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Quantifying risk and uncertainty in macroeconomic forecasts

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Discussion Paper
Series 1: Economic Studies
No 25/2007

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Internet http://www.bundesbank.de

ISBN 978-3–86558–339–0 (Printversion)
ISBN 978-3–86558–340–6 (Internetversion)

Abstract:

This paper discusses methods to quantify risk and uncertainty in macroeconomic forecasts. Both, parametric and non-parametric procedures are developed. The former are based on a class of asymmetrically weighted normal distributions whereas the latter employ asymmetric bootstrap simulations. Both procedures are closely related. The bootstrap is applied to the structural macroeconometric model of the Bundesbank for Germany. Forecast intervals that integrate judgement on risk and uncertainty are obtained.

Keywords: Macroeconomic forecasts, stochastic forecast intervals, risk, uncertainty, asymmetrically weighted normal distribution, asymmetric bootstrap.

JEL-Classification: C14, C53, E37

Non-technical summary

In this paper, procedures for the quantification of risk and uncertainty in macroeconomic forecasts are developed. The focus is on the integration of information about asymmetric developments, upward or downward risks, in the input factors of forecast variables. Parametric as well as non-parametric procedures are discussed.

The parametric approach is based on asymmetrically weighted normal distributions, using a logistic function to obtain a continuous density. This allows to integrate asymmetric information about the distribution of input factors, which may be correlated, and to aggregate them consistently. To generate asymmetry, this procedure requires weaker modifications of the underlying normal distribution than other widely used methods.

More complex forecast models do not allow to determine forecast uncertainty analytically. In these cases stochastic simulation techniques can be applied. This paper uses non-parametric bootstrap procedures as they circumvent the need to make artificial assumption about the distribution of the stochastic shock terms in the model. To generate forecast intervals for the endogenous variables, the bootstrap recurs to the same asymmetric weighting scheme as in the parametric approach.

Finally, the asymmetric bootstrap is applied to the econometric model of the Bundesbank. The Bundesbank model is an empirically estimated, dynamic and non-linear macroeconometric model for Germany with about 180 variables. If the estimated residuals of the model are used asymmetrically in the bootstrap, asymmetric forecast intervals of the endogenous variables are obtained. However, the asymmetry of the shocks is partly absorbed within the model structure such that the endogenous variables of interest, like real growth and inflation, exhibit markedly less skewness than the shocks.

Nicht-technische Zusammenfassung

In der vorliegenden Arbeit werden Methoden zur Quantifizierung von Risiko und Unsicherheit bei der Prognose makroökonomischer Variablen entwickelt. Insbesondere wird untersucht, wie sich Informationen über asymmetrische Entwicklungen, d.h. auf- oder abwärts gerichtete Prognoserisiken, bei den Bestimmungsfaktoren von Prognosevariablen berücksichtigen lassen. Dabei werden sowohl parametrische als auch nicht-parametrische Verfahren diskutiert.

Die parametrischen Verfahren beruhen auf einer asymmetrisch gewichteten Normalverteilung, wobei eine logistische Funktion verwendet wird, um eine stetige Dichtefunktion zu erhalten. Damit lassen sich asymmetrische Informationen über die Verteilung von Bestimmungsfaktoren, die untereinander auch korreliert sein dürfen, abbilden und konsistent aggregieren. Das Verfahren erzeugt Asymmetrie mit einer deutlich schwächeren Modifikation der zugrunde liegenden Normalverteilung als andere verbreitete Methoden.

Komplexere Prognosemodelle lassen eine analytische Bestimmung von Prognoseunsicherheit und Prognoserisiken nicht mehr zu. In diesen Fällen können stochastische Simulationen eingesetzt werden. In dieser Arbeit werden nicht-parametrische Bootstrap-Verfahren verwendet, die keine willkürlichen Annahmen über die Verteilung der stochastischen Schocks des Modells erfordern. Bei den Bootstrap-Ziehungen wird das gleiche asymmetrische Gewichtungsschema wie bei den parametrischen Verfahren benutzt, um Prognoseintervalle für die endogenen Variablen zu schätzen.

Das asymmetrische Bootstrap-Verfahren wird auf das ökonometrische Bundesbankmodell angewandt. Das Bundesbankmodell ist ein empirisch geschätztes, nichtlineares und dynamisches strukturelles Makromodel für Deutschland mit etwa 180 Variablen. Werden die stochastischen Störterme beim Bootstrap asymmetrisch verwendet, so ergeben sich asymmetrische Prognoseintervalle für die endogenen Variablen. Wie sich jedoch zeigt, werden asymmetrische Schocks im Modellzusammenhang teilweise absorbiert, so dass die interessierenden endogenen Variablen wie reales Wachstum und Inflation eine deutlich geringere Schiefe aufweisen.

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1. Introduction

Monetary policy decisions are based on forecasts of inflation, output growth and many other macroeconomic variables. Central banks often rely on deterministic point forecasts, usually supplemented by verbal qualifications. Frequently, baseline forecasts are complemented by alternative scenarios, singled out as likely alternatives to the baseline. The obtained range of point forecasts, however, is not a forecast interval which covers a well-defined probability of outcomes.

Deterministic forecasts do not allow to quantify the associated uncertainty (dispersion of the distribution) and risk (degree of asymmetry) properly. Wallis (2007, 54) points out that *“it is now widely recognised that a point forecast is seldom sufficient for well-informed decision-making in the face of an uncertain future, and that it needs to be supplemented with an indication of the degree of uncertainty.”* Uncertainty intervals underline the inherently uncertain nature of forecasts, they enhance the transparency of a central bank in its communication with the public, and they facilitate the internal discussion by focussing it on the sources of uncertainty and their quantitative importance (Blix and Sellin, 1999). The shape of the uncertainty intervals provides the public with information about the forecast risks. Depending on the loss functions of the public, information about forecast risks can be as important as information about forecast uncertainty. As an example, consider large losses only in case of deflation, and a positive inflation forecast with low uncertainty but a large downward risk.

Central banks do not rely on a single econometric model to generate their forecasts. Usually a suite of models is applied and subjective judgements play an important role. The quantification of forecast risk and uncertainty in such an environment is not a straightforward task. Resulting in the well-known “fan charts”, the Bank of England (Britton, Fisher, and Whitley, 1998) pioneered a parametric procedure to determine the distribution of a linear combination of skewed, yet independent, random input variables. The two-piece normal distribution is utilised to introduce skewness into the forecast input variables. Recently, at the Bank of Portugal a parametric method was developed that achieves skewness by a combination of normal and exponential variates and allows for correlated input variables (Novo and Pinheiro, 2005).

In this paper, we discuss a parametric and a non-parametric procedure for quantification of risk and uncertainty in macroeconomic forecasts, mainly focussing on risk. Whereas the former procedure is based on a generalisation of the normal distribution, the latter relies on bootstrap simulations. Skewness is introduced by an asymmetric weighting scheme.

Section 2 introduces a parametric class of asymmetrically weighted normal (AWN) distributions for constructing forecast intervals. In section 3 a non-parametric asymmetric bootstrap procedure to calculate forecast intervals that take risk into account is discussed. This procedure is closely related to the AWN distributions investigated in section 2. The methods presented in both sections allow to handle skewness without affecting the mean and the variance of the input variables. In section 4 the asymmetric bootstrap is applied to generate forecasts with the structural macroeconometric model of the Bundesbank. Section 5 concludes.

2. Forecast intervals based on AWN distributions

To quantify forecast risks, according to Azzalini (1985, 171): *"it would be ideal to have at hand a class of densities with the following properties: "strict inclusion" of the normal density, mathematical tractability, wide range of the indices of skewness and kurtosis."* Based on the Gaussian normal distribution, we introduce a class of asymmetrically weighted normal (AWN) distributions which comes close to these requirements. AWN distributions include the normal as a special case, they allow to quantify asymmetric risk by a single, easily interpretable parameter, the density of a linear combination of correlated AWN variables can be obtained by standard numerical integration techniques, and for one of the AWN distributions, there is no lower or upper bound for its skewness and no upper bound for its kurtosis.

2.1. Forecasts as linear combinations of input factors

Assume that the deterministic point forecast for a macroeconomic variable $(\tilde{W})$, i.e. inflation or output growth, is a linear combination of input variables $(\tilde{X}_k)$

(1) $$\tilde{w}_h = \alpha_{1,h}\tilde{x}_{1,h} + \ldots + \alpha_{K,h}\tilde{x}_{K,h} \quad , \qquad h = T+1, \ldots, T+H$$

where the $\alpha_{k,h}$ are (estimated or calibrated) interim multipliers or elasticities and h denotes the forecast horizon.[1] The forecasts may be the output of an econometric forecasting model, they may be based on expert judgement, or on a combination of both.

We assume that the forecast errors of the input variables, denoted as $Z_{k,t} = X_{k,t} - \tilde{X}_{k,t}$, are normal, with zero mean (unbiasedness) and variance $\sigma^2 > 0$:

(2) $$\varphi(z;\ \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2}$$

We are interested in the density of a linear combination of forecast errors $Y_h = W_h - \tilde{W}_h$:

(3) $$y_h = \alpha_{1,h} z_{1,h} + \ldots + \alpha_{K,h} z_{K,h}$$

The error variables Z_k may be correlated with covariance matrix Σ. The density of their linear combination (3) can be obtained by standard procedures (Fisz, 1976). The resulting density would be symmetric, however.

2.2. The asymmetrically weighted normal (AWN) distribution

Often, in a specific forecasting round, the forecaster may have information which leads him to judge the forecast risks to be asymmetric, tilted upward or

[1] We adopt the common convention to use uppercase letters for random variables and lowercase letters for their realizations.

downward. Hence, led by subjective judgement, he may wish to deviate from the normal distribution by rendering the forecasts asymmetric.

