Abstract

We adapt structural models of default risk to take into account the special nature of bank assets. The usual assumption of log-normally distributed asset values is not appropriate for banks. Bank assets are risky debt claims, which implies that they embed a short put option on the borrowers’ assets, leading to a concave payoff. This has important consequences for banks’ risk dynamics and distance to default estimation. Due to the payoff non-linearity, bank asset volatility rises following negative shocks to borrower asset values. As a result, standard structural models in which the asset volatility is assumed to be constant can severely understate banks’ default risk in good times when asset values are high. Bank equity payoffs resemble a mezzanine claim rather than a call option. Bank equity return volatility is therefore much more sensitive to negative shocks to asset values than in standard structural models.
I Introduction

The distress that many banks experienced during the recent financial crisis has brought renewed emphasis on the importance of understanding and modeling bank default risk. Assessment of bank default risk is important not only from an investor’s viewpoint, but also for risk managers analyzing counterparty risks and for regulators gauging the risk of bank failure. Accurate modeling of bank default risk is also required for valuing the benefits that banks derive from implicit and explicit government guarantees.

In many applications of this kind, researchers and analysts rely on structural models of default risk in which equity and debt are viewed as contingent claims on the assets of the firm. Following Merton (1974), the standard approach (which we call the Merton model) is to assume that the value of the assets of the firm follows a log-normal process. The options embedded in the firm’s equity and debt can then be valued as in Black and Scholes (1973). In some cases, the distance to default computed from this model is used as one of the ingredients to empirically model bank default risk. Recent examples of bank default risk analysis based on the Merton model include Acharya, Anginer, and Warburton (2014) and Schweikhard, Tsesmelidakis, and Merton (2014), who study the value of implicit (too-big-to-fail) government guarantees. There is also an extensive literature that has applied structural models to price deposit insurance, going back to Merton (1977), Marcus and Shaked (1984), and Pennacchi (1987).

The Merton model’s assumption of log-normally distributed asset values may provide a useful approximation for the asset value process of a typical non-financial firm. However, for banks this assumption is clearly problematic. Much of the asset portfolio of a bank consist of debt claims such as mortgages that involve contingent claims on the assets of the borrower. The fact that the upside of the payoffs of these debt claims is limited is not consistent with the unlimited upside implied by a log-normal distribution.

In this paper, we propose a modification of the Merton model that takes into account the debt-like payoffs of bank assets. Our approach to deal with this problem is to apply the log-normal distribution assumption not to the assets of the bank, but to the assets of the bank’s borrowers. More precisely, we model banks’ assets as a pool of loans where borrowers’ loan repayments depend on the value of their assets at loan maturity as in Vasicek (1991). Borrower asset values are subject
Figure 1: Payoffs at maturity in the simplified model with perfectly correlated borrower defaults to common factor shocks as well as idiosyncratic risk. We then view the assets of the bank as contingent claims on borrower assets, and equity and debt of the bank as contingent claims on these contingent claims.

This options-on-options feature of bank equity and debt has important consequences for the implied default risk and equity risk dynamics. To illustrate the main intuition, it is useful to consider the simplified case without idiosyncratic risk in which all borrowers are identical with perfectly correlated defaults. Assume further that the maturity of the (zero-coupon) debt issued by the bank equals the maturity of the (zero-coupon) loans made by the bank. In this case, the payoffs at maturity as a function of borrower asset value are as shown in Figure 1. In this example, the borrowers have loans with face value 0.80 and the bank has issued debt with face value 0.60. Since the maximum payoff the bank can receive from the loans is their face value, the bank asset value is capped at 0.80. Only when borrower assets fall below 0.8 is the bank asset value sensitive to borrower asset values.

Clearly, the bank asset value cannot follow a log-normal distribution (which would imply unlimited upside). Since the bank’s borrowers keep the upside of a rise in their asset value above the loan face value, the bank’s equity payoff does not resemble a call option on an asset with unlimited
upside, but rather a mezzanine claim with two kinks. This nature of the banks’ asset payoffs is in conflict with the assumptions of the standard Merton model.

This mezzanine-like nature of the bank’s equity claim has important consequence for the risk dynamics of bank equity and for the distance-to-default estimation. Due to the capped upside, bank volatility will be very low in “good times” when asset values are high and it is likely that asset values at maturity will end up towards the right side in Figure 1 where the bank’s equity payoff is insensitive to fluctuations in borrower asset values.

A standard Merton-model in which equity is a call option on an asset with constant volatility misses these nonlinear risk dynamics. Viewed through the lens of this standard model it might seem that a bank in times of high asset values is many standard deviations away from default. But this conclusion would be misleading because it ignores the fact that volatilities could rise dramatically if asset values fall. Similarly, the standard model would give misleading predictions about the riskiness of bank assets, equity, and debt.

Going beyond this simple illustrative example, our model incorporates additional features such as idiosyncratic borrower risks, staggered remaining loan maturities, and the replacement of maturing loans with new loans. While these generalizations are important to get a more realistic model that we can use for a quantitative evaluation of bank risk, the addition of these features does not change the basic insight about the mezzanine-claim nature of bank equity and the resulting consequences for risk dynamics and distance to default.

To assess the differences between our modified model and the Merton model, we simulate data from our modified model and ask to what extent an analyst using the Merton model would misjudge the risk-neutral probability of default. We find that this error is particularly stark when asset values are high relative to the face value of the bank’s debt. In this case, bank asset payoffs are likely to stay in the flat region in Figure 1 and bank equity payoffs are also likely in the flat region. As a consequence, equity volatility is low. Based on the Merton model, an analyst observing low equity volatility would infer that asset volatility must be low. Furthermore, since asset volatility is constant in the Merton model, the analyst would (wrongly) conclude that asset volatility will remain low at this level in the future. What the Merton model misses in this case is that asset volatility could rise substantially following a bad asset value shock, because the region of likely asset payoffs would move closer to or into the downward sloping region in Figure 1. As a result,
the Merton model substantially overestimates the distance to default and it underestimates the risk-neutral probability of default.

