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Time-dependent pricing and New Keynesian Phillips curve

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Abstract

This paper explores what can be lost when assuming price adjustment is a time-independent (memoryless) process. I derive a generalized NKPC in an optimizing model with the non-constant hazard function and trend inflation. Memory emerges in the resulting Phillips curve through the presence of lagged inflation and lagged expectations. It nests the Calvo NKPC as a limiting case in the sense that the effects of both terms are canceled out by one another under the constant-hazard assumption. Furthermore, I find lagged inflation always has negative coefficients, thereby making it impossible to interpret inflation persistence as intrinsic to the model. The numerical evaluation shows that introducing trend inflation strengthens the effects of the increasing hazard function on the inflation dynamics. The model can jointly account for persistent dynamics of inflation and output, hump-shaped impulse responses of inflation to monetary shocks, and the fact that high trend inflation leads to more persistence in inflation but not for real variables.

JEL classification: E12; E31
Key words: Intrinsic inflation persistence, Hazard function, New Keynesian Phillips Curve

Non-technical summary

From recent micro data, convincing evidence has been documented for price adjustments, showing that 1) despite persistent aggregate inflation, price adjustment at the firm's level is quiet flexible. And 2) the hazard function of the time-since-last-adjustment is not constant. This evidence brings difficulties for the widely used Calvo model that implies highly rigid prices at the micro level as well as a constant hazard function. In this paper, I construct a generalized time-dependent model of nominal rigidity and derive the generalized NKPC, conditional on a hazard function with an arbitrary shape.

The analytical solution shows that this version of the NKPC nests the Calvo NKPC in the sense that, given a constant hazard function, the effect of lagged inflation exactly cancels the effects of lagged expectation terms. In light of this result, we learn that lagged inflation and lagged expectations are not extrinsic to the time-dependent model. They are missing in the basic Calvo setup, only because the restrictive 'memoryless' pricing assumption cancels them out. More importantly, inflation persistence generated by this optimizing model is not intrinsic. Instead, it results from the additional autoregressive terms of real marginal cost introduced through lagged expectations. The inclusion of these new terms reflects the impact of past economic decisions on current outcomes. This paper formulates this idea in a theoretical model.

In the numerical assessment, this paper makes two methodological contributions. Firstly, in the calibration exercise, I use the parsimonious Weibull duration model to parameterize the hazard function and the average price duration. Secondly, I log-linearize the NKPC around a non-zero inflation steady state. Combining dynamic effects of the increasing hazard pricing and trend inflation is important because without trend inflation, the increasing-hazard model merely changes the timing of pricing behavior. With trend inflation however, relative prices disperse quickly with the elapsed time since the last adjustment and as a result, not only the timing of the price adjustment but also the magnitude of the adjustment are affected by the increasing hazard function. In addition, trend inflation affects all coefficients in the generalized NKPC. Thereby the change in the trend inflation affects the relative importance between the forward-looking and backward-looking terms in the Phillips curve.

When simulating the full-scale general equilibrium model, simulation results show that the model can jointly account for the following stylized facts from the data: 1) the higher the trend inflation is, the more persistent the inflation gap becomes. 2) Trend inflation has a larger effect on the persistence of output and real marginal cost than on the persistence of inflation. 3) The correlation between inflation and real marginal cost decreases further when trend inflation is high. 4) The impulse response of inflation to a nominal money growth shock is hump-shaped.

Nicht technische Zusammenfassung

Aktuelle Mikrodaten bieten einen überzeugenden Beleg dafür, dass beim Preisbildungsprozess erstens die Preise auf der Unternehmensebene recht flexibel angepasst werden, obwohl die aggregierte Inflation sehr persistent ist und dass zweitens die Hazardfunktion für Preisanpassungen seit der letzten Preisänderung nicht konstant ist. Diese Evidenz stellt ein Problem in Bezug auf das weit verbreitete Calvo-Modell dar, welches auf der Mikroebene sehr starre Preise und eine konstante Hazardfunktion unterstellt. In der vorliegenden Abhandlung wird ein verallgemeinertes zeitabhängiges Modell der nominalen Rigidität entwickelt und eine verallgemeinerte Neukeynesianische Phillips-Kurve (NKPC) abgeleitet, die eine Hazardfunktion mit flexiblem Verlauf annimmt.

Die analytische Lösung zeigt, dass die Calvo-NKPC ein Spezialfall der hier abgeleiteten NKCP-Version ist. Bei konstanter Hazardfunktion wird der Effekt eines verzögerten Erwartungsterms vollständig durch den Effekt einer verzögerten Inflation aufgehoben. Bei variabler Hazardrate ist dies nicht der Fall. Angesichts dieses Ergebnisses ist festzustellen, dass verzögerte Inflation und verzögerte Erwartungen bei einem zeitabhängigen Modell nicht extrinsisch sind. Sie fehlen im Basismodell von Calvo nur deshalb, weil sie durch eine restriktive „gedächtnislose“ Preisbildung aufgehoben werden. Noch entscheidender ist, dass die durch dieses Modell generierte Inflationspersistenz auch nicht intrinsisch ist. Sie resultiert vielmehr aus den zusätzlichen autoregressiven Termen realer Grenzkosten, die über die verzögerten Erwartungen einen Einfluss haben. Diese neuen Terme tragen den Auswirkungen in der Vergangenheit getroffener wirtschaftlicher Entscheidungen auf die Gegenwart Rechnung.

Bei der numerischen Umsetzung leistet das Papier zwei methodische Beiträge. Zum einen wird bei der Kalibrierung das einfache Lebensdauermodell von Weilbull verwendet, um die Hazardfunktion und die durchschnittliche Verweildauer von Preisen zu parametrisieren. Zum anderen wird die NKPC um eine langfristig gleichgewichtige Inflationsrate log-linearisiert, die ungleich null ist. Es ist wichtig, die dynamischen Effekte von steigenden Hazardraten und der Trendinflation zusammen zu betrachten, da eine zunehmende Hazardraten ohne Berücksichtigung der Trendinflation lediglich die zeitliche Verteilung der Preisanpassung verändern. Wird hingegen die Trendinflation berücksichtigt, wird nicht nur die zeitliche Verteilung der Preisanpassung, sondern auch das Ausmaß der Anpassung durch die steigende Hazardfunktion beeinflusst. Darüber hinaus wirkt sich die Trendinflation auf alle Koeffizienten der verallgemeinerten NKPC aus. Eine Veränderung der Trendinflation beeinflusst damit die relative Bedeutung vorwärtsgerichteter und rückwärtsgerichteter Terme der Philips-Kurve.

Bei Simulierung des vollständigen allgemeinen Gleichgewichtsmodells belegen die Simulationsergebnisse, dass sich anhand des Modells die folgenden stilisierten Fakten aus den Daten nachweisen lassen: 1) Je höher die Trendinflation ist, desto persistenter wird die Inflationslücke. 2) Die Trendinflation wirkt sich stärker auf die Persistenz der Produktion und der realen Grenzkosten aus als auf die der Inflation. 3) Die Korrelation zwischen Inflation und realen Grenzkosten nimmt ab, wenn die Trendinflation auf hohem Niveau verläuft. 4) Die Impulsantwort der Inflation auf einen Schock der Geldmenge ist buckelförmig.

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1 Introduction

The nature of inflation persistence is a central issue in macroeconomics. For theorists, this is about the rationale of inflation persistence and how to use well micro-founded models to explain this phenomenon. For policy practitioners, it is interesting because the answer has critical implications when designing the optimal monetary policy. In recent years, the new keynesian Phillips curve (NKPC) has become the main analytical tool for examining these issues. Despite its ubiquity in the literature, there is still no clear consensus on the specification of the NKPC and its implication for inflation persistence. For example, the Calvo NKPC based on forward-looking price setting implies that inflation is only driven by current economic conditions and expectations of the future. By contrast, a more empirically plausible specification[2] which amends the Calvo NKPC with lagged inflation, implies that inflation persistence is mainly 'intrinsic'. However, it is generally agreed that neither of these theories give us a satisfactory answer to the nature of inflation persistence because they either fail in the empirical test or are based on a theory lacking plausible microfoundations.

In search of a satisfactory specification of the NKPC, new waves of theory are developing to resolve the tension between the optimizing microfoundations and a fit to the data[3]. One consideration in this line of research is to theoretically justify the presence of lagged inflation in the NKPC and to give a structural interpretation to its coefficient. In another words, what is the nature of inflation persistence? Is it intrinsic or does inflation persistence result from factors which are unaccounted for in the standing theory?

