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Imperfect information about financial frictions and consequences for the business cycle

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Non-technical summary

Research Question

Financial linkages between savers and borrowers are exposed to agency problems which may arise for several reasons. Imperfect information about an investment project can cause moral hazard behavior of borrowers if lenders do not have sufficient information about them. Our aim is to introduce imperfect information in a contract related to a limited enforcement problem. The way agents process and update new information is key for the optimizing behavior of the individual and hence for aggregate macroeconomic variables. In this paper we ask what the consequences for the business cycle are when agents have to learn about the behavior of banks.

Contribution

The idea of our paper is to implement imperfect information into the banking sector of an otherwise standard New-Keynesian model, in which limited enforcement creates an agency problem. In our setting, economic agents learn about the size of changes in the diverting behavior of bankers. Everything else in the economy, i.e. both the structure and the parameter values, is known. Then we contrast the learning approach to full information rational expectations and analyze their respective roles for the business cycle.

Results

For the period during which agents learn about the economy, the whole economy exhibits higher volatility and different outcome paths for macroeconomic variables compared with rational expectations. In particular, the introduction of imperfect information amplifies the responses of variables as the leverage ratio in the economy is higher for a longer period. Output is also higher due to a boost in investment before it undershoots the rational expectations benchmark outcomes. This goes hand in hand with an increase in uncertainty and higher volatility of all macro-variables. Compared with rational expectations output, investment and the leverage ratio display an increase in their respective volatility of between 1% and 8%. Output becomes even slightly more persistent in the learning case.

Nichttechnische Zusammenfassung

Fragestellung

Die Geschäftsbeziehungen zwischen Banken und ihren Finanzmittelgebern sind aus unterschiedlichen Gründen mit einem Prinzipal-Agenten-Problem behaftet. Sowohl ein aus unvollständigen Informationen heraus resultierendes moralisches Wagnis als auch eine beschränkte Vollstreckung können dafür verantwortlich sein. Die Art und Weise wie Wirtschaftssubjekte neue Informationen zudem aufnehmen und verarbeiten ist entscheidend für das Optimierungsverhalten des Einzelnen und damit auch für die makroökonomischen Variablen im Ganzen. In diesem Papier stellen wir die Frage, welche Folgen das Lernen der Wirtschaftssubjekte über das Verhalten der Banken für den Konjunkturzyklus hat, wenn ein Problem der beschränkten Vollstreckung besteht.

Beitrag

Der Beitrag des Papiers ist es, unvollständige Information im Bankensektor in einem ansonsten Standard Neu-Keynesianischen Modell zu integrieren, in dem beschränkte Vollstreckung ein Prinzipal-Agenten Problem erzeugt. In unserem Szenario lernen Haushalte über das Maß der Veränderung im Fehlverhalten der Banken. Alles andere in der Volkswirtschaft, das heißt, sowohl die Struktur als auch die sonstigen Parameterwerte sind den Wirtschaftsagenten bekannt. Daraufhin kontrastieren wir den Ansatz, in dem die Wirtschaftssubjekte lernen, mit vollständig rationalen Erwartungen und analysieren ihren jeweiligen Einfluss auf den Konjunkturzyklus.

Ergebnisse

Im Zeitraum, in dem die Wirtschaftssubjekte über den Zustand der Ökonomie lernen, zeigt die Volkswirtschaft insgesamt eine höhere Volatilität und verglichen mit rationalen Erwartungen unterschiedliche Simulationspfade für die makroökonomischen Variablen. Im besonderen amplifiziert die Einführung von unvollständigen Informationen die Impulsantworten der Variablen, da die Verschuldungsquote länger höher ist. Das Bruttoinlandsprodukt ist anfangs ebenfalls höher wegen eines Anstiegs der Investitionen bevor es unter die Referenzgröße mit rationalen Erwartungen fällt. Dies geht einher mit einer höheren Unsicherheit und Volatilität in der Ökonomie. Verglichen mit rationalen Erwartungen zeigen das BIP, die Investitionen und die Verschuldungsquote einen Anstieg in ihrer jeweiligen Volatilität zwischen 1% and 8%. Das BIP wird auch etwas persistenter, wenn die Haushalte lernen.

Bundesbank Discussion Paper No 07/2015

Imperfect Information about Financial Frictions and Consequences for the Business Cycle*

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Abstract

In this paper, we discuss the consequences of imperfect information about financial frictions on the macroeconomy. We rely on a New Keynesian DSGE model with a banking sector in which we introduce imperfect information about a limited enforcement problem. Bank managers divert resources and can increase the share of diversion. This can only be observed imperfectly by depositors. The ensuing imperfect information generates a higher volatility of the business cycle. Spillovers from the financial sector to the real economy are higher and shocks in general are considerably amplified in the transition period until agents' learning is complete. Volatility and second-order moments also display an amplification under the learning setup compared with the rational expectations framework.

Keywords: DSGE Model, Financial Frictions, Learning

JEL classification: E3, E44, G3

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1 Introduction

Financial linkages between savers and borrowers are exposed to agency problems which may arise for several reasons. Imperfect information about the outcome of an investment project can cause moral hazard behavior of borrowers if lenders do not have sufficient information about them. The moral hazard problem can be resolved if lenders can charge borrowers with the costs of verifying the actual state of their projects (see Townsend, 1979, for instance). Thus, borrowers have no incentive to misbehave unless imperfect information prevails. If a limited enforcement problem exists, lenders usually observe the activity of borrowers but cannot prevent their misbehavior (Kiyotaki and Moore, 1997). In this case, an agency problem exists although information is perfect. Our aim is to introduce imperfect information, which results from the observed misbehavior of borrowers, in this type of contract. We observe the consequences on the macroeconomy if this misbehavior changes and agents cannot observe this change directly. Thus, we investigate imperfect information about a limited enforcement problem between depositors and bankers. Our approach is to focus on information processing beyond rational expectations and introduce imperfect information in a macro-finance model with banking on behalf of households with respect to the fraction bankers can divert.

Business cycles are driven to a considerable extent by news about expected fundamentals (Barsky and Sims, 2011; Blanchard, L'Huillier, and Lorenzoni, 2013; Jaimovich and Rebelo, 2009; Walker and Leeper, 2011).[1] The way agents process and update this new information is key for the optimizing behavior of the individual and hence for aggregate macroeconomic variables (Eusepi and Preston, 2011). The recent crisis put another view on business cycle fluctuations and exemplified even more how close the interaction is between the financial sector and the real economy, and how financial markets also affect the overall business cycle (Christiano, Motto, and Rostagno, 2014; Jerman and Quadrini, 2012). The banking sector, in particular, played an important role in amplifying shocks (Brunnermeier, 2009). Probably led by the great moderation, it has almost been forgotten that the banking sector can be a source of macroeconomic instability. Already two decades ago, Gorton (1988) presented empirical results of how banking panics and financial turmoil led to business cycle fluctuations. One of his conclusions was that "[b]anking panics [...] were systematic responses by depositors to changing perceptions of risk, based on the arrival of new information rather than random events" (Gorton, 1988, p. 752). In a more extreme form, Diamond and Dybvig (1983) find that bank runs are caused by "a shift in expectations, which could depend on almost anything, consistent with the apparently irrational observed behavior of people running banks" (p. 404). The arguments from this early literature demonstrate that agents seem to update information on the key underlying behavior of bankers and react accordingly, which entails consequences for the business cycle on a bigger scale. The concept of updating information is more appropriately described by a learning approach. Our focus is therefore to depart from imperfect information in a purely rational expectations framework and rather to compare these two different ways in which expectations are formed and what implications each of them have on the business cycle overall.

[1] News on factor productivity stood mainly at the center of this strand in the literature. However, news shocks from monetary policy can also play a role in influencing the business cycle (see, for example, Milani and Treadwell (2012)).

The way many researchers deal with questions related to the recent financial crisis is to incorporate a financial sector in standard macro models. The key challenge is to adopt both the methodology and intuition from the rich financial literature, and implement them in dynamic stochastic general equilibrium (DSGE) models (Christiano et al., 2014; Gerali, Neri, Sessa, and Signoretti, 2010; Gertler and Karadi, 2011; Meh and Moran, 2010). In this respect, the main focus is to gain a greater insight into the interlinkages between real and financial variables. One of the seminal finance papers that addresses the question of how financial crises may arise is Calomiris and Kahn (1991). Their notion of bankers being able to divert funds can lead to similar outcomes as in the famous model of Diamond and Dybvig (1983). This absconding motive of bankers can nevertheless be alleviated by depositors investing in information technology and monitoring the banker more carefully. A recent model which draws on a contracting problem in the banking sector following diverting motives of bankers and is close to these ideas is the model of Gertler and Karadi (2011). This is indicative of how essential information is in this moral hazard framework. The real side of the economy is a standard New Keynesian model. The extensions are bankers that collect deposits, give loans to firms and maximize the expected lifetime net worth. The friction and, therefore, the spread between internal and external financing arises due to the bankers' diverting motive and the ensuing moral hazard phenomenon.

The idea of our paper is to implement imperfect information into the banking sector in which limited enforcement creates an agency problem by following the spirit of the contributions cited above. A central feature of the model, which is closely linked to the model developed by Gertler and Karadi (2011), in its basic form is that it abstracts from risk considerations, while limited enforcement provides a propagation mechanism stemming from financial frictions. For our purposes, this is advantageous because we are able to focus on the effects which stem solely from imperfect information about the degree of limited enforcement without the need to consider feedback effects related to default risk.[2] In our setting, economic agents learn about the size of changes in the diverting behavior of bankers. Everything else in the economy, i.e. both the structure and the parameter values, is known. This is a form of bounded rationality where learning is restricted to one particular feature of the model, and the whole setup is therefore very close to actual rational expectations (similar to Marcet and Sargent, 1989a,b). This is also the reason why we contrast both forms of expectations formations and analyze their respective roles for the business cycle. In particular, we simulate the economic outcomes given the model and the two forms of expectations formation where, after a certain period, there is an exogenous positive shift to the fraction of diversion. In the rational expectations framework, agents immediately realize the new value of this variable and all other implied steady states, and react accordingly. An upward shift in the diversion variable implies more difficult external financing, particularly for new entering bankers. This forces old and new bankers to reduce their steady state leverage ratio and therefore affects overall dynamics in the economy. Due to the higher amount of diversion, bankers need to achieve a greater franchise value of the bank.

In the case where agents have imperfect information about the amount of the financial friction, however, it takes some time for households and firms to figure out the new regime. In those periods, agents have to adapt their expectations and base their optimizing

[2] Angeloni and Faia (2013), for instance, develop a macroeconomic model with a banking sector in which banks are subject to runs. In their setting, there is asymmetric information.

behavior on a preceived steady state fraction of diversion that is too low in relative terms. For the period during which agents learn about the economy, the whole economy exhibits higher volatility and different outcome paths for macroeconomic variables compared with rational expectations. In particular, the introduction of imperfect information amplifies the responses of variables as the leverage ratio in the economy is higher for a longer period. Output is higher for a longer period of time due to a boost in investment before it undershoots the rational expectations benchmark outcomes. This goes hand in hand with an increase in uncertainty and higher volatility of all macro-variables. The range of cross-sectional variations (as a measure for uncertainty) rises by about 25% measured in relative standard deviations. Compared with rational expectations output, investment and the leverage ratio display an increase in their respective volatility of between 1% and 8%. Output becomes even slightly more persistent in the learning case.

The paper is structured as follows: Section 2 presents the New Keynesian model and the moral hazard problem in the financial sector. Calibrated parameters that match key moments in the rational expectations benchmark model are presented in section 3. In section 4, we discuss the expectations formation of agents and how they process and update information. Our experiment is higher financial turbulence, which is explained in section 5. We present all baseline results in section 6. Section 8 concludes.

2 Model

Our model setup draws on the model of Gertler and Karadi (2011)(GK). The model belongs to the class of New Keynesian dynamic general equilibrium models in the tradition of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). A central feature of the model is the role of an active banking sector. Households cannot provide their funds directly to the goods-producing sector which is why they place deposits at banks. Banks intermediate these funds to capital-producing firms. Because of limited enforcement, there is a financial contracting problem between households and banks.

The economy consists of households, financial intermediaries (banks), capital producers, intermediate goods producers, and retailers. Banks receive funds from households and lend them to capital producers. Physical capital is used as an input for the production of intermediate goods. Intermediate goods are differentiated by retailers. These differentiated goods are then bundled to obtain the homogenous final good. Retailers have the function of introducing nominal price rigidities. A central bank conducts conventional monetary policy.

2.1 Households

The household sector is populated by a continuum of identical households with a mass of unity. Each household consists of two groups of members. Members of the first group are workers who consume, save, and supply labor to the intermediate goods sector. This group generates earnings from labor income and returns on their financial wealth. The second group is composed of bank managers who operate a bank. The share of workers is constant over time and amounts to f, from which it follows that the share of bankers is given by $1-f$. There is a transfer between both groups because it is assumed that bankers stemming from the first group return to it after being bankers with a probability of $1-\theta$

in every period. Consequently, the share of bankers who return to the group of workers is $(1-\theta) f$. While operating a bank, bank managers accumulate retained earnings to build up net worth. Only if bankers return to the group of workers do they take all remaining resources net of debt back, which then constitutes income for households. Effectively, banks are owned by households but transfers of funds from bankers to workers only take place at the end of a banker's life.

Every household h has preferences over consumption $C_{h,t}$ and labor $L_{h,t}$, and maximizes lifetime utility

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[\ln \left(C_{h,t+i} - h^C C_{h,t+i-1} \right) - \frac{\chi}{1+\varphi} L_{h,t+i}^{1+\varphi} \right] \tag{1}$$

where $\varphi > 0$ is the inverse Frisch elasticity, $\chi > 0$ is a scaling parameter, and β the time preference. Households show habit formation with $0 < h^C < 1$ and hold two financial assets which are both denominated in real terms with a maturity of one period: deposits $D_{h,t}$ and government bonds $B_{h,t}$. The period return (from $t-1$ to t) for both assets is given by the gross real return R_t. For the moment, we assume that government bonds are in zero net supply. The budget constraint can consequently be written as

$$C_{h,t} + B_{h,t} + D_{h,t} = W_t L_{h,t} + R_t \left(B_{h,t-1} + D_{h,t-1} \right) - T_{h,t} + \Pi_{h,t}, \tag{2}$$

where $T_{h,t}$ denotes lump sum taxes and W_t the real wage. The term $\Pi_{h,t}$ reflects net transfers from banks and the real sector (retailers and capital producers) to households.

