

This paper applies the Diamond-Mortensen-Pissarides search and matching theory to financial markets, in the spirit of Wasmer and Weil (2004). First, this allows to capture some non-walrasian characteristics of financial shocks by creating periods of time in which even sound borrowers cannot raise funds following a shock. As long as under-capitalized banks offer less vacant credit lines to entrepreneurs, credit rationing is magnified. Second, assuming that screening credit applications from entrepreneurs is costly in terms of time and/or effort for banks, the search and matching framework allows to depict a potential misalignment between financial investors' and banks' interests following a shock. On the one hand, bankers tend to choose sectors in which productivity may be lower and investments riskier but in which the number of entrepreneurs looking for a loan is high so that finding an individually suitable entrepreneur is easy. On the other hand, capital holders would prefer long-duration credit relationships at the bank, even if they are risk-neutral, because they are not compensated for riskier investments or bank default in case of a sectoral productivity downturn while bear the cost of vacant credit lines. Finally, the Nash bargaining contract allows to get rid of costly state verification effects by expressing the rate of return on capital as a function of the observable surplus of the match, so that bankers' 'greediness' stems from the incentives they face instead of cheating.

In the traditional banking literature, three main concerns have been highlighted, two of them being out of the scope of this paper. The first one is the liquidity problem due to the difference of maturity between banks' assets and liabilities, causing bank runs from depositors (Diamond and Dybvig, 1983), not studied here. The second is information asymmetry, creating both adverse selection and moral hazard distortions. Information shortages may concern either the banks (lack of information on potential borrowers), resulting in credit rationing (Stiglitz and Weiss, 1981), or the capital holders (lack of information about intermediaries' lending opportunities), leading to moral hazard from bankers themselves and sometimes called 'risk-shifting'.[1] While I account for firms' credit rationing, this paper only introduces information asymmetry at the expense of capital holders while banks do identify suitable entrepreneurs. A last issue in the banking literature is optimal contract design, given costly state verification *à la* Townsend (1979). Here bankers' moral hazard inefficiency is due to the sequentiality of bankers' negotiations instead of costly monitoring.

Some recent macro-finance papers have built on this core literature to analyze the macroeconomic implications of financial disruptions. In particular, introducing financial disturbances that endanger the health of the intermediation sector in New Keynesian frameworks allows to discuss the need for alternative

[1]See Allen and Gale (2000) for instance.

monetary policy rules or unconventional interventions. Cúrdia and Woodford (2010) emphasize two sources or "purely financial disturbances" that are able to reproduce the 2007 increase in credit spreads: on the one hand, some real resources are consumed in the process of originating loans, and, on the other hand, a quantity of loans is defaulted upon each period and this increases in the quantity of loans that is provided by the intermediaries. In this literature however, banks cannot default or default for exogenous reasons. In Gertler, Kiyotaki and Queralto (2011) for instance, financial intermediaries may default if they divert assets for personal gains. This requires a huge direct delinquency rate from bankers in steady-state and does not capture default stemming from riskier investment choices instead. Finally, papers on the systemic risk resulting from banks' interdependence generally take the first bank default as exogenous (Gai and Kapadia, 2010, Krause and Giansante, 2011). In addition to account for the two Cúrdia-Woodford sources of financial disturbances here, I provide an analytical framework which is particularly convenient to explore bank solvency issues, bank default, and credit rationing. Moreover, while both banks' liquidity problem and interconnections (and thus systemic risk) are out of the scope of this paper,[2] the search frictions allow to depart from a perfectly competitive and centralized market, in line with the evidence that banks' size, banks' network and limited anonymity matter for trade outcomes in the banking sector.[3]

In the rest of this paper, Section 2 presents the model, Section 3 analyzes the effects of a sectoral productivity shock and states that bank default is only hit if uncertainty and information asymmetry are combined, Section 4 concludes.

2 Model

2.1 Notations and sequence of events

Let us consider three types of agents, namely financial investors, bankers, and entrepreneurs, who are all infinitely-lived and risk-neutral. Let assume that the financial investors (alternatively, 'capital holders' or 'saving banks') are endowed in capital while neither the banks ('intermediaries' or 'lending banks') nor the entrepreneurs have proper wealth at the beginning of the sequence of events considered here. The intermediaries thus have to raise funds from the financial

[2] Margaretic and Pasten (2012) investigate bank default through sequential bank runs resulting from a signal about liquidity concerns in the first bank.

[3] See for instance Gabrieli (2011) for the role of banks' network effects or Afonso and Lagos (2012b) for the role of bilateral trade in the interbank market.

investors before opening credit lines to the entrepreneurs. It is assumed that an entrepreneur needs a unique indivisible credit line from a bank in order to produce. However, a bank contracts with a continuum of entrepreneurs. There is a number k of productive sectors across which entrepreneurs are not mobile.

The credit market is potentially frictional as a pool of entrepreneurs looking for a loan to launch a business and a pool of banks screening credit applications from entrepreneurs coexist at each period of time. The search theory provides a tractable representation of such credit market frictions — allowing to capture all friction degrees, including frictionless markets, and supported by the data.[4] Thus, let assume that a constant returns-to-scale matching function $m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})$ determines the flow of new credit relationships from the number $N_{E^u_{k,t}}$ of entrepreneurs looking for a loan in sector k and the number $N_{C^v_{k,t}}$ of vacant credit lines that bankers open to sector-k entrepreneurs at time t. Therefore, a sector-specific *credit market tightness* is defined as

$$\phi_{k,t} \equiv N_{E^u_{k,t}}/N_{C^v_{k,t}}$$

The instantaneous probability for an entrepreneur to get a loan is thus $q_C(\phi_{k,t}) = m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})/N_{E^u_{k,t}}$ and the instantaneous probability for a bank to fill a sector-k vacant credit line is $\phi_{k,t} q_C(\phi_{k,t}) = m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})/N_{C^v_{k,t}}$, with $\partial q_C(\phi_{k,t})/\partial \phi_{k,t} < 0$.

In addition, let consider that the financial market in which intermediaries raise funds is also characterized by potential search frictions.[5] New financial relationships are determined from a similar matching function $m_F(N_{L^u_t}, N_{C^u_t})$, increasing in the mass $N_{L^u_t}$ of financiers looking for investment opportunities and in the mass $N_{C^u_t}$ of credit lines that bankers would like to finance at time t. This allows to represent a non-walrasian financial market in distressed times — in the sense that there may be no price adjustments immediately able to clear the market — without excluding efficiency (with infinite matching rates) otherwise. The *financial market tightness* is defined as

[4] See Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) for the evidence.

[5] This market can be thought of as an interbank market or as a private financial market through which large investors (re-)capitalize commercial banks. Bilateral trade is relevant to depict such markets since the actors are neither atomic nor anonymous (See Afonso and Lagos, 2012a, 2012b, for the interbank market for instance). The market for bank deposits, which is likely to be more competitive than bilateral, is also characterized by long-term relationships so that search frictions might be relevant (Tripier, 2012). However since the model is expressed in real terms and is not aimed at explaining bank runs, an equity-like fund-raising market seems more relevant than a market for liquidity here.

$$\xi_t \equiv N_{C_t^u}/N_{L_t^u}$$

and gives the instantaneous probabilities $q_F(\xi_t) = m_F(N_{L_t^u}, N_{C_t^u})/N_{C_t^u}$ for a banker to raise funds and $\xi_t q_F(\xi_t) = m_F(N_{L_t^u}, N_{C_t^u})/N_{L_t^u}$ for a financier to capitalize a credit line at a commercial bank, with $\partial q_F(\xi)/\partial \xi < 0$. Note that, unlike the credit market tightness, the financial market tightness is not k-specific because of some information asymmetry at the expense of financial investors, specified further below.

A bank therefore accumulates credit lines, that are in three possible states: 'unfunded' — as soon as the bank finds it valuable to expand but is capital-constrained —, 'open' (or 'vacant') to applications from (sector-specific) entrepreneurs, and 'productive' — once matched with an entrepreneur. Individual productive entities (credit relationships) may separate. In this case, the entrepreneur becomes unmatched, *i.e.* starts looking for a bank again or exit, while the credit line turns 'vacant'. Moreover, both productive and vacant credit lines may terminate if the bank defaults. In this case, all credit lines either turn 'unfunded' — if the bank looks for recapitalization — or are simply destroyed — if the bank is shut down (exit), while the financial investors and entrepreneurs also become unmatched (have to search again or exit). Both the sector-specific separation probability, denoted s_k, and the bank default probability, d, are endogenous variables, associated with an optimal decision rule to described later on. Figure 1 sums up the sequential matching and destruction probabilities.

Let further assume that productive entities generate output flows $A_{k,t}p_{j,k,t}$, where $A_{k,t}$ is sector-specific productivity, and $p_{j,k,t}$ is idiosyncratic productivity of the entrepreneur j in sector k, once he/she has obtained a credit line. $p_{j,k,t}$ is drawn every period and in advance of production from a time-invariant cumulative distribution function $F(\cdot)$ with positive support and density $f(\cdot)$. This output is used to reimburse the banker at a rate $\psi_{i,j,k,t}$ that is determined by a Nash bargaining rule that maximizes the surplus created by the match between the entrepreneur j and the bank i, in which $0 < \delta_C < 1$ is the bargaining power of the bankers.[6] The commercial bank receiving $\psi_{i,j,k,t}$ in turn pays back a return on capital services $\rho_{i,t}$, negotiated at the time of the match with the financial investor according to a similar Nash bargaining rule with bargaining powers $0 < \delta_F < 1$ and δ_F for capital owners and intermediaries respectively.

