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Banks' net interest margin and the level of interest rates

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Non-technical summary

Research Question

Net interest income is the main source of income for banks. As a result of carrying out maturity transformation, banks' net interest income is affected in the short run by the dynamics in the term structure. Therefore, as a rule, an increase in interest rates leads to a decline in net interest income in the following years. However, little is known about the medium and long-term effects of changes in the interest rate level. Anecdotal evidence suggests that net interest income benefits in the medium and long term if interest rates increase. This paper also investigates the effects of the low interest rate environment on the margin for customer deposits.

Contribution

The short-run effect of a change in the interest rate level is widely discussed and empirically investigated in the literature. The medium and long-term effects are harder to detect, however, because short-term developments mask this effect and the applied datasets often cover a period of only up to 20 years. In this paper, the authors use a dataset that covers more than 40 years and a model which nevertheless allows them to differentiate between the short- and long-run effects of a rise in the interest rate level.

Results

The paper shows that banks' net interest income benefits over the medium to long-term horizon if the interest rate level increases. An increase of 100 basis points in the interest rate level leads to an increase in the net interest margin of about 7 basis points. It is therefore possible to demonstrate empirically that the short-term effect and medium to long-term effects on banks' net interest margin are diametrically opposed. Furthermore, the authors find that the margins for retail deposits have declined by up to 97 basis points owing to the low interest rate environment.

Nichttechnische Zusammenfassung

Fragestellung

Das Zinsergebnis ist die wichtigste Einnahmequelle der Banken. Dadurch, dass Banken Fristentransformation betreiben, ist das Zinsergebnis in der kurzen Frist von der Dynamik der Zinsstruktur betroffen. Ein Ansteigen der Zinsen führt dadurch in der Regel zu abnehmenden Zinsergebnissen in den Folgejahren. Über die mittel- und langfristige Wirkung von Änderungen des Zinsniveaus ist dagegen wenig bekannt. Anekdotische Evidenz legt nahe, dass das Zinsergebnis mittel- und langfristig profitiert, wenn das Zinsniveau ansteigt. Auch wird in dem Papier die Wirkung untersucht, die das gegenwärtige Niedrigzinsumfeld auf die Margen der Kundeneinlagen hat.

Beitrag

Der kurzfristige Effekt einer Veränderung der Zinsstrukturkurve wird in der Literatur ausgiebig diskutiert und empirisch untersucht. Mittel- und langfristige Entwicklungen dagegen sind schwerer nachzuweisen, weil kurzfristige Entwicklungen den Effekt überlagern und die verwendeten Zeitreihen in der Regel nur bis zu 20 Jahre betragen. In diesem Papier verwenden die Autoren einen Datensatz, der mehr als 40 Jahre umfasst, und ein entsprechend spezifiziertes Modell, das es ihnen erlaubt, zwischen den kurzfristen Effekten einer Erhöhung des Zinsniveaus auf der einen Seite und deren langfristigen Effekten auf der anderen Seite zu unterscheiden.

Ergebnisse

Das Papier zeigt, dass das Zinsergebnis der Banken mittel- bis langfristig profitiert, wenn sich das Zinsniveau erhöht, und zwar steigt die Zinsspanne um ungefähr 7 Basispunkte je 100 Basispunkte Zinsniveauerhöhung. Damit kann empirisch gezeigt werden, dass der kurzfristige und der mittel- bis langfristige Effekt gegensätzlich sind in ihrer Wirkung auf das Zinsergebnis der Banken. Auch finden die Autoren, dass die Margen für die Kundeneinlagen durch das Niedrigzinsumfeld um bis 97 Basispunkte zurückgegangen sind.

BUNDESBANK DISCUSSION PAPER NO 16 /2015

Banks' Net Interest Margin and the Level of Interest Rates[1]

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Abstract

An increase in the level of interest rates is said to have a negative impact on banks' net interest margins in the short run. Using a time series of more than 40 years for the German banking system, we show that the opposite effect exists in the long run, where an increase in the level of interest rates by 100 basis points leads to an estimated increase of 7 basis points in the banks' net interest margin. In addition, we analyze the consequences of the low-interest rate environment and find that banks' interest margins for retail deposits, especially for term deposits, have declined by up to 97 basis points.

