DEUTSCHE
BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 21/2012

Saving and learning: theory and evidence from saving for child's college

Junyi Zhu

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0
Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

ISBN 978–3–86558–837–1 (Printversion)
ISBN 978–3–86558–838–8 (Internetversion)

Abstract:
This paper analyzes the main uncertainty of college saving - the child's ability - in the context of the saving with learning model. The first section develops a dynamic model combining asset accumulation and learning to explain the parents' forward-looking saving behavior when they are confronted with the real option of college choice due to uncertainty of their child's ability. The model infers that, with enough time spent learning, information can improve parents' welfare. This can be accomplished by improving the allocation of the consumption to accommodate the burden of college cost given both asset status and the child's true ability. Next, I test the implications of the model from the Panel Study of Income Dynamics/Child Development Supplement & Transition into Adulthood (PSID/CDS & TA) (1997-2005) in the second section. This empirical study investigates college saving behavior when learning is present. Data suggest pessimistic and/or rich parents might reduce the level of college saving, which confirms the interaction of wealth and learning effects predicted by this model. The result also supports the state dependence of parents' college expectations and diminishing persistence over time due to learning. Finally, a number of policy improvements on ESA (Education Saving Account) are proposed to encourage parents to learn about their childs ability.

Keywords:
Education saving, search, learning, intertemporal consumption, real option, dynamic panel

JEL-Classification:
D83, D91, C23, I22.

Non-technical summary

When altruistic parents consider the financial preparation for their child's future higher education, they are confronted with the tradeoff between consumption smoothing and the college premium. There are uncertainties involved with the magnitude of and access to this premium: mainly, parents might only have a noisy estimate of their child's ability, which is the key determinant of the college premium.[1] However, college cost (including tuition, room, board and other expenditures) is expensive in the US and thus it takes time for parents to make adequate financial preparations before the child reaches college age.[2] When the outcome of college entrance is unexpected, there might be redundant saving for college if the child cannot go to college, or insufficient saving if she is eligible for college.[3] Either case will cause welfare loss in terms of either sacrificing consumption in earlier periods or passing up chance to improve the child's career return. To reduce these risks, naturally, parents can utilize observations of the child's various performances to well gauge her ability and thus decrease uncertainty. I adopt this angle to develop the model by linking consumption smoothing and learning in an attempt to explain several empirical characteristics of college saving and provide an insight into policy related to college saving.

The data imply that parents do assess and learn about their child's ability when financially preparing for the future college cost. Whenever this consideration is controlled, the college saving turns out to be positive for many households in our sample, which contrasts with the saving disincentive in the traditional argument on the implicit tax induced by needs-based financial aid. Also, the empirical finding agrees with the model prediction: pessimistic and/or rich parents reduces education saving.

[1] Besides the traditional roles in deciding the scholiastic performance during the college years and predicting the post-college earning power in the market, ability has the impacts on the premium such as the suitability with college education and precollege academic/non-academic performances, which are the important factors involved with the possibility and quality of college admission.

[2] This study assumes that this is the same age for everyone. It is at the age of 18, which is true for the majority of college-goers in the US. Earlier college enrollment is excluded.

[3] Throughout the paper, this eligibility means that the child's college premium determined by her revealed ability is worth the college investment.

Nicht-technische Zusammenfassung

Wenn sich umsichtige Eltern mit der Finanzplanung für die spätere Hochschulausbildung ihres Kindes beschäftigen, gilt es zwischen Konsumglättung und der "college premium" (dem höheren Verdienst von Hochschulabsolventen gegenüber Menschen ohne Hochschulabschluss) abzuwägen. Hinsichtlich der Höhe dieses Mehrverdienstes sowie der Frage, ob das Kind ihn tatsächlich erzielen kann, bestehen gewisse Unsicherheiten, die vor allem damit zusammenhängen, dass Eltern die Fähigkeiten ihres Kindes möglicherweise noch nicht einschätzen können, diese aber entscheidend dafür sind, ob es eine höhere Ausbildung absolvieren und folglich mehr Einkommen erzielen können wird.[4] In den Vereinigten Staaten geht ein Universitätsbesuch jedoch mit sehr hohen Kosten einher (Studiengebühren, Ausgaben für Miete, Verpflegung etc.), sodass die Eltern Zeit brauchen, angemessene finanzielle Vorkehrungen zu treffen, bevor ihr Kind das Universitätseintrittsalter erreicht.[5] Kommt es im Hinblick auf den Universitätsbesuch zu einem unerwarteten Ergebnis, dann entsteht entweder ein Sparüberschuss (wenn für einen Universitätsbesuch angespart wurde, der nicht stattfindet) oder ein Ersparnismangel (wenn nicht angespart wurde, das Kind aber doch für einen Universitätsbesuch geeignet wäre).[6] In beiden Fällen kommt es zu Wohlfahrtsverlusten, entweder durch die Einschränkung der Konsumausgaben im Vorfeld oder durch die verpasste Möglichkeit, das Einkommen des Kindes zu verbessern. Um diese Risiken zu minimieren, können Eltern ihr Kind natürlich genau beobachten und damit die Unsicherheit bezüglich seines Leistungsvermögens senken; so können sie aus den Leistungen, die das Kind in verschiedenen Bereichen erbringt, eine allgemeine Einschätzung seiner Fähigkeiten ableiten. Ausgehend von dieser Perspektive wird durch Herstellung eines Zusammenhangs zwischen Konsumglättung und Lernen ein Modell entwickelt, das darauf abzielt, gewisse empirische Eigenschaften des Sparens für die Ausbildung eines Kindes zu erklären und daraus nützliche Erkennt-

[4]Die Fähigkeiten des Kindes sind nicht nur maßgeblich für die fachlichen Leistungen während der Studienzeit und die anschließenden Verdienstmöglichkeiten auf dem Arbeitsmarkt, sondern beeinflussen auch die "college premium", da Faktoren wie die Eignung für eine Hochschulausbildung sowie die akademischen und nichtakademischen Leistungen im Vorhochschulalter die Möglichkeiten und Qualität des Hochschuleintritts maßgeblich beeinflussen.

[5]Die vorliegende Untersuchung geht von einem einheitlichen Hochschuleintrittsalter von 18 Jahren aus (dies ist bei der Mehrzahl der US-amerikanischen Studienanfänger der Fall). Eine frühere Aufnahme des Studiums wird nicht berücksichtigt.

[6]Im vorliegenden Papier bedeutet diese Eignung, dass der durch das Leistungsvermögen bestimmte Mehrverdienst, den das Kind bei Eintritt ins Berufsleben erzielt, die Investition in das Universitätsstudium rechtfertigt.

nisse für politische Maßnahmen auf diesem Gebiet zu gewinnen.

Die Daten lassen den Schluss zu, dass Eltern die Fähigkeiten ihres Kindes bei der Kostenplanung für ein eventuelles Hochschulstudium tatsächlich mit einbeziehen und beurteilen. Wird für diese Erwägung kontrolliert, ergibt sich für viele Haushalte unserer Stichprobe eine positive Ersparnis für den Hochschulbesuch. Im Gegensatz dazu setzt das traditionelle Argument der impliziten Besteuerung bei bedarfsorientierten studentischen Finanzhilfen in den USA eher einen negativen Sparanreiz. Überdies stehen die empirischen Befunde im Einklang mit der Modellannahme: bei pessimistischen und/oder wohlhabenden Eltern fällt die Ersparnis für Ausbildungszwecke niedriger aus.

Contents

List of Tables

List of Figures

Saving and learning: theory and evidence from saving for child's college[7]

1 Introduction

This paper begins by taking a look at an empirical finding on education saving. Figure 1 reveals the skewness of a distribution, described as a probability (college saving level when child is in 12th grade | college saving level in 8th grade), with respect to college saving level in 8th grade from the National Education Longitudinal Study of 1988 (NELS88):

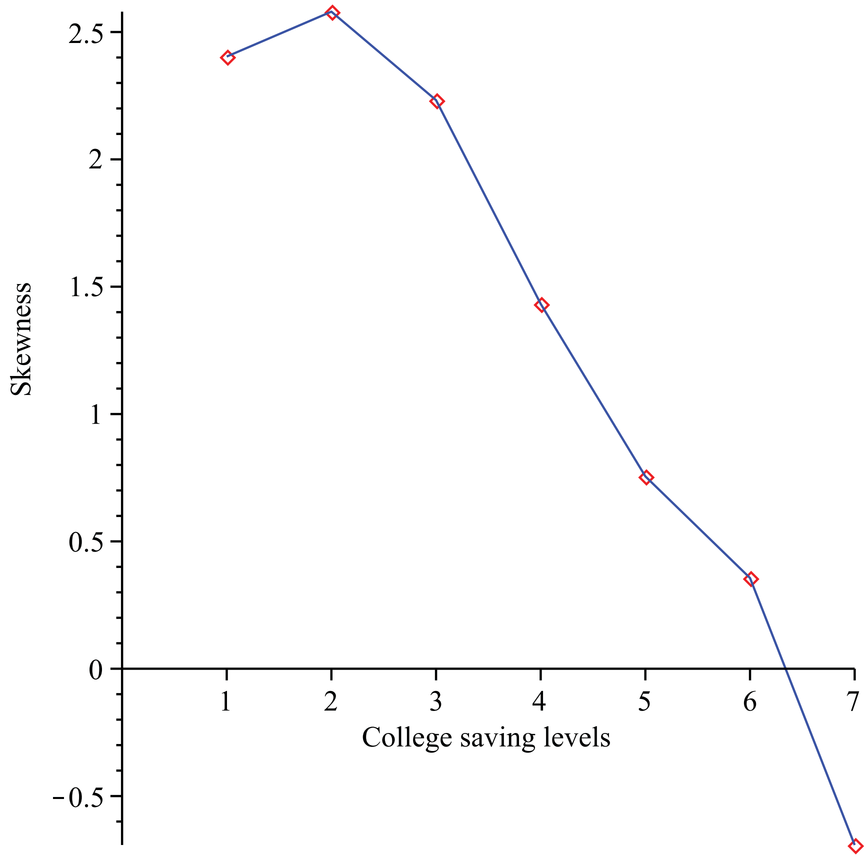
There is striking evidence that skewness decreases with the initial education saving level. Bayesian learning can provide an unsurprising interpretation: parents with a prior lower than the child's true ability are not confident enough to save much at 8th grade. But they have further occasions to boost rather than suppress their beliefs because they are more likely to receive better signals than their priors. The same line of logic can deduce that parents with a prior higher than the true one would achieve the opposite state. The naturally positive relationship between belief and college saving can produce this observation.

Building on this insight, section 2 introduces both learning and wealth effects on the investment in risky assets (college saving) in the search with saving model.[8] Value of learning, though under full self-enforcement and availability of information in my model, can be restricted only by physical constraints, which is different from other typical learning models where only unwillingness or inaccessibility can distort information acquisition.

The empirical study in section 3 using the Income Dynamics/Child Development Supplement & Transition into Adulthood (PSID/CDS & TA) (1997-2005) tests important predictions from the theoretical model. This is the first investigation on the existence of learning behavior and its interaction with wealth in the context of college

[7]I am grateful to Sílvio Rendon, Hugo Benítez-Silva, Warren Sanderson, Mark Montgomery, Heinz Herrmann as well as the seminar participants in EEA, MEA and Deutsche Bundesbank for useful comments. Additionally, I would like to express my appreciation to Elizabeth Hawker and Adriana Calabrese who offered the excellent help in proofreading and translation. The views expressed in this paper are those of the author and should not necessarily be interpreted as those of the Deutsche Bundesbank. Contact: Economic Research Centre, Wilhelm-Epstein-Str. 14, 60431 Frankfurt, Germany. Email: junyi.zhu@bundesbank.de.

[8]Miao and Wang (2006) propose a similar framework with the notion of real option instead of search. But they focus the wealth effect on the survival rate of entrepreneur investment.



Source: NELS88 (see Appendix A.1 for details on college saving questions from this survey). The college saving level is an ordered variable represented by the tickmarks in the horizontal axis. The ranges represented are:

8th grade ($)- **1**: none, **2**: <1,000, **3**: 1,001-3,000, **4**: 3,001-6,000, **5**: 6,001-10,000, **6**: 10,001-15,000, **7**: >15,000;

12th grade ($)- **1**: none, **2**: <1,000, **3**: 1,001-5,000, **4**: 5,001-10,000, **5**: 10,001-15,000, **6**: 15,001-30,000, **7**: >30,000.

Figure 1: Skewness of the raw distribution: probability (college saving level when child is in 12th grade) conditional on college saving level in 8th grade

saving. The estimated holdings of college saving are around $1,200 during 1997-2005 for a family with a child of average ability and zero total net wealth in the sample.

Finally, Section 4 evaluates the economic efficiency of education saving accounts (ESAs) and provides some policy suggestions. Section 5 summarises.

2 Asset accumulation with learning: a theoretical model of college saving

I construct a finite-horizon saving model where the saving decision dependent on asset level and belief in a child's ability over time. In the last period, parents decide whether to send this child to college conditional on their wealth status and the child's revealed true ability level, which creates an option value. By observing the child's performance each period, parents use a Bayesian rule to update their belief and direct their saving strategy by choosing whether to orient towards an additional constant tuition cost. This model is among the first efforts to incorporate learning into a search with saving model.

The main implications from the college saving model with learning expounded in this section are:

- Optimistic parents tend to save more in terms of total assets and extra contributions towards tuition cost.
- Richer and/or optimistic parents set a more lenient standard in terms of the child's performance when making college-bound saving decisions.
- There might be a disincentive to invest in college saving as parents become richer.
- Given enough learning time, information can improve parents' welfare by inducing the poorer and/or pessimistic parents to start college saving earlier for their children who merit a college education and helping richer and/or optimistic parents to reduce the unnecessary college saving for their children who are not as deserving.[9]

Section 2.1 summarizes the related literature. Section 2.2 presents the model setup and implications. Section 2.3 illustrates the policy rules and the contribution of learning in this model.

[9]Simply speaking, parents' intertemporal consumption-saving choice is closer to the optimal state when uncertainty of the child's ability disappears.

2.1 Literature

This section draws the ideas from the research on saving with search and search with learning and explains my contribution to various aspects of this literature. The reliance of the optimal stopping rule about ability belief on the wealth level creates the dynamic interaction between saving and learning behavior in my model, which is not present in the rest of the search models.[10] Studies on saving with search appear in Danforth (1979), Rendon (2006) and Hansen and İmrohoroğlu (1992). Danforth (1979) and Rendon (2006) show the property that the richer the worker, the more selective he is meaning that his reserved wage is higher. Similarly, my model claims that richer parents are more lenient meaning that their lowest belief standard of the child's ability declines. Jovanovic (1979) is a seminal contribution on how the decision maker searches for the best offer with uncertainty while learning about his return ability, which determines his offer. It explains the evolution of policy rules by a dynamic search model with learning and uses them to interpret features of labor market data.

My model contributes to the real option literature by introducing learning, the wealth effect and their joint impact on option value. Few studies have incorporated learning and an intertemporal consumption-saving motive into a real option model. Miao and Wang (2006, 2007) introduce a precautionary saving motive and learning into an entrepreneur's investment decision regarding real option.[11] Volatility can increase both the option and learning values, which might, however, be offset by the precautionary saving motive. Campanale (2004) also uses a consumption-saving model with learning to suggest values from learning and the option of exiting self-employment can explain why an agent might remain an entrepreneur even if return is below the one on the stock upon entry. Many papers focus on the application of a real option model in human capital investment. Heckman et al. (2006) show both theoretically and empirically that the existence of a sequential option of schooling choice and uncertainty of the child's future return can account for the downward bias in a static Mincer equation estimation.[12] Hogan and Walker (2007) and Jacobs

[10]The parents have to seek the optimal stopping rule about the belief updated each period to determine whether they should stay and save in the riskier but better paid saving state towards college or a non-college-bound state with a predetermined saving strategy but less payoff.

[11]Dixit et al. (1994) pioneer the concept of real option.

[12]The others include Belzil and Hansen (2002); Cunha et al. (2005); Keane and Wolpin (2001) and Heckman and Navarro (2007).

(2007) apply real option theory in human capital investment to discuss the timing of college/working choice.