Despite the importance of risk in macroeconomic forecasts, there is no established procedure for quantification of these risks. According to Machina and Rothschild (1987), there are two basic requirements for a measure of risk. First, the measure of risk must be related to the probability distribution of the underlying random variable, e.g. inflation or output growth. Second, the risk measure should be linked to preferences of the forecasting agent. Often, quadratic loss functions are used in order to represent such preferences, as pointed out by Woodford (2003, ch. 6). However, loss functions can take many functional forms. Kilian and Manganelli (2007), for example, proposed an asymmetric loss function for the risk of deflation and excessive inflation.

Because there is no generally accepted specification of a loss function, in this paper we express the risk assessment of the forecaster simply as a probability. An upward (downward) risk in the forecast of an input factor is measured by the probability of a positive (negative) forecast error.

Assume that for the forecast period an upward forecast risk with probability ω is expected. To take this into account, the random variable Z is transformed according to $Z \equiv J(\omega)|Z| - (1 - J(\omega))|Z|$, where $J(\omega)$ is an indicator variable which takes the value 1 with probability ω and the value 0 with probability $1 - \omega$. The density function of the variable Z is defined as

(4) $$f(z;\sigma,\omega) = \begin{cases} 2(1-\omega)\varphi(z;\sigma) & \text{if } z < 0 \\ 2\omega\,\varphi(z;\sigma) & \text{if } z \geq 0 \end{cases}$$

with $0 \leq \omega \leq 1$ denoting the probability of an upward risk.[2] The (risk–adjusted) random variable Z has an asymmetrically weighted normal (AWN) distribution with density $f(z;\sigma,\omega)$: $Z \sim AWN(\sigma,\omega)$.

The transformation shifts probability mass from the left hand side of the underlying normal distribution to the right hand side (or vice versa) by

[2] The asymmetric weighting scheme can be applied to any parametric density function. In section 3 we apply the asymmetric weighting scheme non–parametrically in stochastic model simulations.

proportionally scaling the density up or down, respectively. The AWN distribution has two parameters, σ as a measure of uncertainty and ω as a measure of risk (asymmetry). For ω = 0.5 the normal distribution with zero mean is obtained as a special case. For $\omega \to 1\,(\to 0)$ the so called half-normal distribution results, which may be regarded as extreme asymmetry. The mode of the AWN is zero. The AWN density jumps at z = 0, the absolute size of the jump being $|2\omega - 1|\, 2\,\varphi(0)$. For the interval $[a,b] \in \mathbb{R}^+$, with $a < b$, we obtain:

(5) $$\frac{P(z \geq 0)}{P(z < 0)} = \frac{\omega}{1-\omega} = \frac{P(a \leq z \leq b)}{P(-b \leq z \leq -a)}$$

Hence, the asymmetric weighting scheme does not distort the relative probabilities of the underlying normal distribution $\varphi(z)$. This is a desirable feature because no a priori knowledge with regard to more or less likely sub-intervals for upward or downward risks is implied. The mean of the AWN is

(6) $$m = (2\omega - 1)\sqrt{\frac{2\sigma^2}{\pi}} \quad = \quad 0.798\,(2\omega - 1)\sigma$$

and its higher (central) moments turn out as:

(7) $$\begin{aligned} V &= \sigma^2 - m^2 \\ S &= m\,[2\,m^2 - \sigma^2] \\ W &= 3[\sigma^4 - m^4] - 2m^2\sigma^2 \end{aligned}$$

where V denotes the second, S the third and W the fourth central moment. For ω = 0.5 the moments of the normal distribution are obtained: m = 0, V = σ^2, S = 0, W = $3\sigma^4$.

The forecaster may not wish that the risk assessment changes the mean and variance of an input variable. In this case the following modification of the density function (4) can be applied:

(4’) $$f(z;\sigma,\,\omega) = \begin{cases} 2(1-\omega)\varphi(z;\sigma_1) & \text{if } z < 0 \\ 2\omega\,\varphi(z;\sigma_2) & \text{if } z \geq 0 \end{cases}$$

with $\sigma_1 = \sigma\sqrt{\frac{\omega}{1-\omega}}$, $\sigma_2 = \sigma\sqrt{\frac{1-\omega}{\omega}}$

Similar to the two-piece normal distribution (John, 1982), the underlying normal distribution has variance σ_1 (σ_2) for negative (positive) values of Z. The probability for positive outcomes for the variable Z is $P(z>0)=\int_0^\infty 2\omega\varphi(z,\sigma_2)dz = 2\omega/2=\omega$, as desired. Moreover, its mean and variance are zero and σ^2, respectively:

(7')
$$E(z)=\int_{-\infty}^0 2(1-\omega)z\varphi(z,\sigma_1)dz+\int_0^\infty 2\omega z\ \varphi(z,\sigma_2)dz = \\ =2(1-\omega)[-\frac{1}{2}\sqrt{2\sigma_1^2/\pi}]+2\omega[\frac{1}{2}\sqrt{2\sigma_2^2/\pi}]=0$$

(7'')
$$V(z)=\int_{-\infty}^0 2(1-\omega)\ z^2\ \varphi(z,\sigma_1)dz+\int_0^\infty 2\omega z^2\varphi(z,\sigma_2)dz = \\ =2(1-\omega)[\frac{\sigma_1^2}{2}]+2\omega[\frac{\sigma_2^2}{2}]=\sigma^2$$

Hence, despite skewness, the mean and the variance of the distribution remain unchanged. Also note that no new parameter was introduced to get this property. The skewness s(z) and the kurtosis w(z) are given by

(7''')
$$s(z)=2\sqrt{\frac{2}{\pi}}\frac{1-2\omega}{\sqrt{(1-\omega)\omega}} \quad \text{and} \quad w(z)=\frac{3}{(1-\omega)\omega}-9.$$

Obviously, there is no upper or lower limit for skewness and no upper limit for kurtosis. The lowest kurtosis equals 3 and is obtained with ω = 0.5, i.e. in the case where Z is normally distributed.

2.3. The logistic asymmetrically weighted normal (LAWN) distribution

In principle, the density of the linear combination (3) of several AWN–random variables could be calculated. However, due to the discontinuity, this exercise is burdensome because a rapidly increasing number of cases has to be treated separately. For this reason we apply a continuous approximation of the AWN.

Consider the logistic function

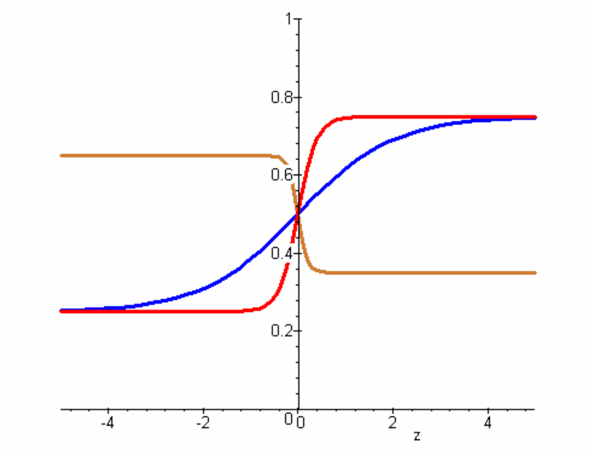
(8) $$H(z) = \frac{e^z}{1+e^z}, \qquad -\infty \le z \le \infty; \;\; 0 \le H(z) \le 1$$

Its derivative H'(z) is a density, which is symmetric about zero. The logistic asymmetric weight function is now defined as

(9) $$G(\lambda z;\, \omega) = (1-\omega)(1 - H(\lambda z)) + \omega H(\lambda z)$$

where $0 \le \omega \le 1$, and $\lambda > 0$ is a technical coefficient that controls the closeness of the approximation to the step function (4). Chart 1 shows the logistic weight function (9) for upward risks [$\omega = 0.75, \lambda =$ (1, 10)], and downward risk: [$\omega = 0.35$, $\lambda = 10$].

Chart 1: The logistic weight function



With increasing λ the approximation to the weighting scheme of the AWN becomes closer. In the limit we get:

(10) $$\lim_{\lambda\to\infty} G(\lambda z;\omega) = \begin{cases} \omega, & z \ge 0 \\ 1-\omega, & z < 0 \end{cases}$$

We define the logistic asymmetrically weighted normal (LAWN) distribution for the random Variable Z as:

(11) $$f(z;\, \omega,\sigma,\lambda) = 2G(\lambda z;\omega)\, \varphi(z;\sigma)$$

where $\varphi(z;\sigma)$ is the density of a normal random variable Z with zero mean and variance σ^2: $Z \sim \text{LAWN}(\sigma,\omega,\lambda)$.