Keywords: Net interest margin, level of interest rates

JEL-Classification: G 21

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1. Introduction

Structural changes in a bank's net interest income – or, equivalently, in its net interest margin – have a huge impact on its profitability and are likely to lead to changes in the bank's behavior, for instance in its risk taking. As a bank's net interest margin results from a mix of interest-bearing products and as the rates of these products are differently linked to (market) interest rates, the structural impact of changes in the (market) interest rate level on this margin is not obvious. The aim of this paper is to empirically establish this important relationship. This issue is especially relevant in a low-interest rate environment or in an environment of structurally falling (or rising) interest rates, because, in normal times, cyclical interest rate movements may mask the structural effect of changes in the interest rate levels; when interest rates fluctuate in an interval between, say, 3% and 6%, it is hard to tell whether the net interest margin is affected by structural changes in the level of interest rates or merely by a cyclical fluctuation of the yield curve.

If all the interest-bearing assets and liabilities are directly linked to market rates and if there is no gap between the volumes of interest-bearing assets and liabilities, then, in the long run, a bank's net interest margin will not be affected by (parallel) shifts in the interest rate level. However, in the short run, the net interest margin may fluctuate as a consequence of shifts in the interest rate level, even if all assets and liabilities are completely linked to market rates. To illustrate this point, we take the example of a bank that recursively invests in long-term government bonds and that finances this investment by issuing short-term bonds. In the short run, its net interest income fluctuates when there is a parallel shift in the term structure of interest rates. This is so, because the assets have a longer maturity than the liabilities, which means that, in a given time span, a portion of the assets is adjusted to the new interest rates which is smaller than the portion of liabilities that is adjusted. In the long run, however, the net interest margin of this bank will be unaffected by parallel shifts in the term structure, because all the assets and all the liabilities will then be adjusted to the new interest rates. In this paper, we will use this as the definition of the net interest margin being independent of the level of interest rates.

In contrast to the example above, there are bank products with no or only weak links to market rates, such as current accounts that are not remunerated. These deposits cause no interest expenses for the bank (although they do lead to administrative costs, which are, however, not part of a bank's interest expenses) and it makes a great difference whether a bank invests these funds in loans at 3 or 10 percent. In cases where the deposits are mainly used as a substitute for money (i.e. payment purposes as the main usage), banks' interest income is comparable to the seigniorage income of central banks, which is proportional to the central bank's policy rate.

To sum up, there is the possibility that, in the long run, a bank's net interest margin is independent of the level of interest rates, for instance in the special cases where all product rates are completely tied to market rates or where the incomplete linkages to market rates on the asset and liability sides are just cancelling each other out. However, based on anecdotal evidence, one can guess that the long-run relationship is positive, meaning that the net interest margin increases when the level of interest rates rises.

In this paper, we address two topics that are connected with the issue raised above. First, using a time span of more than 40 years of data on the German banking system, we separate long-term effects from cyclical fluctuations in the term structure. We find that, in the long run, there exists an economically relevant positive relationship between a bank's net interest margin and the level of interest rates. An increase of 100 basis points in the interest rate level leads to a widening of about 7 basis points in the net interest margin. Second, special attention is given to the low-interest rate environment which we have observed especially in Germany in the recent years. We apply data from the monthly interest rate statistics, where German banks' rates for different products are collected, to the question of whether retail bank rates are set differently in a low-interest rate environment. To do this, we forecast the retail bank rates, based on model parameters from the time before the low-interest rate period, and compare them with the actual retail bank rates in the low-interest rate environment. We find that the margins of retail deposits, especially those of term deposits, have declined by up to 97 basis points.

This paper is structured as follows. In Section 2, we give a brief review of the literature in this field. Section 3 is about the empirical models and Section 4 about the data. Section 5 gives the results, and Section 6 concludes.

2. Literature

Memmel (2011) empirically analyzes the short-term effects of changes in the term structure on the banks' net interest margin. Constructing a passive trading strategy in risk-free government bonds and scaling its return with a bank's exposure to interest rate risk, he finds that this scaled return explains a significant part of the changes in the banks' net interest margins. However, the long-term effects of parallel shifts in the term structure are zero by construction, because he implicitly assumes that all interest-bearing positions in the banks' balance sheet are completely linked to market interest rates.

Whereas Memmel (2011, 2014) finds that there is a close connection between a bank's present value of the banking book and its net interest margin, meaning that an increase in the interest rate level leads to a temporary decline in the banks' net interest rate margins, the results of Banca d'Italia (2013) do not support this view. Instead, it is found that the present value effects of a parallel shift in the term structure are only loosely connected with the corresponding changes in the banks' net interest income in the following year. What is more, for eight out of the 11 Italian banks in the sample, the effect of the upward shift in the interest rate level would be beneficial to their net interest income. Bolt et al. (2012) find as well that the level of market interest rates has a positive impact on the banks' net interest margin. However, they do not distinguish between short-term and long-term effects. English et al. (2014) distinguish between long-run and short-run effects of an increase in the short-term interest rates, but their empirical model does not allow for different signs of the short-term and long-term effect. They find that the short-term effect is significantly positive, but far smaller than the long-run effect. To sum up, concerning the relationship between market interest rates and the banks' net interest margins, there seems to be a tendency for the long-run effect to be more strongly positive than the short-run effect, where the empirical results, especially with respect to the short-run effect, are mixed even with respect to the direction of the effect.

