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Can capacity constraints explain asymmetries of the business cycle?

Malte Knüppel

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1
Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank,
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Abstract:

In this paper, we investigate the ability of a modified RBC model to reproduce asymmetries observed for macroeconomic variables over the business cycle. In order to replicate the empirical skewness of major U.S. macroeconomic variables, we introduce a capacity constraint into an otherwise prototypical RBC model. This constraint emerges due to the assumption of kinked marginal costs of utilization, where the kink is located at a utilization rate of 100 percent. We find that a model with a suitably calibrated cost function reproduces the empirical coefficients of skewness remarkably well.

Keywords: Capacity utilization, capacity constraints, asymmetry, RBC model

JEL-Classification: E32

Non-technical summary

The analysis of business cycles has a long history in economic research. Already in the early days of this research area, many economists were of the opinion that the business cycle phases – expansions and recessions – are not simply mirror images of each other. These phases were considered to differ, for example, with respect to duration and shape. If such asymmetries exist, the underlying reasons might have important consequences for our understanding of the economy and the effects of economic policy measures.

Most modern business cycle models are linearized models, implying that, for example, the reaction to a shock is independent of the current business cycle phase. This linearity also causes positive and negative shocks of identical size in absolute terms to have opposite effects which are exactly equal in absolute terms. Actually, if the shocks hitting such a model economy are symmetrically distributed, none of the model's variables can exhibit asymmetry. However, empirical research has documented many cases of asymmetry in macroeconomic time series, thereby challenging the appropriateness of linearized business cycle models.

In the recent economic literature, only few attempts have been made to investigate non-linear business cycle models with respect to their ability to reproduce empirical asymmetries. With our study, we contribute to fill this gap. We concentrate on a certain type of asymmetry, namely skewness. Skewness is present in several major U.S. macroeconomic time series, as we document in the study. We set up a prototypical business cycle model augmented with a capacity constraint in order to reproduce the empirical coefficients of skewness. The capacity constraint is motivated by the assumption that it is very costly to overuse capital, because this results in very high capital depreciation. In linearized business cycle models, the elasticity of marginal utilization costs with respect to the utilization rate is independent of the utilization rate. In our model, this elasticity depends on whether the utilization rate is smaller or larger than 100%. If the utilization rate equals 100%, a further increase becomes extremely costly, because capital then depreciates at a very high rate.

We find that our model reproduces the empirical coefficients of skewness remarkably well. Moreover, the introduction of the capacity constraint hardly affects the ability of the prototypical business cycle model to mimic empirical volatilities and comovements between the time series under study. We therefore conclude that the existence of capacity constraints gives a plausible explanation for the asymmetries observed in U.S. macroeconomic variables. This would imply that, for example, economic agents have more flexibility when reacting to policy measures if these measures are implemented during recessions, because during expansions, capacity constraints can be binding.

Nicht-technische Zusammenfassung

Die Analyse von Konjunkturzyklen ist bereits seit Langem Bestandteil der Wirtschaftsforschung. Schon zu Beginn der Konjunkturforschung waren viele Ökonomen der Auffassung, dass die Phasen des Konjunkturzyklus – Expansion und Rezession – nicht einfach Spiegelbilder ihres jeweiligen Widerparts sind. Stattdessen wurde angenommen, dass sich diese Phasen zum Beispiel im Hinblick auf Dauer und Gestalt voneinander unterscheiden. Falls solche Asymmetrien vorliegen, könnten die zugrunde liegenden Ursachen wichtige Implikationen für unser Verständnis des Wirtschaftsgeschehens und der Auswirkungen wirtschaftspolitischer Maßnahmen besitzen.

Die meisten neueren Konjunkturmodelle sind linearisierte Modelle. Dies bedeutet zum Beispiel, dass die Reaktionen auf Schocks unabhängig von der aktuellen Phase des Konjunkturzyklus sind. Diese Linearität führt auch dazu, dass betragsmäßig gleich große positive und negative Schocks entgegen gesetzte Effekte besitzen, die wiederum betragsmäßig genau gleich groß sind. Wenn die Schocks, denen ein linearisiertes Konjunkturmodell ausgesetzt ist, symmetrisch sind, so kann keine der Modellvariablen Asymmetrien aufweisen. Die empirische Wirtschaftsforschung hat jedoch viele Fälle von Asymmetrien makroökonomischer Zeitreihen belegt, wodurch die Eignung linearisierter Konjunkturmodelle zur Konjunkturanalyse in Frage gestellt wird.

In der neueren volkswirtschaftlichen Literatur sind nur wenige Versuche unternommen worden, nicht-lineare Konjunkturmodelle im Hinblick darauf zu untersuchen, inwiefern sie in der Lage sind, empirische Asymmetrien zu reproduzieren. Im Rahmen unserer Untersuchung versuchen wir, dazu beizutragen, diese Lücke zu füllen. Wir konzentrieren uns dabei auf eine bestimmte Art von Asymmetrie, nämlich die Schiefe. Schiefe liegt bei mehreren wichtigen makroökonomischen Zeitreihen der USA vor, wie wir in unserer Studie zeigen. Wir stellen ein um die Möglichkeit des Auftretens von Kapazitätsbeschränkungen erweitertes prototypisches Konjunkturmodell auf, um die empirischen Schiefekoeffizienten zu reproduzieren. Die Kapazitätsbeschränkungen entstehen im Modell auf Grund der Annahme, das es sehr kostspielig ist, Kapital übermäßig zu beanspruchen, da eine solche

Beanspruchung zu einem äußerst hohen Kapitalverschleiß führt. In linearisierten Konjunkturmodellen ist die Elastizität der marginalen Auslastungskosten in Bezug auf den Aulastungsgrad unabhängig vom Auslastungsgrad selbst. In unserem Modell hängt diese Elastizität davon ab, ob der Auslastungsgrad niedriger oder höher als 100% ist. Wenn der Auslastungsgrad bei 100% liegt, so wird eine weitere Steigerung äußerst kostspielig, da der Kapitalstock dann sehr schnell verschleißt.

Es zeigt sich, dass unser Modell die empirischen Schiefekoeffizienten bemerkenswert gut reproduzieren kann. Zudem wirkt sich die Einführung der Kapazitätsbeschränkungen kaum auf die Fähigkeit des prototypischen Konjunkturmodells aus, empirische Volatilitäten und dynamische Beziehungen zwischen den einzelnen untersuchten Zeitreihen nachzubilden. Wir kommen daher zu der Schlussfolgerung, dass das Vorliegen von Kapazitätsbeschränkungen eine plausible Erklärung für die in makroökonomischen Zeitreihen der USA auftretenden Asymmetrien darstellt. Dies würde zum Beispiel implizieren, dass Wirtschaftsakteure bei ihrer Reaktion auf wirtschaftspolitische Maßnahmen mehr Flexibilität besitzen, wenn diese Maßnahmen während einer Rezession getroffen werden, da während einer Expansion Kapazitätsbeschränkungen vorliegen können.

Contents

Can Capacity Constraints Explain Asymmetries of the Business Cycle?[1]

1 Introduction

The notion that macroeconomic variables exhibit asymmetry over the business cycle has a long history in economics. The existence of such asymmetries was claimed early by Mitchell (1927, p. 290) who stated that "Business contractions appear to be a briefer and more violent process than business expansions". Also Keynes (1936, p. 314) observed that "The substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning-point when an upward is substituted for a downward tendency". Many results of empirical research point at the importance of asymmetries for macroeconomic variables, as for instance those reported in Goodwin (1993) for output measures and in Neftci (1984) for unemployment, although their importance is not undisputed.[2]

To the best of our knowledge, only relatively few attempts have been made to investigate dynamic stochastic general equilibrium models with respect to asymmetries and to compare the resulting asymmetries to those observed in the data. Nieuwerburgh & Veldkamp (2004) study a real business cycle (henceforth RBC) model where productivity follows a symmetric Markov-switching process whose state cannot be observed by economic agents. Due to the additive nature of the productivity shock in their model, the signal-extraction problem the agents face is characterized by a pro-cyclical signal-to-noise ratio, giving rise to several types of asymmetries. Hansen & Prescott (2005) consider an RBC model where there is an upper bound to the number of plants that can be operated.

[1]Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main, Germany. E-Mail: malte.knueppel@bundesbank.de.
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[2]See, e.g. Bai & Ng (2005) or DeLong & Summers (1986).