In this paper, I tackle these questions by constructing a general time-dependent model of nominal rigidity a la Wolman (1999) and derive the generalized NKPC conditional on an arbitrary hazard function. This version of the NKPC involves components including lagged inflation, forward-looking expectations and lagged expectations of inflations and real marginal cost. It nests the standard NKPC in the sense that, given a constant hazard function, the effect of lagged inflation exactly cancels the effects of lagged expectation terms, so that only current variables and forward-looking expectations remain in the expression as in the Calvo NKPC. In light of this result, we learn that lagged inflation and lagged expectations are not extrinsic to the time-dependent nominal rigidity model. They are missing in the Calvo setup, only because the 'memoryless' pricing assumption cancels them out. By relaxing this assumption, we do not need new microfoundations to introduce lagged inflation into the Phillips curve.

Furthermore, I find that coefficients of the lagged inflations are always negative, while coefficients of expectations are always positive. Note that I derive these results without specifying any functional form of the hazard function, meaning they should hence be robust to the shape of

[1] I am grateful to Michael Burda, Heinz Herrmann, Mathias Hoffmann, Florian Kajuth, Michael Krause and other seminar participants at the Deutsche Bundesbank and in Berlin for helpful comments. A special thank to Thomas Laubach, Alexander Wolman for very helpful comments and discussions. I acknowledge the support of the Deutsche Bundesbank and the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk". All errors are my sole responsibility. Address: Institute for Economic Theory, Humboldt University of Berlin, Spandauer Str. 1, Berlin, Germany +493020935667, email: yaofang@rz.hu-berlin.de.

[2] See: e.g. Gali and Gertler (1999) and Christiano et al. (2005).

[3] See: Woodford (2007) and Sheedy (2007) for a summary.

the hazard function. According to these results, inflation persistence generated by this optimizing model is not intrinsic - defined as inflation driven by its own lags with positive coefficients. Instead, more persistent inflation in this model results from the presence of the lagged expectations. While the sticky information model by Mankiw and Reis (2002) also emphasized the role of lagged expectations in propagating inflation persistence, this model includes lagged expectations along with lagged inflation due to the more general time-dependent pricing setup. The inclusion of both terms reflects the impacts of the past price decisions on current inflation. The fact that past expectations affect the current economic outcome can be observed in daily newspapers. For example, when car makers expect the price of steel to rise, they stock it up in the fear that steel becomes more expensive in the future. After a shock turns the price downward, we do not immediately observe a rise in demand of steel because car makers still have inventory in their warehouses as a result of the past speculation. In this paper, I formulate this idea in a theoretical model.

In the numerical assessment, this paper makes two methodological contributions. Firstly, in the calibration, I use the parsimonious Weibull duration model to parameterize the hazard function. By definition, it is a function with two parameters. One parameter is the scale parameter, which controls the average duration of the price adjustment. The other is the shape parameter that determines the monotonic property of the hazard function. By changing the value of the shape parameter, this hazard function enables the incorporation of a wide range of hazard profiles. Secondly, motivated by the seminal paper by Ascari (2004), I log-linearize the NKPC around a non-zero inflation steady state. Combining dynamic effects of the increasing hazard pricing and trend inflation is important because without trend inflation, the increasing-hazard model merely changes the timing of pricing behavior. By contrast, with trend inflation, relative prices disperse quickly with the elapsed time since the last adjustment and as a result, not only the timing of the price adjustment but also the magnitude of the adjustment, are affected by the increasing hazard function. In addition, trend inflation affects all coefficients in the generalized NKPC, thereby the change in the trend inflation affects the relative importance between the forward-looking and backward-looking terms in the Phillips curve. As a result, trend inflation has a more significant impact on inflation dynamics in the increasing-hazard model than in the Calvo setup.

When simulating the full-scale general equilibrium model, I combine the generalized NKPC with a standard IS curve and an exogenous nominal money growth process. The simulation results show that, even without trend inflation, the increasing hazard pricing helps to increase both persistence of inflation and the output gap. It also helps to reduce the correlation between inflation and real marginal cost. When loglinearizing around a steady state with non-zero trend inflation, the model can jointly account for the following stylized facts: 1) the higher the trend inflation is, the more persistent the inflation gap becomes. 2) Trend inflation has a larger effect on the persistence of output and real marginal cost than on the persistence of inflation. 3) The correlation between inflation and real marginal cost decreases further when trend inflation is high. 4) Impulse response of inflation to a nominal money growth shock is hump-shaped.

The remainder of the paper is organized as follows: section 1 provides a literature review of related papers; in section 2, I introduce the model with generalized time-dependent pricing at the firm's level and derive the New Keynesian Phillips curve; section 3 shows some analytical results to give the structural interpretation of the coefficients of the generalized NKPC; in section 4, I introduce the calibration strategy of the model's parameters and present the simulation results; section 5 contains some concluding remarks.

2 Related literature

A large part of sticky price theory can usefully be classified around the concept of the hazard function. A hazard function gives the conditional probability of the price adjustment. In the literature, there are two kinds of hazard functions. One is the time-dependent hazard function, which is the adjustment probability conditional on time since the last price adjustment[4]. The other hazard function is state-dependent, i.e. the likelihood of the adjustment depends on the deviation from the optimal target in the economy[5]. While the state-dependent hazard function is more theoretically rigorous and micro-founded, the time-dependent hazard function is more popular in the literature due to its tractability. Despite these differences, one can argue that there is no sharp dichotomy between these two kinds of hazard functions. Dotsey et al. (1999) showed that a more general time-dependent specification is formally a first-order approximation to a richer state-dependent pricing model. Woodford (2008) constructs a more general model of state-dependent pricing motivated by the 'rational inattention' assumption[6], which nests both the standard state-dependent pricing model and the Calvo model as limiting cases. He finds that, given small shocks, the time-dependent model is a reasonably accurate approximation of the exact equilibrium dynamics. Therefore, a general form of the time-dependent hazard function is a useful analytical apparatus to help investigate the effects of price stickiness. For this reason, I use a generalized time-dependent hazard model in this paper to study effects of nominal price rigidity on inflation dynamics, and hence, in this section, I only focus on the part of the literature that explores the role played by the nominal price stickiness models with time-dependent hazard functions.

The time-dependent hazard models take price adjustment probability as given. In extreme cases, Calvo (1983) assumed probabilities of nominal price adjustment to be constant and independent over time, while the hazard rates in the staggered-contract model of Taylor (1980) are either zero during the spell of the contract or one at the end of the contract. The more general models take the view that the time profile of hazard rates can be more flexible than those limiting cases. For example, Wolman (1999) studied some simple examples of the general staggered pricing model and found that inflation dynamics are sensitive to different pricing rules. Mash (2003) constructed a specific hazard pricing model that nests both 'hybrid' Calvo model and the Taylor model, and he found that implications for optimal monetary policy and monetary delegation based on limiting cases are not robust to the change in the hazard function.

More recently there is a debate on the rationale for the inclusion of lagged inflation in the new Keynesian Phillips curve and the role played by the lagged inflation in inflation persistence. Mash (2004) derived the generalized NKPC with an increasing hazard function. He shows some general results that this version of the Phillips curve can replicate a large part of persistence in inflation and output gap dynamics, despite the fact that lagged inflations enter the NKPC

[4] See: e.g. Calvo (1983), Taylor (1980), Wolman (1999)

[5] See: e.g. Caplin and Spulber (1987), Dotsey et al. (1999), Caballero and Engel (2007) Golosov and Lucas (2007).

[6] See: Sims (1998) and Sims (2003)

with negative coefficients. However, he did not give explicit interpretations on those negative coefficients on lagged inflations. Whelan (2007) rejects these kinds of models based on this result, and argues that a good price rigidity model should be able to generate the key feature of reduced-form Phillips curve regression: the positive dependence of inflation on its lags. However, this argument is very vulnerable in light of the evidence presented by Dotsey (2002), who shows that the positive reduced-form coefficients themselves could be spurious due to omitted variables in a misspecified regression model. This argument is supported by Cogley and Sbordone (2006), who find that when correctly accounting for the time-varing trend inflation, the purely forward-looking model explains the persistence of the inflation deviation from its trend quite well. They, therefore, identify an independent source of inflation persistence. On the other hand, Mash (2007) proves theoretically that, when agents are allowed to chose the extent of partial indexing optimally, the Nash equilibrium between firms and the policy maker is characterized by zero indexation, therefore having no micro-foundation for the positive dependence of inflation on its lags.

The most closely related paper in the literature is Sheedy (2007), who parameterizes the hazard function in such a way that the resulting NKPC has a positive coefficient on lagged inflation given that the hazard function is upward sloping. My paper however, uses a different parameterization strategy based on the statistical duration theory. This specification of the hazard function allows me to solve for the NKPC more tractably, hence leading to more intuitive interpretations of its coefficients.