We then calibrate our modified model and the standard Merton model to quarterly bank panel data from 2002 to 2012. We choose the value and volatility of bank assets (in the case of the Merton model) or borrower assets (in case of our modified model) to match the observed market value of equity and its volatility. Even though both models are calibrated to the same data, their implied risk-neutral default probabilities are strikingly different. In line with the simulations we discussed above, the differences are particularly big in the years before the financial crisis when equity values were high and volatility low. Based on the Merton model, the risk-neutral default probability of the average bank in 2006 over a 5-year horizon is less than 10%. In contrast, the risk-neutral default probability from our modified model is two to three times as high. Translated into credit spreads, this would imply an annualized spread of around 10 basis points in the Merton model and close to 100 basis points according to our modified model.

Once the financial crisis hit in 2007/08, the models’ predictions are not so different anymore. At this point, bad asset value shocks had moved banks into the downward-sloping asset payoff region in Figure 1. In this region, the kink in the asset payoff becomes less relevant and the predictions from our modified model are close to those from the standard Merton model. In periods of the most extreme distress, Merton model default probabilities can even exceed those from our modified model.

Thus, the key problem with applications of standard structural models to banks is that they understate the risk of default in “good times.” This is an important issue, for example, for the estimation of the value of explicit or implicit government guarantees. Based on a standard Merton model calibrated to equity value and volatility data from 2006 (i.e., pre-crisis times), one may be lead to the conclusion that the value of a guarantee is almost nil when, in fact, the value is a lot higher if one takes into account that banks’ asset volatility will go up when asset values fall.

We further investigate the plausibility of our modification of the Merton model by comparing the models’ predictions about bank equity volatility following a bad shock to the bank’s asset value. We calibrate both models to match data on equity values and volatility in 2006Q2. We then add a negative shock to borrower asset values based on the change in U.S. house prices from 2006Q2 to subsequent quarters. The negative shock to borrower asset values then translates, in our modified
model, into a shock to the bank’s asset value. We then apply an asset value shock of the same magnitude in the Merton model. In the Merton model, the consequences are quite benign. From 2006Q2 to 2009, the average bank’s equity volatility rises, conditional on this shock, by about a quarter. In contrast, in our model, average bank equity volatility doubles because the model takes into account not only the drop in the bank’s asset value, but also the rise in asset volatility. This is still not big enough to match the even bigger rise in actual observed equity volatility, but it goes some way in explaining the gap. This exercise illustrates that application of a standard structural model with constant asset volatility can severely understate the sensitivity of bank equity risk to negative asset value shocks.

Our modified structural model still omits many features—e.g., liquidity concerns, complex capital structure, and government guarantees—that would be necessary for realistic modeling of bank default risk. For default prediction in practice, a reduced form model rather than a structural one may be the preferred method. But for reduced-form models, too, our results have important implications. Many reduced form models use a Merton model distance-to-default as one of the state variables driving default intensity (e.g., Duffie, Saita, and Wang (2007), Bharath and Shumway (2008) Campbell, Hilscher, and Szilagyi (2008)). Our analysis suggests that for banks the default probability from our modified model may be better suited as a default predictor.

Our paper relates to several strands in the literature. Gornall and Strebulaev (2014) model bank assets as loan portfolios in similar ways as we do, albeit without staggering of loan maturities. Their focus is on modeling bank’s capital structure choices in equilibrium, while we focus on implications for default risk estimation and valuation of bank’s securities. Duffie, Jarrow, Purnanandam, and Yang (2003) develop a reduced-form pricing approach for pricing of deposit insurance. The reduced-form approach permits a lot of flexibility to obtain realistic default risk estimates, but the structural approach that we pursue here is useful for understanding the economic drivers of default risk (which may in turn be useful to develop well-specified reduced-form models.) Kelly, Lustig, and Van Nieuwerburgh (2011) estimate the value of implicit government guarantees for the banking system by comparing prices of options on a banking index and a portfolio of options on individual bank stocks. Their method involves fitting models with stochastic volatility and jumps to option prices. In these models, the correlation between returns and shocks to volatility (the “leverage” effect) is a reduced-form parameter. Our structural model of bank risk predicts a specific (non-
linear) relation between bank equity returns and bank equity volatility.

The rest of the paper is organized as follows. Section II presents our modified model and simulations to illustrate the key differences to the standard Merton model. In Section III we apply the model to empirical bank panel data and we analyze the resulting estimates of default risk. Section IV discusses implications for reduced form models of default risk. Section V concludes.

II Structural Model of Default Risk for Banks

Unlike the simplified case in Figure 1, we now set up a model in which borrower asset have idiosyncratic risk and banks issue loans with staggered maturities. Both of these additional features are important because they lead to some smoothing of the bank asset payoff function in Figure 1.

Consider a setting with continuous time. A bank issues zero-coupon loans with maturity $T$. Loans are issued in staggered fashion to $N$ cohorts of borrowers. Cohorts are indexed by $\tau = T, T(N - 1)/N, ..., 1/N$, the remaining maturity on their loans at $t = 0$. These loans were issued at $\tau - T$ with face value $F_1$. Each cohort is comprised of a continuum of borrowers indexed by $i \in [0, 1]$ with mass $1/N$.

Let $A^{\tau,i}_t$ denote the collateral value of a borrower $i$ in cohort $\tau$ at time $t$. Under the risk-neutral measure, the asset value evolves according to the stochastic differential equation

$$
\frac{dA^{\tau,i}_t}{A^{\tau,i}_t} = (r - \delta)dt + \sigma(\sqrt{\rho}dW_t + \sqrt{1 - \rho}dZ^{\tau,i}_t),
$$

where $W$ and $Z^{\tau,i}$ are independent standard Brownian motions, $\delta$ is a depreciation rate, and $r$ is risk-free rate. The $Z^{\tau,i}$ processes are idiosyncratic and independent across borrowers. This is a one-factor model of borrower asset values as in Vasicek (1991). The parameter $\rho$ represents the correlation of asset values that arises from common exposure to $W$.