There have been many studies exploring the interrelationship between consumption smoothing and college investment or intra-family interaction due to learning. But these studies do not connect financial preparation with learning, which is emphasized in my model. Keane and Wolpin (2001) and Morris (2003) build and estimate the parents' consumption-saving model when financially preparing for a child's college education with her future return uncertainty. Cudmore (2005) develops a theoretical model where parents make investments in the human capital of their children and save for post secondary education expenses. This model captures the fact that the potential opt-out penalty will reinforce the incentive to invest earlier in a child's human capital when college saving accounts have been chosen.[13] Learning about a child's ability is heavily documented in the psychological literature on the parent-child relationships and child development (Maccoby and Martin 1983; Osofsky and Connors 1979). Akabayashi (2006) proposes a dynamic model of a child's human capital development with the introduction of an endogenous time preference (Becker and Mulligan 1997) and imperfect monitoring on the basis of parent-child behavioral interaction. The author analyzes the impact of belief differences on how parents learn about the child's potential. The solutions are used to disentangle the causes of maltreatment.

2.2 Model

Above all, I present the main model of saving with multi-period learning, which bears a mixture of impacts from wealth and learning on policy rules. To disentangle the wealth and learning effects, particulary due to the presence of an option value in college-bound choice each period, I will then introduce one auxiliary model without uncertainty and another with uncertainty and one-period learning.[14] The implications are then drawn from the contrast study between the main model and the others.

[13]This section shares the same intuition by showing there is a motive to avoid loss in terms of consumption smoothing due to insufficient saving as long as parents have made contributions to college saving previously. This is actually self-enforcing so that the presence of college-saving accounts is not required.

[14]The former has neither wealth or learning effects and the latter has almost only the wealth effect. They are actually simplified versions of the main model constructed for illustrative purposes.

2.2.1 Main model

Belief updating Consider a college saving model with $T+2$ periods. A child is born at the beginning of period 0 with ability parameter θ and parents are endowed with a prior of θ: μ.[15] The true θ is not revealed until the beginning of the $(T+1)$th period. At the beginning of each period from 0 to T, parents observe a noisy signal $y_t = \theta + u_t$, such as academic performance, where u_t is the random noise. Then, they use y_t to infer the true θ by applying the Bayesian rule. After the updated belief in ability m_t is estimated, parents incorporate it in making a choice regarding extra saving for their child's college tuition in addition to the motive to smooth consumption. At the end of period $T+1$, altruistic parents retire and determine whether their child will be sent to college given the assets accumulated and the true ability revealed.[16] The tradeoff lies between a constant tuition cost C and the child's discounted college premium between white-collar and blue-collar careers. The return from the child's career, as viewed by the parents, is simply defined as a multiplicity of one fixed factor and her ability parameter: the college-educated child will become white-collar and receive $w\theta$, and the child without college education will become blue-collar and receive $b\theta$, where $w > b > 0$ (both w and b also contain the altruism parameter).[17] Parents also receive an income stream $\{I_t\}$ from period 0 to $T+1$.[18]

I assume that θ and u_t are independently distributed with $\theta \sim N(\mu, \sigma_0^2)$ and $u_t \sim N(0, \sigma_\mu^2)$.[19] For $t \geq 0$, define posterior at period t: $m_t = \mathrm{E}(\theta | y^t)$, prior at period 0: $m_{-1} = \mu$, and variance $\Sigma_{t+1} = \mathrm{E}(\theta - m_t)^2$. m_t and Σ_{t+1} can be computed via the

[15] In this study, the belief in a child's ability is often also interpreted as the parent's confidence level.

[16] The assumption that the uncertainty of ability is resolved at the end of period $T+1$ reflects the fact that offer decisions from a range of colleges can actually reveal the true ability related to job market return. Namely, there is ability sorting across schools. Note I assume that the college premium is exogenously uniform across schools.

[17] The range for θ is the whole real line. Therefore, the more accurate interpretation for the career return is the deviation from a baseline constant. I use this specification to acknowledge that the child might not suit the college education so that they can have a negative θ. Since the baseline constant return is unrelated to the solution, I simply ignore it.

[18] At the current stage, I introduce income mainly as a reference for calibrating other variables, such as assets and tuition. I can propose the income dynamics in the future development.

[19] To introduce the learning process, I follow the description from Sections 5.6.2 and 6.7 of Ljungqvist and Sargent (2004) and also use the recursive projection technique (the Kalman filter) from Section 5 of this book.

Kalman filter:

$$m_t = (1 - K_t)m_{t-1} + K_t y_t \tag{1a}$$

$$K_t = \frac{\Sigma_t}{\Sigma_t + R} \tag{1b}$$

$$\Sigma_{t+1} = \frac{\Sigma_t R}{\Sigma_t + R}, \tag{1c}$$

where $R = \sigma_u^2$ and $\Sigma_0 = \sigma_0^2$ the unconditional variance of θ. The recursions can be initialized from $m_{-1} = \mu$ given Σ_0. From the parents' *ex ante* perspective at $t = 0, \ldots, T-1$, by using all the information $y^t = y_0, \ldots, y_t$, m_{t+1} is drawn from a normal distribution $F(\cdot|m_t, t)$ with mean m_t and variance $K_{t+1}\Sigma_{t+1}$. Similarly, at $t = T$, $m_{T+1} = \theta$ is drawn from a normal distribution $F(\cdot|m_T, T)$ with mean m_T and variance Σ_{T+1}.

Problem for the parents At each period t, parents make a decision about the best asset level for the next period A_{t+1}, based on the updated belief m_t and assets A_t. As for the same initial prior m_{-1}, initial asset A_0 is endowed. There is a constant rate of return r for both saving and borrowing.[20] The discount factor is $\beta \in (0, 1)$. The problem for parents is described by a Bellman equation:

$$V_t(A_t, m_t) = U(A_t + I_t - \frac{A_{t+1}}{R}) + \beta V_{t+1}(A_t, m_t), \tag{2}$$

for $t = 0, \ldots, T$. At period $T+1$, given asset A_{T+1}, revealed ability parameter θ and income I_{T+1}, $V_{T+1}(A_{T+1}, \theta) = \max[V^w_{T+1}(A_{T+1}, \theta), V^b_{T+1}(A_{T+1}, \theta)]$, where $V^w_{T+1}(A_{T+1}, \theta)$ and $V^b_{T+1}(A_{T+1}, \theta)$ are defined as follows:

- White-collar state:

$$V^w_{T+1}(A_{T+1}, \theta) = U\left(A_{T+1} + I_{T+1} - C\right) + \beta w\theta. \tag{3}$$

  Parents pay tuition C and send this child to college to receive a discounted altruistic payoff $\beta w\theta$ from the child's post-graduation job market earnings as a white-collar worker.[21]

[20] I use $R = 1 + r$ for the rest of the section.

[21] This is to admit that parents have to wait for one more period (until the child's graduation) to harvest the return. A constant tuition cost C represents the fact there is always an entry cost for college no matter what level it is in the US. The child's career return $w\theta$ can be viewed as a Mincer equation ln(wage of child). By this interpretation, my specification is actually an intra-household utility.

- Blue-collar state:

$$V^b_{T+1}(A_{T+1}, \theta) = U(A_{T+1} + I_{T+1}) + \beta b \theta. \tag{4}$$

Parents do not send this child to college and receive a discounted altruistic payoff $\beta b \theta$ from the child's job market earnings as a blue-collar worker.[22]

Recursive formulation Parents are all faced with a natural borrowing limit B_t, which means they can borrow up to the discounted value of their income streams (Hakansson 1970; Ljungqvist and Sargent 2004; Miller 1974). The existence of a liquidity constraint for a household is consistent with most previous research.[23] Any borrowing constraint below B_t is redundant with a utility function satisfying the Inada condition (Rendon 2006). To make the white-collar state feasible at period $T+1$, A_{T+1} chosen at period T must be larger than $B^C_T = C - I_{T+1}$. Consequently, there is a tuition budget constraint $B^C_t = -\sum_{s=t}^{T+1} \frac{I_s}{R^{s-t}} + \frac{C}{R^{T+1-t}}$ at $t = 0, \ldots, T$. If A_t is below this constraint, parents ultimately cannot afford tuition even if they keep consuming nothing indefinitely. Bearing this in mind, the value functions confronted by the parents at $t = 0, \ldots, T$, can be divided into two:

- Blue-collar state:

$$\begin{aligned} V^b_t(A_t, m_t) = \max_{B_{t+1} \leq A_{t+1}} \Big\{ & U(A_t + I_t - \frac{A_{t+1}}{R}) \\ & + \beta \int V^b_{t+1}(A_{t+1}, m_{t+1}) \, dF(m_{t+1} | m_t, t) \Big\}, \end{aligned} \tag{5}$$

which represents a state where parents drop their college-bound expectations entirely and maintain a typical finite-horizon saving motive for consumption smoothing only. They certainly receive a blue-collar career return from the child, though the magnitude is still uncertain. The best saving strategy is A^b_{t+1}, which depends on A_t only.

[22] Actually, the discount factor β is unnecessary since a blue-collar career can start immediately after the end of $T+1$. I maintain it for consistency, which only requires a reinterpretation of b.

[23] For instance, Keane and Wolpin (2001).

- White-collar state:

$$V_t^w(A_t, m_t) = \max_{B_{t+1}^C \le A_{t+1}} \Big\{ U(A_t + I_t - \frac{A_{t+1}}{R}) + \beta \int \max\left[V_{t+1}^b(A_{t+1}, m_{t+1}), V_{t+1}^w(A_{t+1}, m_{t+1})\right] dF(m_{t+1}|m_t, t) \Big\}, \tag{6}$$

which represents a state where parents still hold college-bound expectations with extra savings put aside the positive possibility paying tuition. The best saving strategy is A_{t+1}^w, which depends on both A_t and m_t.

Both are defined backwardly from T to 0 and $V_{T+1}^b(A_{T+1}, m_{T+1})$ and $V_{T+1}^w(A_{T+1}, m_{T+1})$ are defined in (4) and (3) respectively. This is a dynamic programming (DP) problem with a finite horizon $T+2$. At each period, there are two state variables asset A_t and updated belief m_t. Policy rule is A_{t+1}, which is either A_{t+1}^w or A_{t+1}^b depending on the state chosen. At $t = 0, \ldots, T+1$, given asset A_t, parents settle the tradeoff between values from two states by determining an ability threshold $\bar{m}_t(A_t) = \{m_t | V_t^w(A_t, m_t) = V_t^b(A_t, m_t)\}$. This is the lowest belief in ability for the white-collar state to be acceptable.

The optimal value function as well as the ability threshold can be classified into three cases depending on A_t: (1) $V_t(A_t, m_t) = V_t^b(A_t, m_t)$ and $\bar{m}_t(A_t) = \infty$ if $A_t < B_t^C$, which means the blue-collar state is the only affordable one; (2) $V_t(A_t, m_t) = V_t^w(A_t, m_t)$ and $\bar{m}_t(A_t) = -\infty$ if $A_{t+1}^b(A_t) \ge B_{t+1}^C$, which means the white-collar state is always dominant;[24] (3) $V_t(A_t, m_t) = \max[V_t^w(A_t, m_t), V_t^b(A_t, m_t)]$ and $\bar{m}_t(A_t) \in (-\infty, \infty)$ if $A_t \ge B_t^C$ and $A_{t+1}^b(A_t) < B_{t+1}^C$, which is between the cases (1) and (2) when both states might be chosen with a positive possibility. Therefore, the blue-collar state is chosen by either very poor parents or averagely rich parents whose incentive to smooth consumption dominates. The white-collar state is picked by averagely rich parents whose optimism in investment in the child's ability dominates or very rich parents.

Whether the impact of learning can play a role is determined by the state chosen. Parents choosing the white-collar state might switch to the blue-collar state later and end up with the blue-collar career though those really staying in the blue-collar state

[24]This results from the fact that V_t^b is never better than V_t^w given this asset status by their definitions in (5) and (6).

cannot switch to white-collar due to the tuition budget constraint.[25] In this sense, the white-collar state bears an option to exit to the blue-collar state any time in the future. Parents might opt out of this white-collar state when they receive a bad enough shock about the child's ability. Thus, the learning value towards this option still exists. On the other hand, the blue-collar state represents a state where parents' willingness to learn about the child's ability disappears since this is an absorbing state.[26] Consequently, the saving choice is no longer dependent on of the effect of belief m_t and all the new information from then on.

Throughout this section, I define *college saving* by subtracting the alternative blue-collar saving level from the best choice among parents in the white-collar state: $A^w_{t+1}\big(A_t, m_t\big) - A^b_{t+1}\big(A_t\big)$, for $m_t \geq \bar{m}_t(A_t)$. This corresponds to the extra saving motive and consumption smoothing borne by parents in preparation for the child's college cost.

2.2.2 Two contrast models

In these alternative models, parents' choices at period $T+1$ are still the same as those specified in equations (3) and (4). If parents know exactly the true ability parameter θ at period 0, by dropping the uncertainty about θ, this is a model without effects of learning and option. Then this problem turns into two saving choices with everything same except endowment at period $T+1$ and salvage values: a college-committed state receives less income in the last period due to tuition cost but higher salvage value from the child's white-collar career return; but the other non-college-committed state (this is just the blue-collar state with a certain payoff from the child) leads to a lower blue-collar career return, though ultimately without tuition cost. An ability threshold $\bar{\theta}(A_0)$ depending on initial asset A_0 can be solved by equalizing the return differential and differences between two present discounted utilities of consumptions from both states. This is the lowest ability for parents to choose the white-collar state. Parents can then pick the better state and corresponding saving strategies and stick with them from period 0. There is no option value

[25]The empirical observation supports the disallowance of this switch: both NELS88 and PSID/CDS & TA show a very low frequency of this pattern of transition for each period (3% in NELS88 between two waves and on average 6.2% among three waves in PSID/CDS & TA), which is far lower than the other three patterns of transitions for a binary college-bound choice. This pattern can be included by introducing income or preference shocks; however, the main model implications are unchanged.

[26]This lack of learning is forced by the budget constraint if $A^b_{t+1}(A_t, m_t) < B^C_{t+1}$.

from period one onwards.

On the basis of the case without uncertainty, I then introduce the ability uncertainty with one-period learning. Parents always evaluate the uncertainty by the initial belief before they are confronted with the same choices as those specified in equations (3) and (4) at $t = T + 1$. The value functions become[27]

$$\begin{aligned} V_T^{OL}(A_T, m_{-1}) = \max_{B_{T+1} \leq A_{T+1}} \Big\{ & U(A_T + I_T - \frac{A_{T+1}}{R}) \\ & + \beta \int \max \left[V_{T+1}^b(A_{T+1}, \theta), V_{T+1}^w(A_{T+1}, \theta) \right] dF(\theta | m_{-1}) \Big\} \end{aligned} \tag{7}$$

at period T and

$$V_t^{OL}(A_t, m_{-1}) = \max_{B_{t+1} \leq A_{t+1}} \left\{ U(A_t + I_t - \frac{A_{t+1}}{R}) + \beta \int V_{t+1}^{OL}(A_{t+1}, m_{-1}) \right\} \tag{8}$$

at $t = 0, \ldots, T-1$, where θ is inferred from a normal distribution $F(\cdot | m_{-1})$ with initial prior m_{-1} as a mean for the entire time. The policy rules as well as the blue and white collar states are defined in the same way as those in the main model except the optimal stopping rule, which is always defined on the space of the initial prior: $\bar{m}_{-1}(A_t) = \{m_{-1} | V_t^w(A_t, m_{-1}) = V_t^b(A_t, m_{-1})\}$.[28] Furthermore, the value function for the white-collar state at period T embedded in equation (7) can be shown as

$$\begin{aligned} V_T^{OLw}(A_T, m_{-1}) = \max_{B_{T+1}^C \leq A_{T+1}} \Big\{ & \beta^2 (w - b) \int_{\bar{\theta}(A_{T+1})}^{+\infty} F_c(\theta | m_{-1}) \, d\theta \\ & + V_T^{OLb}(A_{T+1}; A_T, m_{-1}) \Big\}, \end{aligned} \tag{9}$$

where $F_c(\cdot | m_{-1}) = 1 - F(\cdot | m_{-1})$, $V_T^{OLb}(A_{T+1}; A_T, m_{-1}) = U(A_T + I_T - \frac{A_{T+1}}{R}) + \beta V_{T+1}^b(A_{T+1}, m_{-1})$ and V_{T+1}^b is specified in equation (3), and $\bar{\theta}(A_{T+1}) = \frac{U(A_{T+1} + I_{T+1}) - U(A_{T+1} + I_{T+1} - C)}{\beta(w-b)}$, *i.e.* the ability threshold in the last period.[29] The main model can be viewed as built from models without uncertainty and one-period learning by sequentially introducing the option value and multi-period learning.

[27]For distinguishing purposes, I use superscript OL to denote the model of one-period learning as below; and the main model is then labeled main model.

[28]To avoid redundant notations, I only present below the value functions of the blue and white collar states at period T.

[29]V_T^{OLb} is the same as value function (5) of the blue-collar state at T in the main model except that the initial prior m_{-1} is instead used to infer θ.