3 The model

In this section, I introduce the generalized time-dependent model of nominal rigidity a la Wolman(1999). The most important components of the model are 1) monopolistic competitive firms who set their prices according to the demand condition and the probabilities for re-optimizing their prices, and 2) firms cannot adjust their price whenever they want, instead, the opportunities for re-adjusting their prices depend on exogenous hazard rates, which based on the length of time since the last adjustment. I summarize this limited price adjustment scheme using an arbitrary hazard function $h(j)$, where j denotes the period of time elapsed since the last price adjustment $j \in \{0, J\}$. Firms in the same vintage (j) have the same probability ($h(j)$) of adjusting their prices.

3.1 Monopolistic competition firms

I consider an economy with a continuum of monopolistic competitive firms, which are differentiated with respect to the type of worker they use, indexed by $i \in \{0, 1\}$. The final goods sector is perfectly competitive and produces a single final good Y_t with all intermediate goods with a CES aggregate production function (Dixit and Stiglitz, 1977)

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

Given this aggregate production function and the market structure, the profit maximization problem of the final-good firm solves the demand function for intermediate goods,

$$Y_{i,t}^{d} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} Y_t, \tag{2}$$

Where $P_{i,t}$ denotes the nominal price of good i, and P_t is the aggregate price for one unit of final good Y_t. It follows that the welfare-based aggregate price index is obtained by the following expression:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\eta} di\right)^{\frac{1}{1-\eta}} \tag{3}$$

3.2 Generalized time-dependent price stickiness

In this economy, firms are also heterogeneous with respect to the time since their last price adjustment. I call them vintages (j) which indicate that those prices are j periods old ($j \in \{0, J\}$), where J denotes the maximum number of periods in which a firm can fix its price. At the end of each period, those firms that reoptimize their prices in the current period are labelled by the 'vintage 0', while the other firms move to the next vintage $j+1$ because their prices age by one-period[7]. Table (1) summarizes key notations concerning the dynamics of vintages.

Vintage	Hazard Rate	Non-adj. Rate	Survival Rate	Distribution
j	$h(j)$	$\alpha(j)$	$S(j)$	$\theta(j)$
0	0	1	1	$\theta(0)$
1	$h(1)$	$1-h(1)$	$\alpha(1)$	$\theta(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	$h(j)$	$\alpha(j) = 1 - h(j)$	$S(j) = \prod_{i=0}^{j} \alpha(i)$	$\theta(j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	$h(J) = 1$	$\alpha(J) = 0$	$S(J) = 0$	$\theta(J)$

Table 1: Notations of the dynamics of price-vintage-distribution.

3.2.1 Dynamics of the vintage distribution

In order to aggregate the economy, we need to track the distribution of firms according to the age of their prices. Before a price decision is made, at the beginning of each period, the distribution of price vintages is $\Theta_t = \{\theta_t(0), \theta_t(2) \cdots \theta_t(J-1)\}$. Next, firms get the opportunity to adjust their prices, based on the hazard function $h(j)$. After the price adjustment, the ex post distribution $\Theta'_t = \{\theta'_t(0), \theta'_t(2) \cdots \theta'_t(J-1)\}$ is obtained by

$$\theta'_t(j) = \begin{cases} \sum_{i=1}^{J} h(i)\theta_t(i) \text{ , when } j = 0 \\ \alpha(j)\theta_t(j) \text{ , when } j = 1 \cdots J-1 \end{cases} \tag{4}$$

[7] In this model, in contrast to Carvalho (2005), firms are not fixed in any price-stickiness vintage groups, instead they move among vintage groups according to an underlying stochastic price adjustment process.

When period t is over, this ex post distribution Θ_t' becomes an ex ante distribution for the new period Θ_{t+1}.

3.2.2 The stationary distribution

As long as the hazard rates are well defined, distribution dynamics can be viewed as a Markov process with an invariant distribution Θ, obtained by solving $\theta_t(j) = \theta_t'(j-1) = \theta_{t+1}(j)$. As a result, the stationary vintage distribution $\theta(j)$ can be shown as a function of the non-adjustment rates $\alpha(j)$:

$$\theta(j) = \frac{\prod_{i=0}^{j} \alpha(i)}{\sum_{n=0}^{J-1} \prod_{i=0}^{n} \alpha(i)} = \frac{S(j)}{\sum_{n=0}^{J-1} S(n)} \text{ , for } j = 0, 2 \cdots J-1 \tag{5}$$

Let's assume the economy converges to this invariant distribution fairly quickly, so that regardless of the initial vintage distribution, I only consider the economy with the above invariant distribution of vintages, given the hazard function $h(j)$. For any stationary distribution $\theta(j)$, the aggregate price index (3) can be rewritten as a distributed sum of all vintage prices, reflecting the fact that all firms setting prices in the same period should choose the same price, assuming no other heterogeneity affects the firms' price decisions.

The optimal price is defined as P_{t-j}^*, set j periods ago. It allows for the aggregate price index to be obtained by the weighted sum of the past optimal prices as follows:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P_{t-j}^{*1-\eta} \right)^{\frac{1}{1-\eta}} \tag{6}$$

3.3 The optimal pricing

In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be adjusted in the near future. Consequently, adjusting firms choose an optimal price that maximizes the discounted sum of real profits over the time horizon during which the new price is expected to hold. The probability that the new price is fixed is given by the survival function $S(j)$ defined in Table (1). The maximization problem is obtained by

$$\max_{P_t^*} \quad \sum_{j=0}^{J-1} S(j) E_t \{ Q_{t,t+j} [Y_{t+j|t}^d - TC_{t+j} / P_{t+j}] \}$$

Where E_t denotes the conditional expectation operator based on the information set at period t, and $Q_{t,t+j}$ is the stochastic discount factor which is appropriate for discounting real profits from time $t+j$ to time t. $Y_{t+j|t}^d$ denotes real output demand in period $t+j$ for a firm that resets its price in period t. I implicitly assume here that firms have no monopolistic power in individual labor markets, so that firms do not consider the possibility of their price decisions affecting future real wages and hence future marginal costs[8]. As a result, the optimal price has no direct effect on the future cost.

The discounted sum of real profits is maximized subject to the demand function given an optimal price P_t^*

$$Y_{t+j|t}^d = \left(\frac{P_t^*}{P_{t+j}}\right)^{-\eta} Y_{t+j},$$

Then, the first order condition of this maximization problem is obtained by

$$\begin{aligned} P_t^* &= \left(\frac{\eta}{\eta-1}\right) \frac{\sum_{j=0}^{J-1} S(j) E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1} MC_{t+j}]}{\sum_{j=0}^{J-1} S(j) E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]}, \\ &= \left(\frac{\eta}{\eta-1}\right) \sum_{j=0}^{J-1} \frac{S(j) E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]}{\sum_{j=0}^{J-1} S(j) E_t[Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]} E_t\left[MC_{t+j}\right] \end{aligned} \tag{7}$$

Where MC_{t+j} denotes nominal marginal cost. One can see that, in the zero inflation steady state, price level and real variables are constant, allowing this equation to reduce to the static pricing equation, which expresses the optimal price as the nominal marginal cost multiplied by a constant markup ($\frac{\eta}{\eta-1}$). One can see from the second line that in the dynamic environment however, the optimal price is equal to the markup multiplied by the weighted sum of future nominal marginal costs. The weight depends on the survival rate. In addition, the maximum time horizon J depends on the speed at whick the survival function goes to zero. In the Calvo case, where $S(j) = (1-\alpha)\alpha^j$, survival rates approach zero as j increases, but never reach, thereby making the decision horizon infinite in this case.

3.4 Derivation of the New Keynesian Phillips curve

In this section, I derive the New Keynesian Phillips curve for this generalized model. I first log-linearize equations (6) and (7) around the steady state with the trend inflation ($\bar{\pi}$). This is motivated by King and Wolman (1996) and Ascari (2004), who show that it is not innocuous to derive a Phillips curve based on a log-linearization around the zero inflation steady state. Trend inflation affects both the long-run and the short-run dynamic properties of the Phillips curve. Furthermore, Cogley and Sbordone (2006) show that trend inflation could be an independent source of inflation persistence. Based on this evidence, it is important to integrate this feature into the model.