Resulting from the maximization problem, we obtain the first-order condition for consumption with ϱ_t as the marginal utility of consumption

$$\varrho_t = \left(C_t - h^C C_{t-1} \right)^{-1} - \beta h^C E_t \left(C_{t+1} - h^C C_t \right)^{-1} \tag{3}$$

the first-order condition for labor

$$\varrho_t W_t = \chi L_t^{\varphi} \tag{4}$$

and the Euler equation

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \tag{5}$$

with

$$\Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t}. \tag{6}$$

Since the first-order conditions are equal for all individuals, we can drop the subscripts.

2.2 Intermediate goods firms

In a market of perfect competition, firms produce intermediate goods with the help of physical capital K_{t+1}. bought at the end of the period t, and labor by applying a Cobb-Douglas production technology

$$Y_t = A_t \left(U_t \xi_t K_t \right)^{\alpha} L_t^{1-\alpha}. \tag{7}$$

In Eq. (7), A_t controls the total factor productivity which follows an autoregressive process after realizing an i.i.d. shock ϵ_t^A

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A. \tag{8}$$

The term α is the share of utilized capital in production. While U_t denotes the capital utilization rate, the variable ξ_t reflects a shock with disturbance ϵ_t^ξ on the quality of physical capital, i.e. the so-called capital quality shock as discussed in Gertler and Karadi (2011), which follows an autoregressive process

$$\log(\xi_t) = \rho^\xi \log(\xi_{t-1}) + \epsilon_t^\xi. \tag{9}$$

Intermediate goods producers maximize their profits at time t by choosing the utilization rate and the labor input taking the price for intermediate goods P_{mt}, the real wage, and the price for capital as given. The demand for physical capital arises as

$$P_{mt}\alpha\frac{Y_t}{U_t} = \delta'(U_t)\,\xi_t K_t \tag{10}$$

and the demand for labor as

$$P_{mt}(1-\alpha)\frac{Y_t}{L_t} = W_t. \tag{11}$$

The depreciation rate δ is a function of the capital utilization rate U_t, whereas δ' denotes the first derivative. Since intermediate goods firms operate on zero profits, they allocate their ex post return on capital to their owners. The return on capital can be defined as

$$R_{kt+1} = \frac{\left[P_{mt+1}\alpha\frac{Y_{t+1}}{\xi_{t+1}K_{t+1}} + Q_{t+1} - \delta(U_{t+1})\right]\xi_{t+1}}{Q_t}. \tag{12}$$

The term ξ_t is the capital quality shock and Q_t is the price for capital. As can be seen from Eq. (12), the capital quality shock affects the effective quantity of capital and the return on capital at the same time. As highlighted by Gertler and Karadi (2011), its future path also affects the price of capital.

2.3 Capital producers

Capital-producing firms combine depreciated physical capital at the end of period t with new investment goods to produce the new stock of physical capital K_t. Net investment (I_{nt}) is linked to adjustment costs, whereas the related function satisfies $f(1) = f'(1) = 0$ and $f''(1) > 0$. Net investment is investment (I_t) not used to replace depreciated capital $\delta(U_t)\,\xi_t K_t$. In a market of perfect competition, capital producers maximize profits

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Lambda_{t,\tau}\left\{(Q_\tau - 1)\,I_{n\tau} - f\left(\frac{I_{n\tau}+I_{ss}}{I_{n\tau-1}+I_{ss}}\right)(I_{n\tau}+I_{ss})\right\}, \tag{13}$$

which are redistributed to households, where I_{ss} is the steady state level of investment.[3] Resulting from profit maximization, the latter arises as

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \beta \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right). \tag{14}$$

The law of motion for capital is given by

$$K_{t+1} = \xi_t K_t + I_t - \delta\left(U_t\right)\xi_t K_t.$$

2.4 Retail firms

In order to allow for nominal price rigidities in the model, retail firms are introduced. Retail firms, belonging to a continuum with a mass of unity, buy the intermediate goods and differentiate them. The final good is produced with the help of a CES bundling technology in which the output of retailers Y_f enters.

$$Y_t = \left[\int_0^1 Y_{ft}^{(\epsilon-1)/\epsilon} df\right]^{\epsilon/(\epsilon-1)}$$

Since retailers operate in a market of monopolistic competition, they can principally have market power that depends on the degree of substitutability among retailers' output, denoted by ϵ. Nominal price rigidities are introduced by assuming that each firm can only set the price for its goods optimally with a probability of $1-\gamma$. When a firm is not able to set the price freely, it follows an indexation rule in which the lagged rate of inflation π_t enters. Retailers consequently maximize their profits by setting the optimal price P_t^* taking the demand function for its good and the corresponding price as given

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \left(1+\pi_{t+k-1}\right)^{\gamma_p} - P_{mt+i}\right] Y_{ft+1}, \tag{15}$$

where γ_p is a measure of price indexation. The first-order condition results as

$$E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \left(1+\pi_{t+k-1}\right)^{\gamma_p} - \mu P_{mt+i}\right] Y_{ft+1} = 0$$

with $\mu = \frac{1}{1-1/\epsilon}$ as the price markup. The overall price level emerges as a weighted average of the optimal price and price indexation

$$P_t = \left[\left(1-\gamma\right)\left(P_t^*\right)^{1-\epsilon} + \gamma\left(\Pi_{t-1}^{\gamma_p} P_{t-1}\right)^{1-\epsilon}\right]^{1/(1-\epsilon)}.$$

The final good is found by minimizing the cost of producing it thus enabling the

[3] Since adjustment costs arise from net investment rather from gross investment, capital producers earn non-zero profits (Gertler and Karadi, 2011).

demand for each retailer's good with related price P_{ft} to be obtained

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t$$

in conjunction with

$$P_t = \left[\int_0^1 P_{ft}^{1-\epsilon} df\right]^{1/(1-\epsilon)}.$$

2.5 Banking sector

2.5.1 The general setup

Since households cannot directly provide their funds to the goods-producing sector, there is an active role for a financial intermediary, which can be understood to be a bank. There is a continuum of banks with a mass of unity in which each bank j is operated by a bank manager. Bank managers take external funds from households and combine them with internal funds, i.e. net worth N_{jt}, to buy claims S_{jt}^b on capital-producing firms at price Q_t. In the absence of public financial intermediation, the claims are completely backed by the physical amount of capital, i.e. $S_{jt}^b = S_{jt} = K_{jt}$, which automatically means that the price for claims and for physical capital is identical. Consequently, capital production is completely financed by banks which, in turn, is financed by intermediate households' funds. Thus

$$Q_t S_{jt} = Q_t K_{jt+1}.$$

The term K_{jt} denotes the share of the capital stock that is implicity held by banks. Since the claims on capital-producing firms are the only assets that banks hold, the balance sheet constraint of the banks arises as

$$Q_t S_{jt} = N_{jt} + D_{jt}. \tag{16}$$

Net worth is built up by retained earnings, which is simply the difference between the gross returns on claims and the gross costs of external funds. Based upon this consideration and in conjunction with the balance sheet constraint, one obtains the law of motion for a banker's individual net worth

$$N_{jt+1} = (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} N_{jt}. \tag{17}$$

Bank managers survive with a probability of θ so that their survival rate becomes $1/(1-\theta)$. Thus, bank managers have a long-run perspective, from which it follows that their objective is to maximize the franchise value of the bank by determining the volume of assets held and by choosing the required external funds, taking the expected return on capital and the risk-free rate as given. The franchise value of the bank can be expressed as

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[(R_{kt+1+i} - R_{t+1+i}) Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}\right]. \tag{18}$$

Internal funds are needed in addition to external funds because of a costly enforcement problem facing households. Bank managers can potentially divert a fraction λ_{jt} of their total assets and distribute the resources back to the households they came from. In this case, depositors cannot recover these diverted resources but only the remaining fraction of total assets $1-\lambda_{jt}$, in which case a bank's activities are terminated. Consequently, the incentive-compatible constraint for no-running becomes

$$V_{jt} \geq \lambda_{jt} Q_t S_{jt}, \tag{19}$$

where we allow the share of diversion to vary over time (see, for example, Dedola, Karadi, and Lombardo, 2013). The discussion of this time variation is reserved for the next section. Under full information, households can observe λ_{jt}. Since they do not want to lose their savings, they constrain the provision of funds following Eq. (19) and implicitly force bank managers to achieve an expected terminal wealth of the bank.

Without loss of generality, Eq. (18) can be expressed linearly in quantities

$$V_{jt} = \nu_t Q_t S_{jt} + \eta_t N_{jt} \tag{20}$$

with

$$\nu_t = E_t \left[(1-\theta) \beta \Lambda_{t,t+1} \left(R_{kt+1} - R_{t+1} \right) + \beta \Lambda_{t,t+1} \theta x_{t,t+1} \nu_{t+1} \right] \tag{21}$$

$$\eta_t = E_t \left[(1-\theta) + \beta \Lambda_{t,t+1} \theta z_{t,t+1} \eta_{t+1} \right]. \tag{22}$$

The variables $x_{t|t+i}$ and $z_{t|t+i}$ are the gross growth rates for total assets and for net worth, respectively, from period t to period $t+i$.

$$z_{j,t|t+1} = \frac{N_{j,t+1}}{N_{jt}} = \left(R_{kt-1} - R_{t+1} \right) \phi_{jt} + R_{t+1}$$

$$x_{j,t|t+1} = \frac{Q_{t+1} S_{j,t+1}}{Q_t S_{jt}} = \frac{\phi_{j,t+1}}{\phi_{jt}} z_{j,t|t+1},$$

where the term ϕ_{jt} is the leverage ratio. From solving the maximization problem, one can obtain a positive dependence between a bank's total assets and its net worth if the incentive constraint is binding. The dependence is determined by the expected discounted marginal gain of expanding total assets ν_t, the expected discounted value of extending net worth η_t, and the share of diversion λ_{jt}.

$$Q_t S_{jt} = \frac{\eta_t}{\lambda_{jt} - \nu_t} N_{jt} = \phi_{jt} N_{jt}, \tag{23}$$

As opposed to Gertler and Karadi (2011), the leverage ratio in our setting, Eq. (23), depends on individual characteristics, namely on λ_{jt}. In order to facilitate aggregation, we assume that all bankers behave alike regarding their diversion behavior. Following from this assumption, the leverage ratio no longer depends on individual factors. We can thus drop the index j and easily aggregate over all entities. Since the model contains two different sorts of bankers, surviving and new, both types exhibit different laws of motion for net worth. The net worth of surviving bankers N_{ot} (o for old) is the remaining part of retained earnings that is not returned to households because bankers leave the banking sector, i.e.

$$N_{ot} = \theta \left[\left(R_{kt} - R_t \right) \phi_t + R_t \right] N_{t-1}. \tag{24}$$

New bank managers receive an endowment N_{nt} from their households, which is a fraction ω of total claims valued at the period's t price

$$N_{nt} = \omega Q_t S_{t-1}. \tag{25}$$

The aggregate law of motion for a bank's net worth consequently arises as the sum of both components

$$\begin{aligned} N_t &= N_{ot} + N_{nt} \\ &= \theta \left[\left(R_{kt} - R_t \right) \phi_t + R_t \right] N_{t-1} + \omega Q_t S_{t-1}. \end{aligned}$$

2.5.2 Introducing imperfect information in the GK framework

Our aim is to investigate the consequences if agents learn about misbehavior in the banking sector. For this reason, we draw on the share of diversion appearing in the incentive constraint (Eq. (19)). As mentioned, this parameter controls the share of total assets which bank managers can divert and which cannot be recovered by depositors. Dedola et al. (2013) link a shock to this parameter to generate financial stress that originates in the banking sector compared with the capital quality shock, which affects the banking and the real sector simultaneously. In their model, the share is temporarily increased and it returns to its steady state level (following the underlying autoregressive process). The interpretation is a confidence shock departing from households. In our case, the change in the (steady state) diversion share originates in the banking sector. An increase in the share of diversion constrains the provision of deposits at the given leverage ratio. Households are only willing to provide funds if banks' leverage ratio drops. If bankers can increase the diversion share while household provide them with the same amount of funds, bankers have an incentive to take the funds and run.

In our setting, we assume that bankers are able to increase the steady state value of diversion and households only have imperfect information about bankers' behavior. Furthermore, the diversion share is directly observable as long as no runs occur. Although household can receive the signal that bankers have changed their behavior, they do not know to what extent. For this reason, they need to learn about bankers' behavior by exploiting the current economic outcome. The law of motion for the incentive parameter in the incentive constraint can be defined as

$$log \left(\lambda_t \right) = \left(1 - \rho^{\lambda} \right) log \left(\bar{\lambda}_t \right) + \rho^{\lambda} log \left(\lambda_{t-1} \right) + \epsilon_t^{\lambda} \tag{26}$$

which is close to Dedola et al. (2013). Thus, we allow for transitory deviations from the steady state value $\bar{\lambda}_t$. A persistent change in the diversion behavior of bankers is induced in Eq. (26) by altering the steady state value, which is denoted by the time subscript. The diversion parameter suddenly changes at time T

$$\bar{\lambda}_t = \begin{cases} \bar{\lambda}^{old} & for\, t < T \\ \bar{\lambda}^{new} & for\, t \geq T \end{cases}$$

which is neither anticipated nor observed exactly by the households. The term ϵ_t^{λ} reflects i.i.d. shocks, which can be interpreted as confidence shocks, i.e. agents believe that bankers' behavior has changed. This setting means that households observe a signal but do not know whether bankers transitorily (shock) or permanently (change in $\bar{\lambda}_t$) increase their amount of diversion.[4]

The incentive constraint from Eq. (19) enters the maximization problem of banks because households implicitly force bank managers to realize at least the corresponding franchise value of the bank, otherwise households would reduce their supply of funds. If bankers can divert more and households observe this behavior, the incentive constraint will immediately be affected by providing less funds to bankers. Thus, households would cut the provision of external funds if bankers cannot obtain a greater franchise value of the bank. However, if information about bankers' behavior is imperfect and households only (slowly) learn about bankers' behavior, a lower franchise value (compared with the new one) can be kept for a specific period while bankers can potentially divert more funds. New entering (and old) bankers have therefore an advantage of saving internal funds. There is an incentive for bankers not run as long as these savings are greater than resources required to increase the leverage ratio as demanded by funders. A formal description can be found in the appendix. As a result, a surplus might arise for the bankers in this situation. By neglecting transitory shocks, this surplus can be expressed as the difference between the expected terminal wealth which would be needed under rational expectations (RE) and under learning (LE), i.e. the difference between the true and the actual terminal wealth

$$\Delta^{LE} V_t \geq \left(\lambda_t^{true} - \lambda_t^{actual}\right) Q_t S_t.$$