[6] The credit market is thus formally closer to an equity-like rather than to a debt-like contract. Thereby, Nash-bargaining is a reduced-form for an optimal contracting problem, not developed here for simplicity but still preventing from costly state verification issues.

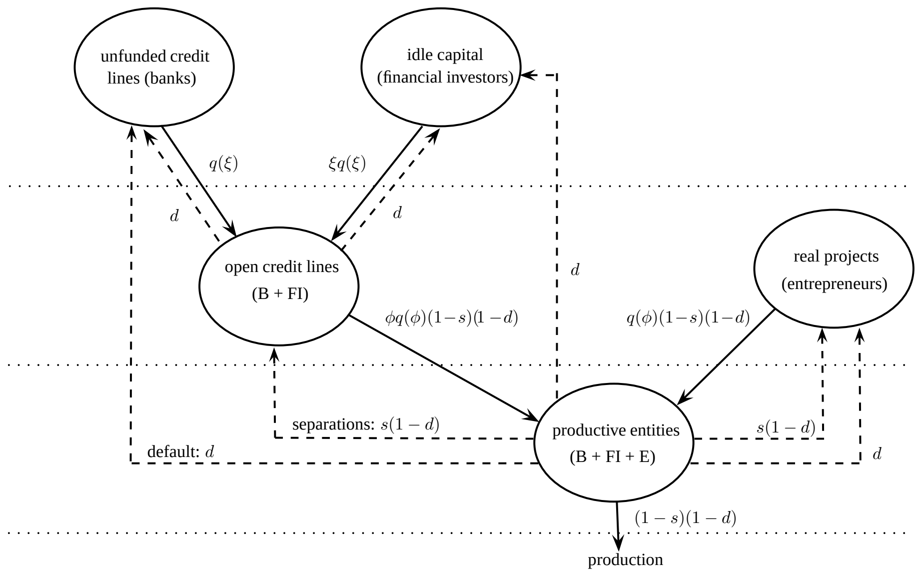

Figure 1: transition probabilities

At the time of the negotiation between the banker and the financier, the additional credit line is not associated with a particular entrepreneur yet, so that the financial repayment rate is not indexed by j, nor with a particular sector k which depends on the banker' later choice.

Finally, some pecuniary and non pecuniary flow costs are associated with search activities as follows. While financial investors are looking for suitable banks, they bear a flow cost c_I. This is the opportunity cost for — hence determines the willingness of — financiers capitalizing commercial banks instead of keeping capital idle, given discounting at a riskfree rate r and the transition probabilities.[7][8] Meanwhile banks bear a flow cost c_B while searching capital, which is non pecuniary since they do not have proper wealth ex ante. Symmetrically, entrepreneurs pay a non pecuniary flow cost c_E per period while looking for a loan. Finally, when a financier encounters a suitable bank, he/she finances c per period for a vacant credit line, while the effective application screening cost of this vacant credit line is $c'_{i,k,t}$. This cost depends on both the entrepreneurial sector and the size of bank i at time t, as banks' ability to screen

[7]Other asset types in which financial investors could invest in could be easily introduced without changing the results. Since we focus on the role of intermediaries' capital constraints here, the financiers just decide to capitalize bank activities or to exit without loss of generality.

[8]With a model in real terms, c_I can be either pecuniary or a utility cost interchangeably.

credit applications may vary with their number of sector-k credit relationships.[9]

2.2 Particular case: constant idiosyncratic productivity and exogenous destruction rates

This economy follows Wasmer and Weil (2004) in that there are three agent types and sequential interactions into two search-and-matching frictional markets.[10] However a number of additional characteristics are also included here so as to stress out the behavior and the role of financial intermediaries:

(i) time-varying idiosyncratic productivity (p) and two optimal destruction rules for individual separations (s) and a bank default rate (d),

(ii) two Nash bargaining rules, *i.e.* the two endogenous repayments ρ and ψ,

(iii) several relationships at each bank so that bank optimal size is key,

(iv) entrepreneurs' heterogeneity and information asymmetry between banks and financiers so that the amount given by the financiers while financing vacant credit lines c is not equal to the effective cost of vacancies at the bank c'_k. A possible interpretation is that only banks observe the characteristics of the entrepreneurial sectors — *i.e.* their productivity and/or the tightness, hence the search duration before a conclusive match — so that financiers have to choose between capitalizing bankers given this information asymmetry or keeping their capital idle. Alternatively, the sectoral characteristics are common knowledge but the financiers cannot control for bankers' sectoral choices while opening new credit lines once the financial match is done (no specific contract).[11]

In a deterministic economy, with constant productivity and exogenous destruction rates, but maintaining (ii)-(iv), Wasmer and Weil (2004)'s methodology can be extended quite easily to solve the model such that, given free entry,

(i) the equilibrium financial and credit market tightnesses are given by

[9] If c'_k is concave, banks tend to specialize in some entrepreneurial activities, whereas if c'_k is convex they tend to diversify their vacancies across sectors. Both cases are considered here.

[10] While Wasmer and Weil have bankers, entrepreneurs, and workers, with frictions in the credit and labor markets, I focus on the behavior of bankers as financial intermediaries, so that the frictions are on the financial and credit markets here.

[11] In this case $f(\cdot)$ can be known by financial investors as long as they are not able to discriminate individual entrepreneurs so that they have to intermediate their investments.

$$\bar{\xi} = \frac{1-\delta_F}{\delta_F}\frac{c_I}{c_B} \quad \text{(financial market tightness), and recursively} \tag{1}$$

$$\bar{\phi}_k = \frac{1-\delta_C}{\delta_C}\Big[\frac{c_B}{c_E}\frac{r+d}{q_F(\bar{\xi})} + \frac{c'_k - c}{c_E}\Big] \quad \text{(credit market tightness)} \tag{2}$$

(ii) the equilibrium repayment rates are the solution to the pair of equations

$$\psi = \frac{\delta_C A A_k p_k - (\rho + c - c'_k)[r + s_k(1-d) + d](1-\delta_C)}{r + s_k(1-d) + d + \delta_C \phi_k q_C(\phi_k)(1-s_k)(1-d)} \tag{3}$$

$$\rho = \delta_F - (\delta_F c'_k - c)\frac{r + d + s_k(1-d)}{\phi_k q_C(\phi_k)(1-s_k)(1-d)} \tag{4}$$

(iii) an equilibrium condition for each agent type — a credit creation condition for banks, a bank capitalization condition for financiers, and a search condition for entrepreneurs — can be derived analytically (see Appendix)[12]

Equation (1) expresses that the financial market tightness — *i.e.* the ratio of the number of banks willing to raise funds over the number of units of capital provided financial investors to the banking sector — increases in financial investors' costs c_I relatively to banks' costs c_B and in banks' (relatively to financial investors') bargaining power $(1-\delta_F)$. The credit market tightness in (2) similarly depends on entrepreneurs' and banks' search costs and bargaining powers. It also increases in the average time that banks need to raise funds on the financial market $(1/q_F(\bar{\xi}))$, in the bank default rate d as this corresponds to fewer banks for a given number of entrepreneurs, and in the riskfree rate r which is the opportunity cost associated with vacant credit lines.[13]

Finally, note that bankers make a trade-off when deciding the sector to which they will open new credit lines. In equilibrium a no-arbitrage condition must hold so that the asset value of opening new credit lines must be equalized across sectors. However, following a negative sectoral productivity shock, bankers may prefer to reallocate their vacant credit lines to the sector hit by the shock even it this sector has low productivity because entrepreneurs looking for a loan in this sector are numerous — hence the sectoral credit market tightness higher and bankers' search duration shorter — instead of investing in high-productivity sectors. However, this will not be sufficient to observe a misalignment between bankers' and financial holders' interests as both find it profitable that the credit

[12]Similarly to firms' job creation condition in the standard labor market literature.

[13]If there were only one entrepreneurial sector in the economy, no bank default nor variable size, equation (2) would be the same than in my first chapter, and would further collapse to Wasmer and Weil (2004)'s credit market tightness if there were only one Nash rule.

line becomes productive rapidly (see Section 3). Thus introducing some source of uncertainty in the model is necessary to make bankers' moral hazard emerge.

2.3 Random idiosyncratic productivity and endogenous separations and default rates

Let us now write the full model and characterize the equilibrium when idiosyncratic productivity is random and the destruction rates are endogenous. Time is discrete. Figure 2 describes the timing of events for capitalized banks: firms' productivity is drawn every morning, then potential separations from existing credit relationships at the bank and potential bank defaults are determined, according to optimality rules which will be described later on. Production then occurs according to the number of remaining filled credit lines. New credit relationships at the end of the day become productive from the next day onward.