3.4.1 Non-zero Inflation steady state

I assume that if the steady state trend inflation is equal to the growth rate of nominal money stock (g), then the steady state is characterized by constant real variables and a growth of all nominal variables at the gross rate g. Because the aggregate price level increases with trend

[8] Here I assume the firm type is not the same as the labor type, thereby, in each labor market, all intermediate firms demand some labor from it. As a result, Iabor markets are competitive, and there is no difference between real wages among individual firms. See Woodford (2003).

inflation in the steady state, firms need to keep their relative prices close to the optimal ratio specified below. If we define $\overline{X}$ as the steady state value of variable X, then the optimality condition (7) can be rewritten as:

$$\bar{p}_t^* = \frac{\eta}{\eta-1}\frac{\sum_{j=0}^{J}\beta^j S(j)\bar{Y}\bar{P}_{t+j}^{\eta}}{\sum_{j=0}^{J}\beta^j S(j)\bar{Y}\bar{P}_{t+j}^{\eta-1}}mc = \frac{\eta}{\eta-1}\frac{\sum_{j=0}^{J}\beta^j S(j)\bar{Y}\bar{P}_t^{\eta}g^{\eta j}}{\sum_{j=0}^{J}\beta^j S(j)\bar{Y}\bar{P}_t^{\eta-1}g^{(\eta-1)j}}mc$$

$$\frac{\bar{p}_t^*}{\bar{P}_t} = \frac{\eta}{\eta-1}mc\left[\frac{\sum_{j=0}^{J}\beta^j S(j)g^{\eta j}}{\sum_{j=0}^{J}\beta^j S(j)g^{(\eta-1)j}}\right] \tag{8}$$

As seen in Equation (8), the optimal relative price ratio is equal to the constant markup multiplied by the real marginal cost along with an extra term, which reflects how fast the aggregate price grows. When the gross inflation rate equals one, this term is also equal to one. In this case we have the familiar static price setting relation. However, when trend inflation is greater than one, this term is also greater than one. It means that, given non-zero trend inflation, the adjusting firms want to hedge the risk that they may not adjust again in the near future, so they adjust their prices more than in the case of zero inflation. Consequently, this higher relative price ratio leads to lower steady state output and hence induces an additional welfare loss caused by steady trend inflation.

3.4.2 Log-linearizing the equilibrium equations

Next I log-linearize equilibrium equations around the steady state defined above. To do that, I define variables with a hat as the log deviation from its non-stochastic steady state, such as $\hat{x}_t = logX_t - log\bar{X}$. Then the log-linearized optimal price equations are obtained by

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{(\beta g^{\eta})^j S(j)}{\Omega}\left(\widehat{mc}_{t+j} + \hat{p}_{t+j}\right)\right], \quad where \;\; \Omega = \sum_{j=0}^{J-1}(\beta g^{\eta})^j S(j) \tag{9}$$

$$\hat{p}_t = \sum_{k=0}^{J-1}\tau(k)\;\hat{p}_{t-k}^*, \qquad where \quad \tau(k) = \frac{\theta(k)g^{(\eta-1)k}}{\sum_{k=0}^{J-1}\theta(k)g^{(\eta-1)k}} \tag{10}$$

3.4.3 New Keynesian Phillips curve

To reveal the implication of the NKPC for the inflation dynamics, I derive the generalized NKPC from equation (9) and (10). To keep the equation as simple as possible, I first derive it without trend inflation, i.e. $g = 1$. After some tedious algebra, we obtain the New Keynesian Phillips curve as follows[9]:

[9] Log-linearization of price equations and the detailed derivation of NKPC can be found in the technical Appendix (A).

$$\hat{\pi}_t = \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right)$$
$$- \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1} S(j)}{\sum_{j=1}^{J-1} S(j)}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S(j) \tag{11}$$

At first sight, this Phillips curve is quite different from the one derived in the Calvo model. It involves not only lagged inflation but also lagged expectations that were built into pricing decisions in the past. All coefficients in the NKPC are derived from deep parameters which are either the stationary distributional parameters or the preference parameter. In particular, coefficients before lagged inflation and lagged expectations ($\frac{\theta(k)}{1-\theta(0)}, \Phi(k)$) consist of only distribution parameters, representing the compositional proportion of each dated price in aggregate inflation, while other coefficients reflect how much impact each factor exerted on current inflation. Next I give an example, where $J = 3$, then the NKPC is obtained by the following form:

$$\begin{aligned}
\hat{\pi}_t = & \frac{1}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_t + \frac{\alpha_1}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_{t-1} + \frac{\alpha_1\alpha_2}{(\alpha_1 + \alpha_1\alpha_2)\Psi} \widehat{mc}_{t-2} - \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2} \hat{\pi}_{t-1} \\
& + \frac{1}{\alpha_1 + \alpha_1\alpha_2} E_t \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_{t+2} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+2} \right) \\
& + \frac{\alpha_1}{\alpha_1 + \alpha_1\alpha_2} E_{t-1} \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_t + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_{t+1} + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_t + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t+1} \right) \\
& + \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_1\alpha_2} E_{t-2} \left(\frac{\beta\alpha_1}{\Psi} \widehat{mc}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \widehat{mc}_t + \frac{\beta\alpha_1 + \beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi} \hat{\pi}_t \right)
\end{aligned}$$

In this example, we see more clearly how current inflation depends on marginal costs, lagged inflation and a complex weighted sum of lagged expectations. All coefficients are expressed in terms of hazard rates ($\alpha_j = 1 - h_j$) and a preference parameter β. It is natural to ask why these lagged terms are absent in the Calvo NKPC. Are there new insights of the NKPC that can be gained by relaxing the constant hazard assumption? The answer is yes. In the next section, I use a proposition to prove this point.

4 Analytical Results

4.1 Derive the Calvo NKPC from the generalized NKPC

Proposition 1 *When reducing the generalized NKPC (11) to the standard Calvo Phillips curve, it implies the following equation must hold:*

$$\hat{\pi}_t = E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i_{t+i} \widehat{mc}_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \tag{12}$$

Proof. *: see Appendix (B)* ■

If one iterates Equation (12) backwards, the following equations hold

$$\hat{\pi}_{t-1} = E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$
$$\hat{\pi}_{t-2} = E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$
$$\vdots$$

In light of these results, we learn that the generalized NKPC nests the Calvo Phillips curve in the sense that, given the constant hazard function, the effects of lagged inflation terms exactly cancel effects of lagged expectations, leaving only current variables and forward-looking expectations in the expression as in the Calvo NKPC. Moreover, lagged inflation and lagged expectations are not extrinsic to the time-dependent nominal rigidity model. They are missing in the Calvo setup, only because the constant hazard assumption causes them to be canceled out. By relaxing this assumption, we do not need new microfoundations to introduce lagged inflation into the Phillips curve.

4.2 Interpretation of lagged inflation in the NKPC

The next question is how to interpret the coefficients of lagged inflations in this NKPC. Can they be interpreted as intrinsic persistence? The answer is no. Given any positive value between zero and one for non-adjustment rates (α_j), they are always negative. Note that I derive this result without specifying any functional form for the hazard function, and hence it should be robust to the shape of the hazard function. Again using the sample Phillips curve, when $J = 3$, I can rewrite it into the following compact form[10], where $F_{t-k} = E_{t-k}\left(\sum_{j=0}^{J-1}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} w_2(i)\hat{\pi}_{t+i-k}\right)$.

$$\hat{\pi}_t = -\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}\hat{\pi}_{t-1} + \frac{1}{\alpha_1+\alpha_1\alpha_2}E_t\left[F_t\right] + \frac{\alpha_1}{\alpha_1+\alpha_1\alpha_2}E_{t-1}\left[F_{t-1}\right] + \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}E_{t-2}\left[F_{t-2}\right] \quad (13)$$

The alphas are the probability of non-adjustment, and lie between zero and one. We see the coefficient on $\hat{\pi}_{t-1}$ always has a negative sign regardless of the relative magnitudes of the alphas (hence the shape of the hazard function). Since a negative coefficient on lagged inflation works against inflation persistence, this theory does not provide a micro-foundation for the intrinsic inflation persistence. Instead, the structural interpretations to those coefficients should be based on the stationary distribution of price vintages derived in the model. This becomes clearer when I rewrite (13) again into the following form:

$$(S(1)+S(2))\hat{\pi}_t = -S(2)\hat{\pi}_{t-1} + S(0)E_t\left[F_t\right] + S(1)E_{t-1}\left[F_{t-1}\right] + S(2)E_{t-2}\left[F_{t-2}\right]$$

$$(S(1)+S(2))\hat{p}_t = S(1)\hat{p}_{t-1} + S(2)\hat{p}_{t-2} + S(0)E_t\left[F_t\right] + S(1)E_{t-1}\left[F_{t-1}\right] + S(2)E_{t-2}\left[F_{t-2}\right]$$

[10] In the more general cases, the pattern of signs on coefficients are the same, only the magnitudes change accordingly.

The coefficients in this equation are then re-expressed in terms of the survival function $S(j)$. According to the definition of the survival function, $S(0), S(1), S(2)$ represent the probability that a price is fixed for $0, 1, 2$ periods respectively. I set $J = 3$, which means a newly reset price can be fixed for a maximum of 2 periods. Intuitively, only the set of surviving prices still exerts influence on the current dynamics, so coefficients in the Phillips curve simply determine the portion of the past prices still affecting the current price.