2.2.3 Wealth effect

In this and the next subsections, I present the wealth and learning effects by mainly using a one-period learning model due to its simplicity. Wealth effects discuss how one state variable asset is related to the policy rules, and learning effects summarize how the other state variable prior determines the policy rules by itself and together with the assets. Note that it is implicitly assumed that the other state variable is fixed when all of the wealth effects and learning effects I & II are introduced below. Since the one-period learning model also bears the option value, which wealth effects are concerned with, all of the wealth effects are shared by our main model. The first two learning effects are shared by both models and the last two are not, which will be expounded in Section 2.2.4.

The second term in value function (9) bears the usual saving motive to smooth consumption. The first term shows the motive of saving additionally for college. This actually represents the option value of the college choice structure in this model which does not exist in the model without uncertainty. The college saving defined at the end of Section 2.2.1 can be interpreted as the investment in this risky option value. Then, the first and second order derivatives of this term with respect to A_{T+1} are

$$-F_c(\bar{\theta})\bar{\theta}' \tag{10}$$

and

$$f(\bar{\theta})(\bar{\theta}')^2 + F_c(\bar{\theta})(-\bar{\theta}''), \tag{11}$$

where $f(\bar{\theta}) = F'(\cdot|m_{-1})$, $\bar{\theta}' < 0$ and $\bar{\theta}'' > 0$ by the definition of $\bar{\theta}(A_{T+1})$ and property of CRRA utility function. The first order derivative catches the wealth effect on option, which is positive. A marginal increase in wealth may avoid the hazard of losing a lump-sum value, the total discounted college premium $\beta(w-b)\theta$, since richness further relaxes their ability threshold. These two factors interactively determine

- ***Wealth effect I**: richer parents allow a lower ability threshold in selecting an acceptable child for a white-collar state over time.*[30]

The first term in (11), the second order derivative of the wealth effect on option value, produces the convexity so that we have

[30]The optimal stopping rule is defined on the other state variable, m_t. This adds the other line of dynamics because the interaction of two state variables on the value function plays a significant role in determining the threshold.

- ***Wealth effect II****: there is an incentive to "oversave" meaning extra sacrifice of consumption smoothing to cover beyond the fixed "entry cost" of tuition and this motive increases with the assets.*

Likewise, parents carry on this additional saving motive across all periods. This intuition is actually shared by the discussion of puzzles in the private equity premium of entrepreneur and human capital investment where people extend the duration of self-employment or schooling with an irreconcilable observed return (Campanale 2004; Hogan and Walker 2007; Jacobs 2007; Miao and Wang 2007). These studies use the typical results of a real option model to explain these behaviors: the agent can capture the upside gains by investing and limit the downside losses by simply waiting until the option is sufficiently "in the money". But the perspective is different in my model, which instead focuses on the wealth effect: wealth can increase risk-taking behavior, which has been proposed by Miao and Wang (2006) and Cressy (2000).

However, the second term in equation (11) reveals the other aspect of the wealth effect on option which is not discussed in these studies: the marginal return from the child's ability realization already "in the money" (above threshold) decreases with wealth. This concavity of the wealth effect on option value creates

- ***Wealth effect III****: there is disincentive to increase the contribution for college cost as the assets grow.*,

The total wealth effect on the college saving defined in Section 2.2.1 depends on which term in (11) dominates.[31]

2.2.4 Learning effect

The learning effect is four-fold. The more optimistic parents (with a higher prior) will gain more in saving for the child's chance to be sent to college, which can be reflected by the positive marginal effect of a prior on (10):

- ***Learning effect I****: optimism lowers the ability threshold.*

[31] In the case of the one-period learning model, m_t in college saving defined at the end of Section 2.2.1 is replaced by m_{-1} for the entire period.
Strictly speaking, we can make the statement that the stronger effect II must mean a positive relationship between wealth and college saving. And a negative relationship between wealth and college saving must imply the dominance of effect III. Further statements about the sufficiency and necessity might not be true because they are also involved with a wealth effect on consumption smoothing.

The second one is on the role of belief. It is straightforward to show that the motive to raise college saving increases with m_{-1}:

- ***Learning effect II***: *optimism boosts college saving.*

Both effects are also shared by the main model: the ability threshold is lower and college saving in any period is increasing when prior m_t for any t is higher. The statement about college saving can be seen from its definition at the end of Section 2.2.1 where m_t has only a positive impact on total saving in the white-collar state $A^w_{t+1}(A_t, m_t)$ instead of the blue-collar state saving $A^b_{t+1}(A_t)$.[32] The third one is drawn from the fact that belief distribution degenerates as learning evolves, which exists only in the main model. This means that with enough learning and $\theta > 0$, the ability threshold $\bar{m}_t$ (including the final one $\bar{\theta}$) converges to m_t (including m_T), which leads to the dominance of the convexity of the wealth effect as shown in the first term of equation (11) since $f(\bar{\theta})$ diverges and $F_c(\bar{\theta})$, $\bar{\theta}'$, and $\bar{\theta}''$, given A_{T+1}, are bound.[33] The implication is that

- ***Learning effect III***: *with enough learning and* $\theta > 0$*, the positive relationship between wealth and college saving dominates in the main model.*

This is not certain in the model with one-period learning only. The fourth one links learning with option structure. In the one-period learning model, the college-bound option is open only in the end. None would choose to exit or enter the white-collar state during in-between periods. Otherwise, they should have done so in the very beginning to avoid the cost on consumption smoothing because they can fully forecast this action in period zero given that no new information appears before the last period. However, the main model with multi-period learning provides the chance of entry or exit conditional on the quality of the signal:

[32] The positive relationship between m_t and $A^w_{t+1}(A_t, m_t)$ can be produced by applying Topkis's theorem (Topkis 1978).

[33] The mechanism is that the variance of the prior is smaller over time and ability threshold concentrates around the mean of the prior. Therefore, the cross-sectional ability threshold over different wealth levels becomes flatter and converges around the true ability as learning evolves. The condition $\theta > 0$ can guarantee the convergence since it is certain that $\bar{\theta} > 0$. This actually represents the fact that the college premium is positive, which is true for almost every child in reality according to most studies. The alternative approach to introducing this learning effect is to assume truncated belief distribution, which might complicate the updating rule without bringing any additional insight.

- ***Learning effect IV***: *with enough learning and $\theta > 0$, parents with a high belief and/or high assets reduce the degree of oversaving, and those with a low belief and/or assets can be induced to start college saving, which does not happen in an environment where there is a lack of learning.*

The reason is that they can defer more saving until enough good signals arrive or opt out of the white-collar state given a bad signal. The following sections expand on these issues by showing numerical and empirical evidence.

2.3 Solutions

This problem does not attain an analytical solution due to non-convexity. Since this is a mixture of the optimal stopping and the optimal control problem, the behavior of the value function turns out to be highly unpredictable. Regular polynomial approximations on value function do not yield satisfactory results. I discretize the continuous state spaces to solve it numerically (see Appendix B).[34] The utility specified is a constant relative risk aversion (CRRA) function $U(C) = C^{1-\rho}/(1-\rho)$, where ρ is the coefficient of risk aversion. All the distributions are approximated by truncated normal distribution.

2.3.1 Characterization of policy rules

Figures 2, 3 and 4 present the interactive impacts of the initial prior and the asset on the policy rules of the main model at period one as described in Section 2.2.1.[35] Table 1 reports the parameter values used. The values of the interest rate R, discount factor β and risk aversion coefficient ρ are typical in the literature. Tuition cost C is calculated approximately as a present value of four years' college expenditures with the annual amount equal to the median yearly college expenditure in 2005-\$10,112-considering a growth rate of 6% drawn from the real change between 1997 and 2005 as in my PSID/CDS & TA sample.[36] The income I_t is picked from our target subsample as a median value across waves, which is assumed to be constant in the current stage.

[34] An efficient global optimization routine combined with polynomial approximation is under consideration to enable the model to incorporate more complications.

[35] For this exercise, I set $T = 1$.

[36] See Appendix A.2.5. This growth rate matches those commonly cited in the literature describing tuition change during this period, *e.g.*, Reyes (2007).

And the remaining values are chosen ad hoc in a reasonable range.[37] I plot the policy rules across one state variable while fixing the other one in two contrasting levels. Figure 2(a) shows that saving rises with wealth level in the white-collar state. This is driven by the typical incentive to smooth consumption. Figure 2(b) illustrates that saving is also an increasing function of prior m_{-1}, which is driven by ***learning effect II***. As discussed above, parents with a more confident belief have a stronger motive to save further to secure their college saving. The interactive effect of asset and prior on saving is also positive, as shown in both figures.

Table 1: Baseline parameters used in the exercises for Figures 2, 3, 4, 5 and 6

Parameter	R	β	w	b	C	ρ	σ_u	σ_0	θ	I_t
Baseline value	1.015	0.98	4	1	4.3\15.3[ab]	1.4	2	2	2	5.5[bc]

[a] C is $43,000 in the exercise for Figures 2, 3 and 4 and $153,000 for Figures 5 and 6.
[b] $1,000.
[c] I_t is constant for $t = 0, \dots, T+1$.

Figure 3(a) displays the ability threshold $\bar{m}_0(A_0)$ and the expected threshold for the next period $\mathrm{E}(\bar{m}_1(A_0, m_0 \geq \bar{m}_0(A_0))|m_{-1})$ across the different asset A_0 given two comparing priors m_{-1}. They all decreases in assets, which matches with wealth effect I. In addition, when the wealth reaches a certain level, the threshold drops to minus infinity or, in other words, disappears.[38] Figure 3(b) depicts that the ability threshold does not increase with prior m_1 when belief updating starts, namely, at $t > 0$. This corresponds to ***learning effect I***. Both figures suggest the positive interactive effect of the asset and the prior on ability threshold.

Figures 4(a) and 4(b) demonstrate the impact of asset A_0 and prior m_{-1} on the expected value of college saving $\mathrm{E}(A_1^w(A_0, m_0 \geq \bar{m}_0(A_0)) - A_1^b(A_0))|m_{-1})$. The college saving, as shown in Figure 4(a), seems to have a decreasing trend with asset A_0 when it is large enough. But this pattern is uncertain when the asset level is small. This follows the discussion on ***wealth effects III and IV*** about the ambiguity of the wealth effect.[39] Figure 4(b) has a similar outcome to that in Figure 2(b) and they

[37]The selection of parameters is not the emphasis in this section, which simply showcases policy rules.

[38]They are rich enough so that a saving strategy in the white-collar state must dominate because it always includes the best saving choice from the blue-collar state in their choice set. See the discussion below equation (6).

[39]This outcome could result from the insufficiency of the learning effect particularly when there are only two periods of learning in this numerical exercise.

are both driven by ***learning effect II***.

2.3.2 Evolution of policy rules

This section presents a contrast study between the main model and the auxiliary models defined in Section 2.2.2 in order to extend the discussion of the wealth effect on college saving and the implication of learning as described in ***learning effects III and IV*** .[40]

Figures 5 and 6 illustrate how college savings and ability thresholds evolve overtime in both one-period learning and multi-period saving with learning model under different combinations of initial states. Figure 5(a) shows true ability $\theta = 2$ and its threshold $\bar{\theta}(A_0)$ at period 0 when true ability is known initially. This reveals that, under the setting of parameters in this exercise, it is best to stay in the white-collar state from the beginning and ultimately send this child to college for parents with either high or low A_0 since their ability thresholds are below true ability.

2.3.3 Wealth effect on college saving

Figure 5(b) demonstrates the evolution of college saving in the white-collar state with one-period learning and college-committed state without uncertainty. They all grow over time because the need to save for college becomes more pressing as time approaches the final period. Parents with a high initial prior oversave for the entire duration (***wealth effect II***): their college saving exceeds the level of annualized tuition trajectory in the college-committed state.[41] Parents with a low initial prior but a high initial asset also choose the white-collar state. But parents with a low initial prior and a low initial asset decide not to send this child to college from the beginning.[42] Therefore, given this low prior, wealth is positively related to college saving. However, given a higher prior, low asset parents make greater preparations

[40]The simulations in this section share the same parameters as the exercises in Section 2.3 (see Table 1) except tuition cost, which is $\$153,000$. This much higher figure is to create illustrative outcomes. In fact, the tuition and fees for ivy league schools in 2008-2009 ranged from $\$34,290$ to $\$37,526$ (source: Integrated Postsecondary Education Data System (IPEDS)). Thus, this figure of $\$153,000$ can be considered a present value of tuition and fees for ivy league schools by the same calculation proposed previously.

[41]The level of oversaving can be measured by integrating over time the distance between trajectory in college-committed state and the ones with high initial priors.

[42]See the caption below Figure 5(b) for the interpretation of this type of parents.

in terms of college saving,[43] which again shows evidence of existence/dominance of ***wealth effect III***: concavity of the marginal effect of wealth on option value of college choice.

Figure 5(c) reveals the evolution of college saving when multi-period learning is allowed. Now the positive relationship between wealth and college saving is consistent and strong across different priors. This supports ***learning effect III*** that learning tends to reinforce convex wealth effect on option value. Section 3.3.2 will revisit the impact of learning on wealth effect using empirical results.

2.3.4 Implications of learning on college saving

The value of learning can be particularly characterized in the following two scenarios. Figure 5(b) shows that, in the low prior case, even exposure to information at the last period improves some parents' college-bound decision since it introduces the option value, which is worth more for parents with an asset advantage than those poorer given this low prior: richer ones are thus induced to save and bet on a good realization of θ.[44]

Figure 5(c) illustrates exactly the same scenario described by ***learning effect IV***. There is welfare improvement from introducing learning in the viewpoints of either reducing oversaving behavior among parents optimistic about child's scholastic capability or lifting the college preparation rate among parents pessimistic about their child's scholastic capability.

Besides, Figures 6(a) and 6(b) also confirm both ***wealth effect I*** and ***learning effect I*** about their relationships with the ability threshold in a dynamic context.

[43] It is discernible only at initial periods from this figure. I verified that this is not due to computational errors. There are more distinctive results among other parameter specifications. However, to maximize the visual effect jointly across all the figures, this group of parameters is chosen.

[44] Imagine that parents have to make a college decision based on the same prior $m_{-1} = -1.1$ when no information will be available at any time. Then the expected return differential between a white-collar career and a blue-collar career is negative from the beginning. Therefore, they would definitely not send this child to college no matter what wealth level they have.

3 Empirical evidence on college saving with learning

This empirical examination focuses on how wealth affects the college-bound choice with the presence of learning and how pessimism/optimism determines college saving over time. This is the first investigation on the existence of learning behavior and its interaction with wealth in the context of college saving.

The main prediction about the impact of wealth on college-bound choice from the theoretical model in section 2 is that richness A_t can relax parents' college-bound standard (ability threshold $\bar{m}_t(A_t)$) so that there is a greater chance to observe parents expecting and saving for their child's college at $t+1$. This is ***wealth effect I***. This effect is identified as follows: to identify the college-bound probability change due to wealth, the prior has to be controlled since it also has a positive effect on this probability via ***learning effect I***. Also, this cross-sectional wealth advantage will diminish over time when learning reduces the uncertainty as to the child's ability.[45] Additionally, these wealth effects can only play a role when parents do not opt out of the white-collar state in the previous period. Finally, we should also control the ability signal because a child with a better ability signal should amplify the wealth effect in improving the probability of selecting a white-collar state. In other words, I should test the positive wealth effect on the college-bound probability when state dependence of parents' college-bound choice, convergence of the wealth effect and the ability signal are under control.[46]

The other control variable in my model is saving. I will investigate the validity of the other important property predicted by the model: saving for college should increase with ability prior, namely ***learning effect II***. Simultaneously, the wealth effect on college saving will also be explored.

Tests for these two predictions fall into the models of discrete and continuous variables respectively. Naturally, to account for the prior, we should include the lagged dependent variable since the outcome from the last period should also be the choice based on the prior. The panel technique is preferred to handle the heterogeneity.

[45]Keeping other factors constant, parents should demand a smaller increase in the ability threshold in order to balance the loss of consumption utility given that the unit of wealth decreases by the same amount every subsequent period. Namely, the ability threshold becomes flatter over time. See Footnote [33].

[46]The ability prior is borne by the lagged college-bound choice, which will be explained shortly.

Consequently, the dynamic panel models for both discrete and continuous variablees are taken for this empirical study. Section 3.1 presents the data used. Section 3.2 analyzes the test specifications and empirical barriers. Section 3.3 discusses the results.

