

# Subjective Housing Beliefs, Falling Natural Rates and the Optimal Inflation Target

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Kaplan, Mitman Violante (2020), Piazzesi and Schneider (2009)
- Housing price beliefs deviate from Full Info RE:  
Case, Shiller & Thompson (2012), Armona, Fuster & Zafar (2018),  
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  - Understand **monetary policy implications** of subj. HP beliefs

# Preview of Main Findings

- Households' subjective housing beliefs in Michigan survey data:
  - capital gain beliefs **revised too sluggishly** (by a factor 3 - 4)
  - beliefs **initially undershoot**, followed by **delayed overshooting**
  - exp. capital gain **procyclical**, actual gains **countercyclical (NEW)**

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  - exp. capital gain **procyclical**, actual gains **countercyclical (NEW)**
- Simple model featuring **(weak) capital gain extrapolation** replicates
  - housing price behavior: large & persistent swings
  - observed patterns of beliefs deviations  
(quantitatively!)

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Insert capital gain extrapolation into

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- Ramsey optimal MP

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average inflation  $\uparrow$  under Ramsey optimal MP: 1%  $\uparrow$  for 3%  $\downarrow$



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- Lower bound constraint more relevant:  
average inflation  $\uparrow$  under Ramsey optimal MP: 1%  $\uparrow$  for 3%  $\downarrow$
- Inflation effect absent with rat. HP expectations:  
**Subj. housing beliefs quant. important for Ramsey optimal MP**

# Structure of Talk

- ① Deviations from rational HP expectations in survey data
- ② Simple housing model: quantitative match
- ③ Full monetary policy model & policy implications

# Subjective Housing Beliefs: Data

- Survey of Consumers, University of Michigan (2007 - 2021)
- 1-year-ahead house price expectations
- Use mean and median expectations
- Case/Shiller Price Index for actual house prices

# Subjective Housing Beliefs: Sluggish Updating

- Following Coibion & Gorodnichenko (2015):

$$q_{t+4} - E_t^{\mathcal{P}} [q_{t+4}] = a^{CG} + b^{CG} \cdot \left( E_t^{\mathcal{P}} [q_{t+4}] - E_{t-1}^{\mathcal{P}} [q_{t+3}] \right) + \varepsilon_t.$$

where  $q_t$  is nominal/real housing price.

Under FIRE  $b^{CG} = 0$ , but find  $b^{CG} > 0$

# Subjective Housing Beliefs: Sluggish Updating

Table:

	Mean Expectations	Median Expectations
<u>Nominal HP</u>		
$\hat{b}^{CG}$	2.22*** (0.507)	2.85*** (0.513)
<u>Real HP</u>		
$\hat{b}^{CG}$	2.00*** (0.332)	2.47*** (0.366)

# Subjective Housing Beliefs: Wrong Cyclicality

Following Adam, Marcet & Beutel (2017):

$$E_t^{\mathcal{P}} \left[ \frac{q_{t+4}}{q_t} \right] = a + c \cdot PR_{t-1} + u_t \quad (1)$$

$$\frac{q_{t+4}}{q_t} = \mathbf{a} + \mathbf{c} \cdot PR_{t-1} + \mathbf{u}_t. \quad (2)$$

- With FIRE:  $H_0 : c = \mathbf{c}$
- We find  $c > 0$  and  $\mathbf{c} < \mathbf{0}$

# Subjective Housing Beliefs: Wrong Cyclicity

Table:

	$\hat{c}$ (in %)	$\hat{c}$ (in %)	$p$ -value $H_0 : c = c$ (small sample corrected)
<u>Nominal HP</u>			
Mean	0.033 (0.008)	-0.102 (0.007)	0.000
Median	0.014 (0.001)	-0.102 (0.007)	0.000
<u>Real HP</u>			
Mean	0.030 (0.017)	-0.113 (0.009)	0.000
Median	0.010 (0.004)	-0.113 (0.009)	0.000

