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## Public debt and changing inflation targets

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## Non-technical summary

What are the effects of a higher central bank inflation target on the burden of real public debt? Large increases in government deficits during the economic crisis of 2008 and 2009 initiated a debate on whether the real value of public debt should be reduced by raising inflation, at least temporarily. For example, it has been argued by Kenneth Rogoff (2010) and others that increasing the U.S. inflation rate by four percentage points for a couple of years would significantly help the public (as well as private) deleveraging process. The same proposal potentially applies to the Eurozone, which suffers from even larger public debt problems. We consider the question in a New Keynesian model with a maturity structure of public debt and an imperfectly observed inflation target.

In OECD countries the average maturity of debt ranges between about 4 to 10 years. Thus, a large fraction of debt is susceptible to increases in inflation even when inflation expectations and thus long-term nominal interest rates on newly-issued debt adjust. How inflation expectations themselves evolve depends crucially on the public's perception of the central bank's inflation target. With changes in the target not perfectly observed, the speed with which economic agents revise their perception of the target rate can be understood as capturing the credibility of a previously communicated inflation target.

Simulating the model calibrated to the U.S. situation shows that a temporary four-percentage-point increase of the inflation target has a minor effect on real public debt. The reason is that the time span for inflation to erode outstanding debt that was priced under low inflation expectations needs to be large. Thus only a persistent increase in the inflation target can reduce a significant part of the debt accrued during the crisis. We show further that there is no additional benefit in fooling the public by not communicating a change in the inflation target, unless the average maturity of debt is implausibly low.

The quantitative framework presented here can be applied to any other country, by an appropriate choice of the average maturity and level of public debt. As our analysis suggests, countries with higher average maturities stand to gain more strongly, yet again, only if the change in inflation is associated with a permanent change in the inflation target. From this perspective, the temptation to raise the inflation rate seems highest for the U.K. and lowest for Japan. Of course, a government would have to weigh the benefits of a somewhat improved budget surplus through lower debt servicing with the cost in terms of lost reputation. Once inflation expectations have adjusted to a higher level, and the public has doubts about the independence of the central bank, future interest rates will incorporate a risk premium for inflation. This in turn would affect a government's incentive later on to reduce inflation back to a lower level, as the real value of outstanding debt would be higher than expected. The modeling of the dynamics of reputation, the risk of losing credibility and other game-theoretic aspects are left to future research.

## Nicht-technische Zusammenfassung

Wie wirkt sich die Erhöhung des Inflationsziels einer Zentralbank auf die reale Staatsverschuldung aus? Der enorme Anstieg der öffentlichen Defizite im Zuge der Wirtschaftskrise in den Jahren 2008 und 2009 löste eine Diskussion darüber aus, ob sich der reale Wert der staatlichen Verschuldung durch eine höhere Inflation zumindest zeitweise senken ließe. So argumentieren beispielsweise Kenneth Rogoff (2010) und andere mit Blick auf die Vereinigten Staaten, dass eine Anhebung der angestrebten Teuerungsrate um vier Prozentpunkte für einige Jahre den Schuldenabbau im öffentlichen (wie auch im privaten) Sektor erheblich erleichtern würde. Die gleiche Annahme dürfte auch für den Euroraum gelten, wo die Probleme der Staatsverschuldung noch gravierender sind. Wir untersuchen diese Frage anhand eines neukeynesianischen Modells mit einer Laufzeitstruktur der staatlichen Schuldtitel und einem nur unvollkommen beobachtbaren Inflationsziel.

In den OECD-Ländern beträgt die durchschnittliche Laufzeit staatlicher Schuldtitel etwa vier bis zehn Jahre. Folglich ist ein Großteil der Verschuldung auch dann inflationssanfällig, wenn sich die Inflationserwartungen und somit die langfristigen Nominalzinsen neu emittierter Schuldtitel anpassen. Wie sich die Inflationserwartungen ihrerseits entwickeln, hängt maßgeblich davon ab, wie die Öffentlichkeit das Inflationsziel der Zentralbank wahrnimmt. Aufgrund der unvollkommenen Beobachtbarkeit des Inflationsziels kann das Tempo, mit dem die Marktteilnehmer ihre Wahrnehmung der Zielrate anpassen, als Ausdruck der Glaubwürdigkeit eines zuvor bekannt gegebenen Inflationsziels gedeutet werden.

Eine Simulation des auf die Lage in den Vereinigten Staaten kalibrierten Modells zeigt, dass eine vorübergehende Anhebung des Inflationsziels um vier Prozentpunkte einen geringen Einfluss auf den realen öffentlichen Schuldenstand hat. Grund hierfür ist, dass es sehr lange dauert, bis die Teuerung die ausstehenden Schulden (die zu einer Zeit bewertet wurden, als eine niedrige Inflation erwartet wurde), reduziert hat. Dementsprechend ist ein signifikanter Abbau der während der Krise angehäuften Schulden nur durch eine dauerhafte Erhöhung des Inflationsziels möglich. Des Weiteren wird gezeigt, dass eine Täuschung der Öffentlichkeit, die darin bestünde, die Anpassung des Inflationsziels zu verheimlichen, keinen weiteren Vorteil mit sich bringt, es sei denn, für die Schuldtitel wird eine unplausibel kurze durchschnittliche Laufzeit angesetzt.

Der hier gewählte quantitative Rahmen lässt sich durch eine entsprechende Auswahl der durchschnittlichen Laufzeit und Höhe der Staatsschulden auf jedes beliebige andere Land anwenden. Die durchgeführte Analyse legt den Schluss nahe, dass sich Länder, deren Verbindlichkeiten im Durchschnitt eine längere Fristigkeit besitzen, durch Inflation besserstellen, jedoch ebenfalls nur bei einer dauerhaften Anhebung des Inflationsziels. So gesehen scheint der Anreiz, den Preisauftrieb anzukurbeln, für das Vereinigte Königreich am größten und für Japan am geringsten zu sein. Natürlich müsste eine Regierung die Vorteile einer leichten Verbesserung der Haushaltslage aufgrund des niedrigeren Schuldendienstes gegen die Gefahr eines Reputationsverlusts abwägen. Haben sich die Inflationserwartungen erst einmal auf einem höheren Niveau gefestigt und zweifelt die Bevölkerung an der Unabhängigkeit der Zentralbank, werden die Zinssätze in Zukunft eine Inflationssrisikoprämie enthalten. Dies würde wiederum den Anreiz der Regierung, die Inflation später wieder auf ein normales Niveau zurückzuführen, verringern, da der reale Wert ihres Schuldenstands dann höher wäre als erwartet. Die Modellierung der Entwicklung von Reputation, des Risikos eines Glaubwürdigkeitsverlusts und weitere spieltheoretische Aspekte bleiben zukünftigen Studien überlassen.

# Public Debt and Changing Inflation Targets\*

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## Abstract

What are the effects of a higher central bank inflation target on the burden of real public debt? Several recent proposals have suggested that even a moderate increase in the inflation target can have a pronounced effect on real public debt. We consider this question in a New Keynesian model with a maturity structure of public debt and an imperfectly observed inflation target. We find that moderate changes in the inflation target only have significant effects on real public debt if they are essentially permanent. Moreover, the additional benefits of not communicating a change in the inflation target are minor.

**Keywords:** Public debt, learning, inflation target, callable perpetuity, debt maturity

**JEL classification:** E31, E52, H63.

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# 1 Introduction

Large increases in government deficits during the economic crisis of 2008 and 2009 initiated a debate on whether the real value of public debt should be reduced by raising inflation, at least temporarily. For example, it has been argued that increasing the U.S. inflation rate by four percentage points for a couple of years would significantly help the public (as well as private) deleveraging process. It would thus also reduce the impact of a possibly contractionary fiscal consolidation.<sup>1</sup> The same proposal potentially applies to the Eurozone, which suffers from even larger debt problems. At the same time, a proposal was made by [Blanchard, Dell’Ariccia and Mauro \(2010\)](#) to increase the Eurozone’s inflation target from two to four percent, albeit to reduce the risk of reaching the zero-lower bound on interest rates during severe contractions.

Two factors determine the effectiveness of raising the targeted inflation rate in reducing public debt: the behavior of long-term inflation expectations and the maturity structure of public debt. On the one hand, inflation expectations affect current inflation through forward-looking price setting, and affect long-term nominal interest rates through the pricing of newly-issued debt. On the other hand, the maturity structure of debt determines the fraction of outstanding debt that can be inflated away, to the extent that it remains priced under the lower long-term inflation expectations that prevailed before the onset of an unanticipated inflationary policy.<sup>2</sup> How inflation expectations themselves evolve depends crucially on the perception of the central bank’s inflation target and its credibility. For example, if the central bank does not announce a policy change and also enjoys a high credibility of its established target, inflation expectations may continue to stay low in spite of rising inflation. Then there would be less need to compensate lenders with higher nominal interest rates, and the real debt burden may fall more strongly.

This paper quantitatively assesses the role of debt maturity and inflation expectations for the evolution of real public debt by analysing the effects of an increase in a central bank’s targeted inflation rate. To this end, we develop a New Keynesian monetary business cycle model with two non-standard features. First, we assume that agents do not directly observe the current inflation target, but have to infer it from the monetary authority’s behavior. The speed with which economic agents revise their perception of the target rate is meant to capture the credibility of a previously communicated inflation target.<sup>3</sup> Technically, we assume that agents face a signal extraction problem as to whether observed changes in monetary policy are due to transitory shocks or due to persistent changes in the inflation target. As in [Erceg and Levin \(2003\)](#), this problem is solved by means of a Kalman Filter, and the associated gain is seen as a measure of the credibility of the central bank.

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<sup>1</sup>Most notably, Kenneth [Rogoff](#) in Project Syndicate has at the end of 2008 and 2010 suggested to allow some 6 to 7 percent inflation for a short time. See [Miller \(2009\)](#) for more details. See also [Rajan \(2011\)](#), who discusses a number of arguments for and against inflating away debt, raising informally some of the issues made explicit here.

<sup>2</sup>For example, if all debt outstanding were of 10 year maturity, so that only a tenth of this debt becomes due each year, then the remaining 90 percent will have nominal interest rates that are set based on the lower inflation expectations of the past.

<sup>3</sup>For an early application of this conceptualization of credibility, see [Cukierman and Meltzer \(1986\)](#).

In OECD countries the average maturity of debt ranges between about 4 to 10 years. Thus, a large fraction of debt is susceptible to increases in inflation even when inflation expectations and thus long-term nominal interest rates on newly-issued debt adjust. We model the maturity structure of long-term debt by means of callable perpetuities of which a given fraction matures each period. Introducing this new stylized type of bond offers a simple mean to calibrate a realistic average maturity of public debt. Furthermore, tracking the average interest rate on outstanding debt reveals countervailing effects of a change in the inflation target not at work in standard models. In all other respects, the model is a standard dynamic stochastic general equilibrium model with monopolistic competition and sticky prices, and with policy described by a Taylor-type interest rate rule and a fiscal tax rule.

In our simulation exercise we proceed in two steps. We first choose an initial level of government debt in excess of a steady-state level to which the tax rule would guide debt in the long-run. The former can be seen as a post-crisis level of debt, and the difference as a ‘crisis’ debt shock. In the second step, we change the inflation target in the interest rate rule, and trace the resulting evolution of the real value of public debt. For concreteness, we calibrate our benchmark economy to the average maturity of debt for the United States, about 4 1/2 years, and set the debt shock to about 70 percent of steady-state debt. We then compare different outcomes for the main parameters of interest.