Furthermore, new insights are gained from generalizing the time-dependent pricing setup. The presence of lagged inflation and lagged expectations have opposite effects on the current inflation due to past fixed prices. Past pricing decisions have positive effects on current inflation through the lagged expectations. Because the price is sticky, the lagged expectations are fixed in the sticky prices. Therefore expectations have a long lasting influence on the economy. The higher the expectations of marginal costs, the higher the current inflation. Contrastingly, past inflation has a negative impact on current inflation due to the "front-loading" effect. Because prices are sticky, firms adjust more than necessary to hedge against the risk that they might not be allowed to re-optimize again in the near future and would therefore be unwilling to react to a current economic condition. The 'front-loading' pricing therefore deters the price adjustment needed in the future. Due to this front-loading effect, a high level of past inflation hinders the ability of current inflation to continue to be high. In the general setting, both effects work against each other, i.e. one strengthens inflation persistence, while the other weakens it. In the Calvo model however, these two effects just cancel each other out.

4.3 The NKPC with trend inflation (g)

If I derive the NKPC by log-linearizing pricing equations around a non-zero trend inflation steady state, one can show that the resulting Phillips curve has the exact same structure as the one without trend inflation. However, trend inflation affects the magnitudes of all coefficients in the NKPC. Again using the example with $J = 3$, we obtain

$$
\begin{aligned}
\hat{\pi}_t &= -\gamma_3 \hat{\pi}_{t-1} + \frac{1}{\Psi} mc_t + \frac{1}{\Psi} mc_{t-1} + \frac{1}{\Psi} mc_{t-2} \\
&+ \gamma_1 E_t \left(\frac{\beta \alpha_1 g^{\eta}}{\Psi} mc_{t+1} + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} mc_{t+2} + \frac{\Psi - 1}{\Psi} \hat{\pi}_{t+1} + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} \hat{\pi}_{t+2} \right) \\
&+ \gamma_2 E_{t-1} \left(\frac{\beta \alpha_1 g^{\eta}}{\Psi} mc_t + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} mc_{t+1} + \frac{\Psi - 1}{\Psi} \hat{\pi}_t + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} \hat{\pi}_{t+1} \right) \\
&+ \gamma_3 E_{t-2} \left(\frac{\beta \alpha_1 g^{\eta}}{\Psi} mc_{t-1} + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} mc_t + \frac{\Psi - 1}{\Psi} \hat{\pi}_{t-1} + \frac{\beta^2 \alpha_1 \alpha_2 g^{2\eta}}{\Psi} \hat{\pi}_t \right) \qquad (14) \\
\gamma_1 &= \frac{1}{\alpha_1 g^{\eta-1} + \alpha_2 \alpha_1 g^{2\eta-2}}, \qquad \gamma_2 = \frac{\alpha_1 g^{\eta-1}}{\alpha_1 g^{\eta-1} + \alpha_2 \alpha_1 g^{2\eta-2}} \\
\gamma_3 &= \frac{\alpha_1 \alpha_2 g^{2\eta-2}}{\alpha_1 g^{\eta-1} + \alpha_2 \alpha_1 g^{2\eta-2}}, \qquad \Psi = 1 + \beta \alpha_1 g^{\eta} + \beta^2 \alpha_1 \alpha_2 g^{2\eta}
\end{aligned}
$$

As seen in this example, trend inflation (g) enters every coefficient in the Phillips curve, and hence, not only has a significant impact on the steady state, but also affects the inflation dynamics in a complex way. In general, γ_1 and γ_2 are decreasing in g, while γ_3 is increasing in g, so changes in trend inflation alter the relative importance between the forward-looking and backward-looking terms in the Phillips curve. To study these important effects more precisely, I simulate this Phillips curve in a general equilibrium framework and report the numerical results of the model in the next section.

5 Numerical experiments

5.1 The general equilibrium model

In the numerical experiment, I study the behavior of inflation dynamics in a general equilibrium setting. For this purpose, I close this model by adding an IS curve, an equation defining marginal cost in terms of log deviation of output and the technology shock, a real money demand equation and a nominal money stock growth rule[11]. The log-linearized equilibrium equations are summarized here:

$$
\begin{aligned}
\hat{\pi}_t &= \sum_{k=0}^{J-1} W_1(k,g) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j,g) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i,g) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k,g) \hat{\pi}_{t-k+1} \\
\widehat{mc}_t &= \frac{\phi+\sigma+a}{1+\eta\phi+\eta a} \hat{y}_t - \frac{1+\phi}{1+\eta\phi+\eta a} \hat{z}_t \\
\sigma E_t[\hat{y}_{t+1}] &= \sigma \hat{y}_t + (\hat{\imath}_t - E_t[\hat{\pi}_{t+1}]) \\
\hat{m}_t &= \sigma \hat{y}_t - \frac{\beta}{1-\beta} \hat{\imath}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \Delta m_t \\
\hat{z}_t &= \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \;\; \epsilon_t \backsim N(0, 0.007^2) \\
\Delta m_t &= \rho_m * \Delta m_{t-1} + \varepsilon_t \quad where \;\; \varepsilon_t \backsim N(0, 0.0025^2)
\end{aligned}
$$

5.2 Calibration

Instead of using micro data sets, which generate conflicting results with regard to the shape of the empirical hazard function[12], I use a novel strategy for parameterizing the hazard function. Since the hazard function in this model is defined in terms of the time-since-last-adjustment, it is reasonable to base its calibration on the well-established statistical theory of duration analysis. In particular, the functional form I apply to parameterize the hazard function is based on the Weibull distribution with two parameters[13].

$$
h(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda} \right)^{\tau-1} \tag{15}
$$

λ is the scale parameter, which controls the average duration of the price adjustment, while τ is the shape parameter to determine the monotonous property of the hazard function. It enables the incorporation of a wide range of hazard functions by using various values of the

[11] All of these equations are derived from a standard New Keynesian framework. The complete model is written in a technical note, which is available upon request from author.

[12] In the literature, mixed evidence has been provided using micro-level data. See, e.g. Dhyne et al. (2006), Alvarez (2007) and references cited therein.

[13] In Appendix(C), I give an introduction to the Weibull distribution.

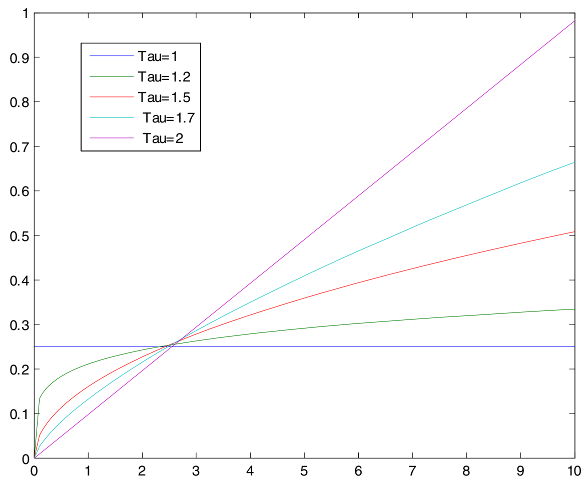

Figure 1: Shapes of the hazard function with various shape parameter values.

shape parameter. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve the Poisson process from the Weibull distribution.

In this numerical experiment, I choose λ, such that it implies an average price duration of 2 quarters, which is consistent with the mean price duration of 6.6 months documented by Bils and Klenow (2004), and the shape parameter is set in the interval between one and two, which covers a wide range of shapes of the hazard function[14]. As for the rest of the structural parameters, I intentionally use some common values in the literature to facilitate comparison between results. In the calibration of the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\sigma = 1$, implying log utility of consumption. I choose a unitary Frisch elasticity of labor supply ($\phi = 1$), values commonly found in the business cycle literature. As for the technology parameters, I set labor's share to be 1 ($a = 0$) and the elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%. Finally, I choose the autocorrelation coefficient parameter of the monetary shock $\rho_m = 0.5$, and choose the innovation to nominal money growth rate to have a standard deviation of 25 basic points per quarter. For the aggregate technology shock, I choose $\rho_z = 0.95$ and innovations with a standard deviation of 0.007, which are commonly used in the RBC literature, for example King and Rebelo (2000).

[14] This range only covers increasing hazard functions because it makes the maximum number of price duration J well defined.

5.3 Simulation results

To evaluate the quantitative performance of the model, I apply the standard algorithm to solve for the log-linearized rational expectation model[15].

5.3.1 Aggregate dynamics of the varying hazard function

In the first experiment, I study the effects of varying the shape parameter on the equilibrium dynamics around the zero-inflation steady state. In Table (2), I report second moments generated by the time-dependent pricing models, which are different with respect to the shape of the hazard function. Because I use the Weibull hazard function to calibrate the model, I can change the shape of the hazard function by varying the value of the shape parameter τ. In this experiment, I focus on the comparison between the baseline Calvo case, corresponding $\tau = 1$, and the increasing hazard models, where τ falls in the range of 1.2 to 2. In all cases, the moments are for a Hodrick-Prescott filtered time series. For each of these hazard functions, two sets of statistics are reported: first, the first-order autocorrelation coefficient of deviations of inflation, real marginal cost and output from steady state; and second, contemporaneous correlation coefficients between inflation and real marginal cost.