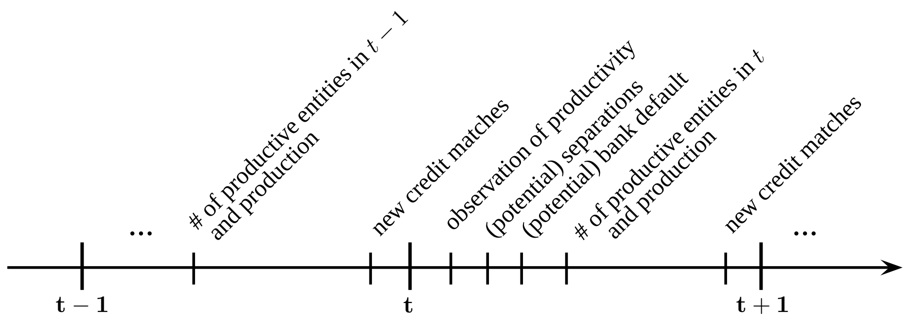

Figure 2: Timing of Events

2.3.1 Surplus sharing

The credit repayment rate that shares the (gross) surplus created by the match between an entrepreneur and a vacant credit line is given by a Nash bargaining rule (dropping i and j subscripts) as

$$\psi_{k,t} = \arg\max(C^p_{k,t} - C^v_{k,t})^{\delta_C}(V^p_{k,t} - V^u_{k,t})^{(1-\delta_C)} \tag{5}$$

where $V^p_{k,t}$, respectively $V^u_{k,t}$, denotes the value function of a sector-k entrepreneur who is producing, respectively looking for a loan (unmatched), at time t, $C^v_{k,t}$, respectively $C^p_{k,t}$, is the value function of a credit line which is vacant, respectively productive, and where $0 < \delta_C < 1$, respectively $1 - \delta_C$, is the bargaining power of the bank, respectively of the entrepreneur, on the credit market.[14]

Given the search costs and the transition probabilities given in Section 2.1, the Bellman equations standing for sector-k entrepreneurs — while unmatched and matched/productive respectively — are given by

$$V_{k,t}^{u} = -c_E + \beta E_t \left\{ q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{V_{k,t+1}^{p}(p) f(p)}{1-F(p^R)} dp \right\} + \beta E_t \left\{ [1 - q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1})] V_{k,t+1}^{u} \right\}$$

$$V_{k,t}^{p}(p) = A_{k,t} p_{k,t} - \psi_{k,t} + \beta E_t \{ [s_{k,t+1}(1-d_{t+1}) + d_{t+1}] V_{k,t+1}^{u} \} + \beta E_t \left\{ (1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{V_{k,t+1}^{p}(p) f(p)}{1-F(p^R)} dp \right\}$$

where p_{ikt}^R is the optimal reservation level for sector-k entrepreneurs' idiosyncratic productivity chosen by bank i at time t, such that matches producing $p_{ijkt} < p_{ikt}^R$ are not profitable and terminate (the optimal separation rule is made explicit further below).[15] The value functions for (sector-k) credit lines, which are respectively unfunded, vacant, and productive, are given by

$$C_{k,t}^{u} = -c_B + \beta q_F(\xi) C_{k,t+1}^{v} + \beta [1 - q_F(\xi)] C_{k,t+1}^{u}$$

$$C_{k,t}^{v} = c - c'_{k,t} + \beta E_t \{ [1 - \phi_{k,t} q_C(\phi_{k,t})(1-s_{k,t+1})](1-d_{t+1}) C_{k,t+1}^{v} + d_{t+1} C_{k,t+1}^{u} \} + \beta E_t \left\{ \phi_{k,t} q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{C_{k,t+1}^{p}(p) f(p)}{1-F(p^R)} dp \right\}$$

$$C_{k,t}^{p}(p) = \psi_{k,t} - \rho_t + \beta E_t \left\{ (1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{C_{k,t+1}^{p}(p) f(p)}{1-F(p^R)} dp \right\} + \beta E_t \{ s_{k,t+1}(1-d_{t+1}) C_{k,t+1}^{v} \} + \beta E_t \{ d_{t+1} C_{k,t+1}^{u} \}$$

[14]This Nash bargaining rule gives tractable analytical results because the agents are risk-neutral here, however the same steady-state could be obtained with risk-averse agents by replacing this equation by a "surplus-splitting bargaining rule" ensuring that the agents exactly get δ_C and $1-\delta_C$ percent of the surplus at any point in time (Merz (1995), Andolfatto (1996), Cooley and Quadrini (1999)).

[15]Similar Bellman equations when idiosyncratic productivity is stochastic, the separation rate is endogenous and time is discrete can be found in Krause and Lubik (2007) for instance.

Symmetrically, the repayment rate ρ_t that shares the financial surplus from investors' and bankers' match is given by a second Nash rule as

$$\rho_t = \arg\max(I_t^v - I_t^u)^{\delta_F}(C_{kt}^v - C_t^u)^{(1-\delta_F)} \tag{6}$$

where δ_F is the bargaining power of investors in the financial market[16], and where the value functions I_t^u and I_t^v of financial investors, respectively looking for banking opportunities and financing vacant credit lines, are

$$I_t^u = -c_I + \beta\xi q_F(\xi)I_{t+1}^v + \beta[1-\xi q_F(\xi)]I_{t+1}^u$$

$$I_t^v = -c + \beta E_t\{d_{t+1}I_{t+1}^u + (1-d_{t+1})[\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1})I_{t+1}^p + (1-\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1}))I_{t+1}^v]\}$$

In addition, investors' value function while credit lines get productive is

$$I_t^p = \rho_t + \beta E_t\{d_{t+1}I_{t+1}^u + (1-d_{t+1})[s_{kt+1}I_{t+1}^v + (1-s_{kt+1})I_{t+1}^p]\}$$

Again, the amount c that is given by the financiers to bankers in order to finance a vacant credit line is not sector-specific since application screening is the competence of commercial banks and the motive for intermediation here. Because of the information asymmetry, c is generally different from the effective application screening cost c'_k derived below. Moreover, because bankers' sectoral choice is taken after the financial bargaining, neither the financial repayment rate, ρ, nor the financiers' value functions are indexed by k. However, the credit surplus is observable so that bankers cannot cheat on the financial repayment once the credit line becomes productive.[17]

2.3.2 Bank screening technology and optimal size

A bank wants to expand as soon as creating new productive credit lines increases its expected present-discounted profits net of search costs. Since every credit line is 'vacant' before being 'productive' here, a sufficient condition to determine an optimal bank size is to consider that the marginal cost of vacant credit lines

[16]Since financial investors and bankers decide how to share the financial surplus at the time they meet, *i.e.* before the banker-entrepreneur negotiation, financial investors know that they will get a fraction $\tilde{\delta}_F$ of the net total surplus, and negotiate backward a share $\delta_F = \tilde{\delta}_F/[1-(1-\tilde{\delta}_F)(1-\delta_C)]$ of the financial surplus.

[17]Without costly state verification issues, bankers' 'greediness' differs from delinquency.

depends on the size of bank i. Let us assume that the application screening cost is increasing in the number $N_{C^v_{ikt}}$ of sector-k vacant credit lines at bank i as

$$c_k = \kappa (N_{C^v_{ikt}})^\epsilon$$

so that banks are more and more efficient in screening credit applications from sector-k entrepreneurs if $\epsilon < 1$, less and less otherwise.[18] Given the law of motion for $N_{C^v_{ikt}}$, further derivation (detailed in Appendix) gives the equilibrium flow cost of an additional vacant credit line opened to sector-k entrepreneurs as

$$c'_{kt} = \frac{\kappa}{q(\phi_{kt})} \frac{\epsilon N^{1-\epsilon} (N_{C^v_{kt}})^\epsilon}{(1 - s_{kt+1})(1 - d_{t+1}) N_{E^u_{kt}}} \tag{7}$$

where N is the total number of banks.

2.3.3 Optimal destruction rules

- Individual credit relationship separations

It is optimal for banks to terminate the credit relationship with a particular productive entrepreneur if the continuation value of remaining matched is smaller than the continuation value of having a vacant credit line, *i.e.* if

$$C^p_{kt}(p) < C^v_{kt}$$

Following the labor market literature, this rule is equivalent to determine an endogenous reservation threshold for idiosyncratic productivity that banks require from the entrepreneurs, such that matches producing $p_{ijkt} < p^R_{ikt}$ are not profitable and separate. After some computations (given in Appendix), substituting the time-varying expression for the Nash-bargained credit repayment rate into $C^p_{ikt}(p) - C^v_{kt} = 0$, the optimal threshold is

$$p^R_{ikt} = \frac{1}{A_{kt}} \left[\rho_{it} + (c - c'_{ikt}) - \frac{c_E}{q_C(\phi_{kt})} \frac{1 - \delta_C \phi_{kt} q(\phi_{kt})}{1 - \delta_C} \right] \tag{8}$$

As one would expect, this threshold decreases in the sectoral productivity level A_{kt}: bankers require from entrepreneurs a higher idiosyncratic productivity to compensate for a low sectoral productivity, everything else equal. It increases in the financial rate ρ that bankers have to pay back to investors. It decreases in

[18] Similarly, Rotemberg (2006) considers that firms have recruitment costs that are concave in the number of job vacancies and defines the equilibrium as if the costs were convex.

both search cost c'_{ikt} and c_E on the credit market as a rise in these costs increases the continuation value of remaining matched today. It increases in the sectoral credit market tightness since, when there are many searching entrepreneurs relatively to bankers, the search duration for bankers is short and this tends to increase the minimum idiosyncratic productivity required from entrepreneurs.