Alessandri and Nelson (2014) provide a theoretical model with a positive relationship between the interest rate level and the banks' net interest margin. They assume that a bank's mark-up on loans is a constant multiple of the market interest rate. Under this assumption, given some market power of the bank, indeed, the bank's net interest margin increases with the level of interest rates. However, one shortcoming of the model is that the interest margin on the asset side would increase by more than the change in the market interest rates, which is not found empirically. By contrast, in the theoretical model of Dell'Ariccia et al. (2014), an increase in the interest rates leads to a decline in the net interest margin, which then has an impact on the banks' risk taking.

Our contribution to the literature is to present an empirical model for the banks' net interest margin that is, at the same time, parsimonious and makes it possible to distinguish between short-run and long-run effects of changes in the interest rate level. This means that our approach is flexible enough to allow for different signs of the short-run and long-run effect of a change in the interest rates. This model is an advancement of the empirical model of Busch et al (2014). In contrast to their study and most other empirical studies in this field, we do not carry out statistical inference by using a sample with a large number of banks and a short time period, but by investigating a long period of more than 40 years. This long period allows us to tell short-run from long-run effects.

As mentioned above, the second part of our paper deals with the additional complications due to the low-interest rate environment. In this context, it is necessary to decompose the bank rates for the different retail products into the appropriate risk-free interest rate and the remaining margin. In the literature, there are different methods of performing this split-up. One method consists in subtracting the market interest rates from the bank rate (see, for instance, HSBC Global Research, 2006). The market interest rate is chosen according to the legal maturity of the retail product. For daily callable accounts, for instance, the overnight market interest rate is used. This method is quite robust and no estimation needs to be carried out. However, this method neglects the fact that the actual duration of retail products largely tends to differ from the legal one. In the example above, the actual empirical duration of daily callable accounts tends to be several years, although the customers have the right to withdraw their money without prior notice. Another approach in the literature (see European Central Bank, 2006) takes into account the fact that the actual and the legal durations of retail products may differ. In this approach, the correlation between the product interest rate and the market interest rates of various maturities is calculated and the maturity for which the correlation is maximal is chosen. Our contribution is to suggest an alternative approach. Our approach consists in determining a portfolio of risk-free bonds of different maturities. It can be shown that our approach is equivalent to the approach of the European Central Bank (2006) insofar as our approach is restricted to exactly one maturity of bonds (See Appendix 3). Our approach has two advantages over that of the European Central Bank (2006): First, our approach also gives the weights of the reference portfolio, while the ECB approach only states which maturity to choose. Second, our approach is applicable to two or more interest rates of different maturities. In this sense, our approach is a generalization of the ECB approach, because we choose the portfolio of bonds of different maturities, and not only the maturity of a single interest rate, whose correlation to the bank rate is maximal (See Appendix 4).

In the literature, the pass-through from market rates to bank rates is often modeled by explaining the bank rate as a linear combination of own lagged values and past and present interest rates (See Kleimeier and Sander (2006) for an overview). For our purposes, the approach in this paper has several advantages over the approach used in this strand of literature. First, the approach in this paper yields the composition of an actual tracking portfolio, so that it could be implemented by the banks, whereas there is no feasible strategy behind the coefficients estimated by the approach from the literature. Second, the proposed approach makes use of interest rates of very many – in principle infinite – different maturities, while, in the approach from the literature, one or at most two different maturities are used.

3. Empirical Models

3.1. Normal times

In the setting from which we derive our empirical model, we analyze a bank that is engaged purely in traditional commercial banking: on the asset side there are customer loans, and on the liability side there are customer deposits. The difference in the volumes of loans and deposits is equalized by interbank lending or borrowing:

$$NI_i = R_{L,i} \cdot L_i - R_{D,i} \cdot D_i - r \cdot \left(L_i - D_i \right) \tag{1}$$

with NI_i bank i ‘s net interest income, L_i the loan volume to the real economy and $R_{L,i}$ the corresponding interest rate, D_i and $R_{D,i}$ are the corresponding variables for the deposits. The variable r

denotes the interest rate for interbank lending and borrowing, which we interpret as the level of interest rates.