At the time of the initial loan issue, $\tau - T$, borrowers’ collateral is normalized to $A^{\tau,i}_t = 1$. Thus, the initial loan-to-value ratio is

$$
\ell = F_1 e^{-\mu T},
$$

with $\mu$ as the promised yield on the loan (that we will solve for below). In line with standard structural models of credit risk, we assume that borrowers default if the asset value at maturity is
lower than the amount owed. The payoff at maturity $t = \tau$ received by the bank from an individual borrower then is

$$L_{\tau}^{\tau,i} = \min(A_{\tau}^{\tau,i}, F_1).$$

(3)

To analyze the payoff to the bank from the whole loan portfolio, it is useful to first solve for the aggregate value of collateral in cohort $n$, which is,

$$A_{\tau}^{\tau} = \frac{1}{N} \int_{0}^{1} A_{\tau,j}^{\tau} dj$$

$$= \frac{1}{N} \exp \left\{ (r - \delta)T - \frac{1}{2} \rho \sigma^2 T + \sigma \sqrt{\rho} (W_\tau - W_{\tau - T}) \right\},$$

(4)

and the aggregate log asset value, which is

$$a_{\tau}^{\tau} = \frac{1}{N} \int_{0}^{1} \log A_{\tau,j}^{\tau} dj$$

$$= \frac{1}{N} (r - \delta)T - \frac{1}{2} \sigma^2 T + \sigma \sqrt{T} \sqrt{\rho} (W_\tau - W_{\tau - T}).$$

(5)

Since idiosyncratic risk fully diversifies away with a continuum of borrowers in each cohort, the stochastic component of the aggregate asset value a cohort depends only on the common factor realization $W_\tau - W_{\tau - T}$.

We now obtain the payoff that the bank receives at maturity from the portfolio of loans given to cohort $\tau$ as

$$L_{\tau}^{\tau} = \frac{1}{N} \int_{0}^{1} L_{\tau,j}^{\tau} dj$$

$$= \frac{1}{N} \int_{0}^{1} A_{\tau,j}^{\tau} dj - \frac{1}{N} \int_{0}^{1} \max(A_{\tau,j}^{\tau} - F_1, 0) dj$$

$$= \frac{1}{N} \left[ A_{\tau}^{\tau} \Phi(d_1) + F_1 \Phi(d_2) \right],$$

(6)

where the last equality follows from the properties of the truncated log-normal distribution, $\Phi(.)$.
denotes the standard normal CDF, and

\[
d_1 = \frac{\log F_1 - a^\tau}{\sqrt{1 - \rho \sqrt{T}\sigma}} - \sqrt{1 - \rho \sqrt{T}\sigma},
\]

(7)

\[
d_2 = -\frac{\log F_1 - a^\tau}{\sqrt{1 - \rho \sqrt{T}\sigma}}.
\]

(8)

The max\((A^{\tau,j}_t - F_1, 0)\) term in (6) reflects the option value for the borrower, i.e., the upside of the collateral value that is retained by the borrower. Conditional on \(W^\tau - W^\tau_{-T}\), there are some borrowers in cohort \(\tau\) for whom this option is in the money, and others for whom it is not, depending on the realization of the idiosyncratic shock. This is why \(d_1\) and \(d_2\) are functions of idiosyncratic risk \(\sqrt{1 - \rho \sqrt{T}\sigma}\). Thus, while idiosyncratic risk is diversified away in the aggregate borrower asset value, it matters for loan payoffs, because borrower default depends on idiosyncratic risk.

We assume that loans are priced competitively, and so the promised yield on the loan can now be found as the \(\mu\) that solves

\[
F_1 e^{-\mu T} = e^{-r T} E_Q^{T\tau}[L^n],
\]

(9)

where \(E_Q^{T\tau}[.]\) denotes a conditional expectation under the risk-neutral measure. The risk-neutral expectation on the right-hand side can be evaluated by simulating the distribution of \(L^n_{\tau}\), based on (6) and the simulated distribution of \(A^n_{\tau}\), under the risk-neutral measure.

At \(t = \tau\), the bank fully reinvests the proceeds, \(L^n_{\tau}\), from the maturing loan portfolio of cohort \(\tau\) into new loans, with uniform amounts, to members of the same cohort. The new loans carry a face value of

\[
F_2 = L^n_{\tau} e^{\mu T}.
\]

(10)

We assume that the bank keeps the time-of-issue loan-to-value ratio at the same level, i.e., \(\ell\), as for the initial round of loans. Borrowers reduce or replenish collateral assets accordingly: the asset value of each member of cohort \(\tau\) is reset to the same value

\[
A^{\tau,i}_{\tau+} = \frac{L^n_{\tau}}{\ell}
\]

(11)

an instant after the re-issue of the loans. The cohort-level aggregates \(A^n_{\tau}\) and \(a^n_{\tau}\) for \(t > \tau\) are based
on these re-initialized asset values.

The aggregate payoff of the portfolio of loans of cohort $\tau$ at the subsequent maturity date $\tau + T$ then follows, along similar lines as above, as

$$L_{\tau+T} = \frac{1}{N} \int_0^1 L_{\tau+Td} \, dj$$

$$= \frac{1}{N} \int_0^1 A_{\tau+Td} \, dj - \frac{1}{N} \int_0^1 \max(A_{\tau+Td}, 0) \, dj$$

$$= \frac{1}{N} \left[ A_{\tau+T} \Phi(d_3) + F_2 \Phi(d_4) \right]$$

(12)

where

$$d_3 = \frac{\log F_2 - a_{\tau+T}}{\sqrt{1 - \rho \sqrt{T} \sigma}} - \sqrt{1 - \rho \sqrt{T} \sigma},$$

(13)

$$d_4 = -\frac{\log F_2 - a_{\tau+T}}{\sqrt{1 - \rho \sqrt{T} \sigma}}.$$

(14)

Thus, after the roll-over into new loans, there are two state variables to keep track of: First, the change of the common factor since roll-over, $W_t - W_\tau$, and second, $L_\tau$, which in turn is driven by $W_\tau - W_{\tau - T}$.

The payoffs in (12) and (6) together allow us to describe the distribution of the bank’s assets. Consider, for example, the aggregate value of the bank’s loan portfolio at $t = H$, where $H < T$. Aggregating across all loans outstanding at this time, we get

$$V_H = \sum_{\tau < H} e^{-r(\tau + T - H)} E_H^Q[L_{\tau+T}] + \sum_{\tau \geq H} e^{-r(\tau - H)} E_H^Q[L_\tau],$$

(15)

where the first term aggregates over cohorts whose loans have been rolled over into a second round, while the term aggregates over cohorts that still have the initial first-round loans outstanding. Substituting in from (6) and (12) yields an expression in which the only source of stochastic shocks is the common factor $W$. Therefore, by simulating $W$ we can simulate the distribution of $V_H$ and price contingent claims whose payoffs are functions of $V_H$.