We find that a key prerequisite for inflating away a significant portion of public debt is a high persistence of the change in the inflation target. For the U.S. calibration, the persistent four-percentage point change in the target reduces after ten years about a third of the additional debt accrued during the crisis.<sup>4</sup> A temporary change similar to what, for example, Rogoff (2008, 2010) suggested has much smaller effects. The intuition is that even though the rise in inflation deflates some of the outstanding debt, the fraction of debt that is newly issued commands a higher interest rate proportional to the changed inflation expectations. This results in a persistent increase in total interest costs that continues well after inflation has returned to the initial target. Only when higher inflation is long lasting, does the devaluing effect on the remaining outstanding debt dominate.<sup>5</sup> Finally, these effects crucially depend on the maturity structure of debt: with only short-term debt, as standard in New Keynesian models, the effect of inflation on debt is negligible.<sup>6</sup>

In general, a higher average maturity of outstanding public debt – or, conversely, a smaller fraction of debt maturing each period – always increases the amount of real debt inflated away within a given time period, taking as given the way expectations are formed. Thus a country with a higher average maturity can be regarded as facing a larger temptation to increase inflation.<sup>7</sup> Another important factor is how a change in the

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<sup>4</sup>Persistence is measured by the autoregressive parameter of the process describing the evolution of the inflation target.

<sup>5</sup>As will become clear in the technical part, our effects present an upper bound since the maturity is assumed not to change after the target change. If the maturity structure would change, and debt immediately repriced, the ‘gain’ from inflation would be even lower.

<sup>6</sup>We also find that a permanent change in the inflation target from two to four percent, as suggested by Blanchard, Dell’Ariccia and Mauro (2010) as an insurance against hitting the zero-lower bound on interest rates, would be too low to have a dramatic effect on real public debt in the transition.

<sup>7</sup>With a maturity of roughly 13 years as in the U.K. in 2010, rather than the four and a half years

inflation target is interpreted by the public. At very low average maturities, a large drop in real debt can only be achieved if the public falsely believes in a low inflation target, so that newly-issued debt is priced at interest rates too low for appropriately compensating lenders. For longer, empirically relevant, maturities, the credibility of a target does not affect the degree to which higher inflation can reduce real government debt.

There are two studies that analyze the possibility of the U.S. government to inflate away its real debt. [Aizenman and Marion \(2011\)](#) carefully document the evolution of debt, inflation and the maturity of debt since World War II. In a simple partial equilibrium model with a fixed interest rate they show that the incentives to inflate to reduce debt are large. In contrast, by including endogenous interest rates and forward-looking expectations, we show that the consequences of raising inflation may be much lower. In a paper about the measurement of interest paid on government debt, [Hall and Sargent \(2011\)](#) also show that under their measures, the fraction of U.S. real debt inflated away was lower than previously estimated. Furthermore, they emphasize, in line with [Giannitsarou and Scott \(2008\)](#), that instead high real GDP growth made the largest contribution to real debt reduction, and not inflation.<sup>8</sup>

[Davig, Leeper and Walker \(2011\)](#) explain how increases in public debt may endogenously lead to a switch in the monetary policy regime when debt reaches a ‘fiscal limit’. In other words, high debt may trigger the central bank to be passive while fiscal policy actively determines the price level, in the terminology of [Leeper \(1991\)](#). Once a fiscal limit is reached, monetary policy switches stochastically to a passive stance, and inflation serves to bring the public debt back to a sustainable level. In our paper, we focus on the role of debt maturity and the public’s beliefs about the inflation target in a regime where monetary policy remains active and fiscal policy passive. A recent study by [Bianchi \(2012\)](#) introduces beliefs about the likelihood of different regimes to interpret past inflationary periods such as the 1970s, which he finds to be one of low credibility.

The paper now proceeds to the following section where we give a brief overview of the current fiscal situation of advanced G7 economies, including details on the maturity structure of debt.<sup>9</sup> Section 3 develops the model, and introduces the long-term stochastic bond designed to approximate a realistic maturity structure of government debt, and specifies the signal extraction problem regarding the long-term inflation target of the central bank. In section 4, the analysis is presented followed by a deeper discussion focusing on debt maturity and the persistence of the inflation target. In Section 5 we will try to answer the following question, How sensitive is real public debt to changing inflation targets? Finally, section 6 concludes.

## 2 Public debt and maturity in advanced economies

Since the onset of the economic and financial crisis in 2008, advanced economies have experienced rising levels of public debt due to financial sector rescue packages, fiscal

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as in the U.S., the amount of additional real debt reduced after ten years is about 42 percent for a four percentage point increase in the inflation target.

<sup>8</sup>See [Persson, Persson and Svensson \(1996\)](#) for an attempt to address this issue for the Swedish case.

<sup>9</sup>We do not discuss in depth the determinants of the debt structure, or its optimality.

stimuli, and falling tax revenues. On average, net debt in G7 countries is going to increase from 52 percent of GDP in 2006 to almost 80 percent in 2011, an average increase of 51.7 percent, as reported by the IMF. Debt is projected to increase further in the coming years. The corresponding gross debt percentages are 83 and 115 percent, respectively.<sup>10</sup> The last four columns of Table 1 show net government debt in percent of GDP and the percentage change of net debt between 2008 and 2011, as well as the same measures relating to gross debt.

Tab. 1 - PUBLIC DEBT AND MATURITY STRUCTURES

	avg. maturity in years	% of debt maturing	net debt % of GDP	% change from 2006	gross debt % of GDP	% change from 2006
Canada	5.6	15.9	35.1	+33.5	84.2	+19.8
France	6.5	16.9	79.2	+33.6	85.0	+33.6
Germany	6.0	10.2	54.7	+3.8	80.1	+18.5
Italy	6.7	21.2	100.6	+12.0	120.3	+12.9
Japan	5.2	54.2	127.8	+51.6	229.1	+19.8
U.K.	12.8	8.6	75.1	+97.6	83.0	+92.6
U.S.	4.4	21.2	72.4	+70.0	99.5	+62.8
Average	6.7	21.2	79.5	+51.7	114.9	+38.6

*Note:* G7 Advanced Economies, 2011 (projected). IMF Fiscal Monitor April 2011, May 2010. Average maturities are reported for 2010.

The average maturity of public debt varies across countries, but is largely in the range of four to seven years. A notable outlier is the U.K. with an average maturity of 12.8 years, which however includes about 20 percent indexed debt. Nonetheless, a large fraction of the real value of U.K. debt remains to be affected by rising inflation, before higher interest rates on rolled-over debt would increase the government's interest expenses. In the wake of the crisis, countries have adjusted the maturity of newly issued debt, in order to manage their future repayment obligations. For example, the U.S. had relatively low average maturity in 2009, due to large issuance of short-term paper to finance crisis-related expenses, but aims at increasing its average maturity.<sup>11</sup> Overall, there is typically

<sup>10</sup>Gross debt comprises government debt held by the government, such as the social security trust fund in the U.S. Net debt excludes such government assets and thus a better measures of what the government owes the private sector. In the U.S. this is reported as *debt held by the public*.

<sup>11</sup>As of April 2009. Taken from [US Department of the Treasury \(2009\)](#) Report to the Secretary of the Treasury from the Treasury Borrowing Advisory Committee of the Securities Industry and Financial Markets Association, April 29, 2009. See also the Report from August 5, 2009 which reports that "... the average maturity of issuance now exceeds the average maturity of marketable debt outstanding. This suggests that the decline in the average maturity of debt outstanding that that we have witnessed over the past seven years – from a high of approximately 70 months in 2000 to a low of approximately 50

some variation over time in the average maturity of public debt, depending on the history of issuance.

The variation in the fraction of debt maturing across countries is larger, which results from differences in the distribution of maturities. For example, while Japanese debt has an average maturity of 5.2 years rather close to the 4.4 years in the U.S., the fraction of debt maturing in 2011 is 54.2 percent, which is much higher than the 21.2 percent for the U.S. This drives up the average in Table 1. To achieve the higher average maturity of debt, there must be a larger fraction of relatively long-term debt outstanding for Japan. For the U.K., the relative numbers are more in line with intuition, in that a higher average maturity of debt than in the U.S. implies a lower fraction of debt maturing.

### 3 A model of long-term debt and inflation

The model is a standard New Keynesian framework, with the addition of long-term bonds with stochastic maturity and of learning about the inflation target. The central bank follows a Taylor rule which sets the short-term nominal interest rate as a function of the inflation rate gap and the output gap; there is a one-period government bond priced at that interest rate, in addition to the newly introduced stochastic bond. Firms are monopolistic competitors selling differentiated products at prices that are allowed to adjust in a stochastic fashion as in Calvo (1983).<sup>12</sup> Consumers maximize lifetime utility from consumption, labor input, and real money holdings. Government budget dynamics are determined by a fiscal rule.

#### 3.1 The maturity structure of public debt

The central element of the model is an approximation of the maturity structure of public debt in terms of a stochastic, long-term, bond. Each period, an individual bond of this type pays the interest determined when the bond was issued and, with a given probability, matures. In this case it also pays back the principal. Technically, the bond is a callable perpetuity with stochastic call date, which is independent across bonds. Since the government issues a large number of these bonds each period, the fraction of bonds maturing is identical to the call probability. Private agents are assumed to hold the representative portfolio of the bonds.<sup>13</sup>

With probability  $\alpha$  the stochastic bond matures, and with probability  $1 - \alpha$  it survives into the next period. Denote the total value of the stock of long-term bonds with  $B_t^L$ , and the more familiar risk-free one-period bond simply with  $B_t$ . The total stock of long-term

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months earlier this year should be arrested and begin to slowly lengthen going forward."

<sup>12</sup>Our results are robust to the inclusion of sticky wages à la Erceg, Henderson and Levin (2000), thus we stick to the simplest model.

<sup>13</sup>We exclude the possibility of explicit government default, other than implicitly by inflation. See Hatchondo and Martinez (2009) or Arellano and Ramanarayanan (2012) for recent examples. Also, we do not explore inflation risk premia and term structure implications of our model. On this, see for example Rudebusch and Swanson (2008). These authors use, as in Woodford (2001), an assumption on declining payment streams on consols, which in the aggregate shows some similarities with the bond structure developed here.

bonds then evolves as

$$B_t^L = (1 - \alpha)B_{t-1}^L + B_t^{L,n}, \quad (1)$$

where  $B_t^{L,n}$  denotes the amount of newly issued bonds, while  $(1 - \alpha)B_{t-1}^L$  is the value of bonds not maturing. Every period, the government is assumed to issue new debt, to replenish the depleted debt and reduce or increase the total amount of outstanding debt. There is always a stock of bonds that was not redeemed, and all ages of bonds are present in the market.

Let the interest rate of bonds newly issued in period  $t$  be given by  $i_t^{L,n}$ , and the average interest rate of all current and previously issued stochastic bonds by  $i_t^L$ . Then the latter is given by

$$i_t^L = \frac{B_t^{L,n}}{B_t^L} i_t^{L,n} + (1 - \alpha) \frac{B_{t-1}^{L,n}}{B_t^L} i_{t-1}^{L,n} + (1 - \alpha)^2 \frac{B_{t-2}^{L,n}}{B_t^L} i_{t-2}^{L,n} + \dots$$

The weights on the interest rates of previously issued bonds depend on the fraction of those bonds that has survived until date  $t$  and the value of these bonds relative to that of the current stock of long-term debt. Thus the average interest rate on outstanding long-term debt can conveniently be tracked in recursive form

$$i_t^L B_t^L = (1 - \alpha) i_{t-1}^L B_{t-1}^L + i_t^{L,n} B_t^{L,n}. \quad (2)$$

The interest rate  $i_t^{L,n}$  is priced according to an appropriate arbitrage condition between the one-period and the stochastic bonds, derived below from the households first-order conditions.

