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Non-technical summary

Up to now, specifying and estimating a dynamic stochastic general equilibrium model with both reasonable asset pricing and macroeconomic implications is a challenging task. In recent years, considerable advances have been made by investigating novel and cleverly crafted model specifications to overcome the common problems. In particular, it has been shown that, e.g., including frictions on the labor market, modifications regarding the households' preferences, or examining the consequences of well-chosen tail risk distributions can be helpful in this respect.

However, even if a model is crafted for the purpose of jointly delivering macroeconomic as well as asset pricing implications, estimation of such models may not necessarily lead to parameters allowing to deliver on both. We therefore propose a complementary strategy. We suggest constraining the estimation to deliver a particular posterior distribution for some implied variable of interest, such as the Sharpe ratio, i.e., the market price for risk. To demonstrate our methodology, we apply it to a model specification proposed by Uhlig (2007). This model can deliver on some key macroeconomic and asset pricing observations for particular parameter configurations. We show that the application of our methodology produces a quantitative model with both reasonable asset-pricing as well as business-cycle implications.

Nicht-technische Zusammenfassung

Bis heute fällt es der ökonomischen Literatur schwer, dynamische stochastische Gleichgewichtsmodelle zu spezifizieren und zu schätzen, die sowohl die Eigenschaften wichtiger makroökonomischer Zusammenhänge replizieren als auch wichtige stilisierte Fakten von Vermögenspreisen widerspiegeln können. In den letzten Jahren sind wichtige Fortschritte gemacht worden, und durch sorgfältigere Formulierungen der Modelle konnten eine Reihe von Ungereimtheiten überwunden werden. So hat sich z.B. gezeigt, dass die Modellierung von Friktionen auf Arbeitsmärkten, Modifikationen der Nutzenfunktionen der Haushalte oder die Betrachtung spezifischer Risikoverteilungen hier helfen können.

Allerdings führt die sorgfältigere Spezifizierung der Modelle nicht notwendigerweise dazu, dass eine Schätzung dieser letztendlich zu zufriedenstellenden Ergebnissen führt. Wir verfolgen in diesem Papier eine ergänzende Strategie: Die Modellschätzungen werden insofern beschränkt, als dass für ausgewählte stilisierte Fakten, z.B. die Sharpe-Ratio, bestimmte posteriore Verteilungen impliziert werden. Diese Methode wird auf das Modell von Uhlig (2007) angewendet, das grundsätzlich in der Lage ist bei bestimmten Parameterkonfigurationen sowohl stilisierte Fakten von Konjunkturzyklen als auch von Vermögenspreisen zu erklären. Tatsächlich zeigt sich, dass unsere Methode in diesem Fall ein quantitatives Modell liefert, welches Vermögenspreise und Konjunkturzyklen in gleichem Maße erklären kann.

Bundesbank Discussion Paper No 37/2013

Bayesian Estimation of a DSGE Model with Asset Prices *

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Abstract

This paper presents a novel Bayesian method for estimating dynamic stochastic general equilibrium (DSGE) models subject to a constrained posterior distribution of the implied Sharpe ratio. We apply our methodology to a DSGE model with habit formation in consumption and leisure, using an estimate of the Sharpe ratio to construct the constraint. We show that the constrained estimation produces a quantitative model with both reasonable asset-pricing as well as business-cycle implications.

Keywords: Bayesian estimation, stochastic steady-state, prior choice, Sharpe ratio.

JEL classification: C11, E32, E44, G12.

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1 Introduction

This paper presents a novel Bayesian method for estimating dynamic stochastic general equilibrium (DSGE) models subject to constraints on the posterior distribution of the implied Sharpe ratio. Starting from an initial, unconstrained prior, we construct a constrained prior so that the resulting implied posterior for some variable of interest (the Sharpe-ratio, for instance) coincides with some a priori chosen distribution, and such that the constrained prior is proportional to the original prior, conditional on that variable. We apply our methodology to a DSGE model with habit formation in consumption and leisure, real wage rigidities, and capital adjustment costs. We use a density centered at the estimated Sharpe ratio to construct our constraint. We show that the estimation subject to this constraint produces a quantitative model with both reasonable asset pricing as well as business cycle implications.

It can be challenging to specify and estimate a dynamic stochastic general equilibrium (DSGE) model with reasonable asset pricing implications. Considerable advances have been made in recent years by investigating novel and cleverly crafted model specifications or by examining the consequences of well-chosen tail risk distributions. Examples of examining tail risk distributions and disaster risks include Barro (2006), Gabaix (2012) and Gourio (2012). Examples for investigating well-crafted model specifications often fall in one of two branches. One branch of the literature exploits Epstein-Zin preference specifications developed by Epstein and Zin (1989, 1991), and includes Tallarini (2000), Rudebusch and Swanson (2012) or Guvenen (2009). The additional role of long-run risk, see e.g. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008), is exploited in Piazzesi and Schneider (2007), for example. The other branch of the literature has pursued habit formation specifications, see e.g. Abel (1990), Ljungqvist and Uhlig (2000), Campbell and Cochrane (1999), Boldrin, Christiano, and Fisher (2001) and Uhlig (2007). For a variety of reasons, it appears that the latter branch raises more challenges than either an Epstein-Zin-based approach or an approach based on disaster risks.

Even if a model is cleverly crafted for the purpose of jointly delivering macroeconomic as well as asset pricing implications, estimation of such models may not necessarily lead to parameters allowing to deliver on both. In essence, the practical problem appears to boil down to having just a single observation on the size of the risk premium, while there are many observations helping to identify parameters crucial for the macroeconomic dynamics of the model.

We therefore propose a complementary strategy. We propose to constrain the estimation to deliver a particular posterior distribution for some implied variable of interest, such as the Sharpe ratio, the market price for risk. Our procedure adds to the existing literature on endogenous prior choice for Bayesian estimation of DSGE models, see e.g. Del Negro and Schorfheide (2008) and Christiano, Trabandt, and Walentin (2011).

To demonstrate our methodology and to set ourselves a bit of a challenge, we purposely apply it to a habit-formation model specification, extending Uhlig (2007) to include additional shocks. While that model can deliver on some key macroeconomic and asset pricing observations for particular parameter configurations, as shown in Uhlig (2007), an estimated version is desirable to obtain a best fit, for assessing uncertainty and for model comparisons. We solve the model around its stochastic steady state rather than around its deterministic steady state: this may be important at the considerable level of risk premia which we seek to match, see also Coeurdacier, Rey, and Winant (2011). The constrained estimation features a high degree of risk aversion with respect to short-term fluctuations in consumption and thereby a low degree of intertemporal substitution, in contrast to the suggestions by, say, Hall (1988) or Vissing-Jørgensen (2002). This is not surprising and is a result of employing a habit-formation model and seeking to match risk premia on asset markets.

We show that our implementation of the unconstrained estimation[1] fails to deliver reasonable asset pricing implications, while the constrained estimation delivers both reasonable asset pricing as well as business cycle implications. In essence, the estimation procedure seeks the least costly compromise between the two. We find that it does so by increasing the persistence of the response of the investment-to-output ratio, using the capital stock as a long-run buffer to smooth out short-term consumption fluctuations. While the effect on the estimated volatility of the shocks as well as the HP-filtered macroeconomic moments of the model is negligible for practical purposes, there is a noticeable difference in the unconditional (read: long-run) volatility of the investment-output ratio. Put differently, the estimated habit formation model turns short-run shocks into long-run risks, pointing to an interesting connection to the long-run risk literature for Epstein-Zin preferences, see Bansal and Yaron (2004) and Hansen et al. (2008). The estimates suggest that labor rigidities like the labor wedge and a small Frisch elasticity rather than external habits in consumption play an important role for producing our results. These insights are in line with Uhlig (2007), who has shown that inelasticity of labor supply, a smaller elasticity of leisure substitution, and wage rigidities can help to explain the risk premium. Remarkably, the volatility of the risk-free return is similar in size to what is observed in the data and the 'risk-free rate puzzle' (Weil, 1989) is avoided. The Bayesian posterior odds ratio between the unconstrained and the constrained model is around 12, demonstrating that the constrained model is not "obviously false" from the unconstrained perspective.

Of course, our procedure has limitations. If the model does not allow to get reasonably close to matching macroeconomic as well as asset pricing implications regardless of the parameter configurations, our estimation procedure will still find the "best compromise", but that compromise may not be appealing. In order to develop insights into these limits, we therefore also apply our procedure to a plain-vanilla real business cycle model with capital adjustment costs, essentially stripping away the habit-formation features in the benchmark model described above. For numerical reasons, we target a Sharpe ratio half as large as the Sharpe ratio used in our benchmark exercise above. We now find that the constrained estimation procedure needs to set the investment-specific shock as well as the labor supply shock four to six times as high as the unconstrained procedure, and that there is now considerably more long-run volatility in both the investment-to-output ratio as well as in employment. The Bayesian posterior odds ratio is very high, making the constrained model look non-credible from the unconstrained perspective. Remarkably, though, the HP-filtered macroeconomic moments still look reasonable, and the Sharpe ratio is half as large as in the data. Put differently, while we would rather recommend the

[1] We recognize that one could alternatively try to add a sufficient number of financial observations as data during the estimation procedure, until the model also delivers on the financial implications. One way to understand our procedure is that it provides a faster and perhaps more transparent method to implement this.

model from our benchmark constrained-estimation exercise for the purpose of matching both macroeconomic and asset pricing facts, this small scale model does not do "too badly" and may be suitable for teaching purposes, for example. We believe that it would have been more difficult to find this specification without the help of our estimation procedure.

The paper is organized as follows. Section 2 introduces our methodology. Section 3 introduces the model, with section 4 providing the analysis regarding the asset pricing implications and the approximation around the stochastic steady state. Section 5 presents the estimation methodology, characterizes the data, and describes the choice of the unconstrained prior and the choice for the constraint. Section 6 presents the estimation results and reports the asset pricing and business cycle characteristics of the model. Section 7 examines our procedure when applied to a simpler real business cycle model with capital adjustment costs, but without habit formation. Section 8 concludes the paper. Several appendices provide auxiliary results.

2 Constructing the Constrained Prior

Our methodology can broadly be understood as a Bayesian estimation subject to a constraint. In our application, we shall estimate a dynamic stochastic general equilibrium (DSGE) model, using standard prior specifications. As is well known, many estimated DSGE models typically have poor asset pricing implications, unless designed otherwise or — and that is the perspective in this paper — unless the estimation pays particular attention to these aspects. From the perspective of model applications, it is interesting to know the properties of the model, if the estimation procedure is constrained to deliver certain asset pricing implications. In our specific example, we seek to constrain our prior to deliver particular implications regarding the Sharpe ratio, i.e. the market price for risk. A standard estimation-subject-to-constraints procedure would impose some equality or inequality constraint on the parameter of interest during estimation. We extend this procedure by imposing a particular posterior shape on a variable of interest.

From a methodological perspective, one might fear that our procedure amounts to "using the data twice". However, the choice of the imposed implied posterior for the variable of interest is entirely up to the researcher employing this methodology and simply a generalization of textbook estimation-subject-to-constraint. From a pure methodological perspective, it should not matter much, whether such constraints are imposed "a priori" in order to express strong beliefs about the properties of some parameter, or "a posteriori" in order to learn more about certain features of a model. From the perspective of econometrics as conversation and rhetoric, see McCloskey (1983), we view this as a practical and appealing way to explore and communicate model properties within a range of specifications of interest, rather than being forced to examine model estimates, that deliver non-credible implications for key variables. For these practical reasons, one may then wish to obtain a data-driven implied posterior for the variable of interest. While one does then use the data twice, the risk of misleading posterior probabilities should be small, if the variable of interest typically is not estimated to be within a reasonable range, if the data was used only once with a standard prior specification.

Since the strategy can be applied more generally than just for the Sharpe ratio, we shall describe it in general terms. Suppose one wishes to impose the constraint (or a priori belief) that a value ω, which can be expressed as a function of the parameters $\omega = \omega(\theta)$, lies within bounds a and b. A straightforward modification of some unconstrained prior would then be calculated by constraining its domain to this set,

$$p(\theta|X) \propto \begin{cases} p(\theta)\, p(X|\theta) & \text{if} \quad \omega(\theta) \in Q \\ 0 & \text{if} \quad \omega(\theta) \notin Q \end{cases} \tag{1}$$

$$\text{where} \qquad Q \equiv \{\theta: \ a \leq \omega(\theta) \leq b\} \tag{2}$$

This, of course, is a straightforward application of Bayesian estimation subject to a constraint.