It has yet to be shown that (11) is indeed a density. Azzalini (1985, 172) proved the following

> ***Lemma:*** *Let φ be a density function symmetric about zero, and Ω an absolute continuous distribution function such that Ω' is symmetric about 0. Then $2\Omega(\lambda z)\varphi(z)$ $(-\infty < z < \infty)$ is a density function for any real λ.*

From this lemma we deduce the

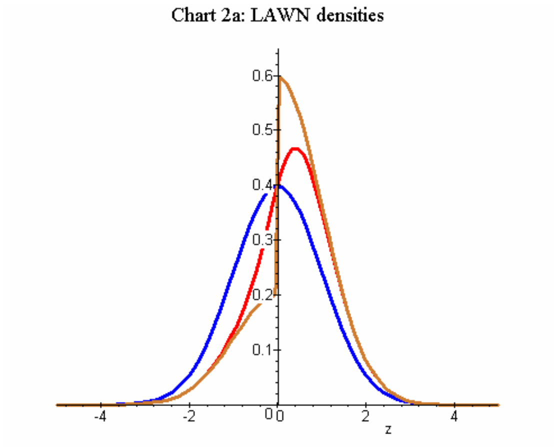
> ***Corollary***: *If $2\Omega(\lambda z)\varphi(z)$ is a density, and $G(\lambda z)$ is the logistic weighting function (9) then $2G(\lambda z)\varphi(z)$ is a density function as well.*
>
> ***Proof:*** *The logistic function H in (8) satisfies the requirements of Ω in the lemma. Since $G(\lambda z)$ in (9) can be written as $(2\omega-1)H(\lambda z)+1-\omega$, the following holds:*
>
> $$\begin{aligned}&\int_{-\infty}^{\infty} 2G(\lambda z)\varphi(z)dz\\ &= (2\omega-1)\int_{-\infty}^{\infty} 2H(\lambda z)\varphi(z)dz + 2(1-\omega)\int_{-\infty}^{\infty}\varphi(z)dz\\ &= (2\omega-1)+2(1-\omega)=1\end{aligned}$$
>
> *Since, in addition, all other conditions for density functions are fulfilled by (11), $f(z;\,\omega,\sigma,\lambda) = 2G(\lambda z;\omega)\ \varphi(z;\sigma)$ is a density function.*

If the forecaster wants to preserve the mean and variance of the risk-adjusted forecast, a modification similar to (4') can be applied

(11') $$f(z;\sigma,\,\omega,\,\lambda) = 2(1-\omega)(1-H(\lambda z))\varphi(z;\sigma_1) + 2\omega H(\lambda z)\varphi(z;\sigma_2)$$

where σ_1, σ_2 are defined in (4').

Chart 2a shows the density function (11) for $[\sigma = 1, \omega = 0.5, \lambda = 0]$ (standard normal density), $[\sigma = 1, \omega = 0.75, \lambda = 3]$ and $[\sigma = 1, \omega = 0.75, \lambda = 100]$. Chart 2b shows the same distributions with mean and variance preserved according to (11').



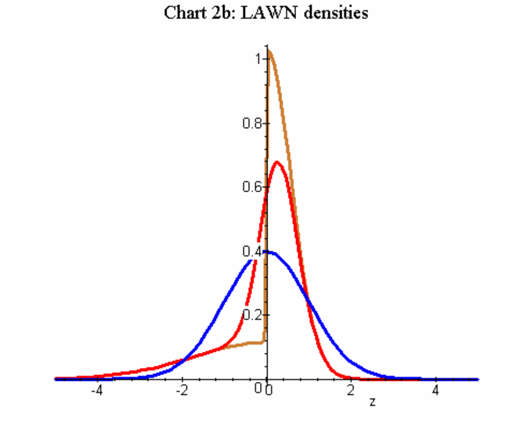


Table 1 provides numerical moments of the LAWN–distribution. For ω = 0.5 (any λ) or for λ = 0 (any ω) the normal distribution is obtained as a special case. With increasing λ the moments of the LAWN distribution rapidly approach to those of the AWN distribution.

Table 1: Moments of LAWN - distributions

ω	λ	**P(z > 0)**	**m**	**s**	**V**	**w**
0.50	any	0.50	0	0	1	3
any	0	0.50	0	0	1	3
0.75	5	0.70 (0.64)	0.38 (-0.04)	-0.29 (-1.73)	0.86 (1.00)	3.52 (6.72)
0.75	10	0.72 (0.69)	0.39 (-0.01)	-0.33 (-1.80)	0.85 (1.00)	3.64 (6.90)
0.75	100	0.75 (0.74)	0.40 (-0.00)	-0.35 (-1.84)	0.84 (1.00)	3.68 (7.00)
0.75	$\to \infty$	0.75 (0.75)	0.40 (-0.00)	-0.35 (-1.84)	0.84 (1.00)	3.69 (7.00)

σ = 1 ; mean = m = E(Z), variance = V = $E(Z\text{-}m)^2$, skewness = s = $E(Z\text{-}m)^3/V^{3/2}$, kurtosis = w = $E(Z\text{-}m)^4/V^2$; mean and variance preserved values in brackets

As the figures in brackets of Table 1 show, applying the mean- and variance-preserving modification effectively fixes the mean and the variance of the LAWN distribution at 0 and 1, respectively. However, skewness as well as kurtosis increase sharply, indicating a stronger deviation from the underlying normal.

2.4. Comparison to other skewed distributions

In this section we briefly discuss three alternative skewed distributions, all based on the normal, and compare them to the LAWN.

Azzalini (1985) defines the *"skew-normal distribution" (SN)* as[3]

(12) $$f(z;\, \sigma, \alpha) = 2\Phi(\alpha z)\varphi(z;\sigma)$$

where $$\varphi(z;\, \sigma) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2}, \quad \Phi(\alpha z) = \int_{-\infty}^{\alpha z} \varphi(t)dt$$

are the density and the distribution function of the normal, respectively. The shape parameter α generates skewness. For $|\alpha| \to \infty$ the SN distribution converges to the half-normal. The normal is included as a special case for α = 0.

At the Bank of England, Britton, Fisher and Whitley (1998) use the *"two-piece normal distribution"* (TPN) with density

(13) $$f(z;\, \sigma_1, \sigma_2) = \begin{cases} \dfrac{2\sigma_1}{\sigma_1 + \sigma_2}\, \varphi(z;\, \sigma_1), & z < 0 \\ \dfrac{2\sigma_2}{\sigma_1 + \sigma_2}\, \varphi(z;\, \sigma_2), & z \geq 0 \end{cases}$$

Here, $\varphi(z;\sigma_j)$, j=1,2, denotes the density of the normal distribution with zero mean and variance σ_j^2. The TPN distribution is also discussed by Blix and Sellin (1998, 1999). As noted by Wallis (2007, 23) "*… the asymmetric distribution has no convenient multivariate generalisation*".

At the Bank of Portugal, Novo and Pinheiro (2005) developed the *"skewed generalized normal distribution" (SGN)* as a linear combination of two independent random variables $Z = \theta_1 + \theta_2 Z_1 + \theta_3 Z_2$, where $\theta_1, \theta_3 \in \mathbb{R}$, $\theta_2 > 0$. Here, Z_1 is a standard normal variable and Z_2 follows an exponential distribution. The density is

[3] See also: A. Azzalini (2005-11-23) : http://azzalini.stat.unipd.it/SN/index.html

$$(14)\qquad f(z;\theta_1,\theta_2,\theta_3)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\,\theta_2}e^{-\frac{1}{2}(\frac{z-\theta_1}{\theta_2})^2} & \theta_3=0\\ e^{\eta}\dfrac{2^{1/3}}{|\theta_3|}e^{-\frac{2^{1/3}}{\theta_3}}\Phi_{\upsilon,\theta_2}\left(\dfrac{\theta_3}{|\theta_3|}z\right) & \theta_3\neq 0\end{cases}$$

where $\eta=-1+2^{1/3}\theta_1/\theta_3+2^{-1/3}(\theta_2/\theta_3)^2$ and $\upsilon=2^{-1/3}(\theta_3\eta+2^{-1/3}\theta_2^2/\theta_3)$ are constants. The function $\Phi_{\upsilon,\theta_2}(.)$ represents the distribution function of a normal variable.

Chart 3.1 shows the LAWN(1, 0.388, 10) and the LAWN(1, 0.4, 100) distribution. Charts 3.2 to 3.4 show the SN(1, -0.325), the TPN(1.5, 1), and the SGN(0, 1, 3) distribution, all parameterised such that $P(z>0)=\omega=0.4$.

Chart 3.1: LAWN densities

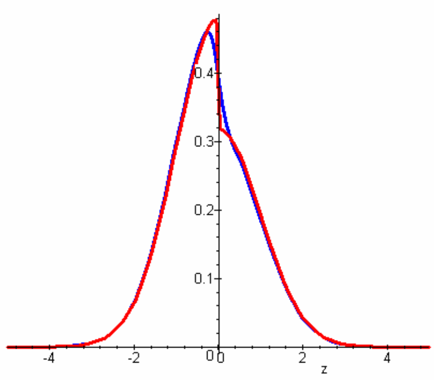


Chart 3.2: SN density

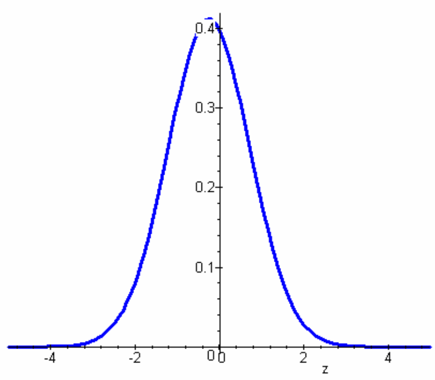


Chart 3.3: TPN density

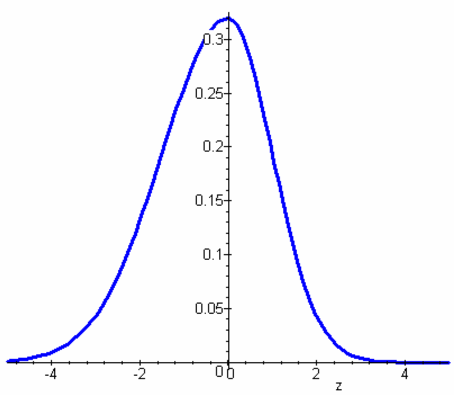


Chart 3.4: SGN density

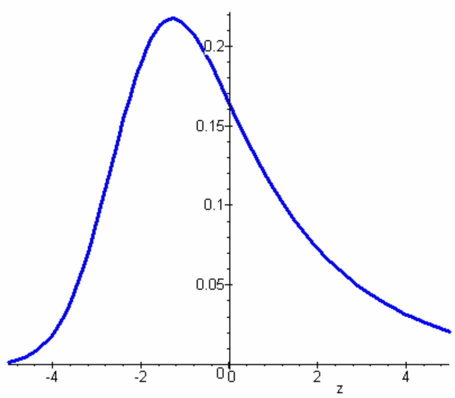


Table 2 provides the moments of the asymmetric distributions shown in Charts 3. The final column gives the Jarque-Bera (JB)–statistic for testing normality:

(15) $$JB = \frac{n}{6}\left[s^2 + \left(\frac{w-3}{2}\right)^2\right]$$

where the sample size was arbitrarily set to n = 100. The widely used JB–statistic, which is independent of σ^2, tests whether a linear combination of skewness and kurtosis deviates from the values implied by the normal distribution (s = 0, w = 3).[4] Measured by the JB–statistic, the LAWN distribution yields the smallest distortion of the underlying normal distribution, closely followed by Azzalini's SN distribution. Achieving the same upside risk of ω = 0.40 with the TPN distribution results in a somewhat bigger distortion of the normal. However, in all three cases the deviation from the normal would not be statistically significant at the 5 % level.