The parameter  $\alpha$  determines not only the fraction of debt maturing each period, but also the average maturity of outstanding debt,  $1/\alpha$ . We calibrate the latter to match the actual average maturity of debt in the data. Below we show that this choice of  $\alpha$  also gives a good approximation of the observed fractions of debt maturing, which is important for the effects of changes in inflation on the real value of outstanding debt.

## 3.2 Households

The representative household is assumed to maximize the present value of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma_c}}{1-\sigma_c} + \chi \frac{(M_t/P_t)^{1-\sigma_m}}{1-\sigma_m} - \varphi \frac{N_t^{1+\phi}}{1+\phi} \right),$$

with  $C_t$  consumption,  $M_t/P_t$  real money balances, and  $\beta$  the discount factor,  $\sigma_c$  the inverse of the inter-temporal elasticity of substitution (and the inverse of risk aversion),  $\sigma_m$  governs the interest elasticity of money demand, and  $\chi$  a utility weight. Labor services provided enter negatively, with  $\phi$  the inverse of the Frisch elasticity of labor supply, and  $\varphi$  scales labor disutility. The consumption good is an aggregate of a continuum of differentiated products  $C_t(z)$ , and given by the function  $C_t = \left( \int_0^{\infty} C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}$ , with  $\epsilon > 1$  a constant elasticity of substitution. The individual goods are supplied by a continuum of monopolistically competitive firms at price  $P_t(z)$  for each firm  $z$ .

Maximization takes place subject to the evolution of the interest rate on the portfolio of bonds (2), and the budget constraint

$$\begin{aligned} \frac{B_t}{P_t} + \frac{B_t^{L,n}}{P_t} + \frac{M_t}{P_t} + C_t = & (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + (\alpha + i_{t-1}^L) \frac{B_{t-1}^L}{P_t} + \frac{M_{t-1}}{P_t} \\ & + (1 - \tau_t) \frac{W_t}{P_t} N_t + \int_0^1 \frac{\Pi_t(z)}{P_t} dz, \end{aligned}$$

where  $W_t/P_t$  is the real wage and  $\tau_t$  is a proportional tax rate on labor income.  $\Pi_t(z)$  is nominal income from dividends of monopolistically competitive intermediate firms – indexed  $z$  – owned by households. As mentioned, the one period bond issued in period  $t$  is denoted by  $B_t$  and pays interest  $i_t$  in the following period. In contrast, only a fraction  $\alpha$  of long-term bonds  $B_{t-1}^L$  is redeemed each period, and a quantity  $B_t^{L,n}$  of bonds are newly issued.

Combining equation (1) with equation (2) and the budget constraint, the representative household maximizes its intertemporal utility with respect to  $C_t$ ,  $B_t$ ,  $B_t^L$ ,  $i_t^L$ ,  $M_t$ ,  $N_t$ , and  $C_t(z)$ . Note at this point that, from the perspective of an individual household, while the market-determined long-term interest rate  $i_t^{L,n}$  is taken as given, the average interest rate  $i_t^L$  depends on the composition of newly-issued relative to outstanding bonds that the households chooses to hold. This must be taken into account when solving the household's optimization problem.

Consumption smoothing and the holdings of the two types of bonds are guided by the familiar Euler equation for short-term bonds,

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + i_t], \quad (3)$$

and a similar Euler equation for long-term bonds

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left[ 1 + i_t^{L,n} - \mu_{t+1} (1 - \alpha) \Delta i_{t+1}^{L,n} \right], \quad (4)$$

where  $\lambda_t$  is the marginal utility of wealth, which must be equal to the marginal utility of consumption

$$\lambda_t = C_t^{-\sigma_c}. \quad (5)$$

The second Euler condition deserves further comment. It relates the nominal stochastic discount factor  $\beta(\lambda_{t+1}/\lambda_t)P_t/P_{t+1}$  to the interest rate on newly-issued long-term debt,  $i_t^{L,n}$ , corrected for its expected change,  $\Delta i_{t+1}^{L,n} = i_{t+1}^{L,n} - i_t^{L,n}$ . The change is valued by the Lagrange multiplier on (2), which is the price of the stochastic bond, and must follow

$$\mu_t = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} [1 + (1 - \alpha)\mu_{t+1}], \quad (6)$$

for an expected quarterly payout of one.<sup>14</sup> Thus  $-\mu_{t+1}\Delta i_{t+1}^{L,n}$  in (4) is the capital loss (or gain) in period  $t + 1$  incurred from a rise (or fall) in the long-term interest rate. The

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<sup>14</sup>In steady state, the price is  $\mu = \frac{1}{i+\alpha}$  which gives the familiar price equations for the one-period bond when  $\alpha = 1$ , and for a consol when  $\alpha = 0$ .

intuition of (4) is that an expected capital loss on long-term bonds reduces the incentives to invest in long-term bonds today, and thus requires the long-term interest rate today,  $i_t^{L,n}$ , to be higher in order to ensure that agents are indifferent to investing in short-term bonds.

The remaining optimality conditions are the familiar conditions for money demand,

$$\frac{M_t}{P_t} = \left[ \chi C_t^{\sigma_c} \frac{1+i_t}{i_t} \right]^{1/\sigma_m}, \quad (7)$$

labor supply,

$$\varphi N_t^\phi = C_t^{-\sigma_c} (1 - \tau_t) \frac{W_t}{P_t}, \quad (8)$$

and the demand for differentiated products,  $C_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} C_t$ , where the price level is defined as the cost of the minimum-expenditure combination of the  $C_t(z)$  to obtain a given value of  $C_t$ ,  $P_t \equiv \left( \int_0^\infty P_t(z)^{1-\epsilon} dz \right)^{1/(1-\epsilon)}$ .

### 3.3 Firms

Firms are monopolistic competitors each facing iso-elastic demand for their differentiated products derived above, and demand labor to produce. Production is linear in labor,  $Y_t(z) = AN_t(z)$ , where  $A$  is the aggregate productivity level. Prices are sticky in that each period, following Calvo (1983), only a fraction  $(1 - \theta)$  of firms is able to optimally adjust prices. If a firm cannot re-optimize its price, the nominal price evolves according to the indexation rule  $P_t(z) = \pi_t^* P_{t-1}(z)$ , where  $\pi_t^*$  is the actual inflation target. Under imperfect information, as introduced later,  $\pi_t^*$  will have to be replaced by the perceived inflation target. Thus  $\pi_t = P_t/P_{t-1}$  is the gross aggregate inflation rate. The inclusion of the actual or perceived inflation target in the indexing rule is crucial for the issue at hand, because we deal with potentially permanent changes in inflation, and want to ensure that long-run monetary neutrality holds.<sup>15</sup>

Taking into account that it might not be able to set its price optimally in a near future, a firm  $z$  chooses the optimal price,  $P_t^*(z)$ , by maximizing intertemporal profits subject to the demand it faces and taking into account the indexing rule. The first-order condition for this program is

$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}}$$

where

$$\mathcal{Z}_{1,t} = \lambda_t m c_t C_t + \theta \beta E_t \left[ \left( \frac{\pi_{t+1}}{\pi_{t+1}^*} \right)^{-\epsilon} \mathcal{Z}_{1,t+1} \right] \quad (9)$$

and

$$\mathcal{Z}_{2,t} = \lambda_t C_t + \theta \beta E_t \left[ \left( \frac{\pi_{t+1}}{\pi_{t+1}^*} \right)^{1-\epsilon} \mathcal{Z}_{2,t+1} \right], \quad (10)$$

<sup>15</sup>We also experimented the more general indexation scheme  $P_t(z) = \tilde{\pi}_t P_{t-1}(z)$ , where we allowed  $\tilde{\pi}_t = \pi_{t-1}^\xi \pi_t^{*(1-\xi)}$  to depend on both lagged actual inflation and the actual (or perceived) inflation target  $\pi_t^*$ . We found that our main results are barely affected by this assumption.

which is the same for all firms that can adjust their price in period  $t$ . Real marginal costs are given by  $mc_t = (W_t/P_t)/A$  and  $\lambda_t$  is the marginal utility of consumption, which appears by the assumption of perfect capital markets. The aggregate price index can be shown to evolve according to

$$1 = \theta \pi_t^{*(1-\epsilon)} \pi_t^{-(1-\epsilon)} + (1-\theta) \left( \frac{\epsilon}{\epsilon-1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} \right)^{1-\epsilon}. \quad (11)$$

### 3.4 The fiscal and monetary authorities

The fiscal authority follows a fiscal rule that adjusts the tax rate depending on the deviation of real debt from a long-run level of real debt, assumed as given. The tax rule is given by

$$\tau_t - \tau = \rho_\tau (\tau_{t-1} - \tau) + \phi_\tau \widehat{d}_t, \quad (12)$$

where  $\tau$  is the steady-state tax rate and  $\widehat{d}_t$  is the percent deviation of total real short- and long-term debt,  $d_t = (B_t + B_t^L)/P_t = b_t + b_t^L$ , with  $b_t = B_t/P_t$  and  $b_t^L = B_t^L/P_t$ , from its long-run steady-state level. The tax smoothing parameter  $\rho_\tau$  prevents excessive jumps in the tax rate, and  $\phi_\tau$  determines the responsiveness of the tax rate to variations in real debt.

Aggregate public debt evolves then according to the consolidated budget constraint of the public sector, written here in real terms as

$$\tau_t w_t N_t + m_t - \frac{m_{t-1}}{\pi_t} + b_t + b_t^{L,n} = g + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} + (\alpha + i_{t-1}^L) \frac{b_{t-1}^L}{\pi_t}. \quad (13)$$

Government revenue consists of tax revenue  $\tau_t w_t N_t$ , seignorage revenue, where  $m_t = M_t/P_t$ , and newly issued debt,  $b_t + b_t^{L,n}$ , while expenditure consists of (exogenous) real government spending  $g$ , redeemed bonds  $b_{t-1} + \alpha b_{t-1}^L$  and real interest paid on bonds  $(i_{t-1} b_{t-1} + i_{t-1}^L b_{t-1}^L)/\pi_t$ . Rewriting the evolution of long-term debt (1) in real terms

$$b_t^L = (1 - \alpha) \frac{b_{t-1}^L}{\pi_t} + b_t^{L,n},$$

shows how much of the last period's outstanding debt,  $b_{t-1}^L$  after accounting for the possible effects of inflation,  $\pi_t$ , is carried over to the current period. In the following, we assume that the one-period bond is issued at an infinitesimal quantity  $b_t = b$ , for it to be merely relevant for the pricing of assets via the short-term policy interest rate  $i_t$ . The monetary authority follows an interest rate rule given by

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ i + \widehat{\pi}_t^* + \phi_\pi (\widehat{\pi}_t - \widehat{\pi}_t^*) + \phi_y (\widehat{Y}_t - \widehat{Y}_t^n) \right] + \eta_t, \quad (14)$$

with  $i$  the steady-state value of the nominal interest rate,  $\widehat{\pi}_t^*$  the time-varying inflation target,  $\widehat{Y}_t$  is actual output and  $\widehat{Y}_t^*$  is the natural rate of output being defined as the level of output that would prevail under fully flexible prices, all expressed as deviations from steady state. Hence,  $i + \widehat{\pi}_t^*$  is the variation of the nominal rate that is governed by changes in the inflation target. The policy interest rate adjusts with inertia, as given by  $\rho_i$ . The

interest rate rule is additionally subject to a monetary policy white noise disturbance  $\eta_t$  with variance  $\sigma^2$ . The policy rule as perceived by agents under imperfect information is introduced later.