We wish to refine this approach for several reasons. First, since the model will typically favor parameters with a low Sharpe ratio, the approach above will lead to a pile-up of the constrained parameters at the lower bound a: the results will then be rather sensitive to specifying that lower bound. Second, our a priori beliefs regarding the Sharpe ratio are more appropriately formulated as a probability distribution, or, more precisely, given by an uncertain estimate of the Sharpe ratio, given observed data, rather than imposing it to lie within some interval. Finally, the procedure becomes both more flexible and more reasonable. We argue that it is more appealing to learn the implications of the model, if the Sharpe ratio is constrained to be high with considerable probability, rather than imposing it to be within some interval.

Consequently, we use the following approach. The model generates data X: the particular observation at hand is the realization $\bar{X}$. We start from some "unconstrained prior" $p(\theta)$ in the parameter vector θ. We then calculate a "constrained prior" $\tilde{p}(\theta)$, such that the following two properties hold:

P1: the implied posterior for the Sharpe ratio $\omega = \omega(\theta)$ coincides with some a priori given distribution $F(\omega)$, with density $f(\omega)$ at the given observation $\bar{X}$.

P2: the constrained prior is proportional to the unconstrained prior, given any ω.

This approach should be viewed as a modification or refinement of the standard approach of Bayesian estimation subject to a constraint, described above. Rather than imposing an inequality constraint on a parameter or function of the parameters, we impose a probability density.

More formally, we start from

1. a proper prior $p(\theta)$, which we call the "unconstrained prior".

2. a likelihood function $\ell(X \mid \theta)$.

3. A mapping $\Omega(\theta) = \omega \in \mathbf{R}$.

4. A differentiable probability distribution function $F(\omega)$ on $\mathbf{R}$, i.e. an increasing function with $\lim_{\omega\to-\infty} F(\omega) = 0$, $\lim_{\omega\to\infty} F(\omega) = 1$. Define the probability density $f(\omega) = F'(\omega)$.

Given any data X, calculate the implied unconstrained posterior for ω as well as its density,

$$G(\omega \mid X) \;=\; \int_{\theta|\Omega(\theta)\leq\omega} p(\theta \mid X) d\theta \tag{3}$$

$$g(\omega \mid X) \quad = \quad G'(\omega \mid X) \tag{4}$$

Define the "transformation function"

$$h(\omega) \quad = \quad \frac{f(\omega)}{g(\omega \mid \bar{X})} \tag{5}$$

Note that h depends on the particular observation $\bar{X}$. One could make the dependence of $h(\cdot)$ on the observation $\bar{X}$ more explicit by writing $h(\omega \mid \bar{X})$: however, we will not calculate $h(\cdot)$ except at $\bar{X}$. It is important, that h does not vary with the data, when defining the constrained prior below in equation (6), i.e., $h(\omega)$ is a function of ω only from here onwards.

We claim that the following prior has the two desired properties listed above. Define the constrained prior $\tilde{p}$ as

$$\tilde{p}(\theta) = C^{-1} p(\theta) h(\Omega(\theta)) \tag{6}$$

where

$$C = \int p(\theta) h(\Omega(\theta)) d\theta \tag{7}$$

is the integration constant. Note that (6) implies the second of the desired properties, i.e. that the constrained prior is proportional to the unconstrained prior for any given ω. We need to verify the first desired property. The implied posterior for ω, given some observation X is given by

$$\tilde{G}(\omega \mid X) = \int_{\theta \mid \Omega(\theta) \leq \omega} \tilde{p}(\theta \mid X) d\theta \tag{8}$$

We need to show that $\tilde{G}(\cdot \mid \bar{X}) = F(\cdot)$.

Let $\tilde{g}(\omega \mid X) = \tilde{G}'(\omega \mid X)$. Substituting the explicit expression for the posterior, it follows that

$$\tilde{g}(\omega \mid X)\Delta \quad \approx \quad \int_{\theta \mid \omega \leq \Omega(\theta) \leq \omega + \Delta} \left(\frac{p(\theta) h(\Omega(\theta)) \ell(X \mid \theta)}{\int p(\theta) h(\Omega(\theta)) \ell(X \mid \theta) d\theta} \right) d\theta \tag{9}$$

$$\approx \quad \frac{h(\omega)}{\int p(\theta) h(\Omega(\theta)) \ell(X \mid \theta) d\theta} \int_{\theta \mid \omega \leq \Omega(\theta) \leq \omega + \Delta} p(\theta) \ell(X \mid \theta) d\theta \tag{10}$$

$$\propto \quad \frac{f(\omega)}{g(\omega \mid \bar{X})} \int_{\theta \mid \omega \leq \Omega(\theta) \leq \omega + \Delta} p(\theta) \ell(X \mid \theta) d\theta \tag{11}$$

Likewise,

$$g(\omega \mid X)\Delta \propto \int_{\theta \mid \omega \leq \Omega(\theta) \leq \omega + \Delta} p(\theta) \ell(X \mid \theta) d\theta \tag{12}$$

Therefore, at $X = \bar{X}$,

$$\tilde{g}(\omega \mid \bar{X})\Delta \quad \propto \quad f(\omega)\Delta \tag{13}$$

Integrating and recognizing that both sides of this equation are probability densities delivers

$$\tilde{G}(\omega \mid \bar{X}) = F(\omega)$$

as claimed.

Numerically, we proceed as follows. We approximate $f(\omega)$ and $g(\omega)$ by a Gamma distribution, imposing that the standard deviation of $g(\omega)$ is larger than the standard deviation of $f(\omega)$ to ensure that the tails of h die out and that we therefore obtain a proper prior. In our application, it turns out that we thus need to impose some $f(\omega)$, which is considerably tighter than would be justified on estimation uncertainty of the Sharpe ratio alone. Since our main aim is a flexible implementation of estimation subject to constraint rather than the imposition of a particular data-driven prior, we do not view this as a substantial drawback. Note that we implicitly impose that the Sharpe ratio is positive: this seems reasonable on economic grounds. The following algorithm implements our methodology:

1. Estimate the model by sampling from the unconstrained posterior $p(\theta|X)$
2. Approximate the implied unconstrained probability density function $g\left(\omega \mid \bar{X}\right)$
3. Calculate the transformation function $h(\omega)$ per (5).
4. Estimate the model with the constrained prior by sampling from $\tilde{p}(\theta|X)$.
5. For Bayesian posterior odds ratio calculations and model comparisons, for example, calculate the normalization constant C of equation (7). We utilize a Laplace approximation for this calculation.

3 The Model

We apply our methodology to a dynamic stochastic general equilibrium model with external habit formation or "catching-up with the Joneses" (see Abel, 1990) in both consumption and leisure, building on Uhlig (2007). We extend the model by adding shocks, to permit estimation, as well as solving the model around the stochastic steady state. We briefly describe it here for the sake of completeness.

Output y_t is produced with capital k_{t-1} in place from the previous period as well as labor n_t per the Cobb-Douglas production function,

$$y_t = k_{t-1}^{\theta} \left(e^{z_{P,t}} n_t\right)^{1-\theta}. \tag{14}$$

where $z_{P,t}$ is a productivity or technology parameter. We assume it to follow a random walk with drift

$$z_{P,t} = \gamma + z_{P,t-1} + \epsilon_{P,t}, \tag{15}$$

with γ reflecting the trend. We assume that $\epsilon_{P,t}$ is i.i.d. normal with standard error σ_P.

Output is produced by a competitive sector of firms. The usual first order conditions imply wages :

$$w_t = \frac{(1-\theta)\, y_t}{n_t} \tag{16}$$

and capital rental rates or dividends

$$d_t = \frac{\theta y_t}{k_{t-1}}. \tag{17}$$

There is a representative household with the utility function

$$U = E\left[\sum_{t=0}^{\infty}\beta^t \frac{\left((c_t - H_t)\left(A + \left(e^{z_{L,t}} l_t - F_t\right)^{\upsilon}\right)\right)^{1-\eta} - 1}{1-\eta}\right]. \tag{18}$$

The discount factor β and A, ν, η are parameters, which we assume to satisfy $\nu > 0$ and $\eta > \nu/(\nu+1)$ in order to assure monotonicity and concavity, see Uhlig (2007). The variables c_t and l_t denote consumption and leisure of the particular household. The utility depends on the economy-wide average level of consumption habit and leisure habit, H_t and F_t, evolving according to

$$H_t = e^{\gamma}\left((1-\rho_c)\chi C_{t-1} + \rho_c H_{t-1}\right), \tag{19}$$
$$F_t = (1-\rho_l)\psi L_{t-1} + \rho_l F_{t-1}, \tag{20}$$

where C_t and L_t are aggregate average levels of consumption and leisure: in equilibrium, $C_t = c_t$ and $L_t = l_t$. The parameters ρ_c, ρ_l, χ and ψ determine the persistence and importance of the habit features. The variable $z_{L,t}$ represents a labor supply shock. We assume it to follow an AR(1)process,

$$z_{L,t} = \pi_L z_{L,t-1} + \epsilon_{L,t} \tag{21}$$

where $\epsilon_{L,t}$ is i.i.d. normal with standard error σ_L. Total time endowment is normalized to unity, so that total labor supply is

$$n_t = 1 - L_t.$$

The budget constraint of the agent is

$$c_t + x_t + T_t = d_t k_{t-1} + w_t n_t. \tag{22}$$

Capital accumulation is affected by a depreciation rate δ and investment adjustment costs $g(\cdot)$,

$$k_t = \left(1 - \delta + g\left(e^{z_{I,t}}\frac{x_t}{k_{t-1}}\right)\right)k_{t-1}. \tag{23}$$

Following Jermann (1998), we assume the adjustment cost function $g(\cdot)$ to satisfy

$$g\left(\tilde{\delta}\right) = \delta + e^{\gamma} - 1, \quad g'\left(\tilde{\delta}\right) = 1, \quad g''\left(\tilde{\delta}\right) = -\frac{1}{\zeta} \qquad \forall \quad \zeta > 0,. \tag{24}$$

where $\tilde{\delta}$ is defined as $\tilde{\delta} = \exp(\gamma) + \delta - 1$, to adjust for trend growth. Adjustment costs are affected by the parameter $z_{I,t}$, following the AR(1) process

$$z_{I,t} = \pi_I z_{I,t-1} + \epsilon_{I,t}, \tag{25}$$

where $\epsilon_{I,t}$ is i.i.d. normal with standard deviation σ_I. Given initial capital k_{-1}, the household maximizes its utility by choosing leisure l_t, consumption c_t, and investments x_t subject to the constraints (22) and (23), taking as given the exogenous habits H_t and F_t and their aggregate evolution, real wages w_t, dividends d_t and lump-sum taxes T_t.

The agent's first-order condition for labor supply yields the frictionless wage or the marginal rate of substitution,

$$w_t^f = \frac{U_L}{U_c} = \frac{e^{z_{L,t}} \upsilon \left(c_t - \chi c_{t-1}\right)}{A \left(e^{z_{L,t}} l_t - \psi l_{t-1}\right)^{1-\nu} + e^{z_{L,t}} l_t - \psi l_{t-1}}, \tag{26}$$

As motivated in Uhlig (2007), we assume real wage rigidities as postulated by e.g. Hall (2005), Shimer (2005), and Blanchard and Galí (2007). More precisely, we assume that

$$w_t = \left(e^{\gamma} w_{t-1}\right)^{\mu} \left(e^{\varpi + \varepsilon_{W,t}} w_t^f\right)^{1-\mu}. \tag{27}$$

The parameter $\varpi > 0$ represents an average wage markup to ensure that $w > w^f$ locally around the steady state, and that therefore the labor market is (typically) demand constrained. The wage markup $\varepsilon_{W,t}$ follows an AR(1) process

$$\varepsilon_{W,t} = \pi_W \varepsilon_{W,t-1} + \epsilon_{W,t}, \tag{28}$$

where $\epsilon_{W,t}$ is a normally i.i.d. with standard deviation σ_W. The parameter μ reflects the degree of real wage stickiness. In the special case of $\mu = \alpha = \varepsilon_W = 0$, there are no frictions and wages are fully flexible.

Finally, there is a government, financing an exogenously given stream of expenditures g_t with lump sum taxes,

$$g_t = T_t \tag{29}$$

We assume that

$$g_t = \bar{g} e^{z_{P,t-1}} e^{g_t} \tag{30}$$

where $e^{z_{P,t-1}}$ appears to assure a stationary spending-to-output ratio and where g_t is assumed to follow the AR(1) process

$$g_t = \pi_G g_{t-1} + \epsilon_{G,t}, \tag{31}$$

with ϵ_G i.i.d. normal with standard deviation σ_G.