This upper bound is due to a minimum labor requirement per plant and the existence of immobile capital. This capital can be idle in recessions, so that capital's income share is pro-cyclical. Both studies yield satisfactory results concerning the reproduction of the empirical asymmetries investigated.

In this work, we investigate a reason for the existence of asymmetries that is similar to the one in Hansen & Prescott (2005), i.e. related to the utilization of capital. We study how the assumption of an upper bound for capital services affects the symmetry properties of the model's variables. However, this upper bound does not emerge due to the existence of two types of capital where the supply of one type is bounded from above as in Hansen & Prescott (2005). Instead, we investigate an upper bound for capital utilization which is motivated by a kink in the marginal costs of utilization. This kink relies on the assumption that the elasticity of marginal cost with respect to utilization jumps to a higher level once capacity utilization reaches 100 percent. In standard RBC models as e.g. described in King & Rebelo (1999), this elasticity is assumed to be constant, implying that no upper bound for utilization exists. We introduce the kinked marginal cost function into an otherwise prototypical RBC model with variable capacity utilization and compare the asymmetries emerging from this model to the asymmetries found in the data. We also study the impact of capacity constraints on second moments by investigating the differences between standard deviations and cross-correlations in models with and without capacity constraints.

The paper is set up as follows. In Section 2, we conduct an investigation of the asymmetries of a set of macroeconomic variables, where asymmetries are measured by third standardized moments, i.e. skewness. We also report results for second moments. In Section 3, we present and calibrate the model with capacity constraints. In Section 4, we report simulation results for the model and compare them to the empirical results. We also compare the simulation results to the outcome of models without capacity constraints. Moreover, we analyze the capacity constraint's impact on stochastic steady-state values, and we investigate the reasons for differing magnitudes of skewness among the model's variables. Finally, an extensive sensitivity analysis is performed in order to verify the robustness of the simulation results. Section 5 concludes.

2 Stylized Facts

Asymmetry in our study will be measured by the coefficient of skewness, i.e. by the standardized third moment.[3] A symmetrically distributed variable always has zero skewness. Thus, if non-zero skewness is present, the variable must have an asymmetric distribution.

If a time series has negative skewness, then there are often less observations below the mean than there are observations above the mean, and on average the former are larger in absolute value. In the context of a stochastic process with symmetric shocks, negative skewness can emerge if the effects of these shocks are dampened when the realizations of the process lie above its mean, or if the effects of these shocks are amplified when the realizations of the process lie below its mean.

2.1 The Data

Our study is based on post-war U.S. macroeconomic per capita data. The macroeconomic variables investigated here are output, consumption, investment, labor, capital, the real wage, labor productivity and total factor productivity. The data for each variable except capital cover the sample period from the first quarter of 1954 (henceforth denoted 1954:1) through 2002:2 and are seasonally adjusted except for the population series. The series are taken from the National Income and Product Accounts of the Bureau of Economic Analysis if not otherwise stated.

Consumption (henceforth denoted as C) is measured as the sum of real personal consumption expenditures for services and non-durable goods and real government consumption. Investment (X) equals the sum of real private consumption of durable goods, real

[3] It is understood that there are several other types of asymmetry that can be of interest, including steepness and sharpness as considered e.g. by Clements & Krolzig (2003). The type of asymmetry that is associated with skewness is sometimes labeled *deepness* in the literature, a term coined by Sichel (1993).

gross private domestic investment and real government investment. Output (Y) is measured by real gross domestic product (GDP). Labor (H) is total number of man-hours in non-agricultural establishments. This series is taken from the data set used in Ireland (2004). The real wage (W) is constructed as the ratio of compensation of employees to the product of labor with the consumer price index for all urban consumers, where the latter series comes from the Federal Reserve Economic Data database. Data for the capital stock (KA) are available on an annual basis only, and the sample considered ranges from 1954 to 2001.

All series mentioned except for the real wage are denoted in per capita terms which are obtained by dividing the series by the civilian non-institutional population. Many studies, as e.g. Ireland (2004) or King et al. (1988) directly use the civilian non-institutional population series provided by the Bureau of Labor Statistics. This series, however, has been revised several times and contains sharp jumps at the revision dates. For example, due to revisions, the civilian non-institutional population increased by more than 0.5% in the first quarters of 1972 and 1990, respectively. While the dimension of these jumps is small enough to pass unnoticed for volatile series like investment, they possess a considerable impact on more stable series like capital and, to a smaller extent, consumption. Moreover, these jumps might have negligible effects on second moments, but could affect third moments to a larger extent.[4] We therefore consider it necessary to smooth the population series before we divide macroeconomic variables by it. In order to do so, we apply the HP-filter and use the resulting trend as the population series.[5]

We consider two productivity series, labor productivity and total factor productivity. Labor productivity (LP) is defined as the ratio of GDP to labor. The measure for total factor productivity (TFP) we construct is based on three assumptions. We assume that output is produced by a Cobb-Douglas production function, that quarterly changes of the capital stock are approximately zero[6] and that the utilization of the capital stock

[4] When we speak of third moments, we always refer to third *standardized* moments, i.e. skewness.

[5] We set the smoothing parameter to the standard value of 14400 for the monthly population series. Then we construct quarterly values by taking averages of the resulting trend.

[6] With this assumption, we follow Cooley & Prescott (1995).

is constant over time. By virtue of these assumptions, total factor productivity can be computed as

$$\ln TFP_t = \ln Y_t - \alpha \ln \bar{K}_t - (1-\alpha) \ln H_t \tag{1}$$

where α is the elasticity of output with respect to capital, and the series $\ln \bar{K}_t$ is simply a linear trend. For the calculation, α is set to 1/3 which is an often-encountered value in the literature.[7]

In order to induce stationarity and to isolate fluctuations associated with business cycle frequencies, we apply the HP-filter with the smoothing parameter set to 1600 to the logarithm of all variables except capital. For the annual capital series, we use the common value of 100. All HP-filtered variables multiplied by 100 are displayed in Figure 1. The quarterly capital series (K) is constructed simply by inserting the annual value for every quarter of that year.

2.2 Third and Second Moments

The skewness of each variable as well as the standard deviation, the relative standard deviation with respect to GDP and the correlation with GDP are presented in Table 1. No correlation is displayed for capital since we report results for the annual capital stock KA.

Concerning the coefficients of skewness, we find that capital is the only variable having positive skewness with a value of about 0.1. The least skewed variables are consumption and the real wage with coefficients close to -0.1. GDP exhibits a coefficient around -0.4, and both productivity measures have coefficients of about -0.35. The most skewed variables are given by labor with a coefficient close to -0.5 and investment with a coefficient of almost -0.7.

It is interesting to investigate whether the mentioned coefficients are significantly different from zero in order to evaluate the importance of asymmetries for macroeconomic

[7] A short survey of the different possibilities to calculate α can be found in Christiano (1988). The value 1/3 is inter alia employed by King & Rebelo (1999).