The percentage deviation of the inflation target from steady state is assumed to evolve according to follow the AR(1) process

$$\widehat{\pi}_t^* = \rho_\pi \widehat{\pi}_{t-1}^* + \eta_t^\pi$$

with  $\eta_t^\pi$  a white noise process with variance  $\sigma_\pi^2$ . The persistence parameter  $\rho_\pi$  is between zero and one so that variations of the target are potentially very persistent. The shocks are i.i.d. normal. To reflect a high degree of credibility of an existing target in the imperfect information scenario, the variance of  $\eta_t^\pi$  is assumed substantially lower than that of  $\eta_t$ .

### 3.5 Market clearing and equilibrium

Aggregate demand is given by total private and government consumption:

$$Y_t = C_t + g, \quad (15)$$

and the market clearing condition on goods market is given by:

$$\Delta_{p,t} Y_t = AN_t, \quad (16)$$

where  $N_t = \int_0^1 N_t(z) dz$  is aggregate labor input and the term  $\Delta_{p,t} = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} dz$  measures the price dispersion arising from staggered price setting. Similar to the aggregate price index, the price distortion has a law of motion that can be shown to be:

$$\Delta_{p,t} = \theta \Delta_{p,t-1} \left( \frac{\pi_t}{\pi_t^*} \right)^\epsilon + (1 - \theta) \left( \frac{\epsilon}{\epsilon - 1} \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} \right)^{-\epsilon}. \quad (17)$$

The competitive equilibrium of our model is a set of stationary processes  $B_t, B_t^L, B_t^{L,n}, C_t, \Delta_{p,t}, i_t, i_t^L, i_t^{L,n}, \lambda_t, M_t, \mu_t, N_t, \pi_t, \tau_t, W_t, Y_t, \mathcal{Z}_{1,t}, \mathcal{Z}_{2,t}$ , satisfying the relations (1) to (17) and  $b_t = b$ , given the exogenous stochastic processes  $\eta_t, \eta_t^\pi$  and  $Y_t^*$ , and the initial conditions  $B_{-1}, B_{-1}^L, i_{-1}, i_{-1}^L, \Delta_{p,-1}$  and  $\pi_{-1}$ .

### 3.6 Imperfect information and credibility

We allow for the possibility that private agents do not have perfect knowledge of the central bank's objectives. That is, agents cannot distinguish movements in the inflation target from movements in the monetary policy shock, but only receive a signal on an aggregate monetary policy shock, defined as

$$\varepsilon_t^\pi \equiv (1 - \rho_i)(1 - \phi_\pi) \widehat{\pi}_t^* + \eta_t. \quad (18)$$

The signal extraction problem entails backing out the two components  $\widehat{\pi}_t^*$  and  $\eta_t$  in the Taylor rule (14). Formally, given their knowledge about the driving process of the shocks

and of the standard deviation of the inflation target and policy shock, agents use a simple Kalman filter to extract the optimal estimates of the two unobserved components of  $\varepsilon_t^\pi$ .<sup>16</sup>

The optimal estimate of the inflation target evolves according to:

$$\tilde{E}_t \hat{\pi}_t^* = \tilde{E}_{t-1} \hat{\pi}_t^* + \frac{k}{\rho_\pi} (\varepsilon_t^\pi - \tilde{E}_{t-1} \varepsilon_t^\pi); \quad (19)$$

where  $k = \frac{\rho_\pi(1-\rho_i)(1-\phi_\pi)\mathcal{P}}{((1-\rho_i)(1-\phi_\pi))^2\mathcal{P}+\sigma^2}$  is the Kalman gain parameter of the steady-state Kalman filter, with  $\mathcal{P}$  solving  $\mathcal{P}^2 + [(1-\rho_\pi^2)\sigma^2 / ((1-\rho_i)(1-\phi_\pi))^2 - \sigma_\pi^2] \mathcal{P} - (\sigma_\pi\sigma / ((1-\rho_i)(1-\phi_\pi)))^2 = 0$ . Note that a higher variance of the monetary policy shock relative to the variance of the target shock implies a lower Kalman gain. Correspondingly, the optimal estimate of the monetary policy shock is

$$\tilde{E}_t \eta_t = \varepsilon_t^\pi - (1-\rho_i)(1-\phi_\pi)\tilde{E}_{t-1}\hat{\pi}_t^*. \quad (20)$$

Then the optimal forecasts of the future inflation targets and monetary policy shock can be obtained:

$$\begin{bmatrix} \tilde{E}_t \hat{\pi}_{t+i}^* \\ \tilde{E}_t \eta_{t+i} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ 0 & 0 \end{bmatrix}^i \begin{bmatrix} \tilde{E}_t \hat{\pi}_t^* \\ \tilde{E}_t \eta_t \end{bmatrix}$$

It is important to keep in mind that, if the intention of a higher inflation rate is announced and believed, expectations of future inflation are correct in the sense that agents would not be making systematic errors in predicting inflation. In other words, the signal extraction problem would be absent, as this is the full information case. The signal extraction problem is used to capture different degrees of credibility of the central bank's established inflation target. Under imperfect information, i.e., when the central bank does not announce a changed inflation target – or agents do not believe an announcement – agents repeatedly make forecast errors, since over many periods their perception of the target differs from the actual realization of the target. Slow learning about the true increasing target reflects a high credibility of the a previously prevailing – low – inflation target. Changes in  $\varepsilon_t^\pi$  will be ascribed mainly to the transitory shocks, and inflation expectations for a longer time will remain anchored at a low level. Our analysis is thus the opposite to the case considered in [Erceg and Levin \(2003\)](#), who analyze slowly revised perceptions about a reduction in the Federal Reserve's inflation target, as it took place during the Volcker disinflation in the 1980s.

### 3.7 Parametrization and solution procedure

The model is parametrized at the quarterly frequency, with a discount factor of  $\beta = 0.99$ , which implies a steady state annual real interest rate of about 4%. The intertemporal elasticity of substitution is governed by  $\sigma_c = 1.5$  following the estimates in [Smets and Wouters \(2007\)](#), and the disutility of labor is determined via  $\varphi = 2$  in line with [Domeij and Floden \(2006\)](#). We set the money demand elasticity  $\sigma_m$  to 2.56 as in [Chari, Kehoe](#)

<sup>16</sup>Examples of such kind of imperfect information mechanism can be found in [Erceg and Levin \(2003\)](#), [Melecky, Palenzuela and Söderström \(2009\)](#), [Fève, Matheron and Sahuc \(2010\)](#) or [Darracq-Pariès and Moyen \(2012\)](#).

and McGrattan (2000), while the scale factor  $\chi$  is set to match the long-term ratio of the monetary base to output in the U.S. The monopolistic markup factor is set to 20 %, resulting from a demand elasticity for the differentiated products of  $\epsilon = 6$ . The average level of hours worked is set to one third.

Tab. 2 - BASELINE CALIBRATION

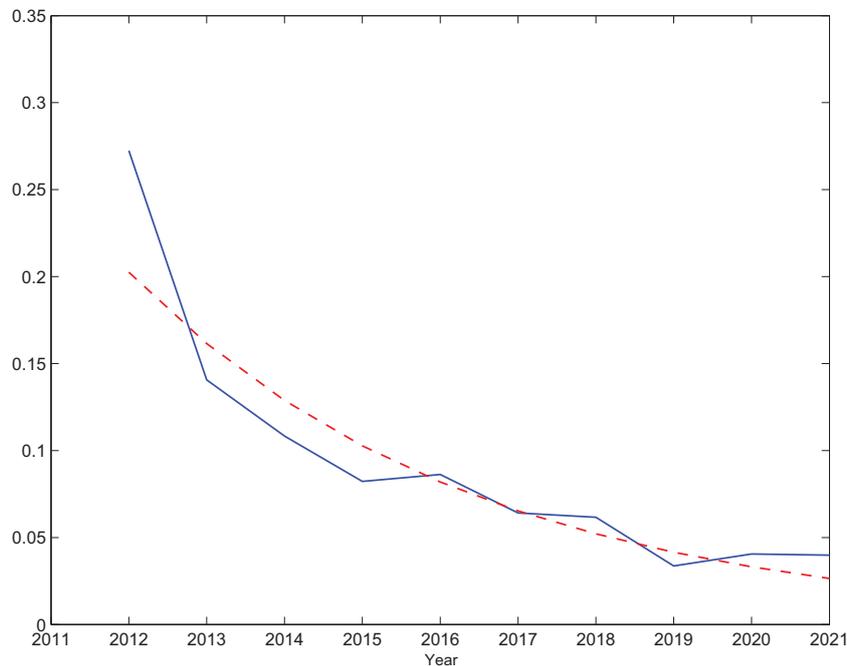
Parameter	Value	Description
<u>Preferences</u>		
$\beta$	0.99	Time discount factor
$\sigma_c$	1.5	Intertemporal elasticity of substitution
$\sigma_m$	2.56	Inverse of the interest elasticity of money demand
$\chi$	$5.2 \times 10^{-6}$	Scale factor to utility of money balances, targets $m/Y = 0.07$
$\phi$	2.00	Inverse of the Frish of labor supply
$\varphi$	35.94	Scale factor to disutility of work, targets $h = 1/3$
<u>Bonds market</u>		
$\alpha$	0.055	Quarterly probability of maturing debt
<u>Firms</u>		
$\epsilon$	6	Price markup of 20%
$\theta$	0.75	One year price contracts
<u>Monetary policy</u>		
$\rho_i$	0.75	Interest rate smoothing parameter
$\phi_\pi$	1.5	Response of interest rate to inflation
$\phi_y$	0.5	Response of interest rate to output gap
<u>Fiscal policy</u>		
$\rho_\tau$	0.5	Tax rate smoothing parameter
$\phi_\tau$	0.0037	Tax feedback to deviations of debt from steady-state

The steady-state debt to GDP ratio is assumed to be 50 percent, as the level to which debt would converge in the long-run. The shock will drive up this debt to slightly above 67 percent. The steady state government spending to GDP ratio is set to 20 percent. Since the short-term bond is only used to determine the stochastic discount factor, its actual quantity is assumed constant and close to zero. So the average maturity essentially depends on the properties of the long-term bond.<sup>17</sup>

The probability of the stochastic bond maturing is set to  $\alpha = 0.055$  to match the average maturity of U.S. debt held by the public of about 4.5 years, or 55 months, in U.S. data. (see Section 2). Recall that  $\alpha$  also determines the fraction of debt maturing each quarter, and the fractions of currently outstanding debt that will mature in future periods. Figure 1 shows as an example how for this value of  $\alpha$  the model gives a good

<sup>17</sup>Allowing for non-infinitesimal quantities of the one-period bond is easy, and would make it possible to calibrate at the same time the average maturity of debt and the fraction of debt maturing. The results of the paper would not be affected.

Fig. 1 - FRACTION OF DEBT MATURING



*Notes:* The solid line shows, for US debt outstanding in mid 2011, the fraction maturing over the next ten years. The dashed line depicts the corresponding fraction implied by the model.

representation of the declining fractions of debt outstanding for the U.S.<sup>18</sup>

The parameters of the monetary policy rule assume the fairly standard values of  $\phi_\pi = 1.5$  and  $\phi_y = 0.5$  and a persistence parameter of  $\rho_i = 0.75$ . Given the private agents' parameters, the values guarantee determinacy of the rational expectations equilibrium in models with balanced budget rules, as well as in models with sufficiently aggressive fiscal policy rules. In the baseline case, the inflation target has a persistence of  $\rho_\pi = 0.99$ .

For the fiscal rule, we assume  $\rho_\tau = 0.5$  and  $\phi_\tau = 0.0037$ , which yields determinate and non-explosive equilibria in all our simulations.<sup>19</sup> The tax response to deviations of debt from the steady-state sustainable level is very mild and thus gives the potentially strongest role for inflation to contribute to debt consolidation. Of course, a high tax responsiveness is possible, and could easily take care of the higher debt. We show its effects below, but this is the very scenario that political constraints will most likely make difficult to follow, and may be avoided by raising inflation instead.<sup>20</sup> We describe the calibration of

<sup>18</sup>The data used to compute the fractions of U.S. debt maturing over the next ten years can be found in Table 2, column 1 in [Bohn \(2011\)](#).