In this case, the difference between the actual and the observed value for λ_t matters. This difference can also be translated into the leverage ratio: the observed leverage ratio is greater than it must be under perfect information

$$\Delta^{LE} \phi_t = \phi_t^{actual} - \phi_t^{true}$$

with

$$\phi_t^{true} = \frac{\nu_t^{true}}{\lambda_t^{true} - \eta_t^{true}}, \quad \phi_t^{actual} = \frac{\nu_t^{actual}}{\lambda_t^{actual} - \eta_t^{actual}}.$$

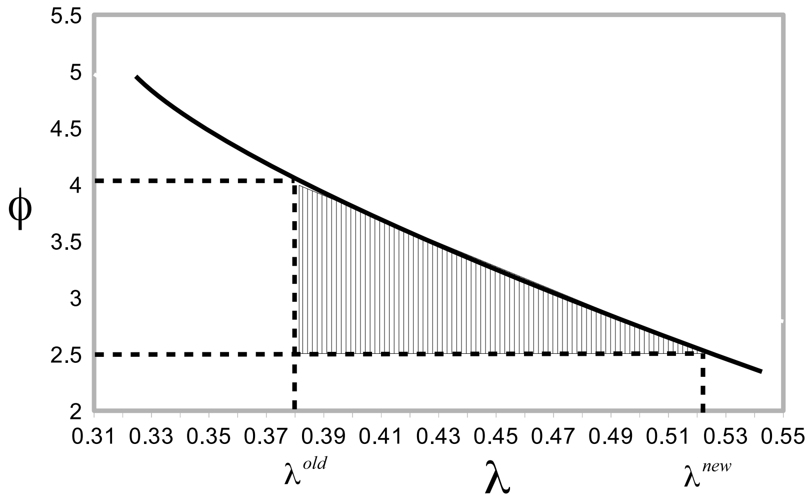
Loosely speaking, the precision of the signal controls how fast this gap is closed. In Figure 1, the potential scope through learning can be made transparent. The solid curve represents the combination of the maximum required leverage ratio given a specific share of diversion if the enforcement constraint is binding. As becomes clear, the negative slope indicates that greater shares of diversion entail lower leverage ratios. Imperfect information allows banks to operate with a greater leverage ratio for a longer period in time while agents are learning.[5] Under rational expectations, the leverage ratio must drop immediately from the old level to the new level (from 4 to 2.5 in our example). Depending on the signal, the shaded area in Figure 1 is then linked to the surplus which can be realized by the bankers.

Above, we predominantly put the focus on the effects of an increase in the steady state

[4]It should be noted that for the transitory shock it is not necessary for bankers to have actually changed their behavior.

[5]The formal derivation of the optimality of the contract is presented in the technical appendix.

Figure 1: Relationship between the leverage ratio and the share of diversion



Note: The figure shows the stylized relationship between the share of diversion λ and the leverage ratio ϕ. The shaded area shows the potential profits for bankers to keep operating the banks while having the opportunity to increase the diversion share.

diversion parameter. This is possible because agents cannot observe the new steady state diversion parameter, from which it follows that they demand a lower franchise value of the bank than would be required under perfect information to ensure that the contract is still feasible. As we demonstrate in the appendix, this mechanism works because households cannot directly observe the franchise value of the bank and there is still an incentive for bankers not to run. The franchise value of the bank arises as expectations about the future path of a bank's profits and are given in Eq. (18). Future profits, in turn, determine the current leverage ratio (Eq. (23)) in which the diverting behavior of bankers also enters because of the enforcement constraint. Given rational expectations, the enforcement constraint as well as the franchise value of the bank are completely observable. However, imperfect information about the diversion parameter creates imperfect knowledge about the franchise value of the firms. Households try to infer the franchise value of the bank by figuring out the diversion share which they learn about consequently. However, the diversion preference cannot be observed directly because diversion does not occur in equilibrium. Nevertheless, the observation of the diversion share is implicitly possible by seeing the share households give to new entering bankers (Eq. (25)), which is the counterpart of changes in the diversion share. Households always take their observation of the diversion share into account when providing funds to bankers. As we demonstrate in the technical appendix, bankers can exploit this period in an intertemporal consideration such that they do not actually divert and run away. Thus, the contract is still feasible.

Implicitly, we assume that households are myopic in forming expectations. It might be possible that households build expectations about the future path of the diversion share. They could observe the signal, learn about the current diversion share and believe that the development will continue to some extent in the future. However, our approach does not allow such a specific forward-looking response to learning. Nevertheless, such learning and expectations formation does not impede our approach as long as the new diversion share expected by households exceeds the actual one. In this case, the contract would indeed be infeasible. Since bankers know how households behave, one can assume that this can be ruled out.

However, the same exercise cannot be conducted for a drop in the steady state diversion parameter. Thus, our approach is asymmetric rather than symmetric. A decrease in the diversion parameter means that bankers would like to divert less and, given perfect information, this translates into a greater leverage ratio because a lower franchise value of the bank is required to compensate the agents. Under imperfect information, bankers cannot choose the new franchise value because the new value would not satisfy the enforcement constraint. Households would still see the old value and take this value into account in their decisions to provide the bank with funds. Hence, banks need to commit trustfully to changing their behavior. Since this issue is beyond the scope of this paper, we will focus on the increase in the steady state share of diversion.

2.5.3 Aggregation and learning

Following Gertler and Karadi (2011), we assume that households cannot place deposits at banks operated by own household members. Furthermore, we assume that all bank managers behave alike regarding their diverting behavior. In such a setting, the bank manager could inform its other household members that he would divert more resources. Since households know that bankers behave identically, they could deduce a greater degree of diversion from the behavior of their own bankers. Similar to reality, we accordingly assume that bank managers are obliged to maintain confidentiality. For this reason, households do not know the exact degree of diversion. However, bank managers might indicate changes in the banking sector in reality. This is what we understand as a signal. Households consequently know that λ_t is about to change in the banking sector but they have to learn about the exact changes in the economy. Despite the learning approach, aggregation does not change.

2.6 Public sector

The central bank conducts monetary policy by controlling the policy rate i_t obeying a Taylor (1993)-type monetary policy rule with interest-rate smoothing

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[i + \kappa_\pi \pi_t + \kappa_y \left(\log Y_t - \log Y_t^* \right) \right] + \epsilon_t^i. \tag{27}$$

In Eq. (27), the term ρ_i denotes the smoothing parameter with $0 < \rho_i < 1$ while Y_t^* is the natural level of output, and the parameters κ_π and κ_y control the responsiveness to inflation and the output gap, respectively, Furthermore, the variable ϵ_t^i represents an unexpected monetary policy shock. The link between the nominal and the real interest rates can be established with the help of the Fisher equation

$$1 + i_t = R_{t+1} \frac{E_t P_{t+1}}{P_t}.$$

2.7 Market clearing

Aggregate demand, which consists of consumption, investment, public expenditure, and investment adjustment costs, equals the output level so that the resource constraint of the economy arises as

$$Y_t = C_t + I_t + f\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right) + G_t.$$

The government budget constraint is assumed to be equilibrated every period as the amount of government spending G_t equals lump sum taxes.

3 Calibration

The calibration for steady state values and deep parameters follows Gertler and Karadi (2011). Since for a simulation of the model variable outcomes become important, we need to calibrate the shock processes such that we are able to match the moments of real data. For this reason, we base our calibration for the four shocks monetary policy shock, total factor productivity, capital quality shock, and incentive shock on estimates from Gelain and Ilbas (2014), who estimate a model with a financial sector identical to Gertler and Karadi (2011) with US data. Based upon these values, we adjust the autoregressive parameters and the standard deviation of the shocks (Table 1) to match the volatility of output growth during the period 1984-2014.[6] In Table 2, we compare the model implied values for the second moments of annual output growth, annual investment growth, and annual inflation with their real world counterparts. Investment and inflation are slightly more volatile in our calibration than in reality. Regarding the correlations of investment growth and inflation to output growth, we are relatively close to real data.

Table 1: Calibration of parameters

Description	Symbol	Value
Inflation coefficient. Taylor rule	κ_π	1.98
Output gap coefficient. Taylor rule	κ_y	-0.125
Interest rate smoothing. Taylor rule	ρ_i	0.8
Autoregressive parameter. capital quality shock	ρ_ξ	0.66
Autoregressive parameter. incentive shock	ρ_λ	0.98
Autoregressive parameter. total factor productivity	ρ_A	0.7
Standard deviation. monetary policy shock	σ_i	0.002
Standard deviation. capital quality shock	σ_ξ	0.008
Standard deviation. incentive shock	σ_λ	0.01
Standard deviation. total factor productivity	σ_A	0.0138

[6]We start in 1984 because we exclude the period of disinflation at the beginning of the 1980s.

Table 2: Second moments of central variables

		USA (Q1-1984 to Q1-2014)	Model
Output growth. annualized	$SD\,(\Delta Y/Y)$	2.48	2.48
Investment growth. annualized	$SD\,(\Delta I/I)$	7.57	9.99
Rate of inflation. annualized	$SD\,(\pi)$	0.97	0.9
Output growth. investment growth	$corr\,(\Delta Y/Y,\,\Delta I/I)$	0.7	0.97
Output growth. inflation	$corr\,(\Delta Y/Y,\,\pi)$	0.12	0.09

Data: U.S. Bureau of Economic Analysis. Output growth, investment growth, and the rate of inflation are measured quarterly and then annualized. Model moments are simulated over a period of 1000 observations.

4 The learning mechanism

Our approach to modelling learning is akin to the work by Cogley, Matthes, and Sbordone (2015). The model is completely known to the agents as they observe all relevant economic outcomes. These outcomes are then used to estimate the single unknown parameter which is the steady state parameter in the AR(1) process of diverting λ. The law of motion for the incentive process exhibits two parameters and the standard deviation of its innovation. Both the standard deviation and the autoregressive parameter ρ^{λ} are known, while the constant which incorporates the autoregressive term and the steady state of λ is uncertain.[7] The rest of the economy, i.e. both the structure and the parameters, is fully known by all agents. On an aggregate level, all private agents share the same beliefs and update their identical information by carrying out inference with the Kalman filter. Equation (26) enters our model in a slightly modified way and the necessary state space system is given by the following observation equation

$$\mathbf{log}\left(\lambda_{\mathbf{t}}\right) = \lambda_t^C + \rho^{\lambda}\mathbf{log}\left(\lambda_{\mathbf{t-1}}\right) + \epsilon_{\mathbf{t}}^{\lambda} \tag{28}$$

and the state equation

$$\lambda_t^C = \lambda_{t-1}^C + \mathbf{1}_t \nu_t \tag{29}$$

with $\lambda_t^C = \left(1-\rho^{\lambda}\right) log\left(\bar{\lambda}_t\right)$. The observation equation is then equivalent to the AR(1) process and the state equation represents the dynamic of the constant, aka the steady state value for λ.[8]

The i.i.d. disturbance of the observation equation is denoted by ϵ^{λ}, which is normally distributed with mean zero and a standard deviation of 0.01. This is known by the agents. The dynamic of the state equation is straightforward by assuming a random walk for λ_t^C which is standard in the literature. The key assumption is now how to specify the variance of this equation. If it is positive over the whole simulation horizon, this means that agents expect a change in the constant in every period and update it accordingly. The way we model this is borrowed from earlier work in this strand of the literature. If the variance in

[7] Note again that we write the whole model in logs and not in deviations from their respective steady state values. A complete overview of all equations of the model and their respective constants is shown in the appendix.

[8] As agents know the functional form of the constant and also the autoregressive parameter, they can infer that the single uncertain parameter is the steady state value.

the state equation $\nu_{\mathbf{t}}$ is zero, then private agents believe that the constant and therefore the steady state of λ undergo no change and they estimate only one unknown constant coefficient. To account for both possibilities, we introduce an indicator function $\mathbf{1}_t$ which serves the purpose laid out by Hollmayr and Matthes (2013) and assume that households only suspect a change in bankers' behavior once they actually start diverting more. By this we give the agents as much information as possible and even allow them to know in what period bankers change their behavior. This is what we understand as the signal which initiates learning. The indicator can therefore take two values. If it is one, the variance is always positive and agents update every period. If there is no change to the unknown parameter, the estimate thereof will fluctuate around the true one. On the other hand, the second alternative is that it is zero throughout and only takes the value 1 in the period when bankers increase the fraction of diversion and is zero afterwards. In other words, the variance is only positive once the actual change in the AR process happens. Otherwise this variance is zero. This does not mean that agents do not update their beliefs throughout the simulation horizon. The variance of $\nu_{\mathbf{t}}$ is calibrated to be a function of the parameter that the agents estimate. In particular, we set it to be $\nu_t = (0.26\lambda_t^C)^2$. This translates to a 35 basis points standard deviation in the spread one year after the break. This is slightly below the standard deviation in the spread between 10-year treasuries and BBB-commercial bonds in the US over the period 1999 to 2012.

Households and firms enter period t with the belief updated in the last period $t-1$ and base all optimization decisions on that belief. Given the belief of the steady state in the AR process, agents assume that this value holds forever and act accordingly. This behavior is called "anticipated utility" and was initially developed by Kreps (1998). In the learning literature, this is a common assumption and many papers rely on it (see, for example, Milani (2007)). Given the optimal decisions, the true steady state value of λ is set or left unchanged. Then all shocks in the economy occur. Given then the outcomes of the economy and especially that of λ_t, the agents try to infer which part of the change comes from the innovation and which from the steady state part. Given the new belief of the steady state, they enter in the next period. In period one, we endow the agents with initial beliefs identical to the true value. As a point estimate, we use the posterior mean of the Kalman filter from the last period for the belief. In order to make sure that a stable perceived law of motion (PLM) exists at all times, the deviations both in the AR process as well as in the estimation step are sufficiently small. This ensures that we do not have to discard any beliefs and that we do not have to rely on a projection facility (see, for example, Cogley et al., 2015).