Local projections of the form

$$X_{t+h} = a^h + b^h \cdot \frac{q_{t-1}}{q_{t-2}} + u_t^h, \quad (3)$$

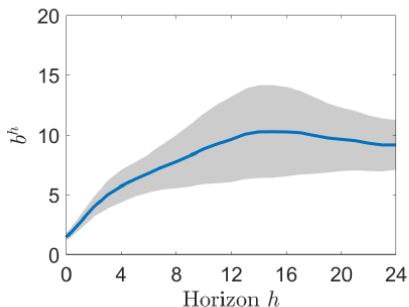
For l.h.s. variable  $X_{t+h}$  we consider:

- cumulative housing capital gain  $q_{t+h}/q_{t+1}$
- 1-year ahead forecast error,  $q_{t+h+4}/q_{t+h} - E_{t+h}^{\mathcal{P}}[q_{t+h+4}/q_{t+h}]$

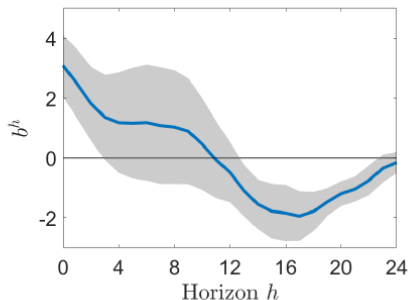


# Subj. Beliefs: Underreaction & Delayed Overshooting

(a) Cumulative Capital Gains



(b) Capital Gain Forecast Errors



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# Simple Model of Subj. Housing Beliefs

(Risk-neutral) household:

$$\max_{\{C_t \geq 0, D^{\max} \geq D_t \geq 0, D_t^R \geq 0\}_{t=0}^{\infty}} E_t^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[ C_t + \zeta_t^d (D_t + D_t^R) \right]$$

$$\text{s.t.} : C_t + (D_t - (1 - \delta)D_{t-1})q_t + R_t D_t^R = Y_t \text{ for all } t \geq 0,$$

Two first-order conditions:

$$R_t = \zeta_t^D$$

$$q_t = \zeta_t^D + \beta(1 - \delta)E_t^{\mathcal{P}} q_{t+1}$$

# Capital Gain Extrapolation from Bayesian Learning

Households perceive capital gains to evolve as

$$\frac{q_t}{q_{t-1}} = b_t + \varepsilon_t$$

with

$$b_t = b_{t-1} + \nu_t$$

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With conjugate prior beliefs, it follows:

$$E_t^{\mathcal{P}} \left[ \frac{q_{t+1}}{q_t} \right] \equiv \beta_t$$

where

$$\beta_t = \min \left\{ \beta_{t-1} + \frac{1}{\alpha} \left( \frac{q_{t-1}}{q_{t-2}} - \beta_{t-1} \right), \beta^U \right\}$$

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with

$$b_t = b_{t-1} + v_t$$

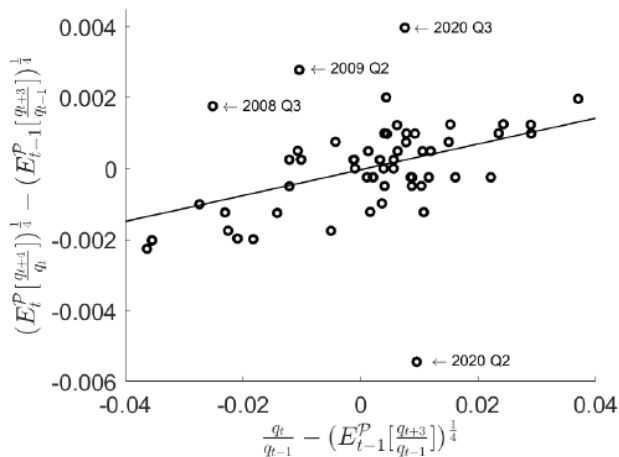
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# Capital Gain Extrapolation in Survey Data



# Equilibrium Housing Price

The equilibrium house price is given by

$$q_t = \frac{1}{1 - \beta(1 - \delta)\beta_t} \zeta_t^D$$

- driven by fundamental shocks  $\zeta_t^D$  and subjective beliefs  $\beta_t$



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- lower natural rate/higher  $\beta \Rightarrow$  more volatile house prices

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- driven by fundamental shocks  $\zeta_t^D$  and subjective beliefs  $\beta_t$
  - lower natural rate/higher  $\beta \Rightarrow$  more volatile house prices
  - Simple model gets very far in terms of *quantitatively* matching
- forecast error patterns
- housing price behavior:

## Simple Calibration:

- $\delta = 0.03/4$  and  $\rho_{\xi} = 0.99$  from Adam & Woodford (2021)
- $\frac{1}{\alpha} = 0.007$  from Adam et al. (2016) for stock prices and  $\beta^U$  to match maximum value of PR
- $\beta$  to match average natural rate of 0.75% and  $\sigma_{\xi}^2$  to match PR volatility

# Quantitative Performance: Housing Prices

	Data	Subjective Belief Model
$std(PR_t)$	8.6	8.6
$corr(PR_t, PR_{t-1})$	0.99	0.99
$std(q_t/q_{t-1})$	0.06	0.04
$corr(q_t/q_{t-1}, q_{t-1}/q_{t-2})$	0.97	0.94

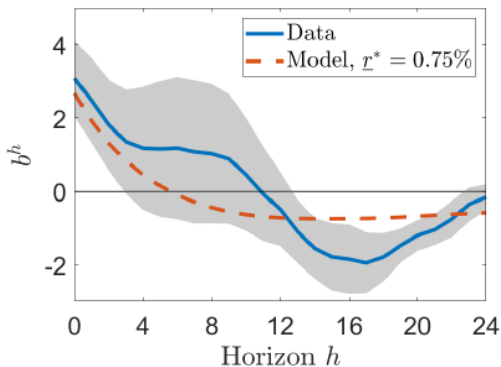
# Quantitative Performance: Housing Prices

	Data	Subjective Belief Model	RE Housing
$std(PR_t)$	8.6	8.6	2.69
$corr(PR_t, PR_{t-1})$	0.99	0.99	0.99
$std(q_t / q_{t-1})$	0.06	0.04	0.003
$corr(q_t / q_{t-1}, q_{t-1} / q_{t-2})$	0.97	0.94	-0.01

# Quantitative Performance: Housing Prices

	Subj. Belief Model	Data	
		Mean	Median
Sluggishness $b^{CG}$	2.09	1.68 (0.355)	2.12 (0.394)
exp. cap gain $c$	0.03	0.030 (0.172)	0.010 (0.043)
actual cap gain $\mathbf{c}$	-0.063	-0.113 (0.009)	-0.113 (0.009)

# Initial Underreaction & Delayed Overshooting



# Lessons Learned from the Simple Model

- Weak extrapolation  $\Rightarrow$ 
  - large & persistent housing price swings
  - patterns of expectational deviations
- Subj. belief dynamics important for HP volatility: 2/3 of std(PR)
- Low real/natural interest rates  $\Rightarrow$  larger housing price volatility

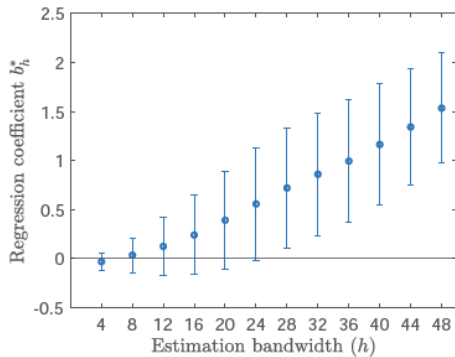


- Consider local projections of the form

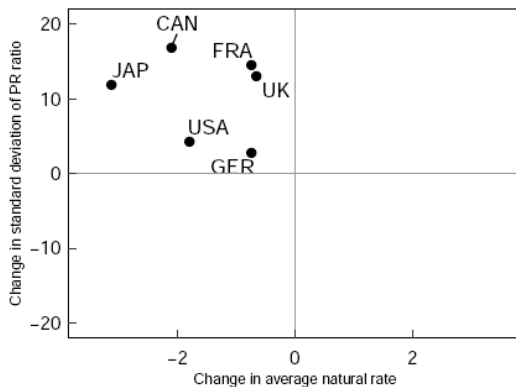
$$\text{Std}(PR_{t-\frac{h}{2}}, \dots, PR_{t+\frac{h}{2}}) = a_h^* - b_h^* \cdot r_t^* + u_{t,h}, \quad (4)$$