The five entries of the shock vector

$$\epsilon_t = \left[\epsilon_{P,t}, \epsilon_{L,t}, \epsilon_{I,t}, \epsilon_{W,t}, \epsilon_{G,t}\right]' \tag{32}$$

are assumed to be independent. Equilibrium is defined as usual.

For the further analysis, define λ_t as the marginal utility of consumption,

$$\lambda_t = U_c\left(c_t, l_t\right) \ , \tag{33}$$

and define the stochastic discount factor

$$M_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \ , \tag{34}$$

Risk premia arise from investigating the Lucas asset pricing equation

$$1 = E_t\left[M_{t+1} R_{t+1}\right]. \tag{35}$$

where R_{t+1} is the one-period return on investing one unit of resources. For investing in capital, for example, let

$$q_t = \left(g' \left(\exp z_{I,t} \frac{x_t}{k_{t-1}} \right) \exp z_{I,t} \right)^{-1} , \tag{36}$$

be the shadow price of a unit of capital. The return for investing in capital is then

$$R_t^k = \frac{\theta \frac{y_t}{k_{t-1}} + \left(1 - \delta + g\left(e^{z_{I,t}} \frac{x_t}{k_{t-1}}\right)\right) q_t - \frac{x_t}{k_{t-1}}}{q_{t-1}} . \tag{37}$$

For the numerical analysis, the variables k_t, y_t, c_t, H_t, w_t, w_t^f, x_t, λ_t, and g_t have to be productivity-detrended to solve the model. This is done by dividing each variable by $\exp(z_{P,t-1})$, except capital k_t, which is detrended with $\exp(z_{P,t})$ and λ_t which is detrended by $\exp(-\eta z_{P,t-1})$. Beside this, l_t, F_t, n_t, q_t, R_t^f, R_t^k, M_t, and d_t are stationary. We use a logarithmic approximation around the stochastic detrended steady state for our computations and solve for the recursive law of motion. In the following, all detrended variables are marked with "$\sim$" and the log-deviations from the detrended variables are marked with "$\wedge$". For details see technical appendix B.

4 Asset Pricing and Stochastic Steady State

Following Campbell (1994) and Uhlig (1999), we log-linearize the detrended model and solve for the recursive law of motion. We solve the model by using the method of undetermined coefficients,

$$\hat{y}_t = A\hat{h}_{t-1} + B\epsilon_t , \tag{38}$$

where $\hat{y}_t = \log(y_t) - \log(y^{ss})$ is a vector containing all log-linearized model variables and $\hat{h}_t = \log(h_t) - \log(h^{ss})$ is the vector containing all log-linearized state variables of the model, with y^{ss} and h^{ss} as their corresponding steady state values. The entries in the matrices A and B can typically be interpreted as elasticities.

Following Lettau and Uhlig (2002) and using the representation (38), we can decompose the log pricing kernel into its conditional expectation and its innovations:

$$\hat{M}_{t+1} = E_t\left[\hat{M}_{t+1}\right] + b_M \epsilon_{t+1} , \tag{39}$$

where b_M indicates the row vector of matrix B with respect to the pricing kernel. Let $\Sigma = \epsilon_t' \epsilon_t$ be the variance-covariance matrix of ϵ_t: we assumed it to be diagonal, but the formulas apply more generally. The conditional variance, σ_M^2 of the pricing kernel is

$$\sigma_M^2 = b_M \Sigma b_M' . \tag{40}$$

Similarly, we can solve for the conditional variances σ_{R^k} of $\hat{R}_t^k$ and other assets. Additionally, the conditional covariance of the pricing kernel and the return on capital, σ_{MR^k}, can be evaluated as

$$\sigma_{MR^k} = b_M \Sigma b_{R^k}' , \tag{41}$$

with b_{R^k} the row vector of matrix B for the return on capital $\hat{R}^k$.

Exploiting the (approximate) joint normality of all variables, as in Lettau and Uhlig (2002), it now follows that the risk premium on, say, the return on capital over the risk free return satisfies[2]

$$\log E_t\left[R^k_{t+1}\right] - \log R^f_t = -\sigma_{MR^k} \ . \tag{42}$$

where "=" is meant to read "up to log-approximation". Likewise, for the Sharpe ratio ω, defined here as in Lettau and Uhlig (2002) as

$$\omega = \frac{\log E_t\left[R^k_{t+1}\right] - \log R^f_t}{\sigma_{R^k}} \tag{43}$$

we obtain

$$\omega = -\frac{\sigma_{MR^k}}{\sigma_{R^k}} \ . \tag{44}$$

This provides the target for our constraint in the constrained estimation procedure. As in Hansen and Jagannathan (1997), Campbell and Cochrane (2000), or Lettau and Uhlig (2002), the highest possible Sharpe ratio is equal to σ_M, by assuming a correlation between the pricing kernel and the return of capital equal to -1.

Following Juillard (2010) and Coeurdacier et al. (2011), we furthermore use (44) to adjust the steady state with this "second-order" correction. For the numerical solution, we impose that

$$E\left[R^f_t\right] = E\left[\exp\left(-\log \bar{M} - E_t\left[\hat{M}_{t+1}\right] - \frac{\sigma^2_M}{2}\right)\right] , \tag{45}$$

as well as

$$E\left[R^k_{t+1}\right] = E\left[\exp\left(-\log \bar{M} - E_t\left[\hat{M}_{t+1}\right] - \frac{\sigma^2_M}{2} - \sigma_{MR^k}\right)\right] . \tag{46}$$

Given $\bar{M}$ as well as variances and covariances, note that the right-hand side can once again be calculated using (38) and the assumption of normally distributed shocks.

Because the conditional second moments depend on the policy function, we get a fixed point problem by solving our model accurately with respect to the stochastic steady state. For this reason, we use an iterative procedure. We start with the nonstochastic steady state to obtain our policy function. The resulting steady state adjustment yields a new set of policy functions, etc.. As discussed in Canton (2002), a few iterations suffice to achieve convergence and to resolve the fixed point problem to a reasonable degree of accuracy.

5 Estimation

5.1 Data

The estimation of the model is based on six time series from 1963:qI to 2008:qII. All data are quarterly and in real terms. For both the unconstrained and constrained estimation, we use the vector of time series $X_t = \left[\Delta\hat{y}_t,\ \Delta\hat{c}_t,\ \hat{n}_t,\ \hat{x}_t - \hat{y}_t,\ \hat{R}^f_t,\ \hat{R}^q_t\right]$, where $\Delta\hat{y}_t$ is the

[2] For example, $E_t[R^k_{t+1}] = E_t\left[\exp\left(E_t\left[\log R^k_{t+1}\right] + b_{R^k}\epsilon_{t+1}\right)\right] = \exp\left(E_t\left[\log R^k_{t+1}\right] + \sigma^2_{R^k}/2\right)$. A few more similar calculations deliver the equations in the text.

first difference of detrended log output per capita, Δc_t is the first difference of detrended log consumption per capita, $\hat{x}_t - \hat{y}_t$ is the demeaned log investment-to-output ratio, $\hat{n}_t$ is de-meaned hours worked per capita, $\hat{R}_t^f$ is the demeaned risk free rate, and $\hat{R}_{q,t}^{obs}$ are the excess returns on an aggregate stock market index. Additionally, for the constrained estimation, we calculate the Sharpe ratio, using total returns on an aggregate stock market index.

We use the quarterly real Gross Domestic Product as a measure of aggregate output.[3] We use civilian noninstitutional population over 16 years from the Bureau of Labor Statistics (BLS) as a proxy for population to calculate per capita time series. We calculate the first differences of the real logarithmic output per capita and afterwards reduce the mean of this time series. This mean is used to calibrate the growth rate γ in the model. Consumption is expenditures on non-durables and services. Private investment is calculated as the sum of nominal gross private investment and personal durable consumption both provided by the Bureau of Economic Analysis (BEA). Both time series, consumption and investment, are transformed into real and per capita terms, by using the GDP deflator and the population series mentioned above. Finally, we calculate the demeaned log-differences of consumption as well as the demeaned logarithm of the investment-output ratio for the estimation. Additionally, we include hours worked into the estimation. In particular, we use quarterly hours worked by employees working in private, non-farm business excluding non-profit business. This series is an updated version of the one used by Francis and Ramey (2009). The final logarithmic time series is demeaned.

As a proxy for the riskless real interest rate, we use the quarterly returns calculated based on the monthly returns of the three month T-Bill returns provided by the Board of Governors of the Federal Reserve System. The returns are calculated in real terms, too, by using the implicit inflation given by the GDP price deflator. Furthermore, the final logarithmic return series is demeaned. Finally, we use also excess returns as observable variable. The excess returns are calculated as the log differences between the total returns of the S&P 500 and the three month T-Bill returns. Because there is no equivalent variable in our model, we define excess returns' log-linear deviations from steady state as follows

$$\hat{R}_t^q = -\sigma_{MR^q} + \frac{1}{1-\Omega}\left(\hat{R}_t^k - \hat{R}_t^f\right) + \epsilon_{Q,t} \ , \tag{47}$$

where Ω is a parameter which can be interpreted as leverage and $\epsilon_{Q,t}$ is an i.i.d. error term and assumed to be uncorrelated with the stochastic discount factor. Similar to Section 4, we derive the mean excess returns as the negative covariance between the pricing kernel and the excess returns, $-\sigma_{MR^q}$. While we can observe the Sharpe ratio of the total returns of the S&P 500 in the data, we cannot observe a Sharpe ratio for the return on economy-wide capital in the data. Hence, we assume that in our economy all assets are priced along the implied market line and therefore excess returns and capital returns share the same Sharpe ratio. Thus, we use $\omega \cdot \sigma_{R^q}$ as a proxy for the mean excess returns. Table 1 summarizes some stylized asset pricing facts:[4]

[3] See appendix A for details on the source and a description of any data used in this paper.

[4] The estimates are based on the maximum likelihood estimation of the following data generating process, $R_t^q = \omega\sigma_{R^q} + \sigma_Q\epsilon_t \ \epsilon_t \sim N(0,1)$. The presented standard deviation are based on inverse Hessian which is a consistent estimator of the covariance matrix of the parameters.

Table 1: Stylized asset pricing facts (quarterly).

	Mean	s.d.
Sharpe ratio	0.2049	0.0753
mean excess returns	0.0149	0.0054
s.d. excess returns	0.0727	0.0038

5.2 Choice of the unconstrained prior

As in Uhlig (2007), we find it useful to describe our unconstrained prior in terms of an economically meaningful transformation of the model's parameter. Specifically, consider the Frisch elasticity of labor supply, defined as the elasticity of labor supply to frictionless wages by holding the marginal rate of consumption constant,

$$\tau = \frac{dn}{dw^f}\frac{w^f}{n}\bigg|_{\bar{U}_c} . \tag{48}$$

Given our preference assumptions, this yields

$$\tau = \frac{U_n}{\bar{n}\left[U_{nn} - \frac{U_{nc}^2}{U_{cc}}\right]} = \frac{1-\bar{n}}{\bar{n}} \cdot \frac{\eta\,(1+\alpha)\,(1-\psi)}{\eta\,(\alpha\,(1-\nu)+1+\nu)-\nu} , \tag{49}$$

where $\alpha = A\,(1-\psi)^{-\nu}\,\bar{l}^{-\nu}$. Therefore, rather than specifying a prior for A or ν we shall specify a prior for τ, and calculate the implied A and ν from τ as well as the other parameters and variables in equation (49). We assume that the steady state level of hours worked is $n = 1/3$. From the first-order conditions,

$$\nu = 1 - (1-\psi)\,\frac{\bar{l}}{1-\bar{l}}\frac{1}{\tau} - \left(2 - \frac{1}{\eta}\right)\frac{1}{(1-\chi)\,\kappa} , \tag{50}$$

with

$$\kappa = \frac{e^{\varpi}}{1-\theta}\frac{1-\bar{l}}{\bar{l}}\frac{\bar{c}}{\bar{y}} , \tag{51}$$

where the $\bar{c}/\bar{y}$ is the steady state consumption share of output.[5] Additionally, we can solve for the remaining preference parameters,

$$\alpha = \frac{\kappa\nu\,(1-\chi)}{1-\psi} - 1 \tag{52}$$

$$A = \alpha\,(1-\psi)^{\nu}\,\bar{l}^{\nu} \tag{53}$$

While the real business cycle literature often assumes a relatively high Frisch elasticity of two or more (Prescott, 1986; King, Plosser, and Rebelo, 1988), recent papers of Bayesian DSGE model estimation found far smaller values for the Frisch elasticity in a New Keynesian model framework. For example, Justiniano and Primiceri (2008) argue

[5] More details regarding steady state calculation can be found in appendix B.2.

for values between 0.25 and 0.5. These findings are in line with some micro-data based studies, which argue for small values in a range between 0 and 0.7, too (see Pistaferri, 2003, and references therein). To that end, we use a prior for the inverse of the Frisch elasticity, which is Gamma distributed with mean 1.0 and a standard deviation of 0.75. This assumption covers the values used in the different strands of the literature.