With respect to the solution and obtaining the perceived and the actual law of motion (ALM) in the economy, we set up a vector of all variables (and a constant intercept) in the model economy and denote it $\mathbb{X}_t$. In order to get the log-linearized perceived law of motion, we stack the log-linearized equilibrium conditions (approximated around the perceived steady state) and the estimated policy rules in the following way:

$$\mathbf{A}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbb{X}_{\mathbf{t}} = \mathbf{B}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbf{E}^*_{\mathbf{t}}\mathbb{X}_{\mathbf{t+1}} + \mathbf{C}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbb{X}_{\mathbf{t-1}} + \mathbf{D}\varepsilon^*_{\mathbf{t}} \tag{30}$$

where $\varepsilon^*_{\mathbf{t}}$ is the perceived shock, i.e. the innovations that agents observe, and contains the actual shock $\tilde{\varepsilon}_{\mathbf{t}}$(the residual in the estimated AR(1) process). $A(\lambda^C_{t-1})$, $B(\lambda^C_{t-1})$, $C(\lambda^C_{t-1})$ and D are matrices that depend on the model's deep parameters according to the linearized model solution that is provided in the appendix. The perceived shock can be expressed as the actual shock plus an additional perceived component

$$\varepsilon^*_{\mathbf{t}} = \tilde{\varepsilon}_{\mathbf{t}} + \left(\lambda^C_t - \lambda^{C,true}_t\right).$$

This means that once the agents' belief converges to the true value, the perceived shock agents observe is also identical to the actual one. The system includes expectations so we solve it with a numerical routine such as gensys (Sims (2001)). We thus obtain the reduced form perceived law of motion:

$$\mathbb{X}_{\mathbf{t}} = \mathbf{S}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbb{X}_{\mathbf{t-1}} + \mathbf{G}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\varepsilon^*_{\mathbf{t}} \tag{31}$$

where $\mathbf{S}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})$ solves the following matrix quadratic equation[9]

$$\mathbf{S}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}) = (\mathbf{A}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}) - \mathbf{B}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbf{S}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}))^{-\mathbf{1}}\mathbf{C}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}) \tag{32}$$

and where $\mathbf{G}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})$ is given by

$$\mathbf{G}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}) = (\mathbf{A}(\lambda^{\mathbf{C}}_{\mathbf{t-1}}))^{-\mathbf{1}}\mathbf{D}. \tag{33}$$

In order to obtain the actual law of motion, we replace the perceived constant of the incentive process in $\overline{\mathbf{C}}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})$ with the actual constant and also use the scalar of the actual innovation in the incentive AR process.

$$\overline{\mathbf{A}}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\overline{\mathbf{Y}}_{\mathbf{t}} = \overline{\mathbf{B}}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\mathbf{E}^*_{\mathbf{t}}\overline{\mathbf{Y}}_{\mathbf{t+1}} + \overline{\mathbf{C}}^{\mathbf{actual}}(\lambda^{\mathbf{C}}_{\mathbf{t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{D}}\varepsilon_{\mathbf{t}}. \tag{34}$$

With the perceived law of motion from above, we can solve the expectations and get:

$$\overline{\mathbf{Y}}_{\mathbf{t}} = H(\lambda^C_{t-1})\overline{\mathbf{Y}}_{\mathbf{t-1}} + G(\lambda^C_{t-1})\varepsilon_t. \tag{35}$$

The actual outcomes are then determined both by the true steady state value for the fraction of diversion as well as by the perceived value of the agents as H is given by:

$$H(\lambda^C_{t-1}) = S(\lambda^C_{t-1}) + \left(A(\lambda^C_{t-1}) - B(\lambda^C_{t-1})S(\lambda^C_{t-1})\right)^{-1}(C^{true}(\lambda^C_{t-1}) - C(\lambda^C_{t-1})) \tag{36}$$

5 Experiment

The central variable in our experiment is a change in the share of diversion, as given by Equation (19) and discussed in section 2.5.2. The change in the steady state value of the amount of diversion affects the computation of the steady state. The steady state leverage ratio, the steady state credit spread, and the steady state share of diversion are directly linked. In GK, the calibration strategy is to set the values for the leverage ratio and the credit spread. This is our benchmark case. Thus, the value for $\bar{\lambda}^{old}$ (and ω) is pinned down by the steady state. For our purposes, we exogenously increase its value to $\bar{\lambda}^{new}$ which must coincide with either changes in the steady state leverage ratio or the

[9]The perceived law of motion can be derived by assuming a VAR perceived law of motion of order 1 and then using the method of undetermined coefficients.

steady state credit spread (or even both) and goes hand in hand with a change in ω.

The learning algorithm, as explained in subsection 4, is concretely applied as follows. Bank managers increase the steady state share of diversion. To obtain a meaningful interpretation of an increase, we raise λ to an amount which corresponds to an increase in the credit spread of 50 basis points by keeping the leverage ratio constant. However, we translate the change of λ into a decrease of the leverage ratio by keeping the credit spread fixed. The reason is twofold: we work, on the one hand, with a number which can easily be interpreted, i.e. a rise of the credit spread, and, on the other hand, with an economic adjustment process that is reasonable, i.e. the adjustment over the leverage ratio. In this respect, we assume that the new value for the share of diversion materializes in a change in the steady state leverage ratio without affecting the steady state credit spread.[10]

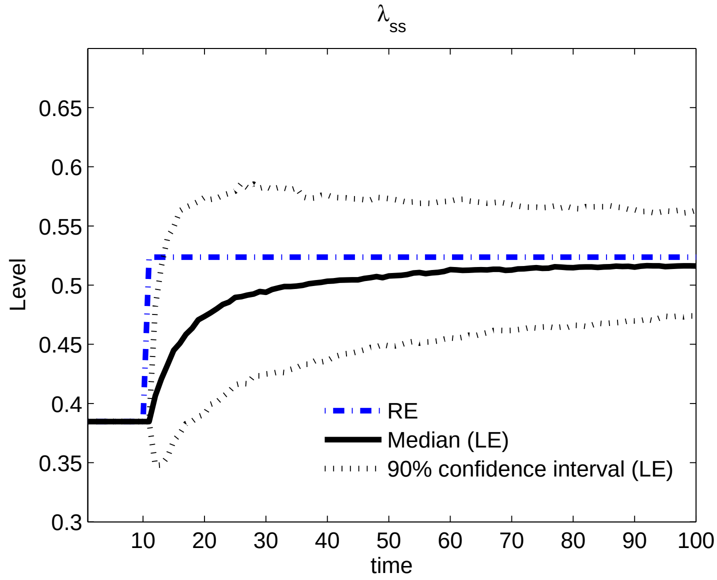
Since information is imperfect, agents need to learn about the change in the steady state value of diversion. Agents can only observe the constant term λ_t^C in Eq. (28) as a whole and need then to disentangle the respective components. Because of our simulation design, a change in the steady state share of diversion is linked to a change in the share that new entering bankers are equipped with (ω in Equation (25)). Since the autoregressive coefficient in the process for λ_t does not change, the agents can infer the steady state value for the share of diversion (and the value for ω). The value for the steady state diversion share cannot be observed directly. Agents need to extract the unobserved value from outcomes. Given these outcomes, the difficulty for agents is then to disentangle transitory shocks, captured by ϵ_t^{λ}, from permanent changes in the constant term. Based upon realizations of observable variables, agents build expectations regarding their future values and conduct utility maximization based on them.

The model economy is simulated a thousand times for 100 periods each. In Figure 2, we show how the share of diversion evolves over time. The bold black line represents the median for the steady state share of diversion across all simulations for every point in time, while the black dotted lines reflect corresponding upper and lower bounds for the 90% confidence interval.[11] The rational expectations solution is given by the blue dashed line with dots. During the first 10 periods, learning is not activated, which means that the median coincides with the rational expectations solution. In period 10, the increase in the steady state share of diversion in induced. In the rational expectations case, the value rises immediately to its new level. The dynamics in the economy for this case have completely changed as a consequence of this new environment. Where information is imperfect, the share of perceived diversion slowly adjusts to its new level (measured by the median). The median approaches the new rational expectations value for the steady state diversion parameter after a horizon of approximately 15 years after the break. However, the confidence bands indicate that there is a broad range of realizations which can be interpreted as uncertainty about outcomes. In the periods after the break in λ, the lower band even falls below its old rational expectations value. This result indicates that agents cannot precisely infer the new steady state value for λ. A significant increase in the

[10]This is our baseline scenario. We can also choose another procedure by letting the adjustment run over the credit spread. However, this would also affect steady state values of the real economy. In our case, we can isolate the financial sector from the real sector. Adjustments only take place within the financial sector. The rationale for this approach is that we can clearly disentangle the effects and can focus on developments from the financial sector.

[11]The lower bound is the 5% percentile while the upper bound is the 95% percentile for each period in time.

Figure 2: Evolution of the steady state share of diversion under rational expectations and under learning (perceived steady state)



Note: The figure shows how the diversion share evolves in the rational expectations case (blue dashed line with dots) and in the learning case (solid black line). The black dotted lines represent the boundaries for the 90% confidence interval in the learning case.

diversion parameter cannot be observed until two years after the break has occurred.

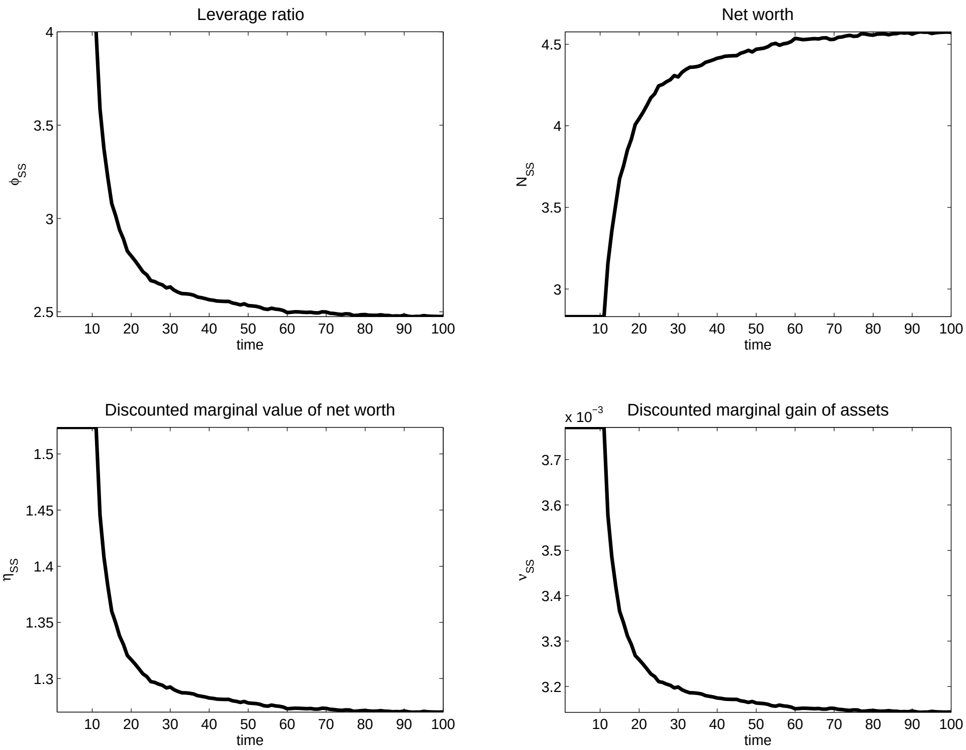
6 Results of simulation

6.1 Effects on steady state values

Changes in the steady state share of diversion have effects on some central financial variables. Because of our simulation design, there are no feedback effects on the steady state values of macroeconomic variables. The change in the steady state share of diversion mainly affects Eq. (23) by keeping the price of capital and the stock of capital constant. Concretely, a change in $\overline{\lambda_t}$ is induced and the system is solved for the corresponding leverage ratio, the expected discounted gain in extending the balance sheet, and the discounted value of expanding net worth. Since the steady state value of total assets does not change, the steady state value for net worth is automatically pinned down. In Fig. (3), we present the evolution of the four aforementioned variables following the structural change over the simulated horizon for the learning case, whereas we draw on the median from the actual law of motion. Because of the design, the trajectory of the four variables resembles the evolution of the steady state value for the share of diversion.

An increase in the steady state share of diversion makes the incentive constraint more binding because households ask for a higher franchise value of the bank given their supply of funds, if they can observe this change. Otherwise, bankers would take the funds and run away. As a result, banks need to delever, which happens by increasing net worth in

Figure 3: Evolution of the median of steady state values (stemming from the actual law of motion)



Note: The figure shows how the steady state value of the leverage ratio, net worth, the discounted marginal value of net worth, and the discounted marginal gain of assets evolve in the learning case.

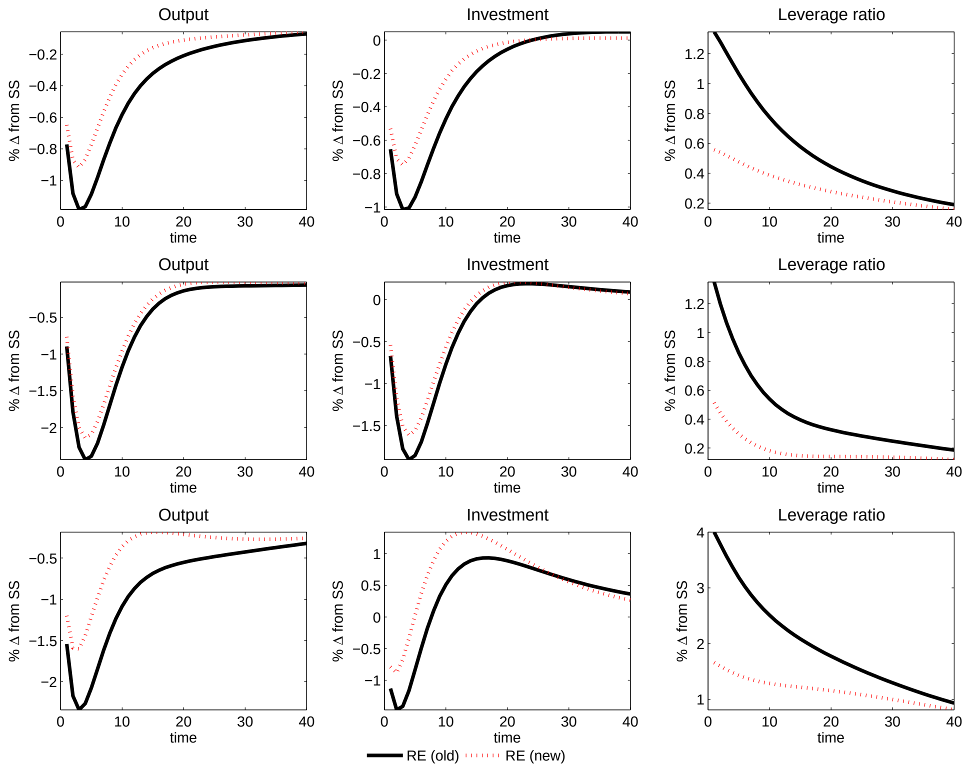
the model.[12] A reduction in the leverage ratio lowers the steady state growth rate of assets x_{ss}, which ultimately decreases the expected discounted marginal gain of expanding total assets. By looking at the median, the new steady state values are achieved approximately ten years after the change in bankers' behavior. These developments are central for the new dynamics in the economy.