- Coefficient  $b_h^*$  has causal interpretation under standard assumptions

# Housing Price Volatility Natural Rates: Unites States



# $\Delta$ PR Volatility vs. $\Delta$ Natural Rate (Pre-/Post-1990)



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- ③ **Full monetary policy model & policy implications**

# Full Monetary Policy Model

Textbook New Keynesian model

- + housing sector with endogenous housing supply
- + subjective housing price beliefs
- + effective lower bound constraint on nominal rates
- + falling natural rates of interest

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- + housing sector with endogenous housing supply
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  - + falling natural rates of interest
- 
- Full model still matches forecast error patterns & HP behavior
  - Study [Ramsey optimal monetary policy](#) in this framework
  - Derive closed-form 2nd order approximation of Ramsey problem

## **New insights generated by the full model:**

Lower average levels of the natural rate

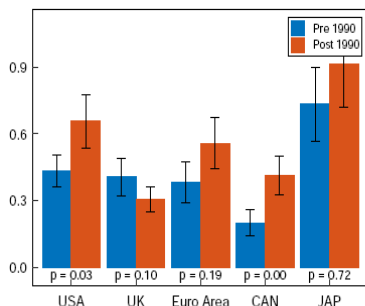
=> increased HP fluctuations

=> increased volatility of natural rate

=> lower bound become more stringent than under RE

=> average inflation increases by more as natural rate falls

Figure 2: Volatility of Natural Rates



Source: Holston et al. (2017) and Fujiwara et al. (2016) (natural rate estimates). The black lines denote the 90%-confidence bands. The reported  $p$ -values are for the null hypothesis that volatility has not changed from pre to post 1990.



# Optimal Policy with Lower Bound Constraint

$$\max_{\{\pi_t, y_t^{gap}, i_t \geq i\}} -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left( \Lambda_{\pi} \pi_t^2 + \Lambda_y (y_t^{gap})^2 + \Lambda_q (\hat{q}_t^u - \hat{q}_t^{u*})^2 \right)$$

s.t. :

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + \underbrace{\kappa_q}_{<0} (\hat{q}_t^u - \hat{q}_t^{u*}) + u_t$$

$$y_t^{gap} = \lim_T E_t y_T^{gap} - \varphi E_t \sum_{k=0}^{\infty} \left( i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right)$$

$$\underbrace{-\frac{C_q}{C_y}}_{>0} (\hat{q}_t^u - \hat{q}_t^{u*})$$

+ Equation(s) determining  $(\hat{q}_t^u - \hat{q}_t^{u*})$

**Rational housing expectations:**  $(\hat{q}_t^u - \hat{q}_t^{u*}) = 0$

# Optimal Policy with Lower Bound Constraint

- The natural rate closing the output gap under subjective beliefs

$$r_t^n \equiv r_t^{n,RE} - \underbrace{\frac{1}{\varphi} \frac{C_q}{C_Y}}_{>0} ((\hat{q}_t^u - \hat{q}_t^{u*}) - E_t(\hat{q}_{t+1}^u - \hat{q}_{t+1}^{u*}))$$

- More volatile housing prices  $\Rightarrow$  more volatile natural rate!

- Bayesian learning about housing price in marginal utility units  $q_t^u$

# Housing Prices and Subjective Beliefs

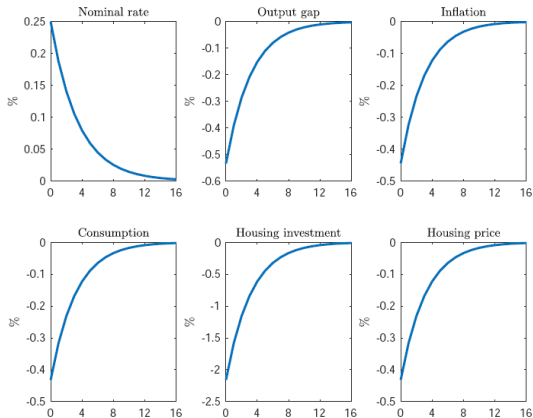
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Can meaningfully discuss optimal policy design!