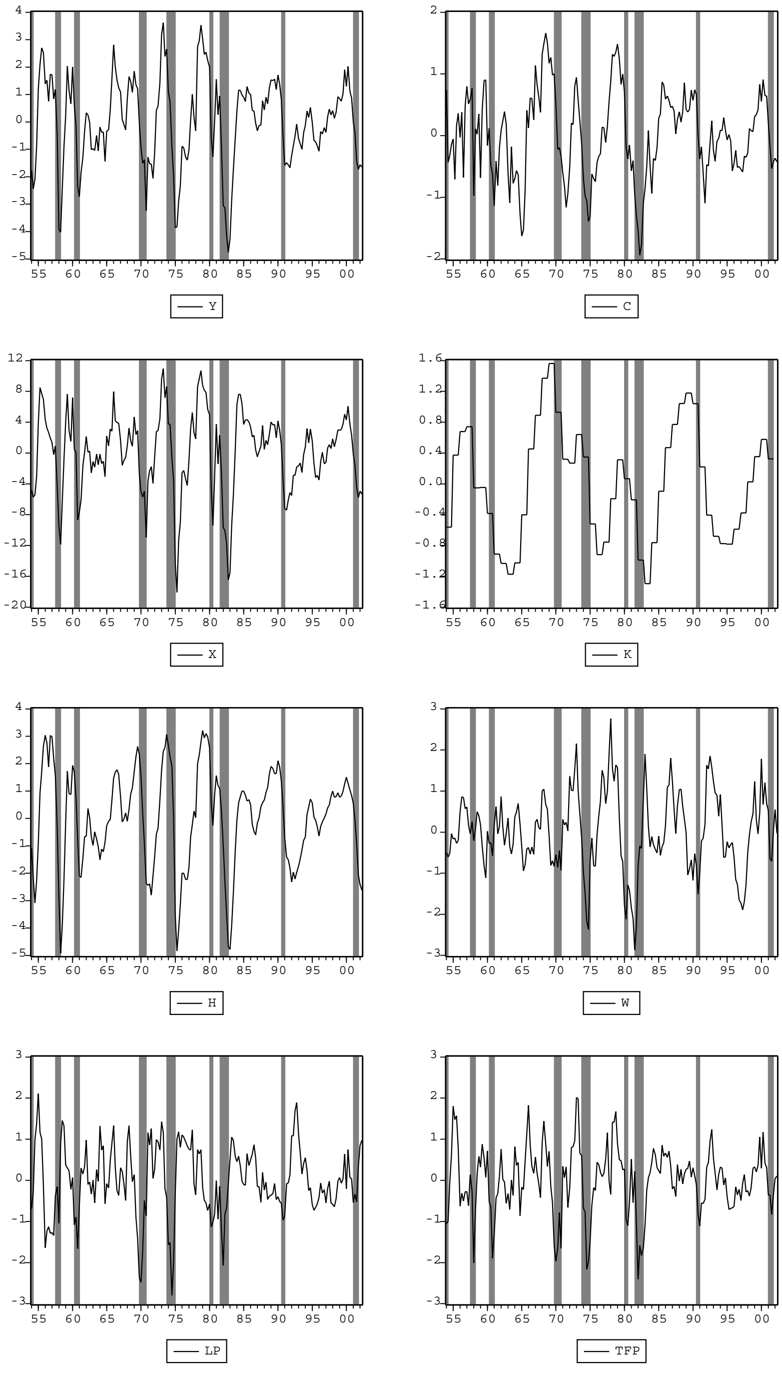

Figure 1: HP-filtered time series. Shaded areas denote recessions as dated by the NBER.

Table 1: Empirical third and second moments

	Y	C	X	KA	H	W	LP	TFP
skewness	-0.42	-0.11	-0.69	0.13	-0.49	-0.10	-0.35	-0.34
Gasser's test								
statistic	-1.74	-0.43	-2.95	0.31	-1.93	-0.42	-1.66	-1.61
p-value	0.08	0.67	0.00	0.76	0.05	0.67	0.10	0.11
triples test								
statistic	-1.92	-0.33	-2.98	0.35	-3.39	0.27	-0.40	-1.57
p-value	0.06	0.74	0.00	0.73	0.00	0.78	0.69	0.12
std. dev.	1.61	0.71	5.22	0.74	1.78	0.95	0.85	0.80
rel. std. dev.	1.00	0.44	3.23	0.46	1.11	0.59	0.53	0.50
corr. with Y	1.00	0.66	0.95		0.88	0.09	0.05	0.71

Note: 'std. dev.' denotes 'standard deviation', 'rel.' stands for 'relative'.

variables. In order to do so, we use two nonparametric tests for symmetry. One was proposed by Gasser (1975) and applied inter alia in Psaradakis & Sola (2003). The other test is the triples test proposed by Randles et al. (1980) and applied inter alia in Razzak (2001).[8] Whereas Gasser's test assumes a marginal normal distribution of the variable under study and is directly based on the coefficient of skewness, the triples test does not require distributional assumptions and is based on all the triples of the sample. The triples test can be expected to be more robust with respect to outliers.[9] However, nonparametric tests for symmetry typically suffer from the drawback of low power in small samples with strong serial correlation, as inter alia emphasized by Bai & Ng (2005). Therefore, the results of these tests should be considered with caution. That is, if these tests do not reject, this outcome might simply be due to their lack of power.

The test results are displayed in Table 1. According to both tests, there are no signs of significant asymmetry for capital, consumption, and the real wage. GDP is significantly asymmetric at the 10% significance level. At the same level, asymmetry of total factor productivity is not significant, but the corresponding p-values exceed 10% by a small

[8] For the triples test we use the GAUSS code provided by Weshah A Razzak.

[9] If the middle observation of a triple is closer to the smallest (largest) observation than it is to the largest (smallest), the value $1/3$ ($-1/3$) is assigned to the triple. If the middle observation of a triple is equally close to the largest and the smallest observation, the value 0 is assigned to the triple. These values from the set $\{-1/3, 0, 1/3\}$ are then used to construct the test statistic. Thus, the test statistic is based on ordinal data, making it more robust to the presence of outliers.

amount only. Symmetry of investment can be rejected even at the 1% significance level. For labor, there is significant asymmetry at a level of 10% according to Gasser's test and of 1% according to the triples test. For labor productivity, the tests deliver strongly contradicting results. While Gasser's test finds asymmetry at the 10% significance level, the p-value of the triples test is close to 0.7. One possible explanation for the contradicting results is the presence of outliers in the labor productivity series which could lead to an exaggerated value for the coefficient of skewness.

Thus, with the exception of labor productivity, the p-values of the tests broadly correspond to the magnitudes of the coefficients of skewness, and the tests indicate that those variables with absolute values of skewness larger than 0.4 are significantly asymmetric at least at the 10% level.

Concerning second moments, we find the well-known results concerning standard deviations and cross-correlations. With respect to volatility this means that the relative standard deviations of consumption and capital are lowest, investment has the largest relative standard deviation, the volatility of labor is of a similar magnitude as the volatility of GDP, and the real wage, labor productivity and total factor productivity are approximately half as volatile as GDP.

The values of the coefficients of skewness might depend to some degree on the sample chosen. While this is also true for second moments, this dependence can be expected to be more pronounced for third moments, because the variance of moment estimators increases with the order of the moment considered. As a kind of robustness check, we vary the sample under study and determine the coefficients of skewness for each sample. We begin with a sample starting in 1954:1 and ending in 1994:2. Then we increase the start and the end of the sample by one quarter. We do so until the resulting sample ends in 2002:2.[10] This implies that the 33 resulting samples are 32 quarters shorter than the original sample.[11] The results of these calculations are presented in Figure 2.

[10]For the quarterly capital series, the last three samples considered all end in 2001:4.

[11]The length of 32 quarters was chosen because, according to Burns & Mitchell (1946), this value corresponds to the maximum length of one business cycle, so that we can be confident to have excluded at least one cycle for the first and the last sample.

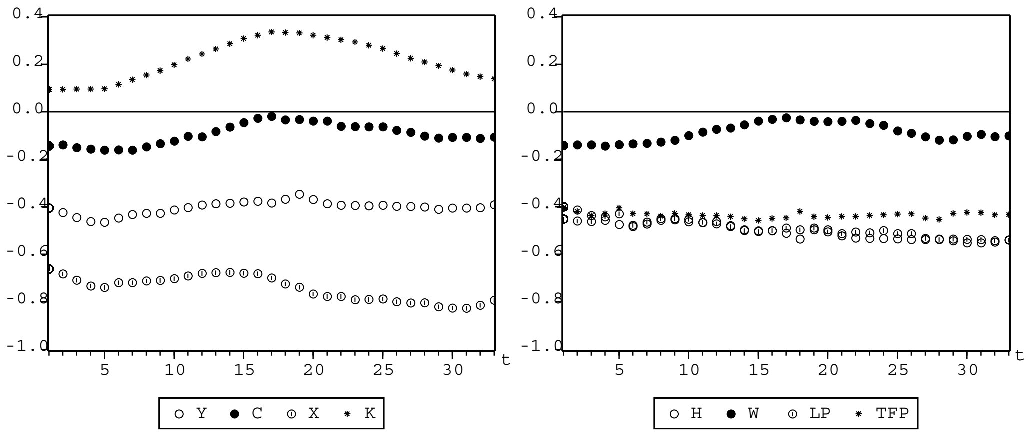

Figure 2: Skewness of variables in samples of 162 observations starting $t-1$ quarters after 1954:1

Obviously, most coefficients broadly equal their respective values reported in Table 1 for all samples. Only the positive skewness of capital increases strongly for certain samples, but this might be explained by the small number of distinct values for this series.[12] Therefore, the results concerning capital should be considered with caution. The skewness of both productivity series is moderately more pronounced in all short samples than in the full sample. Interestingly, the coefficients of skewness of consumption and the real wage are very close to each other in each sample.