6.2 Propagation of shocks

In this section, we compare how the propagation of shocks changes following a permanent increase in the share of diversion, which is translated into a drop in the steady state leverage ratio. In Figure 4, we compare the responses of output, investment and the leverage ratio to the monetary policy shock (first line), the shock on total factor productivity (middle line) and the capital quality shock (last line) for the case of rational expectations. The black lines represent the initial case while the red dotted lines denote the responses after the break. The drop in the steady state leverage ratio as a result of a higher share

[12]Recall that this is rational because bankers can profit from operating longer with a smaller leverage ratio compared to rational expectations.

Figure 4: Comparison of impulse responses under RE following a (a) monetary policy shock (first row), (b) shock on total factor productivity (middle row), and a (c) shock on capital quality (last row)



Note: The figure shows impulse responses to three different shocks (rows) for three different variables (columns) for the rational expectations case. The solid black line represents the situation before the break and the dotted red lines after the break in the steady state value of diversion.

of diversion dampens the responses of the aforementioned variables. In the GK model, banks' leverage ratio controls the amplification of shocks because it is related to financial frictions. Bankers need to increase the franchise value of the bank by building up net worth relative to total assets. A lower leverage ratio relaxes financial frictions because more net worth is available compared with total assets and there is less incentive for the bankers to actually divert. As a result, the amplification of shocks is reduced. Following the three shocks, output falls less after the rise in the steady state diversion parameter compared with its old value. For the monetary policy shocks, this means that output falls by around 0.2 percentage points less in the new case.

To investigate the dynamics in an economy with learning agents, we present the responses of output, investment, the leverage ratio, and net worth for the three aforementioned shocks. Figure 5 shows the monetary policy shock in the first line, the shock in total factor productivity in the middle line, and the capital quality shock in the last line. For all three cases, we report the new situation with rational expectations (red dotted lines) and two situations for the learning case. The bold lines give the responses in the economy in the period after the break while the blue lines with dots reflect 1 year after the break. As can be seen, the introduction of learning amplifies the responses to economic variables compared with the new rational expectations solution. Under learning, net worth drops by a greater amount for all three shocks and this entails a strong increase in the leverage ratio as well. Since banks need to delever more strongly under learning than under rational expectations following the shocks and given the same size of shocks, credit supply is reduced to a greater extent which feeds back into a stronger cut in investment and output. The reason behind the amplification is that the required steady state leverage ratio is still on a higher level compared with rational expectations. Agents cannot observe the new level for the steady state share of diversion, which is in turn related to the steady state leverage ratio. Thus, banks can operate longer with a higher leverage ratio under learning than under perfect information. For the same reason as explained for the two rational expectations cases, amplification is more pronounced while agents have not learned about the new steady leverage ratio because the steady state leverage ratio controls financial frictions. The more agents learn about the new steady state, the smaller the gap between the actual leverage ratio and its new level under rational expectations. Thus, the amplification of shocks induced through learning is dampened the more agents learn about the new steady state level for the share of diversion. In Fig. 5 the blue dotted lines represent the case in which agents have already learned more but not enough about the new steady state.

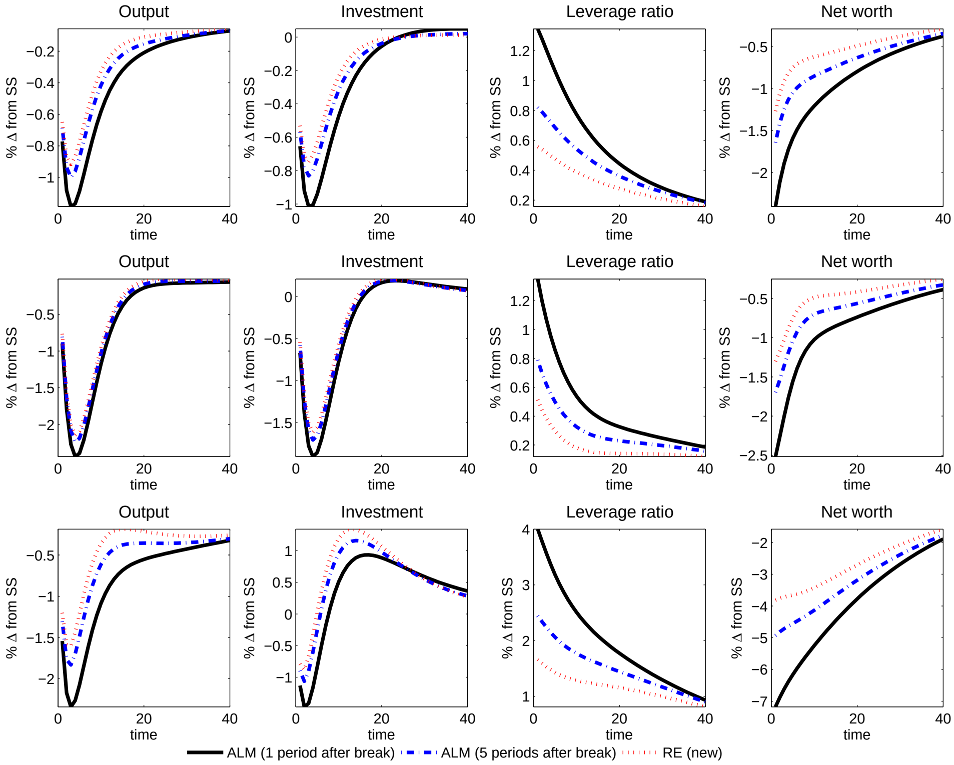
As argued, learning produces a new steady state in every subsequent period following the break in the steady state value for diversion. Hence, in every subsequent period the propagation is different. In Figure 6, we depict these amplification effects for every quarter following the structural change in λ for the monetary policy shock. We take the monetary policy shock as an example. The shapes for the other two shocks are similar. The x-axis gives the periods of the simulation (we simulate 100 periods) while the y-axis gives the quarters after the monetary policy shock occurred. The z-axis measures the differences between the responses under learning and under rational expectations. The figure provides information about the amplification of shocks, i.e. the difference in responses is more negative. We show the relative propagation (y-axis) of a monetary policy shock of the same size while the economy evolves over time (x-axis). The structural break occurred in period 10. In the period before the change in the diversion share took place, there are obviously no differences between the responses. The largest amplification occurs in the first period after the structural change. Regarding output, the shock is not only amplified, the persistency of the shock even increases which stems from the fact that the difference has not returned to zero after 40 quarters in the periods after the break. Again, under learning, the agents take into account a value for the steady state leverage ratio which is close to the old one, while the leverage ratio under rational expectations is already smaller. The amplification crucially depends on how fast the agents update the information on the diversion parameter.

6.3 Consequences of learning on economic outcomes

6.3.1 Amplification of the business cycle

Since learning impacts on the propagation of shocks (subsection 6.2), learning consequently ultimately affects the outcomes of variables. In addition, changes in the steady state value for the share of diversion also affect steady state values of some financial vari-

Figure 5: Impulse responses following a (a) monetary policy shock (first row), (b) shock on total factor productivity (middle row), and a (c) shock on capital quality (last row) for different periods after the break

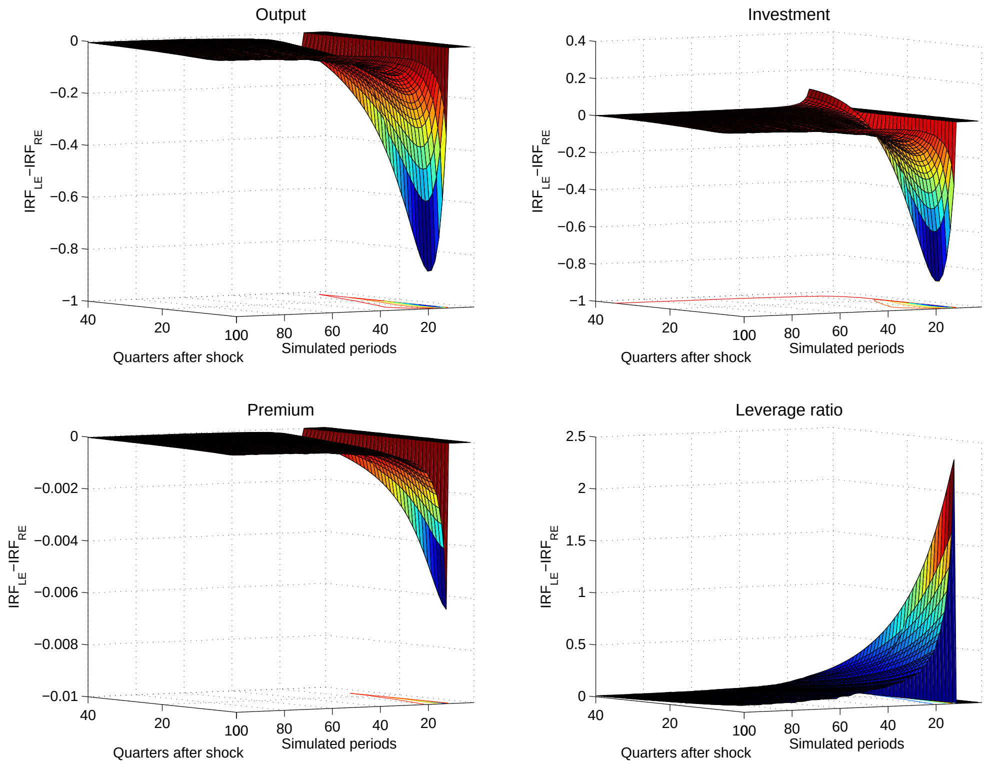


Note: The figure shows impulse responses to three different shocks (rows) for four different variables (columns) for the two learning and one rational expectations cases. The solid black line represents the situation one period after the increase in the steady state value of diversion, while the blue dashed lines with dots are the responses five periods after the break. The red dotted lines are the new rational expectations situation.

ables (subsection 3). How learning changes the outcomes of macroeconomic and financial variables, given a series of shocks (monetary policy shock, shock on total factor productivity, capital quality shock, and the temporary shock on the diversion parameter) and given the increase in the steady state value for the diversion parameter, can simply be shown by expressing the medians of simulated data for the learning case relative to the medians of the rational expectations case. We present the corresponding graph in Figure 7 together with the differences between the percentiles linked to the 90% confidence interval (dotted lines). The outcomes of the learning case are presented in Figure 12 in the appendix. Since no structural change is taking place and agents are not allowed to learn, the black solid line is flat for the first ten quarters, i.e. outcomes are obviously the same in both cases. After the break occurs and agents receive the signal to update new information, both medians deviate.

In the first periods following the rise in the diversion share, the level of output under learning exceeds its value under rational expectations. Apparently, banks' net worth is also higher in the first period after the structural change, which is translated into banks'

Figure 6: Differences between rational expectations and learning for a given period after a monetary policy shock in % of the new steady state



Note: The figure shows the differences between the impulse responses in the learning case and the rational expectations case to a monetary policy shock. The x-axis represents the simulated periods (starting from right to left), the y-axis is the quarters after the shock (from right to left), and the z-axis is the difference in responses relative to the corresponding steady state value.

leverage ratio being lower under learning compared with rational expectations in the first period after the break. Similar to what is explained for the dynamics, the agents cannot observe the new value of diversion under learning, which means that banks can operate with a higher leverage ratio for longer. In the case of rational expectations, banks need to delever immediately which cuts credit supply and induces the economy to contract. Under learning, agents slowly learn about the new share of diversion which is why the economy also contracts but more slowly than in the case of rational expectations. From a relative perspective, banks' borrowing constraints are less strongly binding, which induces the spread to rise by less, i.e. the difference in medians is negative. As a consequence, the price for capital is higher under learning for the first periods due to a relative higher demand for capital thus resulting in a higher level for investment. In the end, output under learning is higher than under rational expectations.

When agents learn about the new diverting behavior of bankers, they force banks to delever, otherwise they would stop or reduce the provision of funds. The levels of output, investment, inflation, the price of capital, and capital under learning fall below their counterparts under rational expectations. At the same time, the credit spread in the learning case exceeds its value for the rational expectations case in the medium and long run. Because of the higher value of diversion, net worth falls short of its value under rational expectations for almost the entire horizon, which is translated into the behavior of the leverage ratio. In the process of deleveraging, the economy under rational expectations outperforms its counterpart under learning.