From Equation (1), we obtain

$$NI_i = \left(R_{L,i} - r\right) \cdot L_i + (r - R_{D,i}) \cdot D_i \,. \tag{2}$$

The first summand can be seen as the net margin on the asset side times the volume of loans, whereas the second summand is the net margin on the liability side times the deposit volume. Differentiated with respect to r, Equation (2) becomes

$$\frac{\partial NI_i}{\partial r} = L_i \cdot \left(\frac{\partial\left[R_{L,i} - r\right]}{\partial r} + \frac{\partial L_i}{\partial r}\Big/ L_i \cdot (R_{L,i} - r) \right) + D_i \cdot \left(\frac{\partial\left[r - R_{D,i}\right]}{\partial r} + \frac{\partial D_i}{\partial r}\Big/ D_i \cdot (r - R_{D,i}) \right) . \tag{3}$$

Equation (3) states that the change in a bank's net interest income due to a change in the interest rate level depends on four quantities: The change of the net interest margins on the asset side and on the liability side, and the volume change of the loans and deposits.

We make two simplifying assumptions. First, the amount of loans is equal to the amount of deposits, i.e.

$$L_i = D_i \,. \tag{4}$$

Second, there are no effects on the loan and deposit volume or they cancel each other out, i.e.

$$\frac{\partial L_i}{\partial r}\Big/ L_i \cdot \left(R_{L,i} - r\right) + \frac{\partial D_i}{\partial r}\Big/ D_i \cdot \left(r - R_{D,i}\right) = 0 \ . \tag{5}$$

The second assumption must be valid for the banking system as a whole, because the banking system's aggregate interbank lending position cannot change (it must remain zero). However, there is empirical evidence that there is a connection between the bank rates and the corresponding volumes (see, for instance, English et al., 2014) and Drechsler et al., 2014).

Under these two assumptions, Equation (3) becomes

$$\frac{\partial NIM_i}{\partial r} = \frac{\partial\left[R_{L,i} - r\right]}{\partial r} + \frac{\partial\left[r - R_{D,i}\right]}{\partial r} \tag{6}$$

with $NIM_i = NI_i / L_i$.

For ease of exposition, we drop the bank identifier i. In accordance with Busch et al (2014), we assume that the banks' loan rate R_L and deposit rate R_D are a function of the level of interest rates r and of their own lagged values:

$$R_{L,t} = \alpha_L + \beta_{L,1} \cdot R_{L,t-1} + \beta_{L,2} \cdot r_t + \varepsilon_{L,t} \tag{7}$$

and

$$R_{D,t} = \alpha_D + \beta_{D,1} \cdot R_{D,t-1} + \beta_{D,2} \cdot r_t + \varepsilon_{D,t} \tag{8}$$

Note that those time series models of interest margins are only a rough description of the real world over a period of decades. In all likelihood, there have been structural breaks and shifts in the composition of the banks' balance sheets. In Subsection 5.1, some of these issues are addressed as robustness checks.

By combining Equations (6), (7) and (8), we obtain expressions for the effect of permanent changes in the market interest rates in the short and the long run. In the short run, the effect will be

$$\frac{\partial NIM_{sh}}{\partial r} = \beta_{L,2} - \beta_{D,2} \tag{9}$$

and, in the long run, i.e. for the infinite future, the expression is

$$\frac{\partial NIM_{\lg}}{\partial r} = \frac{\beta_{L,2}}{1-\beta_{L,1}} - \frac{\beta_{D,2}}{1-\beta_{D,1}} \tag{10}$$

The expression in Equation (10) is closely linked to the definition above of the net interest margin being independent of the level of interest rates. In this case, this expression would equal zero. By contrast, if $\partial NIM_{\lg} / \partial r$ is positive, then there is a positive relationship between the net interest margin and the level of interest rates. In Appendix 1, we give the closed form of the asymptotic standard deviation of the expression (10). There may even be qualitative differences in the impact on a bank's net interest margin in the short run and in the long run. For instance, the short-run impact of a parallel upward shift in interest rates may have a negative impact on the net interest margin, whereas the long-run effect may be positive. The intuition behind this is that, in the short run, due to the usually shorter maturities of the liabilities, a larger portion of the liabilities is adjusted to the interest rate level in a given time. In the long run, this effect vanishes because even the products with the longest maturities will be adjusted to the new interest rate level and the effect of the higher pass-through on the asset side prevails.[2] In technical terms, the above-mentioned effect would be relevant if the degree of persistence for the assets side $\beta_{L,1}$ were sufficiently larger than the one for the liability side $\beta_{D,1}$ so as to offset the stronger short-run effect $\beta_{D,2}$ on the liabilities than on the assets ($\beta_{L,2}$). The appropriate test statistics to be analyzed would be

$$LvsS = \left(\beta_{L,2} - \beta_{D,2}\right) \cdot \left(\frac{\beta_{L,2}}{1-\beta_{L,1}} - \frac{\beta_{D,2}}{1-\beta_{D,1}} \right), \tag{11}$$

where $LvsS$ stands for 'Long-run versus Short-run effect'. (See Appendix 1 for the closed form of the asymptotic distribution of this test statistics.).