Figure 2 shows the simulated bank asset value, $V_H$, based on 10,000 draws of the common factor paths plotted against the aggregate borrower asset value at $t = H$. Parameters are set at $H = 5$, 

Figure 2: Bank asset value at bank debt maturity as a function of aggregate borrower asset value. Simulated bank asset values shown as dots. Dashed line shows the kinked payoff that would result with perfectly correlated borrower asset values and without staggering of loan maturities.

$T = 10$, $\sigma = 0.2$, $\rho = 0.5$, $r = 0.01$, $\delta = 0.005$, and $F_1 = 0.8$. Unlike in Figure 1, there is no sharp kink in the bank’s asset value in Figure 2. There are two reasons for the lack of sharp kink. First, in this case, loans in the bank’s portfolio are not at maturity. For $H < \tau \leq T$, loans have not matured yet, while for $0 \leq \tau < H$, they have been rolled over into new loans. Second, the existence of idiosyncratic borrower risk makes the borrower’s default option more valuable and the loan less valuable to the bank, particularly when the asset value is close to the face value of the debt.

Moreover, unlike in Figure 1, there is dispersion in the bank asset value conditional on the aggregate borrower asset value. The reason is that for loans that have been rolled over into a second generation of loans, the face value of the loan depends on the common factor realization up to the roll-over date $\tau$. For example, if $W_\tau$ is low, there will be more defaults at $\tau$ and hence the face value of new loans will be lower than if $W_\tau$ is high.

$V_0$, the total market value of the bank’s assets at $t = 0$ can be computed as

$$V_0 = e^{-rH}E_Q^0[V_H]$$  \hspace{1cm} (16)
Now suppose the bank has issued zero-coupon debt maturing at $t = H$ with face value $D$. Similar to standard structural models we assume that the bank will pay off its creditors in full if there are sufficient assets available to do so. The bank will default if the asset value at maturity is lower than the face value of debt. To allow for a realistic calibration in our empirical exercise below, we also introduce payouts of the bank to its shareholders. For simplicity, we introduce them as a single certain payment,

$$Y_H = F_1 \gamma H,$$

just before the debt matures. The parameter $\gamma$ determines the payout level. The assets that leave the bank through these payouts are no longer available to pay off the debt holders at $t = H$. The value of the bank’s creditors’ claim at $t = 0$ then is

$$B_0 = e^{-rH}D - e^{-rH}E^Q_t[(D - V_H + Y_H)^+]$$

The bank’s equity value, without the claim to the dividends paid just before maturity, then follows
Figure 4: Instantaneous bank equity volatility as function of current loan portfolio value as

\[ S_0 = V_0 - e^{-rH}Y_H - B_0 \]  \hspace{1cm} (19)

Figure 3 shows simulation results for the relationship between \( S_0 \) and \( V_0 \). We use the same parameters as in Figure 2 and \( D = 0.6, Y_H = 0.008 \). To explore the effect of changes in borrower asset value, we set common factor shocks until \( t = 0 \) to zero, and we then apply a single shock \( dW_0 \) to all cohorts and then simulate \( W \) going forward. We vary \( dW_0 \) from \(-0.8\) to \(0.8\) across simulations. As a consequence, we generate variation in bank asset values across simulations.

As Figure 3 shows, the value of bank equity is concave in bank assets for large values. This is in contrast to the standard Merton model in which the equity value asymptotes towards a slope of one. Thus, we get a similar mezzanine-like payoff for bank equity as in our preliminary analysis in Figure 1, albeit with smoothed kinks.

Figure 4 shows the instantaneous volatility of the bank’s equity. Given knowledge of the parameters, one can numerically compute the instantaneous equity volatility from the first derivative of \( S_0 \) with respect to \( W_0 \). Since common factor shocks are the only source of stochastic shocks to
the bank asset value, this derivative with respect to \( W_0 \) is directly related to the slope of the curve in Figure 3. As Figure 4 shows, equity volatility converges towards zero for high bank asset values. In the Merton model, in contrast, equity volatility would asymptote towards the (strictly positive) volatility of assets.

This very low equity volatility at high asset values arises from the mezzanine-claim nature of bank equity. Positive shocks to asset values raise borrower asset values far above the default thresholds. As a result, bank assets have very low instantaneous risk in good times and bank equity risk resembles the risk of a bond because the region of concavity dominates. Application of a standard Merton model with log-normal asset value would miss this non-linearity in bank’s equity risk. Our modified model suggests that this non-linearity is a key property of bank equity risk dynamics.

In particular, our model makes clear that low instantaneous bank equity volatility in good times can quickly turn into high risk in bad times if asset values fall. In the standard model with a log-normal asset value, the rise in bank equity volatility with a fall in asset values is moderate because bank asset volatility is fixed. In our modified model, bank asset volatility goes up as loans fall in value and become riskier. The rise in equity volatility following bad shocks is therefore more dramatic than in the standard Merton model.

II.A Default Risk Assessment: Comparison with Standard Merton Model

The highly nonlinear relation between bank asset value and bank equity risk due to the short put option embedded in bank assets leads to important consequences for distance to default estimation and empirical assessment of default risk. To illustrate these consequences, we now analyze a setting in which our modified model represents the true data generating process. We simulate from our model with parameter values set to the same values as above in Figure 3. We then study to what extent an analyst applying the (misspecified) standard Merton model would arrive at misleading conclusions about bank default risk.

Figure 5 shows the simulated actual risk-neutral default probabilities in our model (blue) and those estimated based on the Merton model (red) applied to our simulated data. The Merton model default probabilities are obtained by using the simulated equity values and instantaneous volatilities to extract asset values and asset volatilities under the (false) assumption of a log-normal
Figure 5: Risk-neutral default probabilities as function of current loan portfolio value: Merton (red) and modified (blue) asset value process (see Appendix A). This corresponds to the practice of applying the Merton model to recent empirical volatilities and observed equity values. As the Figure shows, the Merton model underestimates the probability of default for moderate (and, in most cases, more realistic) default probabilities. In these good states of the world, the default probabilities are massively understated when the (misspecified) Merton model is applied. If the borrower asset values are relatively high, bank equity volatility is very low because bank asset volatility is very low. However, asset volatility could quickly rise if asset values fall, which means that it is much more likely than one would think based on the Merton model that the default threshold could be reached. Low instantaneous equity volatility hence does not mean that the bank operates at a high distance to default. However, an analyst applying the standard Merton model with constant asset volatility would miss these nonlinear risk dynamics. Within the standard Merton model, the analyst would interpret the low instantaneous equity volatility as a high distance to default and hence low default risk. Thus, particularly in good times, application of the Merton model is likely to lead to severe underestimation of bank default risk. Only in a severely distressed situation, when asset values are
depressed and default is almost certain, the Merton model slightly overstates risk-neutral default probabilities. Here the above effect works in reverse.

