

# Discussion Paper

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## **Banks, markets, and financial stability**

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# Non-technical summary

## Research Question

In the present paper, we analyze effects of the interaction between funding liquidity of banks and financial market liquidity on risk sharing efficiency and asset prices. In addition, we draw conclusions with respect to the appearance of contagion effects and asset price bubbles.

## Contribution

The paper presents a theoretical model where investors' ability to absorb asset fire sales by banks arises endogenously from the interaction of banks, financial markets and investor behavior. Moreover, we assume that there is a commonly known strictly positive ex-ante probability for a crisis at one bank in our economy to allow actors to take future financial crises into account.

## Results

We find that due to endogenous interaction between funding liquidity of banks and financial market liquidity the banking system in our model cannot provide efficient risk sharing and may show contagion effects as well as asset price bubbles. The ex-ante probability of a future financial crisis and the number of wholesale investors jointly determine the strength of these results. Our findings suggest that liquidity regulation defining a mandatory liquidity buffer (although not explicitly modeled) may mitigate contagion and asset price bubbles but at the cost of further reduced risk sharing efficiency.

# Nichttechnische Zusammenfassung

## Fragestellung

Das vorliegende Papier analysiert die Interaktion zwischen der Finanzierungsliquidität von Banken und der Liquidität von Finanzmärkten in ihrer Wirkung auf die Effizienz der Risikoteilung und Marktpreise im Bankensystem. Hierdurch sollen auch Schlußfolgerungen hinsichtlich des Auftretens von Ansteckunseffekten und Preisblasen ermöglicht werden.

## Beitrag

Das Papier präsentiert ein theoretisches Modell, in dem die Fähigkeit der Marktteilnehmer, als Nachfrager an Finanzmärkten aufzutreten, endogen aus der Interaktion zwischen Banken, Finanzmärkten und Investorenverhalten resultiert. Zudem wird davon ausgegangen, dass eine allgemein bekannte (und strikt positive) Wahrscheinlichkeit für eine Krise bei einer der im Modell betrachteten Banken existiert. Die Akteure des Modells können so die Möglichkeit zukünftiger Krisen in ihr Entscheidungsverhalten einbeziehen.

## Ergebnisse

Die Analyse des Modells zeigt, dass die endogene Interaktion zwischen Finanzierungsliquidität der betrachteten Banken und Finanzmarktliquidität zu einer ineffizienten Risikoteilung sowie zu Ansteckungseffekten und Preisblasen innerhalb des betrachteten Bankensystems führt. Die Stärke dieser Effekte hängt maßgeblich von der Höhe der Wahrscheinlichkeit einer künftigen Krise und der Anzahl institutioneller Investoren im System ab. Insgesamt liefern die Ergebnisse Hinweise, dass (obwohl nicht explizit modelliert) eine Liquiditätsregulierung, die einen Mindestliquiditätspuffer für Banken verpflichtend vorgibt, Ansteckungseffekte und Preisblasen abschwächen kann – allerdings zu Lasten der Effizienz der Risikoteilung im Gesamtsystem.

# Banks, Markets, and Financial Stability\*

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## Abstract

We use a Diamond/Dybvig-based model with two banks operating in separate regions connected by a common asset market in which banks and sophisticated depositors invest. We study the effect of a potential run (crisis) and subsequent fire sales on the asset price in both the crisis and no-crisis state. In our model, the two are jointly determined by a cash-in-the-market pricing and a no-arbitrage condition. We find that (i) a higher crisis probability increases the liquidity premium and thus asset prices in the normal and crisis case and (ii) a higher share of sophisticated investors increases market depth and thus the crisis price while it might also raise the asset price in the normal state.

*Keywords:* liquidity risk, financial crises, contagion, asset price bubbles

*JEL classification:* G21, G23, G12

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# 1 Introduction

Liquidity shortages of individual banks and subsequent fire sales led to deteriorating prices during the subprime crisis not only of mortgage-backed securities but also of a broad range of other assets. Similarly, in the sovereign debt crisis tightening refinancing conditions of southern European banks and their offloading of domestic sovereign bonds supposedly contributed to widening spreads between core countries' and crisis countries' sovereign bonds. These asset price drops obviously had severe knock-on effects, for instance, because other financial institutions had to write down their positions held in those assets. However, the extent of these asset price drops strongly depends on cash-in-the-market, i.e. the ability and willingness of market participants to absorb fire sales of troubled financial institutions (Allen and Gale (1998)). Recent literature has largely emphasized the presence of arbitrageurs (Gromb and Vayanos (2002)) or market makers (Brunnermeier and Pedersen (2009)) in those markets and in particular their financial constraints as a key determinant in markets' ability to absorb fire sales. Since asset prices, in turn, affect the financial constraints of those market participants destabilising feedback effects emerge. Those models, however, do not take into account the extent to which the presence of such informed market participants has an effect on the ratio of inside to outside liquidity in the banking system, i.e. on the ratio of assets whose value is common knowledge and state independent (reserves) and claims against the banking sector that might also serve as a medium of exchange but whose value is endogenous (Bolton, Santons, and Scheinkman (2011)).

In order to endogenize investors' ability to absorb fire sales, we set up a Diamond/Dybvig-based economy with two banks operating in two separate regions with a continuum of depositors that are only linked through a common asset market. In this asset market, not only banks but also some sophisticated depositors (i.e. arbitrageurs/market makers/wholesale investors) can invest. For the non-sophisticated part of depositors (i.e. private households/retail investors), direct market participation is not beneficial. The latter prefer using banks' investment abilities. Furthermore, we assume an exogenous positive probability for a run on one bank and study the contagious effect of subsequent fire sales on the other bank and on equilibrium asset prices. Our setting appears quite rich as we consider numerous elements of real-world financial systems which, ultimately, allows us to analyze interactions between those elements and resulting effects for financial system efficiency and stability.

Surprisingly we find that a higher probability of a run not only increases the equilibrium asset price in the normal state (due a liquidity premium) but also leads to a higher asset price in the crisis state. In addition, a higher ratio of sophisticated investors with financial market access may increase financial market depth and improve prices in crisis as well as no-crisis states. We obtain these results because asset prices in crisis and no-crisis states are jointly determined by two effects. On the one hand, arbitrage considerations determine asset prices: a lower asset price in crisis periods implies a higher asset price in no-crisis periods. Investors charge a liquidity premium. On the other hand, market prices are settled by cash-in-the-market constraints: the degree to which market participants hold liquidity relative to the fire sales defines the price in crisis states. In our model, liquidity

holdings are determined by banks' reserves which are, in turn, determined by the optimal deposit contract banks offer. Investors' financial market access restrains banks' ability to provide efficient risk sharing against idiosyncratic liquidity shocks as in [Diamond \(1997\)](#) and [Fecht \(2004\)](#). Better financial market access for investors thus reduces banks liquidity holdings. A higher secondary market price for assets in normal times makes investors' direct investment opportunities less attractive, improves banks' liquidity transformation and increases their liquidity holdings. Higher liquidity holdings of banks reduce the asset price drop due to fire sales in crisis periods.

A higher probability of a run with subsequent fire sales and asset price drops increases the liquidity premium market participants charge in normal times. Thus asset prices in normal times increase. A higher asset price in normal states, however, improves banks' risk sharing capacity as a higher price in normal states makes it less attractive for sophisticated depositors to withdraw their deposits and reinvest in financial markets. It thus fosters banks' liquidity transformation and increases their liquidity holdings, such that asset prices in crisis times improve.

A larger share of sophisticated investors with efficient market access improves financial market depth. Consequently there is more cash in the market which mitigates fire sales. This reduces the required liquidity premium in the normal state and thus the asset price in this state. At the same time, however, a higher share of sophisticated investors and thus more liquid financial markets foster banks' incentives to invest in assets and sell them off in financial markets to gather liquidity rather than maintaining sufficient liquidity reserves *ex ante*. In order to ensure that the no-arbitrage condition holds and banks invest in assets and liquidity, the asset price in normal times must increase for more liquid financial markets. Thus if this second effect prevails, more investors having access to financial markets might actually lead to higher asset prices in no-crisis periods even though the liquidity premium declines.

In sum, our results help explain why banks' funding liquidity as well as financial market liquidity arises endogenously from the interactions between financial system actors. Both types of liquidity are determined by the ratio of inside to outside liquidity in the banking sector. Banks' inside liquidity determines available liquidity of both non-sophisticated as well as sophisticated depositors. Sophisticated investors' available liquidity, however, crucially drives financial market liquidity and hence asset prices in crisis as well as no-crisis states. Liquidity-driven asset price changes then affect banks' funding liquidity which finally explains why there may be contagion in times of financial crisis. Our results also shed some light on the appearance of asset price bubbles in financial markets. In no-crisis times, the liquidity premium charged by sophisticated investors may be interpreted to determine positive price bubbles while fire-sale prices in times of crisis imply negative price bubbles. Our analysis, in this context, shows relationships between these two types of bubbles as a consequence of the interactions mentioned previously.

The rest of the paper is organized as follows. Section 2 relates our paper to the literature. The model is laid out in Section 3. The main analysis is presented in Sections 4 and 5. Section 6 concludes.