3.1 Data

This analysis draws data from the Panel Study of Income Dynamics (PSID), which is a nationally representative sample of households and individuals in the United States. The PSID began in 1968 and presently continues to follow families and individuals. The PSID collects information from both individuals and families, primarily focusing on economics and demographics, including income, employment, family composition, and residential location. I also use data from the Child Development Supplement (CDS) and Transition into Adulthood (TA) to the PSID. With a grant from the National Institute of Child Health and Human Development, researchers for the PSID were able to collect extensive data regarding children's home environment, family, time use at home and school, school and daycare environment, and other cognitive, behavioral, physical, and emotional measures for up to two randomly selected children living in PSID households. Up to three generations of samples are currently available: CDS-I (1997), CDS-II (2002-2003) and TA (2005), which represent wave one to three in this study. CDS-I completed interviews for 3,563 children of 0-12 years of age. CDS-II fielded the data collection by following the same children appearing in CDS-I with a final size of 2,907 children aged 5-18. Finally, TA covers a sample of 760 respondents who were initially in CDS-I, associated with a PSID family in 2005, at least 18 years old and graduated from or no longer attending high school (Mainieri 2006; Stafford et al. 2008).

The empirical study mainly uses the data constructed by linking two waves of Child Development Supplement (CDS) and one wave of Transition into Adulthood (TA) from the Panel Study of Income Dynamics (PSID) (the target subsample has 174 separate families; detailed data description are available in Appendix A). The full sample contains the children/families from the whole TA sample with younger siblings dropped if there are any existed. The observations already enrolled in college at wave two are also excluded. The number of observations available for each ranges from 453 to 630.[47] None of the members in the target subsample have the missing values for any of the measures listed in Table 2 and they follow the same family heads

[47] The raw sample of TA has a sample size of 740.

for all the waves. The sample size for this subsample is 172. Table 2 contains the summary statistics for both the target subsample and the full sample.[48] Appendices A.2 and A.2.5 provide detailed illustration on variable constructions.

As far as the author is aware, this is the only dataset which contains the questions on asset holding, college-bound related choice and child's ability measure when the child is near college age. It covers the state and control variables in my model. The longitudinal nature of this sample can help to overcome the difficulties in measuring parents' belief in a child's ability and to controll unobservable heterogeneity. The former can allow the estimation of learning behavior, which implies a switch in college expectation/outcome and/or change of college saving observed over time in response to different ability signals. The latter can restrain the selection due to a different preference of college choice or college premium. Last but not least, this sample traced the household saving behavior between 1997 and 2005, which is more recent than other data used to capture the growing incentive of college saving due to the fast rise in the price of college.[49]

3.2 Specification and empirical issues

Next, Section 3.2.1 shows the specification for the test on college-bound choice and the presence of the initial condition problem and the solutions involved. Section 3.2.2 covers the specification for the test on college saving level and GMM estimator to resolve the endogeneity issue.

3.2.1 Test on college-bound choice

The specification for a test of wealth effect on college-bound choice is a dynamic binary choice model, where the latent dependent variable y_{it}^* is

$$y_{it}^* = \gamma y_{it-1} + \beta_1 A_{it} + \beta_2 (t-1) A_{it} + \nu(t-1) + \eta s_{it} + x_{it}'\delta + \alpha_i + u_{it}, \tag{12}$$

[48] This target subsample has more children, a lower divorce rate and variation, higher income and wealth, a better test score for the child and other signs which means it contains a more stable and wealthier set of households. Its response to the college expectation and outcome question has a higher mean and a smaller standard deviation. In all of these senses, this subsample with a small sample size should not be expected to pose a systematic bias against my model inquiry about the issue of saving for child's college under learning.

[49] PSID also has a higher response rate of 95-98% and fewer missing data for the wealth information (compared with about 10% of missing values in NLSY wealth data; see Bradley and Corwyn (2002) and Yeung and Conley (2008)).

where $i = 1, \ldots, 174$ and $t = 2, 3$ since the sample has 174 respondents and three waves. y_{it} is the binary choice variable, college-bound choice, defined as $y_{it} = 1$ (white-collar state) if $y^*_{it} > 0$, and 0 (blue-collar state) otherwise. I use a recoded variable of college expectation/outcome from the data as a proxy for y_{it}. y_{it-1} appears to allow the control of the prior since this depends on the posterior formed in the last period, which should be directly related to prior.[50] A_{it} is the asset level for family i at time t. To control the flattening of the ability threshold over time, the joint term of $t-1$ and A_{it} is included together with the time trend $t-1$. The latter might measure the shift in the overall level of belief in a child's ability as well as the college-bound expectation. The ability signal is controlled by a proxy s_{it}, the normalized assessment score. α_i is the individual-specific time-invariant unobservable, which can capture our ability prior, which my model structurally specifies. u_{it} is the error term and $u_{it} \sim N(0, \sigma^2_u)$. All the rest of the control variables appear in the vector x_{it}. Most of these variables attempt to catch the impact of the psychic cost in schooling choice discussed by Heckman et al. (2006), Cunha et al. (2005) and Cunha and Heckman (2008).[51] Keane and Roemer (2009) analyze the cause of psychic costs due to a difference in taste and/or efforts with respect to preparation for college.[52] The other important control variable is the assessment of college expenditure when college attendance probability, average yearly college expenditure for different types of college, state of residence and year of measurement are considered. This is to acknowledge that parents might also use the rational forecast of future college cost when forming their college-bound choice. In equation (12), the main interests lie in β_1 and β_2, which represent the positive wealth effect on college-bound choice and its convergence over time. γ captures the state dependence of college-bound choice and η reveals the impact of new information emerging every period on forming the choice.

The presence of unobservable heterogeneity causes two problems: firstly, the composite error term, $v_{it} = \alpha_i + u_{it}$, is correlated over time due to α_i. For example, the

[50] See Appendix A for related description.

[51] See 1(b) below Table 3 for the coverage of controls. They cover almost all the variables, mentioned by these recent studies, related to psychic costs. The only exclusion is a child's grade because it has perfect collinearity with the college expectation/outcome in the last wave by construction.

[52] Keane and Roemer (2009) discuss, for instance, the relationship between parents' education and college choice: "(i) youth with less educated parents may have less taste for college attendance, partly because it was not inculcated in their youth, partly because it would involve deviating from behavior of their peers, (ii) academic pursuits may be frowned upon in less educated families, (iii) youth with less educated parents may be poorly prepared for college, so attendance would take more effort."

very beginning prior on a child's ability will be carried on when parents repeatedly update their belief and make their college-bound choice. The standard solution is the random effect (RE) model.[53] As documented in Greene (2005), the RE model is much more tractable than the fixed effect (FE) one. Unlike the linear model, the FE binary choice model cannot use differencing or mean deviation transformation to remove the individual dummy. Besides this computational complexity, the notorious incidental parameters problem also arises. In a short panel with small T_i, the number of parameters increases with N. The estimation will be subject to a small sample bias. However, the RE model assumes α_i uncorrelated with all the explanatory variables, which can be too restrictive. Honore and Kyriazidou (2000) propose a fixed effect specification for the dynamic binary choice model. However, the assumption of a single regressor precludes my application. Mundlak (1978) and Chamberlain (1984) allow the correlation between α_i and either the time means of the time-variant explanatory variables or a combination of their lags and leads. Secondly, the combination of unobservable heterogeneity and state dependence can cause an "initial conditions" problem and result in inconsistent estimates of the structural parameters, particularly the state dependence. As documented in Heckman (1981) and Chay and Hyslop (1998), there are two sources of state dependence: one is the structural persistence, as predicted by my model and measured by γ, and the other is heterogeneity. The latter, in my model, includes those observables, such as psychic cost, college cost information, asset and ability signal, and unobservables, such as ability prior and possible serial correlation in error terms. The main issue lies in the correlation between the ability prior contained in α_i and the initial college-bound choice y_{i1}. Heckman (1981), Greene (2003) and Chay and Hyslop (1998) have shown misspecifying the sample initial conditions due to unobservability of pre-sample history is the core problem. The seriousness dramatically increases when the time series is short.

With the strong assumption that initial observations y_{i1} are independent with α_i, *e.g.* ability prior, the model can be estimated by a random effect probit model where the likelihood contribution by individual i at t is

$$\Pr(y_{it}|x_{it}, y_{it-1}, \alpha_i) = \Phi\{(\gamma y_{it-1} + w_{it}'\xi + \alpha_i)(2y_{it} - 1)\},^{54} \tag{13}$$

[53] I apply the RE probit model. The result from a RE logit model is very close.

[54] For a detailed discussion on the estimation of dynamic probit model with RE, see Stewart (2006), from which I borrow most of the notations on model specifications and use his **redprob** and

where w_{it} is a vector including all the rest of regressors in (12) except y_{it-1} and α_i. However, the college-bound choice at wave one is structurally dependent on the prior. I use the approach proposed by Heckman (1981) to specify a reduced-form equation for initial condition $y_{i1}^* = y_{i1}\pi + \tau\alpha_i + u_{i1}$, where y_{i1} is a vector of exogenous instruments (containing x_{i1}).[55] The likelihood function is obtained by

$$\prod_i \int_{\alpha^*} \left[\Phi\{(y_{i1}\pi + \tau\sigma_\alpha\alpha^*)(2y_{i1}-1)\} \prod_{t=2}^{3} \Phi\{(\gamma y_{it-1} + w_{it}'\xi + \sigma_\alpha\alpha^*)(2y_{i1}-1)\} \right] dF(\alpha^*), \tag{14}$$

where $\alpha_i \sim N(0, \sigma_\alpha^2)$ and F is the distribution function of $\alpha^* = \alpha_i/\sigma_\alpha$, the integral is evaluated by Gaussian-Hermite quadrature (Butler and Moffitt 1982). If there is no autocorrelation of u_{it}, Heckman's estimator can render consistent results. Otherwise, this is not true, particularly when the model is exposed to a learning process. Then, I implement another estimator following Hyslop (1999) to extend the Heckman estimator by integrating a high-dimensional normal distribution. Similarly, I use Maximum Simulated Likelihood (MSL) based on the Geweke-Hajivassiliou-Keane (GHK) algorithm (Keane 1994) to approximate the expectations.[56] The AR1 process is specified as follows: $u_{it} = \rho u_{it-1} + \epsilon_{it}$. To explore the learning effect, I also carry out each specification on a contrast case when the normalized assessment score s_{it} is dropped. All the specifications are estimated with both asset with housing, and asset without housing.

Furthermore, the Mundlak correction suggested in the beginning of this section is adopted to explore the possible correlation between individual unobservable and independent variables. This approach simply incorporates the means of the time varying variables into the regressors. I implement this over all of the panel specifications discussed above.

3.2.2 Test on college saving level

The test on both effects of wealth and belief on college saving is given as

$$w_{it} = \pi_1 w_{it-1} + \kappa_1 s_{it} + \zeta c_{it} + \kappa_2 c_{it} s_{it} + \pi_2 c_{it} w_{it-1} + x_{it}'\delta + \alpha_i + u_{it}, \tag{15}$$

redpace module in *Stata* for the implementation of Heckman's estimator and its extension with AR1 errors.

[55]I also use BPI (behavior problem index) measured in wave one since the simple probit estimation shows a significant effect from it.

[56]The GHK algorithm transforms the probability of an observed sequence of random variable as the product of recursively defined conditional probability, which can be much easier for forming simulated expectation (Stewart 2006).

where $i = 1, \ldots, 174$ and $t = 1, \ldots, 3$. For each family i at period t, w_{it} is the log-transformed asset level, s_{it} normalized assessment score, c_{it} college expectation/outcome, x_{it} the vector of control variables, α_i the individual-specific time-invariant unobservable and u_{it} the error term and $u_{it} \sim N(0, \sigma_u^2)$.[57]. As discussed previously, the lagged asset/saving level is included since it has the information on the prior. π_1 measures the regular life-cycle saving motive except the preparation for child's college, where the latter is caught by π_2. To capture the belief effect on college saving, I use the combination of the single regressor about the assessment score s_{it} and its joint term with college expectation/outcome c_{it}. The single term plus other control variables and individual-specific heterogeneity α_i should work together as a control of the ability posterior which determines asset holding w_{it} in my model. After the posterior is controlled, the better quality of the ability signal s_{it} implies the lower ability prior held by pessimistic parents, who view the signal as a big shock. Therefore, κ_2 should measure the inverse effect of our prior on college saving and κ_1 accounts for the effect from posterior. In addition, the combination of ζ, $kappa_2$ and pi_2 estimates the change of saving due to the switch of college-bound choice.

This is a dynamic panel data model where the lagged dependent variable is correlated with the disturbance. This renders the inconsistency by standard estimation of either a fixed or random effect model, which adopts first differences in the first step to remove individual specific α_i:

$$\triangle w_{it} = \pi_1 \triangle w_{it-1} + \kappa_1 \triangle s_{it} + \zeta \triangle c_{it} + \kappa_2 \triangle c_{it} s_{it} + \pi_2 c_{it} \triangle w_{it-1} + \triangle x_{it}' \delta + \triangle u_{it}. \quad (16)$$

But $\triangle w_{it-1}$ is correlated with $\triangle u_{it}$, plus the first difference of the predetermined variables also become endogenous.[58] Arellano and Bond (1991) propose the solution of using the appropriately lagged dependent variables and lagged levels of predetermined variables as instruments to form a generalized method of moments (GMM) estimator. Therefore, in my case, every observation in the third wave would have two observations of the dependent variable as instruments,[59] three observations of the predetermined variables as instruments, and all observations of the exogenous variables. To account for the weak instrument issue of lagged levels for first differences, Arellano and Bover

[57]The log-transformation for the asset is $w_{it} = \ln(\hat{w}_{it}+1)$ if $\hat{w}_{it} \leq 0$ and $-\ln(-\hat{w}_{it}+1)$ otherwise, where $\hat{w}_{it}$ is the raw value. x_{it} shares the same variables as those in equation (12) plus the child's grade. See 1(b) below Table 3 for the coverage.

[58]No. of children, urbanicity, ln total income[t] and normalized assessment score are suspected to be predetermined. They might be related to the previous shocks for the history of a saving decision.

[59]They are from waves zero and one.

(1995) introduced the main level equation (15) into the system of equation estimated. I further specify a two-step estimator to firstly achieve a covariance matrix robust to panel-specific autocorrelation and heteroskedasticity, use it to update the initial weighting matrix and then implement GMM again. Both the asset including housing and the asset excluding housing are tested.

3.3 Results

3.3.1 College-bound choice

Tables 3 and 4 display the estimate result for the model specified by equation 12. They are the estimates of wealth, learning and state dependence on college expectation/outcome under different specifications and asset measures discussed previously.[60] The control variables incorporated are discussed under these tables along with the definition of all the parameters presented and issues of model comparisons (see Tables 3 and 4).

Including the asset excluding housing as the regressor, Table 3 demonstrates that the estimates of the structural parameters under all the specifications exhibit the correct sign as predicted by the model. College expectation $t-1$ bears a positive coefficient γ, illustrating the existence of state dependence. In the context of learning, this is postulated to contain the updated belief up to the last period. The coefficient of the assessment score η is positive too. Parents make college-bound inferences by observing the child's ability signal. All in all, the framework of Bayesian learning can reasonably explain the mechanism of college-bound expectation as the estimates suggest. The positive β_1 illustrates that richness lowers the selectivity as ***wealth effect I*** infers. The joint term of $t-1$ and asset renders a negative coefficient β_2. This assures that the ability threshold tends to flatten out over time due to the convergence of the belief distribution. The negativity of the time trend coefficient ν reveals the plausible initial overestimate of the child's ability in this sample.

In terms of specifications, we can firstly compare different models under the RE assumption. The first pooled probit estimates disallow any cross-correlation between the composite error terms for individual i across different periods. Thus, all the

[60]The number of Gaussian-Hermite quadratures used for evaluating the expectation in Heckman's estimator is 30. That of random number draws for calculating the MSL in the extended AR1 model is 200 for each expectation. I also check the results in another setting: the other number of quadratures for the experiment includes 25, 100 and 200; that of random draws are 100, 300, and 1000. The robustness of the results is confirmed.

estimated coefficients of structural variables are larger in absolute term than the other estimators, except when serial autocorrelation is controlled. The random effect probit estimator reduces the estimates and significance of these structural parameters, particularly γ. The quasi-likelihood-ratio statistic for a test on the existence of the random effect is 4.48, which is significant at 5% levels.[61] The exogeneity measure of the initial condition τ turns out to be zero. A quasi-likelihood-ratio test of imposing $\tau = 0$ can be easily done by comparing the total log-likelihood between RE probit and Heckman's estimators. The outcome is obviously to reject the hypothesis on endogeneity of the initial condition. According to my model, this evidence suggests that all of these control variables used form a good proxy for the ability prior in the beginning of the first wave. When the serial autocorrelation in the error term is controlled in Heckman's estimator, significance of almost all the coefficients increases. The magnitude of state dependence parameter γ rises because of the significantly negative autocorrelation presented by ρ.