Summing up, we have found evidence that macroeconomic variables exhibit different magnitudes of skewness. Capital shows weak positive skewness, whereas consumption and the real wage feature weak negative skewness. GDP, labor productivity, total factor productivity and labor all exhibit at least moderate negative skewness. Finally, investment is strongly negatively skewed.

3 The Model

In this section, we will consider a business cycle model with kinked marginal utilization costs. Except for the kinked cost function, the model is a standard RBC model. We will

[12] The capital series has only 48 distinct values.

present its setup, its calibration and briefly mention computational details concerning its solution.

3.1 Economic Environment

The economy under consideration is populated by many identical infinitely-lived households. Households are assumed to have separable logarithmic preferences over consumption and leisure. We further assume that labor is indivisible and that employment lotteries exist, as suggested by Hansen (1985) and Rogerson (1988). By doing so, we imply that real wages and consumption exhibit the same cyclical behavior. Here this modeling strategy is motivated by the similarity of skewness of these variables found in the previous section.

Because of the assumptions made, the momentary utility function of the stand-in representative household takes the form

$$\tilde{u}\left(c_t, h_t\right) = \ln c_t - \omega h_t, \tag{2}$$

where c_t denotes consumption and h_t denotes the ratio of time worked to total disposable time in period t. Thus, $1 - h_t$ equals the share of leisure time in period t.

The household ranks alternative streams of consumption and leisure according to the criterion function

$$\sum_{t=s}^{\infty} \beta^{t-s} E_s \left[\tilde{u}\left(c_t, h_t\right)\right] \qquad \text{with } 1 > \beta > 0 \tag{3}$$

where β is the discount rate and the operator E_s denotes the expectation conditional on information available at time $t = s$.

The production function is defined by

$$y_t = z_t \left(u_t k_t\right)^{\alpha} h_t^{1-\alpha} \qquad \text{with } 0 < \alpha < 1 \tag{4}$$

where z_t is a stationary random variable which permits temporary changes in total factor productivity (henceforth simply productivity), y_t denotes output, k_t is the stock of capital and u_t is the utilization rate of capital. Obviously, without further restrictions, the household would choose the utilization rate as large as possible. The standard method in order to rule out this possibility is the assumption of utilization costs which are commonly modeled as a convex increasing function of the utilization rate. Many authors as e.g. King & Rebelo (1999) choose the depreciation rate to be determined by this cost function, so that the depreciation rate increases with higher utilization. An alternative approach is used by Christiano et al. (2001) as well as Smets & Wouters (2003) who model utilization costs in terms of foregone output. We decide to pursue the former approach here, so that the resource constraint of the economy is simply given by

$$y_t = c_t + x_t \tag{5}$$

where x_t is gross investment.

The capital stock evolves according to

$$\nu k_{t+1} = (1 - [\delta + g(u_t)])\, k_t + x_t \tag{6}$$

where $\delta + g(u_t)$ is the depreciation rate of capital and $\ln(\nu)$ is the constant growth rate of labor augmenting technical progress. Thus, $g(u_t)$ is the stochastic part of the depreciation rate, and the function $g(u_t)\, k_t$ denotes the costs of utilization. These costs are assumed to be convex and increasing in u_t.

Concerning the exogenous process for z_t, we consider the first-order autoregressive process

$$\ln z_t = (1-\rho)\ln \bar{z} + \rho \ln z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \text{i.i.d. } \mathcal{N}\left(0, \sigma^2\right), \tag{7}$$

where $\bar{z}$ is constant and greater than zero and ε_t is Gaussian white noise.

The share of time spent working cannot exceed total time, so that the household fulfills the time-endowment constraint $h_t \leq 1$. In addition, none of the variables mentioned can become negative except for investment, and the transversality condition given by

$$\lim_{t\to\infty} \beta^t \frac{\partial \tilde{u}\left(c_t, h_t\right)}{\partial c_t} k_{t+1} = 0.$$

must be fulfilled.

Maximizing the criterion function subject to the constraints presented leads to the first-order conditions

$$\omega = \frac{1}{c_t}\left(1-\alpha\right)\frac{y_t}{h_t}, \tag{8}$$

$$\frac{1}{c_t} = \frac{\beta}{\nu} E_t \left[\frac{1}{c_{t+1}}\left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \left[\delta + g\left(u_{t+1}\right)\right]\right)\right] \tag{9}$$

and

$$\frac{\partial g\left(u_t\right)}{\partial u_t} k_t = \alpha \frac{y_t}{u_t} \tag{10}$$

The latter condition states that marginal costs of utilization must equal marginal returns.

3.2 The Non-Stochastic Steady State

The non-stochastic steady state of the economy is given by the values that the variables adopt if the logarithm of productivity equals its unconditional mean $\ln \bar{z}$ for all t with certainty. For utilization and its cost, we impose the non-stochastic steady-state values

$$\bar{u} = 1$$

and

$$g\left(\bar{u}\right) = 0,$$

thereby implying that the non-stochastic steady state of the model is identical to the non-stochastic steady state of an equivalent model with constant capital utilization.

3.3 Capacity Constraints and the Function of Utilization Costs

For an investigation of this economy, additional assumptions about the cost function $g(u_t)\,k_t$ are necessary. If the model is log-linearized in order to solve it, one does not have to specify a functional form for $g(u_t)$, but only needs to determine the elasticity of marginal costs with respect to utilization given by

$$\eta = \frac{u_t \frac{\partial^2 (g(u_t)k_t)}{\partial u_t^2}\Big|_{u_t=\bar{u}}}{\frac{\partial (g(u_t)k_t)}{\partial u_t}\Big|_{u_t=\bar{u}}}.$$

However, since we aim at a numerical solution, we have to rely on a specific functional form of $g(u_t)$ which we choose to be

$$g(u_t) = \alpha \frac{\bar{y}}{\bar{k}} \frac{1}{1+\eta} \left(u_t^{1+\eta} - 1\right) \qquad \text{with } \eta > 0 \tag{11}$$

where η must be positive in order to guarantee convexity of $g(u_t)$. Note that this function fulfills the requirement on $g(u_t)$ with respect to its value at the non-stochastic steady state. In addition, at the non-stochastic steady state with $\bar{u} = 1$, the first derivative of $g(u_t)$ is always equal to $\alpha \frac{\bar{y}}{\bar{k}}$, independently of the value of η. Therefore, we have that the first-order condition with respect to utilization (10) which here becomes

$$\alpha \frac{\bar{y}}{\bar{k}} u_t^{\eta} k_t = \alpha \frac{y_t}{u_t}, \tag{12}$$

is satisfied for all possible values of η at the non-stochastic steady state.

The first-order condition (12) implies that when η goes to infinity, the marginal costs of utilization also go to infinity if u_t exceeds one, and are zero if u_t is lower than one. Thus, in this case, the household always sets u_t equal to one and the model is identical to a model with fixed utilization. If, in contrast, η is close to zero, marginal costs hardly vary with utilization, and u_t exhibits large volatility.

The assumption of a constant elasticity of marginal costs with respect to utilization implies that the utilization rate can become infinitely large, so that even in the short-run, there is no upper bound to the supply of capital services. Thus, if in a certain period a positive shock to productivity occurs, utilization can always increase, independently of the size of the shock and the size of the capital stock. This assumption seems problematic, since there are physical limits to many kinds of capital services that cannot be exceeded. For example, machines employed in production cannot be used for more than 24 hours per day. Upper bounds of the services they can provide are obviously present. The same is true for most other kinds of capital, as for example land for agricultural production, where the amount of crop per unit of land cannot be increased to infinity.

These reasons lead us to impose a non-constant elasticity of marginal costs with respect to utilization. We assume that there are two possible values for this elasticity, and that they differ depending on the magnitude of the utilization rate with respect to its non-stochastic steady-state value. That is, we assume that $g(u_t)$ is described by the function

$$g(u_t) = \begin{cases} \alpha \frac{\bar{y}}{k} \frac{1}{1+\eta_1} \left(u_t^{1+\eta_1} - 1 \right) & \text{if } u_t > 1 \\ \alpha \frac{\bar{y}}{k} \frac{1}{1+\eta_2} \left(u_t^{1+\eta_2} - 1 \right) & \text{if } u_t \leq 1 \end{cases} . \tag{13}$$

This function is related to the capacity constraint mentioned above by the value of η_1. If η_1 goes to infinity, u_t will never exceed one. Since, as stated above, the first derivative of $g(u_t)$ at the steady state is independent of η, the function described by (13) is differentiable for all u_t.[13] In order to illustrate the possible behavior of the costs of utilization, we plot two functions $g(u_t)$ and two functions $\partial g(u_t)/\partial u_t$ in Figure 3. When k_t equals 1, these functions correspond to the costs and marginal costs of utilization, respectively. In each panel one function is characterized by a large increase of the elasticity with respect to utilization if the utilization rate exceeds 100%, while the elasticity used for the other

[13] The conditions for the maximization by value-function iteration which will be employed to solve the model do not require differentiability of the constraints, so that we could also have directly constrained u_t to be smaller or equal to one. However, we prefer the specification presented here, since it is more flexible and nests the case of u_t never being larger than one if η_1 approaches infinity.