<sup>19</sup>Even though  $\phi_\tau$  is small, the long-term response is  $\phi_\tau/(1 - \rho_\tau)$ , constituting passive fiscal policy, in the sense of [Leeper \(1991\)](#).

<sup>20</sup>Throughout, we assume that the model's approximation is far enough from a fiscal limit, where

the shocks' standard deviations in the next sections along with the presentation of our different scenarios.

The calibrated models' rational expectations equilibrium dynamics are derived numerically using Dynare, which utilizes the ideas of Sims (2002). To determine the solutions under imperfect information, the resulting state space solution is then augmented by the evolution of the inferences on the permanent and transitory shocks to the monetary policy rule. That is, rather than simulating the model response to the shocks following the AR(1) processes specified above, the perceived shocks derived from the Kalman filter are fed into the model, and the corresponding impulse responses displayed.

## 4 Analysis

In this section, we analyse the dynamic adjustment of public debt and other key variables with and without an exogenous change in the inflation target. The starting point of the simulations is an increase of public debt of the magnitude observed in the U.S. since the onset of the economic crisis of 2008 and 2009. We first consider two scenarios for the response of tax rates to show how public debt would evolve absent change in the inflation target.<sup>21</sup> Then we subject the inflation target to shocks of varying persistence.

### 4.1 A debt shock

In all the scenarios, debt is assumed to increase by about 70 percent from the current debt-to-GDP ratio. For the U.S., this corresponds to the increase of debt from about 42.5 percent of GDP in 2006 to the projected 72.4 in 2011. Figure 2 shows the subsequent evolution of real government debt. All variables responses are reported as percentage deviations from steady state, except inflation and the interest rates, which are expressed in annualized absolute deviations. The solid line depicts the dynamic response for a fiscal rule where the tax rate adjusts to higher debt just sufficiently to keep debt from exploding. Then real debt barely falls over the following 20 years. The associated tax rate increase is about 0.5 percentage points. Because of the higher stock of debt, of which each period a fraction  $\alpha = 5.5\%$  becomes due, a correspondingly higher amount of new debt is issued each period.

The higher tax rate leads to drop in after-tax *net* real wages for workers, reducing their labor supply and consequently output by about 0.2 percent below steady state. This distortionary effect lasts until debt returns to its long-run level. The inflation rate only initially slightly falls below the long-run target, following a short-lived contractionary effect of the tax rate on labor supply and thus the real wage, which reduces real marginal costs. However, after a few quarters, inflation, real wages, and the real interest rate return back to, or close to, the steady state.

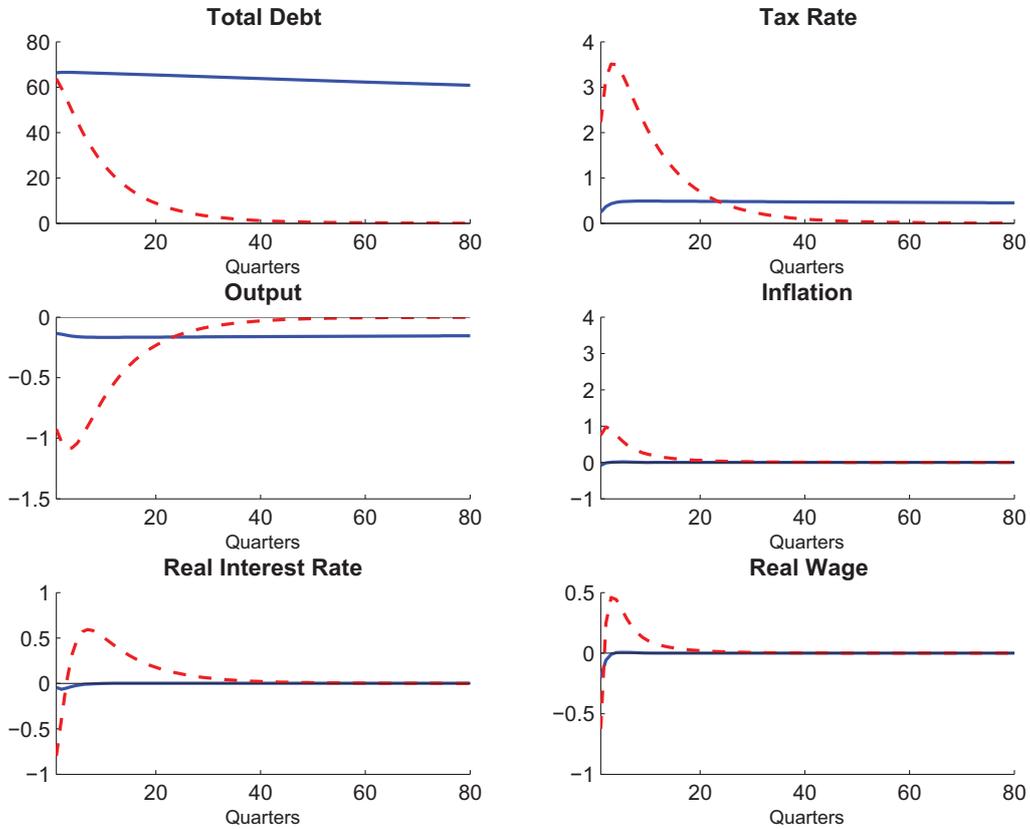
Contrast this with a tax policy that reduces the additional debt within a short amount of time, depicted by the dashed line in Figure 2. This tax policy is represented in the

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further tax increases would lead to falling tax revenue due to Laffer curve effects.

<sup>21</sup>The debt increase can be seen as a helicopter drop of government bonds, as in Leith and Wren-Lewis (2011).

Fig. 2 - DEBT SHOCK



*Notes:* The impulse responses portray selected variables responses to the debt shock described in the text for two different scenarios. The solid line depicts the response of the economy under the baseline calibration while the dashed line illustrates the dynamics under a debt reducing tax policy.

fiscal authority's tax rule by a high coefficient on debt. The resulting tax increase is up to almost 4 percentage *points* above the steady state tax rate, and leads to a much larger drop in output over several periods, almost 1.2 percent in the second quarter. At the same time, inflation rises by over one percentage point, which induces the policy interest rate, and also the real interest rate, to increase after a short drop. Finally, real gross wages rise because of the reduced labor supply after the tax increase.

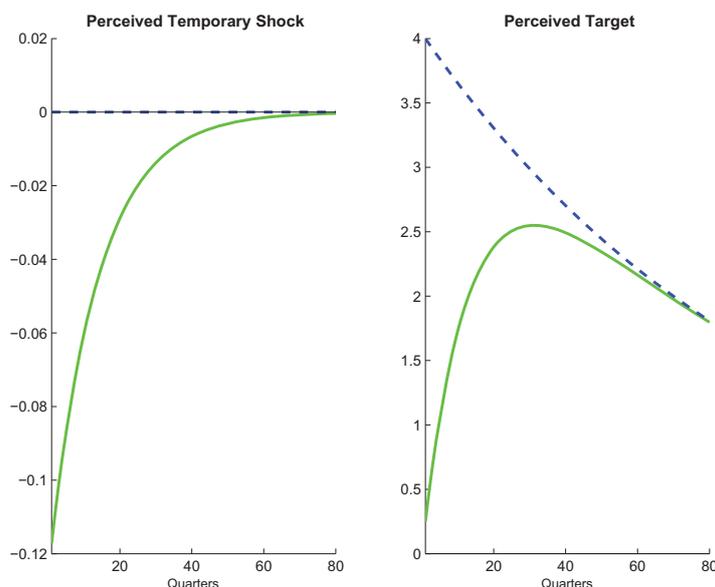
## 4.2 Real debt and changes of the inflation target

We now turn to the question how much a change in the inflation target rather than raising taxes would contribute to a reduction in government debt. That is, we take the baseline change in debt to GDP of 70 percent, the fiscal rule that implies a minimal reaction of

the tax rate, and simulate a persistent change in the annualized inflation target by 4 percentage points from 2 to 6 percent.

The persistence parameter of the target process is set to a high value of  $\rho_\pi = 0.99$ . Later, we consider less persistent changes.<sup>22</sup> Throughout, we compare here the evolution of the economy after changes in the inflation target both when it is perfectly observed and when the public cannot distinguish a change of the target from the transitory monetary policy shock. Recall that we take imperfect information to reflect a high credibility of a previously prevailing low inflation target, when the change in the target has not been communicated to the public. The degree of misperception depends crucially on how volatile the public perceives the inflation target to be, which in turn determines the speed of learning according to the Kalman filter. In this calibration, the relative perceived volatilities of the target and the monetary policy shock are set such that perceived and actual inflation target coincide after 20 years.

Fig. 3 - PERSISTENT TARGET SHOCK



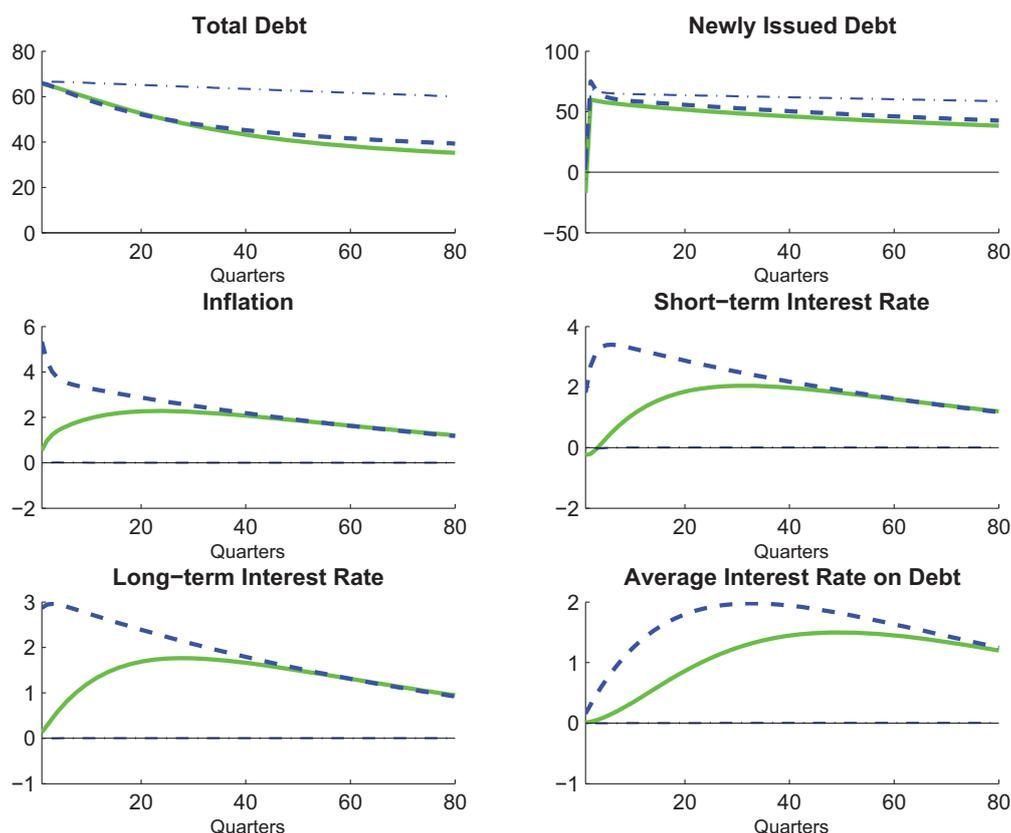
*Notes:* The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning.