Figure 6 presents the implied credit spread of the bank’s 5-year debt. The credit spread reflects the product of the risk-neutral probability of default (as shown in Figure 5) and the loss given default (which equals one minus the recovery rate), shown on an annualized basis. Since the recovery rate in a Merton style model can often be quite high in cases where the asset value at maturity ends up only slightly below the face value of the debt, the implied credit spreads are much lower than the risk-neutral default probabilities. Our main focus is again on how an analyst using the standard Merton model would severely underestimate credit spreads. For example, when borrower asset values are around one, and hence substantially above the face value of their loans, the bank’s credit spread according to the Merton model should be about 10 basis points. In contrast, the true credit spread is close to 1% and hence almost an order of magnitude bigger.

Figure 7 shows how application of the standard Merton model would severely underestimate the value of a government guarantee. For illustration, we suppose that there is a 50% risk-neutral probability that the government will fully bail out the debt holders (and absorb the entire loss given default)
in the event of default. The value of the government guarantee then is 0.5 times the value of the bank’s default option. To interpret the magnitudes in Figure 7, recall that the aggregate face value of the bank’s loans in our simulations is around 0.80. As the figure shows, application of the standard Merton model would severely underestimate the value of the guarantee. For aggregate borrower asset values around one, the value of the guarantee is about 0.0015, i.e., about 0.2% of the face value of the bank’s loan portfolio, while the actual value is almost ten times as big.

### III Empirical Calibration

To find out how much, quantitatively, the standard Merton model and our modified model differ in their predictions about default probabilities and risk dynamics, we now calibrate these models with empirical data on bank’s capital structures and equity volatility.
Table I: Model Inputs

Our sample covers all commercial banks listed in the Federal Reserve Bank of New York's CRSP-FRB linked dataset from 2002-2012 that are also covered by Compustat. Market equity and book value of assets are normalized by $D$, i.e., the numbers shown in the table are the market equity/book debt and the book assets/book debt ratios. Equity volatility is annualized and estimated from daily stock returns over one-year moving windows. Our risk-free interest rate proxy is the Federal Reserve Board’s 10-year Treasury bond yield series.

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Observations 20,823

III.A Data

Our sample covers all commercial banks listed in the Federal Reserve Bank of New York’s CRSP-FRB linked dataset from 2002-2012 that are also covered by Compustat. We obtain equity returns and market value of equity from CRSP and the accounting data from the Compustat Annual files for banks. Since accounting data is available at annual frequency, we linearly extrapolate them over the year to generate accounting numbers for the intermediate months. In our base model, we consider the entire debt face value of bank (including demand and time deposits) as $D$. We compute the bank’s payout ratio, $\gamma$, as the sum of cash dividend on common and preferred equity as a fraction of the book value of assets. Appendix B provides more details on the construction of the bank-level variables.

To calibrate the modified model, we need a risk-free interest rate. The Merton model as well as our modified model abstract from interest-rate risk and a term structure of default-free interest rates. As an approximation, we use the Federal Reserve Board’s 10-year Treasury yield series.

Summary statistics of key inputs used for our estimation are provided in Table I. Market equity and book value of assets are normalized by $D$, i.e., the numbers shown in the table are the market equity/book debt and the book assets/book debt ratios. Equity volatility is computed (in

\footnote{Note that this is different from several earlier papers that estimate distance-to-default based on KMV’s model that only includes short-term debt and half of long-term debt as the face value of $D$. A large proportion of a typical bank’s debt consists of deposits. Some of these deposits do not have any stated maturity (e.g., demand deposits), whereas some of them can be withdrawn anytime with small penalty (e.g., time deposits). Hence we include the bank’s entire debt in $D$. Naturally, this approach provides a higher estimate of the bank’s default rates as compared to the earlier approach.}
Table II: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Borrower Asset Depreciation Rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Bank payout Rate</td>
<td>0.002</td>
</tr>
<tr>
<td>$T$</td>
<td>Bank Loan Maturity</td>
<td>10 years</td>
</tr>
<tr>
<td>$H$</td>
<td>Bank Debt Maturity</td>
<td>5 years</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Borrower asset value correlation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Loan-to-Value Ratio</td>
<td>$0.8e^{(\mu-r)T}$</td>
</tr>
</tbody>
</table>

annualized form) from daily bank stock returns over one-year moving windows.

III.B Model calibration

For both the standard Merton model and our modified model, we set the maturity of bank debt to $H = 5$. It is well known that models with only diffusive shocks do not succeed in delivering realistic default risk and credit spread predictions for short-term debt (Duffie and Lando (2001), Zhou (2001)). Our modified model is no different. At short horizons, therefore, differences between our modified model and the standard model are also relatively small. The non-linearity in banks’ asset payoffs becomes more relevant as the probability distribution of borrower asset values spreads out with longer horizons. Longer horizons may also be relevant even for investors in short-term debt (or guarantors of short-term debt). Solvency problems may not be immediately apparent when bad shocks are realized. Deterioration in asset values may be hidden for a while, perhaps facilitated by regulatory forbearance, and short-term debt may be rolled over even if the bank is actually insolvent. By the time default happens, additional losses may have accumulated.

We calibrate the standard Merton model by simultaneously solving for asset value and asset volatility that deliver the observed values of a bank’s equity and stock return volatility (see Appendix A). This approach has been used by prior researchers such as Campbell, Hilscher, and Szilagyi (2008) and Acharya, Anginer, and Warburton (2014). We solve the model quarterly from 2002 to 2012. We use the equity value as of the beginning of the quarter and compute the standard deviation of equity returns based on trailing 12 months of return data.