Table 2: Moments of asymmetric distributions

Distribution	P(z > 0)	m	s	V	w	JB
LAWN(1, 0.39, 10)	0.40	-0.18	0.17	0.97	3.12	0.53
LAWN(1, 0.4, 100)	0.40	-0.16	0.16	0.97	3.10	0.46
SN(1, -0.325)	*0.40*	*-0.25*	*-0.01*	*0.94*	*3.45*	*0.85*
TPN(1, 3)	*0.40*	*-0.40*	*-0.31*	*1.59*	*3.07*	*1.64*
SGN(0, 1, 3)	*0.40*	*0.00*	*1.57*	*6.67*	*7.34*	*119.28*
LAWN(1, 0.37, 10) *)	0.40	0.01	0.83	1.00	3.82	14.18
LAWN(1, 0.4, 100) *)	0.40	0.00	0.65	1.00	3.50	8.12

*Mean = m = E(Z), variance = V = $E(Z\text{-}m)^2$, skewness = s = $E(Z\text{-}m)^3/V^{3/2}$, kurtosis = w = $E(Z\text{-}m)^4/V^2$; The critical values of the JB–statistic at the 5 % significance level for n = [50, 100, 200] are [5,00, 5,45, 5,73]. *) Mean and variance preserved.*

In contrast, using the SGN distribution to introduce asymmetry yields a huge distortion of the normal. Partly this may be due to the fact that the SGN fixes the mean at zero. The final two rows of Table 2 display the moments of the LAWN distribution calculated under the condition that asymmetry does not change the

4 Asymptotically the JB – statistic has a χ^2 – distribution with 2 degrees of freedom and a critical 5 % value of 5.99; see Thadewald and Büning (2007).

mean and variance. In this case the JB–test rejects normality. However, compared to the SGN distribution, the same degree of asymmetry is obtained with a much smaller deviation from normality.

2.5. The multivariate asymmetrically weighted normal distribution (MLAWN)

To calculate the distribution of a linear combination of several LAWN–distributed input factors, which may or may not be correlated, we generalise the logistic weight function (9) in the following way:

(16) $$G(z;\,\omega,\lambda)=\prod_{i=1}^{K}G_i(z_i;\,\omega_i,\lambda)$$

where $$G_i=\omega_i\frac{e^{\lambda x_i}}{1+e^{\lambda x_i}}+(1-\omega_i)\frac{1}{1+e^{\lambda x_i}}$$

$$z=(z_1,z_2,\ldots,z_k)\quad\text{and}\quad\omega=(\omega_1,\omega_2,\ldots,\omega_k)$$

With $\omega_1=0.75,\ \omega_2=0.75$, charts 4.1 and 4.2 show the bivariate logistic weight function for $\lambda=(3,10)$.

Chart 4.1. Logistic weight function

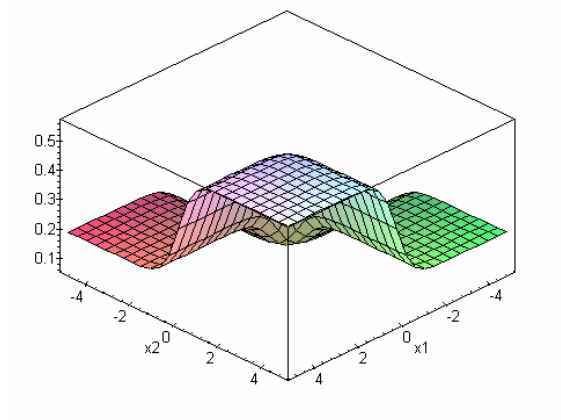
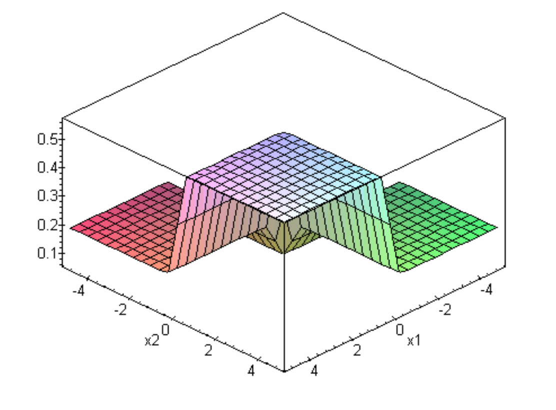


Chart 4.2. Logistic weight function

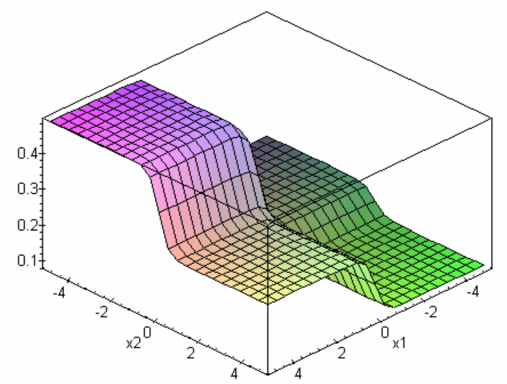


Increasing λ yields a more pronounced step function. The following limits apply:

(17) $$\lim_{\lambda\to\infty}G(z;\omega,\lambda)=\begin{cases}\omega_1\,\omega_2\ldots\omega_k & z_1,z_2,\ldots,z_k\ge 0\\(1-\omega_1)\,\omega_2\ldots\omega_k & z_1\le 0,z_2,\ldots,z_k\ge 0\\\ldots & \\(1-\omega_1)(1-\omega_2)\ldots(1-\omega_k) & z_1,z_2,\ldots,z_k\le 0\end{cases}$$

Chart 4.3 shows the weight function for combined risks (upward risk in Z_1 and downward risk in Z_2): $[\omega_1 = 0.75,\ \omega_2 = 0.35,\ \lambda = 5]$.

Chart 4.3. Logistic weight function



The density of a multivariate logistic asymmetrically weighted normal (MLAWN) distribution is defined as:

(18) $$f(z;\, \omega, \lambda, \Sigma) = \kappa\ G(z;\, \omega, \lambda)\varphi(z; \Sigma)$$

where $\varphi(.)$ is the density of the multivariate normal and κ is a normalising constant. Charts 5.1 to 5.3 show the MLAWN for the parameters $[\omega_1 = 0.75,\ \omega_2 = 0.75,\ \lambda = 5,\ \sigma_1 = \sigma_2 = 1]$ and $\rho = (0,\ 0.8, -0.8)$:

Chart 5.1. MLAWN (rho = 0)

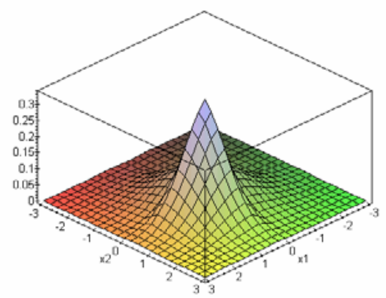


Chart 5.2. MLAWN (rho = 0.8)

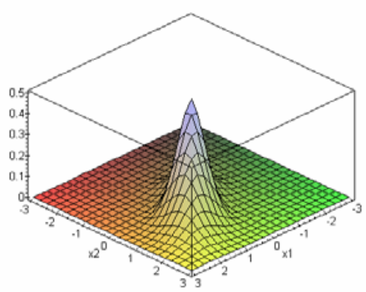
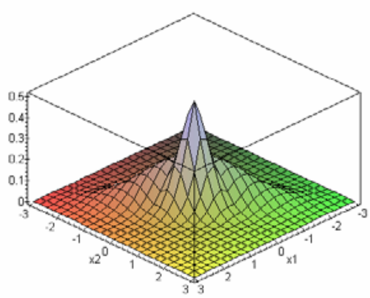


Chart 5.3. MLAWN (rho = -0.8)



In Table 3 it is shown how the weighting scheme influences the probabilities for certain outcomes under the MLAWN compared to the MLAWU uniform distribution in the first row.