	Calvo Model	Increasing-hazard Models				
τ	1	1.2	1.4	1.6	1.8	2
$AR(1)\ \hat{\pi}$	0.583	0.612	0.622	0.628	0.631	0.629
$AR(1)\ \widehat{mc}$	0.586	0.533	0.499	0.447	0.411	0.393
$AR(1)\ \hat{y}$	0.782	0.791	0.804	0.806	0.805	0.804
$Corr(\hat{\pi}, \widehat{mc})$	0.993	0.987	0.982	0.967	0.952	0.943

Table 2: Second moments of the simulated data (HP filtered, lambda=1600)

We can observe that the increasing hazard function helps to increase both persistence of inflation and output. Its effect peaks at a value of around 1.8. On the other hand, persistence of real marginal cost, the driving force of inflation, is decreasing in the shape parameter. In the Calvo case, because inflation persistence is solely determined by dynamics of real marginal cost, inflation persistence cannot exceed persistence of real marginal cost. In the increasing hazard model, however, the autoregressive terms of real marginal cost are brought into the Phillips curve through lagged expectations, and thus in comparison to the Calvo model, my transimission mechanism propagates more inflation persistence. With the shape parameter $\tau = 1.8$, the increasing-hazard model can account for persistent deviations of inflation and output from the steady state, despite the fact that all results are generated by using a modest level of price stickiness (an average duration of 2 quarters). The discussion of the failure of the NKPC, in that it does not provide a complete structural interpretation of inflation persistence was first brought to attention by Fuhrer and Moore (1995). More recently, Fuhrer (2006) presented

[15] I am grateful to Alexander Meyer-Gohde for helping me to calculate the equilibrium with some extreme parameter values, where the computation involves large numbers of lags and leads of expectations. For the details of the algorithm, you can refer to Meyer-Gohde (2007).

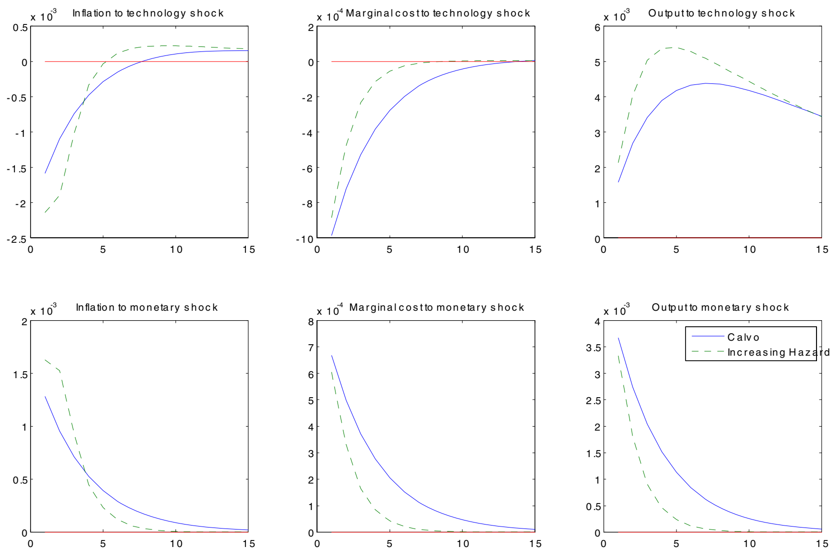

Figure 2: Impulse responses with different hazard functions

empirical evidence showing that it is difficult to have a sizable coefficient on the driving process in the Calvo NKPC and a reduced form shock in the NKPC explains a significant portion of inflation persistence. We can analyze this evidence through the lens of the generalized NKPC. The problem of the conventional NKPC is literally caused by ignoring terms like lagged inflations and lagged expectations. As I show in the analytical result, this is not the case in the more general time-dependent pricing model. The misspecified Phillips curve fails to explain inflation persistence with its limited structure. Consequently, we either need to introduce the ad hoc backward-looking behavior (intrinsic inflation persistence) or a persistent reduced-form shock to achieve a good fit to the data.

In addition, as shown in the last row, the increasing-hazard pricing model also helps to reduce the correlation between inflation and real marginal cost, as it introduces lagged real marginal costs as the driving force for inflation in the NKPC.

Figure 2 shows the impulse responses of the Calvo model and the increasing-hazard model where the shape parameter equals 1.8. The three figures in the upper row depict the impulse responses of deviations on inflation, real marginal cost and output to a positive technology shock. After a persistent technology shock, the responses of output are stronger and more persistent in the increasing-hazard model than in the Calvo case. The dynamics of inflation with an increasing hazard function decreases sharply at the beginning, but reverts quickly to the steady state afterwards, before it changes into a persistent inflation pressure after about 5 quarters. By contrast, the response of real marginal cost is less persistent in the increasing hazard model. This results from the more rapid responses of output to the technology shock in this model. The impulse responses to a 1% increase in the annual nominal money growth rate are shown in the lower row. The nominal money growth shock has fewer real effects on output and marginal cost in the increasing hazard model than those in the Calvo model, because it is less likely that a price is fixed for long periods. However, less price rigidity does not mean less inflation persistence. On the contrary, inflation reacts to the monetary shock in a long-lasting manner. The impulse response has an initial period of persistence before reverting to a quick drop back to the steady state.

5.3.2 Aggregate dynamics around trend inflation

In his seminal paper, Ascari (2004) has shown that trend inflation has important implications for the model's dynamics when the Calvo pricing model is log-linearized around non-zero trend inflation. Here I analyze the dynamic effects of trend inflation in the increasing hazard pricing model. Combining these two features is interesting because, as I have shown in the previous exercise, without trend inflation the increasing hazard model merely changes the timing of the impulse response of inflation and not the general pattern. By contrast, with trend inflation, relative prices disperse quickly over the time elapsed since the last adjustment and, as a result, not only the timing of the price adjustment but also the magnitude of the adjustment are affected by the increasing hazard function. Furthermore, introducing trend inflation affects all coefficients in the generalized NKPC (See Equation 14), and hence it changes the relative importance between the forward-looking and backward-looking terms in the Phillips curve. As a result, trend inflation exerts a larger impact on the dynamics of inflation in the increasing-hazard pricing model than in the Calvo case.

g	$\hat{\pi}$	$\hat{y}$	$\widehat{mc}$	$Corr(\hat{\pi},\widehat{mc})$	U.S. data	$\overline{\pi}$	AC(+1)
1	0.631	0.805	0.411	0.952	96Q1-08Q3	2.2%	0.432
1.02	0.671	0.800	0.427	0.932	84Q1-95Q4	3.2%	0.440
1.05	0.719	0.799	0.448	0.892	70Q1-83Q4	6.4%	0.735

Table 3: Simulated results (HP filtered, lambda = 1600) with varying trend inflation

In Table (3), I report the first-order autocorrelations of $\hat{\pi}$, $\hat{y}$, $\widehat{mc}$ and the correlation between inflation and real marginal cost generated by the model with quarterly trend inflation equaling 0%, 2% and 5% respectively. In addition, I report the average trend inflation and the first-order autocorrelation of the U.S. quarterly Implicit GDP deflator in three sub-periods[16], all detrended using the Hodrick-Prescott filter. Several results stand out. First, I find the trend inflation raises persistence of inflation dynamics. The higher the trend inflation, the more persistent the inflation gap. This pattern is consistent with the one calculated from the U.S. data. Cogley et al. (2008) uses Bayesian methods to study the effects of stochastic volatility on the persistence of the inflation gap and obtains a similar result. Their VAR estimates provide strong evidence of a statistically significant increase in inflation gap persistence during the high inflation periods and a significant decline in persistence in the disinflation periods. Second, trend inflation affects the persistence of output and real marginal cost less than inflation. This finding confirms the evidence presented by Fuhrer (2006), who shows that the estimated persistence of inflation may have declined over the past two decades, but it is not the case of autocorrelation of an output gap and a unit labor cost measure. Third, the correlation between inflation and real marginal

[16]Data source: NIPA table 1.3.4 from BEA.