## 2 Relationship to the literature

Our paper uses [Fecht \(2004\)](#) as a baseline for our analysis which in turn builds on the seminal papers of [Diamond and Dybvig \(1983\)](#), [Jacklin \(1987\)](#), [Diamond \(1997\)](#) and [Allen and Gale \(2004c\)](#). Similar to those models, households are exposed to liquidity risk. Banks are able to provide liquidity insurance to households. Financial intermediaries and financial markets coexist. Financial markets allow for trading – and hence liquidating – claims on long-term investment projects before maturity and may be used by banks as well as households to exchange liquid funds for claims on illiquid (long-term) investment projects. As a result, the model is able to include the aspect of market participation in the analysis of market liquidity.<sup>1</sup>

Our paper extends the model of [Fecht \(2004\)](#) by considering a positive and commonly known probability of a run on either of the two banks in the financial system. Banks as well as households, therefore, make investment decisions taking into account the possibility of a future bank-run. As a result, optimal decisions may be expected to differ from the standard results of the relevant literature which usually assumes that future crises are not anticipated in the decision-making process, i.e. are zero probability events.<sup>2</sup>

Although the assumption is not completely new to the literature<sup>3</sup>, we are – to the best of our knowledge – the first who consider a positive ex-ante crisis probability in a setting where banks are interconnected via asset markets which also may be directly used by households. Market liquidity, then, arises from the joint effect of household behavior and bank behavior. In this way – and in contrast to earlier papers<sup>4</sup> – our setting allows the role of the interaction of funding and market liquidity to be considered as well as conclusions regarding asset price bubbles and financial system stability to be drawn.

These aspects, furthermore, represent the main differences between the present paper and the papers of [Allen and Gale \(2004a\)](#) and [Allen and Gale \(2000a\)](#). Although [Allen and Gale \(2004a\)](#) consider the role of banks regarding the accrual of asset price bubbles, they assume that outside liquidity of speculators is exogenous to the model. Our model overcomes this shortcoming by allowing sophisticated households (which take the role of speculators in our setting) to participate in financial market transactions and to deposit funds with banks.

Our paper is, moreover, quite closely related to recent papers of [Freixas et al. \(2011\)](#) and [Carletti and Leonello \(2011\)](#). In particular [Freixas et al. \(2011\)](#) address an objective similar to ours also assuming a non-zero crisis probability. In contrast to our approach,

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<sup>1</sup> [Huang and Wang \(2008\)](#) analyze the effect of market participation on market liquidity and asset price formation in financial markets in more detail.

<sup>2</sup> For example, the papers of [Allen and Gale \(2000b\)](#), [Allen and Gale \(2004b\)](#), [Fecht \(2004\)](#), and [Caballero and Simsek \(2011\)](#) analyze bank behavior and the propagation of shocks via financial markets. However, all the papers share the common assumption that ex ante the probability of a future financial crisis is zero. As a result, decisions to be made in these papers do not take future financial crises into account.

<sup>3</sup> See, eg, [Cooper and Ross \(1998\)](#), [Ennis and Keister \(2005\)](#) and [Freixas, Martin, and Skeie \(2011\)](#).

<sup>4</sup> See, eg, [Cooper and Ross \(1998\)](#), [Holmstrom and Tirole \(2000\)](#), [Bougheas and Ruis-Porras \(2005\)](#), and [Garleanu and Pedersen \(2007\)](#).

however, they consider direct links between banks via interbank market exposures. In their model, the interbank market redistributes liquidity in the financial system. Moreover, the aggregate amount of liquidity is fixed in Freixas et al. (2011), which is not the case in our model. We consider an asset market (instead of interbank lending) that allows for early liquidation of claims on long-term assets. The market is generally accessible, i.e. households, too, may enter and demand or supply claims on long-term investment projects. As a consequence, in our model, the asset market provides liquidity to market participants who supply claims on long-term assets. And the aggregate amount of liquidity in the market is endogenously determined by the initial decisions of banks and households to invest their funds into short-term or long-term assets.

Just as in our paper, asset market liquidity in Carletti and Leonello (2011) arises endogenously from banks' initial decisions to invest funds into liquid short-term or illiquid long-term assets. In contrast to our model, the asset market in Carletti and Leonello (2011) is a pure interbank market, i.e. households do not directly have access to the market. Moreover, Carletti and Leonello (2011) do not consider a strictly positive ex-ante probability of a future financial crisis. Instead they focus on the question as to whether the strength of credit market competition between banks affects bank behavior and, in turn, financial stability. The aspect of competition between banks is, however, beyond the scope of our paper.

### 3 The Model

Consider a Diamond-Dybvig-style economy with one good, three dates ( $t = 0, 1, 2$ ) and two identical regions  $\{I; II\}$ . In each region, there is a continuum of ex-ante identical households of measure 1. A non-random proportion  $\pi$  of households will prefer to consume early – at time  $t = 1$  – and the complementary proportion  $1 - \pi$  will prefer to consume late – at time  $t = 2$ . Each household is endowed with one unit of goods and has preferences over consumption  $c_t$  at date  $t = 0$  given by

$$U(c_1, c_2) = \begin{cases} u_1(c_1) & \text{with probability } \pi \\ u_2(c_2) & \text{with probability } (1 - \pi) \end{cases} \quad (1)$$

The uncertainty about the preferred consumption date resolves at  $t = 1$ . This means that every household learns at  $t = 1$  whether it is patient (prefers consuming at  $t = 2$ ) or impatient (prefers consuming at  $t = 1$ ). However, the individual realization is private information of the respective household and not publicly observable. There is no aggregate uncertainty regarding the share of patient and impatient households. Therefore, from the law of large numbers, the portion of impatient and patient households in the economy as a whole is given by  $\pi$  and  $1 - \pi$ , respectively. For simplicity, there is no discounting and we assume risk-neutral households, i.e. linear utility functions.

$$U(c_1, c_2) = \begin{cases} x_1 c_1 & \text{with probability } \pi \\ c_2 & \text{with probability } (1 - \pi) \end{cases} \quad (2)$$

$$x_1 > R \quad (3)$$

In the economy, two different production technologies are available. The first is a pure storage technology that yields zero net interest and enables households to transfer units between any two dates. The second production technology is owned by a continuum of entrepreneurs who do not have any initial endowment but offer a long-run investment project to households. Investments have to be made in  $t = 0$  to realize some return in  $t = 2$ . At  $t = 1$ , the entrepreneurs decide whether they spend their entire effort and generate a return of  $R > 1$  at  $t = 2$  for every unit invested at  $t = 0$  or whether they shirk. Entrepreneurs have an incentive to reduce their effort since doing so increases their private benefit. Shirking, however, reduces the return of the long-run project to  $\epsilon = 0$ . If the project is prematurely liquidated, it also yields a return of  $\epsilon = 0$ . Table 1 summarizes the investment options.

	t=0	t=1	t=2
Storage	-1	+1	
		-1	+1
Production			
behave	-1	0	R
shirk	-1	0	0
liquidate	-1	0	0

Table 1: Investment options

In order to invest in the long-run project, households can use a centralized financial market. In  $t = 0$ , households use the primary market to invest in the long-run project by buying financial claims from an entrepreneur. Since funds are assumed to be scarce, competition between entrepreneurs will lead to a promised repayment of  $R$  in  $t = 2$ . Depending on their consumption needs, households may be inclined to trade the claims on the long-run investment project against consumption goods with other agents in a secondary market in  $t = 1$ . At  $t = 2$ , the entrepreneurs pay out the actual return of the project to the final claim holder.

Moreover, households are assumed to be of either two types. A fraction  $(1 - i)$  of households is sophisticated (henceforth Type-A). They are able to monitor entrepreneurs and force them to spend their entire effort for the long-term project. Thus these households can realize a return of  $R$  on financial claims that they own in  $t = 1$ . The complementary fraction  $i$  of households is of the naive type (henceforth Type-B). They are not able to monitor the entrepreneurs and achieve a return of  $\epsilon$  since, then, the entrepreneurs have an incentive to shirk.

Besides the direct investment strategy, households can decide to deposit their funds with a bank. A bank is a financial institution that offers deposit contracts against households' initial endowments. The proceeds from deposit contracts are then used to build up a portfolio containing investments in the storage technology and claims on the long-term production technology. We assume that only one bank operates in each region. But due to the contestability of the deposit market, both banks are forced to offer households a utility-maximizing deposit contract. Like Type-A households, banks are able to monitor the

effort level of entrepreneurs accurately and achieve a return of  $R$  on financial claims. But in contrast to sophisticated households, banks are able, through their deposit contract, to credibly commit to pass on the entire return to naive households.<sup>5</sup> Thus, only banks have the ability to provide naive households with efficient access to the long-run investment opportunity. Moreover, we do not allow for direct interbank transactions, neither in  $t = 0$  nor in  $t = 1$ . Such interbank transactions would anyways not allow banks to share the risk of regional runs. Given that runs are low probability events, banks will not find it efficient to hold excess liquidity that they could offer in an  $t = 1$ -interbank market. Furthermore, following [Bhattacharya and Fulghieri \(1994\)](#) an efficient risk sharing cannot be implemented through an interbank market when bank-specific liquidity needs are unobservable.