If $LvsS$ is negative, there is a qualitative difference concerning the short and long-run effects of a change in the interest rate level. Equations (9) and (10) are the expressions for the change in the net interest

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[2] See, for example, European Central Bank (2009).

margin in the limiting cases, i.e. for a horizon of one year ("short-run") and for an infinite horizon ("long-run"). In the case of an arbitrary horizon k [in years], the expression is

$$\frac{\partial NIM(k)}{\partial r} = \beta_{L,2} \cdot \frac{1-\beta_{L,1}^{k}}{1-\beta_{L,1}} - \beta_{D,2} \cdot \frac{1-\beta_{D,1}^{k}}{1-\beta_{D,1}} \tag{12}$$

In the event that a change in the interest rate level impacts the net interest margin differently in the long and in the short run, there exists a horizon k^* for which the impact due to a change in the interest rate level is zero. Unfortunately, there does not exist a closed-form expression for this horizon, but k^* can be easily determined using numerical methods, and the asymptotic standard errors of the estimated k^* can even be calculated analytically (see Appendix 2).

3.2. Low-interest environment

The interest rate of retail products can be decomposed into two parts: (i) the interest rate for an alternative investment at the capital market, and (ii) the interest margin that banks charge from their customers where, for products on the liability-side, the interest margin lowers the remuneration. This margin is determined by the competition the bank faces and by the costs associated with the retail product. Current accounts, for instance, are relatively costly, because banks have to carry out the payment and liquidity management for the current account holders (see Busch and Memmel, 2014). In addition, this margin contains a liquidity premium arising from the fact that – at least for daily callable accounts – the customers always have the funds at their disposal. In the case of loan products, banks additionally have to charge a premium for the credit risk.

We assume that the margin is constant through time and that changes in the bank rates are driven only by changes in the market rates, but we abstain from the assumption that changes in the market rates are completely passed through to the bank rates. In detail, we look at the following interest rates and yields (t is the time index in months):

- Bank rate: $R_{j,t}$ is the interest rate that banks charge (in case of an asset) or pay (in case of a liability) for the product j.
- Government bonds $G(M)$: $r_t(M)$ is the return of par yield government bonds with maturity M [in months] at time t.
- Strategy $S(M)$: $z_t(M)$ is the return of an investment strategy that consists of investing each month $1/M$ in par yield government bonds with maturity M [in months].[3]

$$z_t(M) = \frac{1}{M} \sum_{i=1}^{M} r_{t-i+1}(M) \tag{13}$$

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[3] See Memmel (2008) for further information on this investment strategy.

- Investment opportunity P: r_P is the return of an investment opportunity with a rate that does not change in the course of time. For our study, it is set equal to $r_P = 4\%$ p.a. Note that the level of this rate does not have any impact on the composition of the reference portfolio.

As stated above, the interest rate of the retail product is compared with an alternative investment at the capital market. In this paper, the alternative investment is a passive strategy, i.e. there exists a mechanical rule to buy and sell government bonds. For instance, such a passive strategy may consist of investing 30% of the funds in strategy $S(36)$ and 70% in investment opportunity P. In our study, the objective is to minimize the timely variation of the margin, i.e. the difference between the product interest rate and the alternative investment at the capital market. In Appendix 4, we show that this is equivalent to maximizing the correlation to a portfolio of government bonds, where the maturity of the bonds and their weights are the parameters for the optimization. If there is the possibility of allocating the assets to investment opportunity P and to two different investment strategies $S(M_1)$ and $S(M_2)$, then the optimization problem has two layers: The outer one is to determine the appropriate maturities M_1 and M_2, the inner one is to obtain the optimal weights (w_1, w_2 and w_P) for the three investments, given the maturities M_1 and M_2. Formally, we can state the optimization problem as

$$\min_{M_1,M_2}\left(\min_{m,w_1,w_2,w_P}\frac{1}{T}\sum_{t=1}^{T}\varepsilon_t^2\right) \tag{14}$$

subject to

$$\varepsilon_t = R_t - \left(m + w_1 \cdot z_t(M_1) + w_2 \cdot z_t(M_2) + w_P \cdot r_P\right) \tag{15}$$