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Can meaningfully discuss optimal policy design!
- At the same time:
  - MP does affect housing demand & housing prices
  - natural rate (& other fundamentals) affect belief dynamics

# Impulse Response to MP Shock: Calibrated Model

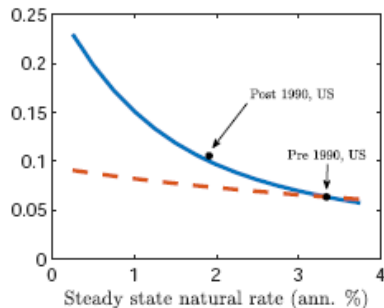


- Calibrate the model to match the pre-1990
  - (1) average natural rate
  - (2) volatility of the natural rate
  - (3) volatility of price-to-rent ratio
- Do this for the RE model and the Subj. Belief model
- What happens as natural rate falls to post-1990 average (or lower):
  - increase in the discount factor  $\beta$
  - may reflect lower steady-state growth

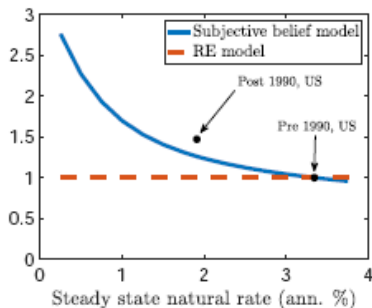


# Model: Non-targeted Moments

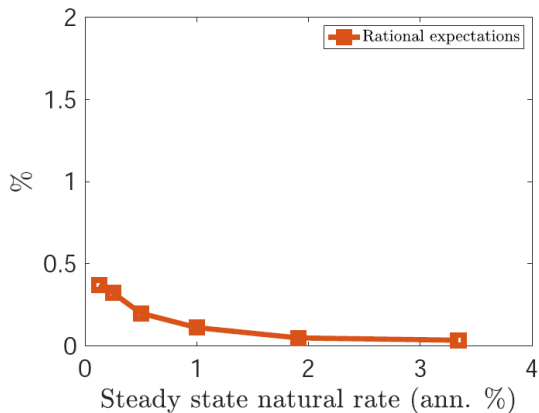
(a) Standard deviation of price-to-rent ratio (relative to corresponding mean)



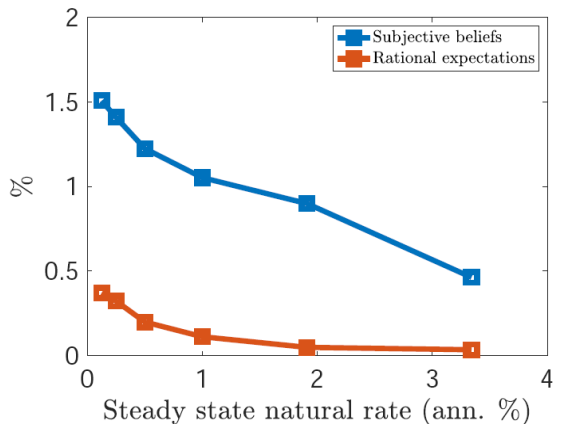
(b) Standard deviation of the natural rate relative to case with  $\underline{r}^{n,RE} = 3.34\%$



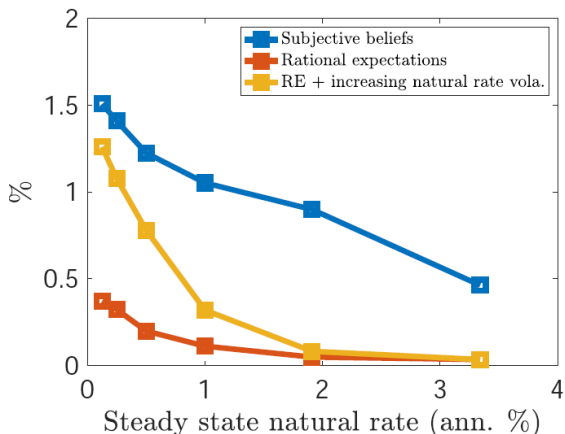
# Average Inflation under Optimal Monetary Policy