These cross-specification comparisons can be applied exactly to FE models with Mundlak correction. The outcome is similar. The Heckman estimators with AR1 considered enjoy the highest total log-likelihood in both FE and RE models. Then a likelihood ratio test strongly favors the FE model. When group means are included in the Mundlak correction, the significance and magnitude of the coefficient of assessment score η drops dramatically. On the other hand, only the group mean of the assessment score is significant and positive among all the group means in the FE estimate with Mundlak correction. This suggests that prior belief plays a vital role in the college-bound choice so that an unexpected shock carried by an abnormally high or low assessment score might only produce a limited impact.

Exactly the same tests are implemented when the asset including housing is inserted instead. All of the results are similar to those of the asset excluding housing except the wealth effect. Table 4 reports the most preferred specification - Heckman estimator with AR1 controlled. The coefficient of asset β_1 becomes negative and that of the joint term β_2 turns out to be positive although they are not significant. Two reasons might explain this: buying a house in a good school district is a popular approach to raising the child's college-bound prospect. But there is strong self-selection such that the parents worrying about the college-bound prospect will be more likely

[61] They are calculated as twice of the difference of total log-likelihood between alternative models listed at the bottom of Table 3 (pooled probit vs. RE probit). The limiting distribution of this statistic is $\chi^2(1)$ (Godfrey (1988) and Andrews (2001)).

to do so. On the other hand, households that are rich due to housing investment might not be truly rich given that the sample periods runs between 1997 and 2005, when US house prices underwent an unprecedented increase.

To further examine the importance of state dependence and convergence of the wealth effect over time, I supply two alternative specifications when y_{it-1} and $(t-1)A_{it}$ together with time trend $t-1$ are dropped respectively from equation (12). Since a Heckman estimator with AR1 errors under FE assumption is preferred by the quasi-likelihood ratio test, the alternative specifications are all modeled by this model. The asset measure is the one excluding housing. Table 5 shows the result.

Two alternative specifications are strongly rejected by the likelihood ratio test, particularly the one ignoring state dependence in column [1]. All the structural parameters become smaller in magnitude and much less significant. This emphasizes that the incentive to learn about the child's cognitive ability highly depends on whether parents hold the college-bound expectation, and a flattening of the ability threshold exists.

3.3.2 College saving

Table 6 reveals the test results of wealth and belief effects on college saving.[62] This is the estimate for the specification in equation 15.

Initially, the asset measure includes housing, which corresponds to the total asset concept. π_2 is significantly negative at the 1% level, which confirms the dominance of ***wealth effect III*** induced by a lower incentive to invest in the college-bound option as the wealth level grows.

The saving decision involving housing can be approximately interpreted by the model of one-period learning when college saving is considered. The household cannot hold real property in a sensitive way with respect to a child's ability shock given its lack of liquidity. As discussed previously, when the learning effect is not strong, ***wealth effect III*** might dominate: equation (11) shows, given the asset level, the first term converges to zero, but not the second term, when the prior is large enough, which is shared by very optimistic parents. There is only an insignificant value to secure the child's college-bound chance by a marginal increase in wealth (decrease in ability threshold) since they are too confident to expect a bad quality child. But the

[62]Neither Arellano and Bond autocorrelation tests for the level equations nor the Sargan test indicate misspecification.

extra expected gain from children "in the money" is too small since they are almost certain in college outcome. It is also very natural to claim that parents holding home equity should be more confident about their child's prospect. And a lot of confident parents buy housing in the hope of using home equity loan to relax the borrowing constraint towards the college cost.[63] All of these factors may explain the existence of ***wealth effect III*** when the effect from the asset including housing is tested, while the test outcome using the asset excluding housing has the reverse sign and is highly insignificant.

The regular saving incentive captured by π_1 is significantly positive at the 3% level. It also suggests a negative asset growth rate, which matches the wealth effect explained by a typical finite-horizon saving model: a richer household (with positive assets) decumulates wealth and a poorer one (with negative assets) accumulates wealth over time so that wealth levels tend to converge. Additionally, the extra wealth effect from college saving π_2 reinforces this convergence dramatically.[64]

Whether the household will accumulate or decumulate wealth is not a uniform answer when there is a switch in the college-bound expectation for the child. For a household with an average child and zero assets in 1997, we expect the ratio of asset holding to be around 1,200 between the parents with college-bound expectation and those without it in our sample.[65] However, when the wealth for this same household rises to the average level in this sample, the same ratio drops to only 0.82.[66] This ratio is larger than one as long as total assets in 1997 are less than $\$14,472$ which is not far away from the average level of $\$18,958$. Thus, there are still a relatively large number of households that are not or are less affected by the aid tax. This finding contributes to the long-time debate on implicit financial aid tax as a saving disincentive (Dick et al. 2003; Edlin 1993; Feldstein 1995). My result suggests a weaker impact of this tax can be induced when considering the heterogeneity involved in the family's college saving behavior, particularly when their college-bound expectation is measured together with the learning aspect. Long (2004), Monks (2004) and Reyes

[63]A booming housing market during the 1997-2005 period covered in our data can strengthen this motive. These families tend to be more stable, better-off in terms of living standards, more educated and invest in a child's human capital more actively and at an earlier stage.

[64]Since the year gap between waves in the sample is different meaning that it is not straightforward to estimate the growth rate.

[65]Namely, $e^{7.09}$. By the construction of normalized assessment score, the average person in the population has this score zero. Zero asset level is at 7.9 percentile in this sample.

[66]Namely, $e^{7.09+9.85\times(-0.74)}$, where 9.85 is the mean of Ln asset incl. housing$[t-1]$ in the sample.

(2007) also reject the evidence of this tax effect from the prior literature by arguing that there have been large changes to the federal and institutional aid system. They suggest incorporating more detailed measures on families' college-bound expectation process which can cause huge disparity in estimations.

Using the asset including housing, the effect from learning is highly insignificant. But the opposite is true when the asset excluding housing is inserted instead. Both estimates of κ_1 and κ_2 significantly obtain the sign predicted by my model. This contrast can be explained by the evidence from the test on college-bound choice that the asset excluding housing should be more liquid to respond to the information observed from the child's ability signal over time.

The other interesting result is that these two tests always render the coefficients and significance in the opposite way. This can support the conjecture that the family is involved with reallocating the asset portfolio to shield it from the possible aid tax or the child's ability risk (Reyes 2007) by hedging among different asset categories. The total college expectation seems significantly negative for the asset excluding housing. Parents might play the aid tax game in another asset category since home equity has been removed from the calculation of a family's ability to pay tuition by Federal Methodology. This determines the distribution of Federal funds, including both grants and loans.

4 Policy implication

This section presents a policy discussion on the empirical findings on ESA (Education Saving Account) by extracting the insights from my main saving with learning model.[67]

4.1 Inefficiency of tax-preferred ESA to encourage ESP

Zhu (2011) empirically discovers from the Survey of Consumer Finance that families contributing to ESA mismatch with those with ESP (Education Saving Propensity), and ESA seems not to be more popular among the wealthy families that have an incentive to financially prepare well for the cost of education compared with those

[67]ESA refers to the tax-favored saving accounts introduced by state and federal governments in the US, e.g. 529 plan and Coverdell Education Savings Account. For a detailed description, see Zhu (2011).

which do not. The penalty structure of ESA contingent on the college-bound propensity might be the cause:

Since the benefit of ESA, such as 529, is conditional on the final college enrollment decision, this actually reduces the marginal wealth effect. If the child does not enter college then, naturally, all the distributions are impossible to be qualified as educational expenses in order to enjoy the waiver of income tax. Plus, a penalty (now 10%) will be imposed on these distributions in most cases. The benefit can be more or less considered as a one-time tuition subsidy and it reduces the gap of asset positions between the binary state of college-bound choice. Therefore, the lowest ability (threshold) to justify the education saving drops and becomes flatter with the assets. The absolute value of option increases but the marginal one with respect to wealth decreases.[68] The former effect might intrigue some incumbents to start opening an ESA but the latter effect influencing the existing holders of ESA could be uncertain and depends on the combinational states of parents' asset and prior. However, those who are very optimistic or pessimistic will be very likely to contribute less.[69] As a result, these parents would shift saving out of ESA to other forms to maintain a higher option value. This also sheds light on the reason why ESA cannot capture enough propensity to save for college. In reality, the population of highly optimistic and/or pessimistic parents can be quite large when not enough intra-family interaction is devoted to exploring the child's ability. On the other hand, for relatively optimistic parents, the one-time increase in absolute option value might intrigue them into a higher level of oversaving, which has negative impact on their welfare.

4.2 Policy suggestion

As the discussion proceeds, the pecuniary benefit introduced by simply changing the option value of college saving conditional on final college enrollment might not lead to a satisfactory outcome, particularly when parents are highly uncertain of their child's ability. This study shows that encouraging the early acquisition of information about their child's ability can improve the linkage between the wealth effect, such as tax benefit, and college saving. Thus, college participation can be much better prepared for financially by parents who are either pessimistic or poor. Plus, there

[68] As evidence, Zhu (2011) shows the overall association between ESA and ESP is significantly non-positive for median and high income groups.

[69] Their marginal effect of wealth on option value of college choice may very possibly decrease.

is evidence of positive psychological and behavioral effects of savings on educational achievement (Destin and Oyserman 2009; Elliott 2009).

The ESA policy of providing an incentive to learn about a child's ability early on can be an effective way to increase the motive to save for college via ESA. Two methods have already been introduced across the states:

- Partnership between GEAR UP and ESA (Clancy and Miller 2009): Gaining Early Awareness and Readiness for Undergraduate Programs (GEAR UP) was established in the Higher Education Amendments of 1998 to increase the low-income family students' and parents' readiness, selection and funding for accessing higher education. In addition to making college more accessible financially, GEAR UP scholarships reward parents' involvement so that they can know a child's prospect better and at an early stage since GEAR UP starts serving the students no later than the seventh grade. State partnership programs match deposits in 529 savings accounts with federally-funded GEAR UP scholarships to induce the parents' learning.

- Multiple states have already provided matching contributions.[70] This actually distributes the value of option across years and lowers the startup requirement for college saving. Therefore, parents are more likely to be induced to start college saving even with a very low level. The spread of option value encourages them to gauge the decision as to whether to take the match every subsequent year sequentially based on the child's ability signal since the exit or entry cost is lower at every period. Additionally, the earlier contribution means more matching. Parents would tend to begin the contribution earlier and thus learn about the child's ability earlier.[71]

The other possible approach is to apply a regressive tax benefit in terms of time, which intends to reward much earlier contributions and punish the later ones. Currently, 529 plans have contributions that grow tax-free over time. Instead, for instance, the benefit enjoyed by contributions close to the child's college enrollment age might be discounted back to those earlier contributions (namely, matching them).[72]

[70] For example, Arkansas, Oklahoma, Colorado, Maine, Michigan, Nebraska and Alabama. The amount ranges from $200 to $500 per year (See `http://www.savingforcollege.com/compare_529_plans/`).

[71] This analysis shares the idea similar to that in ***learning effect IV***.

[72] The other effect involved is to curb oversaving. Given a fully estimated model, I can carry out

5 Conclusion

The saving model with learning in this paper provides an explanation for the important empirical features about college saving observed from NELS88: decreasing skewness of an empirical distribution, probability (college saving level when child is in 12th grade | college saving level in 8th grade), with respect to college saving level at 8th grade. Given enough time of learning, oversaving by optimistic parents or inability to save for college by pessimistic parents might be alleviated due to the positive relationship between option value and learning.

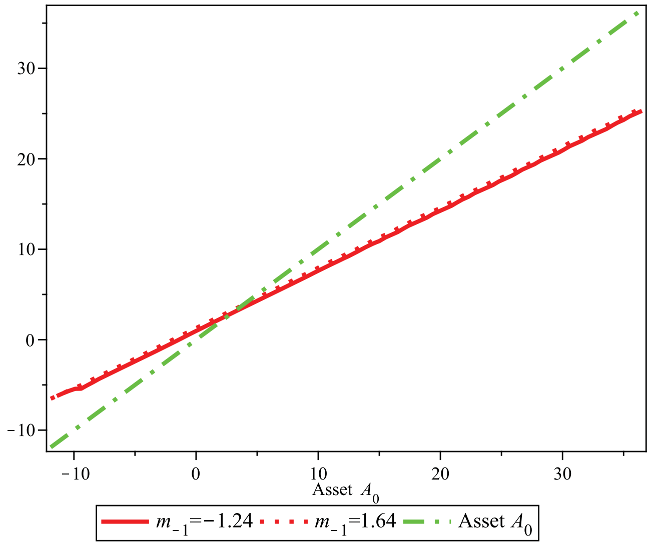
Test on college-bound choice confirms the positive effect from assets and its convergence over time. The state dependence of college-bound choice is significant particularly when autocorrelation due to learning is controlled. There is strong support for the existence of parents' learning behavior when preparing for the child's college cost. Test on college saving level shows confident parents respond more positively by increasing additional saving induced by college-bound expectation when only assets excluding housing are considered.

This study formally pins down the main risk inherent in college saving and introduces important behavioral factors, such as confidence, in the saving model.[73] Data suggest pessimistic and/or rich parents reduce the education saving, the state dependence of parents' college expectation and diminishing persistence over time due to learning.

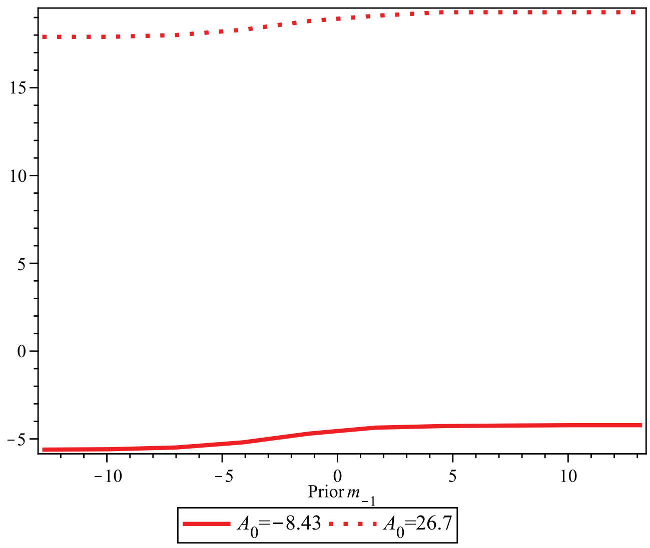
---

some experiments to examine the validity of these policy proposals. This will be the extension in the next step.

[73] There are also uncertainties regarding tuition level and financial aid which play a role in parents' investment in their child's higher education. The model developed at the current stage only captures the main uncertainties regarding ability. Moreover, college enrollment is not only an endogenous decision but also dependent on the exogenous college eligibility controlled by admission offices. The latter bears the uncertainty factor too. All of these risks might be considered in the future extension. Browning and Lusardi (1996) and Keane and Wolpin (2001) suggest the significance of accounting for a whole range of behavioral issues beyond consumption and saving decisions.