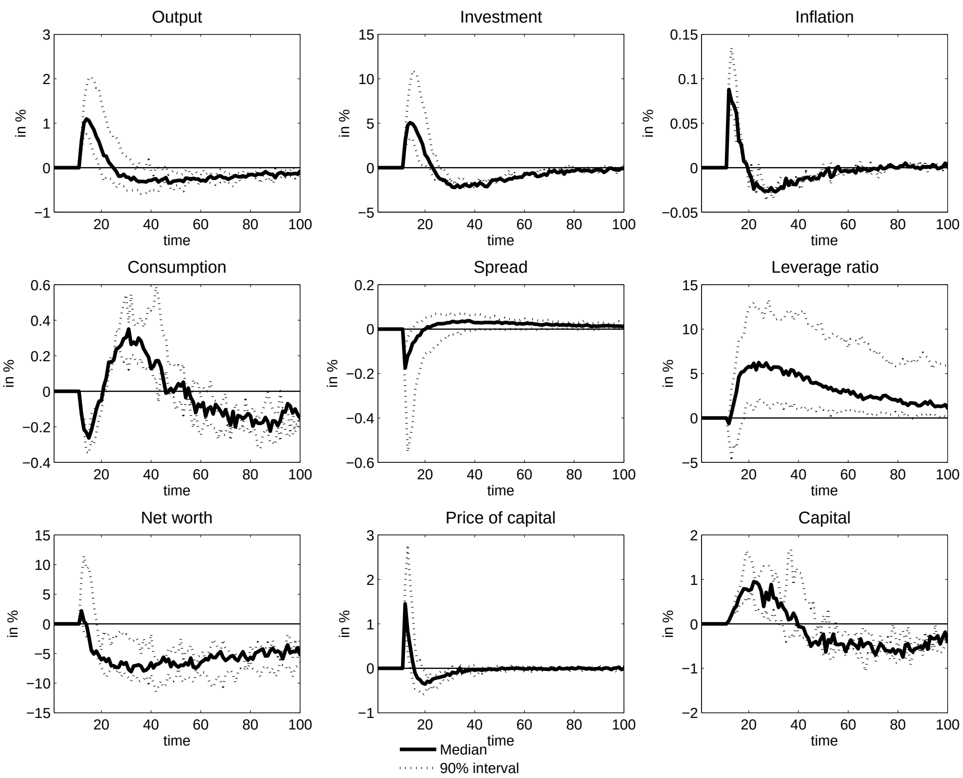
Form this point of view, the economy is stimulated under imperfect information compared with rational expectations if banks increase the amount they would divert if they could. When agents learn about their behavior, the economy performs worse under learning than under rational expectations in the medium run. Hence, a few periods of overshooting are followed by a significant time of undershooting. This means that the learning scenario produces a statistically significant amplification of the macroeconomy with respect to the rational expectations benchmark. In our case, output is more than one percent higher in the periods following the break. Investment is the driving force, which is temporarily 5 percent higher under learning. The overshooting is stronger than the undershooting, which reaches a maximum at around 0.5 percent. However, the undershooting is more persistent than the overshooting. Output is still below the level consistent with rational expectations after 30 years. Obviously, imperfect information about misbehavior in the banking sector creates a less severe recession compared with rational expectations before the economy realizes persistently less output under learning compared with rational expectations. Since only the outcomes of the leverage ratio and related variables are strongly driven by steady state changes (see subsection 3), the effects on macroeconomic variables we presented in this subsection are solely related to the business cycle.[13]

6.3.2 Cross-sectional variations: higher uncertainty

The explanations above have drawn on the relative behavior of medians for the learning and the rational expectations cases. As can be seen from Fig. 2, there is a substantial variation in the value for the actual steady state diversion share. We noticed that this variation can be interpreted as uncertainty about this parameter. Since values in this area also produce remarkably different values for the macroeconomic outcomes, which is also evident in Fig. 7, we can interpret this as uncertainty regarding macroeconomic and financial variables. Thus, the behavior of (median) outcomes goes hand in hand with an increase in uncertainty after the structural change in the diversion share has occurred. As a measure for uncertainty, we take the cross-sectional perspective and calculate the standard deviations across the draws for every (simulated) period in time. This can be done for both cases, learning and rational expectations. Following on from this exercise, we obtain two series of standard deviations which are measures for cross-sectional variations or simply uncertainty. In Figure 8, we present the differences in these standard deviations for every period in time between learning and rational expectations. An increase in this number shows that the bands around the median become wider in the learning case compared with rational expectations. The uncertainty about output is around 25 percent higher under under learning compared with rational expectations while it is nearly 40 percent higher for investment. Since the financial sector in the model yields a very strong prop-

[13]It should be noted that the results are not specific to our draws of the shocks because we repeat the simulation 1000 times and the law of large numbers applies. The bands around the median are quite small for macroeconomic variables.

Figure 7: Difference in medians from simulated data between learning and rational expectations



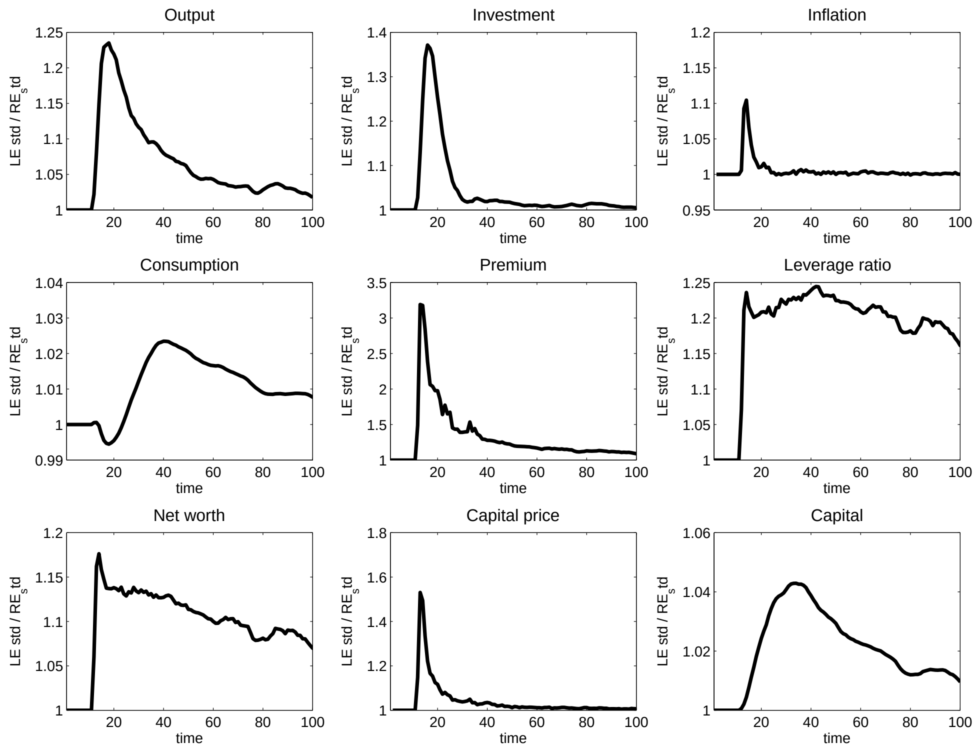
Note: The figure shows the differences between outcomes of the learning and the rational expectations case. The solid lines are based on the median outcome, while the black dashed lines are related to the 90% confidence interval.

agation mechanism, the standard deviation of credit spreads is even 300 percent higher under learning than under rational expectations. With the exception of consumption, the cross-sectional standard deviation is always higher under learning than under rational expectations. From this point of view, the uncertainty about the outcomes of economic variables is greater under learning. In addition to the sign of our indicator, its persistency also differs remarkably across the variables. While the cross-sectional standard deviation of investment, inflation, the credit spread, and the capital price under learning returns quickly to the value under rational expectations, this is not true of consumption, the capital stock, net worth, and the leverage ratio. Obviously, uncertainty about the steady state value of diversion is predominantly translated into uncertainty about net worth and the leverage ratio with effects on capital. Summing up, learning increases the cross-sectional dispersion of variables' outcomes after the occurrence of a structural change that cannot be perfectly observed by the agents.

6.3.3 Business cycle implications: more volatility

Above, we discussed the cross-section of simulations which we interpreted as uncertainty about outcomes induced into the economy by introducing imperfect information. However,

Figure 8: Variation of learning outcomes relative to rational expectations' outcomes across simulated data for each period of time



Note: The figure shows the relation of the cross-sectional standard deviation in the learning case to the rational expectations counterpart for every point in time.

we are also interested in how the volatility of macroeconomic and financial variables and their correlations change over time. The discussion of responses following economic shocks has already indicated that there is an amplification of shocks. This amplification might entail a higher volatility of the economy as a whole. To shed light on these issues, we put our focus on second moments, i.e. we calculate variances for a specific variable for every simulation and take the corresponding medians and 10 and 90 percentiles of this exercise. The same is done for correlations between selected variables. Instead of using the entire simulated period, we start after the break has occurred and stop after period 60, which is slightly more than eleven years. We take this period because we want to focus on the regime in which the bankers have changed their behavior. The results are given in Table 3. By drawing on the medians of standard deviations, we can see that the economy is basically more volatile in the case of learning than in the case of rational expectations.[14] Output is 8% more volatile in the learning case compared with the rational expectations case. Inflation also shows a higher variation in the learning case of about 1%. However, the standard deviation of consumption remains the same while that of net worth is even smaller under learning. The leverage ratio, however, once again exhibits a higher volatility

[14] Although the percentiles are overlapping for both cases. Nevertheless, we draw on the medians.

(about 7% more volatile under learning).

The cross correlations also change slightly by moving from rational expectations to learning. Learning yields an increase of correlations between macroeconomic variables. The correlation between output and investment increases by a factor of 1.02 while the correlation between output and inflation rises by about less than 1%. Interestingly, the bands around the latter are quite large and the value for the 10% percentile is even negative in both cases. A similar situation arises for the correlation between the leverage ratio and capital as well as for the correlation between output and the leverage ratio. However, the corresponding median is negative but the 90% percentile indicates that the correlations can also take positive numbers. While the median correlation between output and the leverage ratio becomes larger under learning, the same is not true of the correlation between the leverage ratio and capital, and between the leverage ratio and net worth.

Table 3: Comparison of simulated moments for period 12 to 60

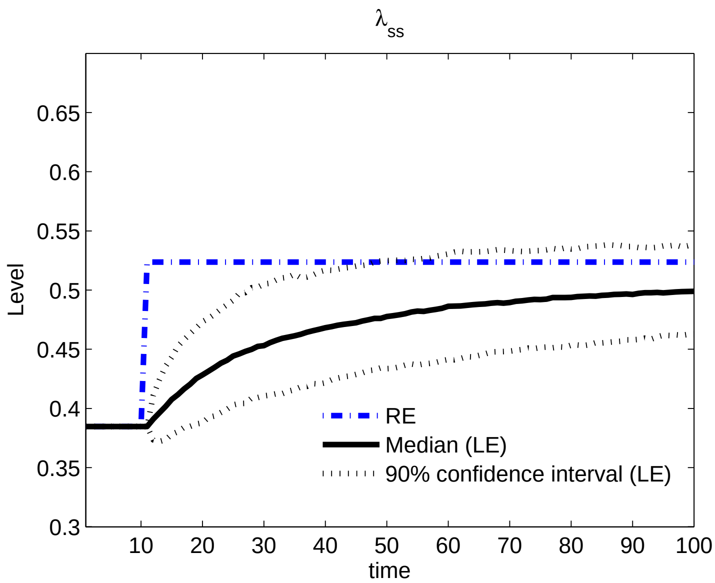
	Rational expectations			Learning		
Variable	10%	Median	90%	10%	Median	90%
			Standard deviations			
Output	0.0132	0.0189	0.0274	0.0139	0.0204	0.0296
Investment	0.0128	0.0196	0.0294	0.0134	0.0201	0.0305
Consumption	0.0067	0.0126	0.0217	0.0067	0.0126	0.0219
Inflation	0.7308	0.8576	1.0131	0.7310	0.8645	1.0107
Capital	0.2940	0.5153	0.8355	0.2998	0.5322	0.8620
Net worth	0.2499	0.4738	0.8268	0.2564	0.4675	0.8262
Leverage ratio	19.65	38.35	61.49	22.22	40.95	67.85
			Cross correlations			
Output, Investment	0.4411	0.8285	0.9534	0.5006	0.8460	0.9581
Output, Inflation	-0.1836	0.1389	0.4250	-0.1858	0.1401	0.4429
Output, Leverage ratio	-0.8333	-0.5104	0.1014	-0.8014	-0.4361	0.1941
Leverage ratio, Capital	-0.8385	-0.2280	0.4705	-0.8629	-0.3019	0.4376
Leverage ratio, Net worth	-0.9793	-0.9473	-0.6876	-0.9819	-0.9518	-0.7560

Regarding the interpretation of our experiment, there is one caveat. As the original model, which is simulated for the first ten periods, is calibrated to match some moments in the data, both the rational expectations and the learning approach after ten periods must be worse in matching the data. As learning sluggishly approximates the new rational expectations equilibrium, for the phase of transition it is closer to the old RE equilibrium. Therefore learning must match the actual moments in the data better by construction. However, as we concentrate on a one-time shift in the bankers' amount of diversion we cannot compare the new regime with data. This is why we focus here on a relative comparison between the two approaches.

7 Robustness: Strength of the Signal

A crucial parameter in the whole setup is the size of the signal that agents receive. As households and firms obtain the signal only once, the strength of the signal determines the speed with which they apprehend how much funds the bankers abscond. In this section, we offer some robustness checks as to how much the amplification of the business cycle is driven by the signal. We simulate both cases 1000 times just as in the benchmark case and thereby offer two additional sets of results where, on the one hand, the signal is half the size of the benchmark case and, on the other hand, with a signal twice the size. This has enormous implications for the adjustment of the perceived steady state parameter of $\bar{\lambda}$ to the true value of the rational expectations benchmark.

Figure 9: Evolution of the steady state share of diversion under learning and rational expectations (perceived steady state) with a lower signal

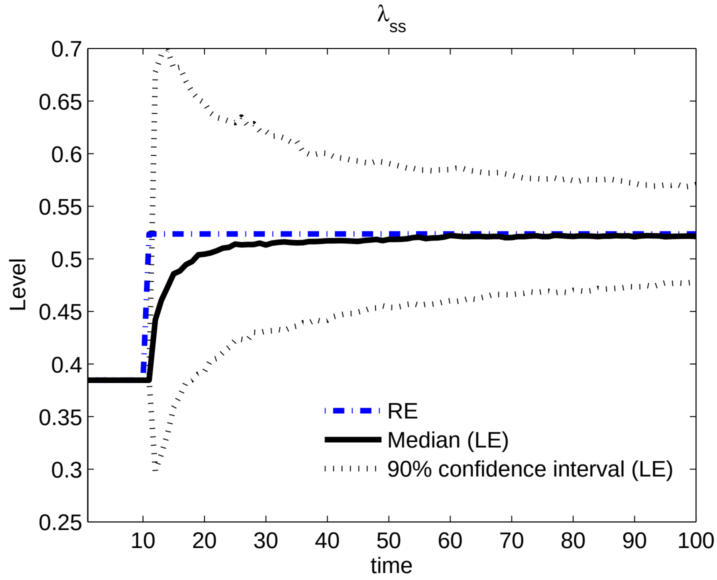


Note: The figure shows how the diversion share evolves in the rational expectations case (blue dashed line with dots) and in the learning case (solid black line) with half the signal compared with the benchmark case. The black dotted lines represent the boundaries for the 90% confidence interval in the learning case.