Table 3: Probabilities under the MLAWN distribution

	$P(z_1>0, z_2>0)$	$P(z_1>0, z_2<0)$	$P(z_1<0, z_2>0)$	$P(z_1<0, z_2<0)$
MLAWU *)	*0.55*	*0.19*	*0.19*	*0.07*
MLAWN (ρ = 0) +)	0.55	0.19	0.19	0.07
MLAWN(ρ = 0,8) +)	0.76	0.07	0.07	0.10
MLAWN(ρ = -0,8) +)	0.24	0.36	0.36	0.04

**) Multivariate logistic asymmetrically weighted uniform distribution with $f(z_i) = 1$ for $-0.5 < z_i < 0.5$ and 0 else; $\omega_1 = 0{,}75$, $\omega_2 = 0{,}75$, $\lambda = 20$*

+) Multivariate logistic asymmetrically weighted normal distribution; $\omega_1 = 0{,}75$, $\omega_2 = 0{,}75$, $\lambda = 20$

We now turn to the distribution of a linear combination of MLAWN–distributed – possibly dependent – random variables, i.e. we want to calculate the aggregate density of Y in (3). Consider the following transformations:

(19)
$$\begin{aligned} Y_1 &= Z_1 \\ &\dots \\ Y_{K-1} &= Z_{K-1} \\ Y_K &= \alpha_1 Z_1 + \alpha_2 Z_2 + \dots + \alpha_k Z_K \end{aligned}$$

Assuming existence of the inverse functions we may write

(19')
$$\begin{aligned} Z_1 &= Y_1 \\ &\dots \\ Z_{K-1} &= Y_{K-1} \\ Z_K &= \frac{1}{\alpha_K} Z_K - \frac{\alpha_1}{\alpha_K} Z_2 - \dots - \frac{\alpha_{K-1}}{\alpha_K} Z_{K-1} \end{aligned}$$

The partial derivatives of (19') are

(20) $$D = [\frac{\partial z_i}{\partial y_j}] = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ -\frac{\alpha_1}{\alpha_K} & -\frac{\alpha_2}{\alpha_K} & ... & \frac{1}{\alpha_K} \end{bmatrix} \Rightarrow \det(D) = \frac{1}{\alpha_K}$$

Hence, the joint density is:

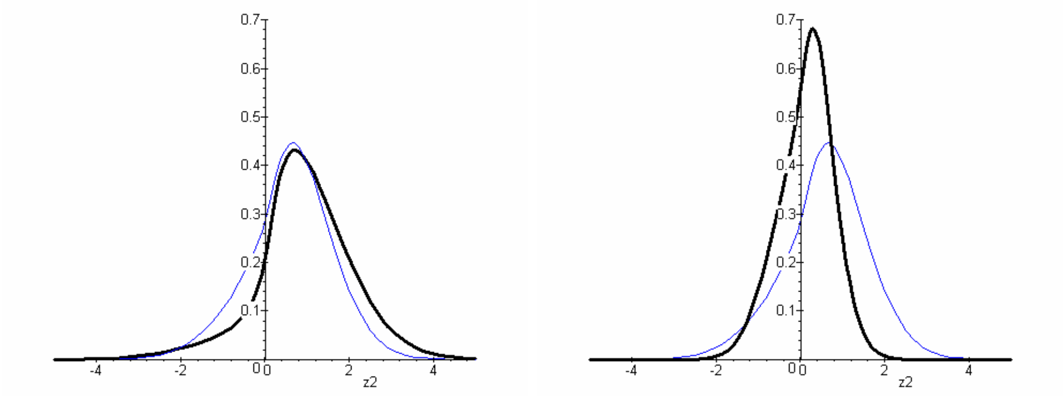
(21) $$h_s(y_1,...,y_K; \omega, \Sigma, \alpha, \lambda) = h\left[y_1,...,y_{K-1}, \frac{y_K - \alpha_1 y_1 - ... - \alpha_{K-1} y_{K-1}}{\alpha_K}; \omega, \Sigma, \alpha, \lambda \right] \left| \frac{1}{\alpha_K} \right|$$

The marginal density of the linear combination (Y_K) can be obtained by integrating out the variables Y_1 … Y_{K-1}:

(22) $$h(y) = \int\int...\int h_s(y_1,...,y_K) dy_1...dy_{K-1}$$

As an example, the density of $y = \alpha_1 z_1 + \alpha_2 z_2$ with $\alpha_1 = 1$, $\alpha_2 = 0.5$ and $[\omega_1 = 0.75, \omega_2 = 0.75, \sigma_1 = \sigma_2 = 1, \lambda = 20]$ is shown, where both input variables are correlated with ρ = (0, 0.8) (chart 6a) and with ρ = (0, -0.8) (chart 6b).

Chart 6a: Density of a sum of MLAWN - Variables (rho = 0.8) Chart 6b: Density of a sum of MLAWN - Variables (rho = -0.8)



The probabilities for upward and downward risks are given in Table 4. Strong upward risks of 75 % in both input factors are to some extent moderated (amplified) in the aggregate if the input factors are negatively (positively) correlated.

Table 4: Aggregate upward and downward risks

ρ	P(y < 0)	P(y > 0)
- 0.8	0.38	0.62
0	0.25	0.75
+ 0.8	0.16	0.84
$\omega_1 = \omega_2 = 0.75$; $\sigma_1 = \sigma_2 = 1$; $\alpha_1 = 1$, $\alpha_2 = 0.5$; $\lambda = 20$		

3. Forecast intervals based on asymmetric bootstrap simulations

Many economic models do not allow for an analytical investigation of forecast risk and uncertainty. This can for example be due to the size of the model or due to non-linearities. In this case, stochastic simulations can be used to obtain estimates of forecast uncertainty. Stochastic simulations require random draws from the model's shocks. While this can be achieved using distributional assumptions, it is also possible to use the distribution-free bootstrap-approach. In the following it is discussed how the bootstrap approach can be modified to incorporate risk and uncertainty assessments into the stochastic simulations. The approach chosen generates shocks which have an AWN distribution, if the residuals of the model are normally distributed.

3.1. Stochastic simulations

Consider the reduced form of a dynamic economic model consisting of g equations given by

(23) $$y_t = F(y_{t-1}, y_t, x_t, u_t; \theta) \qquad t = 1...T.$$

F denotes a g-vector of functions, y_t (y_{t-1}) denotes a g-vector of endogenous (lagged endogenous) variables, x_t denotes a k-vector of exogenous variables, u_t denotes a g-vector of shocks, θ is a vector of coefficients, and t = 1…T denotes the estimation sample of the model. Estimation yields $\hat{\theta}$ and $\hat{u}_t$ for t = 1…T.

The model is simulated M times, using random shocks $\hat{u}_h^m$ in every forecast period h. Starting with $\hat{y}_T^m = y_T$, in simulation m the forecast

$$(24) \qquad \hat{y}_h^m = F(\hat{y}_{h-1}^m, \hat{x}_h, \hat{u}_h^m; \hat{\theta}) \qquad h = T+1,\dots,T+H;\ \ m = 1,\dots,M$$

emerges.[5] The stochastic simulation thus gives samples $\left\{\hat{y}_h^1, \hat{y}_h^2, \dots, \hat{y}_h^M\right\}$ for every forecast horizon h. From these samples, the statistics of interest like mean, variance, skewness or confidence bounds can be computed.

3.2. Bootstrap simulations

For stochastic simulations, random shocks $\hat{u}_h^m$ are needed for every forecast period h and every run m. One way of generating these shocks is drawing random variables from an appropriate distribution. However, often it is unclear what the appropriate distribution is. In this case, it can be convenient to resort to the bootstrap method. The bootstrap method uses the set of the estimated residual vectors $\left\{\hat{u}_1, \hat{u}_2, \dots, \hat{u}_T\right\}$ from which shocks are drawn with replacement.[6] So for every forecast period h and every run m, a number τ from the set $\left\{1, 2, \dots, T\right\}$ is chosen randomly, and the vector $\hat{u}_\tau$ is used as the vector of shocks $\hat{u}_h^m$. The bootstrap method preserves all moments of the empirical residuals like for example variance, skewness and correlations.

3.3. Asymmetric bootstrap

In certain situations, as discussed above, it might be preferable to adjust the moments of the empirical residuals in the stochastic simulations in a certain way. Information beyond those contained in the model might point to important changes in moments. For instance, rising political tensions in major OPEC countries or the expectation of a very active hurricane season in the Gulf of

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5 In principle one could also draw random values for $\hat{\theta}$. However, we abstract from parameter uncertainty in this work.

6 If the residuals have non-zero means, they are recentered prior to resampling as suggested by Berkowitz and Kilian (2000).

Mexico can be expected to increase the level and the volatility of the oil price. It can also be appropriate to model asymmetric shocks, for example due to upcoming elections which will be won by a liberal party with high probability or by a socialist party with low probability. In these cases the standard bootstrap method can be modified in order to incorporate judgement about future shocks.

Suppose that the forecaster wants to add judgement about mean and volatility of the shock z, where z denotes the i-th element of $\hat{u}_h^m$ ($z \equiv \hat{u}_{i,h}^m$), i.e. the shock to equation i for the forecast horizon h in the run m. This can of course be achieved by simply transforming the shock z according to

(25) $$\tilde{z} = a + bz \qquad b > 0$$

where a is the judgemental mean, b is the judgemental volatility factor for shock $\tilde{z}$, and z denotes the original shock. Note that these transformations do not change the correlations with other shocks. Of course, a and b can differ for each forecast horizon h and each equation i, but they are constant for every run m.

If the forecaster wants to incorporate judgement about asymmetric risk, this can be achieved by applying the transformation

(26) $$\tilde{z} = J|z| - (1-J)|z| \qquad \text{with } J = J(q < \omega)$$

where $J(q < \omega)$ is the indicator function that takes the value 1 if the condition $q < \omega$ is satisfied and 0 else; q is the realisation of a random variable which is uniformly distributed over the interval [0,1] and ω denotes the judgemental probability that the shock will be larger than zero.[7] So $J(q < \omega)$ equals 1 in approximately $100 \cdot \omega$ percent of the runs.

If the empirical residuals $\hat{u}_{i,t}$ from which the z's are drawn are normally distributed, equation (26) implies that $\tilde{z}$ has an AWN distribution, i.e. its density is given by (4). Note that with $\omega = 0.5$, $\tilde{z}$ is symmetrically distributed regardless of the symmetry properties of z.