For explaining business cycle facts and asset pricing facts simultaneously, we expect that the discount factor β as well as the power utility parameter η play an important role. The business cycle literature often uses values for the discount factor that are slightly smaller than one to ensure a positive time preference of the representative agent and steady-state risk-free returns comparable to observed returns. However, from an asset pricing perspective, discount factors with much smaller values or values greater than one are postulated. These opposing assumptions are known as the risk-free rate puzzle (see Weil, 1989). However, Kocherlakota (1990) has shown that values for the discount factor above unity can be in line with positive time preference if the economy is growing. For this reason, we use a prior information for the riskless return to ensure positive time preference and solve recursively for the discount factor:

$$\beta = \exp(\eta\gamma - \sigma_M^2/2 - \log(\bar{R}^f)) \tag{54}$$

In particular, we assume that the steady state risk-free real quarterly return is inverted-gamma distributed with mean 0.005 and standard deviation 0.01. This ensures that the mass of the prior is on positive real annual returns which are smaller than 4%. Finally, we assume that the power utility parameter is uniformly distributed between 1 and 200 which implies most of the prior mass on high values. This is in contrast to the common business cycle literature which generally assumes small values and therefore uses quite informative prior, but allows our procedure to consider high degrees of risk aversion when seeking to match asset pricing facts: this is useful when constraining the prior in the next step. The prior for the remaining deep model parameters are chosen in line with the recent literature. An overview of the priors is given in Table 2.

In addition, to the steady state labor supply, we also calibrate the growth rate of the economy γ and the capital share θ. As mentioned in the previous Subsection, we calibrate the growth rate equal to the observed value of 0.0044 per quarter. The capital share is calibrated to 0.33 as common.

5.3 Choice of constrained prior

As described in Section 2, we approximate $f(\omega)$ and $g(\omega)$ using a Gamma distribution, imposing that the standard deviation of $g(\omega)$ is larger than the standard deviation of $f(\omega)$ to ensure that the tails of h die out and that we therefore guarantee a proper prior. Given our application in the present paper, we approximate $g(\omega)$ using a Gamma distribution with mean 0.02 and standard deviation 0.002. By approximating $f(\omega)$, we follow our estimates for the Sharpe ratio (Table 1) and assume that $f(\omega)$ is Gamma distributed with mean 0.2049. We choose a standard deviation of 0.001 for $f(\omega)$. This is considerably tighter than would be justified on estimation uncertainty of the Sharpe ratio alone. Since the aim of the exercise here is to obtain a model estimate in line with the observed Sharpe ratio as well as to illustrate our estimation methodology, we view this

Table 2: Prior distribution for model parameter and additional parameter. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution, while for the Uniform distribution, these values correspond to the lower and upper bounds. The acronym s.s. indicates steady state values.

		Domain	Density	Para(1)	Para(2)
MODEL PARAMETER					
μ	wage rigidity	$[0,1)$	Beta	0.75	0.1
η	power utility parameter	$\mathbb{R}^+$	Uniform	1	200
χ	consumption habit	$[0,1)$	Beta	0.5	0.23
ψ	leisure habit	$[0,1)$	Beta	0.5	0.23
ρ_c	consumption habit	$[0,1)$	Beta	0.5	0.23
ρ_l	leisure habit	$[0,1)$	Beta	0.5	0.23
δ	depreciation rate	$[0,1)$	Beta	0.02	0.005
ζ	investment adjustment costs	$\mathbb{R}$	Normal	4.0	1.0
$1/\tau$	inverse Frisch elasticity (s.s.)	$\mathbb{R}^+$	Gamma	1.00	0.750
$\log \bar{R}^f$	risk-free return (s.s.)	$\mathbb{R}^+$	InvGam	0.005	4.0
Ω	leverage	$[0,1)$	Beta	0.5	0.23
AUTOREGRESSIVE PARAMETER AND S.D. OF SHOCKS					
π_G	AR government shock	$[0,1)$	Beta	0.85	0.1
π_W	AR wage mark-up shock	$[0,1)$	Beta	0.85	0.1
π_I	AR investment shock	$[0,1)$	Beta	0.85	0.1
π_L	AR labor supply shock	$[0,1)$	Beta	0.85	0.1
ϵ_P	s.d. technology shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_W	s.d. wage mark-up shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_I	s.d. investment shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_L	s.d. labor supply shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_G	s.d. government shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_Q	s.d. excess return shock	$\mathbb{R}^+$	InvGam	0.01	4.0

as perfectly adequate: after all, this is part of the prior specification, and not necessarily entirely data-driven.

6 Estimation Results

We estimate the posterior mode of the distribution and employ a random walk Metropolis-Hastings algorithm to approximate the uncertainty distribution of the parameters. We run two chains, each with 300,000 parameter vector draws. The first 75% have been discarded. We provide results for both the unconstrained and the constrained prior.

Table 3 shows detailed posterior statistics, e.g. the posterior mean and the highest probability density (HPD) interval[6], using the range between 5% and 95%. The results

[6]This will be the HPD interval, if the posterior density is symmetric and strictly increasing towards the median, an assumption which appears to be approximately satisfied for the posterior.

indicate that the posterior distributions of all structural parameters are well approximated and different from the prior distribution.

Table 3: MCMC Results

	Unconstrained prior			Constrained prior		
Parameter	Posterior	HPD		Posterior	HPD	
	Mean	5%	95%	Mean	5%	95%
Model parameter						
μ	0.2685	0.1392	0.3948	0.2882	0.1644	0.4209
η	4.9508	2.1058	8.0901	108.18	84.19	134.30
χ	0.8689	0.8096	0.9258	0.8402	0.7777	0.9086
ψ	0.8555	0.7951	0.9167	0.8438	0.7871	0.9022
ρ_c	0.6090	0.4967	0.7237	0.6551	0.5523	0.7646
ρ_l	0.0696	0.0044	0.1344	0.0723	0.0043	0.1367
Ω	0.1188	0.0099	0.2276	0.1073	0.0059	0.2059
ζ	7.8726	6.7565	9.0130	7.8361	6.7952	8.9570
δ	0.0172	0.0136	0.0206	0.0175	0.0139	0.0210
$1/\tau$	5.5308	3.6103	7.4448	7.1055	5.1878	9.0453
$\log(\bar{R}^f)$	0.0047	0.0027	0.0065	0.0032	0.0017	0.0046
autoregressive parameter and s.d. of shocks						
π_G	0.9116	0.8798	0.9450	0.9253	0.8909	0.9611
π_I	0.7065	0.6432	0.7711	0.7003	0.6417	0.7580
π_W	0.6008	0.4787	0.7119	0.9303	0.8909	0.9719
π_L	0.9240	0.8812	0.9693	0.6516	0.5400	0.7670
σ_P	0.0091	0.0083	0.0098	0.0090	0.0083	0.0097
σ_I	0.0135	0.0107	0.0162	0.0132	0.0108	0.0156
σ_L	0.0031	0.0027	0.0034	0.0029	0.0026	0.0032
σ_W	0.0205	0.0149	0.0256	0.0211	0.0155	0.0265
σ_G	0.0197	0.0171	0.0224	0.0190	0.0169	0.0211
σ_Q	0.0765	0.0696	0.0831	0.0738	0.0677	0.0802
Log marginal density	3439.74			3437.25		

By comparing the results of the estimation with unconstrained prior and constrained prior, we find the biggest difference for the power utility parameter η. This result is expected, because introducing the constrained prior decisively shifts the marginal prior distribution with respect to η to high values. Additionally, for this class of preferences, the parameter is directly linked to the agents' relative risk aversion regarding short-term fluctuations in consumption, which can be calculated as:

$$RRA = \frac{\eta}{1-\chi} \, , \tag{55}$$

We calculate the relative risk aversion for every draw from the posterior. The implied posterior statistics can also be found in Table 4. This calculation of the relative risk aversion regarding short-term fluctuations in consumption has been criticized by Boldrin, Christiano, and Fisher (1997), who argue that relative risk aversion with respect to wealth

Table 4: Distributions of implicit model parameter and steady state values.

	Unconstrained prior			Constrained Prior		
Parameter	Posterior	HPD		Posterior	HPD	
	Mean	5%	95%	Mean	5%	95%
β	1.0162	1.0043	1.0292	1.0436	0.9538	1.1388
ν	5.8312	3.4399	8.1339	5.1088	2.9349	7.4408
α	0.5953	0.3922	0.7917	0.6071	0.3247	0.8797
$\bar{x}/\bar{y}$	0.3232	0.2944	0.3513	0.3129	0.2939	0.3328
$\bar{c}/\bar{y}$	0.3968	0.3687	0.4256	0.4071	0.3872	0.4261
RRA	39.20	15.26	63.95	719.25	419.23	1032.3

is more meaningful. Similarly, Swanson (2012) argues that this measure ignores the labor margin which can lead to a inaccurate measure of the household's true attitudes toward risk, especially in the case of habit formation. We do not wish to take a stand here on the debate as to how to measure "risk aversion" in the most meaningful manner: rather, our results intend to illustrates the intuitive fact that high relative risk aversion for short-term consumption fluctuations is needed to explain stylized asset pricing facts.

For both estimations we identify similar volatilities of the exogenous shocks. This means, both economies face the same "economy-wide" risk. Since high economy-wide risk is therefore not at the heart of matching the high Sharpe ratio according to these estimates, a high relative risk aversion (in the sense described above) is unavoidable, see also Rudebusch and Swanson (2008) or Lettau and Uhlig (2002). For habit-based DSGE models as presented in the present paper, the elasticity of intertemporal substitution is the inverse of the relative risk aversion. This elasticity can be calculated as 0.026 and 0.0013 for the estimation with unconstrained prior and constrained prior, respectively. Compared to the findings by Hall (1988) or Vissing-Jørgensen (2002), these are very small, but unavoidably so due to the need for a high degree of relative risk aversion. Some recently estimated DSGE models likewise postulate high parameter values for external or internal habit and therefore also imply small elasticities. As shown by Uhlig (2007), wage rigidities can be a helpful ingredient to explain a high risk premium in habit-based DSGE models. Our estimation results, however, show that the degree of wage rigidity is small and similar for both estimation. Instead, the constrained estimation prefers a smaller Frisch elasticity τ, compared to the unconstrained estimation: the Frisch elasticity of labor supply τ decreases from 0.18 for the unconstrained estimation to 0.14 for the constrained estimation. Both values are in line with findings of the microeconomic literature (see Pistaferri, 2003), but are at odds with some of the macroeconomic literature. We feel comfortable with these results, however, since the model is nonetheless capable of matching aggregate labor fluctuations as shown in Subsection 6.2: it is these aggregate observations which are at the heart of the motivation for the large macroeconomic Frisch elasticities used elsewhere. Intuitively, these estimates indicate high labor market rigidities for the estimation with the constrained prior, in line with the insights of Uhlig (2007), but focus on supply elasticities rather than wage elasticities.

6.1 Implied asset pricing facts

Table 5 shows the implied distribution of the first and conditional second moments for both estimations. In general, the estimation with constrained prior delivers asset price facts similar to those observed in the data. The estimation with unconstrained prior can only explain the moments of the risk-free rate appropriately. Especially, the Sharpe ratio and the risk premium illustrate the well known difficulty of explaining asset pricing facts using standard DSGE models.

Table 5: Implied quarterly asset pricing facts by the estimated models. All values in percent with the exception of the Sharpe ratio.