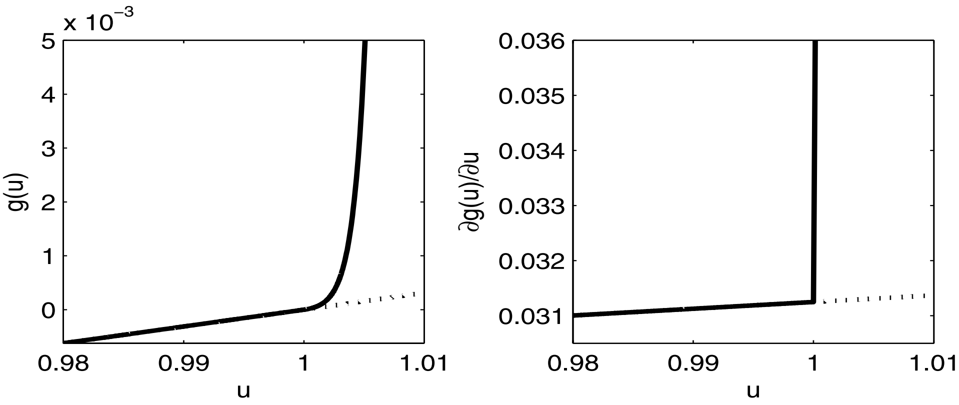

Figure 3: Costs of utilization (left panel) and marginal costs of utilization (right panel) with $\eta_2 = 0.39$ and $\eta_1 = 1000$ (solid lines), and with $\eta_1 = \eta_2 = 0.39$ (dotted lines)

function is constant.[14]

3.4 Calibration

Several parameters are calibrated according to the values reported in King & Rebelo (1999). These include the share of capital income α which is set to 1/3, the growth rate of the economy $\ln(\nu)$ which is set to 0.004, the share of time devoted to work at the non-stochastic steady state $\bar{h}$ set to 20% and the discount factor β determined by $\beta = \nu/(1+r)$ where r is the average quarterly real interest rate and equals $r = 0.065/4$. The quarterly depreciation rate at the non-stochastic steady state δ is set to 0.015 which is in line with the result reported in Stokey & Rebelo (1995). As mentioned in Section 3.3, utilization at the non-stochastic steady state $\bar{u}$ is set to 1. With these values, the capital to output ratio at the non-stochastic steady state equals 10.67, and the consumption to output ratio attains a value of 0.80. Finally, since the value of $\bar{z}$ neither affects the dynamics of the model nor the great ratios ($\bar{c}/\bar{y}$, $\bar{k}/\bar{y}$, $\bar{x}/\bar{y}$), but only the scale of the economy, we normalize it to 1.

The choice of the parameter values of the process for productivity (7) depends on the

[14]The parameter values and the value for the ratio $\bar{y}/\bar{k}$ chosen here correspond to those used later on.

Table 2: Calibrated parameters and non-stochastic steady-state values

α	β	δ	ν	ω	η_1	η_2	ρ	σ	$\bar{h}$
0.333	0.988	0.015	1.004	4.181	1000	0.39	0.985	0.00354	0.2

costs of utilization. We decide to set the persistence parameter ρ equal to 0.985. This choice is based on Table 5 in King & Rebelo (1999), according to which a value of 0.978 corresponds to the case of constant capacity utilization and a value of 0.989 corresponds to an almost costless variation of capacity utilization. So we have chosen an intermediate value for ρ that appears appropriate in the case of moderate marginal costs with respect to utilization. For η_1, we choose a value of 1000, since this value turns out to be large enough in order to be considered equivalent to a constraint that sets an upper limit to utilization. The values of η_2 and σ are then determined by the requirement that the model should replicate the standard deviation and skewness of output measured by GDP. We find that a value of η_2 equal to 0.39 and a value of σ equal to 0.00354 fulfill these requirements. Table 2 summarizes the calibrated parameter and steady-state values.

3.5 Solution

Since the model under study can be expected to exhibit pronounced asymmetries due to the constraint on capacity utilization, a solution by log-linearization would not be appropriate. In fact, any solution method imposing a smooth functional form on the decision rules of the household could be problematic, since the decision rules can be expected to be kinked at the point where the capacity constraint starts to bind. Therefore, we decide to solve the model by value-function iteration, as also done by Hansen & Prescott (2005).

The application of this approach requires k_t and z_t to be discrete-valued variables. In the setting of the maximization problem here, both variables are continuous. This problem is addressed by the common approach of transforming k_t and z_t into discrete-valued variables, i.e. by choosing grids that these variables lie on. The choice of the number of grid-points for k_t and z_t is subject to the trade-off between accurateness and computing time. While the choice of the number of grid-points for k_t is to some extent arbitrary, the range of this grid must be chosen in such a way that it contains the complete ergodic set, i.e. the set that k_t does not leave once it has entered it. In order for such a set to exist, z_t has to be bounded. Concerning the AR(1)-process for z_t, Tauchen (1986) proposes a discrete-valued approximation by an m-state Markov chain. Using a discrete-valued approximation evidently leads to boundedness of z_t. Following Hansen & Prescott (2005), we set m to 15 and choose the values attained by the Markov chain in such a way that they cover $b = \pm 2$ standard deviations of the process for productivity from $\ln(\bar{z})$. The grid for k_t consists of 1200 evenly spaced grid points. The values $\ln z_t$ can adopt, the corresponding states and the transition matrix of the Markov chain are given in Appendix A.

4 Results

4.1 Summary Statistics

In order to find the moments implied by the model economy, we run 50000 simulations, each one yielding 2194 observations. We disregard the first 2000 observations, so that 194 observations remain. For every variable except the depreciation rate and utilization, we take logarithms and apply the HP-filter. In addition to the variables contained in the model, we also report results for total factor productivity measured as if utilization and capital were constant and as if utilization was equal to one, hence as

$$tfp_t = \frac{y_t}{\bar{k}^{\alpha} h_t^{1-\alpha}}.$$

This variable corresponds to the empirical measure of total factor productivity given by (1). Henceforth, in the context of the model economy we will refer to tfp_t as total factor productivity.[15] The variable labor productivity is constructed as $lp_t = y_t/h_t$. This defini-

[15] Remember that z_t is simply labeled productivity.

tion corresponds to the definition used for the calculation of empirical labor productivity. $2/3lp_t$ of course simply equals the real wage w_t. Concerning capital, we construct an annual variable ka_t with $t = 4, 8, \ldots$ by considering only every fourth value of the quarterly capital series k_t in order to make the results from the model comparable to the empirical results. The annual capital series is calculated prior to the application of the HP-filter.

Coefficients of skewness, standard deviations, relative standard deviations and correlations generated by the model economy are displayed in Table 3. For convenience, we again show the respective values found in the empirical data. Standard deviations are multiplied by 100.

Table 3: Third and second moments

	y	c	x	ka	h	w	lp	tfp
skewness								
model	-0.42	-0.12	-0.68	0.07	-0.50	-0.12	-0.12	-0.31
data	*-0.42*	*-0.11*	*-0.69*	*0.13*	*-0.49*	*-0.10*	*-0.35*	*-0.34*
std. dev.								
model	1.61	0.42	6.95	0.70	1.25	0.42	0.42	0.80
data	*1.61*	*0.71*	*5.22*	*0.74*	*1.78*	*0.95*	*0.85*	*0.80*
rel. std. dev.								
model	1.00	0.26	4.31	0.43	0.77	0.26	0.26	0.49
data	*1.00*	*0.44*	*3.23*	*0.46*	*1.11*	*0.59*	*0.53*	*0.50*
corr. with y								
model	1.00	0.90	0.98		0.98	0.90	0.90	0.98
data	*1.00*	*0.66*	*0.95*		*0.88*	*0.09*	*0.05*	*0.71*

Note: 'std. dev.' denotes 'standard deviation', 'rel.' stands for 'relative'.