We further consider two possible states of the world. With strictly positive probability  $\theta$ , either of the two banks is subject to a run due to a coordination failure of depositors.<sup>6</sup> The probability of such a run is common knowledge to market participants who will adjust their expectations accordingly. Let  $m \in M \equiv \{0; 1\}$ , where

$$m = \begin{cases} 1 & \text{with probability } \theta & \text{crisis state} \\ 0 & \text{with probability } (1 - \theta) & \text{normal times state} \end{cases} \quad (4)$$

and  $\theta \in [0, 1]$  is the probability of a coordination failure state  $m = 1$ . We assume that  $m$  is observable but not verifiable and thus contracts cannot be written contingent on the realization of  $m$ . Since we have two banks of which one is subject to a run at a time, the probability of a specific bank to be subject to a run is  $\theta/2$ .

## 4 Financial System Structure and Stability

For an analysis of the structure and stability features of the financial system, let us assume for the moment that  $\theta = 0$ . Since a household's type and realized preference shock is private information, banks are not able to offer contracts contingent on the realization of these characteristics. Thus banks can only offer type-specific deposit contracts if they are self-revealing. A deposit contract specifies depositors' claims  $d_1$  and  $d_2$  on the bank at time  $t = 1$  and  $t = 2$ , respectively. If banks offer a deposit contract that provides naive depositors with an option for consumption smoothing, i.e.  $d_2/d_1 < R$ , then sophisticated households pool with naive households and also choose this contract.<sup>7</sup> Therefore, the optimal deposit contract that banks can offer solves the optimization problem ( $P1$ ).

<sup>5</sup> The assumption can be thought of as reflecting [Diamond and Rajan \(2001\)](#) who argued that the attempt to renegotiate the deposit contract would lead in a run on the bank due to sequential service property of deposit contracts (first-come, first-served).

<sup>6</sup> While we simply assume that such coordination failures occur with an exogenous positive probability, application of global games to [Diamond/Dybvig](#) based models such as [Goldstein and Pauzner \(2005\)](#) show that this can be derived from uncertainty and heterogenous information about fundamentals.

<sup>7</sup> As we shall discuss in detail below, sophisticated households always withdraw in equilibrium in  $t = 1$ . This behavior is obvious for impatient households but it is also true of patient sophisticated households because, at the equilibrium asset price, they withdraw deposits to reinvest directly into asset holdings in  $t = 1$ . Therefore, sophisticated households choose the deposit contract with the highest  $d_1$ . This also implies that the bank cannot offer two different type-specific contracts that would

$$(P1) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = \pi i x_1 d_1 + (1 - \pi) i d_2 \\ \quad \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\ \quad d_1 \leq d_2 \quad (IC_B) \\ \quad d_1 \leq \frac{l + p_n k}{1 - (1 - \pi)^i} \quad (BC_1) \\ \text{s.t.} \quad d_2 \leq \frac{1 - l - k}{(1 - \pi)^i} \cdot R \quad (BC_2) \\ \quad \pi x_1 d_1 + (1 - \pi) \frac{R}{p_n} d_1 \quad (PC) \\ \quad > \max \left\{ \pi x_1 + (1 - \pi) \frac{R}{p_n}, \pi x_1 p_n + (1 - \pi) R \right\} \end{array} \right.$$

Given that the deposit contract provides some insurance against liquidity risks, sophisticated households, too, might find it optimal to invest in bank deposits in  $t = 0$ . But in contrast to naive households, sophisticated depositors can withdraw and reinvest in assets directly in the financial market if they turn out to be patient. While patient naive depositors will have incentives to hold on to their deposits as long as this allows for more consumption at  $t = 2$  (see  $(IC_B)$ ), patient sophisticated households will rather withdraw their deposits to reinvest in financial markets if this increases consumption in  $t = 2$  beyond  $d_2$  (see  $(IC_A)$ ). Given that they plan to withdraw and reinvest if they turn out to be patient, sophisticated households have an ex-ante incentive to invest in deposits rather than hold a portfolio of liquidity (storage technology) and assets (claims against entrepreneurs) in  $t = 0$  and rebalance the portfolio in  $t = 1$  according to their consumption preferences and the participation constraint  $(PC)$ .

Given that only naive patient households keep their deposits until  $t = 2$ , the bank must dispose of sufficient liquidity in  $t = 1$  to refinance the repayment  $d_1$  to all but the patient naive households. Thus the initial liquidity holding  $l$  plus the revenues  $p_n k$ <sup>8</sup> from selling assets in the financial market must suffice to repay  $d_1$  to the fraction  $[1 - (1 - \pi)^i]$  of households. Returns on the long-term asset holdings must suffice to refinance the repayment to patient depositors. Consequently, we have the two budget constraints  $(BC_1)$  and  $(BC_2)$ .

Because the banking market is assumed to be contestable, banks will offer a deposit contract that maximizes naive households' expected utility. Given that  $d_2/d_1 < R$  and that sophisticated households withdraw irrespective of whether they are patient or impatient, the deposit contract involves a cross-subsidization from naive to sophisticated households. Therefore, if a bank does not maximize the expected utility of naive households given this cross-subsidization, a competitor could always offer a deposit contract preferable to the naive households leaving the incumbent bank with only the sophisticated households.

Since we assume that banks act as price takers in the financial market, it is easy to see from  $(BC_1)$  that for  $p_n > 1$  banks will only invest in assets and try to refinance

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induce self-revelation as long as the contract meant for naive households provides some consumption smoothing. For detailed proof, see [Fecht \(2004\)](#).

<sup>8</sup> Note  $p_n$  represents the market price for long-term assets when there is no crisis because due to our assumption  $\theta = 0$ , a possible future crisis is not taken into account by banks and households. Moreover,  $k$  represents the amount of long-term assets dedicated to be sold in the market in order to provide the bank with enough liquidity to meet depositor claims.  $k$  may, hence, be interpreted as a bank's trading portfolio or trading book.

short-term repayments solely with the revenue from asset sales. But this would mean that no liquidity is held in the economy. Thus banks actually could not exchange their assets against liquidity and this cannot be an equilibrium. For  $p_n < 1$ , banks would only hold liquidity. Patient sophisticated depositors receiving liquidity when withdrawing their deposits will not find any supply of assets in the market. Thus banks are indifferent only for  $p_n = 1$  and will sell assets in the financial market while at the same time also investing some of their portfolio in liquidity. Taking this equilibrium asset price into account, it is easy to see that both  $(IC_A)$  and  $(PC)$  hold for any deposit contract with  $d_1 > 1$  that provides some liquidity insurance, i.e.  $d_2/d_1 < R$ .

Assuming a symmetric equilibrium in which all banks hold the same amount of assets in their trading book, we can derive from the no-arbitrage condition  $p_n = 1$  the market clearing condition:

$$k = d_1(1 - \pi)(1 - i) \quad (MC_n)$$

Given the no-arbitrage condition, we can simplify the budget constraints to:

$$1 \geq (1 - (1 - \pi)i)d_1 + (1 - \pi)id_2/R \quad (BC)$$

Consequently, as long as the costs of increasing the short-term repayments in terms of forgone long-term repayment are lower than the marginal rate of substitution between short and long-term repayment for naive households, the bank will choose the maximum incentive compatible with short-term repayment: whenever the budget constraint is flatter than the indifference curve, the bank will choose  $d_1$  such that  $(IC_B)$  holds with equality, i.e.

$$x_1 > \frac{1 - (1 - \pi)i}{\pi i} \cdot R$$

and we have  $d_1 = d_2$ . Reinserting in the budget constraint allows us to derive

$$d^* = d_1 = d_2 = \frac{R}{R - (1 - \pi)i(R - 1)} \quad (5)$$

Thus we have the following proposition:

**Proposition 1** (Bank-dominated financial system). *If the fraction of naive households  $i$  is higher than the threshold level  $\hat{i}$  with*

$$\hat{i} = \frac{R}{\pi x_1 + (1 - \pi)R}$$

*banks offer the same short and long-term repayment  $d^*$  on deposits. While this contract allows naive depositors not only to benefit from the long-term productive investment, it also provides them with a maximum liquidity insurance. While sophisticated households initially deposit their funds with the bank, too, they withdraw their deposits irrespective of whether or not they are patient or impatient. Patient sophisticated households reinvest the*

proceeds in the financial market in  $t = 1$  buying assets from the banks at the arbitrage-free price  $p_n = 1$ .

If, however, the fraction of naive households is small such that

$$i < \frac{R}{\pi x_1 + (1 - \pi)R}$$

the cross-subsidization of sophisticated households becomes too costly for naive depositors. In this case, banks offer a deposit contract

$$\{d_1; d_2\} = \{1; R\}$$

which is only attractive for naive households. While this contract allows naive households to benefit from the productive investment, it does not provide any insurance against liquidity risks. Banks (and sophisticated households) are again indifferent between holding assets or liquidity at the arbitrage-free price  $p_n = 1$ . Thus sophisticated households do not fare better investing directly than holding deposits initially. Thus sophisticated households invest directly in liquidity and assets and rebalance their portfolio according to their consumption preference shock. In this case, banks only hold assets to refinance the repayment to patient naive depositors. They do not hold assets to sell them in the financial market.