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7 ω can differ for each forecast horizon h and each equation i, but it is constant for every run m.

If one intends to preserve the zero-mean property and the variance of the shocks, the formula

$$\tilde{z} = \sqrt{\frac{1-\omega}{\omega}}\ J|z| - \sqrt{\frac{\omega}{1-\omega}}\ (1-J)|z| \tag{27}$$

can be used, which in the case of normally distributed residuals yields the density (4') for $\tilde{z}$. However, in contrast to (4'), one does not have to assume a specific distribution for z. The mean of $\tilde{z}$ equals zero regardless of the distribution of the empirical residuals $\hat{u}_{i,t}$, since the expectation of $\tilde{z}$ is given by

$$\begin{aligned} E[\tilde{z}] &= E\left[\sqrt{\frac{1-\omega}{\omega}}\ J|z| - \sqrt{\frac{\omega}{1-\omega}}\ (1-J)|z|\right] \\ &= E\left[\sqrt{\frac{1-\omega}{\omega}}\omega|z| - \sqrt{\frac{\omega}{1-\omega}}\ (1-\omega)|z|\right] \\ &= 0 \end{aligned} \tag{28}$$

In order to investigate the variance of $\tilde{z}$, it is helpful to note that

$$E\left[J^2\right] = \omega, \quad E\left[J(1-J)\right] = 0, \quad E\left[(1-J)^2\right] = 1-\omega \tag{29}$$

hold. The variance of $\tilde{z}$ is given by

$$\begin{aligned} E\left[\tilde{z}^2\right] &= E\left[\left(\sqrt{\frac{1-\omega}{\omega}}\ J|z| - \sqrt{\frac{\omega}{1-\omega}}\ (1-J)|z|\right)^2\right] \\ &= E\left[\frac{1-\omega}{\omega}\ J^2|z|^2 - 2\ J(1-J) + \frac{\omega}{1-\omega}\ (1-J)^2|z|^2\right] \\ &= E\left[\frac{1-\omega}{\omega}\ \omega|z|^2 + \frac{\omega}{1-\omega}\ (1-\omega)|z|^2\right] \\ &= E\left[|z|^2\right] \end{aligned} \tag{30}$$

If z is symmetric, i.e. if f(z) = f(–z) where f(.) denotes the density function of z, then the variance of z equals the variance of $|z|$. This means that the variance of $\tilde{z}$ equals the variance of the empirical residuals if the empirical residuals are symmetrically distributed.

Of course, (26) and (27) can be combined with (25) to generate shocks with judgemental mean, volatility and asymmetry. In this case, one first uses (26) or (27) to obtain an asymmetric shock $\tilde{z}$ and then applies (25) to this shock in order to obtain an asymmetric shock with mean a and standard deviation bσ, where σ is the standard deviation of $\tilde{z}$.

The correlation of $\tilde{z}$ with all other shocks equals zero. Therefore, the proposed method for generating asymmetries is useful especially if empirical correlations are low in absolute value.[8] In principle, it is possible to modify the approach presented so that correlations can be preserved in many cases. In the Appendix, we show how one can generate $\tilde{z}$ according to (26) and impose correlations with another possibly asymmetric shock. However, this modification becomes very complicated if many shocks are involved. Furthermore, not all asymmetries can be reconciled with all correlations. For example, if two variables are supposed to be greater than zero in a large number of cases, i.e. if both variables have a high ω, this can be incompatible with a negative correlation between these variables.

If the model's residuals have a normal distribution, are independent, and the model F(.) is linear, the bootstrap approach yields uncertainty and risk assessments for the endogenous variables which are identical to those obtained analytically in the previous section with the class of AWN distributions.

4. Stochastic forecasts with the Bundesbank model

The Bundesbank model is a dynamic non-linear structural macroeconometric model for Germany containing about 180 variables of which about 40 are exogenous. The model has 50 behavioural equations. In order to use this model for stochastic forecasts, it is transformed in two ways. First, the exogenous variables are endogenized by specifying equations in which they depend on their own past values and possibly other formerly exogenous variables. The only variables remaining exogenous are dummies, trends and

8 This is the case for the Bundesbank model used in Section 4. In this model, more than 90 percent of the correlations between the residuals do not differ significantly from zero at a significance level of 5%.

tax rates. Second, autoregressive equations are specified for the residuals of all model equations, so that these original residuals become endogenous variables of the model, and the new residuals of the autoregressive equations are the model's residuals. This last step is convenient in order to obtain residuals which are free from autocorrelation.[9]

4.1. Forecast intervals

The stochastic simulations of the Bundesbank model are performed for the period from the first quarter of 2006 (henceforth written as 2006q1) to 2008q4. The residuals are drawn from the period 1992q1 to 2005q4. We conduct 10,000 simulations. Since the results will be compared to those of an asymmetric bootstrap simulation, we use symmetric residuals here. This is achieved by multiplying all residuals of a given run m with a constant λ determined by $\lambda = 2\ J(q < 0.5) - 1$, where again J(q < 0.5) is the indicator function that takes the value 1 if the condition q < 0.5 is satisfied and 0 else, and q is the realisation of a uniformly distributed random variable in the interval [0,1].

Chart 7 shows the resulting forecasts for the four-quarter growth rates of GDP and the consumption deflator. Confidence bands centred on the median and covering 90% of the forecast distributions are displayed. Each confidence band corresponds to a probability mass of 5%. The dotted line indicates the mean.

As one can see for both variables, the mean is almost indistinguishable from the median. This implies that there is at least no apparent sign of asymmetry for the variables under study.[10] The upward shift in the forecast for the consumption deflator in 2007 is caused by the increase in the VAT rate. This tax rate increase also leads to a downward shift of GDP growth in 2007.

[9] One of the reasons why the original residuals can be autocorrelated is the fact that the estimation samples can differ from the sample used for bootstrapping. Since every equation of the Bundesbank model is estimated separately, estimation samples can differ from each other. The sample used for bootstrapping is the largest common estimation sample.

[10] The results would be different for example for the growth rate of the energy component of the HICP which is strongly asymmetric due to large excise taxes on fuels.

Chart 7: Fan charts for growth rates of real GDP and consumption deflator – symmetric shocks

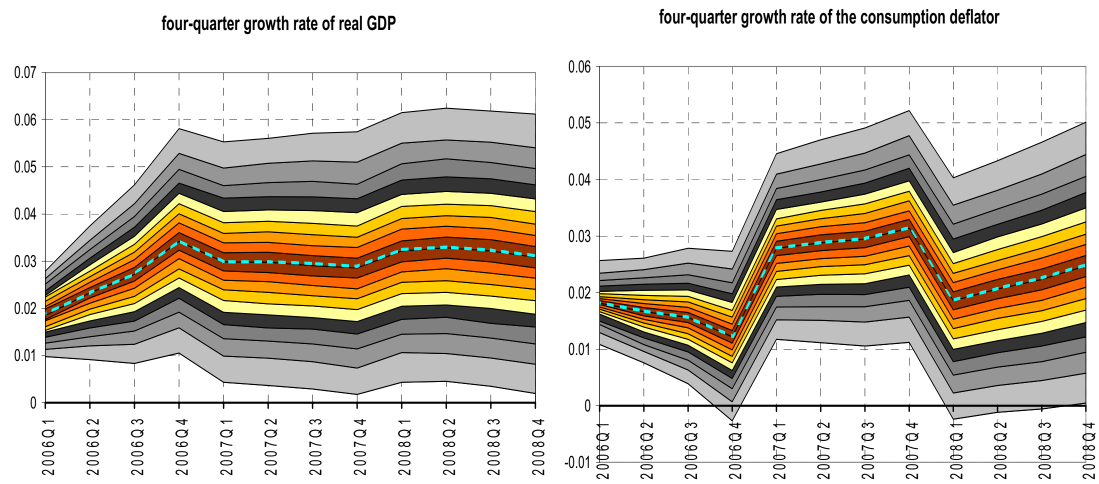


Table 5 shows moments of the stochastic GDP and consumption deflator forecasts. Skewness is presented for three different types of growth rates: four-quarter growth rates, quarterly growth rates and annual growth rates. For none of these growth rates, the coefficient of skewness exceeds 0.1 in absolute value. Thus, there are no indications of asymmetry for the growth rates of both variables under study.

Table 5: Moments of stochastic forecast with symmetric shocks

GDP

growth rates	moment	2006	2007	2008
four-quarter	mean	2.6	3.0	3.2
four-quarter	standard deviation	1.0	1.6	1.8
four-quarter	skewness	0.0	0.1	0.1
quarterly	skewness	0.0	0.0	0.1
annual	skewness	0.0	0.1	0.1

Consumption Deflator

growth rates	moment	2006	2007	2008
four-quarter	mean	1.6	2.9	2.2
four-quarter	standard deviation	0.6	1.1	1.4
four-quarter	skewness	0.0	0.1	0.1
quarterly	skewness	0.0	0.0	0.0
annual	skewness	0.0	0.1	0.1

For four-quarter and quarterly growth rates, the value for a specific year is calculated as the average quarterly moment observed in that year

4.2. Asymmetric bootstrap forecasts

In order to investigate the results of asymmetric shocks, we choose to assume strongly asymmetric shocks for almost all model equations, aiming at creating strongly positively skewed growth rates of GDP.

Using formula (27), we set ω to 0.3 for all equations of the expenditure components of GDP. Moreover, we set ω to 0.7 for all price equations. In almost all other equations, ω is set either to 0.3 or to 0.7, depending on the equation's initial impact on GDP. If a positive shock in the equation under study is supposed to increase GDP growth in the short-run, ω is set to 0.3. If such a shock decreases GDP growth in the short-run, ω is set to 0.7. Due to the structure of the model, where higher prices dampen demand, this approach can also be expected to generate negatively skewed growth rates of the consumption deflator.[11] The number of asymmetric shocks amounts to about 80.