Concerning the coefficients of skewness, the results of the model correspond remarkably well to the empirical results for most variables. The differences of the coefficients of skewness between those from the model and those from the empirical data do not exceed 0.03 for the variables consumption, investment, the real wage, labor, and total factor productivity. Moreover, it turns out that capital is the only positively skewed variable of the model, attaining a coefficient of skewness only 0.06 smaller than its empirical counterpart. The skewness of labor productivity in the model is considerably less pronounced than in the data. However, as mentioned above, the empirical skewness of labor productivity might be overstated by outliers. The skewness of output in the model matches the empirical skewness by construction, i.e. due to the choice of η_2.

Concerning second moments, many variables from the model are less volatile than their empirical counterparts, above all consumption, and the associated variables real wage and labor productivity. In contrast to that, the empirical standard deviations of investment, capital and total factor productivity are fairly well reproduced by the model. The strongest deviations of simulated cross-correlations from their empirical counterparts are observed for the real wage and labor productivity.

In order to investigate to what extent the coefficients of skewness of the model are caused by the capacity constraint, we simulate two models without such a constraint. In one model, we set η_2 to 1000, so that utilization becomes virtually constant. The parameter ρ is set to 0.98 and the parameter σ to 0.00634.[16] All remaining parameters are unchanged. We will refer to this model as model with constant utilization. In the other model, we set η_1 to 0.39, so that utilization can be varied, but the marginal costs of utilization are not kinked at $u_t = 1$, so that utilization can be expected to be approximately symmetric. In this model, the parameter ρ is set to 0.988 and the parameter σ to 0.00292.[17] This model will be labeled as model with symmetric utilization. Results of the simulations with these two models and with the model with capacity constraint are presented in Table 4. Obviously, in the models without capacity constraint the skewness of all variables except for investment is close to zero. The skewness of investment equals about -0.18 in the model with constant utilization and about -0.21 in the model with symmetric utilization. To the best of our knowledge, the skewness of investment in prototypical RBC models has not been documented in the literature yet and seems at least noteworthy.

Hence, for the model with capacity constraint we can conclude that it is indeed only the

[16] As mentioned above, a value for ρ close to 0.98 is suggested in King & Rebelo (1999) for an economy with constant utilization. The value for σ was chosen in order to replicate the standard deviation of GDP.

[17] Again, a value for ρ equal to about 0.988 is suggested in King & Rebelo (1999) for an economy with highly variable utilization, and the value for σ was chosen in order to replicate the standard deviation of GDP.

Table 4: Second and third moments of all models

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z
skewness										
cap. constr.	-0.42	-0.12	-0.68	0.07	-0.50	-0.31	-1.22	-1.21	0.04	0.00
const. util.	-0.01	0.01	-0.18	-0.01	-0.02	0.00	-0.02	0.05	-0.01	0.00
sym. util.	-0.01	0.00	-0.21	-0.01	-0.02	-0.01	0.03	0.05	-0.01	0.00
std. dev.										
cap. constr.	1.61	0.42	6.95	0.70	1.25	0.80	1.95	0.06	0.30	0.49
const. util.	1.61	0.55	6.12	0.92	1.13	0.88	0.00	0.00	0.40	0.87
sym. util.	1.61	0.41	6.59	0.69	1.24	0.80	2.64	0.08	0.30	0.41
rel. std. dev.										
cap. constr.	1.00	0.26	4.31	0.43	0.77	0.49	1.21	0.04	0.19	0.31
const. util.	1.00	0.34	3.80	0.57	0.70	0.55	0.00	0.00	0.25	0.54
sym. util.	1.00	0.25	4.09	0.43	0.77	0.50	1.64	0.05	0.18	0.26
corr. with y										
cap. constr.	1.00	0.90	0.98		0.98	0.98	0.61	0.61	-0.04	0.93
const. util.	1.00	0.91	0.98		0.98	0.98	0.62	0.62	0.05	0.99
sym. util.	1.00	0.93	0.99		0.99	0.99	0.56	0.56	-0.06	0.98

Note: 'std. dev.' denotes standard deviation, 'rel.' stands for relative, 'cap. constr.' stands for the model with capacity constraint, 'const. util.' for the model with constant utilization, and 'sym. util.' for the model with symmetric utilization.

capacity constraint which causes the skewness of all variables except investment and which strongly amplifies the negative skewness of investment. The most pronounced negative skewness of the model with capacity constraint is observed for the variables utilization and the depreciation rate[18] with coefficients of about -1.2. This result is not surprising, since, in contrast to the other variables, these two variables can virtually not exceed their non-stochastic steady-state values.

Second moments are hardly affected by the introduction of a capacity constraint. They rather depend on the possibility of varying capital utilization. Therefore, one often finds noticeable differences between the model with constant utilization and the models with varying utilization whereas, in many cases, the model with capacity constraint produces similar results as the model with symmetric utilization.

[18] The depreciation rate is given by $\delta + g(u_t)$. Since δ is constant, the central moments of the depreciation rate like skewness, standard deviation and cross-correlation are identical to the central moments of $g(u_t)$.

Table 5: Stochastic steady-state and mean values

	nsss	cap. constr.		const. util.		sym. util.	
		sss	mean	sss	mean	sss	mean
	level	deviations from nsss in %					
y	0.65	0.1	-0.2	0.0	0.1	0.0	0.1
c	0.52	0.2	0.1	0.0	0.1	0.0	0.1
x	0.13	-0.1	-1.3	0.0	0.2	0.0	0.0
k	6.97	1.5	1.6	0.0	0.2	0.0	0.3
h	0.20	-0.1	-0.3	0.0	0.0	0.0	0.0
$w, 2/3lp$	2.18	0.2	0.1	0.0	0.1	0.0	0.1
tfp	1.00	0.2	0.0	0.0	0.1	0.0	0.1
u	1.00	-1.0	-1.7	0.0	0.0	0.0	-0.1
$100(\delta + g(u))$	1.50	-2.0	-3.6	0.0	0.0	-0.1	-0.1

Note: 'cap. constr.' stands for the model with capacity constraint, 'const. util.' for the model with constant utilization, and 'sym. util.' for the model with symmetric utilization. 'nsss' denotes the non-stochastic steady state and 'sss' the stochastic steady state.

An interesting feature of the economy with capacity constraint is given by its behavior at the stochastic steady state. In contrast to the non-stochastic steady state, the stochastic steady state is characterized by uncertainty about future values of productivity. That is, the stochastic steady state is reached when productivity is always at its steady-state level, but the household believes that productivity is uncertain and evolves according to (7). Due to uncertainty about future income, precautionary saving in our models leads to a value of the capital stock that is larger than its non-stochastic steady-state value. This increase is unrelated to the existence of a capacity constraint. However, due to the capacity constraint, the stochastic steady-state value of capital further increases. This effect occurs because the household loses the possibility to adjust its production to new levels via the variation of utilization as soon the capacity constraint is reached. Therefore, the household seeks to prevent the constraint from binding, and it does so by accumulating a larger capital stock. In order to assess the quantitative importance of the additional capital accumulation, we report the stochastic steady-state values and the mean values over all simulations for all models as well as the non-stochastic steady-state values of the models' variables in Table 5.

The values at the non-stochastic steady state are identical for all models. The values at the stochastic steady state can be regarded as a kind of median, since in 50% of all cases productivity is larger than at its stochastic steady-state value.[19] As a consequence, the other variables can be expected to lie above or below their stochastic steady-state values in about 50% of all cases.

In the models without capacity constraint, the non-stochastic steady-state values of all variables are very close to their counterparts at the stochastic steady state. The increase of the capital stock due to precautionary saving is below 0.05% in both cases. The existence of a capacity constraint, however, causes an increase of the capital stock by 1.5%. Thus, the additional capital accumulation due to the capacity constraint is much larger than the additional capital accumulation due to the existence of uncertainty.