**Proposition 2** (Market-oriented financial system). *If the fraction of naive households  $i$  is smaller than the threshold level  $\hat{i}$ , banks only provide efficient access for naive households to the long-term investment opportunity. Banks do not sell assets in the financial market. Both naive as well as sophisticated households retain considerable liquidity risk.*

Now consider in this benchmark case the effects of a run on one bank. In a bank-dominated financial system with  $i > \hat{i}$ , the bank affected by the run will not only sell  $k$  assets, this bank is also forced to fire sell its remaining  $(1 - l - k)$  assets. Thus per-capita repayment to depositors is then given by

$$d_c = l + p_c(1 - l) \tag{6}$$

where  $p_c$  represents the fire-sale asset price in the market.

Given that also in a bank-run (i.e. crisis) situation, patient sophisticated depositors will use the repayment to reinvest in the financial market, the market clearing condition would be

$$(d_c + d_1)(1 - \pi)(1 - i) = p_c[k + (1 - l)]$$

assuming that the other bank remained solvent and could still repay  $d_1$  to its patient sophisticated depositors and sell only  $k$  in the financial market. Because  $d_c < d_1$ , the liquidity that patient sophisticated depositors receive from the failing bank and that they use to demand assets in the financial market falls short of the liquidity that they would

provide to the asset market if their bank were solvent. At the same time,  $k < (1 - l)$ . Consequently, due to the fire sales of the failing banks, asset supply increases while, at the same time, asset demand is being reduced. Thus in the zero probability event of a crisis, the asset price drops to  $p_c < 1$  in a bank-dominated financial system. But from  $(MC_n)$  it immediately follows that for  $p_c < 1$  the other bank does not receive sufficient liquidity from asset sales out of its trading book. Since all of the remaining assets are needed to refinance the repayment to patient naive households, the bank cannot sell additional assets to increase the liquidity inflow. Thus an asset price drop due to one bank's fire sales cannot be sustained by the other bank and will always lead to contagion in a bank-dominated financial system. In other words, given a low probability for a financial crisis in a bank-dominated financial system the banks in both regions are connected via the asset market. Contagion, then, happens because without holding liquidity buffers – which was found earlier – the ability of both banks to repay depositors depends on the market value of the respective (illiquid) assets portfolio. This market value, however, drops as the market price deteriorates.

In a market-oriented financial system with  $i \leq \hat{i}$ , banks do not rely on liquidity inflow from the financial market. If a run hits one bank and forces it into fire sales, any detrimental effect on asset prices of these fire sales will not destabilize the other bank.

**Proposition 3** (Stability). *If the run on one bank is a zero probability event, this run with subsequent fire sales of assets will lead to an asset price deterioration. In a bank-dominated financial system, the asset price deterioration is unsustainable for the other bank and will inevitably lead to contagion. In a market-oriented financial system, the asset price drop does not affect other banks.*

Finally, consider the constrained efficient solution in this setting. The social planner that can shut down financial markets but cannot observe the type of an individual household solves the following problem.

$$(P^{sp}) \begin{cases} \max_{l,k} & E[U] = \pi x_1 c_1 + (1 - \pi) c_2 \\ \text{s.t.} & c_1 \leq c_2 \quad (IC) \\ & \pi c_1 + (1 - \pi) \frac{c_2}{R} \leq 1 \quad (BC) \end{cases}$$

He maximizes the overall expected utility of naive and sophisticated households. Taking into account only the budget constraint and the incentive constraint ensures that patient households do not withdraw early. For  $x_1 > R$ , both  $(IC)$  and  $(BC)$  are binding and the constrained efficient consumption allocation is given by

$$\{c_1^{sp}; c_2^{sp}\} = \left\{ \frac{R}{R - (1 - \pi)(R - 1)}; \frac{R}{R - (1 - \pi)(R - 1)} \right\}$$

Thus the level of risk sharing provided by banks in a bank-dominated financial system ( $i > \hat{i}$ ) is optimal:  $d_2/d_1 = c_2/c_1 = 1$ . However, the consumption level of patient and impatient naive households is lower than in the optimal allocation because of the information rent extracted by patient sophisticated investors. The difference between the

optimal consumption level and the level achieved by naive households in a bank-dominated financial system increases in the share of patient sophisticated households. Sophisticated investors bear considerable liquidity risk. Their consumption level is  $d_1$  when patient and  $Rd_1$  when impatient. Only if no investors can invest in the financial market ( $i = 0$ ) is the allocation achieved by a bank-dominated financial system optimal.

Sophisticated investors cannot extract an information rent in a market-oriented financial system ( $i \leq \hat{i}$ ). However, in such a system, neither markets nor banks provide the optimal liquidity insurance. The interest rate from  $t = 1$  to  $t = 2$  is the same in financial markets as in bank deposits. Compared to the constrained efficient allocation, banks underinvest in liquidity in both market-oriented and bank-dominated financial systems.

**Proposition 4** (Efficiency). *For  $i < 1$ , neither the allocation in a market-oriented ( $i \leq \hat{i}$ ) nor in a bank-dominated financial system ( $i > \hat{i}$ ) is constrained efficient. A market-oriented system provides inefficient liquidity insurance. In a bank-dominated financial system, naive households achieve optimal liquidity insurance but pay an information rent to patient sophisticated investors. The larger this information rent, i.e. the larger the share of patient sophisticated investors  $(1 - \pi)(1 - i)$ , the less efficient the allocation in a bank-dominated financial system.*

## 5 Asset Price Bubbles

Consider now the run on one bank as an event that occurs with a small but positive probability. In a bank-dominated financial system, a run on one bank and the resulting fire sales will always induce a liquidity shortage at the other bank unless banks hold liquidity buffers. However, as long as a run on one bank occurs with a sufficiently low probability, expected costs of holding liquidity buffers to avoid contagion, i.e. the reduced repayment on deposits in a no-crisis state, overcompensate the expected benefits from being able to sustain the asset price drop following fire sales of the other bank. Thus for a sufficiently low  $\theta$ , the possibility of a liquidity crisis will only affect asset prices. The resulting price volatility, however, turns out to be more extensive than just an asset price drop in times of crisis. In normal times, the asset price can be shown to include a liquidity risk premium in order to compensate banks for the liquidity risk they incur, i.e. for the expected costs of contagion through financial markets. As a result, the present situation shows that asset price bubbles accrue when banks and households consider a small but strictly positive crisis probability.

Let  $\bar{\theta}$  denote some threshold crisis probability up to which a run on one bank is sufficiently unlikely ( $\theta \leq \bar{\theta}$ ). Then, banks can still provide some consumption smoothing for naive households  $R > d_2 > d_1 > 1$  (i.e. the fraction of naive households is again sufficiently high  $i \geq \bar{i}$ ); banks offer a deposit contract that solves the optimization problem (P2).

$$(P2) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = (1 - \theta) [\pi i x_1 d_1 + (1 - \pi) i d_2] + \theta [\pi i x_1 d_c + (1 - \pi) i d_c] \\ \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\ d_1 \leq d_2 \quad (IC_B) \\ d_1 \leq \frac{l + p_n k}{1 - (1 - \pi)^i} \quad (BC_1) \\ d_2 \leq \frac{(1 - l - k)}{(1 - \pi)^i} \cdot R \quad (BC_2) \\ \text{s.t.} \quad d_c \leq (1 - l) p_c + l \quad (BC_c) \\ (1 - \theta) \left[ (1 - \pi) \frac{R}{p_n} d_1 + \pi x_1 d_1 \right] + \theta \left[ (1 - \pi) \frac{R}{p_c} d_c + \pi x_1 d_c \right] \quad (PC) \\ > \max \left\{ (1 - \pi) \left[ \theta \frac{R}{p_c} + (1 - \theta) \frac{R}{p_n} \right] + \pi x_1, \right. \\ \left. \pi x_1 [\theta p_c + (1 - \theta) p_n] + (1 - \pi) R \right\} \end{array} \right.$$

When designing the optimal deposit contract, banks must also take into account the amount they can repay in a crisis if such an event has a positive probability. Due to their financial market activity, in a bank-dominated financial system not only the bank that directly suffers from a run is forced to liquidate its entire portfolio in the market, the other bank will be liquidated too because of a liquidity shortage given that it does not maintain a liquidity buffer. Consequently, in the depositors' expected utility function that banks maximize, we only need to consider banks' repayment  $d_1$  and  $d_2$  to patient and impatient depositors when there is no crisis as well as the per-capita liquidation return  $d_c$  that both banks can distribute in the crisis.

While the incentive compatibility constraints for naive and sophisticated households ( $(IC_A)$  and  $(IC_B)$ ) and the budget constraints for early and late repayment ( $(BC_1)$  and  $(BC_2)$ ) remain unchanged, we also need to take into account the budget constraint  $(BC_c)$  for the crisis situation. This constraint simply states that the repayment per capita after liquidation equals at most the entire liquidation proceeds whereby all assets are sold off in the financial market at the crisis price  $p_c$ .