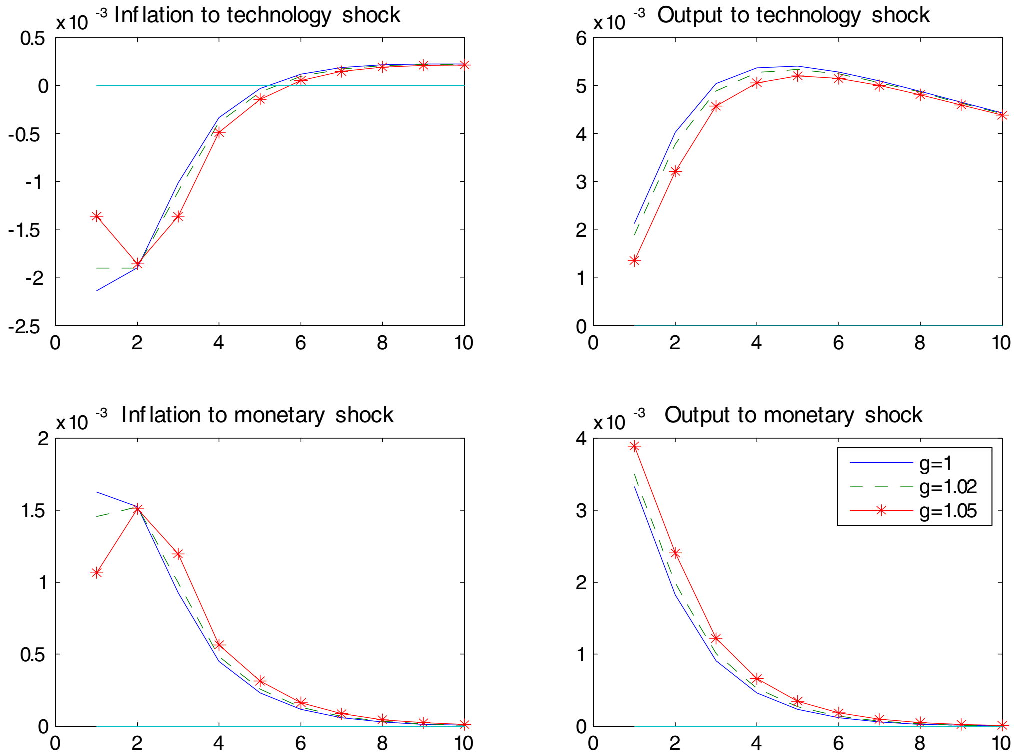

Figure 3: Impulse response function with varying trend inflation

cost decreases even further when trend inflation is high. The intuition behind these results is that trend inflation has no significant impact on real variables because it is predictable by economic agents, although it does significantly affect the mechanism through which inflation is propagated from the underlying real economy.

Hornstein (2007) documents that changes in average inflation are positively associated with changes in inflation persistence, but negatively associated with the correlation between inflation and real marginal cost over time. He also finds that these patterns, however, could not generated by the hybrid NKPC after it is modified for trend inflation. By contrast, the increasing hazard model, loglinearized around trend inflation, can replicate these stylized facts in the data at least qualitatively.

To understand why trend inflation makes the inflation gap more persistent and what the role of the increasing hazard function plays in this transmission process, I plot the impulse responses of output and inflation to the technology and nominal money growth shocks in Figure 3. For both shocks we observe that high trend inflation has more impact on inflation than on output. In particular, inflation responses become hump-shaped when trend inflation is high, while responses of output to the shocks have roughly the same pattern, just with a slightly different magnitude. The reason the combination of high trend inflation and an increasing hazard function gives rise to the hump-shaped responses of inflation is for one, that the increasing hazard function with respect to the time-since-last-adjustment increases the proportion of firms that respond late to a shock. There is only a few firms adjusting their prices immediately after a shock, but more and more adjust later on, thereby making the timing of the adjustment postponed. On the other hand, trend inflation amplifies this effect further, as high trend inflation causes relative prices to disperse quickly and thereby the size of a firm's first adjustment is increasing in the time since the shock occured. Put into another words, in the increasing hazard model, the fact that firms are more likely to adjust when their prices are old means that the average size of firms' adjustments will tend to increase in the first few periods after the shock, leading to a hump-shaped response.

6 Conclusion

The central theme of this study is to pursue the rationale and the effects of the inclusion of lagged inflation and lagged expectations in the new Keynesian Phillips curve. I derive a generalized NKPC in a micro-founded optimizing model, reflecting the non-constant hazard function and non-zero trend inflation. While the standing theory of the Phillips curve has argued that, in order to generate inflation persistence in the data, the NKPC needs to incorporate the lagged inflation with a significant positive coefficient, which is interpreted as 'intrinsic inflation persistence', I however, show that this is not the case in the general time-dependent pricing model.

The generalized NKPC involves components including lagged inflation, forward-looking and lagged expectations of inflations and real marginal cost, which nests the standard Calvo Phillips curve as a limiting case. This enables the introduction of lagged inflation into the Phillips curve without the need for new microfoundations. However, the coefficients of lagged inflations are always negative, regardless of specifications in the hazard function. In light of these results, inflation persistence generated by this optimizing model is not intrinsic. Instead, more persistent inflation in this model results from the 'long-memory' character of the economy introduced through lagged expectations.

In the numerical exercise, I contribute to the literature with a new approach to parameterize the hazard function by using the Weibull distribution. The main advantage of this approach is that it allows for flexible characteristics in the hazard function, which in turn provides an alternative discipline to calibrate the parameters of the distribution of price vintages. More importantly, this parsimonious approach makes the underlying mechanism transparent. The numerical results show that deviations in inflation and output are more persistent in the increasing hazard model than in the Calvo case. Furthermore, introducing trend inflation strengthens the effects of the increasing hazard function on the inflation dynamics. The model can jointly account for persistent dynamics of inflation and output, hump-shaped impulse responses of inflation to monetary shocks, as well as the fact that high trend inflation leads to more persistence in inflation and not for real variables.

To summarize, the generalized Phillips curve generates more persistent inflation dynamics than the benchmark Calvo NKPC, but not because of the 'intrinsic' inflation persistence. The new insights won from relaxing the restrictive 'memoryless' assumption in the standard Calvo model are that both lagged inflation and lagged expectations are inherent in the NKPC, reflecting the long memory of the inflation dynamics.

A Deviation of the New Keynesian Phillips curve

A.1 Log-linearize the optimal price equation

First I log-linearize the optimal price equation (7) around the steady state equation (8)

$$\begin{aligned}\sum_{j=0}^{J-1} S(j)\beta^j E_t[Q_{t,t+j}Y_{t+j}P_{t+j}^{\eta-1}]P_t^* &= \frac{\eta}{\eta-1}\sum_{j=0}^{J-1} S(j)\beta^j E_t[Q_{t,t+j}Y_{t+j}P_{t+j}^{\eta-1}MC_{t+j}] \\ \bar{p}_t^*\bar{Y}\bar{P}_t^{\eta-1}\sum_{j=0}^{J-1}\beta^j S(j)g^{(\eta-1)j} &= \frac{\eta}{\eta-1}mc\bar{Y}\bar{P}_t^{\eta}\sum_{j=0}^{J-1}\beta^j S(j)g^{\eta j}\end{aligned}$$

$$\sum_{j=0}^{J-1} S(j)\beta^j\bar{Y}\bar{P}_t^{\eta-1}g^{(\eta-1)j}\bar{p}_t^* E_t[1+\hat{y}_{t+j}+(\eta-1)\hat{p}_{t+j}+\hat{p}_t^*] =$$
$$\frac{\eta}{\eta-1}\sum_{j=0}^{J-1} S(j)\beta^j\bar{Y}\bar{P}_t^{\eta-1}g_t^{\eta j}\overline{MC}_t E_t[1+\hat{y}_{t+j}+(\eta-1)\hat{p}_{t+j}+\widehat{MC}_t]$$

After using the steady state equation to cancel some terms out, we get

$$\sum_{j=0}^{J-1}\frac{\left(\beta g^{\eta-1}\right)^j S(j)}{\Delta}E_t[\hat{y}_{t+j}+(\eta-1)\hat{p}_{t+j}+\hat{p}_t^*] = \sum_{j=0}^{J-1}\frac{\left(\beta g^{\eta}\right)^j S(j)}{\Omega}E_t[\hat{y}_{t+j}+(\eta-1)\hat{p}_{t+j}+\widehat{MC}_t]$$

$$where, \quad \Delta = \sum_{j=0}^{J-1}\left(\beta g^{\eta-1}\right)^j S(j), \quad \text{and} \quad \Omega = \sum_{j=0}^{J-1}\left(\beta g^{\eta}\right)^j S(j)$$

It follows that

$$\hat{p}_t^* = \sum_{j=0}^{J-1}\frac{\left(\beta g^{\eta}\right)^j S(j)}{\Omega}\widehat{MC}_t + \sum_{j=0}^{J-1}\left(\frac{\left(\beta g^{\eta}\right)^j S(j)}{\Omega} - \frac{\left(\beta g^{\eta-1}\right)^j S(j)}{\Delta}\right)E_t[\hat{y}_{t+j}+(\eta-1)\hat{p}_{t+j}]$$

Given that the gross inflation rate is not high, the term in brackets should be close to zero, resulting in the log-linearized optimal price equation.