Figure 3 shows the evolution of the actual and perceived inflation target after the one time increase of four percentage points. The target follows the process specified above, as depicted by the dashed line in the right-hand-side panel. In contrast, when agents only

<sup>22</sup>Furthermore, for clarity, we assume that all the new debt is priced at the steady-state nominal interest rates, so that the shock to the inflation target occurs after the increase in debt. This avoids confounding effects of the target change with effects from the debt change as such.

slowly learn about the changed inflation trend, it takes long for the difference between actual and perceived target to vanish. This is because, initially, agents assign a large fraction of the change in the nominal interest to the transitory shock, as can be seen clearly in the left panel.

Fig. 4 - PERSISTENT TARGET SHOCK



*Notes:* The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning. Finally, the dashed dotted line represents the benchmark constant target case.

The corresponding evolution of the economic variables of interest are shown in Figures 4 and 5. The former focuses on real government debt held by the public, the realized inflation rate, and the three measures of interest generated by the model. The latter figure shows the evolution of output, real interest and government spending on interest, the real wage, the marginal tax rate, and tax revenue.

Consider first the responses under full information about the inflation target in Figure 4, again depicted by the dashed lines. The addition to total debt that followed the debt shock falls over time, but at a decelerating rate. After ten years, about 29 percent of

this increase in total real debt has been inflated away. Recall that we show here only the additional debt above steady-state debt. Along with total debt, newly-issued debt follows a similar path in percentage deviations from its steady state, since a fraction  $(1 - \alpha)$  of the now higher total debt has to be rolled over. The initial jump in new debt beyond what is needed to roll over is due to a substitution from money holdings to bonds, induced by a higher nominal interest rate that induces to a drop in money demand. The government budget constraint mandates a commensurate increase in bonds to offset the loss in seignorage.<sup>23</sup>

Actual inflation follows the same time path as the target, except for a short-lived initial cost push effect, which follows from the assumed inertial behavior of the short-term, policy, interest rate. Thus the policy interest rate in the fourth panel does not immediately adjust to the higher target. Of course, after a few periods, the Fisher relationship has to hold, and the deviations of the short-term rate from steady state track that of inflation closely. This relationship is exploited in more detail below.

The behavior of the long-term interest rates on newly-issued debt,  $i_t^{L,n}$ , shows the implications of the long-term bond introduced in this paper. To see this most clearly, linearize about the steady state the two Euler equations (3) and (4) for the short- and long-term bonds, to find that, up to first order:  $i_t^{L,n} \approx \alpha^n i_t + (1 - \alpha^n) E_t i_{t+1}^{L,n}$ , or

$$i_t^{L,n} \approx \alpha^n \sum_{s=0}^{\infty} (1 - \alpha^n)^s E_t i_{t+s}, \quad (21)$$

where  $\alpha^n \equiv (\alpha + i)/(1 + i)$ . The long-term interest rate on newly-issued debt is the weighted sum of all future expected short-term nominal interest rates, with declining weights  $(1 - \alpha^n)^s$  as the time horizon  $s$  increases. This is borne out in the bottom left graph.

While the long-term interest rate is an average of future short rates, the average interest rate of outstanding debt is an average of all long-term rates set in the past. Thus the average interest rate paid on debt can only sluggishly follow the evolution of the long-term interest rates on newly-issued debt. This can be made explicit by linearizing equation (2), resulting in the recursion  $i_t^L \approx \alpha^L i_t^{L,n} + (1 - \alpha^L) i_{t-1}^L$  or

$$i_t^L \approx \alpha^L \sum_{s=0}^{\infty} (1 - \alpha^L)^s i_{t-s}^{L,n}, \quad (22)$$

where  $\alpha^L \equiv (1 - (1 - \alpha)/\pi)$ , which is close to  $\alpha$  for the gross inflation rate  $\pi$  close to one. The average interest rate on outstanding debt is thus approximately a weighted average of all past long-term interest rates, with declining weights  $(1 - \alpha^L)^s$ , as the time  $s$  since issuance increases. This explains the slowly rising (dashed) line for the average interest rate on debt. Only after about 30 quarters it begins to fall again. Before turning to other variables of interest, we now discuss the adjustments under imperfect information.

The dynamics under imperfect information are depicted by the solid lines in Figure 4. The perceived inflation target differs from the actual target because agents assign a

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<sup>23</sup>Alternative calibrations of the money demand parameters imply slightly changed initial dynamics, but do not affect our main results.

large fraction of observed interest rate changes to the monetary policy shock, rather than the target. Recall the definition (18) of the signal, which is now filtered according to the Kalman filter. Consequently, actual inflation moves only slowly upwards, since the perceived target  $\tilde{E}_t \hat{\pi}_t^*$  enters firms' price setting, as given by the linearized Phillips curve for full indexation to the perceived target rate:

$$\hat{\pi}_t = \tilde{E}_t \hat{\pi}_t^* + \beta(E_t \hat{\pi}_{t+1} - \tilde{E}_t \hat{\pi}_{t+1}^*) + \kappa \widehat{m}c_t, \quad (23)$$

with  $\kappa$  a nonlinear function of some models' structural parameters. Thus a high credibility of a previously established inflation target lowers the responsiveness of actual inflation to an inflationary change in the central bank's target.

The initial impact on real debt of the change on the inflation target under imperfect information differs only slightly from that under full information. The surprise effect of inflation on outstanding debt is smaller than before. This time, newly issued debt increases by less, because of the slight drop in the nominal interest rate that leads to a small increase in seignorage revenue. The main difference to the full information case bears out over time, however, as agents underprice newly-issued debt, because their inflation expectations are persistently lower than the actual inflation rates. This shows up in the correspondingly slow movements of the short and long-term interest rates.

Figure 5 shows the movements of the remaining variables of interest. Output initially rises under both information scenarios, because the inertial interest rate rule allows real interest rates to drop after the increase in inflation. Over time, output falls below steady state because of the distortionary effect of the higher tax rate. In proportion to the higher debt level, the public sector's spending on interest increases and follows the same dynamics. The behavior of labor tax revenue mirrors the dynamics of output and the tax rate.

### 4.3 Inspecting the mechanism

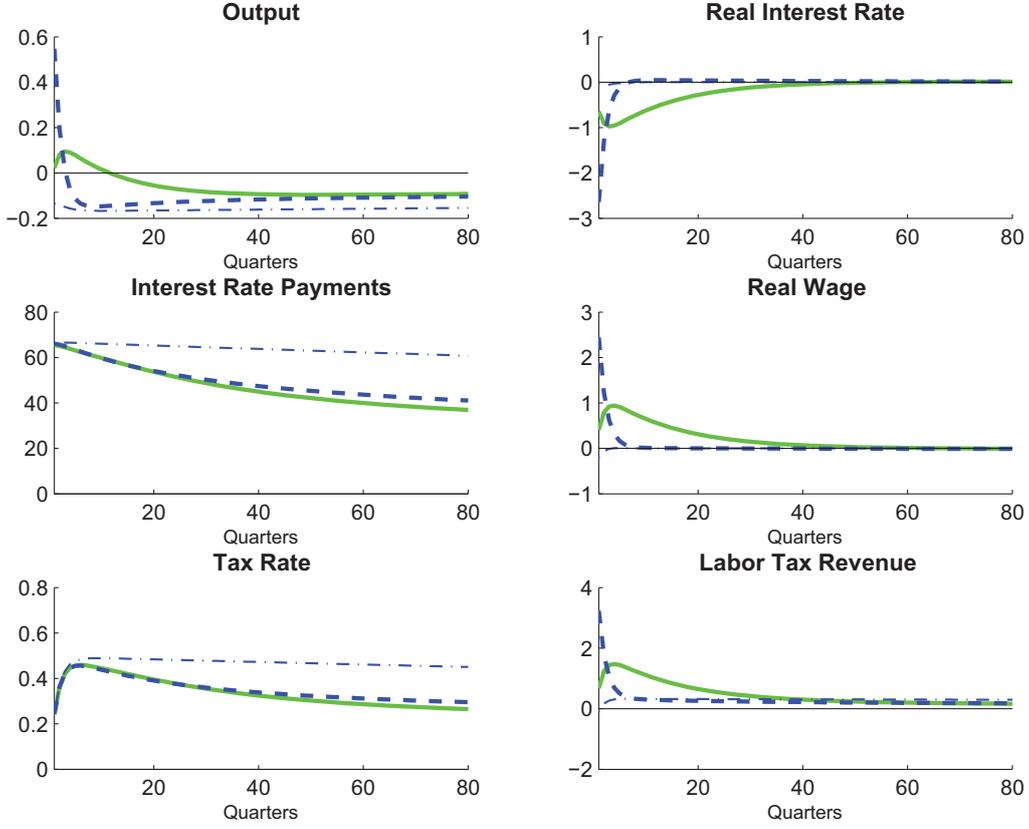
We now turn to a qualitative analysis of how debt maturity and persistence of the inflation target determine the observed evolution of public debt. A few modest simplifications of the model allow us to give transparent analytical representations of the key mechanisms. First, we keep labor supply constant, which removes any feedback effects of taxes on output. Secondly, we ignore seignorage, for simplicity, and because seignorage revenue is known to empirically play only a small role in government revenue dynamics for the ranges of inflation considered here. Finally, we assume perfectly flexible prices.

Under these assumptions, the equation for the evolution of debt combined with the real government budget constraint can be reduced to  $\tau_t wN + b_t^L = g + (1 + i_{t-1}^L) b_{t-1}^L / \pi_t$ . Then, with a slightly simplified fiscal rule  $\tau_t = \tau + \phi_\tau (b_t^L / b^L - 1)$ , we obtain

$$b_t^L = \frac{1}{1 + (\phi_\tau / b^L) wN} \left[ g - (\tau - \phi_\tau) wN + (1 + i_{t-1}^L) \frac{b_{t-1}^L}{\pi_t} \right]. \quad (24)$$

The evolution of debt essentially depends on the coefficient  $(1 + (\phi_\tau / b^L) wN)^{-1}$  and the relative dynamics of  $i_{t-1}^L$  and  $\pi_t$ . The smaller  $\phi_\tau$ , the slower the adjustment of  $b_t^L$  will be to any variations of the right-hand side variables. However, as long as  $i_{t-1}^L$  and  $\pi_t$  do

Fig. 5 - PERSISTENT TARGET SHOCK



*Notes:* The impulse responses portray selected variables responses to a persistent target shock for two different scenarios. The dashed line depicts the response of the economy under full information while the solid line illustrates the dynamics under learning. Finally, the dashed dotted line represents the benchmark constant target case.

not act systematically to stabilize debt,  $\phi_\tau$  must be strictly positive and high enough to guarantee a non-explosive path of debt.<sup>24</sup>

The future dynamics of  $i_t^L$  and  $\pi_t$  depend crucially on the expected path of inflation, which in turn depends on the expected path of the inflation target,  $\pi_t^*$ . This dependence can now be easily made explicit. The assumption of flexible prices implies that the monetary authority directly determines the inflation rate through its control over the

<sup>24</sup>In fact, for  $\phi_\tau = 0$ , the tax rate would be constant. Then, since in steady state,  $g - \tau wN = (1 - (1+i)/\pi)b^L$ , we can linearize equation (24) to get

$$\widehat{b}_t^L = \frac{1+i^L}{\pi} \widehat{b}_{t-1}^L + \frac{b^L}{\pi} \widehat{i}_{t-1}^L - (1+i^L)b^L \widehat{\pi}_t$$

Since the real interest rate rate  $(1+i)/\pi$  is larger than one in steady state, debt would be explosive up to first order.

short-term nominal interest rate. Setting  $\rho_i = \phi_y = 0$  in the Taylor rule (14), and using the consumption-Euler equation (3), inflation can be easily found to follow:<sup>25</sup>

$$\widehat{\pi}_t = \omega \widehat{\pi}_t^* + \frac{1}{\phi_\pi} \eta_t.$$

where we have defined a scale factor  $\omega \equiv (\phi_\pi - 1)/(\phi_\pi - \rho_\pi)$ . The equation holds under full information, and shows that, after a change in the inflation target, future inflation can be expected to evolve directly proportional to the expected inflation target, since  $E_t \widehat{\pi}_{t+s}^* = \rho_\pi^s \widehat{\pi}_t^*$  for  $s \geq 0$ , or

$$E_t \widehat{\pi}_{t+1+s} = \omega \rho_\pi^{1+s} \widehat{\pi}_t^*. \quad (25)$$

Under imperfect information, we have to substitute the target and monetary policy shocks by their respective perceptions at time  $t$ , i.e.,  $\widetilde{E}_t \widehat{\pi}_t^*$  and  $\widetilde{E}_t \eta_t$ , which will be further used below. Then of course, actual and target inflation will not necessarily move closely together.