Figure 9 depicts the true steady state of the rational expectations case and the perceived one with a signal that is half the size of the benchmark case. Two points are worth mentioning: first, the adjustment to the true steady state takes much longer and, even at the end of the simulation horizon, the median has still not reached the true parameter. Secondly, the confidence interval is much narrower over the whole sample and therefore also significantly different from the true value for a prolonged period of time. The volatility of the perceived steady state could be translated into the spread's standard deviation of 10 BP, which is about 70% lower than what we calibrated in our benchmark case. Additionally, Figure 10 plots the information of the perceived steady state with a much stronger signal.

The rational expectations steady state value is again the same, but the median of the perceived steady state is approaching the true parameter much faster and reaches the

Figure 10: Evolution of the steady state share of diversion under learning and rational expectations (perceived steady state) with a higher signal

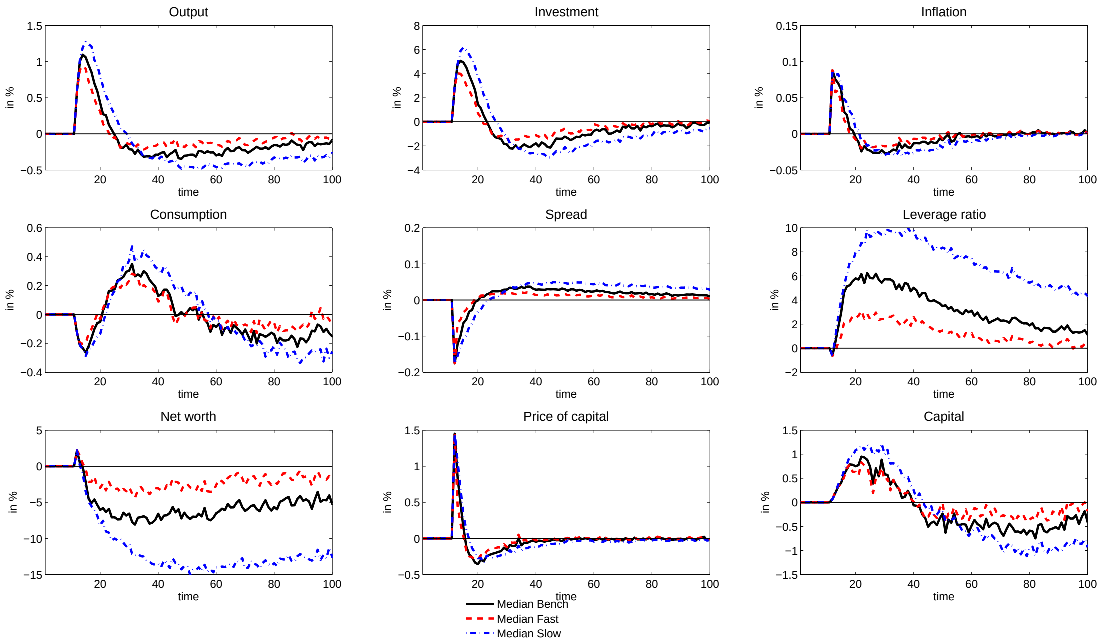


Note: The figure shows how the diversion share evolves in the rational expectations case (blue dashed line with dots) and in the learning case (solid black line) with twice the signal compared with the benchmark case. The black dotted lines represent the boundaries for the 90% confidence interval in the learning case.

true value after only a couple of periods. The volatility is much higher and includes the true value from the first period after the shift occurs. This enormous volatility could once again be translated into the volatility of the spread. Given this signal, the dispersion of the spread would be approximately 67.5 BP which is about 50% higher compared with our benchmark assumption. With those differences in the speed of adjustment, we can look at the actual differences for the amplifications of the business cycle. Figure 11 plots the resulting medians for the benchmark case (see above) and with a strong and weak signal.

As expected, the outcomes in the benchmark case for all variables are in the middle of the two extreme cases. What is rather unexpected, however, is the fact that although the variation of the signal is quite strong and the adjustment of the perceived steady state is very diverse, the amplification of the business cycle is very robust and quantitatively not too different across the regimes. This can best be seen in the outcomes for output and investment where the differences of both extreme cases in the initial peak are only half and two percentage points, respectively. The banking sector variables exhibit the largest deviations, which is also not very surprising as the learning mechanism includes, above all, a financial variable. The leverage ratio increases by around 10% in the case where learning is very slow compared with around 3% with faster learning. Net worth exhibits a similar pattern with a decrease of close to 15% (always relative to the rational expectations benchmark) if the signal is weak and only a slight decrease of around 3% when the signal is strong. Variables, such as inflation, the price of capital and the spread, are not significantly affected by the differences in the signal. Thus, the result of an amplification of the business cycle with respect to rational expectations holds very robustly also when

Figure 11: Different outcomes of learning vs. rational expectations contingent on the strength of the signal



Note: The figure shows how different values for the signal affect the amplification of the business cycle. We contrast the median under the benchmark case with the median when the signal is strong (median fast) and the signal is weak (median slow).

we vary the degree of the signal.

8 Conclusion

We introduced imperfect information into the banking model of Gertler and Karadi (2011). In the modified model, agents need to learn about a permanent change in the misbehavior of bankers. Bank managers like to divert a fraction of bank resources which cannot be perfectly observed. We show that bank managers can exploit the period of learning in the sense that the leverage ratio remains higher for a longer period of time. If learning occurs, deleveraging is initiated. Compared with the new rational expectations solution, amplification as a result of financial frictions is stronger under learning after the structural change has occurred. Imperfect information combined with learning contributes to a rise in economic activity driven by investments in the first year after the structural change in the diverting behavior of bankers. Learning about the new misbehavior of bankers coincides with a contraction in economic activity, i.e. there is a short overshooting and a more persistent undershooting in the learning case compared with rational expectations. However, there is remarkable uncertainty about economic outcomes during the overshooting period, i.e. the realized outcomes can vary significantly across the simulations. Because of the incentive constraint in the model of Gertler and Karadi (2011), the bank is not exposed to default risk. This is the reason why bank runs cannot be treated in this model although we borrow arguments from Diamond and Dybvig (1983), for example. We introduced imperfect information into an agency problem which stems from limited enforcement.

Although we focus on the consequences of learning on business cycle dynamics and facts, we cannot compare our results with real data because of our simulation design. We were interested in investigating the effects following a structural change in factors related to the financial contracting problem that controls financial frictions in the banking sector under learning. For this reason, we apply a structural break that agents must learn about. An econometric analysis dealing with these issues is left for future research.

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A Optimality of the contract under learning

The objective of this section is to discuss the circumstances under which the contract is still feasible in the learning case. In general, the contract is feasible if there is no incentive for the bankers to run. In the model, it is assumed that bankers would like to divert. They have to decide in which cases they have a profit from running and how they profit from staying. If they run, bankers take the resources they divert $\lambda Q_t K_t$ and get what remains after liquidation of the bank. Thus, the value of running becomes

$$\begin{aligned} V_t^{run} &= N_t - Q_t K_t + \lambda Q_t K_t + D_t \\ &= N_t - (1-\lambda) Q_t K_t + D_t. \end{aligned} \tag{37}$$

In the no-default case, bankers accumulate net worth over time, the discounted value of which is

$$V_t^{no-run} = N_t + E_t \Lambda_{t,t+1} V_{t+1}. \tag{38}$$

Consequently, bankers do not run if the value for the no-run/no-default case is at least greater than the value they would realize in the case of default $V_t^{no-run} \geq V_t^{run}$. The well-known incentive constraint follows from taking Equations (37), (38), and the balance sheet constraint into account

$$\begin{aligned} V_t^B &\geq NW_t^B - (1-\lambda) Q_t K_t + D_t \\ &\geq \lambda Q_t K_t. \end{aligned} \tag{39}$$

In the rational expectations case, creditors know the bankers' incentive constraint and provide only funds up to the limit where no defaults occur. In all other cases, bankers have an incentive to run.

Next, we assume that bankers can increase the share of diversion that cannot be observed by households. Hence, a new value for λ must be taken into account. However, bankers only have an incentive to stay if they can generate an intertemporal profit from this situation. In the case of default, bankers exit and return to their household. However, households would replace the exiting bankers with new ones in the new period. Given that all bankers behave in the same way, the new bankers would operate with the new diversion share as creditors take this into account. From this, it follows that bankers' future net worth is affected, i.e. the net worth for the new bankers. In an intertemporal consideration, this effect can be taken into account. Bankers do not have an incentive to run if there is an intertemporal profit from staying, given the new diversion share. Departing from the previous value of the bank V_t^{old}, we assume that a new value V_t^{new} can be achieved, whereas the profit from staying is X_t. Thus, bankers do not run as long as there is an intertemporal profit from staying, given the new diversion share

$$\begin{aligned} V_t^{old} + X_t &\geq N_t - (1-\lambda^{new}) Q_t K_t + D_t \\ V_t^{new} &\geq N_t - (1-\lambda^{new}) Q_t K_t + D_t. \end{aligned} \tag{40}$$

Based upon the new diversion share, a new incentive constraint would arise $V_t^{new} \geq \lambda^{new} Q_t K_t$. Consequently, bankers do not run if the new value of the bank exceeds at least the old one and this difference is greater than the difference between the new and old share of diversion

$$V_t^{new} - V_t^{old} \geq \left(\lambda^{new} - \lambda^{old}\right) Q_t K_t$$

or

$$X_t \;\; \geq \;\; \left(\lambda^{new} - \lambda^{old}\right) Q_t K_t, \tag{41}$$

which simply means that the intertemporal profit from staying must be at least as large as the difference in the new opportunity to divert. Hence, the inequality constraint in Equation (40) can be re-written with the help of Equation (41) (assuming equality for the moment) and becomes

$$\begin{aligned}
V_t^{old} + X_t &\geq N_t - \left(1 - \lambda^{new}\right) Q_t K_t + D_t \\
V_t^{old} + \left(\theta^{new} - \theta^{old}\right) Q_t K_t &\geq N_t - \left(1 - \lambda^{new}\right) Q_t K_t + D_t \\
V_t^{old} + \left(\theta^{new} - \theta^{old}\right) Q_t K_t &\geq N_t - \left(1 - \lambda^{new}\right) Q_t K_t + Q_t K_t - N_t \\
V_t^{old} &\geq -\left(\lambda^{new} - \lambda^{old}\right) Q_t K_t - \left(1 - \lambda^{new}\right) Q_t K_t + Q_t K_t \\
V_t^{old} &\geq \lambda^{old} Q_t K_t. \qquad (42)
\end{aligned}$$

As can be seen in Equation (42), the old incentive constraint remains as long as Equation (41) is satisfied. Since creditors can only observe the old diversion share and believe that its value remains, the perceived incentive constraint of creditors is $V_t \geq \lambda^{old} Q_t K_t$, such that creditors provide funds based on the old diversion share and bankers do not run as long as Equation (41) is not violated.

The relevant question is now to determine X_t. Following the argument that new bankers would be faced with the new diversion share in the next period, they would obtain less funds given a specific value of net worth. By dropping time indices (for the sake of simplicity but without the loss of generality), the old leverage ratio in equilibrium is set in relation to the old diversion share

$$\phi^{old} \;\; = \;\; \frac{(QK)^{old}}{N} = \frac{N + D^{old}}{N} = \frac{\eta}{\lambda^{old} - \nu}.$$

Consequently, the old value of external funds D^{old} implicitly depends *c.p.* on the old diversion share

$$D^{old} = \left(\frac{\eta}{\lambda^{old} - \nu} - 1 \right) N.$$

For the new diversion share, a similar expression can be derived, i.e. the new leverage ratio is linked to the new diversion share

$$\phi^{new} \;\; = \;\; \frac{(QK)^{new}}{N} = \frac{N + D^{new}}{N} = \frac{\eta}{\lambda^{new} - \nu}$$

and a new leverage ratio *c.p.* evolves because less external funds would be available

$$D^{new} = \left(\frac{\eta}{\lambda^{new} - \nu} - 1 \right) N.$$

Thus, the difference between the new and the old level of external funds is *c.p.* related to the difference between the old and the new diversion share.

$$D^{new} - D^{old} \;=\; \eta\left(\frac{\lambda^{old} - \lambda^{new}}{\left(\lambda^{new} - \nu\right)\left(\lambda^{old} - \nu\right)}\right) N \tag{43}$$

As can be seen, the new level of external funds will fall short of the old level if the new diversion share exceeds the old one. Since the new value of total assets is smaller than the old value

$$(QK)^{old} = N + D^{old} > (QK)^{new} = N + D^{new}$$

it follows that the value of the bank, given the new diversion share, will be smaller than the old one

$$V^{new} < V^{old}\, c.p.$$

which creates the incentive to run.

By assuming that creditors cannot observe the new diversion share, there will be savings in funds. Now assume that total assets shall be kept constant, i.e. $(QK)^{old} = (QK)^{new}$. Starting from the new levels of external funds and by keeping net worth constant, this can only be realized if there is an additional source of funds Z, which can simply be expressed as $Z = D^{old} - D^{new}$. If changes in external funds might be related to the change in the diversion share, we can relate Z to Equation (43) from which it follows that

$$Z \;=\; \eta\left(\frac{\lambda^{new} - \lambda^{old}}{\left(\lambda^{new} - \nu\right)\left(\lambda^{old} - \nu\right)}\right) N \;.$$

Hence, there is no incentive to run as long as the generated funds exceed at least the additional profit from diversion $Z \geq \left(\lambda^{new} - \lambda^{old}\right) QK$, which can be expressed as $D^{old} - D^{new} \geq \left(\lambda^{new} - \lambda^{old}\right) QK$. Consequently, we can establish the inequality constraint

$$\begin{aligned} \eta\left(\frac{\lambda^{new} - \lambda^{old}}{\left(\lambda^{new} - \nu\right)\left(\lambda^{old} - \nu\right)}\right) N \;&\geq\; \left(\lambda^{new} - \lambda^{old}\right) QK \\ \frac{\eta}{\left(\lambda^{new} - \nu\right)\left(\lambda^{old} - \nu\right)} \;&\geq\; \frac{QK}{N} = \phi \end{aligned}$$

The last equation gives the conditions under which there is an incentive for the bankers not to run, given the possibility of increasing the share of diversion. This must hold in every period by taking into account that changes in the share of diversion might also affect η and ν in equilibrium. If the bankers take into account that new bankers might get less resources for every unit of net worth, given the new diversion share, the saved funds can be internalized, which affects the decision to run. For this reason the savings for each individual banks must be at least larger than the additional resources households provide to new entering bankers.