Finally, in contrast to the no-crisis case, the participation constraint of sophisticated depositors  $(PC)$  must now take into account that prices in the asset market and the repayment of bank both vary depending on the different states that can occur. Thus it is only preferable for sophisticated depositors to initially invest in deposits if the expected payoff they can realize by withdrawing and consuming if impatient or reinvesting in financial markets if patient is larger than the payoff they realize by investing either only in liquidity or assets and trade in the financial market in  $t = 1$  according to their realized consumption preferences.

In addition it is worth mentioning that from the budget and participation constraints above, one can observe in which way a bank's inside liquidity affects the amount of sophisticated investors' outside liquidity. In a no-crisis situation, a high market price  $p_n$  supports banks' inside liquidity which, in turn, maintains high repayments  $d_1 > d_c$ . As a result, in a no-crisis situation sophisticated households dispose of plenty of liquidity which can be reinvested in the asset market and keeps asset prices high. In contrast, in times of crisis, the low market price  $p_c$  reduces sophisticated investors' funds due to low repayment on deposit contracts  $d_c < d_1$ . This, in turn, limits market liquidity and puts

further strain on market prices.

Therefore, supplementary to the optimal deposit contract solving (P2), the equilibrium with a bank-dominated financial system and an infrequent crisis is characterized by the market clearing condition for the asset market in the good and in the bad state. In the no-crisis state, the market value of a bank's trading portfolio must be equal to the withdrawals of patient sophisticated households who reinvest in financial markets.

$$p_n k = d_1(1 - \pi)(1 - i) \quad (MC_n)$$

In the crisis situation, the market value of the entire asset holding, i.e. the trading book plus the banking book, must equal the cash received by the patient sophisticated households from the liquidation of their respective bank.

$$p_c(1 - l) = d_c(1 - \pi)(1 - i) \quad (MC_c)$$

Because of the higher marginal utility of impatient depositors, depositors' expected utility in the no-crisis state is optimized with a maximum repayment on deposits in the short-run for  $p_n \geq 1$ . Taking the incentive constraints of patient naive households into account, maximum expected utility for the no-crisis period is achieved with  $d_1 = d_2$ . In the crisis state, depositors' utility is maximized with a maximum  $l$  for  $p_c \leq 1$ . Increasing the liquidity holdings beyond the amount required to implement  $d_1 = d_2$  is costly in the no-crisis state because holding such a liquidity buffer would imply that the repayment to patient depositors in the no-crisis state is inefficiently refinanced with proceeds from the storage technology rather than the long-term investment technology. Consequently, it is efficient for the bank not to increase liquidity holdings beyond what is needed to implement the optimal repayments in the no-crisis state if marginal disutility from holding a liquidity buffer in the no-crisis state is not smaller than the benefits in the crisis period:

$$(1 - \theta) i [\pi x_1 + (1 - \pi)] \frac{(R - 1)}{(1 - \pi) i + (1 - (1 - \pi) i) R} \geq \theta i [\pi x_1 + (1 - \pi) i] (1 - p_c)$$

Thus, as long as the crisis probability is lower than a threshold  $\tilde{\theta}$  with<sup>9</sup>

$$\tilde{\theta} = \frac{(R - 1)}{[R - (1 - \pi) i (R - 1)] (1 - p_c) + (R - 1)}$$

banks will not hold excess liquidity and will choose a portfolio to maximize depositors' expected utility in the no-crisis state.

Taking as given that prices  $p_n$  and  $p_c$  adjust such that banks are indifferent between holding liquidity or investing in assets, banks hold in equilibrium exactly enough assets in their trading book such that (MC<sub>n</sub>) holds. The withdrawals of all impatient depositors

<sup>9</sup> Note that this implies that  $p_c \geq 1 - \frac{(1-\theta)}{\theta} \frac{(R-1)}{[R-(1-\pi)i(R-1)]}$ . If  $p_c$  drops below this threshold, banks would find it beneficial to only invest in liquidity. However, if banks only invest in liquidity, a bank-dominated financial system does not emerge and banks are redundant.

must be financed with liquidity holdings and the repayment to patient naive households, who only withdraw their deposits in  $t = 2$ , will be financed out of the banking book, i.e. assets held until maturity. Since  $(IC_b)$  is the binding constant, it will hold with equality. Thus given  $\theta \leq \tilde{\theta}$ , the optimal repayment in no-crisis situations is given by the general budget constraint:

$$(1 - i)(1 - \pi)d^*/p_n + \pi d^* + i(1 - \pi)d^*/R = 1$$

Consequently, the optimal deposit contract is given by:

$$d^{**} = d_1 = d_2 = \frac{p_n R}{(1 - \pi)[(1 - i)R + ip_n] + \pi p_n R} \quad (7)$$

and banks' liquidity holding is

$$l^{**} = \frac{\pi p_n R}{(1 - \pi)[(1 - i)R + ip_n] + \pi p_n R} \quad (8)$$

Obviously, both banks' liquidity holdings as well as their repayments in no-crisis times increase in the asset price in no-crisis states.

Inserting (8) and  $(BC_c)$  from  $(P2)$  into the market clearing condition for the crisis period  $(MC_c)$  gives the following cash-in-the-market equilibrium condition for the asset price in the crisis period<sup>10</sup>

$$p_c = \frac{\pi p_n R}{[(1 - i)R + ip_n]} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))} \quad (\text{CMP})$$

which implies that the asset price in the crisis state increases the price in normal times:

$$\frac{\partial p_c}{\partial p_n} = \frac{\pi}{(1 - (1 - \pi)(1 - i))} \cdot \left( \frac{R(1 - i)}{[(1 - i)R + ip_n]} \right)^2 > 0 \quad (9)$$

The intuition is that the larger the price in the no-crisis state, the larger the general repayment that banks can afford in no-crisis times. To fund the higher repayment for impatient households, banks hold somewhat more liquidity. In the crisis state, a larger liquidity holding reduces the amount of assets thrown on the market and reduces asset price deterioration during the banking crisis.<sup>11</sup>

$$d_c^{**} = \frac{\pi p_n R}{(1 - \pi)[(1 - i)R + ip_n] + \pi p_n R} \cdot \frac{1}{1 - (1 - \pi)(1 - i)} \quad (10)$$

The condition that we have not considered so far but that is required to close the model

<sup>10</sup> See Appendix A for details.

<sup>11</sup> See Appendix A for details.

is the no-arbitrage condition. An equilibrium combination of asset prices in normal and crisis times requires that banks are ex ante indifferent between investing in assets or holding liquidity.

If the bank only held liquidity, it could repay all early withdrawing depositors, both impatient and patient sophisticated ones, with liquidity and use liquidity to buy assets at the no-crisis price  $p_n$  to finance the repayments to impatient naive depositors. Following that strategy, the bank could pay depositors in  $t = 1$  and  $t = 2$ :<sup>12</sup>

$$d = \frac{R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n}$$

Since a bank holding only liquidity could repay  $d_c = 1$  in the crisis, the expected utility a bank following that strategy could provide to naive households is given by

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n} + \theta [\pi x_1 + (1 - \pi)] i \quad (11)$$

A bank that only invests in assets and sells some of them off in  $t = 1$  to finance the short-term repayments would be able to pay

$$d = \frac{R}{(1 - (1 - \pi)i) \frac{R}{p_n} + (1 - \pi)i}$$

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price  $p_c$ , expected utility of naive households depositing at a bank that only invests in asset amounts to:

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{R}{(1 - (1 - \pi)i) \frac{R}{p_n} + (1 - \pi)i} + \theta [\pi x_1 + (1 - \pi)] ip_c \quad (12)$$

Thus from the equality of (11) and (12) follows that banks will be indifferent between holding liquidity and investing in assets given the following no-arbitrage condition

$$p_c = 1 - \frac{(1 - \theta)}{\theta} \left( \frac{(p_n - 1)R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n} \right) \quad (\text{NAC})$$

Following the no-arbitrage condition, the equilibrium asset price in crisis states is a decreasing function of the asset price in no-crisis states:

$$\frac{\partial p_c}{\partial p_n} = - \frac{(1 - \theta)}{\theta} \frac{(1 - (1 - \pi)i)R^2 + (1 - \pi)iR}{[(1 - (1 - \pi)i)R + (1 - \pi)ip_n]^2} < 0 \quad (13)$$

Intuitively, a higher asset price in normal times makes asset holdings more attractive. In order to ensure that banks are indifferent, the price in crisis periods must be lower.

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<sup>12</sup> See Appendix A for details.

Using **CMP** and **NAC** finally allows us to determine the equilibrium asset price in no-crisis and crisis states. From (9) it is easy to see that, according to **CMP**,  $p_c$  is a monotonically increasing concave function in  $p_n \forall p_n \in \mathbb{R}_{\geq 0}$ , while (13) indicates that  $p_c$ , according to **NAC**, is a monotonically decreasing convex function of  $p_n \forall p_n \in \mathbb{R}_{\geq 0}$ . Consequently, there is only one equilibrium combination of asset prices. Figure 1 illustrates the case.