Therefore the sector-specific separation rate s_k is given by

$$s_{kt} = s_k(p^R) = F(p^R) = \int_0^{p^R_{kt}} f(p)dp \tag{9}$$

- Bank default

Symmetrically, a bank defaults if the stockholders' continuation value of remaining matched with this bank is smaller than the continuation value of having idle capital — given the costs of financing vacant credit lines and the expected duration before finding a suitable entrepreneur in particular —, *i.e.* if

$$I_t^v < I_t^u, \quad \text{where } I_t^u = 0 \text{ by free entry.}$$

However it is not possible to determine a similar threshold of idiosyncratic productivity in order to infer the default rate, even though a time-varying expression for the financial repayment rate is determined. Therefore, it is necessary to consider the equilibrium default rate directly, stemming from $I_t^v = 0$ and the Nash rule for the financial repayment rate, so as to obtain

$$E_t d_{t+1} = 1 + \frac{\xi_t q_F(\xi_t)}{c_I}\left[\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}c_E\phi_{kt} - c\right] \tag{10}$$

Bank default decreases in c_I since higher costs while financial investors are searching for a suitable bank increases the value of remaining matched today, *i.e.* not impose on the bank to default. The same rationale holds for the cost c borne by the financiers as long as the credit line is vacant. On the contrary, a higher matching rate $\xi q_F(\xi)$ increases financial investors' outside options so that the value of remaining matched with the same bank decreases, leading more banks to default. The higher the bargaining power $\tilde{\delta}_F$ of the investors on the financial market vis-à-vis the bankers $(1-\tilde{\delta}_F)$, the higher the bank default rate since bargaining with a new bank becomes more profitable. Finally, when the separation rate increases, the bank default rates increases along with the credit market tightness.

2.3.4 Equilibrium credit rationing

Finally, let us define some aggregate quantities that can easily be derived at equilibrium. In particular, credit rationing in a productive sector is defined as the number of entrepreneurs within the sector who are currently — effectively but unsuccessfully — looking for a loan. By normalizing the total number of entrepreneurs to unity and equalizing flows into and out of the pool of unmatched entrepreneurs, we get an equilibrium level of credit rationing in sector k as

$$N_{E_k^u} = \frac{s_k(1-d)+d}{s_k(1-d)+d+q_C(\phi_k)(1-s_k)(1-d)} \tag{11}$$

More separations in sector k or bank default increase credit rationing in sector k whereas a higher matching probability $q_C(\phi_k)$ for sector-k entrepreneurs decreases credit rationing in sector k.[19]

Moreover, the number of productive entities in each sector is $N_{E_k^p} = 1 - N_{E_k^u}$ and is also equal to the number $N_{C_k^p}$ of productive credit lines, given by

$$N_{C_k^v} = \frac{s_k(1-d)+d}{\phi_k q_C(\phi_k)} N_{C_k^p} \tag{12}$$

Recursively, the number $N_{C_k^u}$ of unfunded credit lines (bank projects) is

$$N_{C^u} = \frac{[d+\phi_k q_C(\phi_k)(1-s_k)(1-d)]N_{C_k^v} - s_k(1-d)N_{C_k^p}}{q_F(\xi)} \tag{13}$$

Regarding financial investors, the number of capital units involved in production is equal to the number of productive entities summed up across sectors ($N_{I^p} = \sum_k N_{C_k^p}$), while the number of capital units financing vacant credit lines is equal to the sum of vacant credit lines ($N_{I^v} = \sum_k N_{C_k^v}$). As we know from the transition matrix that the law of motion for $N_{I_{t+1}^v}$ is[20]

$$N_{I_{t+1}^v} = (1-d_{t+1})[1-\phi_{kt} q_C(\phi_{kt})(1-s_{kt})N_{I_t^v} + s_{kt}(1-d_{t+1})N_{I_t^p} + \xi_t q_F(\xi_t) N_{I_t^u}$$

the equilibrium number of financiers looking for a banking investment is

$$N_{I^u} = \frac{[d+\phi_k q_C(\phi_k)(1-s_k)(1-d)]N_{I^v} - s_k(1-d)N_{I^p}}{\xi q_F(\xi)} \tag{14}$$

This is the mass of capital that is available for the banking sector but not imme-

[19] If $d = 0$, this expression reminds the Beveridge curve in the labor market literature.

[20] The law of motion could easily be generalized to allow for other types of financial assets, including bonds with an endogenous riskfree rate r for instance, without changing the results.

diately allocated to a particular bank because of the financial friction stemming from the information asymmetry between the financiers and individual banks.

2.3.5 Equilibrium

Given free entry, *i.e* with $V_k^u = 0$, $C_k^u = 0$, and $I^u = 0$, the Nash bargaining rules (5) and (6) imply that (see Appendix for details)

(i) the equilibrium financial and credit market tightnesses are similar to (1) and (2) in the deterministic case despite time-with varying idiosyncratic productivity and endogenous destruction rates here,

(ii) the agents' equilibrium conditions (for searching for a loan, creating an additional credit opportunity, and capitalizing banks) are also similar to the deterministic case

(iii) in addition to (3) and (4) that still hold in equilibrium, time-varying expressions for the two repayment rates can be derived as

$$\psi_{kt} = \delta_C A_{kt} p_{kt} + (1-\delta_C)(c - c'_{kt} + \rho_t) + \delta_C c_E \phi_{kt} \tag{15}$$

$$\rho_t = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}(A_{kt}p_{kt} - \psi_{kt} + c_E\phi_{kt}) - c \tag{16}$$

Further, since the repayment rate is linear in the idiosyncratic productivity level and since credit relationships producing $p_{jk} < p_{ik}^R$ separate, the average repayment rate observed at time t on the credit market is given by

$$\begin{aligned} E_t[\psi_{ijkt}(p)|p_{jkt} \geq p_{ikt}^R] = \delta_C A_{kt} E_t(p_{jkt}|p_{jkt} \geq p_{ikt}^R) + \delta_C c_E \phi_{kt} \\ +(1-\delta_C)[\rho_t + (c - c'_{ikt})] \end{aligned}$$

Equilibrium is thus characterized by equations (1),(2), (7)–(16) and the set of unknowns $\{\xi, \phi_k, \psi_k, \rho, c'_k, p_k^R, s_k, d, N_{E_k^u}, N_{C_k^v}, N_{C^u}, N_{L^u}\}$. The next Section analyzes the effects of a permanent sectoral productivity shock.

3 Effects of a sectoral productivity shock

3.1 Case 1: No information asymmetry

As a first case, let suppose that there are several entrepreneurial sectors but no information asymmetry in the economy. The financiers know (or can control for) the particular sectors in which the bankers open new credit lines as well as the size of the bank so that they pay the exact amount c'_k that is necessary for each vacancy. For further simplicity, let assume that $c'_k = c$ for all credit lines as bank size becomes irrelevant in this particular case.

Let consider a negative sectoral productivity shock. As A_1 falls, the threshold for idiosyncratic productivity that bankers require from entrepreneurs rises (equation (8)). Since the distribution $f(\cdot)$ is time-invariant, more entrepreneurs in sector 1 fall below the new threshold and more separations occur by (9). Therefore credit rationing increases in sector 1 by (11). However, because the cash flow ψ_1 from the entrepreneurs to the bankers falls with the surplus by (15) — while separations are not constrained — bankers increase the number of vacant credit lines in sector-1 to increase their profits. Thus the credit market tightness in sector 1, ϕ_1, is unaffected by the shock. The financial repayment ρ also decreases (16) so that, given that there is no information asymmetry, the financiers capitalize more vacant credit lines to offset the profit loss on individual relationships. This is the case because they know the cost associated with each vacancy and because there is no costly state verification (the surplus is known every period so that the banks cannot cheat on the repayment to the financiers even though ρ is not sector-specific). The financial market tightness ξ is thus unchanged as well. (See Appendix A.8.)

If the economy is deterministic (constant idiosyncratic productivity), the effects on the two repayment rates ϕ_1 and ρ are identical, however, all the other variables — including the sector-1 credit rationing and credit market tightness — are unaffected by the shock.[21] Thus the response of credit rationing to a sectoral productivity shock is due to the presence of uncertainty, captured by random draws from $f(\cdot)$ here. Since there is no risk premium in the model — that would bankers to increase ψ_1 above its current value following the shock —, bankers rise their reservation threshold, so that credit relationships are more fragile (their duration is subjected to changes in individual productivity) in the

[21] On the contrary, in the (deterministic) Wasmer and Weil (2004)'s economy, a fall in output affects the labor market tightness — and a financial accelerator stemming from the existence of two search frictions magnifies the effect — because workers' wage is exogenous.

stochastic economy. However, as long as there is no information asymmetry at the expense of financial investors, the ability of banks to raise funds is the same than before the shock and bank default is unaffected, even though the turn-over of credit relationships is more important in the sector hit by the shock.