and

$$w_1 + w_2 + w_P = 1 \tag{16}$$

where m is the time-constant margin that the bank earns above its refinancing costs. Using an approach laid down in Kempf and Memmel (2006), we can rewrite the inner minimization problem, i.e. the one between the brackets, as a linear regression and solve it with the ordinary least squares technique:

$$R_t - r_P = \alpha + \beta_1\left(z_t(M_1) - r_P\right) + \beta_2\left(z_t(M_2) - r_P\right) + \varepsilon_t \tag{17}$$

where $m = \alpha$, $w_1 = \beta_1$, $w_2 = \beta_2$ and $w_P = 1 - \beta_1 - \beta_2$. In addition, we impose non-negative constraints for the weights w_1, w_2 and w_P. The non-negative constraints on the weights make the optimization more robust. In particular, we can ease the problem of near-multicollinearity that arises if the regressors are highly correlated (which is the case for returns of investment strategies with similar maturity). The outer minimization problem can be solved by trying out all possible discrete pairs of maturity combinations $\left(M_1, M_2\right)$ and then checking which pair yields the lowest sum of squared residuals.

We fit the parameters for the period from January 2003 to September 2008, the month of the Lehman failure. For the determination of the reference portfolio, we neglect the period of the subsequent low-interest rate environment. Instead, we try to answer the following question: If the composition of the reference portfolio had been unchanged in the low-interest rate environment, what would the margins have been in this environment? Using the composition of the reference portfolios, we can calculate hypothetical bank rates and compare them to the actual bank rates. Note that structural breaks like the Lehman failure may have changed the model parameters, too. Therefore, the estimated change in the margins may be – at least in part – also attributed to changes in model parameters, not only to a change in actual margins.

4. Data

For our analysis, we use publicly available data provided by the Deutsche Bundesbank. Our first data source is aggregated profit and loss data of German universal banks broken down into banking groups.[4] We look at two subsamples: the small banks, which consist of the savings and cooperative banks, and the smaller private commercial banks, and the large banks, which consist of the large commercial banks and the central institutions of the savings and cooperative banks. Here, we get information about interest income, interest expenses, net interest income and total assets for the period 1968-2013 at yearly frequency. Second, we use information about the yields of German government bonds. To be more precise, we use the yield on the outstanding government bonds ("Umlaufrendite"). As the interest rates for different maturities and their yearly changes are highly correlated, we abstain from applying two or more interest rates of different maturities and we interpret the yield of the government bonds outstanding as the interest rate level.[5] We test for unit roots in our time series (interest income to total assets, interest expenses to total assets, and bond yields) using the augmented Dickey-Fuller unit root test. Under the null hypothesis, time series contain a unit root, where under the alternative the time series are stationary. Our test statistics show that, for the relevant time series in levels, the null hypothesis cannot be rejected (see Table 1). This is in line with the findings of Diebold and Li (2006). Furthermore, the tests show that the first differences of the variables can be assumed to be stationary.

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[4] Universal banks are broken down into commercial banks, which can be further divided into big banks and smaller private commercial banks, savings banks, "Landesbanken", credit cooperatives and central institutions of credit cooperatives.

[5] Litterman and Scheinkman (1991) and Bliss et al. (1997) find that the first component of principal component analyses of the US yield curve for different periods usually accounts for more than 80% of the variation and Memmel (2014) finds for Germany a share of even more than 90% of the variation.

Variable		All banks	Small banks	Large Banks
Interest income margin	Level	-0.129	-0.477	0.046
	First difference	-3.178**	-3.189**	-3.203**
Interest expense margin	Level	-0.607	-0.831	-0.312
	First difference	-3.372**	-3.404**	-3.317**
Net interest margin	Level	-0.475	-0.994	-0.745
	First difference	-3.399**	-3.853***	-3.696***
Interest rate	Level	-0.917	-0.917	-0.917
	First difference	-4.651***	-4.651***	-4.651***

Table 1: Test statistics of the augmented Dickey-Fuller-Test, period 1968-2013, two lags are included in all time series, 43 observations in the level specification, 42 observations in the first difference specification. ** and *** denote the 5% and 1% p-value for the null-hypothesis "Time series contains a unit root".