		Unconstrained prior			Constrained prior		
		Posterior			Posterior		
		Mean	5%	95%	Mean	5%	95%
s.d. risk-free return	σ_{R^f}	0.39	0.35	0.42	0.38	0.35	0.42
s.d. return on capital	σ_{R^k}	1.19	0.97	1.41	1.18	0.98	1.37
s.d. excess returns	σ_{R^q}	7.86	7.20	8.54	7.59	6.98	8.22
s.d. pricing kernel	σ_M	4.16	1.90	6.82	92.15	72.13	110.92
Risk premium (R^k/R^f)	$-\sigma_{MR^k}$	0.024	0.015	0.034	0.240	0.200	0.280
Sharpe ratio (R^k)	ω	0.0203	0.0140	0.0270	0.205	0.203	0.206

The conditional second moments of the risk free rate are similar for both estimations and comparable to the data. In particular, the mean of risk free return for the estimation with constrained prior is slightly smaller but still in line with observations (see Table 3). Additionally, the moments of the return of capital for both estimations are comparable with each other. The high risk aversion parameter η delivers the large conditional volatility of the stochastic pricing kernel σ_M, which in turn is needed to obtain the observed Sharpe ratio: this standard deviation is also the maximal Sharpe ratio for any asset and approximately five times as high as the Sharpe ratio for the return on capital. Put differently, the constrained estimate of our model is in principle compatible with Sharpe ratios for other asset classes, which happen to be up to five times as large as those observed for the stock market, and therefore compatible in principle with the findings in Scholl and Uhlig (2008) and Piazzesi and Schneider (2012). By contrast, the implied Sharpe ratio and risk premium for the unconstrained estimates is too low by a factor of 10, compared to observations.

6.2 Implied business cycle facts

In the following subsection, we investigate in more detail the empirical performance of our estimated models with respect to business cycle statistics. Table 6 compares standard deviations and cross-correlations after HP-filtering the data as well as simulated time series, both for the unconstrained and the constrained estimation. The differences between these two types of estimates are remarkably small, and reasonably close to the data.

To better detect where the constrained estimation procedure "compromises" in order to also explain the asset pricing facts, it is useful to analyze unconstrained variances. To

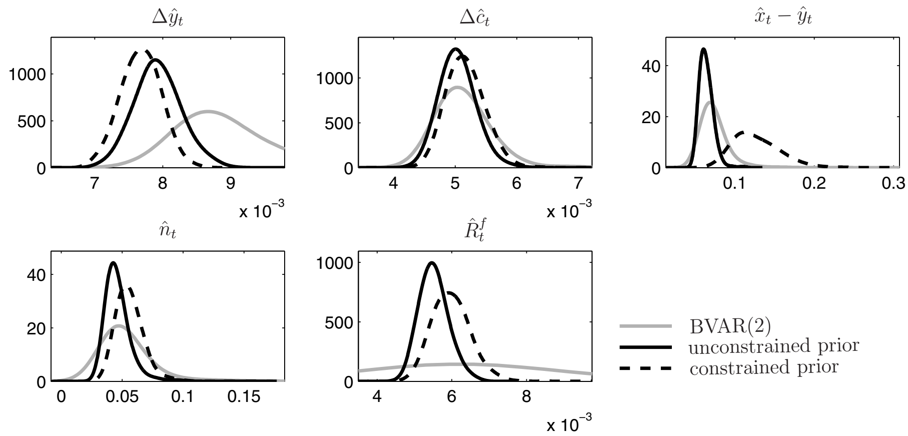
Table 6: HP-filtered ($\lambda = 1600$) theoretical and empirical moments. The theoretical moments are based on 1200 draws from the posterior. The numbers in brackets indicate 5% and 95% probabilities.

		Unconstrained Prior	Constrained Prior	Data
Standard Deviation of Output				
Output	$\hat{y}$	0.0129	0.0126	0.0148
		[0.0120;0.0141]	[0.0118;0.0135]	
Relative Standard Deviation to Output				
Consumption	$\hat{c}$	1.0415	1.0452	0.5516
		[0.9653;1.1162]	[0.9606;1.1324]	
Investment	$\hat{x}$	2.9560	3.1034	3.6632
		[2.7632;3.1738]	[2.9451;3.2903]	
Hours worked	$\hat{n}$	1.1570	1.1299	1.2372
		[1.0931;1.2223]	[1.0621;1.1954]	
Correlation with Output				
Consumption	$\hat{c}$	0.6387	0.6016	0.8210
		[0.5547;0.6929]	[0.5085;0.6660]	
Investment	$\hat{x}$	0.7503	0.7531	0.9226
		[0.7074;0.7863]	[0.7149;0.7902]	
Hours worked	$\hat{n}$	0.7829	0.7763	0.8676
		[0.7435;0.8210]	[0.7372;0.8111]	

do so, we compare the predicted unconditional second moments of the DSGE models with those of a Bayesian vector autoregression (BVAR) with two lags. In particular, we estimate the BVAR with the same set of observable variables as used for the DSGE estimation with the exception of the excess returns. Moreover, we assume a weak Normal-Whishart prior for the coefficients and the covariance matrix of the BVAR. Afterwards, we draw 1200 parameter vectors from the posterior of the BVAR as well as 1200 parameter vectors from the posterior distributions of both estimated DSGE models. For each parameter vector draw, we calculate the unconditional second moments. Figure 1 shows the implied distributions for the standard deviations of the observable variables.

The estimated DSGE models predict similar standard deviations for output growth which are slightly smaller than those predicted by the BVAR(2). Moreover, the predicted standard deviations for consumption growth, hours worked, and the real risk-free interest rate are similar to each other. While they match the standard deviation of the real quantities well, both DSGE models overpredict the standard deviation of the real risk-free interest rate compared to the BVAR (Christiano et al., 2011). The biggest difference can be found for the implied standard deviation of the investment-output ratio of the DSGE models. While the model estimated with the unconstrained prior predicts values close to those of the BVAR, the model estimated with constrained prior predicts a bigger standard deviation. This characteristic is related to a higher autocorrelation of the investment-output ratio in comparison to the BVAR and the benchmark DSGE. Put differently, the estimated habit formation model turns short-run shocks into long-run risks. This

Figure 1: Implied standard deviations of the DSGEs and the BVAR(2) based on 1200 draws from the corresponding posterior.



can also be seen in the considerably more persistent autocorrelations for the investment-output ratio for the constrained estimates compared to the unconstrained estimates or the BVAR (see Figure 5 in the Appendix). This points to an interesting connection to the long-run risk literature for Epstein-Zin preferences, see Bansal and Yaron (2004) and Hansen et al. (2008), which should be explored in future research.

6.3 Model comparison

Our procedure allows a formal comparison of the constrained and the unconstrained estimate. We use the Modified Harmonic Mean estimator by Geweke (1999) to calculate the marginal data density of each model. We find a difference in the marginal data density which implies posterior probabilities of 0.92 vs. 0.08 and a posterior odds ratio of 12.1 in favor of the model estimated with the unconstrained prior. While the unconstrained model is unsurprisingly more probable, when ignoring the asset pricing implications, these calculations show that the constrained estimates are reasonably plausible, from the perspective of the unconstrained model. It therefore turns out that one does not have to strain too much to also explain asset pricing features with this habit-formation model.

7 Application to a simple RBC model

In the following section, we apply our method of a constrained prior to a w8j0o34 version of the model in this paper. First, we assume that households have the following utility function which is separable in consumption and leisure,

$$U = E\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\eta}}{1-\eta} - \Psi_l \frac{e^{z_{L,t}} n_t^{1+1/\tau}}{1+\frac{1}{\tau}}\right], \tag{56}$$

where η is the parameter of relative risk aversion and τ is the Frisch elasticity with respect to labor supply. The variable $e^{z_{L,t}}$ reflects preference fluctuations in labor supply and Ψ_l the scaling parameter with respect to disutility of labor. In comparison to the benchmark model of 3, we exclude habit formation in consumption and leisure. Additionally, we abstract from real wage rigidities by assuming that the real wage is equal to the marginal rate of substitution: as a result, there is no separate exogenous shock to market wages. We set growth to zero and solve the model around its deterministic steady state. Additionally, we assume that the exogenous technology process follows an AR(1)process,

$$z_{P,t} = \pi_P z_{P,t-1} + \epsilon_{P,t} \tag{57}$$

where $\epsilon_{P,t}$ is normally i.i.d. with standard deviation σ_P. Finally, we also abstain from defining excess returns in the model. Our assumption about the government and production of output and capital remain the same, as do our assumptions about the four remaining exogenous stochastic processes, other than the change in (57).

Since we have four rather than six shocks, we reduce the vector of quarterly observables for the unconstrained estimation to $X_t = [\Delta\hat{y}_t,\ \Delta\hat{c}_t,\ \hat{n}_t,\ \hat{x}_t - \hat{y}_t]$. Fewer parameters are estimated. We calibrate the depreciation rate to $\delta = 0.025$ and the steady state risk-free rate to 1.0042, which is the mean of the corresponding times series used in the former part of this paper. This implies a discount rate $\beta = 0.9958$. Additionally, we choose a more diffuse prior distribution for the inverse of the Frisch elasticity, which is now a Gamma distribution with mean 5 and standard deviation of 2, to avoid maximization problems. For the same reason, we soften our additional Sharpe ratio constraint for the estimation with constrained prior. More precisely, we assume that the observed Sharpe ratio is centered around 0.102 instead of 0.204 as in the data. The prior distributions of the remaining parameters are the same.

Table 7: MCMC Results for the simple RBC model

	Unconstrained Prior			Constrained Prior		
Parameter	Posterior Mean	HPD 5%	HPD 95%	Posterior Mean	HPD 5%	HPD 95%
Model parameter						
η	7.6114	4.3446	10.8825	43.5015	29.3441	57.4003
ζ	6.3621	5.0788	7.5694	5.8340	4.5777	7.0856
$1/\tau$	2.9802	1.7344	4.1938	12.1938	10.3318	14.1113
autoregressive parameter and s.d. of shocks						
π_G	0.9072	0.8846	0.9288	0.9270	0.9054	0.9489
π_I	0.9444	0.9219	0.9671	0.7978	0.7549	0.8418
π_P	0.9464	0.9287	0.9648	0.9931	0.9910	0.9953
π_L	0.9933	0.9882	0.9985	0.9964	0.9944	0.9984
σ_P	0.0098	0.0089	0.0106	0.0110	0.0100	0.0120
σ_I	0.0407	0.0270	0.0537	0.1786	0.1609	0.1957
σ_L	0.0532	0.0338	0.0707	0.2726	0.2065	0.3371
σ_G	0.0139	0.0127	0.0151	0.0137	0.0125	0.0149
Log marginal density	2443.91			2393.29		

Table 7 shows detailed posterior statistics. There are similarities to the corresponding Table 3. As should be expected, the distribution for the power utility parameter η is now centered around a value corresponding to high relative risk aversion. Likewise, the Frisch elasticity of labor supply decreases when imposing the constraint, but the decrease is now larger, falling from 0.31 to 0.08. In particular, we can observe the same mechanisms, higher relative risk aversion and higher labor market frictions as before. The estimated standard deviations of the shocks are a key difference. The constrained estimate for the RBC model implies volatilities for the investment-specific shock σ_I as well as the labor supply shock σ_L. Thus, high "economy-wide" risk contributes to match the imposed Sharpe ratio. Due to the separability of the utility function, the maximum Sharpe ratio is just a function of η and the conditional standard deviation of consumption, which also depends on the model parameters,

$$\omega^{max} = \eta\sigma_c \tag{58}$$

An increase of the relative risk aversion raises the desire of households to smooth consumption: the high volatilities σ_I and σ_L help to offset that.

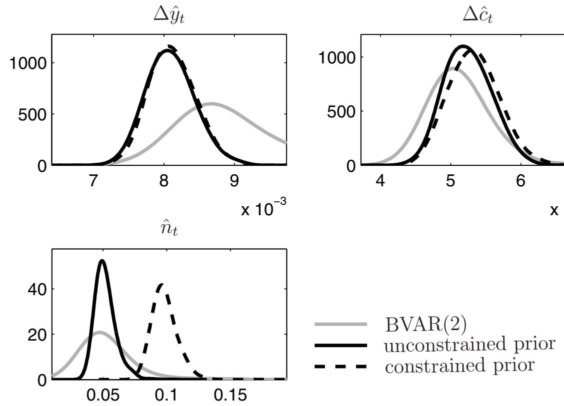
Table 8: Implied quarterly asset pricing facts by the estimated models. All values in percent with the exception of the Sharpe ratio.