3.2 Case 2: information asymmetry and moral hazard

The simplest way to capture information asymmetry in this model is to assume that $c'_k \neq c$ such that the financiers pay c for each vacant credit line whatever the sector. If c'_k differs from c but is exogenous, the effects of a sectoral productivity shock is the same than if $c'_k = c$. Therefore it is assumed that this cost depends on the number of vacant credit lines at the bank, $c_k(N_{C^v_{ikt}}) = \kappa(N_{C^v_{ikt}})^\epsilon$. To this respect, bank size matters but one could imagine other determinants of c'_k so that information asymmetry would hold with constant bank size.

When idiosyncratic productivity is constant (deterministic economy), the effects of a sectoral productivity shock are identical whether information is asymmetric or not and whether there the bank default rate is exogenous or not. More precisely the two repayment rate increase following a negative sectoral productivity shock, while all the other variables — including credit rationing — are not affected (See Appendix A.9.1). Information asymmetry is thus irrelevant because the variation in the credit repayment rate captures the fall in sectoral productivity and, since there is no costly state verification, the sectoral choice made by bankers does not affect the financiers' earnings.

On the contrary, the combination of an endogenous size of banks and an optimal threshold for idiosyncratic productivity (stochastic economy) makes information asymmetry matter, because it allows bankers to benefit from the high profitability of short-duration credit relationships at the expense of the financiers. The sign of the responses differs if bankers' application screening costs are concave ($\epsilon < 1$) or convex ($\epsilon > 1$). In particular, the credit market tightness, ϕ_k, goes up if the marginal cost of opening new vacant credit lines is convex since the rise in c'_k makes bankers better off if they stop searching for new entrepreneurs. On the contrary, if the cost is concave (and low enough), the tightness falls because banks have more incentive to expand their size as long as the entrepreneurs meet the higher idiosyncratic productivity threshold (See Appendix A.9.2). However the financial market tightness ξ is unchanged so that banks' expansion in the sector hit by the shock reduces the number of vacant credit lines for other sectors in the economy (everything else equal).

Overall, bankers benefit from the fact that (*i*) a higher threshold for idiosyncratic productivity increases the credit repayment rate relatively to the deterministic economy, despite lower sectoral productivity, (*ii*) the period during which credit lines are vacant is less costly, either because the match is quicker when relatively more entrepreneurs are looking for a loan (c'_k convex) or because application screening is more and more effective (c'_k concave). Nonetheless, the drawback of a negative sectoral productivity shock is that separations are more frequent (since $f(\cdot)$ is time-invariant but the threshold is higher, less entrepreneurs are viable next periods). As the financiers have to pay a fix amount per vacancy each time credit relationships separate, they would prefer that the banks invest in sectors with long-duration credit relationships instead. Following the shock, bankers thus become "greedy" in the sense that they make investment choices that are neither aligned with the financiers' interests — with a higher risk since the entrepreneurs who are solvent today are more likely to fall below the reservation threshold tomorrow — nor profitable for the economy — as they provide relatively more loans to the low-productivity sector at the expense of the 'good' sectors.[22]

The combination of information asymmetry and uncertainty in the economy creates a 'risk-shifting incentive' for bankers, with major macroeconomic consequences as the initial sectoral productivity shock is transmitted through credit rationing to the other sectors in the economy. Besides, bank default only reacts to sectoral downturns under these two conditions, while it remains unaffected as long as there is either information asymmetry or uncertainty in the economy (Appendix A.9.2). When a bank defaults, the costs of vacant credit lines that never get productive or have become productive for a short period of time are borne by the financiers while the bankers only stop making profits. Since there is no bailout of the defaulting banks here, more of the entrepreneurs who are individually creditworthy or belong to the 'good' sectors of the economy get credit constrained since all credit relationships are destroyed in the banks which are shut down. Finally, in case the application screening costs are concave, the size of the banks which do not default but invest in short-run profitability at the expense of safer borrowers increases following the shock. Therefore bank default not only goes up because of the risk-shifting mechanism but is also likely to increase in turn the inefficiency resulting from bankers' moral hazard.[23]

[22] A social planner's problem would be necessary to assess the welfare loss more precisely.

[23] This may also increase the systemic risk or the cost of a public bailout but these issues are out of the scope of this paper. Bank default inefficiently magnifies credit rationing by

4 Conclusion

This paper develops a multi-frictional yet tractable model in which bankers' behavior crucially affects the macroeconomic outcomes, including credit rationing and bank default. By introducing alternative investment choices for bankers and information at the expense of financial investors, a risk-shifting mechanism arises. Following a sectoral productivity shock, bankers tend to choose investments that are riskier, because they are more profitable in the short run, even if they are not aligned with the financiers' interests nor suitable for the economy as credit constraints become more binding for high-productivity entrepreneurs.

Moreover, it allows to determine an endogenous bank default rate and shows that a sectoral productivity shock affects the equilibrium bank default rate if and only if information asymmetry and uncertainty are combined in the economy. Because of entrepreneurs' heterogeneity and information asymmetry, financial investors cannot observe real investment opportunities and/or control for banks' investment choices. Therefore, if a negative sectoral productivity shock arrives, they may prefer not to (re-)capitalize banks even though the latter are still able to appraise the idiosyncratic productivity of potential borrowers within or outside of the sector hit by the shock. The search and matching environment is especially appropriate to depict financial markets that are almost frictionless in normal times but can remain frictional for an extended period of time in case of a disruption. Hence, distressed times are characterized by a significant slow down of fund-raising from sound borrowers and a magnification of credit rationing for all sectors in the economy.

Some extensions of the model could add to the results presented here. For instance, considering the problem of a social planner that is able to discriminate suitable credit relationships (similarly to banks) but suffers from capital losses in case of default (similarly to capital holders) would allow to assess the size of the inefficiency and to determine the desirability of policy interventions that would provide liquidity access to the banking sector. Furthermore, including depositors in the model is not likely to change the main predictions. In the absence of deposit insurance, threats to the solvency of a particular bank would give a signal for a run, so that the bank would effectively defaults under the conditions presented here. In the presence of a deposit insurance scheme or a

destructing the relationships with viable entrepreneurs at the defaulting banks. An extension of the model in which banks' balance sheets would be interrelated could however provide a rationale to the systemic risk with the same underlying mechanism.

public bank bailout conditional on the departure of the shareholders, a drop in the stock value of the bank would replace the bank default in practice.

A Mathematical appendix

A.1 Equilibrium market tightnesses and repayment rates

Following Wasmer and Weil (2004), the equilibrium financial market tightness is easily derived from the Nash bargaining rule on the financial repayment rate,

$$(1-\delta_F)(I_t^v - I_t^u) = \delta_F(C_{kt}^v - C_t^u),$$

together with the first (backward looking) Bellman equation for banks,

$$C_{k,t}^u = -c_B + \beta q_F(\xi) C_{k,t+1}^v + \beta[1 - q_F(\xi)] C_{k,t+1}^u,$$

considered at equilibrium, and for financial investors,

$$I_t^u = -c_I + \beta \xi q_F(\xi) I_{t+1}^v + \beta[1 - \xi q_F(\xi)] I_{t+1}^u,$$

and given that free entry implies $I^u = 0$ and $C^u = 0$, as equation (1).

Similarly, the Nash bargaining rule for the credit repayment rate

$$(1-\delta_C)(C_{k,t}^p - C_{k,t}^v) = \delta_C(V_{k,t}^p - V_{k,t}^u),$$

with the first (backward looking) Bellman equation for entrepreneurs which is, in the deterministic case,

$$\begin{aligned} V_{k,t}^u = &-c_E + \beta q_C(\phi_{k,t})(1 - s_{k,t+1})(1 - d_{t+1}) V_{k,t+1}^p \\ &+\beta[1 - q_C(\phi_{k,t})(1 - s_{k,t+1})(1 - d_{t+1})] V_{k,t+1}^u, \end{aligned}$$

the first and the second Bellman equation for credit line as

$$\begin{aligned} C_{k,t}^v = c - c_{k,t}' + \beta \phi_{k,t} q_C(\phi_{k,t})(1 - s_{k,t+1})(1 - d_{t+1}) C_{k,t+1}^p + \beta d_{t+1} C_{k,t+1}^u \\ +\beta[1 - \phi_{k,t} q_C(\phi_{k,t})(1 - s_{k,t+1})](1 - d_{t+1}) C_{k,t+1}^v \end{aligned}$$

and free entry, $V^u = 0$, give the expression for the credit market tightness (2).

The two equilibrium repayment rates are the solution to the pair of equations

(3) and (4) which are obtained by substituting into the two Nash rules the forward looking Bellman equations, respectively given by

$$V^p_{k,t} = A_k p_k - \psi_{k,t} + \beta[s_{k,t+1}(1-d_{t+1}) + d_{t+1}]V^u_{k,t+1} \\ +\beta(1-s_{k,t+1})(1-d_{t+1})V^p_{k,t+1}$$

$$C^p_{k,t} = \psi_{k,t} - \rho_t + \beta(1-s_{k,t+1})(1-d_{t+1})C^p_{k,t+1} + \beta d_{t+1}C^u_{k,t+1} \\ +\beta s_{k,t+1}(1-d_{t+1})C^v_{k,t+1}$$

$$I^v_t = -c + \beta d_{t+1}I^u_{t+1} + \beta(1-d_{t+1})[\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1})I^p_{t+1} \\ +\beta(1-\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1})]I^v_{t+1}$$

$$I^p_t = \rho_t + \beta d_{t+1}I^u_{t+1} + \beta(1-d_{t+1})[s_{kt+1}I^v_{t+1} + \beta(1-s_{kt+1})I^p_{t+1}]\}$$

The Bellman equations in the case where idiosyncratic productivity is stochastic and the separation rate endogenous can be reduced to the Bellman equations in the deterministic case so that (1)–(4) hold from the same computation.