B Steady state solution strategy following changes in $\bar{\lambda}$

For the strategy to induce changes in $\bar{\lambda}$ to have an effect on steady state variables, we start from Eq. (36). Agents extract λ_t^C, which is $\lambda_t^C = \left(1-\rho^{\lambda}\right)\log\left(\bar{\lambda}_t\right)$, from outcomes. Consequently, we have $\bar{\lambda}_t = \exp\left(\lambda_t^C / \left(1-\rho^{\lambda}\right)\right)$. Given the deep parameters, we solve for ϕ_{ss}

$$\phi_{ss} = \frac{\eta_{ss}}{\bar{\lambda}_t - \nu_{ss}}$$

given

$$\begin{aligned} x_{ss} &= z_{ss} = \left(R_{k,ss} - R_{ss}\right)\phi_{ss} + R_{ss} \\ \nu_{ss} &= (1-\theta)\,\beta\left(R_{k,ss} - R_{ss}\right)/\left(1-\beta\theta x_{ss}\right) \\ \eta_{ss} &= (1-\theta)\,\beta R_{ss}/\left(1-\beta\theta x_{ss}\right), \end{aligned}$$

where $R_{k,ss}$, R_{ss}, θ, and β are calibrated. By knowing ϕ_{ss}, we can also get the value for ω:

$$\begin{aligned} \omega &= \left(1-\theta\left[\left(R_{k,ss} - R_{ss}\right)\phi_{ss} + R_{ss}\right]/\phi_{ss}\right) \\ &= \left(1-\theta R_{ss}\right)/\phi_{ss} - \theta\left(R_{k,ss} - R_{ss}\right). \end{aligned}$$

As can be seen, $\bar{\lambda}_t$ is related to ω and can be used interchangeably to reflect the behavior of bankers. An increase in the steady state value of diversion decreases the leverage ratio, which means that ω is going up. Following from our calibration strategy, all other steady state values are not affected.

C Log-linearized model

$$
\begin{array}{rcl}
\upsilon_{ss}log(\upsilon_t) & = & Const_{\upsilon} + (1-\theta)\beta(R_{kss}E_t log(r_{kt+1}) - R_{ss}log(r_t)) + (1-\theta)\beta(R_{kss} - R_{ss})E_t log(\Lambda_{t+1}) \\
 & & +\beta\theta xt_{ss}\upsilon_{ss}(E_t log(\Lambda_{t+1}) + E_t log(xt_{t+1}) + E_t log(\upsilon_{t+1})) \\
log(\eta_t) & = & Const_{\eta} + \beta\theta ZT_{ss}(E_t log(\Lambda_{t+1} + E_t log(zt_{t+1} + log(\xi_{t+1})) \\
log(xt_t) & = & Const_{XT} + log(\phi_t) - log(\phi_{t-1}) + log(zt_t) \\
log(q_t) & = & Const_{I_n} + \eta^i(log(i_{nt}) - log(i_{nt})/I_{ss} - \beta\upsilon_{ss}\eta^i(E_t log(i_{nt+1} - log(i_{nt}))/I_{ss} \\
Y_{ss}log(y_t) & = & Const_C + C_{ss}log(c_t) + I_{ss}log(i_t) \\
log(\Lambda_t) & = & Const_{\Lambda} + log(\varrho_t) - log(\varrho_{t-1}) \\
log(\pi_t^*) & = & Const_F + log(f_t) - log(z_t) \\
Z_{ss}log(z_t) & = & Const_Z + Y_{ss}log(y_t) + \beta\gamma\Lambda_{ss}\pi_{ss}(\gamma^p - 1)Z_{ss}(E_t log(\Lambda_{t+1}) + \gamma^p log(\pi_t) - log(\pi_{t-1}) \\
 & & +E_t log(z_{t+1})) \\
log(rn_t) & = & Const_{\Pi} + \rho^i log(rn_{t-1}) + (1-\rho^i)(\kappa_\pi * log(\pi_t) + \kappa_x * log(x_t) + \epsilon_t^i \\
log(r_{kt}) & = & Const_{R_k} + log(\xi_t) + \dfrac{1}{R_{kss}}(\alpha\dfrac{Y_{ss}}{K_{ss}}(-log(x_t) + log(y_t) - log(k_{t-1}) - log(xt_t)) + log(q_t) \\
 & & -\delta_{ss}log(\delta_t)) - log(q_{t-1}) \\
ZT_{ss}log(zt_t) & = & Const_{ZT} + \phi_{ss}I_{ss}(R_{kss}log(rk_t) - R_{ss}log(r_{t-1})) + \phi_{ss}(R_{kss} - R_{ss})log(\phi_{t-1}) + R_{ss}log(r_{t-1}) \\
log(\eta_t) & = & Const_{\Phi} + \dfrac{\phi_{ss}\lambda_{ss}}{\upsilon_{ss}}(log(\phi_t) + log(\lambda_t) + log(\upsilon_t)) \\
log(q_t) + log(k_t) & = & Const_Q + log(\phi_t) + log(n_t) \\
N_{ss}log(n_t) & = & Const_N + NE_{ss}log(ne_t) + NN_{ss}log(nn_t) \\
log(ne_t) & = & Const_{NE} + log(zt_t) + log(n_{t-1}) \\
log(nn_t) & = & Const_{NN} + log(q_t) + log(\xi_t) + log(k_{t-1}) \\
log(y_t) & = & Const_Y + log(a_t) + \alpha * log(u_t) + \alpha * log(\xi_t) + \alpha * log(k_{t-1}) + (1-\alpha) * log(l_t) \\
\dfrac{\delta_{ss}}{(\delta_{ss} - \delta_{ss})}log(\delta_t) & = & Const_\delta + (1+\vartheta)log(u_t) \\
-log(x_t) + log(y_t) - log(u_t) & = & Const_U + \vartheta * log(u_t) + log(\xi_t) + log(k_{t-1}) \\
log(i_{nt}) & = & Const_I + I_{ss}log(i_t) - \delta_{ss}K_{ss}(log(k_{t-1}) + log(\xi_t) + log(\delta_t)) \\
K_{ss}log(k_t) & = & Const_K + K_{ss}log(k_{t-1}) + K_{ss}log(\xi_t) + log(i_{nt}) \\
log(\varrho_t) & = & Const_{\varrho} - \dfrac{1}{((1-\beta h^C)(1-h^C))}(log(c_t) - h^C * log(c_{t-1}) - \beta h^C(E_t log(c_{t+1}) - h^C * log(c_t))) \\
E_t log(\Lambda_{t+1}) + log(r_t) & = & Const_R \\
-log(x_t) + log(y_t) - log(l_t) & = & Const_L - log(\varrho_t) + \varphi * log(l_t) \\
-log(x_t) & = & Const_X + log(p_{mt}) - log(\pi) \\
F_{ss}log(f_t) & = & Const_{P_m} + Y_{ss}P_{mss}(log(y_t) + log(p_{mt})) + \beta\gamma\Lambda_{ss}F_{ss}(E_t log(\Lambda_{t+1}) + log(\pi_t) \\
 & & -log(\pi_{t+1}) + E_t log(f_{t+1})) \\
log(\pi_t) & = & Const_{\Pi^*} + \gamma\gamma^p log(\pi_{t-1}) + (1-\gamma)log(\pi_t^*) \\
log(rn_t) & = & Const_{RN} + log(r_t) + E_t log(\pi_{t+1}) \\
log(IRS_t) & = & Const_{IRS} + E_t log(r_{kt+1}) - log(r_t) \\
log(a_t) & = & Const_A + \rho_a log(a_{t-1}) - \epsilon_t^A \\
log(\xi_t) & = & Const_\xi + \rho_\xi log(\xi_{t.-1}) - \epsilon_t^\xi \\
log(\lambda_t) & = & \lambda^C + \rho_\lambda log(\lambda_{t-1}) - \epsilon_t^\lambda
\end{array}
$$

with the constants given by:

Constant	Expression
$Const_\upsilon$	$-(1-\theta)\beta RK_{ss}log(R_{kss})+(1-\theta)\beta R_{ss}log(R_{ss})-((1-\theta)\beta(R_{kss}-R_{ss})$
	$+\beta\theta XT_{ss}\upsilon_{ss})log(\Lambda_{ss})-\beta\theta XT_{ss}\upsilon_{ss}log(X_{ss})-(\beta\theta XT_{ss}\upsilon_{ss}-\upsilon_{ss})log(\upsilon_{ss})$
$Const_\eta$	$(1-\beta\theta ZT_{ss})log(\xi_{ss})-\beta\theta ZT_{ss}(log(\Lambda_{ss})+log(ZT_{ss}))$
$Const_{XT}$	$log(XT_{ss})-log(ZT_{ss})$
$Const_{I_N}$	$-log(Q_{ss})$
$Const_C$	$-log(Y_{ss})+\frac{C_{ss}}{Y_{ss}}log(C_{ss})+\frac{I_{ss}}{Y_{ss}}log(I_{ss})$
$Const_\Lambda$	$log(\Lambda_{ss})$
$Const_F$	$log(F_{ss})-log(Z_{ss})-log(\Pi^*_{ss})$
$Const_Z$	$-Y_{ss}log(Y_{ss})-\beta\gamma\Lambda_{ss}\Pi_{ss}^{(\gamma_p-1)}Z_{ss}(log(\Lambda_{ss})+(\gamma_p-1)log(\pi_{ss}))$
	$-(\beta\gamma\Lambda_{ss}\Pi_{ss}^{(\gamma_p-1)}Z_{ss}-Z_{ss})log(Z_{ss})$
$Const_\Pi$	$(1-\rho_i)\kappa_\pi log(\Pi_{ss})+(1-\rho_i)\kappa_x log(X_{ss})-(1-\rho_i)log(RN_{ss})$
$Const_{R_k}$	$log(R_{kss})-log(\xi_{ss})-\frac{1}{R_{kss}}(\alpha\frac{Y_{ss}}{K_{ss}})(-log(X_{ss})+log(Y_{ss})-log(K_{ss})$
	$-log(\xi_{ss}))-\frac{1}{R_{kss}}log(Q_{ss})+\frac{1}{RK_{kss}}\delta_{ss}log(\delta_{ss})+log(Q_{ss})$
$Const_\Phi$	$-log(\eta_{ss})+\phi_{ss}\frac{\lambda_{ss}}{\upsilon_{ss}}(log(\phi_{ss})+log(\lambda_{ss})-log(\upsilon_{ss}))$
$Const_{ZT}$	$ZT_{ss}log(ZT_{ss})-\phi_{ss}R_{kss}log(R_{kss})+(\phi_{ss}R_{kss}R_{ss}-R_{ss})log(R_{ss})$
	$-log(\phi_{ss})(\phi_{ss}(R_{kss}-R_{ss}))$
$Const_Q$	$log(Q_{ss})+log(K_{ss})-log(\phi_{ss})-log(N_{ss})$
$Const_N$	$N_{ss}log(N_{ss})-NE_{ss}log(NE_{ss})-NN_{ss}log(NN_{ss})$
$Const_{NE}$	$log(NE_{ss})-log(ZT_{ss})-log(N_{ss})$
$Const_{NN}$	$log(NN_{ss})-log(Q_{ss})-log(\xi_{ss})-log(K_{ss})$
$Const_Y$	$log(Y_{ss})-log(AA_{ss})-\alpha log(U_{ss})-\alpha log(\xi_{ss})-\alpha log(K_{ss})-(1-\alpha)log(L_{ss})$
$Const_\delta$	$\frac{\delta_{ss}}{(\delta_{ss}-\delta_{ss})}log(\delta_{ss})-(1+\vartheta)log(U_{ss})$
$Const_U$	$(-1-\vartheta)log(U_{ss})+log(Y_{ss})-log(X_{ss})-log(\xi_{ss})-log(K_{ss})$
$Const_I$	$I_{ss}log(I_{ss})-\delta_{ss}K_{ss}(log(K_{ss})+log(\xi_{ss})+log(\delta_{ss}))-log(I_{nss})$
$Const_K$	$-log(\xi_{ss})-\frac{I_{nss}}{K_{ss}}log(I_{nss})$
$Const_\varrho$	$log(\varrho_{ss})$
$Const_R$	$log(R_{ss})+log(\Lambda_{ss})$
$Const_L$	$(1+\varphi)log(L_{ss})-log(\varrho_{ss})-log(Y_{ss})+log(X_{ss})$
$Const_X$	$-log(X_{ss})-log(P_{mss})+log(\Pi_{ss})$
$Const_{P_m}$	$log(F_{ss})(-F_{ss}+\beta\gamma\Lambda_{ss}F_{ss})+\beta\gamma\Lambda_{ss}F_{ss}log(\Lambda_{ss})$
	$+Y_{ss}P_{mss}(log(P_{mss})+log(Y_{ss}))$
$Const_{\Pi^*}$	$log(\Pi^*_{ss})(1-\gamma)-(1-\gamma\gamma_p)log(\Pi_{ss})$
$Const_{RN}$	$log(RN_{ss})-log(R_{ss})-log(\Pi_{ss})$
$Const_{IRS}$	$log(IRS_{ss})-log(RK_{kss})+log(R_{ss})$
$Const_A$	$log(A_{ss})(1-\rho_a)$
$Const_\xi$	$log(\xi_{ss})(1-\rho_\xi)$
λ^C	$log(\lambda_{ss})(1-\rho_\lambda)$

D Calibrated parameters

Table 4: Calibrated parameters of the model

Description	Parameter	Value
Discount rate	β	0.99
Relative utility weight of labor	χ	3.409
Habit parameter	h^C	0.815
Inverse Frisch elasticity of labor supply	ϕ	0.276
Effective capital share	α	0.33
Elasticity of substitution	ε	4.167
Elasticity of marginal depreciation wrt utilization rate	ϑ	7.2
Inverse elasticity of net investment to the price of capital	η_i	1.728
Calvo parameter, probability of keeping goods' prices fixed	γ	0.779
Price indexation	γ_p	0.241
Depreciation rate of capital	δ_{ss}	0.025
Steady state capital utilization rate	U	1
Steady state proportion of government expenditure	G_{SS}/Y_{SS}	0.2

E Additional figures

Figure 12: Medians from simulated data under learning learning