For our modified model, we have several additional parameters that we fix exogenously, as shown in Table II. We set the depreciation rate $\delta = 0.005$. We assume that the maturity of loans issued by banks is $T = 10$ and that borrower asset values have a pairwise correlation of $\rho = 0.5$. 18
Table III: Model-Implied Risk-Neutral Probabilities of Default

We calibrate the Merton model and our modified model quarterly from 2002-2012 based on the data summarized in Table I. For each bank in each calibration period, we compute the risk-neutral default probability from the two models. The table reports summary statistics for these risk-neutral default probabilities for the whole panel of banks over the full sample period.

<table>
<thead>
<tr>
<th>Model</th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton Model RN prob(default)</td>
<td>0.26</td>
<td>0.27</td>
<td>0.01</td>
<td>0.05</td>
<td>0.15</td>
<td>0.39</td>
<td>0.89</td>
</tr>
<tr>
<td>Modified Model RN prob(default)</td>
<td>0.34</td>
<td>0.21</td>
<td>0.00</td>
<td>0.17</td>
<td>0.35</td>
<td>0.52</td>
<td>0.70</td>
</tr>
</tbody>
</table>

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This correlation assumption is not crucial. A lower $\rho$ implies more smoothing of the kinks in the payoff function in Figure 2, but otherwise its effects are limited because any change in $\rho$ requires an offsetting change in $\sigma$ when we calibrate the model to match observed equity value and volatility. We set $\gamma$ equal to the observed payout rate of the bank.

We normalize $D = 1$. The face value of the loans in the bank’s portfolio represents, roughly, the book assets of the bank. To take into account the zero-coupon nature of the loans, we adjust the book value for the time until loan maturity and we set $F_1$ equal to the inverse of the observed book leverage times $\exp(10r)$. We then look for values of $dW_0$ and $\sigma$ that allow us to match the empirically observed equity value and equity volatility (over a 12-month backward looking window) of the bank with the model implied value.

### III.C Model-implied risk-neutral default probabilities

Table III gives the summary statistics of the two calibrated models’ implied risk-neutral default probabilities (RNPD) for our panel of banks. We calibrate each model quarterly from 2002-2012. The average RNPD is slightly higher in our modified model compared with the standard Merton model. However, the Merton model RNPD is much more positively skewed. The reason is that when a bank is not in distress, the implied RNPD in the Merton model is very low because it is based on the assumption that the bank’s assets have constant volatility. In contrast, our model takes into account that bad shocks to asset values in the future would drive up the volatility of the bank’s assets (as borrower’s default options move into the money), which drastically shrinks the distance to default and hence raises the RNPD in times when asset values are relatively high.
Figure 8: Comparison of calibrated risk-neutral default probabilities (5-year horizon, cumulative)
Figure 8 further illustrates the different behavior of the RNPD from the two models in good times and bad times. The figure shows the average RNPD across all banks each quarter from 2002-2012. The modified model’s RNPD is two to three times as high as Merton RNPD during the time before the financial crisis. Expressed in terms of annualized credit spreads, these RNPD in 2006 would correspond, roughly, to 10 basis points for the Merton model and close to 100 basis points for our modified model.

This behavior of the relative RNPDs is in line with the intuition that bank assets have nonlinear debt-like payoffs. In good times, the Merton model’s assumption of constant asset volatility produces very high distance to default and low RNPD. In our model, the analysis recognizes that the volatility of bank assets in good times (in the flat part of the concave asset payoff region in Figure 2) is low, but that it can quickly rise after a bad shock (when the bank gets into the downward sloping asset payoff region). Once the asset value has suffered a sufficiently big bad shock, the asset payoff is dominated by the linear downward sloping region and the kink is not playing much of a role. In this case, the predictions of the Merton model and our modified model are relatively similar. This is why after the onset of the financial crisis in 2007, the difference between the RNPD shrinks and eventually inverts in 2008.

We further illustrate this cyclical behavior of the RNPD differences between the Merton model and our modified model with the following panel regression

$$\log \frac{RNPD_{it}^{Modified}}{RNPD_{it}^{Merton}} = \alpha_i + \beta_1 \times LowVIX_t + \beta_2 \times \log E_{it} + \beta_3 \times LowVIX_t \times \log E_{it} + \epsilon_{it} \quad (20)$$

The dependent variable is the log difference in default probabilities for a bank $i$ in quarter $t$. VIX is the CBOE index of implied volatilities on S&P500 index options. LowVIX equals one for quarters with below median VIX, zero otherwise. $E$ measures each bank’s market equity (normalized by total debt, as in Table I). The regression results in Table IV show that the modified model RNPD is significantly higher during “quiet” periods, and for high equity banks. Column (IV) further shows the interaction effect: The standard Merton model delivers a lower default probability for banks with higher equity during low VIX periods.

Table V presents results of a similar regression model, but with the level of the S&P500 index during our sample period as the measure of good/bad times.
Table IV: Differences in Model-Implied Risk-Neutral Default Probability: Comparison Between High- and Low-VIX Periods

The dependent variable is the log risk-neutral default probability from our modified model minus the log risk-neutral default probability from the standard Merton model for our panel of banks from 2002-2012. Explanatory variables include a dummy for quarters with below median VIX index, and the bank’s log market equity (normalized by to total debt, as in Table I).

\[
\log \frac{RNPD_{it}^{Modified}}{RNPD_{it}^{Merton}} = \alpha_i + \beta_1 \times HighS&P_t + \beta_2 \times \log E_{it} + \beta_3 \times HighS&P_t \times \log E_{it} + \epsilon_{it} \tag{21}
\]

*HighS&P* equals one for quarters with above median level of S&P500 index, zero otherwise. The regression results show that the modified model RNPD is significantly higher during periods with high stock market valuation, and for high equity banks. Column (IV) further shows the interaction effect: The standard Merton model implies a lower default probability for banks with higher equity during periods in which the S&P500 index is high.

### III.D Model-implied equity risk dynamics

The motivation for our modification of the standard Merton model is based on a priori reasoning that the nature of bank asset payoffs is fundamentally inconsistent with a log-normal process. Improving the model on this dimension seems of first-order importance and should lead to an
Table V: Differences in Model-Implied Risk-Neutral Default Probability: Comparison Between High- and Low-Stock Market Index Periods

The dependent variable is the log risk-neutral default probability from our modified model minus the log risk-neutral default probability from the standard Merton model for our panel of banks from 2002-2012. Explanatory variables include a dummy for quarters with below median S&P 500 Index and the bank’s log market equity (normalized by to total debt, as in Table I).