For the retail deposits rates, we use data from the German part of the MFI interest rate statistics. Since January 2003, all member states of the European Monetary Union have been carrying out a monthly survey among the banks in their countries, surveying the interest rates for various retail products (for the German data of the MFI interest rate statistics, see Deutsche Bundesbank, 2004). The retail deposits are broken down into six different categories: daily callable accounts (sight deposits), three kinds of term deposits (up to one year, more than one year to two years, more than two years), and two kinds of savings accounts (period of notice of up to three months, periods of notice of more than three months). For the purpose of presentation, daily callable accounts and the savings accounts are subsumed under the term 'non-maturing accounts'. We restrict ourselves to retail deposits and ignore retail loans for two reasons. First, the rates for loan products also contain a mark-up for credit risk, which cannot be easily assumed to be constant through time as we assume with the remaining margin. Second, the rates for deposits products are usually lower than the loan products and the market interest rates. Therefore, the zero lower bound in a low-interest rate environment tends to be more quickly binding for these products, so that a noticeable effect can be expected to be seen especially here.

The returns of German government bonds are taken from Deutsche Bundesbank. The Bundesbank estimates for each trading day the term structure of listed German government bonds using the Svensson (1994) approach, which is an extension to the Nelson/Siegel method (see Schich, 1997). Table 2 shows summary statistics of the return for the investment strategy $S(M)$ and of the return of government bonds $r(M)$ for different maturities M.

Maturity M [in months]	Strategy S(M)		Government bonds r(M)	
	mean (p.a.)	stand. dev. (p.a.)	mean (p.a.)	stand. dev. (p.a.)
6	1.73%	1.40%	1.68%	1.43%
12	1.89%	1.36%	1.77%	1.43%
18	2.06%	1.29%	1.85%	1.42%
24	2.24%	1.21%	1.95%	1.40%
30	2.44%	1.13%	2.04%	1.37%
36	2.63%	1.06%	2.14%	1.35%
42	2.81%	1.00%	2.24%	1.32%
48	2.98%	0.93%	2.34%	1.29%
54	3.14%	0.86%	2.43%	1.27%
60	3.29%	0.79%	2.53%	1.24%
66	3.42%	0.72%	2.62%	1.21%
72	3.55%	0.67%	2.70%	1.19%
78	3.66%	0.64%	2.78%	1.16%
84	3.77%	0.63%	2.86%	1.14%
90	3.87%	0.63%	2.93%	1.11%
96	3.97%	0.63%	3.00%	1.09%
102	4.08%	0.65%	3.07%	1.06%
108	4.18%	0.67%	3.13%	1.04%
114	4.28%	0.68%	3.19%	1.02%
120	4.37%	0.68%	3.24%	1.00%

Table 2: Summary statistics for the returns of the strategies S(M) and of the returns of German government bonds, which were issued at par, for different maturities M. Period: January 2003 to April 2014, 136 monthly observations.

We see that the mean returns of the strategies $S(M)$ and the mean return of the government bonds $r(M)$ increase monotonously with the maturity M of the underlying government bonds. During our observation period from January 2003 to April 2014, the mean return for the strategy of investing revolvingly in bonds with a maturity of six months is 1.73% compared to the mean return of 4.37% for strategy $S(120)$. The respective figures for the return of government bonds are 1.68% and 3.24%.

5. Empirical Results

5.1. Normal times

The Breusch-Pagan test shows that the errors in the two Equations (7) and (8) are not independent. As we need the joint distribution of the estimated coefficients of the interest income and interest expense (Eq. (7) and Eq. (8)), we estimate Equation (18) as a panel specification, which considers the correlated error structure. Here, the cross sectional dimension consist of two units, namely the interest income margin (R_L) and the interest expense margin (R_D) (N = 2; T = 44). In addition, there is autocorrelation in the two error terms. We opt for the following panel specification:

$$\Delta R_{i,t} = \alpha_L \cdot D_L + \beta_{L,1} \cdot D_L \cdot \Delta R_{L,t-1} + \beta_{L,2} \cdot D_L \cdot \Delta r_t + \alpha_D \cdot D_D + \beta_{D,1} \cdot D_D \cdot \Delta R_{D,t-1} + \beta_{D,2} \cdot D_D \cdot \Delta r_t + \varepsilon_{i,t} \quad (18)$$

with $i = L, D$ and, D_L and D_D are dummy variables that take on the value one in the event that $i = L$ and $i = D$, respectively.[6] We estimate in first differences, because the interest margins do not seem to be stationary (See Table 1). In addition, our estimator accounts for autocorrelation of order 1 in the error terms. For reasons of clarity, the results in Table 3 are displayed as if they were derived from univariate regressions, although they are estimated from the panel specification (18). The results are given for the sample of all universal banks and broken down into small and large banks.