A.2 Individual equilibrium conditions

A.2.1 Banks

An equilibrium credit creation condition for banks can be obtained by equalizing the forward and backward values of C^v and C^p from the three Bellman equations for credit lines and free entry ($C^u = 0$) (see Wasmer and Weil, 2004), as

$$\frac{c_B}{q_F(\xi)} = \frac{(\psi_k(p)-\rho)(1-s_k)(1-d)\phi_k q_C(\phi_k) + (c-c'_k)[r+d+s_k(1-d)]}{(r+d)[r+d+s_k(1-d)+\phi_k q_C(\phi_k)(1-s_k)(1-d)]}$$

where the left-hand side is the flow cost of fundraising c_B times the average duration $1/q_F(\xi)$, and the right-hand side is the expected present-discounted profits earned from productive credit lines depending on the search costs and duration of application screening (vacant credit lines), the separation and default rate, and the riskfree rate $r = 1/\beta - 1$.

An alternative method to obtain this equilibrium condition is to maximize over $N_{C^u_{i,t}}$, $N_{C^v_{i,k,t+1}}$, $N_{C^p_{i,k,t+1}}$, and $p^R_{i,k,t}$, bank i's profits given by

$$E_0 \sum_0^{\infty} \beta^t \left\{ \Psi_{i,t} - \rho_{i,t} N_{C^p_{i,t}} + c N_{C^v_{i,t}} - c(N_{C^v_{i,t}}) - c_B N_{C^u_{i,t}} \right\}$$

subject to

$$N_{C^p_{i,k,t+1}} = (1 - s_{i,k,t+1})(1 - d_{i,t+1})[N_{C^p_{i,k,t}} + N_{C^v_{i,k,t}} \phi_{k,t} q_C(\phi_{k,t})],$$

$$N_{C^v_{i,k,t+1}} = (1 - d_{i,t+1})[1 - \phi_{k,t} q_C(\phi_{k,t})(1 - s_{i,k,t+1})] N_{C^v_{i,k,t}} + q_F(\xi_t) N_{C^u_{i,t}} + (1 - d_{i,t+1}) s_{i,k,t+1} N_{C^p_{i,k,t}}$$

$$\text{and} \quad \Psi_{i,k,t} = N_{C^p_{i,k,t}} \int_{p^R}^{\infty} \frac{\psi_{k,t}(p) f_k(p)}{1 - F(p^R_{i,k,t})} dp,$$

where the first two constraints are the laws of motion for filled and vacant credit lines respectively, to be summed up across sectors with $N_{C^p} = \sum_k N_{C^p_k}$, $N_{C^v} = \sum_k N_{C^v_k}$, and where the third equation is the sum of the repayment rates at the bank (total instantaneous earnings) obtained from Nash bargaining with individual entrepreneurs given their idiosyncratic productivity draws (with $\Psi = \sum_k \Psi_k$).[24] The first-order conditions for this problem are

$$(N_{C^u_{i,t}} :) \quad \lambda_t = \frac{c_B}{q_F(\xi_t)}$$

$$(N_{C^v_{i,k,t+1}} :) \quad \lambda_t = \beta E_t \{ c - c'_{ikt} + \lambda_{t+1}(1 - d_{it+1})[1 - \phi_{kt} q_C(\phi_{kt})(1 - s_{ikt+1})] + \mu_{t+1} \phi_{kt} q_C(\phi_{kt})(1 - s_{ikt+1})(1 - d_{it+1}) \}$$

$$(N_{C^p_{i,k,t+1}} :) \quad \mu_t = \beta E_t \left\{ \frac{\partial \Psi_{ikt}}{\partial N_{C^p_{ikt}}} - \rho_{it} + \mu_{t+1}(1 - s_{ikt+1})(1 - d_{it+1}) + \lambda_{t+1} s_{ikt+1}(1 - d_{it+1}) \right\}$$

$$(p^R_{ijkt} :) \quad \frac{\partial \Psi_{ikt}}{\partial p^R_{ijkt}} = (\mu_t - \lambda_t) \frac{\partial s_{ikt}}{\partial p^R_{ijkt}} (1 - d_{it})[N_{C^p_{ikt-1}} + \phi_{kt-1} q(\phi_{kt-1})) N_{C^v_{ikt-1}}]$$

[24] If all credit lines had the same productivity, total earnings at time t at bank i would simply be $\psi_{ikt} N_{C^p_{ikt}}$.

where λ and μ are the Lagrange multipliers associated with the constraints. Solving for the first three equations at equilibrium would give the same credit creation condition for bankers.

A.2.2 Financial investors

With free entry, $L^u = 0$, the first Bellman equation gives the forward value $I^v = \frac{c_I}{\beta \xi q_F(\xi)}$. The second and third Bellman equations can be solved together at equilibrium to obtain the backward value of I^p and I^v. Equalizing the forward and backward values of I^v finally gives financial investors' condition as

$$\frac{c_I}{\xi q_F(\xi)} = \frac{\rho\, \phi_k q_C(\phi_k)(1-s_k)(1-d) - c[r+d+s_k(1-d)]}{(r+d)[r+d+s_k(1-d)+\phi_k q_C(\phi_k)(1-s_k)(1-d)]}$$

On the left hand side is the cost of entering the financial market that depends on the periodic cost times the duration before a conclusive match. On the right hand side are the expected gains that depend on the periodic return on capital ρ received from the banks minus the cost paid while the bank is screening entrepreneurs' applications, given the discount rate and the transition rates.

A.2.3 Entrepreneurs

The free entry condition, $V^u = 0$, and the first Bellman equation, gives the forward value for V^p. Free exit and the last Bellman equation give the backward value for V^p. Hence the equilibrium condition is as follows

$$\frac{c_E}{q(\phi_k)} = \frac{[AA_k p_k - \psi_k(p)](1-s_k)(1-d)}{r+d+s_k(1-d)}$$

On the left hand side are expected costs for sector-k entrepreneurs while seeking a loan (the flow cost c_E time the duration of the search $1/q(\phi_k)$). On the right hand side are the expected profit flows (value of production minus credit repayments), discounted by the riskfree rate r, the destruction rates s_k and d.

A.3 Bank screening technology and optimal size

Following the derivation in Rotemberg (2006) for the labor market, let derive the non-linear cost $c_{ikt} = c_k(N_{C^v_{ikt}})$ that is paid while bank i is screening credit applications from (sector-k) entrepreneurs and that helps to determine the size of bank i. In particular, assuming that $c_k(N_{C^v_{ikt}}) = \kappa N^{\epsilon}_{C^v_{ikt}}$ and given that

$\phi_{kt} q(\phi_{kt}) \equiv q(\phi_{kt}) \frac{N_{E^u_{kt}}}{N_{C^v_{kt}}}$, we can reexpress the law of motion for lending relationships at bank i as

$$N_{C^v_{i,k,t}} = \left[\frac{N_{C^p_{i,k,t+1}}}{(1 - s_{i,k,t+1})(1 - d_{i,t+1})} - N_{C^p_{i,k,t}} \right] \frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}}$$

From $c_k(N_{C_{ikt}}) = \kappa \left\{ \left[\frac{N_{C^p_{i,k,t+1}}}{(1-s_{ikt+1})(1-d_{it+1})} - N_{C^p_{i,k,t}} \right] \frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}} \right\}^{\epsilon}$, the marginal cost of creating lending relationships for bank i becomes

$$c'_{ikt} = \frac{\kappa \epsilon}{(1 - s_{ikt+1})(1 - d_{it+1})} \left[\frac{N_{C^p_{i,k,t+1}}}{(1 - s_{ikt+1})(1 - d_{it+1})} - N_{C^p_{i,k,t}} \right]^{\epsilon - 1} \left[\frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}} \right]^{\epsilon}$$

With symmetric banks, $\frac{N_{C^v_{ikt}}}{N_{C^v_{kt}}} q(\phi_{kt}) N_{E^u_{kt}} = \frac{1}{N} q(\phi_{kt}) N_{E^u_{kt}}$, where N is the number of banks, such that the equilibrium marginal cost is given by equation (7).

A.4 Time-varying credit repayment rate

The time-varying expression for the (sector-k) credit repayment rate ψ_{kt} will allow to compute the optimal reservation threshold that determines the the separation rate thereafter. Let derive it from the first Nash bargaining rule, $V^p_{kt}(p) - V^u_{kt} = \frac{\delta_C}{1-\delta_C}(C^p_{kt}(p) - C^v_{kt})$, as follows.