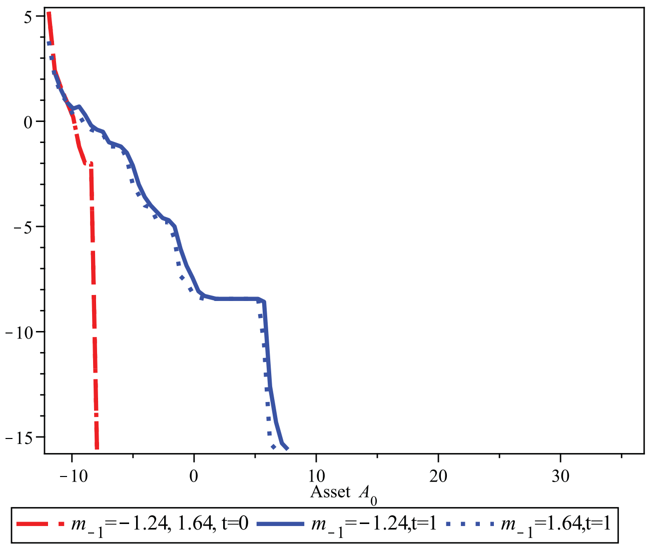


(a) Asset next period in the white-collar state: $E(A_1^w(A_0, m_0 \geq \bar{m}_0(A_0))|m_{-1})$ with prior m_{-1} fixed

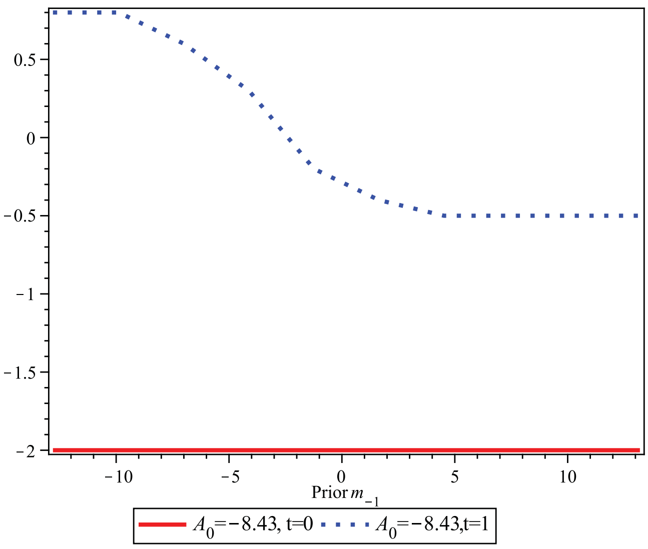


(b) Asset next period in the white-collar state: $E(A_1^w(A_0, m_0 \geq \bar{m}_0(A_0))|m_{-1})$ with asset A_0 fixed

Figure 2: Asset next period in the white-collar state at $t = 1$ (asset unit: \$10,000)

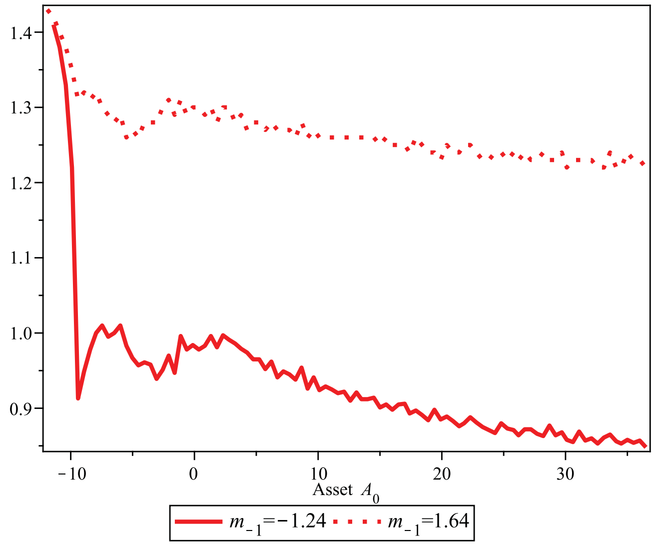


(a) Evolution of ability thresholds: $\bar{m}_0(A_0)$ and $\mathrm{E}(\bar{m}_1(A_0, m_0 \geq \bar{m}_0(A_0))|m_{-1})$ with prior m_{-1} fixed

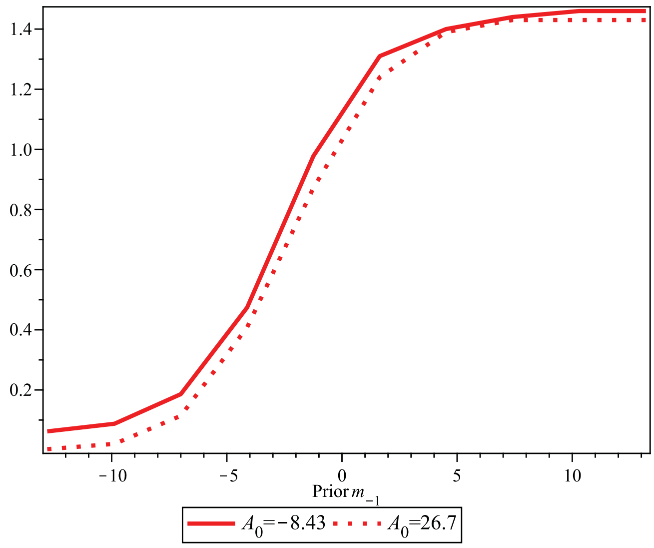


(b) Evolution of ability thresholds: $\bar{m}_0(A_0)$ and $\mathrm{E}(\bar{m}_1(A_0, m_0 \geq \bar{m}_0(A_0))|m_{-1})$ with asset A_0 fixed(parents with high assets have no thresholds at either period yet)

Figure 3: Evolution of ability thresholds starting from $t = 1$ (asset unit: \$10,000)

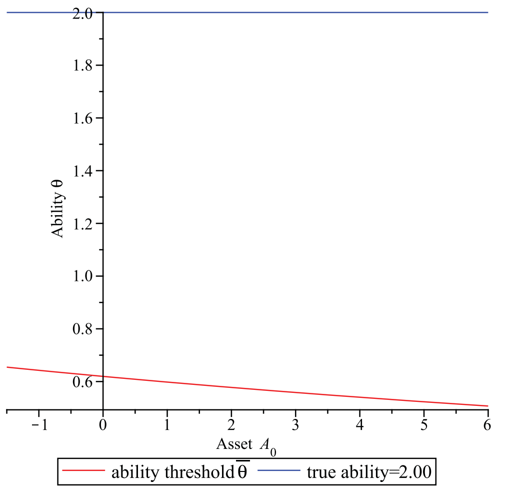


(a) Education saving next period in the white-collar state: $\mathrm{E}(A_1^w(A_0, m_0 \geq \bar{m}_0(A_0)) - A_1^b(A_0)|m_{-1})$ with prior m_{-1} fixed

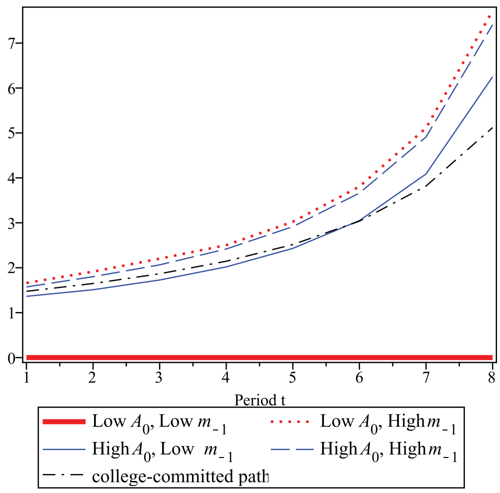


(b) Education saving next period in the white-collar state: $\mathrm{E}(A_1^w(A_0, m_0 \geq \bar{m}_0(A_0)) - A_1^b(A_0)|m_{-1})$ with asset A_0 fixed

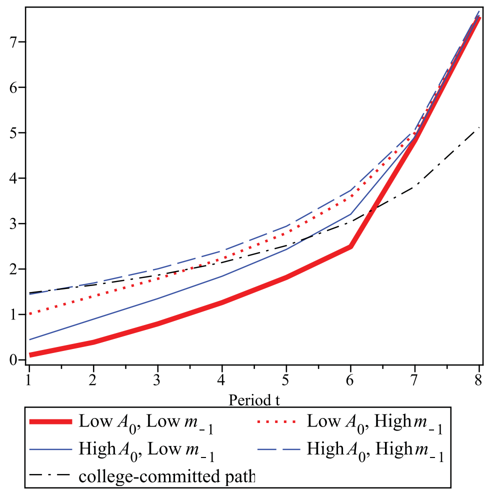
Figure 4: Education saving next period in the white-collar state at $t = 1$ (asset unit: $\$10,000$)



(a) Initial ability threshold under full information vs. true ability



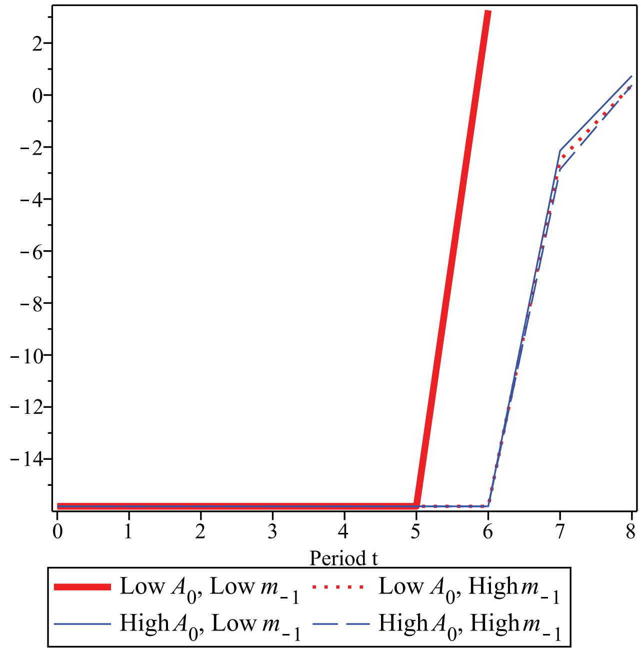
(b) One-period learning: college saving in the white-collar and college-committed states (parents with low A_0 and low m_{-1} actually have no college saving; they are kept for showing the legend and for comparison purposes)



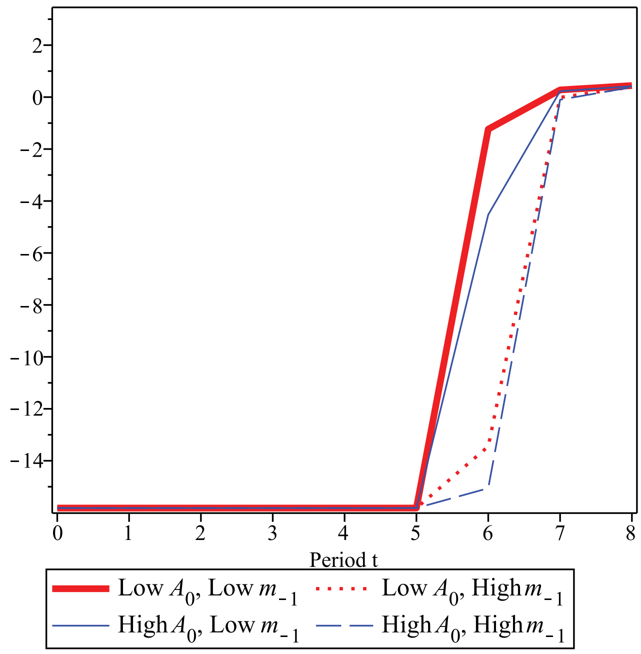
(c) Learning: college saving in the white-collar and college-committed states

College savings under different combinations of initial state variables (asset unit: \$10,000) and initial ability threshold under full information compared with true ability (High $A_0 = 5$, Low $A_0 = -1$, High $m_{-1} = 1.7$ and Low $m_{-1} = -1.1$)

Figure 5: Evolutions of college saving by different models



(a) One-period learning: ability thresholds



(b) Learning: ability thresholds

Ability thresholds under different combinations of initial state variables (asset unit: \$10,000) and initial ability threshold under full information compared with true ability (High $A_0 = 5$, Low $A_0 = -1$, High $m_{-1} = 1.7$ and Low $m_{-1} = -1.1$)

Figure 6: Evolutions of ability threshold by different models

Table 2: Summary statistics of PSID/CDS & TA (1997-2005)*

	Target subsample		*Full sample*	
	Mean	Std Dev	Mean	Std Dev
Demographics				
Child's race (1=white 0=black)	0.67	0.47	0.54	0.5
Child's age (wave 1)	10.59	0.95	10.83	1.04
Child's grade (wave 1)	4.16	0.89	4.59	1.11
Child's grade (wave 2)	9.93	0.83	9.89	0.89
Child's grade (wave 3)	11.63	1.16	11.28	1.44
Mother's yr of education (wave 3)	13.74	2.19	13.07	2.47
Mother's age (wave 1)	39.45	5.19	38.62	5.45
# of children (wave 1)	2.45	0.86	2.45	1.05
# of children (wave 2)	2.51	1.32	2.35	1.68
# of children (wave 3)	3.01	2.45	2.48	2.2
Parents divorced (wave 1)	0.09	0.29	0.13	0.33
Parents divorced (wave 2)	0.09	0.29	0.16	0.36
Parents divorced (wave 3)	0.09	0.29	0.17	0.37
Financial characteristics				
Total income $ (wave 1)	77,270	72,132	61,003	72,470
Total income $ (wave 2)	97,089	110,701	69,481	82,044
Total income $ (wave 3)	97,787	124,497	67,141	84,500
Asset excluding housing $ (wave 0)	116,640	430,833	80,870	345,573
Asset excluding housing $ (wave 1)	253,884	1,411,523	135,900	794,734
Asset excluding housing $ (wave 2)	391,738	2,527,072	181,047	1,374,728
Asset excluding housing $ (wave 3)	334,533	2,335,689	170,033	1,286,806
Asset including housing $ (wave 0)	162,508	486,237	114,506	373,190

Continued on next page

Table 2: Summary statistics (continued)

	Target subsample		*Full sample*	
	Mean	Std Dev	Mean	Std Dev
Asset including housing $ (wave 1)	306,683	1,460,308	177,863	828,142
Asset including housing $ (wave 2)	479,540	2,623,473	242,813	1,431,619
Asset including housing $ (wave 3)	443,431	2,415,851	241,448	1,338,525
Others				
Normalized assessment score (wave 1)	0.14	0.76	-0.11	0.82
Normalized assessment score (wave 2)	-0.03	0.9	-0.37	0.92
Normalized assessment score (wave 3)	-0.04	1.09	-0.14	1.04
Urbanicity (wave 1; 10 = most rural, 1 = most urban)	2.7	2.2	2.71	2.22
Urbanicity (wave 2; 10 = most rural, 1 = most urban)	3.31	2.31	3.39	2.42
Urbanicity (wave 3; 10 = most rural, 1 = most urban)	3.32	2.28	3.42	2.42
Yearly college expenditure $ (wave 1)	8,325	2,899	8,428	3,120
Yearly college expenditure $ (wave 2)	10,072	3,071	10,042	3,220
Yearly college expenditure $ (wave 3)	10,585	3,212	10,611	3,393
Child's behavior problem index	38.38	6.98	40.78	8.98
College expectation/outcome (wave 1; 1=college 0=non-college)	0.86	0.35	0.77	0.42
College expectation/outcome (wave 2; 1=college 0=non-college)	0.93	0.26	0.75	0.43
College expectation/outcome (wave 3; 1=college 0=non-college)	0.85	0.36	0.66	0.47

Source: PSID/CDS & TA

* See Appendix A for details.

Table 3: Estimated effects of state dependence, wealth and learning on college expectation/outcome[1]
- I. Asset regressor excludes housing

	Pooled probit	Random effect probit		Heckman's estimator		With AR(1) errors	
	(SE)	(SE)		(SE)		MSL (SE)	
Variable[2]		RE[3]	FE	RE	FE	RE	FE
College exp $[t-1]$: γ	1.055***	0.653**	0.411	0.628**	0.377	1.568***	1.472***
	(0.245)	(0.362)	(0.492)	(0.376)	(0.519)	(0.353)	(0.388)
Asset: β_1	0.103**	0.087*	0.070	0.08*	0.065	0.112**	0.119**
	(0.051)	(0.061)	(0.075)	(0.061)	(0.076)	(0.060)	(0.070)
$(t-1)\times$Asset: β_2	-0.072**	-0.062**	-0.051	-0.057*	-0.047	-0.075**	-0.072**
	(0.034)	(0.040)	(0.048)	(0.040)	(0.048)	(0.041)	(0.047)
$t-1$: ν	-0.020	-0.049	-0.260	-0.073	-0.297	-0.005	-0.148
	(0.384)	(0.472)	(0.636)	(0.482)	(0.658)	(0.466)	(0.534)
Assessment score: η	0.223**	0.194**	0.004	0.192**	0.009	0.225**	0.068
	(0.098)	(0.124)	(0.174)	(0.126)	(0.177)	(0.126)	(0.171)
Random effect: λ		0.395**	0.581**	0.419*	0.605***	0.359**	0.39***
		(0.233)	(0.229)	(0.235)	(0.229)	(0.143)	(0.143)
Exogeneity measure: τ				0.000	0	0.000	0.000
				(0.001)	(0.001)	(0.000)	(0.000)
AR(1) coefficient: ρ						-0.713***	-0.685***
						(0.138)	(0.144)
No. of obs.	348	348	348	522	522	522	522
Model log-likelihood	-94.42	-93.13	-86.54	-141.36	-135.00	-137.26	-132.83
Total log-likelihood	-144.89	-143.60	-137.01	-141.36	-135.00	-137.26	-132.83

[1] (a) significance level: *** 1%, , ** 5% and * 10%; (b) the control variables include mother's yrs of edu, child's age, child's race, mother's age, no. of children, urbanicity, yearly college expenditure, parents divorced and ln total income$[t-1]$; Continued below Table 4.