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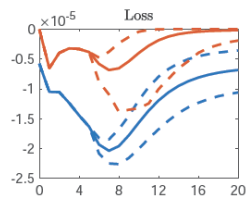
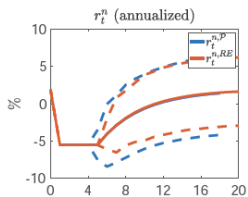
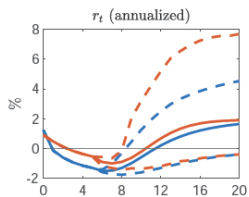
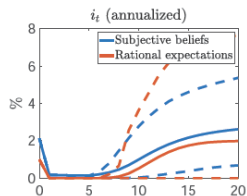
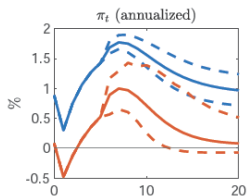
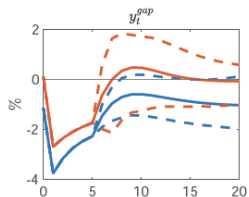


- MP implications of falling natural rates critically depend on the nature of housing price expectations
- Observed forecast errors & housing price behavior is consistent with capital gain extrapolation
- Justifies targeting higher inflation as natural rate falls than under RE
- Alternative approaches:
  - teach HHs how to make more accurate predictions
  - reverse unfavorable macro trends causing fall in average natural rate

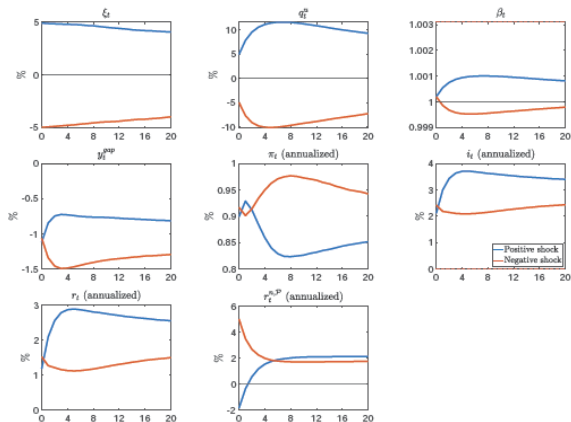
# Impulse Response Analysis for ZLB Event

- Start economy in period 0 at ergodic mean of state variables
- 6 quarters negative natural rate that puts RE economy to ZLB & no other shocks
- After quarter 6: all shocks move again according to their stochastic laws of motion
- Show the mean response: average over all paths
- Show the 1% and 99% percentile of the response distribution
- Put the same shocks into the subjective belief model

# Impulse Response Analysis for ZLB Event



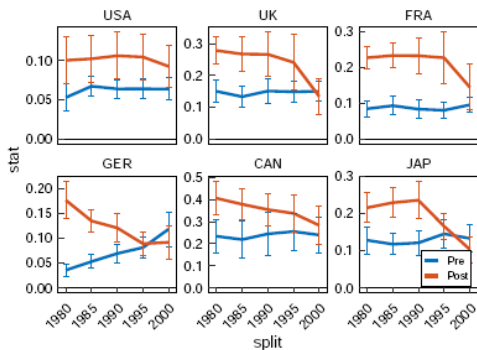
# (Asymmetric) Leaning Against Housing Demand Shocks





# Std. Deviation of the Price-to-Rent Ratio

(a) Standard Deviation of the Price-to-Rent Ratios for Different Sample Splits.



- Representative HH:

$$\max E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[ \tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{\omega}(D_t + D_t^R; \xi_t) \right]$$

s.t. :

$$C_t + B_t + (D_t - (1 - \delta)D_{t-1})q_t + k_t + R_t D_t^R = \\ \tilde{d}(k_t; \xi_t)q_t + \int_0^1 w_t(j)H_t(j)dj + \frac{B_{t-1}(1 + i_{t-1})}{\Pi_t} + \frac{\Sigma_t + T_t}{P_t}$$

- $\mathcal{P}$  : subjective housing price beliefs, otherwise rational beliefs

# NK Model with Housing

- Model formulated in terms of growth-detrended variables
- Discount rate  $\beta$  jointly captures:
  - pure time preference rate  $\tilde{\beta} \in (0, 1)$
  - the **steady-state growth rate  $g_c$**  of consumption

$$\beta \equiv \tilde{\beta} \frac{\tilde{u}_C(C(1 + g_c))}{\tilde{u}_C(C)},$$

- **Growth rate  $g_c$  falls  $\Rightarrow$  discount rate  $\beta$  increases**
- **Decline in growth & natural rate captured via increase in  $\beta$**