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>High S&amp;P Level</td>
<td>0.135∗∗∗</td>
<td>0.101∗∗∗</td>
<td>0.683∗∗∗</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(.)</td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>0.873∗∗∗</td>
<td>0.868∗∗∗</td>
<td>0.730∗∗∗</td>
<td>-0.069</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.34)</td>
<td></td>
</tr>
<tr>
<td>High S&amp;P Level x Equity</td>
<td>0.290∗∗∗</td>
<td>0.595∗∗∗</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.362∗∗∗</td>
<td>2.182∗∗∗</td>
<td>2.120∗∗∗</td>
<td>1.841∗∗∗</td>
<td>-0.161</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>YQ Fixed Effect</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>19,297</td>
<td>19,297</td>
<td>19,297</td>
<td>19,297</td>
<td>19,297</td>
</tr>
<tr>
<td>R²</td>
<td>0.204</td>
<td>0.259</td>
<td>0.260</td>
<td>0.262</td>
<td>0.325</td>
</tr>
<tr>
<td>Absorbed FE</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
</tr>
<tr>
<td>Clustered by</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
<td>Bank</td>
</tr>
</tbody>
</table>

*p*-values in parentheses

* p < 0.10, ** p < 0.05, *** p < 0.01

improvement in the empirical performance of the model. At the same time, it is clear that even the modified model, in this simple form, is likely to miss important features of a bank’s capital structure and of how a distressed bank enters into default. Among other things, our modified model does not take into account the presence of implicit and explicit government guarantees. Furthermore, bank capital structures are a lot more complex than our simple model allows for. A direct comparison of the model implied RNPD with bank CDS rates or credit spreads would therefore difficult to interpret.

At this point, we do not focus on refining the model to account for these additional complexities, but rather to evaluate, with a broad brush, the plausibility of our modification of the Merton model. To do so, we focus on the dynamics of bank equity risk. The risk of the equity claim should be less sensitive to government guarantees and interventions than default risk measures, as their main effect is on the more senior claims in a bank’s capital structure.

The modified model and the standard model differ starkly in their predictions of how bank
equity risk responds to asset value shocks. To study these differences, we subject bank asset values to a realistic negative asset value shock. We then calculate the models’ equity volatility predictions, conditional on this shock, and we compare these predictions with actual data on the trajectory of bank equity volatilities going into the financial crisis around 2008/09. In the standard model, a shock to a bank’s asset value leaves the asset volatility unchanged. In contrast, in our modified model a shock to a bank’s asset value is associated with a shock to the asset volatility in the opposite direction. As a consequence, the volatility of the bank’s equity return rises more in response to a negative asset value shock than in the standard model.

We start by calibrating both models to fit the pre-crisis data in 2006Q2 for each bank, as we did in our earlier analyses above. In our modified model we then apply a shock to the asset value of the bank’s borrowers by modifying $dW_0$. To get realistic magnitudes of the shock, we use the cumulative log change in the Federal Housing Finance Agency (FHFA) quarterly house price index (purchases only) from 2006Q2 until a subsequent quarter $t$. Leaving all other parameters unchanged at the 2006Q2 values, we then re-calculate the risk-neutral probability distribution of the bank’s loan portfolio payoffs, the loan portfolio value, and then the bank’s equity value and equity volatility.

For the Merton model, we start by calibrating the model to fit the pre-crisis data in 2006Q2 and we then subject the model to the asset value shock. To compare the Merton model and the modified model on an equal footing, we use the same shock to bank asset values, i.e., we use the post-shock loan portfolio value from our modified model as the post-shock bank asset value in the Merton model. The crucial difference is that the Merton model features a constant asset volatility. Thus, an analyst making predictions about equity volatility conditional on a shock to the bank’s asset value would be led to assume that the asset volatility will remain at its 2006Q2 level.

Panel (a) in Figure 9 shows the trajectories of model-implied equity volatilities from the two models, averaged across all banks in the data set. The differences are quite stark. Even though the drop in the banks’ asset values is the same in both models, equity volatility rises only by roughly a quarter in the Merton model while it doubles in the modified model. In the Merton model, equity volatility rises only because of the leverage effect: the drop in banks’ asset values leads to higher leverage, moving the bank’s equity call option on the assets further out of the money. In our modified model there is an additional effect. Since the fall in asset values originates from a fall
Figure 9: Bank Equity Volatility After a Negative Shock to Borrower Asset Values
in borrower asset values, the loan portfolio becomes more risky and hence banks’ asset values not only fall, but also become riskier.

Panel (b) repeats the same plot, but now with an additional line for the actual realized (annualized) equity volatility of the banks in the data set. Realized volatilities went up even more than predicted by our calibration of the modified model. This is not surprising, as our modified model does not take into account liquidity problems, runs, systemic risks, fire sales, and various other factors that may have led to strongly elevated levels of volatility at the peak of the financial crisis in 2008/09. The fact that our model takes into account the options-on-options feature of bank assets however goes some way towards explaining the high volatility of bank stocks during the crisis.

IV Implications for reduced form models

The insights gained from our analysis are useful beyond the narrow confines of structural modeling of default risk. It is clear from the prior literature that reduced form models outperform structural models in terms of default prediction performance and in matching market pricing of default risk. As Jarrow and Protter (2004) and Duffie and Lando (2001) argue, part of the reason is that the structural approach assumes, implausibly, that market participants observe a firm’s asset value continuously. In many practical applications, a reduced form model is therefore the preferred approach.

In reduced form models, a firm’s default intensity depends on a vector of state variables. The model is silent about the nature of these state variables. In applications, modelers typically choose covariates relating to the state of the economy and various balance sheet variables and other firm-level predictors of default as elements of the state vector. As Duffie, Saita, and Wang (2007) demonstrate, distance-to-default estimates obtained from structural models can be a useful default predictor within a reduced form model (see, also, Bharath and Shumway (2008) Campbell, Hilscher, and Szilagyi (2008)).