Taking a closer look at **CMP** and **NAC**, furthermore, shows that besides fundamentals  $R$  and  $\pi$  of the model, the level of the ex-ante crisis probability  $\theta$  together with the share of naive households  $i$  in particular determine the unique equilibrium combination of asset prices  $(p_n, p_c)$ .

Consider first the role of  $\theta$  and note that **CMP** is independent of  $\theta$ . Thus, as depicted in Figure 1, **CMP** expressing  $p_c$  as a function of  $p_n$  does not change if  $\theta$  varies. **NAC**, to the contrary, is affected by a change in  $\theta$ . An increase in  $\theta$  increases the coefficient of  $p_n$  in **NAC**. Thus, in Figure 1, **NAC** expressing  $p_c$  as a function of  $p_n$  is turned to the upper right moving asset prices  $(p_{n,1}, p_{c,1})$  in crisis as well as normal states to a higher level  $(p_{n,2}, p_{c,2})$ . Thus, as depicted in Figure 1 and panel a) of Figure 3, an increase in the crisis probability ( $\theta$ ) leads to soaring prices in both the crisis as well as the no-crisis state.

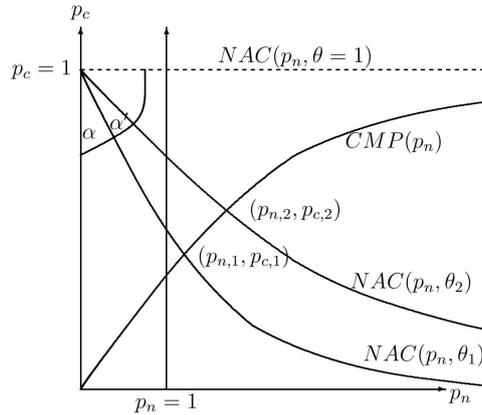


Figure 1: Impact of a change in  $\theta$  on equilibrium prices

Intuitively, for a given asset price in crisis times, an increase in the crisis probability increases the required liquidity premium. Thus the asset price in normal times must rise (shift of **NAC**). However, a shift of the asset price in normal times improves banks' ability to meet the asset demand of patient sophisticated depositors. This, in turn, increases overall repayments on deposits and banks' liquidity holdings. As a consequence, the asset price in crisis periods drops less sharply than in the case of less frequent crises.

**Proposition 5** (Asset Price Bubbles). *For  $\theta \leq \min\{\bar{\theta}, \tilde{\theta}\}$  there is only one equilibrium related to one combination of asset prices  $(p_n, p_c)$ . The larger  $\theta$  in the considered interval, the larger are both  $p_n$  and  $p_c$ .*

**Proof 1.** *See Appendix B.*

In sum, in a bank-dominated financial system, a higher probability of a run on one bank with associated fire sales and contagion of the other bank leads to overall higher asset

prices in normal times due to a higher required liquidity premium. A higher level of  $\theta$  and, hence, a higher asset price increases the overall value of liquidity which, in turn, induces banks to hold more liquidity. Thus more can be paid for assets both in the crisis as well as in the no-crisis state.

From (7) and (10) it is straight forward to see that a higher asset price in the no-crisis state increases the repayment investors receive. The intuition is that with a higher asset price, the information rent that patient sophisticated investors can extract is lower. Therefore, the repayment that banks can provide to depositors is higher. Banks have invested fewer resources ex ante in assets that are only held to be sold to patient sophisticated investors in the market. Thus the liquidity insurance provided by the banking sector in the no-crisis state becomes more effective and approaches the constant efficient allocation. Thus the threat of a crisis with the fostered incentives to withhold liquidity improves efficiency of the deposit contract and the allocation achieved if banks are stable.

The proposition also sheds some light on the role of banks' liquidity transformation in the accrual and strength of asset price bubbles. Banks withhold reserves, i.e. *outside liquidity*, to repay impatient depositors. But they also create *inside liquidity*. They issue claims, deposits, that can be used as a medium of exchange. In particular, patient sophisticated depositors use their deposits to pay for the assets they buy from banks in the normal state. However, in a banking crisis inside liquidity loses its value and claims against banks are only worth the reserves held by banks to back their deposits. Thus while in normal times sophisticated investors can use the full value of their deposits to buy assets, in crisis states their ability to absorb fire sales of assets is determined by the outside liquidity of the banking sector. When there is no crisis, there is ample liquidity in the system maintaining asset prices at very high levels. In a crisis situation, declining asset prices reduce the inside liquidity in the banking system and scarcity of liquid funds deteriorates asset prices even further. This downward spiral leads to financial contagion and financial instability.

The impact of  $i$  on the equilibrium combination of asset prices, however, is less straightforward. Because both CMP and NAC depend on  $i$ , a change in the share of naive households affects both functions. Nevertheless, it is still straightforward to see that  $p_c$  decreases with an increase of naive households. First, observe that NAC as well as CMP decrease with  $i$ .

$$\begin{aligned}\frac{\partial NAC}{\partial i} &= -\frac{(1-\theta)}{\theta} \left( \frac{(1-\pi)(R-p_n)(p_n-1)R}{((1-(1-\pi)i)R + (1-\pi)ip_n)^2} \right) < 0 \\ \frac{\partial CMP}{\partial i} &= \frac{(\pi p_n R)(R-p_n)}{[(1-i)R + ip_n]^2} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} \\ &\quad - \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{1}{(1-(1-\pi)(1-i))^2} < 0\end{aligned}$$

Since both functions decrease with  $i$ , the equilibrium asset price  $p_c$  has to decrease as well. The impact of  $i$  on  $p_n$  is ambiguous which prevents a clear conclusion about the relationship between  $i$  and the equilibrium combination of asset prices  $(p_n, p_c)$ . In [Figure 2](#), we plot the three possible consequences of change in the fraction of naive households on the