Table 4: Estimated effects of state dependence, wealth and learning on college expectation/outcome[1]
- II. Asset regressor includes housing

	With AR(1) errors	
	MSL (SE)	
Variable[2]	RE[3]	FE
College exp.[$t-1$]: γ	1.482***	1.327***
	(0.365)	(0.415)
Asset: β_1	-0.052	-0.005
	(0.106)	(0.127)
$(t-1)$·Asset: β_2	0.027	0.027
	(0.064)	(0.078)
$t-1$: ν	-1.018	-1.195
	(0.733)	(0.880)
Assessment score: η	0.266**	0.063
	(0.127)	(0.176)
Random effect: λ	0.384**	0.434***
	(0.153)	(0.152)
Exogeneity measure: τ	0.000	0.000
	(0.000)	(0.000)
AR(1) coefficient: ρ	-0.676***	-0.653***
	(0.152)	(0.161)
No. of obs.	522	522
Model log-likelihood	-140.06	-133.87
Total log-likelihood	-140.06	-133.87

[1] (a-b) See 1 below Table 3; (c) to account for the different normalizations on error term between pooled probit and all the random effect models, this table shows all the rescaled estimates by multiplying the raw results by $\sqrt{1-\lambda}$ except those from pooled probit.

[2] The first five structural parameters are specified in equation (12). λ is the ratio between the variance of individual-specific RE effect α_i and that of composite error: $\frac{\sigma_\alpha^2}{\sigma_\alpha^2+\sigma_u^2}$, τ is specified as exogeneity measure in equation (14) and ρ is AR1 coefficient for u_{it}.

[3] RE: only the random effect individual unobservable is modeled; FE: fixed effect is also controlled by Mundlak correction (Mundlak 1978).

[4] For the purpose of cross-specification comparison, the total log-likelihood of either pooled probit or random effect probit models is the sum of the model log-likelihood at $t \geq 2$ (348 observations) and log-likelihood of a simple probit from reduced form specification at $t = 1$ (174 observations). Critical value of $\chi^2(1)$: $1\% - 5.412$, $5\% - 2.706$, $10\% - 1.642$.

Table 5: Alternative specifications: existence of state dependence and convergence of wealth effect on college expectation/outcome*

Variable	Baseline (SE)	[1] (SE)	[2] (SE)
College exp.$[t-1]$: γ	1.472***		1.268***
	(0.388)		(0.387)
Asset: β_1	0.119**	0.055	0.015
	(0.070)	(0.090)	(0.034)
$(t-1)\cdot$Asset: β_2	-0.072**	-0.041	
	(0.047)	(0.057)	
$t-1$: α	-0.148	-0.336	
	(0.534)	(0.757)	
Assessment score: η	0.068	-0.009	0.036
	(0.171)	(0.194)	(0.171)
No. of obs.	522	522	522
Model log-likelihood	-132.83	-138.62	-137.85
Total log-likelihood	-132.83	-138.62	-137.85

* (a) All the specifications are implemented by the Heckman estimator with AR1 errors. The baseline model is exactly the last column in Table 3. Column [1] corresponds to the model with state dependence effect omitted and [2] to the model with convergence of wealth effect and time trend omitted; (b) refer to 1 and 2 below Tables 3 and 4 for the issues of significance levels, control variables, rescaling and definition of parameters and 3 for the calculation of total log-likelihood which is also shared in this table.

Table 6: Estimated effects of wealth and belief on college saving[1]

	Asset incl. housing[2]		Asset excl. housing[3]	
Variable	Coef. (SE)	P value	Coef. (SE)	P value
Asset $[t-1]$: π_1	0.67	0.03	-0.35	0.28
	(0.30)		(0.32)	
Assessment score: κ_1	-0.18	0.78	3.52	0.01
	(0.64)		(1.39)	
College exp.: ζ	7.09	0.01	-3.92	0.11
	(2.83)		(2.41)	
College exp.×assessment score: κ_2	0.01	0.99	-3.98	0.02
	(0.74)		(1.65)	
College exp.×asset $[t-1]$: π_2	-0.74	0.01	0.10	0.76
	(0.28)		(0.34)	

[1] See 1(b) below Table 3 for the coverage of most control variables. The only extra one is child's grade.

[2] Asset measure is asset including housing. The value is logarithmized. After experimenting, the classification of instruments is: the endogenous variables are asset $[t-1]$ and assessment score, the predetermined ones are mother's yrs of edu, child's age, child's race, mother's age, no. of children, urbanicity, yearly college expenditure, parents divorced, child's grade, ln total income$[t-1]$, college exp., college exp.×assessment score and college exp.×asset $[t-1]$, and the strictly exogenous one is time dummies. Arellano and Bond autocorrelation tests for the level equations: $z = 0.43$ and $\Pr > z = 0.699$ for AR(1) test and $z = 1.31$ and $\Pr > z = 0.190$ for AR(2) test. The Sargan test shows $\chi^2(42) = 77.00$ and $\Pr > \chi^2(42) = 0.001$.

[3] Asset measure is asset excluding housing. The value is logarithmized. After experimenting, the classification of instruments is: the endogenous variables are asset $[t-1]$, assessment score, mother's yrs of edu, mother's age, no. of children, parents divorced and urbanicity, the predetermined ones are child's grade, ln total income$[t-1]$, college exp., college exp.×assessment score and college exp.×asset $[t-1]$, and the strictly exogenous ones are time dummies, child's age, child's race and yearly college expenditure. Arellano and Bond autocorrelation tests for the level equations: $z = 1.18$ and $\Pr > z = 0.240$ for AR(1) test and $z = 1.84$ and $\Pr > z = 0.065$ for AR(2) test. The Sargan test shows $\chi^2(42) = 68.70$ and $\Pr > \chi^2(42) = 0.000$.

Appendix

A Data description

This appendix covers the variable constructions in the samples from NELS88 and PSID/CDS & TA.

A.1 Variables from NELS88

I draw the empirical evidence from the National Education Longitudinal Study of 1988 (NELS88) to expose unique features on saving for college. In this survey, a nationally representative sample of eighth-graders were first surveyed in the spring of 1988. A subsample of these respondents were then resurveyed through four follow-ups in 1990, 1992, 1994, and 2000. 3,703 respondents (with their parents) are included who do not have missing values in questions about college saving and cognitive test scores when they were in 8th and 12th grade.

The questionnaire contains a question on how much parents have saved for their child's future college education for two waves when their child was in the 8th and 12th grades (Ingels 1994b). The college saving categories are obtained from the question: "money respondent set aside for child's future education", asked when the child was in at 8th and 12th grades. The parents are not asked this question if they answered "No" to the other two screening questions appearing before this college saving question:

1. "do you expect that your eighth grader will go on to additional education beyond high school?" (at the wave of 8th grader); "does teen plan to continue education?" (at the wave of 12th grader)

2. "have you or your spouse/partner done anything specific in order to have some money for your eighth grader's education after high school?" (at the wave of 8th grader); "grade teen in when respondent started saving" (at the wave of 12th grader; "Not Begun" is the choice in this question corresponding to "No" in others.)

Similar to CDS, this structure assures that parents have seriously considered their child's college-bound chance (e.g. by measuring her ability) and actual financial actions before answering the level of college saving.

These data maintain a nationwide cognitive test on multiple disciplines administered and standardized over all the student respondents who are still enrolled, which is by design highly comparable across waves. There is also a quartile measure for each wave, which I take to decide which half of the sample the child's standardized test score is in. (Ingels 1994a).

A.2 Variables from PSID/CDS & TA

A.2.1 Asset/income

Household asset information is drawn from the wealth supplement of PSID's family data. It contains household net worth of wealth by adding separate values for a business, checking or saving accounts, real estate, stocks and mutual funds, and other assets and subtracting credit card and other debt. Both the values with and without home equity are available and used in this study. Since the wealth supplement is not collected each year, I pick the values closest to the end of each wave of CDS & TA data because this research focuses on the saving behavior of parents after observing their child's ability signal. Therefore, the years of asset measures from the first to the last waves are 1999, 2003 and 2005 respectively. Additionally, wave zero corresponds to 1994 when the last wealth supplement interview occurred before CDS-I (1997).

Total income is a continuous variable in the PSID adding total household income from the previous tax year including all taxable income, transfer income, and social security income for anyone in the household. Since PSID has been collected every two years recently, I also have to form the values representing the year closest to the end of each wave of CDS & TA data. To account for the well-documented measurement error issue in income data, I apply the averaging approach. The timing strategy is formed as follows: the average of 1996 and 1998 for wave one, the average of 2000, 2002 and 2004 for wave two, and the average of 2002 and 2004 for wave three.

Finally, all of these values and other dollar measures in this study are converted to the 1997's value.

A.2.2 Signal of child's cognitive ability

A child's cognitive ability, conceived broadly to include language skills, literacy, and problem-solving skills, was assessed in CDS-I/II through the W-J Achievement Test C Revised (Woodcock and Johnson 1989). Two age-standardized scores jointly available in both waves of data are used: Applied Problems (AP) and Broad Reading (BR). The former is a proxy of a child's math skill and the latter covers her capacity and mastery of passage comprehension and letter-word (Mainieri 2006).

This study produces a normalized assessment score for each wave. The first two waves of this score are calculated from AP and BR scores from W-J Achievement Tests. I firstly use the age normalized scores of these two tests available from the data. Then, I form a random sample by dropping the observations from over-sampled low-income/black families in each wave of the CDS sample. For comparison purposes, I re-normalize the scores by subtracting the mean and dividing by the standard deviation from the random sample. This re-normalized score has a mean of zero and standard deviation of one for the random sample of respondents. The average of the normalized AP and BR scores is then re-normalized by the same method within the random sample to achieve the final normalized assessment scores. The last wave of scores is calculated from high school GPA. The relative scale measure is firstly obtained through dividing the raw score by the highest GPA in the high school of the respondent. I apply the same random sample selection and re-normalization approach again on this scale measure to create the normalized assessment score of wave 3.

A.2.3 College expectation/outcome

This is a binary variable with one (or yes, holding college expectation or entering college) or zero (or no, abandoning college expectation or not entering college). Parents' expectations for children attending college are constructed from the question asking heads of households in the CDS how much schooling they expect their child to complete. Response categories include: (1) eleventh grade or less, (2) graduate from high school, (3) post-high school vocational training, (4) some college (5) graduate from a two-year college, (6) graduate from a four-year college, (7) master's degree, or (8) MD, LAW, PhD, or other doctoral degree. The reference group with college expectations equal to one consists of parents who responded by selecting numbers 4, 5, 6, 7, or 8. In CDS-II, the question for college expectation is: "Sometimes children do not get as much education as we would like. How much schooling do you expect that CHILD will really complete?" And parents were asked a screening question before this one: "In the best of all worlds, how much schooling would you like CHILD to complete?" From this context, we have reason to believe the college expectation question actually captures the parents' serious choice of their child's college-bound preparation instead of only aspiration for college after taking all the constraints, *e.g.*

their child's ability and family financial situation, into account.[74]

College outcome information is collected from the TA questionnaire, which asked whether the respondent ever attended college.

A.2.4 Others from PSID/CDS & TA

Urbanicity is measured by the Beale-Ross Rural-Urban Continuum Code for residence at the time of each interview (Mainieri 2006). These codes are based on matches to the FIPS state and county codes, which range from 1/most urban to 10/most rural.

The child's behavior problem index (BPI) is drawn from CDS-I. A higher score on this measure implies a greater level of behavior problems.

A.2.5 Yearly college expenditure from NCES

The NCES Digest of Education Statistics provides state-by-state data on college attendance and tuition. These data are used to determine the probability that a freshman attends three types of college based on his state of residence: four-year public, two-year public and four-year private in 1997, 2003 and 2005. Additionally, the average yearly college expenditure for these three types of college in a specific state is also drawn from these data for these three years. This expenditure covers undergraduate tuition, fees, room, and board charged for full-time students. Each child's yearly college expenditure expected by the parents in each wave is a function of four variables: the probability of attendance in each type of college, average yearly college expenditure for each type of college, state of residence and year of each wave.

B Numerical algorithm

1. Setting:

   - Assets - $A(i,t)$: gridpoint, $i = 1..N_a(t)$. Let $A(1,t) = B_t^C$, since this study is concerned with the area where there is ability threshold and blue-state value function can be evaluated by analytical expression. $N_a(0)$ can be assigned arbitrarily. Then, $N_a(t+1)$ is determined such that the range

[74] The college expectation question in CDS-I is "How much schooling do you expect that (CHILD) will complete?" There was no screening question before this one. The parents' college expectation for their child should be the same as their aspiration considering CDS-I interviewed at an early stage of the child's development when she had not yet started high school.

between $A(1,t)$ and $(A(1,t)+I(t))R$ contains the required number of grids for the purpose of sufficient refinement.

- Ability - $m(j)$: gridpoint, $j = 1..N_m$. The end points represent lower and upper bound. $\triangle_m = \frac{m(N_m)-m(1)}{N_m}$
- Discretized markov transition proability: $f(p,q,t) = \frac{\Phi\left(\frac{m(p)+\triangle_m/2-m(q)}{\sigma_{t-1}}\right)-\Phi\left(\frac{m(p)-\triangle_m/2-m(q)}{\sigma_{t-1}}\right)}{\Phi\left(\frac{m(N_m)+\triangle_m/2-m(q)}{\sigma_{t-1}}\right)-\Phi\left(\frac{m(1)-\triangle_m/2-m(q)}{\sigma_{t-1}}\right)}$

2. Numerical solution:

- $t = T+1$: for each $i = 1$ to $N_a(t)$ and $j = 1$ to N_m, I evaluate the values of blue-collar and white-collar states: $\hat{V}^b[i,j,t]$ and $\hat{V}^w[i,j,t]$, and determine the value function:
  $\hat{V}[i,j,t] \leftarrow \max\{\hat{V}^b[i,j,t], \hat{V}^w[i,j,t]\}$.
- $t = 0,\ldots,T+1$: Starting from $t = T$ and solving recursively back to 1, for each $i = 1$ to $N_a(t)$ and $j = 1$ to N_m, I calculate:
  (a) locate $\bar{i}$ such that $A(\bar{i},t)$ is the closest grid to $R(A(i,t)+I(t))$
  (b) evaluate $\hat{V}^w[i,j,t]$:
  $$\hat{V}^w[i,j,t] \leftarrow \max_{1\le q\le\bar{i}} \left\{ u\left(A(i,t)+I(t)-\frac{A(q,t+1)}{R}\right) + \beta\sum_{k=1}^{N_m}\hat{V}[q,k,t+1]f(k,j,t+1)\right\}$$
  (c) evaluate $\hat{V}^b[i,j,t]$ by calling its analytic expression
  (d) $V[i,j,t] \leftarrow \max\left\{\hat{V}^b[i,j,t], \hat{V}^w[i,j,t]\right\}$ and pick the maximizer $k^*(i,j,t)$
  (e) ability threshold: $j^*(i,t) \leftarrow \{j|\text{the first } j \text{ such that } \hat{V}^w[i,j,t] \ge \hat{V}^b[i,j,t]\}$

References

H. Akabayashi. An equilibrium model of child maltreatment. *Journal of Economic Dynamics and Control*, 30(6):993–1025, 2006.

D.W.K. Andrews. Testing when a parameter is on the boundary of the maintained hypothesis. *Econometrica*, pages 683–734, 2001.

M. Arellano and S. Bond. Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *The Review of Economic Studies*, pages 277–297, 1991.

M. Arellano and O. Bover. Another look at the instrumental variable estimation of error-components models. *Journal of econometrics*, 68(1):29–52, 1995.

G.S. Becker and C.B. Mulligan. The Endogenous Determination of Time Preference. *The Quarterly Journal of Economics*, 112(3):729–758, 1997.

C. Belzil and J. Hansen. Unobserved ability and the return to schooling. *Econometrica*, pages 2075–2091, 2002.

R.H. Bradley and R.F. Corwyn. Socioeconomic status and child development. *Annual Review of Psychology*, 53(1):371–399, 2002.

M. Browning and A. Lusardi. Household saving: Micro theories and micro facts. *Journal of Economic Literature*, 34(4):1797–1855, December 1996.

J.S. Butler and R. Moffitt. A computationally efficient quadrature procedure for the one-factor multinomial probit model. *Econometrica*, pages 761–764, 1982.

C. Campanale. Learning and the Return to Private Equity. Working paper, University of Rochester, 2004.

G. Chamberlain. *Panel data*, volume 1 of *Handbook of Econometrics*, chapter 7, pages 307–458. Amsterdam: North-Holland, 1984.

K.Y. Chay and D.R. Hyslop. Identification and estimation of dynamic binary response panel data models: empirical evidence using alternative approaches. Working paper, Center for Labor Economics, UC Berkeley, 1998.

M. Clancy and R. Miller. Creative partnerships between GEAR UP and State College Savings Plans: Experience and policy potential. *CSD Policy Brief*, 09-23, 2009.