To determine the expected evolution of different nominal interest rates, we can make use of the two relationships (21) and (22) in combination with the consumption Euler equation (3) and the expected evolution of the inflation target,  $E_t \widehat{\pi}_{t+s}^* = \rho_\pi^s \widehat{\pi}_t^*$ . Since there are no movements in the simplified model's natural real rate of interest,  $1/\beta - 1$ , the evolution of the short-term nominal interest rate is solely determined by the expected inflation rate. Then the Euler equation implies  $E_t i_{t+s} - i = E_t \widehat{\pi}_{t+1+s}$ . Inserting (25) yields

$$E_t i_{t+s} - i = \omega \rho_\pi^{s+1} \widehat{\pi}_t^*.$$

Using the approximation (21) for  $i_t^{L,n}$ , the interest rate on newly-issued long-term debt can now be written as

$$i_t^{L,n} - i \approx \frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \omega \widehat{\pi}_t^*. \quad (26)$$

Furthermore, inserting this into (22) delivers the average interest rate on outstanding debt as a function of the process for the inflation target

$$i_t^L - i \approx \frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \alpha^L \sum_{s=0}^{\infty} (1 - \alpha^L)^s \omega \widehat{\pi}_{t-s}^*, \quad (27)$$

which is the second expression needed to characterize the evolution of government debt.<sup>26</sup>

To gain some further intuition, consider first the factor in front of  $\omega \widehat{\pi}_t^*$  in equation (26). It reflects the relevant aspects of the forward-looking nature of long-term nominal interest rates: maturity of debt and expected evolution of inflation. When  $\rho_\pi = 0$ , that interest rate will be equal to the steady-state rate in all periods,  $i_t^{L,n} = i$ , since target inflation is i.i.d. about its steady state. In contrast, for an inflation target close to a random walk,  $\rho_\pi \approx 1$ , the interest rate follows the same process as the target. In fact, then also  $\omega \approx 1$ . Only for intermediate values of  $\rho_\pi$  does the maturity structure exert its

<sup>25</sup>To obtain this relationship, combine the simplified interest rate rule and the Euler equation and solve forward. Ruling out explosive paths gives the stated (unique) solution to the inflation rate.

<sup>26</sup>Recall that  $\alpha^n = \frac{\alpha+i}{1+i}$  and  $\alpha^L = 1 - \frac{1-\alpha}{\pi}$ , which are close to  $\alpha$  for low steady-state values of  $i$  and  $\pi$ .

influence on the long-term interest rate, which then on average compensates for future inflation rates. In contrast, for an economy with only short-term bonds,  $\alpha = \alpha^n = 1$ , as in the standard New Keynesian model, the nominal interest rate only needs to compensate for one period-ahead inflation.  $i_t^{L,n} - i \approx \rho_\pi \omega \widehat{\pi}_t^*$ , which is of course the short-term nominal interest rate. For (27), the shorter the average maturity, i.e.,  $\alpha$  and thus  $\alpha^L$  close to 1, the smaller the weights on past inflation rates will be. Then, again, the average rate tends to equal the long-term rate and the short-term rate as well. It is clear that under full information, unless  $\alpha$  and  $\rho_\pi$  are at extreme values, the average long-term interest rate will be unable to compensate even for fully and correctly anticipated inflation that follows a change in the inflation target.

The equation for the evolution of real debt, (24), can now be rewritten in deviations from its steady-state level:

$$\widehat{b}_t^L = \Phi \left[ \frac{\alpha^n \rho_\pi}{1 - (1 - \alpha^n) \rho_\pi} \alpha^L \sum_{s=1}^{\infty} (1 - \alpha^L)^{s-1} \omega \widehat{\pi}_{t-s}^* - \widehat{\pi}_t + \widehat{b}_{t-1}^L \right]$$

with  $\Phi = 1/(1 + (\phi_\tau/b^L)\omega N)$ . This equation shows most directly the role of inflation persistence and average debt maturity  $\alpha$ , and gives a simple characterizations for the evolution of debt for different values for the parameters  $\alpha$  and  $\rho_\pi$ . To further understand the role of the determinants of real debt dynamics, assume again that there is no long-term debt, i.e.,  $\alpha = 1$ . Then  $\alpha^n = \alpha^L = 1$ , and debt follows

$$\widehat{b}_t^L = \Phi \left[ E_{t-1} \widehat{\pi}_t - \widehat{\pi}_t + \widehat{b}_{t-1}^L \right] \quad (28)$$

since  $E_{t-1} \widehat{\pi}_t = \rho_\pi \widehat{\pi}_{t-1}$ . Under full information, a one-time increase in the inflation target would have an effect on real debt only in the period of the change, since future nominal rates adjust to compensate for the predictable path of inflation. That is, if  $E_{t-1} \widehat{\pi}_t = 0$ , then a rise in inflation deflates real debt by  $-\widehat{\pi}_t$ . But absent further shocks, there will be no expectational errors, since  $E_t \widehat{\pi}_{t+1} - \widehat{\pi}_{t+1} = \omega (E_t \widehat{\pi}_{t+1}^* - \widehat{\pi}_{t+1}^*) = \omega (\rho_\pi \widehat{\pi}_t^* - \rho_\pi \widehat{\pi}_t^*) = 0$ . In other words, without long-term debt, and under full information, only inflation surprises can affect the real value of the stock of outstanding debt. This is different under imperfect information.

Recall that the inflation rates  $\widehat{\pi}_t$  themselves are the outcome of realizations of the inflation target  $\widehat{\pi}_t^*$  or the beliefs  $\widetilde{E}_t \widehat{\pi}_t^*$  under imperfect information. In the latter case, when agents slowly learn the true inflation target, they will make repeated expectational errors, even if all debt matures after one period. Thus real debt will be affected by inflation even when no further surprise shock to the target rate occurs. This explains the differences between the impulse responses under full information and under imperfect information in the previous section. To be explicit, we can write the evolution of agents' optimal estimate of the target as <sup>27</sup>

$$\widetilde{E}_t \widehat{\pi}_{t+1}^* = \rho_\pi \widetilde{E}_{t-1} \widehat{\pi}_t^* + k' (\widehat{\pi}_t^* - \widetilde{E}_{t-1} \widehat{\pi}_t^*) \quad (29)$$

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<sup>27</sup>Again,  $k' = \frac{\rho_\pi (1 - \phi_\pi) \mathcal{P}'}{(1 - \phi_\pi)^2 \mathcal{P}' + \sigma^2}$  is the Kalman gain parameter, where  $\mathcal{P}'$  solves the equation  $\mathcal{P}'^2 + [(1 - \rho_\pi^2) \sigma^2 / (1 - \phi_\pi)^2 - \sigma_\pi^2] \mathcal{P}' - (\sigma_\pi \sigma / (1 - \phi_\pi))^2 = 0$ .

The object of interest here are the future expectational errors for the inflation target  $\tilde{E}_{t+s-1}\hat{\pi}_{t+s}^* - \hat{\pi}_{t+s}^*$  which, up to the factor  $\omega$ , determine future expectational errors for inflation  $E_{t+s-1}\hat{\pi}_{t+s} - \hat{\pi}_{t+s}$  in the equation for debt (28). It is easy to show that the expectational error for inflation must evolve according to

$$\tilde{E}_{t+s-1}\hat{\pi}_{t+s} - \hat{\pi}_{t+s} = (\rho_\pi - k')^s \left[ \tilde{E}_{t-1}\hat{\pi}_t - \hat{\pi}_t \right]$$

after a one-time surprise increase in the target.<sup>28</sup> Under imperfect information, this expectational error only slowly declines as agents update their perception of the inflation target.

## 5 The sensitivity of debt to changing the inflation target

In this section, we return to the full model and explore the quantitative sensitivity of the results to variations in the parameters governing the persistence of the inflation target and the maturity structure of debt. It turns out that unless the deviation of the inflation target from its steady-state is highly persistent, a significant effect on real public debt can only be achieved when the initial target change is very large. Also a high average maturity alone does not give a temporary but moderate change in inflation sufficient bite to have a large effect on debt. We also find that with mainly short-term debt, that the mispricing of newly-issued debt arising from imperfect information amplifies the effect of higher inflation on debt.

To proceed, we need to decide on a metric that summarizes the effects of changing inflation targets on public debt. Recall Figure 4, where a large part of the debt reduction was achieved after 40 quarters, and the difference between full and imperfect information was noticeable. Therefore, we compute the relative percentage-point difference between the real debt level under the changed inflation target and its level in the absence of that change, at a ten-year horizon. Formally, this is expressed by the following debt multiplier (DM hereafter) measure,

$$DM(h) = (\hat{d}_{t+h}^{TS}/\hat{d}_{t+h} - 1) \times 100,$$

with  $h = 40$  quarters, and where  $\hat{d}_{t+h}^{TS}$  and  $\hat{d}_{t+h}$  are, respectively, the percent-deviations from steady state of the levels of real debt after the target shock and the level of debt under no target shock. Thus the measure basically compares the difference between the dashed-dotted line and the solid or dashed lines in Figure 4.

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<sup>28</sup>The future evolution of the perception after a one-time target shock is

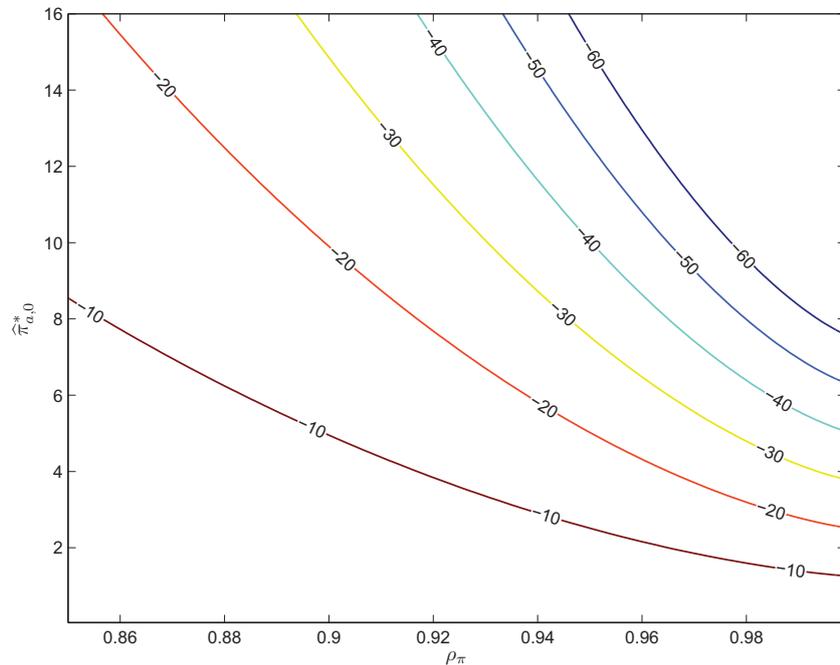
$$\tilde{E}_{t+s-1}\hat{\pi}_{t+s}^* = \rho_\pi^s \hat{\pi}_t^* + (\rho_\pi - k')^s \left[ \tilde{E}_{t-1}\hat{\pi}_t^* - \hat{\pi}_t^* \right]$$

while the actual evolution of the target after the shock at time  $t$  is  $\hat{\pi}_{t+s}^* = \rho_\pi^s \hat{\pi}_t^*$ . For persistent target shocks and noisy signals,  $\rho_\pi$  close to one and  $k'$  small,  $(\rho_\pi - k')^s$  will decline monotonically in  $s$ .