- Internally rational households & firms (Adam&Marcet (JET, 2011))
- HHs choose  $\{C_t, H_t(j), D_t, D_t^R, k_t, B_t\}_{t=0}^{\infty}$  to maximize utility subject to the budget constraints
- Beliefs about variables beyond their control given by  $\mathcal{P}$  :  
 $\{P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t\}$ , where  
 $q_t^u \equiv q_t \tilde{u}_C(C; \xi_t)$  is housing price in marginal utility units

# NK Model: Household Optimality Conditions

- Set of standard FOCs: labor-leisure choice, cons. Euler EQ
- 3 new optimality conditions:

$$\text{Optimal housing demand} \quad : \quad q_t^u = \tilde{\zeta}_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} q_{t+1}^u$$

$$\text{Optimal housing investment:} \quad k_t = \left( A_t^d q_t^u \frac{C_t^{\tilde{\sigma}-1}}{\tilde{C}_t^{\tilde{\sigma}-1}} \right)^{\frac{1}{1-\tilde{\alpha}}}$$

$$\text{Purchase vs. renting margin:} \quad \tilde{\zeta}_t^d = R_t \tilde{u}_C(C_t, \tilde{\zeta}_t)$$

- Supply side standard:
  - differentiated goods with Calvo price stickiness  $\alpha \in (0, 1)$
  - Dixit-Stiglitz aggregation
- Standard firm FOCs for optimal reset price: Phillips curve
- **New feature:** wage/marginal costs depend on housing prices

# Structure of Presentation

- ① New Keynesian model with housing & lower bound constraint
- ② **Optimal policy problem & economic mechanisms**

# Nonlinear Optimal Policy Problem

$$\begin{aligned}
 & \max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t) \\
 \left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} &= \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta \phi w_T(j)}{\eta-1} \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} (1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \\
 (P_t/P_{t-1})^{\eta-1} &= (1 - (1-\alpha)(p_t^*/P_t)^{1-\eta})/\alpha \\
 \Delta_t &= h(\Delta_{t-1}, P_t/P_{t-1}) \\
 w_t(j) &= \lambda \frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\bar{\sigma}-1}} \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C(Y_t, q_t^u, \xi_t)^{\bar{\sigma}-1} \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \\
 \tilde{u}_C(C(Y_t, q_t^u, \xi_t); \xi_t) &= \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[ \tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1+i_{t+k}}{P_{t+k+1}/P_{t+k}} \right] \\
 q_t^u &= \xi_t^d + \beta(1-\delta) E_t^{\mathcal{P}} q_{t+1}^u
 \end{aligned}$$



# Optimal Policy with Lower Bound Constraint

- Can derive insightful LQ approx. to nonlinear policy problem
- Helps understanding stabilization trade-offs for output & inflation

# Model Calibration

Parameter	Value	Source/Target
<i>Preferences and technology</i>		
$\beta$	0.9917	Average U.S. natural rate pre 1990
$\varphi$	1	Adam and Billi (2006)
$\kappa_y$	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_r}$	0.007	Adam and Billi (2006)
$\kappa_g$	-0.0023	Adam and Woodford (2020)
$\frac{C_g}{C_Y}$	-0.29633	Adam and Woodford (2020)
$s^d$	15%	Adam and Woodford (2020)
$\delta$	0.03/4	Adam and Woodford (2020)
<i>Exogenous shock processes</i>		
$\rho_{r^n}$	0.8	Adam and Billi (2006)
$\sigma_{r^n}$	0.2940% (RE)	Adam and Billi (2006)
	0.1394% (subj beliefs)	
$\rho_{\xi^d}$	0.99	Adam and Woodford (2020)
$\sigma_{\xi^d}$	0.0233 (RE)	Std. dev. of price-to-rent ratio pre 1990
	0.0165 (subj. beliefs)	
<i>Subjective belief parameters</i>		
$\alpha$	1/0.007	Adam et al. (2016)
$\beta^U$	1.0031	Max percent deviation of PR-ratio from mean