R. Cressy. Credit rationing or entrepreneurial risk aversion? An alternative explanation for the Evans and Jovanovic finding. *Economics Letters*, 66(2):235–240, 2000.

E. Cudmore. Saving for post secondary education in Canada. Department summer paper, The University of Western Ontario, 2005.

F. Cunha and J. Heckman. A new framework for the analysis of inequality. *Macroeconomic Dynamics*, 12(S2):315–354, 2008.

F. Cunha, J. Heckman, and S. Navarro. Separating uncertainty from heterogeneity in life cycle earnings. *Oxford Economic Papers*, 57(2):191–261, 2005.

J.P. Danforth. On the Role of Consumption and Decreasing Absolute Risk Aversion in the Theory of Job Search. *Studies in the Economics of Search*, pages 109–31, 1979.

M. Destin and D. Oyserman. From Assets to School Outcomes: How Finances Shape Children's Perceived Possibilities and Intentions. *Psychological Science*, 20(4):414–418, 2009.

A.W. Dick, A.S. Edlin, and E. Emch. The savings impact of college financial aid. *Contributions to Economic Analysis and Policy*, 2(1), 2003.

A.K. Dixit, R.S. Pindyck, and G.A. Davis. *Investment under uncertainty*. Princeton University Press Princeton, NJ, 1994.

A.S. Edlin. Is College Financial Aid Equitable and Efficient? *The Journal of Economic Perspectives*, pages 143–158, 1993.

W. Elliott. Children's college aspirations and expectations: The potential role of children's development accounts (CDAs). *Children and Youth Services Review*, 31 (2):274–283, 2009.

M. Feldstein. College scholarship rules and private saving. *The American Economic Review*, pages 552–566, 1995.

L.G. Godfrey. *Misspecification tests in econometrics*. Cambridge Univ. Press, 1988.

W.H. Greene. *Econometric analysis*. Prentice Hall Upper Saddle River, NJ, 2003.

W.H. Greene. Censored data and truncated distributions. Working Papers 05-08, New York University, Leonard N. Stern School of Business, Department of Economics, 2005.

N.H. Hakansson. Optimal investment and consumption strategies under risk for a class of utility functions. *Econometrica*, pages 587–607, 1970.

G.D. Hansen and A. İmrohoroğlu. The role of unemployment insurance in an economy with liquidity constraints and moral hazard. *Journal of Political Economy*, pages 118–142, 1992.

J.J. Heckman. The incidental parameters problem and the problem of initial conditions in estimating a discrete time-discrete data stochastic process. *Structural analysis of discrete data with econometric applications*, pages 179–195, 1981.

J.J. Heckman and S. Navarro. Dynamic discrete choice and dynamic treatment effects. *Journal of Econometrics*, 136(2):341–396, 2007.

J.J. Heckman, L.J. Lochner, and P.E. Todd. *Earnings Functions, Rates of Return and Treatment Effects: The Mincer Equation and Beyond*, volume 1 of *Handbook of the Economics of Education*, chapter 7, pages 307–458. Elsevier, November 2006.

V. Hogan and I. Walker. Education choice under uncertainty: implications for public policy. *Labour Economics*, 14(6):894–912, 2007.

B.E. Honore and E. Kyriazidou. Panel data discrete choice models with lagged dependent variables. *Econometrica*, pages 839–874, 2000.

D.R. Hyslop. State dependence, serial correlation and heterogeneity in intertemporal labor force participation of married women. *Econometrica*, pages 1255–1294, 1999.

S.J. Ingels. *National Education Longitudinal Study of 1988: second follow-up: parent component data file user's manual.* US Dept. of Education, Office of Educational Research and Improvement, National Center for Education Statistics, 1994a.

S.J. Ingels. *National Education Longitudinal Study of 1988: second follow-up: student component data file user's manual.* US Dept. of Education, Office of Educational Research and Improvement, National Center for Education Statistics, 1994b.

B. Jacobs. Real options and human capital investment. *Labour Economics*, 14(6): 913–925, 2007.

B. Jovanovic. Job matching and the theory of turnover. *The Journal of Political Economy*, 87(5):972, 1979.

M.P. Keane. A computationally practical simulation estimator for panel data. *Econometrica*, pages 95–116, 1994.

M.P. Keane and J.E. Roemer. Assessing policies to equalize opportunity using an equilibrium model of educational and occupational choices. *Journal of Public Economics*, 2009.

M.P. Keane and K.I. Wolpin. The effect of parental transfers and borrowing constraints on educational attainment. *International Economic Review*, pages 1051–1103, 2001.

L. Ljungqvist and T.J. Sargent. *Recursive macroeconomic theory.* MIT press, 2004.

M. Long. The impact of asset-tested college financial aid on household savings. *Journal of Public Economics*, 88(1-2):63–88, 2004.

E.E. Maccoby and J.A. Martin. *Socialization in the context of the family: Parent-child interaction.* Wiley, 1983.

T. Mainieri. *The panel study of income dynamics child development supplement: User guide for CDS-II.* Institute for Social Research, University of Michigan, 2006.

J. Miao and N. Wang. Learning, Investment, and Entrepreneurial Survival: A Real Options Approach. mimeo, Boston University, 2006.

J. Miao and N. Wang. Investment, consumption, and hedging under incomplete markets. *Journal of Financial Economics*, 86(3):608–642, 2007.

B.L. Miller. Optimal consumption with a stochastic income stream. *Econometrica*, pages 253–266, 1974.

J. Monks. An empirical examination of the impact of college financial aid on family savings. *National Tax Journal*, 57(1):189–208, 2004.

M.D. Morris. The impact of grants, tax credit and education savings account on parental contributions to college expenses and the educational attainment of children. Working paper, University of New Orleans, Department of Economics and Finance, February 2003.

Y. Mundlak. On the pooling of time series and cross section data. *Econometrica*, 46 (1):69–85, 1978.

J.D. Osofsky and K. Connors. *Mother-infant interaction: An integrative view of a complex system*, pages 519–48. New York, John Wiley, 1979.

S. Rendon. Job search and asset accumulation under borrowing constraints. *International Economic Review*, 47(1):233–263, 2006.

J.W. Reyes. College financial aid rules and the allocation of savings. *Education Economics*, 99999(1):1–25, 2007.

F.P. Stafford, J. Eccles, R. Schoeni, K. McGonagle, and W.J. Yeung. *The panel study of income dynamics child development supplement: User guide for Transition into Adulthood 2005.* Institute for Social Research, University of Michigan, 2008.

M.B. Stewart. Maximum simulated likelihood estimation of random-effects dynamic probit models with autocorrelated errors. *Stata Journal*, 6(2):256, 2006.

D.M. Topkis. Minimizing a submodular function on a lattice. *Operations Research*, pages 305–321, 1978.

R.W. Woodcock and M.B. Johnson. *Tests of achievement, standard battery [form B]*. Chicago, IL: Riverside (Houghton Mifflin), 1989.

W.J. Yeung and D. Conley. Black-white achievement gap and family wealth. *Child Development*, 79(2):303–324, 2008.

J.Y. Zhu. Education saving propensity and education saving accounts: are tax incentives for education saving really exploited for education? Working paper, Research Center, Deutsche Bundesbank, 2011.

The following Discussion Papers have been published since 2012:

01	2012	A user cost approach to capital measurement in aggregate production functions	Thomas A. Knetsch
02	2012	Assessing macro-financial linkages: a model comparison exercise	Gerke, Jonsson, Kliem Kolasa, Lafourcade, Locarno Makarski, McAdam
03	2012	Executive board composition and bank risk taking	A. N. Berger T. Kick, K. Schaeck
04	2012	Stress testing German banks against a global cost-of-capital shock	Klaus Duellmann Thomas Kick
05	2012	Regulation, credit risk transfer with CDS, and bank lending	Thilo Pausch Peter Welzel
06	2012	Maturity shortening and market failure	Felix Thierfelder
07	2012	Towards an explanation of cross-country asymmetries in monetary transmission	Georgios Georgiadis
08	2012	Does Wagner's law ruin the sustainability of German public finances?	Christoph Priesmeier Gerrit B. Koester
09	2012	Bank regulation and stability: an examination of the Basel market risk framework	Gordon J. Alexander Alexandre M. Baptista Shu Yan
10	2012	Capital regulation, liquidity requirements and taxation in a dynamic model of banking	Gianni De Nicolò Andrea Gamba Marcella Lucchetta
11	2012	Credit portfolio modelling and its effect on capital requirements	Dilek Bülbül Claudia Lambert

12	2012	Trend growth expectations and U.S. house prices before and after the crisis	Mathias Hoffmann Michael U. Krause Thomas Laubach
13	2012	The PHF: a comprehensive panel survey on household finances and wealth in Germany	Ulf von Kalckreuth Martin Eisele, Julia Le Blanc Tobias Schmidt, Junyi Zhu
14	2012	The effectiveness of monetary policy in steering money market rates during the financial crisis	Puriya Abbassi Tobias Linzert
15	2012	Cyclical adjustment in fiscal rules: some evidence on real-time bias for EU-15 countries	Gerhard Kempkes
16	2012	Credit risk connectivity in the financial industry and stabilization effects of government bailouts	Jakob Bosma Micheal Koetter Michael Wedow
17	2012	Determinants of bank interest margins: impact of maturity transformation	O. Entrop, C. Memmel B. Ruprecht, M. Wilkens
18	2012	Tax incentives and capital structure choice: evidence from Germany	Thomas Hartmann-Wendels Ingrid Stein, Alwin Stöter
19	2012	Competition for internal funds within multinational banks: foreign affiliate lending in the crisis	Cornelia Düwel Rainer Frey
20	2012	Fiscal deficits, financial fragility, and the effectiveness of government policies	Markus Kirchner Sweder van Wijnbergen
21	2012	Saving and learning: theory and evidence from saving for child's college	Junyi Zhu

The following Discussion Papers have been published since 2011:

Series 1: Economic Studies

01	2011	Long-run growth expectations and "global imbalances"	M. Hoffmann M. Krause, T. Laubach
02	2011	Robust monetary policy in a New Keynesian model with imperfect interest rate pass-through	Rafael Gerke Felix Hammermann
03	2011	The impact of fiscal policy on economic activity over the business cycle – evidence from a threshold VAR analysis	Anja Baum Gerrit B. Koester
04	2011	Classical time-varying FAVAR models – estimation, forecasting and structural analysis	S. Eickmeier W. Lemke, M. Marcellino
05	2011	The changing international transmission of financial shocks: evidence from a classical time-varying FAVAR	Sandra Eickmeier Wolfgang Lemke Massimiliano Marcellino
06	2011	FiMod – a DSGE model for fiscal policy simulations	Nikolai Stähler Carlos Thomas
07	2011	Portfolio holdings in the euro area – home bias and the role of international, domestic and sector-specific factors	Axel Jochem Ute Volz
08	2011	Seasonality in house prices	F. Kajuth, T. Schmidt
09	2011	The third pillar in Europe: institutional factors and individual decisions	Julia Le Blanc
10	2011	In search for yield? Survey-based evidence on bank risk taking	C. M. Buch S. Eickmeier, E. Prieto

11	2011	Fatigue in payment diaries – empirical evidence from Germany	Tobias Schmidt
12	2011	Currency blocs in the 21st century	Christoph Fischer
13	2011	How informative are central bank assessments of macroeconomic risks?	Malte Knüppel Guido Schultefrankenfeld
14	2011	Evaluating macroeconomic risk forecasts	Malte Knüppel Guido Schultefrankenfeld
15	2011	Crises, rescues, and policy transmission through international banks	Claudia M. Buch Cathérine Tahmee Koch Michael Koetter
16	2011	Substitution between net and gross settlement systems – A concern for financial stability?	Ben Craig Falko Fecht
17	2011	Recent developments in quantitative models of sovereign default	Nikolai Stähler
18	2011	Exchange rate dynamics, expectations, and monetary policy	Qianying Chen
19	2011	An information economics perspective on main bank relationships and firm R&D	D. Hoewer T. Schmidt, W. Sofka
20	2011	Foreign demand for euro banknotes issued in Germany: estimation using direct approaches	Nikolaus Bartzsch Gerhard Rösl Franz Seitz
21	2011	Foreign demand for euro banknotes issued in Germany: estimation using indirect approaches	Nikolaus Bartzsch Gerhard Rösl Franz Seitz

22	2011	Using cash to monitor liquidity – implications for payments, currency demand and withdrawal behavior	Ulf von Kalckreuth Tobias Schmidt Helmut Stix
23	2011	Home-field advantage or a matter of ambiguity aversion? Local bias among German individual investors	Markus Baltzer Oscar Stolper Andreas Walter
24	2011	Monetary transmission right from the start: on the information content of the eurosystem's main refinancing operations	Puriya Abbassi Dieter Nautz
25	2011	Output sensitivity of inflation in the euro area: indirect evidence from disaggregated consumer prices	Annette Fröhling Kirsten Lommatzsch
26	2011	Detecting multiple breaks in long memory: the case of U.S. inflation	Uwe Hassler Barbara Meller
27	2011	How do credit supply shocks propagate internationally? A GVAR approach	Sandra Eickmeier Tim Ng
28	2011	Reforming the labor market and improving competitiveness: an analysis for Spain using FiMod	Tim Schwarzmüller Nikolai Stähler
29	2011	Cross-border bank lending, risk aversion and the financial crisis	Cornelia Düwel, Rainer Frey Alexander Lipponer
30	2011	The use of tax havens in exemption regimes	Anna Gumpert James R. Hines, Jr. Monika Schnitzer
31	2011	Bank-related loan supply factors during the crisis: an analysis based on the German bank lending survey	Barno Blaes

32	2011	Evaluating the calibration of multi-step-ahead density forecasts using raw moments	Malte Knüppel
33	2011	Optimal savings for retirement: the role of individual accounts and disaster expectations	Julia Le Blanc Almuth Scholl
34	2011	Transitions in the German labor market: structure and crisis	Michael U. Krause Harald Uhlig
35	2011	U-MIDAS: MIDAS regressions with unrestricted lag polynomials	C. Foroni M. Marcellino, C. Schumacher

Series 2: Banking and Financial Studies

01	2011	Contingent capital to strengthen the private safety net for financial institutions: Cocos to the rescue?	George M. von Furstenberg
02	2011	Gauging the impact of a low-interest rate environment on German life insurers	Anke Kablau Michael Wedow
03	2011	Do capital buffers mitigate volatility of bank lending? A simulation study	Frank Heid Ulrich Krüger
04	2011	The price impact of lending relationships	Ingrid Stein
05	2011	Does modeling framework matter? A comparative study of structural and reduced-form models	Yalin Gündüz Marliese Uhrig-Homburg
06	2011	Contagion at the interbank market with stochastic LGD	Christoph Memmel Angelika Sachs, Ingrid Stein
07	2011	The two-sided effect of financial globalization on output volatility	Barbara Meller
08	2011	Systemic risk contributions: a credit portfolio approach	Klaus Düllmann Natalia Puzanova
09	2011	The importance of qualitative risk assessment in banking supervision before and during the crisis	Thomas Kick Andreas Pfingsten
10	2011	Bank bailouts, interventions, and moral hazard	Lammertjan Dam Michael Koetter
11	2011	Improvements in rating models for the German corporate sector	Till Förstemann

12	2011	The effect of the interbank network structure on contagion and common shocks	Co-Pierre Georg
13	2011	Banks' management of the net interest margin: evidence from Germany	Christoph Memmel Andrea Schertler
14	2011	A hierarchical Archimedean copula for portfolio credit risk modelling	Natalia Puzanova
15	2011	Credit contagion between financial systems	Natalia Podlich Michael Wedow
16	2011	A hierarchical model of tail dependent asset returns for assessing portfolio credit risk	Natalia Puzanova
17	2011	Contagion in the interbank market and its determinants	Christoph Memmel Angelika Sachs
18	2011	Does it pay to have friends? Social ties and executive appointments in banking	A. N. Berger, T. Kick M. Koetter, K. Schaeck

Visiting researcher at the Deutsche Bundesbank

The Deutsche Bundesbank in Frankfurt is looking for a visiting researcher. Among others under certain conditions visiting researchers have access to a wide range of data in the Bundesbank. They include micro data on firms and banks not available in the public. Visitors should prepare a research project during their stay at the Bundesbank. Candidates must hold a PhD and be engaged in the field of either macroeconomics and monetary economics, financial markets or international economics. Proposed research projects should be from these fields. The visiting term will be from 3 to 6 months. Salary is commensurate with experience.

Applicants are requested to send a CV, copies of recent papers, letters of reference and a proposal for a research project to:

Deutsche Bundesbank
Personalabteilung
Wilhelm-Epstein-Str. 14

60431 Frankfurt
GERMANY