The Bellman equation standing for the surplus of the credit match is

$$CS_{kt}(p_k) = V^p_{kt}(p_k) - V^u_{kt} + C^p_{kt} - C^v_{kt}$$

With $V^u_{kt} = 0$ by free entry, replacing by the (time-varying) Bellman equations for the credit lines and entrepreneurs gives, after some simplification,

$$\begin{aligned} CS_{kt} = A_{kt} p_{kt} + c'_{kt} - c - \rho_t + \beta E_t \{ (1 - s_{kt+1})(1 - d_{t+1}) [(V^p_{kt+1} + C^p_{kt+1} \\ - C^v_{kt+1}) - \phi_{kt} q_C(\phi_{kt}) (C^p_{kt+1} - C^v_{kt+1})] \} \end{aligned}$$

Since $(V^p_{kt+1} + C^p_{kt+1} - C^v_{kt+1}) = (CS)_{kt+1}$ and $(C^p_{kt+1} - C^v_{kt+1}) = \delta_C (CS)_{kt+1}$,

$$CS_{kt} = A_{kt} p_{kt} + c'_{kt} - c - \rho_t + \beta E_t \{ (1 - s_{kt+1})(1 - d_{t+1}) [1 - \phi_{kt} q_C(\phi_{kt}) \delta_C] (CS)_{kt+1}$$

Then, from the first Bellman equation for entrepreneurs, we know that

$$V^p_{kt+1} = \frac{c_E}{\beta q_C(\phi_{kt}) E_t[(1-s_{kt+1})(1-d_{t+1})]}$$

and since $V^p_{kt+1} = (1-\delta_C)(CS)_{kt+1}$, we have

$$CS_{kt} = (A_{kt}p_{kt} + c'_{kt} - c - \rho_t) + \frac{1-\phi_{kt}q_C(\phi_{kt})\delta_C}{1-\delta_C}\frac{c_E}{q_C(\phi_{kt})} \quad \text{(A)}$$

From the last Bellman equation for entrepreneurs (10), we also have

$$V^p_{kt} = A_{kt}p_{kt} - \psi_{kt} + \beta E_t[(1-s_{kt+1})(1-d_{t+1})V^p_{kt+1}]$$

so that,

$$CS_{kt} = \frac{A_{kt}p_{kt} - \psi_{kt}}{1-\delta_C} + \frac{c_E}{(1-\delta_C)q_C(\phi_{kt})} \quad \text{(B)}$$

Equalizing (A) and (B) finally gives

$$\psi_{ikt} = \delta_C A_{kt}p_{jkt} + (1-\delta_C)(c - c'_{ikt} + \rho_{it}) + \delta_C c_E \phi_{kt} \quad \text{(C)}$$

At equilibrium

$$\bar{\psi}_k = \delta_C \bar{A}\bar{A}_k\bar{p}_k + (1-\delta_C)(c - \bar{c}'_k + \bar{\rho}) + \delta_C c_E \bar{\phi}_k$$

The credit repayment rate depends on the relative bargaining powers of entrepreneurs and bankers in the credit market (δ_C), the productivity of the match, the costs involved by the credit search period (c'_k and c_E), the sectoral credit market tightness (ϕ_k), and the rate of return on capital (ρ).[25]

A.5 Separation rule and optimal reservation threshold for idiosyncratic productivity

The time-varying expression for the credit repayment is further used to compute the threshold as follows. By definition, the credit relationship terminates if its asset value for the bank is negative, $C^p_{kt}(p) - C^v_{kt}(p) < 0$. The reservation level

[25] In the labor market search literature, a similar equation gives the wages as a function of the bargaining powers, the productivity, the search costs, and the labor market tightness. However, ρ has no counterpart and is due to the multi-search framework considered here.

for idiosyncratic productivity is such that

$$C^p_{kt}(p^R) - C^v_{kt} = 0$$

From (B) and given that $C^p_{kt} - C^v_{kt} = \delta_C(CS)_{kt}$, we have

$$A_{kt}p_{kt} - \psi_{kt}(p^R) + \frac{c_E}{q_C(\phi_{kt})} = 0$$

Replacing $\bar{\psi}_k$, we get the time-varying threshold required by banks for entrepreneurs' idiosyncratic productivity as

$$p^R_{ikt} = \frac{1}{A_{kt}}\Big[\rho_{it} + (c - c'_{ikt}) - \frac{c_E}{q_C(\phi_{kt})}\frac{1-\delta_C\phi_{kt}q(\phi_{kt})}{1-\delta_C}\Big]$$

which further determines the credit separation rate at bank i.

A.6 Time-varying financial repayment rate

The sequence of events is such that the shares that each agent type effectively gets from the net surplus (NS) at the end is as follows: $(1-\delta_C)(1-\tilde{\delta}_F)$ for entrepreneurs, $\delta_C(1-\tilde{\delta}_F)$ for bankers, and $\tilde{\delta}_F$ for financial investors. As bankers know that future bargaining with entrepreneurs on the credit market will determine their effective share of the net surplus, they take this effect into account at the time they bargain with financial investors, so that where

$$\delta_F = \frac{\tilde{\delta}_F}{1-(1-\tilde{\delta}_F)(1-\delta_C)}$$

The net surplus is given by

$$NS_t = C^p_{kt} - C^v_{kt} + V^p_{kt} - V^u_{kt} + I^p_t - I^v_t = CS_{kt} + I^p_t - I^v_t$$

Replacing by the Bellman equations for financial investors we have

$$NS_t = CS_{kt} + \rho_t + c + \frac{c_E}{q_C(\phi_{kt})}\frac{1-\phi_{kt}q_C(\phi_{kt})}{1-\delta_C}\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F} \quad \text{(D)}$$

From the fact that $I^p_t - I^v_t = \tilde{\delta}_F NS_t$, we also have

$$NS_t = \frac{\rho_t + c}{\delta_F} + \frac{c_E}{q_C(\phi_{kt})}\frac{1-\phi_{kt}q_C(\phi_{kt})}{1-\delta_C}\frac{1}{1-\tilde{\delta}_F} \quad \text{(E)}$$

Therefore, equalizing (D) and (E), with CS_{kt} given by (B), we get

$$\rho_t = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}[A_{kt}p_{kt} - \psi_{kt} + c_E\phi_{kt}] - c$$

A.7 Bank default

It is more profitable for the financial investors to impose default on banks if the continuation value of remaining matched is less than the continuation value of being unmatched, *i.e.* if $I_t^v - I_t^u < 0$. Since $I_t^u = 0$ by free entry, $I_t^v = 0$ gives

$$-c + \beta E_t\{(1-d_{t+1})\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1})(I_{t+1}^p - I_{t+1}^v) + (1-d_{t+1})I_{t+1}^v\} = 0$$

Since $I_{t+1}^p - I_{t+1}^v = \tilde{\delta}_F NS_{t+1}$, $NS_{t+1} = \frac{CS_{t+1}}{1-\tilde{\delta}_F}$, and $I_{t+1}^v = \frac{c_I}{\beta\xi_t q_F(\xi_t)}$, we get

$$E_t d_{t+1} = 1 + \frac{\xi_t q_F(\xi_t)}{c_I}\left[\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}c_E\phi_{kt} - c\right]$$

A.8 Equilibrium conditions: no information asymmetry

Here is the loglinearized system of equilibrium conditions in the particular case where bankers' application screening cost is linear in the number of vacant credit lines and exactly equal to the amount provided by financiers: $c = c_k'$, ie. the case where there is no information asymmetry. It is also assumed for simplicity that $\bar{r} = 0$ and $\bar{A} = 1$. The system consists in 10 equations with the following set of variables: $\{\xi, \phi_k, \psi_k, \rho, p_k^R, s_k, d, N_{E_k^u}, N_{C_k^v}, N_{C_k^p}\}$.

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi}_k = \hat{c}_B + \hat{d} + \eta_F\hat{\xi}$$

$$\bar{A}_k\bar{p}_k^R(\hat{A}_k + \hat{p}_k^R) = \bar{\rho}\hat{\rho} - \frac{c_E}{q_C(\bar{\phi}_k)}\frac{\eta_C - \delta_C\bar{\phi}_k q_C(\bar{\phi}_k)}{1-\delta_C}\hat{\phi}_k$$

$$\hat{s}_k = \sigma_k\hat{p}_k^R$$

$$\bar{d}\hat{d} = \frac{1-\delta_F}{\delta_F}\frac{q_F(\bar{\xi})}{\bar{c}_B}\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{c_E\bar{\phi}_k}{1-\delta_C}(-\eta_F\hat{\xi} - \hat{c}_B + \hat{\phi}_k)$$

$$\hat{N}_{E_k^u} = \hat{\phi}_k + \hat{N}_{C_k^v}$$

$$\frac{\bar{N}_{C_k^v}\bar{\phi}_k q_C(\bar{\phi}_k)}{\bar{N}_{C_k^p}}[\hat{N}_{C_k^v} - \hat{N}_{C_k^p} + (1-\eta_C)\hat{\phi}_k] = \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{\psi}_k\hat{\psi}_k = \delta_C\bar{A}_k\bar{p}_k\hat{A}_k + (1-\delta_C)\bar{\rho}\hat{\rho} + \delta_C c_E\bar{\phi}_k\hat{\phi}_k$$

$$\bar{\rho}\hat{\rho} = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}(\bar{A}_k\bar{p}_k\hat{A}_k - \bar{\psi}_k\hat{\psi}_k + c_E\bar{\phi}_k\hat{\phi}_k)$$

$$\bar{N}_{C_k^p}\hat{N}_{C_k^p} = -\bar{N}_{E_k^u}\hat{N}_{E_k^u}$$

where an overbar indicates the equilibrium value of a variable, a hat indicates the log-deviation from equilibrium of a variable, and where η_F, η_C, and σ_k are respectively the elasticities of $q_F(\bar{\xi})$, $q_C(\bar{\phi}_k)$, and $s_k(p_k^R)$ with respect to their argument. Below are the effects of a sectoral productivity shock when $c_k' = c$.