$$\hat{p}_t^* = \sum_{j=0}^{J-1}\frac{\left(\beta g^{\eta}\right)^j S(j)}{\Omega}\widehat{MC}_t$$

A.2 Derivation of New Keynesian Phillips curve

Here I derive the NKPC for $g = 1$, Starting from 9

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right] \tag{16}$$

$$= E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{p}_{t+j}\right] \tag{17}$$

The last term can be further expressed in terms of future rates of inflation

$$\begin{aligned}
\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{p}_{t+j} &= \frac{1}{\Psi}\hat{p}_t + \frac{\beta S(1)}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S(J-1)}{\Psi}\hat{p}_{t+J-1} \\
&= \frac{1}{\Psi}\hat{p}_t + \frac{\beta S(1)}{\Psi}\hat{p}_t + \frac{\beta S(1)}{\Psi}\left(\hat{p}_{t+1}-\hat{p}_t\right) + \cdots + \frac{\beta^{J-1}S(J-1)}{\Psi}\hat{p}_{t+J-1} \\
&= \left(\frac{1}{\Psi}+\frac{\beta S(1)}{\Psi}\right)\hat{p}_t + \sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+j} + \frac{\beta^2 S(2)}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S(J-1)}{\Psi}\hat{p}_{t+J-2} \\
&= \left(\frac{1}{\Psi}+\frac{\beta S(1)}{\Psi}+\frac{\beta^2 S(2)}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+j} + \sum_{j=2}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+j-1} \\
&\quad + \frac{\beta^3 S(3)}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S(J-1)}{\Psi}\hat{p}_{t+J-2} \\
&\quad \vdots \\
&= \left(\frac{1}{\Psi}+\frac{\beta S(1)}{\Psi}+\cdots+\frac{\beta^{J-1}S(J-1)}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+j} \\
&\quad + \sum_{j=2}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+j-1} + \ldots + \frac{\beta^{J-1}S(J-1)}{\Psi}\hat{\pi}_{t+1} \\
&= \hat{p}_t + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i}
\end{aligned}$$

The optimal price can be expressed in terms of inflation rates, real marginal cost and aggregate prices.

$$\hat{p}_t^* = \hat{p}_t + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i}\right] \tag{18}$$

Next I derive the aggregate price equation as the sum of past optimal prices. I lag equation

18 and substitute it for each $\hat{p}^*_{t-j}$ into equation 10

$$\begin{aligned}
\hat{p}_t &= \theta(0)\ \hat{p}^*_t + \theta(1)\ \hat{p}^*_{t-1} + \cdots + \theta(J-1)\hat{p}^*_{t-J+1} \\
&= \theta(0)\left[\hat{p}_t + E_t\left(\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j}\right) + E_t\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i}\right)\right] \\
&+ \theta(1)\left[\hat{p}_{t-1} + E_{t-1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j-1}\right) + E_{t-1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i-1}\right)\right] \\
&\vdots \\
&+ \theta(J-1)\left[\hat{p}_{t-J+1} + E_{t-J+1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j-J+1}\right) + E_{t-J+1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i-J+1}\right)\right]
\end{aligned}$$

$$\hat{p}_t = \sum_{k=0}^{J-1}\theta(k)\left[\hat{p}_{t-k} + \underbrace{E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i-k}\right)}_{F_{t-k}}\right] \qquad (19)$$

Where F_t summarizes all current and lagged expectations formed at period t.
Finally, we derive the New Keynesian Phillips curve from equation 19.

$$\begin{aligned}
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} + \underbrace{\sum_{k=0}^{J-1}\theta(k)F_{t-k}}_{Q_t} \\
\hat{\pi}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + \theta(0)\hat{p}_{t-1} + \theta(1)\hat{p}_{t-1} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + (\theta(0)+\theta(1))\,\hat{p}_{t-1} + \theta(2)\hat{p}_{t-2} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \underbrace{\theta(0)}_{W(0)}\hat{\pi}_t + \underbrace{(\theta(0)+\theta(1))}_{W(1)}\hat{\pi}_{t-1} + (\theta(0)+\theta(1)+\theta(2))\,\hat{p}_{t-2}\cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&\vdots \\
&= W(0)\ \hat{\pi}_t + W(1)\hat{\pi}_{t-1} + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{W(J-1)}_{=1}\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= W(0)\ \hat{\pi}_t + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{\hat{p}_{t-J+1} - \hat{p}_{t-J+2}}_{-\hat{\pi}_{t-J+2}} + \hat{p}_{t-J+2} - \cdots + \underbrace{\hat{p}_{t-2} - \hat{p}_{t-1}}_{-\hat{\pi}_{t-1}} + Q_t \\
(1-W(0))\hat{\pi}_t &= -(1-W(2))\hat{\pi}_{t-1} - \cdots - (1-W(J-1))\hat{\pi}_{t-J+2} + Q_t \\
\hat{\pi}_t &= -\sum_{k=2}^{J-1}\frac{1-W(k)}{1-\theta(0)}\hat{\pi}_{t-k+1} + \sum_{k=0}^{J-1}\frac{\theta(k)}{1-\theta(0)}F_{t-k}
\end{aligned}$$

The generalized New Keynesian Phillips curve is:

$$\hat{\pi}_t = \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right)$$
$$- \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1} S(j)}{\sum_{j=1}^{J-1} S(j)}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S(j) \tag{20}$$

B Proof for Proposition 1

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Let's denote the constant hazard as $h = 1 - \alpha$. We can rearrange the NKPC 11 in the following way:

$$\begin{aligned}
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} &= (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left((1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-k} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right) \\
\hat{\pi}_t + \alpha \hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} + \cdots &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+ \alpha^2 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots
\end{aligned} \tag{21}$$

Then iterating this equation one period forwards,

$$
\begin{aligned}
\hat{\pi}_{t+1} + \alpha\hat{\pi}_t + \alpha^2\hat{\pi}_{t-1} + \alpha^3\hat{\pi}_{t-2}\cdots &= E_{t+1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i+1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i+1}\right) \\
&+ \alpha E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \\
&+ \alpha^2 E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right) \\
&\vdots \\
\hat{\pi}_{t+1} + \alpha(\hat{\pi}_t + \alpha\hat{\pi}_{t-1} + \alpha^2\hat{\pi}_{t-2}\cdots) &= E_{t+1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i+1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i+1}\right) \\
&+ \alpha E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \\
&+ \alpha^2 E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right) \\
&\vdots
\end{aligned}
$$

Then substitute Equation 21 for the term in the brackets on the left hand side of this equation,

$$
\begin{aligned}
&\hat{\pi}_{t+1} + \alpha E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \\
&+\alpha^2 E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right) \\
&+\alpha^3 E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-2} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-2}\right) \\
&\vdots \\
&= E_{t+1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i+1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i+1}\right) \\
&+\alpha E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \\
&+\alpha^2 E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right) \\
&\vdots
\end{aligned}
$$

After canceling out equaling terms from both sides of the equation, we obtain the following equation:

$$\hat{\pi}_{t+1} = E_{t+1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i+1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i+1}\right)$$

Iterate this equation backwards and rearrange it, we get the familiar NKPC of the Calvo model.

$$\begin{aligned}
\hat{\pi}_t &= E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \\
\hat{\pi}_t &= (1-\alpha)(1-\alpha\beta)mc_t + (1-\alpha)\hat{\pi}_t + \alpha\beta E_t\left(\hat{\pi}_{t+1}\right) \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}mc_t + \beta E_t\left(\hat{\pi}_{t+1}\right)
\end{aligned}$$

C The Weibull distribution

The PDF of Weibull distribution is given by the following expression:

$$Pr(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1} exp\left(-\left(\frac{j}{\lambda}\right)^{\tau}\right)$$

and the cumulative probability function is:

$$F(j) = 1 - exp\left(-\left(\frac{j}{\lambda}\right)^{\tau}\right)$$

The parameters that characterize the Weibull distribution are the scale parameter λ and the shape parameter τ. The shape parameter determines the shape of the Weibull's pdf function, e.g. when $\tau = 1$, it reduces to an exponential case; while with $\tau = 3.4$, the Weibull amounts to the normal distribution. The scale parameter defines the characteristic life of the random process that amounts to the time at which 63.2% of the firms adjust their labor. This can be seen with the evaluation of the cdf function of the Weibull distribution at j equaling the scale parameterλ. Then we have, $F(\lambda) = 1 - e^{(-1)} = 0.632$.

Note that it relates to the mean duration $\bar{j}$ according to the following equation:

$$\bar{j} = \frac{1}{\bar{\alpha}} = \lambda\Gamma(\frac{1}{\tau} + 1), \tag{22}$$

where $\Gamma()$ is the Gamma function.

It follows that the hazard function of Weibull distribution is:

$$h(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1}$$

Note that this hazard is constant when the shape parameter τ equals one, and increasing when τ is greater than one.

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