## 5.1 The process of the inflation target

In the baseline scenario, the annual inflation target was increased by four percentage points and very slowly reverted to the low-inflation, steady-state target. This essentially amounts to a permanent departure from a previously communicated monetary policy strategy. However, a decision-maker may be contemplating a policy where inflation increases only temporarily, which corresponds to the proposals made by Rogoff (2008, 2010) and others. We therefore analyze here the role of the persistence and size of the innovation to the inflation target, mainly focusing on the full information case.

Fig. 6 - INFLATION TARGET PROCESS AND SHOCK SIZE



*Notes:* Difference in debt reduction  $DM(40)$  contour plot as a function of the parameters  $\hat{\pi}_{a,0}^*$  and  $\rho_\pi$ . Keeping the other parameters fixed, these parameters are varied in the reported range.

Figure 6 depicts those combinations of  $\rho_\pi$  and  $\hat{\pi}_{a,0}^*$  that yield a given relative reduction  $DM(40)$  in real public debt after ten years (40 quarters). Here,  $\hat{\pi}_{a,0}^* = 4 \times \hat{\pi}_0^*$  denotes the annualized increase of the target at time  $t = 0$ . The baseline scenario is given by  $\rho_\pi = 0.99$  and  $\hat{\pi}_{a,0}^* = 4$ , which results in a drop of about 30 percent in the real value of debt. Larger initial target increases for a given persistence yield proportionately stronger effects on real debt. A smaller increase, of only two percentage points, to the four-percent inflation target proposed by Blanchard et al. (2010) would therefore only reduce real debt by 15 percent after ten years. In contrast, when the persistence is decreased, the effect of a given target change declines more than proportionately. Thus for a persistence of,

say,  $\rho_\pi = 0.95$ , a 30-percent reduction in real debt would only be achieved if the target instead jumps by  $\widehat{\pi}_{a,0}^* = 8$  annualized percentage points above the steady-state target.<sup>29</sup>

In general, if a policy maker wanted to achieve a particular reduction in real debt by raising inflation temporarily, as advocated by Rogoff and others, the effect would be moderate unless the initial jump in the target is very large. An obvious reason is that there is less time for inflation to affect outstanding debt. But also, actual inflation increases by somewhat less than the target itself, due to the scale factor  $\omega$ , which is declining with  $\rho_\pi$ . Finally, in the model, interest rates on newly-issued debt are set to compensate higher inflation on average. Thus, even though inflation may have fallen, some of the debt issued after the target change will still require a higher interest service. As can be gathered from the impulse responses in Figure 4, the corresponding effects on real debt would only be slightly higher under imperfect information.

## 5.2 Debt maturity and Credibility

The left panel of Figure 7 highlights the importance of the maturity structure of public debt for the susceptibility of real debt to a higher inflation target. The change in the target is the four-percentage-point increase considered before, and the vertical line indicates the baseline value  $\alpha = 0.055$  for the fraction of debt that matures each period. Recall that  $1/\alpha$  is then the average expected maturity of debt, in this case four and a half years. The intersections of the vertical line with the dashed and solid curves show the debt reductions after ten years under full and imperfect information, respectively. While under full information, the debt reduction is a little under 30 percent, under learning it is 34 percent. Since debt barely changed even after 10 years when the inflation target was held constant, these values almost fully correspond to the reduction of debt due to the changed inflation target.

Consider first higher values of  $\alpha$ , which imply lower average maturities of debt. As  $\alpha$  increases, the gains from the baseline increase in the inflation target shrink, but do not vanish. In the extreme case with one-quarter bonds only, when  $\alpha = 1$ , the reduction in real debt after a fully perceived increase in the target falls to about 10 percent.<sup>30</sup> In contrast, if a change in inflation is only slowly perceived to be due to a change in the target, then inflation expectations are too low. Then also interest rates on rolled-over debt are repeatedly set too low, and real debt can be deflated by more than 23 percent after ten years. Thus the shorter the average maturity of public debt, the higher is the role of more firmly-anchored past inflation expectations on the sensitivity of real debt to higher actual inflation.

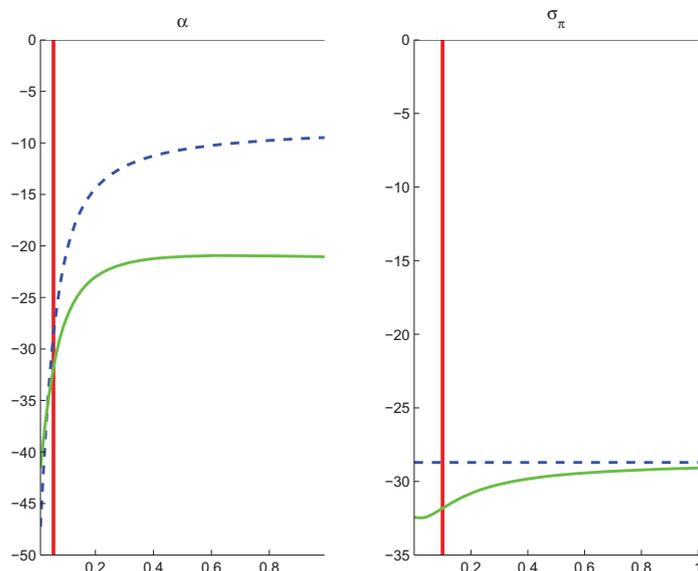
In contrast, a higher maturity implied by values of  $\alpha$  below 0.055 further facilitates inflating away debt. For lower values of  $\alpha$ , that is, higher average maturity of debt, the possible real debt reduction from the changed inflation target increases up to a maximum

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<sup>29</sup>For  $\rho_\pi = 0.95$ , the average annual target inflation rate over ten years would be 3.48 percentage above the steady-state target after the 8 percentage point shock. For  $\rho_\pi = 0.99$ , the inflation target would in this case be on average 6.62 percentage points above steady state.

<sup>30</sup>In fact, when  $\alpha = 1$ , and the target change is fully-perceived, the drop in real debt is solely achieved by the initial surprise jump in inflation that deflates the existing debt before it is rolled over at a higher nominal interest rate.

Fig. 7 - AVERAGE MATURITY AND CREDIBILITY



*Notes:* Difference in debt reduction  $DM(40)$  as a function of the parameter on top of each panel. Keeping the other parameters fixed, that parameter is varied in the reported range. The vertical solid bar indicates its baseline value. The dashed line and the solid line respectively depict the full information and the learning case.

beyond 40 percent. Consider a ‘British’ scenario, with an average maturity of about 13 years ( $\alpha \approx 0.019$ ), much higher than the close to four and a half years for the U.S. In that case, after 10 years, inflation would reduce the additional debt by more than 40 percent, irrespective on whether there is full information or not. The difference between full information and imperfect information remains relatively small, as in the baseline. Thus for realistic maturity structures, the credibility of a previously established inflation target does not strongly affect the degree to which pushing up inflation can reduce real government debt.

The right panel of Figure 7 shows the role of the perceived volatility  $\sigma_\pi$  of the inflation target, which enters the Kalman gain calculation, on the reduction of debt after a given target change of four percentage points. That is, we illustrate here for the case of imperfect information how much the effect of inflation on debt is changing when agents interpret a change in the signal  $\varepsilon_t^\pi$  more or less strongly as a change in the inflation target. The perceived volatility of the inflation target relative to the perceived volatility of the monetary policy shock determines the gain in the Kalman gain, and thus the speed of learning. Again, the vertical line in the graph depicts the baseline case, which corresponds to the dynamics of learning as shown in Figure 4. The more of a change in the signal  $\varepsilon_t^\pi$  is assigned to a change in the target, that is, the higher  $\sigma_\pi$ , the lower is the gain from not

fully communicating the target changes. Conversely, the lower  $\sigma_\pi$ , the less agents believe that a change in the target is possible, and stick to their previous inflation expectations. Then the effect on real debt is much larger, since nominal interest rates compensate to little for subsequently higher inflation.<sup>31</sup>

## 6 Conclusion

This paper investigated to what extent and under which conditions a higher inflation target reduces the real value of public debt. We take the proposals made by Rogoff (2008, 2010) as a starting point, who suggested a two to three year increase in U.S. inflation of about four percentage points, with the aim of alleviating private and public sector balance sheets. To reiterate our main finding obtained in a New Keynesian model with long-term debt and changing inflation targets: a permanent increase of the targeted inflation rate from 2 to 6 percent results in a reduction of about 29 percent of the additional real government debt accrued after the crisis. In contrast, the proposed temporary changes in inflation have substantially smaller effects. For realistic average maturities of public debt, the credibility of a previously established inflation target does not strongly affect the extent to which pushing up inflation can reduce real government debt. That is, not much can be gained from misleading the public by attempting an inflationary policy without revealing it.

We conducted our numerical simulations in the context of the U.S. budgetary situation which appears dire by many accounts.<sup>32</sup> But our theoretical framework equally applies to any other country. Many developed countries have experienced large increases in government debt, and face various degrees of temptation to increase inflation. This can be easily modeled in our framework by appropriate choice of the average maturity and level of public debt. As our analysis suggests, countries with higher average maturities stand to gain more strongly, in particular the U.K. As of now, no country appears to have yielded to the temptation.

We leave it to future research and the reader to decide whether the effect of a 29 percent reduction is to be judged large or small relative to the cost paid in terms of persistently higher inflation. To answer this normative question would require embedding a mechanism that assigns costs to particular levels of debt and inflation. Because of our theoretical requirement that monetary policy be neutral in the long-run, implemented through an indexation assumption, there is no explicit cost to any inflation target change other than the small distortion in monetary holdings induced by a nominal interest rate above the Friedman rule of zero. Only if we allowed for price dispersion or recurrent price adjustment or information costs, would a cost of higher inflation targets be incorporated. Similarly, debt as such has no particular cost, other than the distortion brought about by the proportional labor tax. Future research ought to shed light on these and other trade-offs faced by policy-makers. This notwithstanding, we expect the magnitudes and mechanics of debt dynamics presented here to prevail.

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<sup>31</sup>Melecky et al. (2009) estimate the volatilities of the inflation target and the policy shock, and interpret a higher perceived volatility as an overestimate by households.

<sup>32</sup>See Congressional Budget Office (2010).

In our scenario with imperfect information and learning, the slow learning of the inflation target by the public can be interpreted as a high credibility of the previously established low inflation target. A government would have to weigh the benefits of an increased budget surplus through lower debt servicing with the cost in terms of reputation. Once inflation expectations have adjusted to a higher level, and the public has doubts about the independence of the central bank, future interest rates will incorporate a risk premium for inflation risk.<sup>33</sup> This in turn would affect a government's incentive later on to reduce inflation back to a lower level, as the real value of outstanding debt would be higher than expected. The modeling of the dynamics of reputation, the risk of losing credibility and other game-theoretic aspects is beyond the scope of the paper.

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<sup>33</sup>Rajan (2011) argues along similar lines.

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