$$\frac{\partial\hat{N}_{E_k^u}}{\partial\hat{A}_k} = \frac{\partial\hat{N}_{C_k^v}}{\partial\hat{A}_k} = -\frac{\sigma_k\bar{s}_k(1-\bar{d})\bar{N}_{C_k^p}(1-\tilde{\delta}_F\bar{p}_k/\bar{p}_k^R)}{\bar{s}_k(1-\bar{d})+\bar{d}} \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{\psi}_k}{\partial\hat{A}_k} = \frac{\bar{A}_k\bar{p}_k}{\bar{\psi}_k}[\delta_C(1-\tilde{\delta}_F)+\tilde{\delta}_F] > 0$$

$$\frac{\partial\hat{\rho}}{\partial\hat{A}_k} = \frac{\tilde{\delta}_F\bar{A}_k\bar{p}_k}{\bar{\rho}} > 0$$

$$\frac{\partial\hat{s}_k}{\partial\hat{A}_k} = -\sigma_k\left(1-\tilde{\delta}_F\frac{\bar{p}_k}{\bar{p}_k^R}\right) \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{p}_k^R}{\partial\hat{A}_k} = -1+\tilde{\delta}_F\frac{\bar{p}_k}{\bar{p}_k^R} \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{\xi}}{\partial\hat{A}_k} = \frac{\partial\hat{\phi}_k}{\partial\hat{A}_k} = \frac{\partial\hat{d}}{\partial\hat{A}_k} = 0$$

If the economy is deterministic (constant idiosyncratic productivity and exogenous separations), and whether bank default is exogenous or not, $\partial\hat{\rho}/\partial\hat{A}_k$ and $\partial\hat{\psi}_k/\partial\hat{A}_k$ are the same than in the stochastic case, however $\partial\hat{N}_{E_k^u}/\partial\hat{A}_k = 0$.

A.9 Equilibrium conditions: information asymmetry

In the presence of information asymmetry, it must be that $c'_k \neq c$ and bank size matters to determine c'_k. For simplicity that, $\bar{r} = 0$, $\bar{A} = 1$, and $\bar{N} = 1$. The system consists in 11 equations in $\{\xi, \phi_k, \psi_k, \rho, p_k^R, s_k, d, c'_k, N_{E_k^u}, N_{C_k^v}, N_{C_k^p}\}$.

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi}_k = \hat{c}_B + \hat{d} + \eta_F\hat{\xi} + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{xi})}{\bar{d}}\hat{c}'_k$$

$$\hat{c}'_k = \eta_C\hat{\phi}_k + \epsilon\hat{N}_{C_k^v} - \hat{N}_{E_k^u} + \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{A}_k\bar{p}_k^R(\hat{A}_k + \hat{p}_k^R) = \bar{\rho}\hat{\rho} - \frac{c_E}{q_C(\bar{\phi}_k)}\frac{\eta_C - \delta_C\bar{\phi}_k q_C(\bar{\phi}_k)}{1-\delta_C}\hat{\phi}_k$$

$$\hat{s}_k = \sigma_k\hat{p}_k^R$$

$$\bar{d}\hat{d} = \frac{\bar{\xi}q_F(\bar{\xi})}{\bar{c}_I}\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{c_E\bar{\phi}_k}{1-\delta_C}[(1-\eta_F)\hat{\xi} - \hat{c}_I + \hat{\phi}_k)$$

$$\hat{N}_{E_k^u} = \hat{\phi}_k + \hat{N}_{C_k^v}$$

$$\frac{\bar{N}_{C_k^v}\bar{\phi}_k q_C(\bar{\phi}_k)}{\bar{N}_{C_k^p}}[\hat{N}_{C_k^v} - \hat{N}_{C_k^p} + (1-\eta_C)\hat{\phi}_k] = \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{\psi}_k\hat{\psi}_k = \delta_C\bar{A}_k\bar{p}_k\hat{A}_k + (1-\delta_C)\bar{\rho}\hat{\rho} + \delta_C c_E\bar{\phi}_k\hat{\phi}_k - (1-\delta_C)\bar{c}'_k\hat{c}'_k$$

$$\bar{\rho}\hat{\rho} = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}(\bar{A}_k\bar{p}_k\hat{A}_k - \bar{\psi}_k\hat{\psi}_k + c_E\bar{\phi}_k\hat{\phi}_k)$$

$$\bar{N}_{C_k^p}\hat{N}_{C_k^p} = -\bar{N}_{E_k^u}\hat{N}_{E_k^u}$$

A.9.1 Effects of a shock to A_k in the deterministic economy

When idiosyncratic productivity is constant, the effects of a sectoral productivity shock are identical whether information is asymmetric or not and whether there the bank default rate is exogenous or not. More precisely we get

$$\partial \hat{N}_{E_k^u}/\partial \hat{A}_k = 0,\ \partial \hat{d}/\partial \hat{A}_k = 0,\ \partial \hat{\phi}_k/\partial \hat{A}_k = 0,\ \partial \hat{c}'_k/\partial \hat{A}_k = 0,$$

$$\frac{\partial \hat{\psi}_k}{\partial \hat{A}_k} = \frac{\bar{A}_k \bar{p}_k}{\bar{\psi}_k}[\delta_C(1-\tilde{\delta}_F) + \tilde{\delta}_F] > 0,$$

$$\frac{\partial \hat{\rho}}{\partial \hat{A}_k} = \frac{\tilde{\delta}_F \bar{A}_k \bar{p}_k}{\bar{\rho}} > 0$$

A.9.2 Effects of a shock to A_k in the stochastic economy (endogenous threshold and separation)

- Exogenous bank default

$$\frac{\partial \hat{N}_{E_k^u}}{\partial \hat{A}_k} = -\frac{\sigma_k \bar{s}_k (1-\bar{d}) N_{C_k^p}(1-\tilde{\delta}_F p_k/p_k^R)\, B}{D}$$

$$\frac{\partial \hat{s}_k}{\partial \hat{A}_k} = -\frac{\sigma_k (1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)[\bar{s}_k(1-\bar{d}) + \bar{d}]\, F}{D}$$

$$\frac{\partial \hat{p}_k^R}{\partial \hat{A}_k} = -\frac{(1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)[\bar{s}_k(1-\bar{d}) + \bar{d}]\, F}{D}$$

$$\frac{\partial \hat{c}'_k}{\partial \hat{A}_k} = -\frac{\sigma_k \bar{s}_k (1-\bar{d})(1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)\, G}{D}$$

$$\frac{\partial \hat{\phi}_k}{\partial \hat{A}_k} = -\frac{\sigma_k \bar{s}_k (1-\bar{d})(1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)\frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\, G}{D}$$

$$\text{where} \quad B = 1 + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\left[\epsilon - \eta_C(1-\bar{s}_k)(1-\bar{d})\right],$$

$$\text{where} \quad F = 1 + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\left[\epsilon(1-\eta_C) - \eta_C(1-\epsilon)\bar{N}_{E_k^u}\right],$$

$$\text{where} \quad G = \bar{s}_k(1-\bar{d}) + \bar{d} - (1-\epsilon)(1-\bar{N}_{E_k^u}),$$

$$\text{and} \quad D = [\bar{s}_k(1-\bar{d}) + \bar{d}]\, F + \left\{ \frac{\delta_C}{1-\delta_C} \frac{q_F(\bar{\xi})}{c_B \bar{d}} \frac{c_E}{q_C(\bar{\phi}_k)(1-\delta_C)} \Big[-\tilde{\delta}_F \right.$$
$$\left. + \eta_C - \bar{\phi}_k q_C(\bar{\phi}_k)(\delta_C + \tilde{\delta}_F(1-\delta_C)) \Big] \right\} \frac{\bar{c}_k' \sigma_k \bar{s}_k(1-\bar{d})}{\bar{A}_k \bar{p}_k^R}\, G$$

The sign of these expressions depends on the range for $\tilde{\delta}_F$ and for ϵ. In particular having concave ($\epsilon < 1$) or convex ($\epsilon > 1$) application screening costs matters.

- Endogenous bank default

The expressions of the derivatives including all of the effects disentangled so far can hardly be interpreted. A numerical estimation could be helpful, but because the parameters to be included in the calibration are generally unknown, this would only serve as an exercise to simulate the effects discussed above. However, it is found that it is the only case where $\partial \hat{d} / \partial \hat{A}_k \neq 0$.