Thus, based on our analysis in this paper, one would expect that default probabilities obtained from our modified model should be a better predictor of bank default than the distance to default obtained from the standard Merton model. The extent to which this makes a difference should also
depend on economic conditions. As we showed earlier, the differences in implied RNPD between our modified model and the Merton model are particularly stark in good times when asset values are high.

Of course, for actual bank default prediction, one must also consider the presence of explicit and implicit government guarantees, including too-big-to-fail (TBTF) subsidies. Ideally, one would want to extend the modified model to allow for the possibility of bailouts and deposit insurance. But even without this extension, the modified model could be useful in reduced-form analyses of the government’s role. For example, Acharya, Anginer, and Warburton (2014) use distance to default within a reduced form model to predict counterfactual no-TBTF credit spreads of large banks by extrapolating, based on the estimated model, from smaller banks. The value of the subsidy then follows from the difference between this counterfactual and the actual credit spread. One of the state variables in their reduced form model is the Merton model distance to default. Our analysis here suggests that using the default probabilities from our modified model should deliver a more accurate assessment of the counterfactual default risk.

V Conclusion

The standard assumption that firms have a log-normally distributed asset value is not appropriate when applying structural models of default risk to banks. Banks’ assets are risky debt claims with capped upside and hence the asset payoff is nonlinear, with embedded optionality. As a consequence, bad shocks to bank asset values lead to a rise in asset volatility, unlike in the standard model where asset volatility is constant. A bad shock to asset values therefore reduces the distance to default much more than it would in the standard model. Our modification of the standard model takes this effect into account and leads to substantially different assessment of distance to default and bank risk dynamics. In good times, when asset values are high, the standard model substantially understates risk-neutral default probabilities because it ignores the options-on-options nature of bank equity and debt. For the same reasons, the standard model understates the value of implicit or explicit government guarantees in good times. Furthermore, the standard model also understates the degree to which banks’ equity risk rises in response to an adverse shock to asset values.

Our focus in this paper is on the fundamental issue that a bank’s asset value cannot be log-
normally distributed. As we have shown, this issue has first-order consequences for default risk evaluation. Our modified structural model is useful for understanding the economic drivers of bank default risk. Of course, a simple structural model of the kind we use here still omits many additional features that would be necessary to realistically describe banks’ default risks. Extensions of the model could explore jumps in asset values, default due to liquidity problems, complex maturity and seniority structures of banks’ debt, and various forms of explicit and implicit government support.
References


Crosbie, Peter J., and Jeffrey R. Bohn, 2001, Modeling default risk, KMV LLC.


Appendix

A Distance to default estimation in the standard Merton model: Simulations

The firm’s zero-coupon debt has face value $D$ and matures at $T$. The asset value $V$ evolves according to

$$\frac{dV_t}{V_t} = (r - \gamma)dt + \sigma_v dB_t$$

(22)

where $\gamma$ is the cash payout rate. The value of the firm’s equity then is

$$S_t = C(V_t, D, r, \gamma, T - t, \sigma_v) + (1 - \exp[-\gamma(T - t)])V_t$$

(23)

where $C(.)$ is the Black-Scholes call option price,

$$C(V_t, D, r, \gamma, T - t, \sigma_v) = V_t \exp[-\gamma(T - t)]N(d_1) - D \exp[-r(T - t)]N(d_2)$$

(24)

and

$$d_1 = \log V_t - \log D + (r - \gamma + \frac{\sigma_v^2}{2})(T - t)\frac{\sigma_v \sqrt{T - t}}{\sigma_v \sqrt{T - t}}$$

$$d_2 = d_1 - \sigma_v \sqrt{T - t},$$

while equity volatility follows from the leverage ratio of the call option replicating portfolio as

$$\sigma_{s,t} = \frac{V_t \exp[-\gamma(T - t)]N(d_1)}{S_t} \sigma_v.$$  

(25)

In our simulations in Section II.A where we apply the Merton model (as a misspecified model) to data generated from our modified model, we use the simulated values of $S_t$ and the instantaneous equity volatility $\sigma_{s,t}$ to solve equations (23) and (25) for $V_t$ and $\sigma_v$. Based on $V_t$ and $\sigma_v$ we can then compute the risk-neutral (RN) distance-to-default as

$$DD_{BSM} = \log V_t - \log D + (r - \gamma - \frac{\sigma_v^2}{2})(T - t)\frac{\sigma_v \sqrt{T - t}}{\sigma_v \sqrt{T - t}}$$

The corresponding implied RN default probability, also called the expected default frequency (EDF), can be computed as follows:

$$EDF_{BSM} = \Phi \left( -\log V_t + \log D - (r - \gamma - \frac{\sigma_v^2}{2})(T - t) \right) \frac{\sigma_v \sqrt{T - t}}{\sigma_v \sqrt{T - t}}$$

where $\Phi(.)$ is the standard normal CDF.

There are two approaches to empirically implement this model. In the first approach, one can take empirically observed equity values and equity volatility estimates to solve equations (23) and (25) simultaneously, as we do in our simulations. We adopt the same method for obtaining the expected default frequency based on empirically observed data. The second approach involves an iterative process to estimate the asset value and asset volatilities from equity market data. This
approach has been adopted by previous researchers such as Crosbie and Bohn (2001) and Bharath and Shumway (2008). As expected both these approaches provide broadly similar results. We prefer the first method since it is in line with the simulation results we provide in the earlier part of the paper.

B Data construction

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Market Equity Value of Bank</td>
<td>CRSP</td>
<td>shrcc x shrout</td>
</tr>
<tr>
<td>sE</td>
<td>Stock Return Volatility</td>
<td>CRSP</td>
<td>Stock return volatility over the past year</td>
</tr>
<tr>
<td>F</td>
<td>Face Value of Borrower’s Loan</td>
<td>Compustat</td>
<td>(BV of bank debt + equity + loan loss reserve) × e^{rf×10}</td>
</tr>
<tr>
<td>r</td>
<td>Risk-free rate</td>
<td>FRB</td>
<td>log 1-year risk-free rate</td>
</tr>
<tr>
<td>D</td>
<td>Face Value of Bank Debt</td>
<td>Compustat</td>
<td>short-term debt + long-term debt + deposits</td>
</tr>
</tbody>
</table>

Notes:
All book values are obtained from Annual Compustat Files for Banks.
Book values are linearly interpolated between the annual year end dates.
Stock return volatility is based on daily returns.