In contrast to the stochastic steady-state values, the means of the capital stock in the models without capacity constraint moderately exceed the non-stochastic steady-state value, namely by 0.2 to 0.3%. Given that stochastic steady-state values can be thought of as median values and given that the skewness of capital is close to zero according to the results in Table 4, this result might appear puzzling at first sight. However, the reason is simply that in Table 5, we consider levels, while in Table 4 we consider log-levels of all variables except for the depreciation rate and utilization. Since the logarithm is a convex function, variables in levels tend to have larger values of skewness than variables in log-levels. However, the effect of the log function on skewness appears to be rather small for the variables considered here. In the model with capacity constraint, the mean values of all variables that exhibit negative skewness according to the results in Table 4 are lower than their respective values at the non-stochastic steady state.

Apart from capital, strong deviations from the non-stochastic steady state in the model with capacity constraint are only found for utilization and, consequently, for the depreciation rate. In order to prevent the capacity constraint from binding too often, at the stochastic steady state the household chooses a utilization rate that is 1% lower than at the non-stochastic steady state, leading to a decrease in the depreciation rate by

[19] For productivity, the stochastic steady state is identical to the non-stochastic steady state.

2%. On average, utilization is 1.7% lower and the depreciation rate is 3.6% lower than at the non-stochastic steady state, so that the average depreciation rate equals about 1.45% instead of 1.5%.

4.2 Differing Magnitudes of Skewness

According to the results in Table 3 some variables are more skewed than others. As mentioned above, it is straightforward to give an explanation why the depreciation rate and utilization are strongly negatively skewed. They are virtually bounded above in contrast to the other variables of the model. However, arguing why, for example, output exhibits stronger skewness than consumption or why capital has positive skewness is less evident.

One possible reason why consumption is less skewed than most other variables could be given by the preferences of the household. Since its utility is logarithmic in consumption, its expected utility from consumption can be approximated by

$$E\left[\ln c_t\right] \approx \ln \bar{c} + \frac{E\left[c_t - \bar{c}\right]}{\bar{c}} - \frac{E\left[\left(c_t - \bar{c}\right)^2\right]}{2\bar{c}^2} + \frac{E\left[\left(c_t - \bar{c}\right)^3\right]}{3\bar{c}^3}.$$

According to this approximation, the household has a preference for positively skewed consumption.

In order to investigate whether preferences can explain why consumption is less skewed than most other variables, we simulate the model with capacity constraint using a modified momentary utility function. Instead of (2) we employ the approximation

$$\tilde{u}\left(c_t, h_t\right) = \ln \bar{c} + \frac{\left(c_t - \bar{c}\right)}{\bar{c}} - \frac{\left(c_t - \bar{c}\right)^2}{2\bar{c}^2} + \Phi\frac{\left(c_t - \bar{c}\right)^3}{3\bar{c}^3} - \frac{\left(c_t - \bar{c}\right)^4}{4\bar{c}^4} - \omega h_t.$$

For the parameter Φ, we consider the values 1, 0 and -1. The value of 1 is used to check the validity of the approximation. The coefficients of skewness emerging from these utility functions are presented in Table 6. We also show the coefficients of skewness obtained with the original utility function (2).

Table 6: Skewness with modified utility functions

utility	y	c	x	ka	h	w, lp	tfp	u	$g(u)$	k
original	-0.42	-0.12	-0.68	0.07	-0.50	-0.12	-0.31	-1.22	-1.21	0.04
$\Phi = 1$	-0.42	-0.12	-0.68	0.07	-0.50	-0.12	-0.31	-1.21	-1.20	0.04
$\Phi = 0$	-0.43	-0.14	-0.69	0.06	-0.52	-0.11	-0.32	-1.21	-1.20	0.04
$\Phi = -1$	-0.44	-0.16	-0.71	0.05	-0.53	-0.10	-0.32	-1.20	-1.19	0.03

The approximation with $\Phi = 1$ appears to be sufficiently exact. Differences with respect to the results with the original utility function are hardly observable. When the value of is Φ lowered to zero, i.e. when the household is indifferent with respect to skewness of consumption instead of preferring positive skewness, the skewness of consumption indeed attains a lower value. However, the decrease is fairly small. Instead of -0.12, the coefficient equals -0.14. When Φ is set to -1, so that the household has preferences for negatively skewed consumption, the coefficient of skewness decreases to -0.16. Thus, one can conclude that preferences play at best a minor role for the explanation of the skewness of consumption. Moreover, they cannot explain differences among the magnitudes of skewness of different variables. When Φ is set to values lower than 1, not only the skewness of consumption, but also the skewness of output, investment, capital and total factor productivity become marginally smaller.

A well-known statistical reason for differing magnitudes of skewness of variables which are exposed to identical shocks is given by differing degrees of persistence. Variables subject to more persistent processes have distributions which are closer to normal. A simple example is given by the process $Z_t = \theta Z_{t-1} + \epsilon_t$ with $\theta \in [0, 1)$ and $E\left[\epsilon_t^2\right] = 1$. It is easy to show that the skewness of Z_t equals $\left(\left(1-\theta^3\right)^{-1} / \left(1-\theta^2\right)^{-3/2}\right) \cdot E\left[\epsilon_t^3\right]$ and is hence falling in θ. If θ equals 0, the skewness of Z_t is identical to the skewness of ϵ_t. But the more persistent the process is, i.e. the larger the value of θ, the smaller is the skewness of Z_t. If θ equals 1, the asymptotic distribution of Z_T (T denoting the sample size) is normal and therefore unskewed, regardless of the distribution of ϵ_t.[20]

Therefore, we conduct an investigation of the persistence of each variable in order to

[20] cf. Hamilton (1994), p. 480.

Table 7: Persistence prior to HP-filtering

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k
persistence	0.93	0.99	0.91	0.96	0.91	0.96	0.89	0.89	1.00

see whether persistence is related to skewness in our model. Persistence is measured as the sum of the coefficients of an autoregressive process as suggested by Andrews & Chen (1994). We estimate an AR(4)-process for each variable before HP-filtering. The results of this investigation are displayed in Table 7.

It turns out that consumption, the real wage, labor productivity and the quarterly capital stock[21] are the most persistent variables. Before HP-filtering, these variables almost have a unit root. The least persistent variables are investment, labor, utilization and the depreciation rate. Output is less persistent than total factor productivity.

If we order all variables with respect to their persistence, this order corresponds to the magnitudes of their coefficients of skewness after HP-filtering. That is, the quarterly capital stock is the most persistent variable and has the largest coefficient of skewness. Consumption, the real wage and labor productivity are the second most persistent variables and have the second largest coefficients of skewness. This ordering continues with total factor productivity, output, labor, investment, utilization and the depreciation rate.

If skewness is mainly determined by persistence, then one of the weaknesses of the RBC model with respect to second moments, namely the excessively smooth behavior of consumption contributes to the success of replicating this variable's skewness. However, while differences in persistence appear to be the main reason for differences in skewness, they cannot explain all phenomena observed. For example, investment and labor have almost identical persistence, but investment is considerably more skewed. Moreover, it is not clear why capital is the only positively skewed variable. Unfortunately, we cannot offer explanations for these issues.

[21] Persistence is related to frequency, so that the persistence of the annual capital stock is not suitable for comparisons with the persistence of quarterly variables. Its value is reported for the sake of completeness only.

4.3 Sensitivity Analysis

In the following, we will investigate whether the reproduction of empirical third moments by the model hinges on certain parameter values. In order to check the robustness of our results, we vary model parameters as well as parameters related to the numerical solution method. For every altered parameter, we compute the decision rules of the economy and simulate in the same manner as described before.

For the depreciation rate, we consider an alternative value of 0.025 as found e.g. in King & Rebelo (1999). The parameter α is set to 0.36 for the robustness check. This value is employed e.g. in Altig et al. (2005). Based on the same study, we consider a value of the yearly interest rate of 3%. Concerning labor at the non-stochastic steady state, a value 10% higher than previously and hence equal to $\bar{h} = 0.22$ is employed. We also consider a model with no growth and thus ν equal to 1.[22] In another model, the standard deviation of productivity σ is increased by 10%. In the next model, ρ is increased such that the standard deviation of productivity before HP-filtering increases by 10%. Finally, we relax the capacity constraint and set η_1 to 10 so that exceeding an utilization rate of 100% is still relatively expensive but becomes much less costly than in the original model.

With respect to the solution method, we consider four modifications. In one modification, we increase b from 2 to 2.5, so that the discrete process for productivity covers ± 2.5 standard deviations of the continuous process. We also consider a larger number of states of the Markov chain for productivity by increasing m from 15 to 20. For the third modification, we combine both modifications. Finally, we increase the number of grid points (gp) for capital from 1200 to 1400.