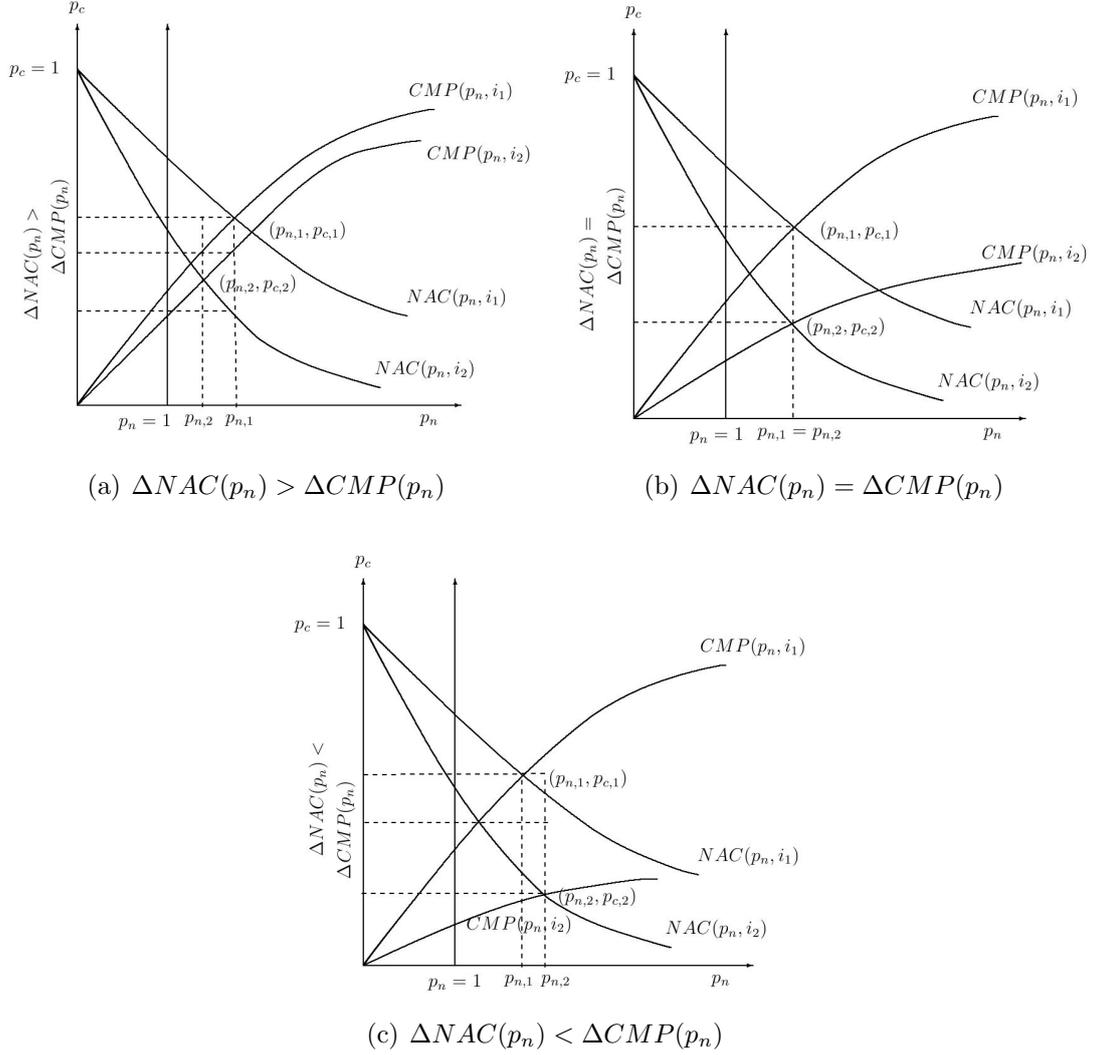


Figure 2: Comparative statics of equilibrium asset prices with respect to  $i$

equilibrium asset prices. In each plot we increased the fraction of naive households from  $i_1$  to  $i_2$ . The single difference between these plots is the impact of  $\Delta i$  on NAC and CMP, where we define  $\Delta NAC(p_n) = \text{abs}(NAC(p_{n,1}, i_2) - NAC(p_{n,1}, i_1))$  and  $\Delta CMP(p_n) = \text{abs}(CMP(p_{n,1}, i_2) - CMP(p_{n,1}, i_1))$ . In panel (a) of Figure 2, the impact of an increase in  $i$  on NAC is larger in comparison to the impact on CMP,  $\Delta NAC(p_n) > \Delta CMP(p_n)$ , while in panel (c) the opposite is true,  $\Delta NAC(p_n) < \Delta CMP(p_n)$ . Finally, panel (b) illustrates the case  $\Delta NAC(p_n) = \Delta CMP(p_n)$ . In each case,  $p_{c,2}$  is smaller than  $p_{c,1}$  supporting the conclusion that  $p_c$  is a decreasing function with respect to  $i$ . In sharp contrast, the effect on  $p_n$  critically depends on difference  $h(p_n) := \Delta NAC(p_n) - \Delta CMP(p_n)$ . If  $h(p_n) > 0$ ,  $p_{n,1} > p_{n,2}$ ; hence  $p_n$  is a decreasing function of the fraction of naive households. This situation is depicted in panel (a) of Figure 2. If the opposite is true and  $h(p_n) < 0$ ,  $p_n$  increases with  $i$ , which is shown in panel (c). Finally, if  $h(p_n) = 0$ ,  $p_n$  is not affected by an increase in the fraction of naive households (panel b). This brings the following

conclusion:<sup>13</sup>

$$\frac{\partial p_n}{\partial i} = \begin{cases} > 0 & \text{if } \frac{\partial NAC(p_n)}{\partial i} > \frac{\partial CMP(p_n)}{\partial i} \\ < 0 & \text{if } \frac{\partial NAC(p_n)}{\partial i} < \frac{\partial CMP(p_n)}{\partial i} \\ = 0 & \text{if } \frac{\partial NAC(p_n)}{\partial i} = \frac{\partial CMP(p_n)}{\partial i} \end{cases}$$

The sign of  $\frac{\partial p_n}{\partial i}$  is determined by the fraction of impatient households that determine  $\left[\frac{\partial NAC}{\partial i}\right]_{\pi=0} < 0$  while  $\left[\frac{\partial CMP}{\partial i}\right]_{\pi=0} = 0$ , hence  $\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=0} < 0$ . On the contrary,  $\left[\frac{\partial NAC}{\partial i}\right]_{\pi=1} = 0$  while  $\left[\frac{\partial CMP}{\partial i}\right]_{\pi=1} < 0$ , hence  $\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=1} > 0$ . Consequently, the total effect of  $i$  on  $p_n$  depends critically on the fraction of early consuming households  $\pi$ . This is shown graphically in panel (b) and (c) of Figure 3. Panel (b) shows a financial system with a relatively small fraction of impatient households,  $\pi = 0.2$ . The same financial system is depicted in panel (c), yet with a high fraction of impatient households  $\pi = 0.7$ . It turns out that in the first financial system  $p_n$  is not a monotonic increasing function with respect to  $i$ , while in the second financial system  $p_n$  is a strictly increasing function.

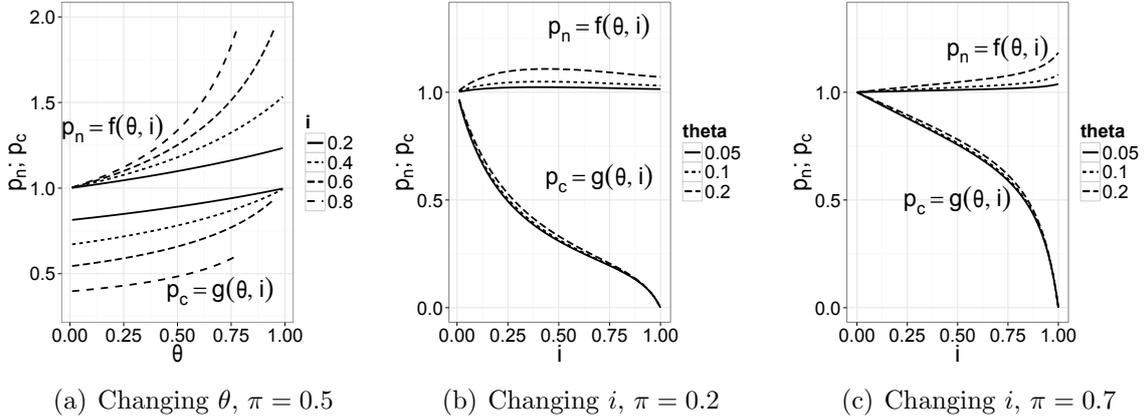


Figure 3: Comparative statics of equilibrium asset prices

**Proposition 6** (Asset Price Bubbles). *For  $\theta \leq \min\{\bar{\theta}, \tilde{\theta}\}$  the unique equilibrium asset price in crisis  $p_c$  declines in  $i$ , while a change in  $i$  has an ambiguous effect on  $p_n$ . For a sufficiently high ratio of impatient households the asset price in normal times  $p_n$  also declines.*

**Proof 2.** See Appendix B.

The intuition for this result is somewhat subtle. For a given  $p_n$ , an increase in the fraction of naive households decreases financial market depth, exacerbates the cash-in-the-market constraint and contributes to a lower asset price during a crisis (CMT turns to the lower right). This effect is stronger the larger the ratio of patient households. At the same time, for a given asset price in normal times  $R > p_n > 1$  it is less preferable for banks to invest in assets and sell them off in financial markets to acquire the liquidity needed to repay

<sup>13</sup> For formal proof, see Appendix B.

patient sophisticated and impatient depositors the smaller their share of those households in the population. Since the smaller their share, the relative arbitrage profit they can make with this strategy compared to investing in liquidity in  $t = 0$  is smaller. Therefore, the smaller the share of (patient) naive households, the more banks could increase the repayment on deposits in normal times. Thus for a given  $p_n$ , the incentives to invest only in assets decrease in  $i$ . To compensate for this, the price drop in crisis periods must be smaller (NAC turns to the upper right).

## 6 Conclusion

In this paper we endogenize the liquidity risk sharing provided by the banking sector and the ratio of outside (reserves) to inside liquidity (claims against the banking sector) held in the financial system. Outside liquidity held by the financial system is a key determinant of sophisticated investors' (arbitrageurs or speculators) ability to absorb fire sales of run-prone financial institutions. At the same time, sophisticated investors' ability to buy assets in financial markets affects banks' risk sharing and their holdings of inside relative to outside liquidity. Thus asset prices in crisis and no-crisis states are jointly determined by a cash-in-the-market pricing and a no-arbitrage condition.

This set-up allows us to show that a higher probability of a run on one bank increases the asset prices in the crisis as well as in the normal state. This result is driven by the interaction of the cash-in-the-market pricing and the no-arbitrage condition: a higher crisis probability increases the probability that a bank has to sell off its assets at a fire-sale price. Thus to be compensated for these losses, the no-arbitrage condition requires a higher no-crisis price for the assets – a liquidity premium. But a higher asset price in normal times improves banks' ability to provide liquidity insurance. Thus they hold more outside liquidity, which fosters the ability of sophisticated investors (arbitrageurs) to absorb fire sales. The cash-in-the-market pricing is alleviated.

Better access to financial markets increases market depth. Consequently, the price in the crisis state increases with the fraction of sophisticated households. This reduces the liquidity premium charged in the normal state. However, more liquid financial markets also foster banks' incentives to sell off asset holdings in financial markets in order to gather liquidity instead of holding sufficient liquidity *ex ante*. Thus the no-arbitrage condition requires that prices in the normal state increase. Hence improved financial market access has an ambiguous effect on the price in the normal state.

Our results shed some light on the potential value of regulatory liquidity requirements – although not explicitly considered in the model. In our model liquidity regulation, which defines some minimum liquidity buffer, may be expected to strengthen financial stability at the cost of a less efficient allocation of funds in normal times. A sufficiently large mandatory liquidity buffer might enable banks that are not affected by a run to sustain asset price drops resulting from fire sales of other banks. Moreover, liquidity requirements will alleviate the cash-in-the-market constraint. At the same time, though, a higher liquidity requirement directly impairs the efficiency of banks' liquidity insurance. Moreover, a lower liquidity premium will lead to a lower asset price in normal times which

further deteriorates the efficiency of banks' liquidity transformation.