		Unconstrained prior			Constrained prior		
		Posterior			Posterior		
		Mean	5%	95%	Mean	5%	95%
s.d. risk-free return	σ_{R^f}	0.27	0.17	0.37	3.26	2.90	3.63
s.d. return on capital	σ_{R^k}	3.05	2.00	4.09	13.83	12.58	14.95
s.d. pricing kernel	σ_M	3.87	2.33	5.39	22.25	16.43	27.93
Risk premium	$-\sigma_{MR^k}$	0.063	0.029	0.097	1.41	1.28	1.53
Sharpe ratio (R^k)	ω	0.020	0.0149	0.0253	0.102	0.100	0.104

Table 8 shows the implied asset pricing facts of the model for both estimation approaches. We are successful in matching the Sharpe ratio constraint, which was set at half of the observed value. The model estimated with the constrained prior fails in matching the low volatility of the risk-free rate. In comparison with the former model, the present model estimation delivers a high volatility of the return on installed capital which is higher than the volatility of equity observed in the data, and thus contributing to a risk premium higher than should be expected given the reduced Sharpe ratio. However, as Gomme, Ravikumar, and Rupert (2011) have shown, the observed volatility of the return to installed capital is smaller than the observed volatility of stock market returns.

Figure 2 illustrates the implied distributions of the unconditional moments of the observed variables for both estimation approaches and compares them with those of a BVAR with two lags. Similar to the benchmark model and the results in Figure 1, we obtain a higher implied standard deviation for the investment-output ratio for the constrained-prior estimation, along with a high autocorrelation of this variable. In contrast to the benchmark model and the results in Figure 1, hours worked likewise now have considerably higher unconditional variance and persistence. Both are, again, low-frequency phenomena: these effects disappear when calculating HP-filtered moments, as Table 9 in the appendix shows. Finally, the posterior odds ratio is $9.64e^{21}$ to one in favor of

Figure 2: Implied standard deviations of the of the more simple DSGEs and the BVAR(2) based on 1200 draws from the corresponding posterior.



the model estimated with the unconstrained prior, compared with 12.1 for the former model. The Bayesian posterior odds ratio therefore makes the constrained model look non-credible from the unconstrained perspective. The application to this simpler RBC model therefore shows the limitations of our procedure: while it tries hard to push this model to explain both asset pricing facts and macroeconomic facts, larger gaps between the quantitative model results and the observations emerge.

Then again, one may also consider the glass "half-full". Since the HP-filtered macroeconomic moments still look reasonable, the risk premium is roughly in line with the data and the Sharpe ratio, at half of its observed value, comes within the range of observations, one may wish to consider this a decent fit or a good starting point for other uses and explorations. Put differently, while we would rather recommend the model from our benchmark constrained-estimation exercise for the purpose of matching both macroeconomic and asset pricing facts, this small scale model does not perform "too badly" and may be suitable for teaching purposes, for example. We believe that it would have been hard to find this specification without the help of our estimation procedure.

8 Conclusion

We have presented a novel Bayesian method for estimating dynamic stochastic general equilibrium (DSGE) models subject to the constraining the posterior distribution of the implied Sharpe ratio. We first presented our methodology in more general terms. Starting from an initial, unconstrained prior, we construct a constrained prior so that the resulting implied posterior for some variable of interest (the Sharpe-ratio, say) coincides with some a priori chosen distribution, and such that the constrained prior is proportional to the original prior, conditional on that variable.

We then applied our methodology to a DSGE model with habit formation in consumption and leisure, real wage rigidities and capital adjustment costs, building on Uhlig (2007). We use a density centered at the estimated Sharpe ratio to construct our constraint. We show that the estimation subject to this constraint produces a quantitative model with both reasonable asset-pricing as well as business-cycle implications, thus offering more hope than the somewhat more pessimistic message in Rudebusch and Swanson (2008, 2012) regarding habit-formation models.

To understand the limitations of our procedure, we also apply our methodology to a simpler RBC model, which does not feature habit formation or real wage rigidities. We show that the discrepancies between observations and model simulations become more apparent, even at the best fit. However, even here one may consider the glass "half-full". Since the HP-filtered macroeconomic moments still look reasonable, the risk premium is roughly in line with the data and the Sharpe ratio, at half of its observed value, comes within the range of observations. We believe that it would have been hard to find this specification without the help of our estimation procedure.

A Data

In this paper, we use several macro and financial time series. This appendix describes some modifications and, in particular, the source of the raw data.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This series is *BEA NIPA table 1.1.5 line 1 (A191RC1)*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of the respective nominal values of the *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* and finally deflated by the deflator mentioned above.

Private Investment: Total real private investment is the sum of the respective nominal values of the series Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* and finally deflated by the deflator mentioned above.

Hours worked: The series measures the hours worked of employees working in private non-farm business excluding non-profit business. This series is an updated version of the one used by Francis and Ramey (2009) and is available on the authors' website. Source: http://weber.ucsd.edu/~vramey/

Civilian Population: This series is calculated from monthly data of civilian noninstitutional population over 16 years (CNP16OV) from the U.S. Department of Labor: Bureau of Labor Statistics.

S&P 500: The total returns of the S&P 500 are calculated by the monthly values from the S&P price index and dividends calculated by Robert J. Shiller and provided on his website. Source: http://www.econ.yale.edu/~shiller/data.htm

Risk-free Rate: The quarterly risk-free return is calculated from monthly returns of the 3-Month Treasury Bill: Secondary Market Rate provided by the Board of

Governors of the Federal Reserve System. The real returns are calculated with the implicit inflation rate of the price deflator series above.

B Model solution

B.1 FONC

The economy described in the paper follows the trend γ. To write the equilibrium conditions in stationary terms, the set of variables has to be detrended by z_{t-1} as follows:

$$\tilde{c}_t = \frac{c_t}{e^{z_{P,t-1}}}, \qquad \tilde{y}_t = \frac{y_t}{e^{z_{P,t-1}}}, \qquad \tilde{w}_t = \frac{w_t}{e^{z_{P,t-1}}}, \qquad \tilde{w}_t^f = \frac{w_t^f}{e^{z_{P,t-1}}} \tag{B-1}$$

$$\tilde{k}_{t-1} = \frac{k_{t-1}}{e^{z_{P,t-1}}}, \qquad \tilde{x}_t = \frac{x_t}{e^{z_{P,t-1}}}, \qquad \tilde{H}_t = \frac{H_t}{e^{z_{P,t-1}}}, \qquad \tilde{\lambda}_t = \frac{\lambda_t}{e^{-\eta z_{P,t-1}}}$$

Following, the set of the stationary first order necessary conditions of the equilibrium can be rewritten as:

$$n_t = 1 - l_t \tag{B-2}$$

$$R_t^k q_{t-1} = \frac{\theta \tilde{y}_t}{\tilde{k}_{t-1}} + \left(1 - \delta + g\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) q_t - \frac{\tilde{x}_t}{\tilde{k}_{t-1}} \tag{B-3}$$

$$q_t e^{z_{I,t}} = \frac{1}{g'\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)} \tag{B-4}$$

$$E\left[R_t^f\right] = E\left[\exp\left(-\log \bar{M} - E_t\left[\hat{M}_{t+1}\right] - \frac{\sigma_M^2}{2}\right)\right] \tag{B-5}$$

$$E\left[R_{t+1}^k\right] = E\left[\exp\left(-\log \bar{M} - E_t\left[\hat{M}_{t+1}\right] - \frac{\sigma_M^2}{2} + \sigma_{MR^k}\right)\right] \tag{B-6}$$

$$M_t = \beta \frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t-1}} \exp\left(-\eta\left(\gamma + \epsilon_{P,t-1}\right)\right) \tag{B-7}$$

$$\tilde{\lambda}_t = \left(\tilde{c}_t - \tilde{H}_t\right)^{-\eta} \left(A + \left(e^{z_{L,t}} l_t - F_t\right)^{\nu}\right)^{1-\eta} \tag{B-8}$$

$$\exp\left(\epsilon_{P,t-1}\right) \tilde{H}_t = \left(1 - \rho_c\right) \chi \tilde{c}_{t-1} + \rho_c \tilde{H}_{t-1} \tag{B-9}$$

$$F_t = \left(1 - \rho_l\right) \psi l_{t-1} + \rho_l F_{t-1} \tag{B-10}$$

$$\tilde{w}_t^f = \frac{e^{z_{L,t}} \upsilon \left(\tilde{c}_t - \tilde{H}_t\right)}{A\left(e^{z_{L,t}} l_t - F_t\right)^{1-\nu} + e^{z_{L,t}} l_t - F_t} \tag{B-11}$$

$$\tilde{w}_t = \frac{\left(1 - \theta\right) \tilde{y}_t}{n_t} \tag{B-12}$$

$$\exp\left(\mu \epsilon_{P,t-1}\right) \tilde{w}_t = \left(\tilde{w}_{t-1}\right)^{\mu} \left(e^{\varpi + \varepsilon_{W,t}} \tilde{w}_t^f\right)^{1-\mu} \tag{B-13}$$

$$\tilde{y}_t = \left(\tilde{k}_{t-1}\right)^{\theta} \left(\exp\left(\gamma + \epsilon_{P,t}\right) n_t\right)^{1-\theta} \tag{B-14}$$

$$\exp\left(\gamma + \epsilon_{P,t}\right) \tilde{k}_t = \left(1 - \delta + g\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) \tilde{k}_{t-1} \tag{B-15}$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \bar{g}e^{g_t} \tag{B-16}$$

The equilibrium is defined together with the exogenous variables $z_{L,t}$, $z_{I,t}$, $\varepsilon_{W,t}$ and g_t.

B.2 Steady-state

To calculate the steady state we take the following as given:

$$\bar{z}_L = \bar{z}_I = 1 \quad \text{and} \quad \bar{q} = 1 \ , \tag{B-17}$$

as well as that the steady-state ratio of government expenditures to output is 28%:

$$\frac{\bar{g}}{\bar{\tilde{y}}} = 0.28 \tag{B-18}$$

Furthermore, we can calculate the real depreciation rate:

$$\tilde{\delta} = e^{\gamma} + \delta - 1$$

Remembering the previous discussion about the asset pricing implications, we know that the Euler equation has to hold for any asset. This implies that (eq. B-5) is equal to (eq. B-6). Given a value for $\bar{R}^f$ and σ^2_M, we can solve for steady state pricing kernel:

$$\bar{M} = \exp\left(-\log\left(\bar{R}^f\right) - \frac{\sigma^2_M}{2}\right) \tag{B-19}$$

The return on capital is equal to:

$$\bar{R}^k = \frac{1}{\bar{M}\exp\left(\frac{\sigma^2_M}{2} + \sigma_{MR^k}\right)} \ . \tag{B-20}$$

Now, we can also solve for the discount rate:

$$\beta = \bar{m}\exp\left(\eta\gamma\right) \tag{B-21}$$

Now, we can also solve for:

$$\frac{\bar{\tilde{x}}}{\bar{\tilde{y}}} = \frac{\theta\tilde{\delta}}{\bar{R}^k + \delta - 1} \tag{B-22}$$

and

$$\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} = \frac{\bar{R}^k + \delta - 1}{\theta} \tag{B-23}$$

and because $\bar{\tilde{x}}/\bar{\tilde{k}} = \tilde{\delta}$ for:

$$\frac{\bar{\tilde{c}}}{\bar{\tilde{k}}} = \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} - \frac{\bar{\tilde{x}}}{\bar{\tilde{k}}} - \frac{\bar{g}}{\bar{\tilde{y}}} \cdot \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}. \tag{B-24}$$

Given the assumption that steady state leisure is twice as high as labor, $\bar{l} = 2/3$ and

$$\bar{n} = 1 - \bar{l}, \tag{B-25}$$

we can solve for the steady state capital:

$$\bar{\tilde{k}} = \left[\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}\right]^{\frac{1}{\theta-1}} \bar{n}e^{\gamma} \ , \tag{B-26}$$

this allows now to solve for steady-state value $\bar{\tilde{y}}$, $\bar{\tilde{x}}$, $\bar{\tilde{g}}$, and $\bar{\tilde{c}}$.

As shown in section 5.2 we use the condition of the Frisch elasticity (τ) to resolve for the remaining steady states and parameters. In the case of wage rigidities, the following steady-state relationship between the market wage and the frictionless wage (marginal rate of substitution) holds:

$$\bar{\tilde{w}} = \bar{\tilde{w}}^{f} e^{\varpi} \ , \tag{B-27}$$

where the market wage is determined by the condition:

$$\bar{\tilde{w}} = (1-\theta)\frac{\bar{\tilde{y}}}{\bar{n}} \tag{B-28}$$

Now we define the parameter κ as:

$$\kappa = \frac{e^{\varpi}}{1-\theta}\frac{1-\bar{l}}{\bar{l}}\frac{\bar{c}}{\bar{y}} \tag{B-29}$$

Given the Frisch elasticity τ the following has to hold:

$$\Upsilon = \frac{\bar{l}}{1-\bar{l}}\frac{1}{\tau} - \left(2-\frac{1}{\eta}\right)\frac{1}{(1-\chi)\,\kappa} \tag{B-30}$$

Afterwards, we can resolve for the remaining parameters by solving the equation:

$$\nu = 1-(1-\psi)\,\Upsilon \tag{B-31}$$

$$\alpha = \frac{\kappa\nu\,(1-\chi)}{1-\psi} - 1 \tag{B-32}$$

$$A = \alpha\,(1-\psi)^{\nu}\,\bar{l}^{\nu} \ . \tag{B-33}$$

Given these remaining parameters, we can solve for the steady state values of the remaining variables.