The results of these simulations are reported in Table 8. The coefficients of skewness exhibit considerable robustness to all modifications employed. Minor changes are observed for the skewness of investment and of labor. It would, however, not be correct to conclude that, e.g., zero growth of the economy induces per se a more pronounced skewness of investment. This conclusion is misleading since it does not take into account the

[22] When r or ν are modified, β is of course changed accordingly.

Table 8: Sensitivity analysis of skewness

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z
original	-0.42	-0.12	-0.68	0.07	-0.50	-0.31	-1.22	-1.21	0.04	0.00
$\delta = 0.025$	-0.42	-0.11	-0.63	0.08	-0.53	-0.29	-1.22	-1.21	0.05	0.00
$\alpha = 0.036$	-0.42	-0.12	-0.66	0.06	-0.50	-0.33	-1.22	-1.20	0.03	0.00
$r = 0.03/4$	-0.41	-0.13	-0.60	0.06	-0.47	-0.32	-1.21	-1.20	0.04	0.00
$\bar{l} = 0.22$	-0.42	-0.12	-0.68	0.06	-0.50	-0.31	-1.22	-1.20	0.04	0.00
$\nu = 1$	-0.42	-0.12	-0.78	0.07	-0.50	-0.31	-1.21	-1.20	0.04	0.00
$\sigma = 0.003894$	-0.41	-0.12	-0.70	0.07	-0.50	-0.31	-1.21	-1.19	0.04	0.00
$\rho = 0.98762$	-0.41	-0.12	-0.67	0.06	-0.51	-0.30	-1.21	-1.20	0.04	0.00
$\eta_1 = 10$	-0.40	-0.11	-0.66	0.07	-0.48	-0.29	-1.17	-1.16	0.04	0.00
$b = 2.5$	-0.42	-0.12	-0.67	0.06	-0.51	-0.30	-1.21	-1.20	0.04	0.00
$m = 20$	-0.41	-0.12	-0.66	0.07	-0.50	-0.31	-1.21	-1.20	0.04	0.00
$b = 2.5, m = 20$	-0.41	-0.12	-0.65	0.06	-0.49	-0.30	-1.20	-1.18	0.03	0.00
$gp = 1400$	-0.42	-0.12	-0.68	0.07	-0.51	-0.31	-1.22	-1.20	0.04	0.00

Table 9: Sensitivity analysis of standard deviation

	y	c, w, lp	x	ka	h	tfp	u	$g(u)$	k	z
original	1.61	0.42	6.95	0.70	1.25	0.80	1.95	0.06	0.30	0.49
$\delta = 0.025$	1.48	0.45	5.21	0.77	1.09	0.77	1.66	0.07	0.35	0.49
$\alpha = 0.036$	1.61	0.41	6.48	0.65	1.25	0.82	1.99	0.06	0.28	0.49
$r = 0.03/4$	1.81	0.38	5.85	0.64	1.48	0.84	2.35	0.05	0.27	0.49
$\bar{l} = 0.22$	1.61	0.42	6.95	0.70	1.25	0.80	1.95	0.06	0.30	0.49
$\nu = 1$	1.61	0.42	9.02	0.68	1.24	0.80	1.95	0.06	0.30	0.49
$\sigma = 0.003894$	1.78	0.46	7.75	0.77	1.38	0.88	2.15	0.07	0.33	0.54
$\rho = 0.98762$	1.58	0.43	6.67	0.67	1.19	0.80	1.91	0.06	0.29	0.50
$\eta_1 = 10$	1.63	0.42	6.99	0.71	1.26	0.80	1.99	0.06	0.31	0.49
$b = 2.5$	1.61	0.45	6.79	0.69	1.21	0.82	1.97	0.06	0.30	0.51
$m = 20$	1.54	0.40	6.57	0.66	1.18	0.76	1.86	0.06	0.29	0.47
$b = 2.5, m = 20$	1.53	0.43	6.45	0.66	1.15	0.78	1.88	0.06	0.28	0.49
$gp = 1400$	1.61	0.42	6.94	0.70	1.24	0.80	1.95	0.06	0.30	0.49

changes of standard deviations caused by the alternative parameter values. Considering the volatilities of the variables in each simulation, as presented in Table 9, indeed suggests that it might be higher volatility of investment that gives rise to more pronounced skewness of investment.[23] Yet, the results strongly support the suggestion that the remarkable reproduction of the empirical coefficients of skewness is a finding which almost entirely hinges on the kink in marginal costs of utilization, and which does not depend on the values of other model parameters or the discrete approximation used to obtain the solution. In this context, it also appears noteworthy that third moments exhibit more robustness than second moments. This could be related to the fact that, in contrast to the second moments, the third moments investigated are standardized moments.

[23] This sugggestion is supported by simulation results with even higher values of σ, not reported here. Probably this is due to the fact that with high volatility, investment can become very close to 0, leading to extremely low values of its logarithm.

5 Summary and Concluding Remarks

In this work, we have analyzed the consequences of the existence of capacity constraints for the asymmetries emerging from an otherwise prototypical RBC model. The capacity constraint originates from the assumption of an upper bound to the utilization of capital, motivated by kinked marginal utilization costs. We have compared the asymmetries caused by a model with such a constraint to the asymmetries present in the data, and we have found that the model can replicate the asymmetries of most variables, i.e. of output, consumption, investment, capital, labor, the real wage and (measured) total factor productivity very well. The skewness of labor productivity is more pronounced in the data than in the model, but this might be due to outliers.

In order to verify that it is the capacity constraint which causes the model's asymmetries, we have simulated two alternative models without constraint and found that only investment exhibits noteworthy skewness. Comparing the model with capacity constraint to the alternative models, we find that the existence of the capacity constraint leads to increased capital accumulation and lower utilization. Comparing the models among each other, we have also discovered that the introduction of a capacity constraint has negligible effects on standard deviations and cross-correlations.

Investigating the reason for differing magnitudes of skewness, we have found that these differences appear to be related to differences in persistence. However, not all the phenomena observed can be explained by differences in persistence.

The sensitivity analysis reveals that the asymmetries of all variables are robust to changes of the model's parameters, as long as the model continues to feature a strong increase in the elasticity of marginal costs with respect to utilization when the utilization rate exceeds 100%. In addition, the results concerning asymmetries are found to be robust to modifications of the approximations employed in order to obtain the model's solution.

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A Parameters of Discrete Process for Productivity

The values $\ln z_t$ can attain are given by

$$100 \ln z_t = \begin{cases} 4.103 & \text{if } s_t = 1 \\ 3.517 & \text{if } s_t = 2 \\ 2.931 & \text{if } s_t = 3 \\ 2.345 & \text{if } s_t = 4 \\ 1.758 & \text{if } s_t = 5 \\ 1.172 & \text{if } s_t = 6 \\ 0.586 & \text{if } s_t = 7 \\ 0.000 & \text{if } s_t = 8 \end{cases}, \quad 100 \ln z_t = \begin{cases} \text{-}0.586 & \text{if } s_t = 9 \\ \text{-}1.172 & \text{if } s_t = 10 \\ \text{-}1.758 & \text{if } s_t = 11 \\ \text{-}2.345 & \text{if } s_t = 12 \\ \text{-}2.931 & \text{if } s_t = 13 \\ \text{-}3.517 & \text{if } s_t = 14 \\ \text{-}4.103 & \text{if } s_t = 15 \end{cases}$$

with s_t being the state of the Markov chain with transition matrix $\mathbf{P}$ given by

$$\mathbf{P} = \begin{bmatrix} .74 & .16 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .25 & .59 & .17 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .01 & .24 & .59 & .17 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .00 & .01 & .23 & .59 & .18 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .00 & .00 & .01 & .22 & .59 & .18 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .01 & .22 & .59 & .19 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .00 & .01 & .21 & .59 & .20 & .01 & .00 & .00 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .01 & .20 & .59 & .20 & .01 & .00 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & .01 & .20 & .59 & .21 & .01 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .19 & .59 & .22 & .01 & .00 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .18 & .59 & .22 & .01 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .18 & .59 & .23 & .01 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .01 & .17 & .59 & .24 & .01 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .17 & .59 & .25 \\ .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .16 & .74 \end{bmatrix}.$$

Element $\mathbf{P}_{i,j}$ of this matrix contains the transition probability $\Pr(s_{t+1} = j \,| s_t = i)$.

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