Chart 8 shows the forecasts resulting from the asymmetric shocks. The mean now lies above the median for the GDP forecast and below the median for the consumer price inflation forecast. However, the differences between mean and median are very small. Asymmetries of the confidence bands are not too evident either. Only the outer two confidence bands appear to differ in size. While the lowest confidence band for GDP seems to be slightly smaller than the highest one, the lowest confidence band for the consumption deflator appears somewhat wider than the highest one.

[11] Of course, there are several equations where a shock with a positive impact on GDP growth also has a positive impact on inflation. The wage equation is an example, where a positive shock temporarily leads to higher GDP growth via higher demand, but also to higher prices via the production cost channel. With the approach chosen, such an equation would cause positively skewed inflation. However, one can expect the shocks to the price equations to dominate the skewness of inflation.

Chart 8: Fan charts for growth rates of real GDP and consumption deflator – asymmetric shocks

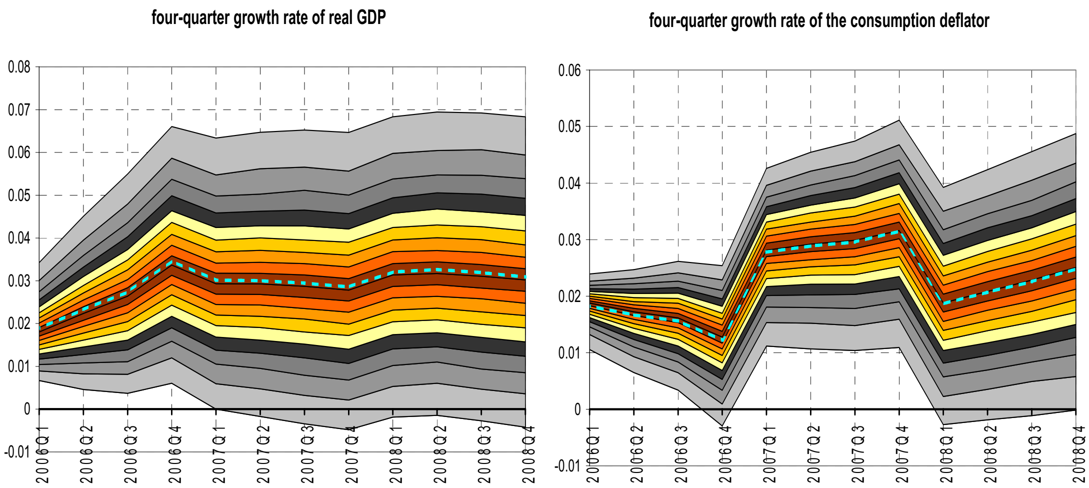


Table 6 displays moments of the stochastic forecasts with asymmetric shocks. Again, we consider three types of growth rates. It turns out that the means of the forecasts are not affected by the asymmetry of the shocks. Since the asymmetry of the shocks does not change their means, this result can be interpreted as another indication for the almost linear behaviour of real GDP and consumption deflator growth in the Bundesbank model. The standard deviations of consumer price inflation also remain unchanged with respect to the simulation with symmetric shocks. Those of GDP growth, however, increase by 20 to 40 %. The increase is strongest in the first year and weakest in the third. While the method used for generating asymmetric shocks leaves their standard deviations unchanged, the asymmetric shocks are assumed to be independent. In the case of real GDP growth, ignoring the interdependencies between the shocks apparently leads to higher volatility.

Table 6: Moments of stochastic forecast with asymmetric shocks

Real GDP

growth rates	moment	2006	2007	2008
four-quarter	mean	2.6	3.0	3.2
four-quarter	standard deviation	1.4	2.0	2.2
four-quarter	skewness	0.5	0.3	0.2
quarterly	skewness	0.5	0.4	0.4
annual	skewness	0.4	0.2	0.2

Consumption Deflator

growth rates	moment	2006	2007	2008
four-quarter	mean	1.6	2.9	2.2
four-quarter	standard deviation	0.6	1.1	1.4
four-quarter	skewness	-0.6	-0.2	-0.1
quarterly	skewness	-0.7	-0.5	-0.4
annual	skewness	-0.5	-0.2	-0.1

For four-quarter and quarterly growth rates, the value for a specific year is calculated as the average quarterly moment observed in that year

The skewness of the variables under study clearly differs from zero. While the skewness of real GDP growth is positive, consumer price inflation is negatively skewed. Evidently, the coefficients of skewness depend on the growth rates used. Annual and four-quarter growth rates exhibit less skewness in absolute value than quarterly growth rates. The reason is that the former growth rates strongly depend on the sum of four quarterly shocks, whereas quarterly growth rates are rather determined by the shocks of a specific quarter.[12] For the same reason, the degree of asymmetry decreases over time. In the third year, due to the dynamics of the model, shocks from all three years affect the simulation results, whereas in the first year, only the shocks of the first year matter.

In general, the skewness of aggregates in large interdependent models can always expected to be considerably smaller than the skewness of the shocks. This is related to the following property of skewness

[12] The sum of several independent equally asymmetric shocks is less asymmetric than the individual shocks, because the sum approaches normality as stated by the central limit theorem.

(31) $$s\left(\sum_{i=1}^{n} z_i\right) = \frac{\bar{s}}{\sqrt{n}} \qquad \text{with } \bar{s} = s(z_i) \text{ for } i \in \{1,2,...,n\}$$

where the z_i's are i.i.d. random variables. For the purpose of illustration, it might be helpful to consider an aggregate that is affected by the sum of 80 i.i.d. shocks which is the number of asymmetric shocks used in the stochastic simulation. The skewness of this aggregate would be about 9 times smaller in absolute value than the skewness of the shocks.

While the skewness of shocks with ω set to 0.3 equals 1.4, the skewness of GDP growth only equals 0.5 in the first year. Similarly, while the skewness of shocks with ω set to 0.7 equals –1.4, the skewness of consumer price inflation attains only –0.7 in the first year. These results indicate that a considerable amount of asymmetry is indeed absorbed in the aggregation and propagation of the shocks. However, given that the absolute skewness of the TPN distribution and of the SGN distribution, for example, cannot exceed 1 and 2, respectively,[13] the coefficients of skewness observed in the first year can still be regarded as pronounced at least in absolute terms.

For four-quarter and annual growth rates in the second and third year, the absolute values of skewness appear relatively small given the large asymmetry of the shocks.

In order to inspect the nature of the asymmetries in detail, it is interesting to look at the distribution for a single forecast horizon. Consider the four-quarter growth rates of real GDP and consumer price inflation in 2007q1. With the asymmetric stochastic simulations, the former has a skewness of 0.24, and the latter of –0.24. One would thus expect the densities of both variables to look broadly like mirror images of each other in this case. With the symmetric simulations, both coefficients of skewness equal 0.05.

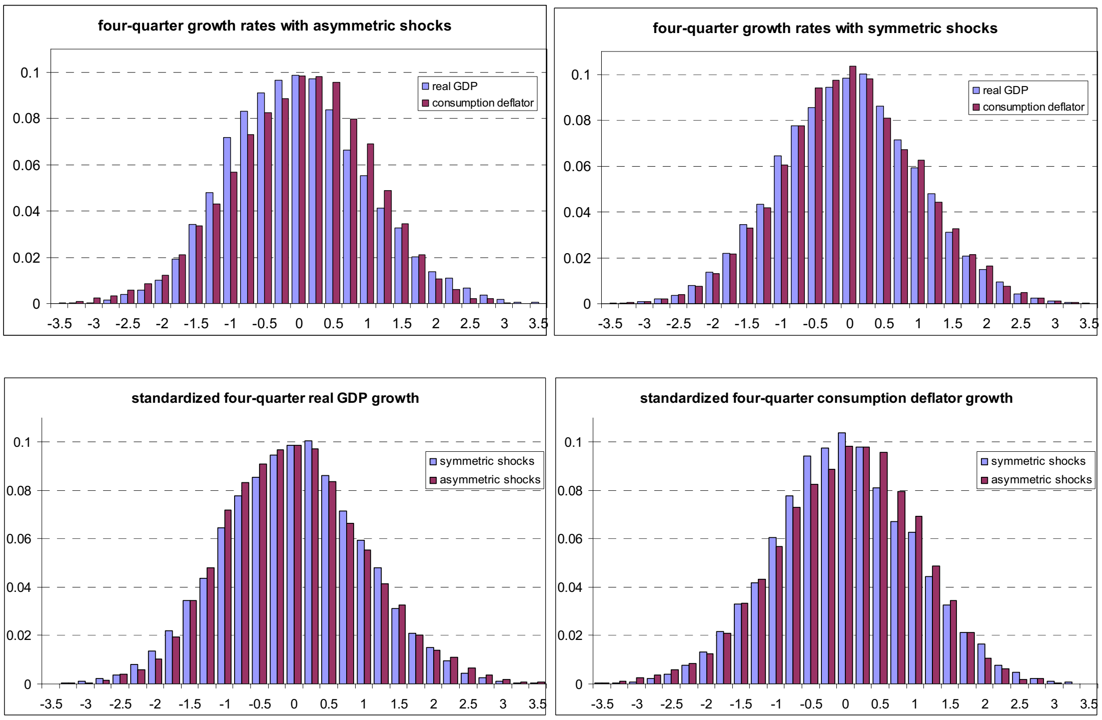
In Chart 9, histograms containing the mentioned growth rates of 2007q1 in their standardized form, i.e. after mean subtractions and division by their standard deviations, are displayed. The upper panels contain one histogram for the

[13] For a proof, see Novo and Pinheiro (2005).

simulation with asymmetric shocks and one histogram for the simulation with symmetric shocks. The lower panels contain the same data as the upper panels, but here one histogram contains only the growth rates of GDP and the other only the growth rates of the consumption deflator.