B.3 Log-linearization

$$\hat{l}_t = -\frac{\bar{n}}{1-\bar{n}}\hat{n}_t \tag{B-34}$$

$$\hat{r}^k_t + \hat{q}_{t-1} = \left[\frac{\bar{R}^k - 1 + \delta}{\bar{R}^k}\right]\left(\hat{\tilde{y}}_t - \hat{\tilde{k}}_{t-1}\right) + \frac{e^{\gamma}}{\bar{R}^k}\hat{q}_t + \frac{\tilde{\delta}}{\bar{R}^k} z_{I,t} \tag{B-35}$$

$$\hat{q}_t = \frac{1}{\zeta}\hat{\tilde{x}}_t + \left(\frac{1}{\zeta}-1\right) z_{I,t} - \frac{1}{\zeta}\hat{\tilde{k}}_{t-1} \tag{B-36}$$

$$\hat{\tilde{w}}_t = \hat{\tilde{y}}_t - \hat{n}_t \tag{B-37}$$

$$\hat{\tilde{w}}^f_t = z_{L,t} + \hat{c}^d_t + \left[\frac{\nu\alpha}{1+\alpha} - 1\right]\hat{l}^d_t \tag{B-38}$$

$$\hat{\tilde{\lambda}}_t = -\eta \hat{c}_t^d + \left[\frac{\nu (1-\eta)}{1+\alpha} \right] \hat{l}_t^d \tag{B-39}$$

$$\hat{\tilde{H}}_t + \epsilon_{P,t-1} = (1-\rho_c) \hat{\tilde{c}}_{t-1} - \rho_c \hat{\tilde{H}}_{t-1} \tag{B-40}$$

$$\hat{F}_t = (1-\rho_l) \hat{l}_{t-1} - \rho_l \hat{F}_{t-1} \tag{B-41}$$

$$(1-\chi) \hat{c}_t^d = \hat{\tilde{c}}_t - \chi \hat{\tilde{H}}_t \tag{B-42}$$

$$(1-\psi) \hat{l}_t^d = z_{L,t} + \hat{l}_t - \psi \hat{F}_t \tag{B-43}$$

$$0 = E_t \left[\hat{r}_{t+1}^k + \hat{M}_{t+1} \right] \tag{B-44}$$

$$0 = E_t \left[\hat{M}_{t+1} \right] + \hat{r}_t^f \tag{B-45}$$

$$\hat{M}_t = \hat{\tilde{\lambda}}_t + \hat{\tilde{\lambda}}_{t-1} - \eta \epsilon_{P,t-1} \tag{B-46}$$

$$\hat{\tilde{y}}_t = \theta \hat{\tilde{k}}_{t-1} + (1-\theta) \hat{n}_t + (1-\theta) \epsilon_{P,t} \tag{B-47}$$

$$\hat{\tilde{w}}_t = \mu \hat{\tilde{w}}_{t-1} + (1-\mu) \hat{\tilde{w}}_t^f + (1-\mu) \varepsilon_{W,t} - \mu \epsilon_{P,t-1} \tag{B-48}$$

$$e^{\gamma} \hat{\tilde{k}}_t = (1-\delta) \hat{\tilde{k}}_{t-1} + \tilde{\delta} \hat{\tilde{x}}_t + \tilde{\delta} z_{I,t} - e^{\gamma} \epsilon_{P,t} \tag{B-49}$$

$$\bar{\tilde{y}} \hat{\tilde{y}}_t = \bar{\tilde{c}} \hat{\tilde{c}}_t + \bar{\tilde{x}} \hat{\tilde{x}}_t + \bar{g} g_t \tag{B-50}$$

$$\varepsilon_{W,t} = \pi_W \hat{\varepsilon}_{W,t-1} + \epsilon_{W,t} \tag{B-51}$$

$$z_{L,t} = \pi_L \hat{z}_{L,t-1} + \epsilon_{L,t} \tag{B-52}$$

$$z_{I,t} = \pi_I \hat{z}_{I,t-1} + \epsilon_{I,t} \tag{B-53}$$

$$g_t = \pi_G \hat{g}_{t-1} + \epsilon_{G,t} \tag{B-54}$$

C Model solution of the more simple RBC model

C.1 FONC

$$n_t = 1 - l_t \tag{C-1}$$

$$R_t^k q_{t-1} = \frac{\theta y_t}{k_{t-1}} + \left(1 - \delta + g \left(e^{z_{I,t}} \frac{x_t}{k_{t-1}} \right) \right) q_t - \frac{x_t}{k_{t-1}} \tag{C-2}$$

$$q_t e^{z_{I,t}} = \frac{1}{g' \left(e^{z_{I,t}} \frac{x_t}{k_{t-1}} \right)} \tag{C-3}$$

$$1 = E_t \left[R_t^f M_{t+1} \right] \tag{C-4}$$

$$1 = E_t \left[R_{t+1}^k M_{t+1} \right] \tag{C-5}$$

$$M_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \tag{C-6}$$

$$\lambda_t = c_t^{-\eta} \tag{C-7}$$

$$w_t = \Psi_l e^{z_{L,t}} \frac{l_t^{\frac{1}{\tau}}}{\lambda_t} \tag{C-8}$$

$$w_t = \frac{(1-\theta) y_t}{n_t} \tag{C-9}$$

$$y_t = (k_{t-1})^{\theta} \left(e^{z_{P,t}} n_t\right)^{1-\theta} \tag{C-10}$$

$$k_t = \left(1 - \delta + g\left(e^{z_{I,t}} \frac{x_t}{k_{t-1}}\right)\right) k_{t-1} \tag{C-11}$$

$$y_t = c_t + x_t + \bar{g} e^{g_t} \tag{C-12}$$

$$\tag{C-13}$$

The equilibrium is defined together with the exogenous variables $z_{L,t}$, $z_{I,t}$, $z_{P,t}$ and g_t.

C.2 Steady-state

To calculate the steady state, we follow the same steps as for the foregoing model with the exception of wages. Given that steady state labor supply is calibrated to $\bar{n} = 1/3$, wages are calculated as before:

$$\bar{w} = (1 - \theta) \frac{\bar{y}}{\bar{n}} \tag{C-14}$$

The marginal rate of substitution implies also that

$$\bar{w} = \Psi_l \frac{\bar{l}^{\frac{1}{\tau}}}{\bar{c}^{-\eta}}. \tag{C-15}$$

To clear the labor market, we have to solve for the scaling factor

$$\Psi_l = (1 - \theta)\, \bar{y} \frac{\bar{c}^{-\eta}}{\bar{n}^{1/\tau}} \tag{C-16}$$

C.3 Log-linearization

$$\hat{l}_t = -\frac{\bar{n}}{1 - \bar{n}} \hat{n}_t \tag{C-17}$$

$$\hat{r}_t^k + \hat{q}_{t-1} = \left[\frac{\bar{R}^k - 1 + \delta}{\bar{R}^k}\right] \left(\hat{y}_t - \hat{k}_{t-1}\right) + \frac{1}{\bar{R}^k} \hat{q}_t + \frac{\delta}{\bar{R}^k} z_{I,t} \tag{C-18}$$

$$\hat{q}_t = \frac{1}{\zeta} \hat{x}_t + \left(\frac{1}{\zeta} - 1\right) z_{I,t} - \frac{1}{\zeta} \hat{k}_{t-1} \tag{C-19}$$

$$\hat{w}_t = \hat{y}_t - \hat{n}_t \tag{C-20}$$

$$\hat{\tilde{w}}_t = z_{L,t} + \frac{1}{\tau} \hat{l}_t - \hat{\lambda}_t \tag{C-21}$$

$$\hat{\lambda}_t = -\eta \hat{c}_t \tag{C-22}$$

$$0 = E_t \left[\hat{r}_{t+1}^k + \hat{M}_{t+1}\right] \tag{C-23}$$

$$0 = E_t \left[\hat{M}_{t+1}\right] + \hat{r}_t^f \tag{C-24}$$

$$\hat{m}_t = \hat{\lambda}_t + \hat{\lambda}_{t-1} \tag{C-25}$$

$$\hat{y}_t = \theta \hat{k}_{t-1} + (1 - \theta)\, \hat{n}_t + (1 - \theta)\, z_{P,t} \tag{C-26}$$

$$\hat{k}_t = (1 - \delta)\, \hat{k}_{t-1} + \delta \hat{x}_t + \delta z_{I,t} \tag{C-27}$$

$$\bar{y} \hat{y}_t = \bar{c} \hat{c}_t + \bar{x} \hat{x}_t + \bar{g} g_t \tag{C-28}$$

$$z_{P,t} = \pi_P z_{P,t-1} + \epsilon_{P,t} \tag{C-29}$$
$$z_{L,t} = \pi_L z_{L,t-1} + \epsilon_{L,t} \tag{C-30}$$
$$z_{I,t} = \pi_I z_{I,t-1} + \epsilon_{I,t} \tag{C-31}$$
$$g_t = \pi_G g_{t-1} + \epsilon_{G,t} \tag{C-32}$$

D Tables and Figures

D.1 Additional results benchmark model

Figure 3: Prior and posterior distribution of the model with unconstrained prior. Vertical dashed line indicates the posterior mode.

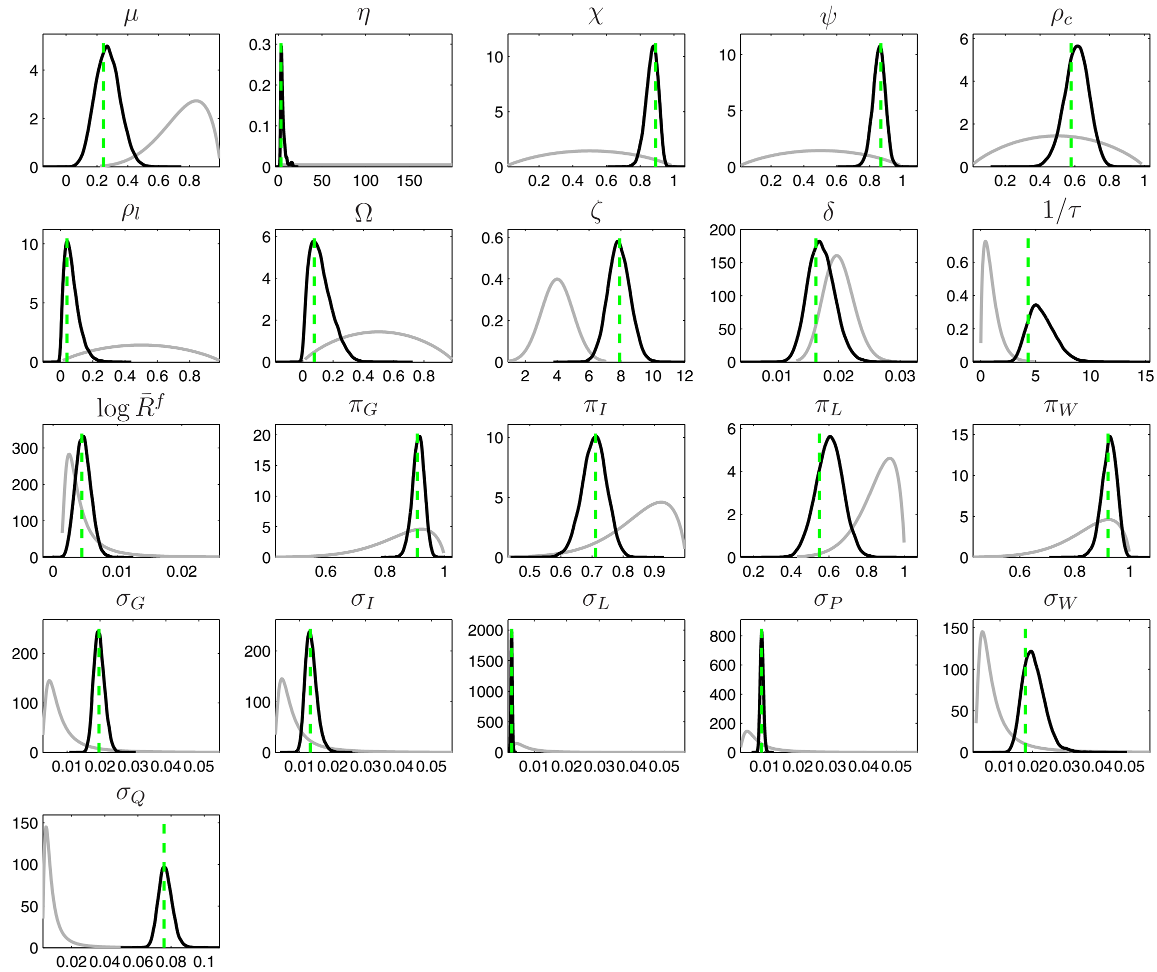


Figure 4: Prior and posterior distribution of the model with constrained prior. Vertical dashed line indicates the posterior mode.