Moreover, central bank interventions, such as the *Bernanke Put* in the financial crisis of 2007-2009, help to stabilize the financial system by mitigating asset price drops. Conditional liquidity injections will relax cash-in-the-market constraints. Hence asset prices in the crisis state will increase and prices in normal times will decline due to a lower liquidity premium. This, however, again impairs banks' ability to provide efficient risk sharing and thus bears some efficiency losses.

## Appendix A

### Equilibrium conditions given infrequent crisis

**Deriving the crisis price:** Inserting 8 and  $(BC_c)$  from  $(P2)$  in the market clearing condition for the crisis period  $(MC_c)$  yields:

$$\begin{aligned} p_c(1-l) &= [(1-l)p_c + l](1-\pi)(1-i) \\ p_c(1-l)(1-(1-\pi)(1-i)) &= l(1-\pi)(1-i) \\ p_c &= \frac{l}{(1-l)} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} \end{aligned} \quad (14)$$

$$\frac{l}{(1-l)} = \frac{\frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R}}{\frac{(1-\pi)[(1-i)R + ip_n] + \pi p_n R - \pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R}} = \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n]} \quad (15)$$

Inserting 15 in 14 gives

$$\begin{aligned} p_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n]} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} \\ p_c &= \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} \end{aligned}$$

**Deriving the per-capita repayment in crisis:** Inserting  $(CMP)$  and (8) in  $(BC_c)$  from  $(P2)$  yields

$$\begin{aligned} d_c &= \left( 1 - \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \right) \frac{\pi p_n R}{[(1-i)R + ip_n]} \\ &\quad \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\ d_c &= \frac{(1-\pi)[(1-i)R + ip_n]}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} + \\ &\quad + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\ d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\ d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \left( 1 + \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} \right) \\ d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{1}{1-(1-\pi)(1-i)} \end{aligned}$$

**Deriving the no-arbitrage condition:** If the bank only holds liquidity, it could repay all early withdrawing depositors – impatient and patient sophisticated ones – with liquidity and use liquidity to buy assets at the no-crisis price  $p_n$  to refinance the repayments to impatient naive depositors. As a consequence, when only holding liquidity, a bank would face the budget constraint:

$$\begin{aligned}\pi d + (1 - \pi)(1 - i)d + (1 - \pi)i\frac{p_n}{R}d &= 1 \\ \left[ \pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R} \right] d &= 1\end{aligned}$$

Following that strategy, the bank could pay depositors in  $t = 1$  and  $t = 2$

$$d = \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}}$$

Since holding only liquidity permits the bank to pay  $d_c = 1$  in the crisis period, the expected utility that a bank could provide to naive households would be

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} + \theta [\pi x_1 + (1 - \pi)] i$$

A bank that only invests in assets and sells off some of them in  $t = 1$  to refinance the short-term repayments would be able to repay

$$d = \frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}}$$

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price  $p_c$ , the expected utility of naive households depositing at a bank that only invests in assets amounts to:

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}} + \theta [\pi x_1 + (1 - \pi)] ip_c$$

Thus banks will be indifferent between holding liquidity and investing in assets if

$$(1 - \theta) \frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}} + \theta p_c = (1 - \theta) \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} + \theta$$

$$p_c = \frac{(1 - \theta)}{\theta} \left( \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} - \frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}} \right) + 1$$

$$p_c = \frac{(1 - \theta)}{\theta} \left( \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} - \frac{p_n}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} \right) + 1$$

$$p_c = 1 - \frac{(1 - \theta)}{\theta} \left( \frac{p_n - 1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} \right)$$

## Appendix B

### Partial differentials of CMP and NAC

#### CMP

$$CMP := p_c = \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} \quad (\text{CMP})$$

This gives us the following partial derivatives:

$$\begin{aligned} \frac{\partial CMP}{\partial p_n} &= \frac{\pi}{(1-(1-\pi)(1-i))} \cdot \left( \frac{R(1-i)}{[(1-i)R + ip_n]} \right)^2 > 0 \\ \frac{\partial CMP}{\partial \theta} &= 0 \\ \frac{\partial CMP}{\partial i} &= \frac{(\pi p_n R)(R - p_n)}{[(1-i)R + ip_n]^2} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} \\ &\quad - \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{1}{(1-(1-\pi)(1-i))^2} < 0 \end{aligned}$$

#### NAC

$$NAC := p_c = 1 - \frac{(1-\theta)}{\theta} \left( \frac{(p_n - 1)R}{(1-(1-\pi)i)R + (1-\pi)ip_n} \right) \quad (\text{NAC})$$

This gives us the following partial derivatives:

$$\begin{aligned} \frac{\partial NAC}{\partial p_n} &= -\frac{(1-\theta)}{\theta} \frac{(1-(1-\pi)i)R^2 + (1-\pi)iR}{[(1-(1-\pi)i)R + (1-\pi)ip_n]^2} < 0 \\ \frac{\partial NAC}{\partial \theta} &= \frac{1}{\theta^2} \left( \frac{(p_n - 1)R}{(1-(1-\pi)i)R + (1-\pi)ip_n} \right) > 0 \\ \frac{\partial NAC}{\partial i} &= -\frac{(1-\theta)}{\theta} \left( \frac{(1-\pi)(R - p_n)(p_n - 1)R}{((1-(1-\pi)i)R + (1-\pi)ip_n)^2} \right) < 0 \end{aligned}$$

Note that  $\frac{\partial NAC}{\partial i} < 0$  since  $p_n \leq R$ , otherwise nobody would buy assets in  $t = 1$  and we would end up with a different financial system (i.e. banks hold excess liquidity).

#### Summary

$$\begin{aligned} \frac{\partial CMP}{\partial p_n} &> 0; \quad \frac{\partial CMP}{\partial \theta} = 0; \quad \frac{\partial CMP}{\partial i} < 0; \\ \frac{\partial NAC}{\partial p_n} &< 0; \quad \frac{\partial NAC}{\partial \theta} > 0; \quad \frac{\partial NAC}{\partial i} < 0 \end{aligned}$$

## Comparative static analysis $\theta$

Showing that  $p_n$  is increasing with  $\theta$

Total differential:

$$\begin{aligned} dp_c = dNAC &= \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta \\ dp_c = dCMP &= \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta \end{aligned}$$

In equilibrium  $dNAC = dCMP$ ; hence:

$$\frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta$$

Solving for  $dp_n$  and setting  $\frac{\partial CMP}{\partial \theta} = 0$  leads to:

$$dp_n = \frac{\frac{\partial NAC}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} \cdot d\theta$$

Since

$$\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n} > 0$$

and

$$\frac{\partial NAC}{\partial \theta} > 0$$

we have

$$\frac{\frac{\partial NAC}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} > 0.$$

Consequently  $p_n$  increases with the crisis probability  $\theta$ .

Showing that  $p_c$  is increasing with  $\theta$

Total differential:

$$\begin{aligned} dp_c = dNAC &= \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta \\ dp_c = dCMP &= \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta \end{aligned}$$

Setting  $dp_n = dp_n$ , replacing  $\frac{\partial CMP}{\partial \theta} = 0$  and solving for  $dp_c$  leads to:

$$dp_c = \frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} \cdot d\theta$$

It is again straightforward to see that

$$\frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} > 0,$$

hence  $p_c$  increases with the crisis probability  $\theta$ .

## Comparative static analysis $i$

Showing that  $p_c$  is decreasing with  $i$

Total differential:

$$\begin{aligned} dp_c = dNAC &= \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial i} \cdot di \\ dp_c = dCMP &= \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial i} \cdot di \end{aligned}$$

Gives the solution:

$$dp_c = \frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} \cdot di$$

Again the denominator is positive and  $\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i}$  is negative. Obviously  $-\frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i} < 0$  and  $\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} < 0$  since  $\frac{\partial NAC}{\partial i} < 0$  and  $\frac{\partial CMP}{\partial p_n} > 0$ . Thus we conclude  $p_c$  is decreasing with  $i$ .

## Ambiguous effect of $i$ on $p_n$

Total differential:

$$\begin{aligned} dp_c = dNAC &= \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial i} \cdot di \\ dp_c = dCMP &= \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial i} \cdot di \end{aligned}$$

Solving for  $p_n$ :

$$dp_n = \frac{\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} \cdot di$$

The denominator is positive, but  $-\frac{\partial CMP}{\partial i} > 0$  and  $\frac{\partial NAC}{\partial i} < 0$ . Thus the sign of the numerator is ambiguous which translates to the total effect of  $i$  on  $p_n$ . Further note that  $\left[\frac{\partial NAC}{\partial i}\right]_{\pi=0} < 0$  while  $\left[\frac{\partial CMP}{\partial i}\right]_{\pi=0} = 0$ , hence  $\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=0} < 0$ . On the contrary,  $\left[\frac{\partial NAC}{\partial i}\right]_{\pi=1} = 0$  while  $\left[\frac{\partial CMP}{\partial i}\right]_{\pi=1} < 0$ , hence  $\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=1} > 0$ . Consequently, the total effect of  $i$  on  $p_n$  depends critically on the fraction of early consuming households  $\pi$ .

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