The histogram for the simulation with asymmetric shocks, shown in the upper left panel, shows that the upward skewed growth rate of GDP has relatively few moderately positive and extremely negative observations, but relatively many moderately negative and extremely positive observations. For the consumption deflator, indeed a mirror image emerges. The histogram in the upper right panel, displaying the results of the simulation with symmetric shocks, does not reveal obvious differences between both variables.

Chart 9: Histograms of 2007q1



Turning to the lower left panel for the comparison of the real GDP growth rates in the symmetric and asymmetric case, it is striking that differences between both cases seem to be very small. In the asymmetric case, slightly more extremely positive and moderately negative values and slightly less extremely negative and moderately positive values appear to be observed. In contrast to that, for consumer price inflation the differences between the asymmetric and the symmetric case are much larger, especially for moderately positive or negative observations. This might be explained by the fact that the skewness of real GDP growth in the asymmetric case increases only by about 0.2 with respect to the symmetric case, whereas the skewness of consumer price inflation decreases by about 0.3.

In any case, it seems fair to say that the strong asymmetries of the shocks do not yield strongly asymmetric four-quarter growth rates in the fifth quarter after the beginning of the forecast. Since in subsequent periods, asymmetries of the four-quarter growth rates are generally even less pronounced, it can be concluded that when focusing on four-quarter growth rates of real GDP and the consumption deflator, asymmetries are of minor importance in the medium to long term, even if shocks are strongly asymmetric. The interdependent structure and transmission mechanisms of the model appear to level out asymmetries to a large extent.

5. Conclusions

In this work, we have discussed a parametric and a non-parametric method to quantify risk and uncertainty in macroeconomic forecasts, mainly focussing on risk quantification. Both methods can be applied such that the incorporation of asymmetric risk does not affect the mean and variance of the input variables.

The parametric method is based on a class of asymmetrically weighted normal distributions. It was shown how this class relates to other asymmetric distributions and how to consistently aggregate the risks and uncertainty of input factors with asymmetrically weighted normal distributions in a linear model. The non-parametric method presented also relies on asymmetric weights but uses the bootstrap procedure to generate forecast intervals. Both approaches are closely related and give identical input factors if the model's residuals are normally distributed. If, in addition, the model's residuals are independent and the model is linear, both approaches yield identical uncertainty and risk assessments.

The asymmetric bootstrap is used to generate stochastic forecasts with the structural macroeconometric model of the Bundesbank for Germany. It turns out that asymmetries matter for real GDP growth and consumer price inflation mainly in the short run. In the short run or with quarterly growth rates, asymmetries of real GDP growth and consumer price inflation can be rather pronounced if shocks are strongly asymmetric. However, the propagation mechanisms of the model absorb a substantial share of the shocks' asymmetries, so that the endogenous variables considered are far less asymmetric than the shocks. In the medium to long run, asymmetries tend to be smoothed out at least if four-quarter and annual growth rates are considered.

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Appendix

Asymmetric shocks generated according to (27), i.e. according to

$$\tilde{z} = \sqrt{\frac{1-\omega}{\omega}} J|z| - \sqrt{\frac{\omega}{1-\omega}}(1-J)|z| \qquad \text{with } J = J(q < \omega)$$

can be correlated by using the same uniformly distributed random variable q for their construction.

Suppose that we have two asymmetric shocks

$$\tilde{z}_1 = \sqrt{\frac{1-\omega_1}{\omega_1}} J_1|z_1| - \sqrt{\frac{\omega_1}{1-\omega_1}}(1-J_1)|z_1| \qquad \text{with } J_1 = J_1(q, \omega_1, \mathrm{Cov}(z_1, z_2))$$

$$\tilde{z}_2 = \sqrt{\frac{1-\omega_2}{\omega_2}} J_2|z_2| - \sqrt{\frac{\omega_2}{1-\omega_2}}(1-J_2)|z_2| \qquad \text{with } J_2 = J_2(q, \omega_2, \mathrm{Cov}(z_1, z_2))$$

where $\mathrm{Cov}(z_1,z_2)$ denotes the covariance of z_1 and z_2. The question now is how to construct J_1 and J_2, so that $\tilde{z}_1$ and $\tilde{z}_2$ also have a covariance equal to $\mathrm{Cov}(z_1,z_2)$.

In order to achieve this, consider the following table containing joint probabilities and the unconditional probabilities of the indicator functions J_1 and J_2.

	$J_1 = 1$	$J_1 = 0$	uncond. prob.
$J_2 = 1$	$\omega_1 \cdot \omega_2 + \alpha$	$(1-\omega_1) \cdot \omega_2 - \alpha$	ω_2
$J_2 = 0$	$\omega_1 \cdot (1-\omega_2) - \alpha$	$(1-\omega_1) \cdot (1-\omega_2) + \alpha$	$1-\omega_2$
uncond. prob.	ω_1	$1-\omega_1$	

For example, the joint probability of $J_1 = 1$ and $J_2 = 1$ equals $\omega_1 \cdot \omega_2 + \alpha$. The unconditional probabilities are independent of the parameter α.

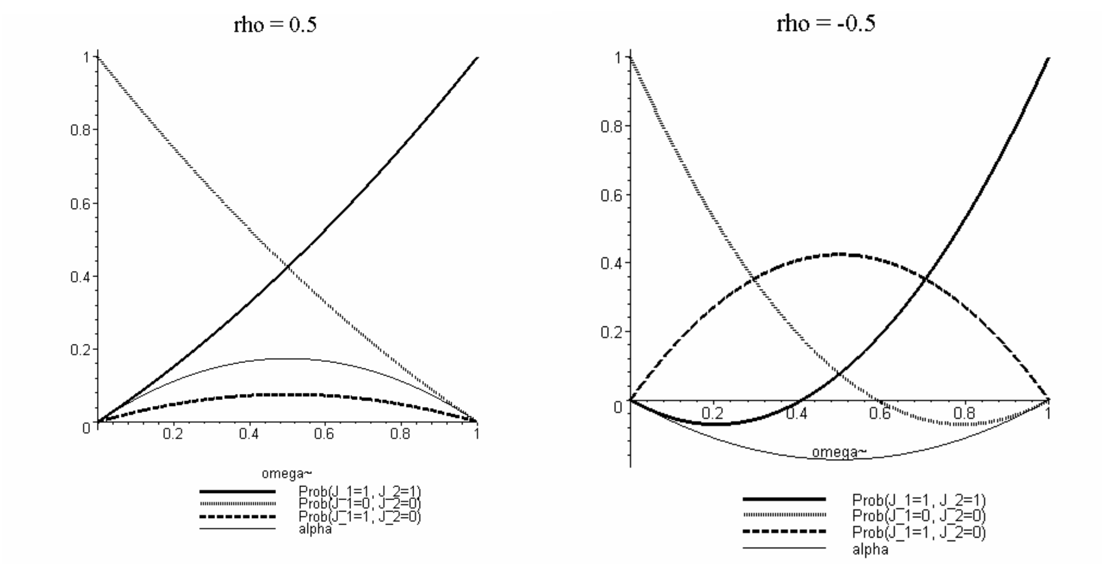
From this setup, it follows that the covariance of $\tilde{z}_1$ and $\tilde{z}_2$, denoted as $\mathrm{Cov}(\tilde{z}_1, \tilde{z}_2)$ is given by

(A.1) $\text{Cov}(\tilde{z}_1, \tilde{z}_2) = \dfrac{\alpha \cdot E[|z_1 z_2|]}{\sqrt{\omega_1(1-\omega_1)} \cdot \sqrt{\omega_2(1-\omega_2)}}$.

Given a joint distribution of z_1 and z_2 and probabilities ω_1 and ω_2, one can thus try to set α in such a way that Cov($\tilde{z}_1, \tilde{z}_2$) equals Cov(z_1,z_2).

In order to inspect under which conditions this approach works, it is helpful to set ω_1 equal to ω_2 and to use a bivariate standard normal distribution with correlation coefficient ρ for z_1 and z_2. Since the standard deviations of z_1 and z_2 equal 1, the correlation is equal to the covariance. Chart A shows the joint probabilities of $J_1 = 1$ and $J_2 = 1$, $J_1 = 0$ and $J_2 = 0$, and $J_1 = 1$ and $J_2 = 0$. The latter is identical to the joint probability of $J_1 = 0$ and $J_2 = 2$, because of $\omega_1 = \omega_2$. The chart also displays the value of α following from (A.1). Values are shown for $\rho = 0.5$ in the left chart and for $\rho = -0.5$ in the right chart; ω denotes the value of ω_1 and ω_2.

Chart A: Joint probabilities and α



Evidently, there is no problem to replicate the correlation of z_1 and z_2 if ρ equals 0.5. α must simply be set to values between 0 and 0.2, depending on the value of ω. However, if z_1 and z_2 are negatively correlated, only for certain values of ω the correlation of z_1 and z_2 can be replicated for $\tilde{z}_1$ and $\tilde{z}_2$. For ω smaller than

0.4 or larger than 0.6, the joint probabilities of $J_1 = 1$ and $J_2 = 1$, and $J_1 = 0$ and $J_2 = 0$ implied by the value of α become negative. Experimenting with other values of ρ leads to the conclusion that the more negatively z_1 and z_2 are correlated, the closer ω has to be to 0.5 in order to be able to replicate this negative correlation with $\tilde{z}_1$ and $\tilde{z}_2$. If ω_1 and ω_2 are allowed to differ from each other and z_1 and z_2 are positively correlated, this can lead to the same problem, especially if the correlation of z_1 and z_2 as well as the difference between ω_1 and ω_2 are large.

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