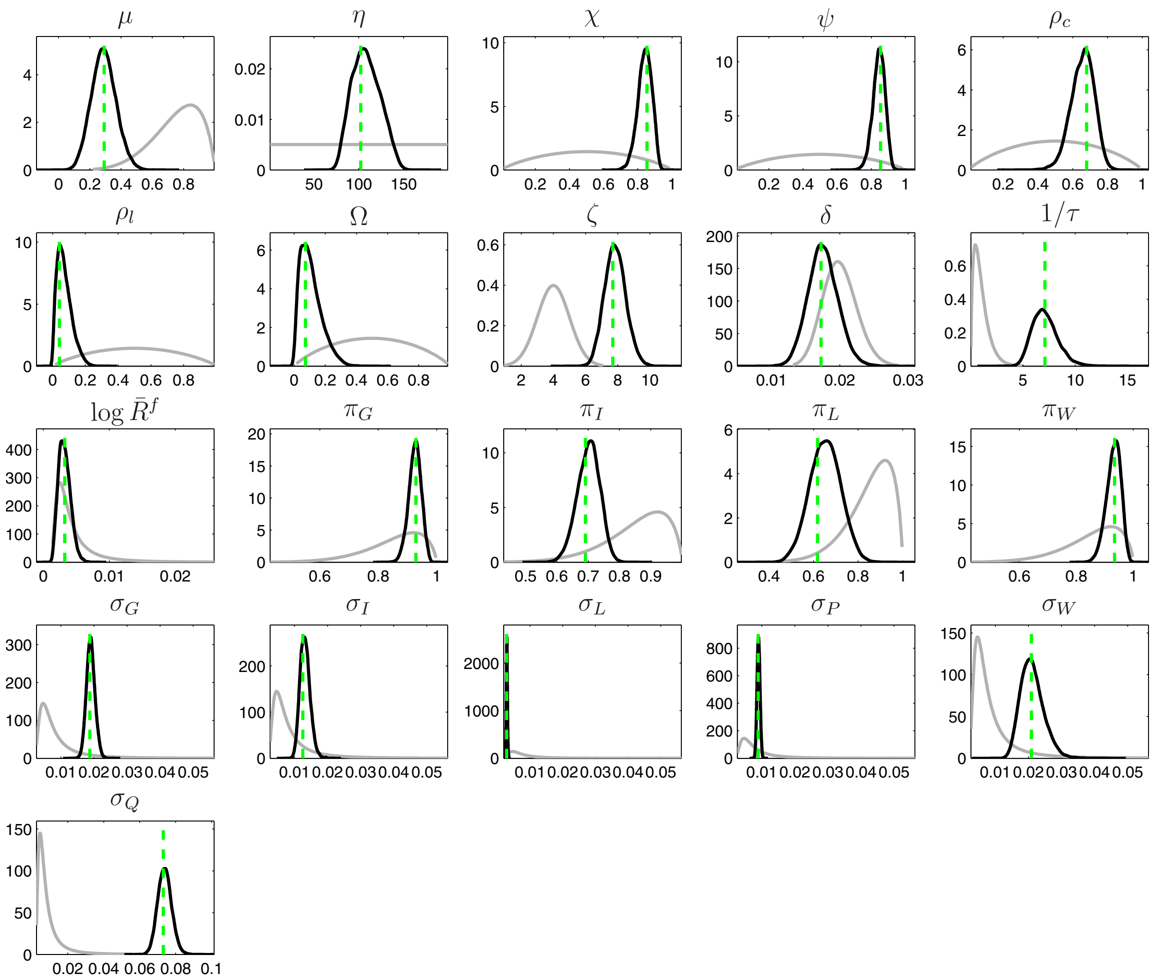
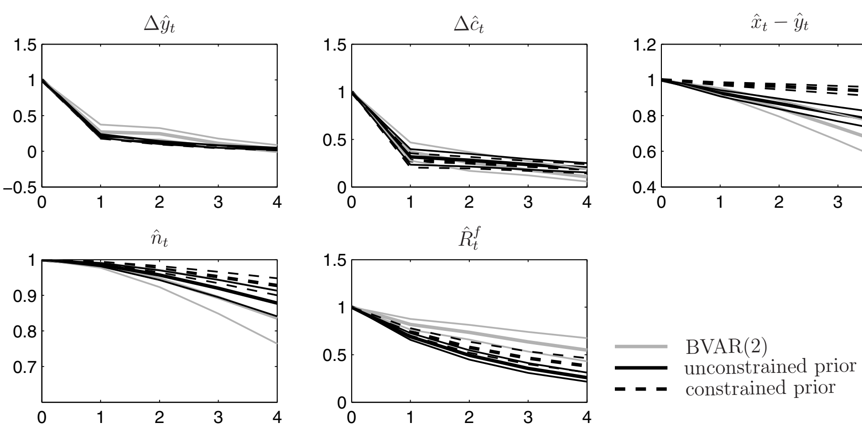


Figure 5: Implied autocorrelation of observable variables of the DSGEs and the BVAR(2).



D.2 Additional results of the more simple RBC model

Figure 6: Prior and posterior distribution of the more simple RBC model with unconstrained prior. Vertical dashed line indicates the posterior mode.

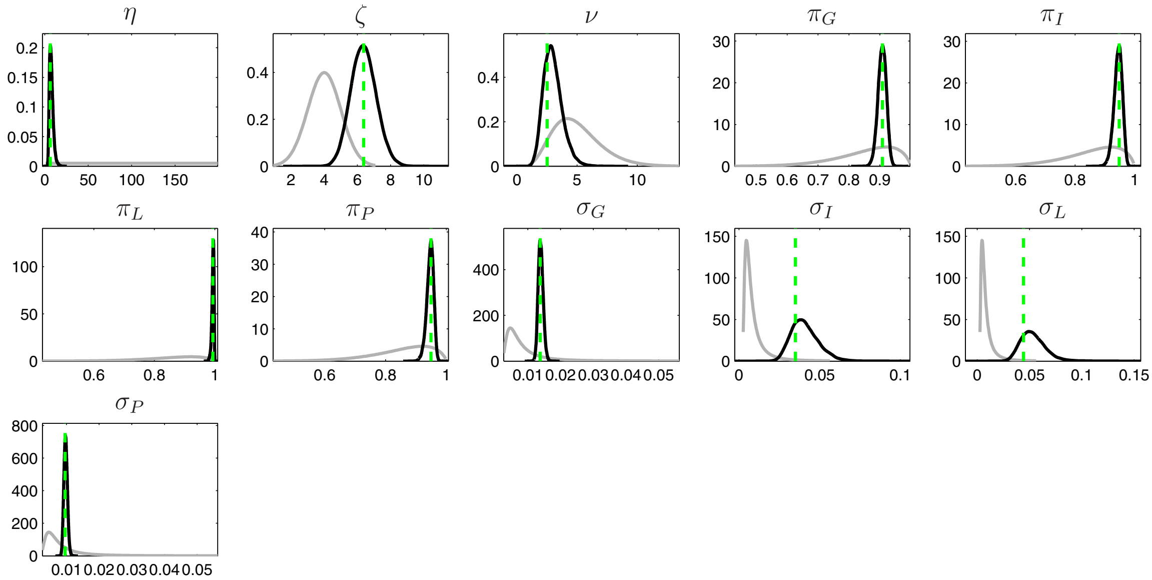


Figure 7: Prior and posterior distribution of the more simple RBC model with constrained prior. Vertical dashed line indicates the posterior mode.

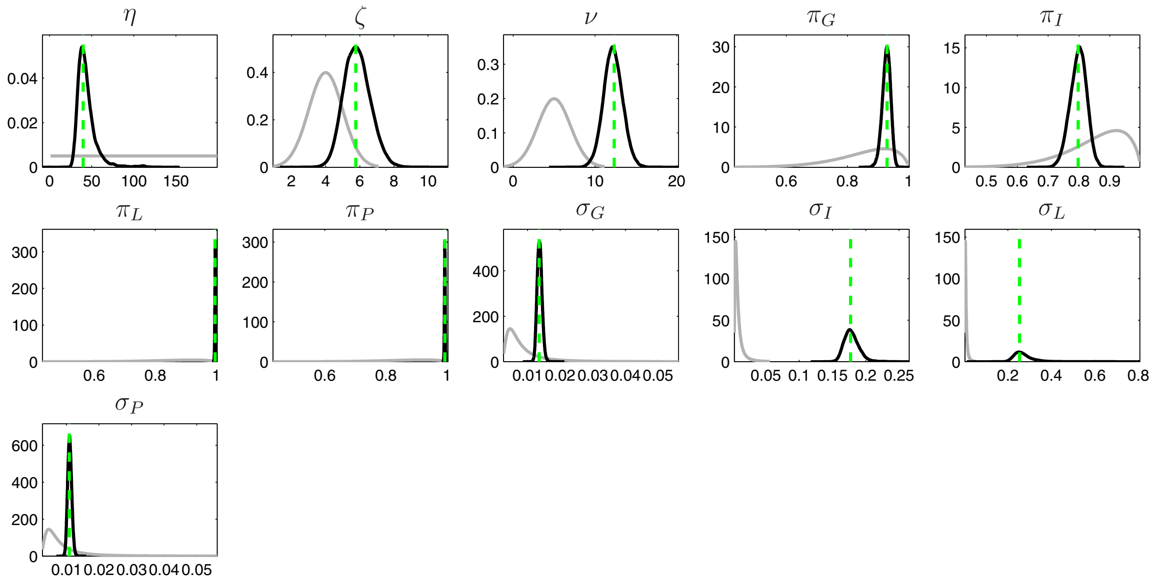


Figure 8: Implied autocorrelation of observable variables of the more simple DSGEs and the BVAR(2).

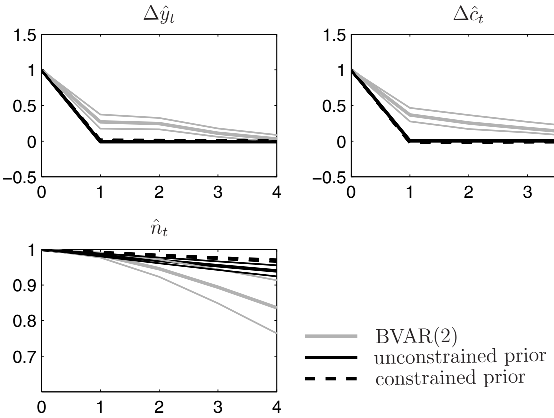


Table 9: HP-filtered ($\lambda = 1600$) theoretical and empirical moments for the more simple RBC model. The theoretical moments are based on 1200 draws from the posterior. The numbers in brackets indicate 5% and 95% probabilities.

		Unconstrained Prior	Constrained Prior	Data
Standard Deviation of Output				
Output	$\hat{\hat{y}}$	0.0104	0.0105	0.0148
		[0.0098;0.0111]	[0.0098;0.0112]	
Relative Standard Deviation to Output				
Consumption	$\hat{\hat{c}}$	0.6503	0.6416	0.5516
		[0.5829;0.7240]	[0.5737;0.7124]	
Investment	$\hat{\hat{x}}$	3.3759	3.4671	3.6632
		[3.2632;3.4857]	[3.3546;3.5902]	
Hours worked	$\hat{\hat{n}}$	1.1032	1.5276	1.2372
		[1.0128;1.1960]	[1.4233;1.6405]	
Correlation with Output				
Consumption	$\hat{\hat{c}}$	0.4250	0.4457	0.8210
		[0.3332;0.5092]	[0.3395;0.5354]	
Investment	$\hat{\hat{x}}$	0.8567	0.8542	0.9226
		[0.8313;0.8787]	[0.8256;0.8769]	
Hours worked	$\hat{\hat{n}}$	0.5639	0.4424	0.8676
		[0.4911;0.6360]	[0.3365;0.5346]	

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