Variable	All banks		Small banks		Large Banks	
	Int. income	Int. exp.	Int. income	Int. exp.	Int. income	Int. exp.
Lagged dep. variable	0.3162*** (0.0650)	0.2081*** (0.0682)	0.3122*** (0.0671)	0.1981*** (0.0694)	0.2293*** (0.0710)	0.1361* (0.0730)
Interest rate level	0.5355*** (0.0582)	0.5617*** (0.0612)	0.5262*** (0.0361)	0.5462*** (0.0599)	0.5477*** (0.0613)	0.5872*** (0.0647)
Constant	0.0001 (0.0005)	0.0002 (0.0005)	0.0001 (0.0005)	0.0002 (0.0005)	0.0000 (0.0005)	0.0002 (0.0005)
Number of years	44	44	44	44	44	44

Table 3: The relationship between interest rate level and interest income/interest expenses. Dependent variables are “Int. income” (interest income over total assets=interest income margin) and “Int. exp.” (interest expense over total assets=interest expense margin). Yearly data 1968-2013. See Equations (7) and (8), and for the actual estimation Equation (18). Robust standard errors in brackets. * and *** denote significance at the 10% and 1% level.

The interest income and expenses are significantly positively related to changes in the level of interest rates. The same is true of the impact of the lagged dependent variable. The impact of the lagged dependent variable is larger for small banks than for large banks. This can be interpreted to mean that the maturities on the balance sheet of small banks are greater than those of the large banks.

For all samples, we see in Table 4 that the short-run impact of an increase in the interest rate level is highly positive for the interest income and expenses, ranging between 0.52 and 0.55. The short-run impact on the interest expenses is larger than on the interest income, leading to a negative net effect (as can be seen in the column “NIM”), which is statistically significant for the samples of all banks and of the large banks (see Appendix 1 for the derivation of the test statistics). By contrast, in the long run, the net effect of an increase in the interest rate level is positive, which is significant for the samples of all banks and of small banks.

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[6] The STATA command *xtgls*, which we use in our study, allows for heteroskedatic and autocorrelated error structures. Furthermore, we allow panels to be correlated and choose the option “panels(correlated)”. Philipps and Sul (2007) show that the bias in the autoregressive coefficient can be neglected if the number of times series observation is relatively large.

Sample	Impact of a 1 percentage point increase in the interest rate level			
	Horizon	Int. income	Int. expenses	NIM
All banks	short-run	0.5355*** (0.0650)	0.5617*** (0.0682)	-0.0261** (0.0133)
	long-run	0.7831*** (0.0985)	0.7093*** (0.0922)	0.0738*** (0.0275)
Small banks	short-run	0.5262*** (0.0671)	0.5462*** (0.0694)	-0.0200 (0.0149)
	long-run	0.7650*** (0.1006)	0.6811*** (0.0914)	0.0839*** (0.0310)
Large banks	short-run	0.5477*** (0.0710)	0.5872*** (0.0730)	-0.0396*** (0.0136)
	long-run	0.7106*** (0.0891)	0.6797*** (0.0847)	0.0309 (0.0216)

Table 4: Pass-through (in percentage points) of a 1 percentage point increase in the interest rate level. Robust standard errors in brackets (See Appendix 1). ** and *** denote significance at the 5% and 1% level, respectively. “Int. income” is the interest income over total assets. The same standardization applies for “Int. exp.” (= interest expenses). “NIM” is the net interest margin, i.e. the net interest income over total assets. Yearly data 1968-2013.

For the sample of all banks, we see that, in the short run, the banks’ net interest margin goes down by 2.6 bp after a 1 percentage point increase in the interest rate level and, in the long run, it will be increasing by 7.4 bp. This qualitative difference, as laid down in Equation (11), is significant for the sample of all banks at the 1% level (see Table 5 and Appendix 1). This means that the theoretical predictions of Dell’Ariccia et al. (2014) hold for the short-term horizon, but not for the long run.

	All banks	Small Banks	Large Banks
LvsS x1000	-1.930*** (0.794)	-1.682* (1.017)	-1.221* (0.669)
Horizon k* [in years]	1.464*** (0.256)	1.325*** (0.259)	1.814*** (0.427)

Table 5: Long-run versus short-run effects and time horizon k*. “LvsS” is the test statistics defined in Equation (11), “k*” is the horizon where the change in the level of interest rates has no effect on the banks’ net interest margin (See Equation (20)). Robust standard errors in brackets (See the Appendices 1 and 2). * and *** denote significance at the 10% and 1% level, respectively.

Table 5 also shows the horizon where the different effects of an increase in the interest rate level exactly offset each other, i.e. before this critical horizon k^* the net effect is negative and, after this point in time, it is positive. It turns out that this critical horizon k^* is less than two years; for the sample of small banks, the estimate is 1.3 years, for the large banks 1.8 years, and 1.5 years for the whole sample. The standard errors (see Table 5 and the Appendix 2) range between a quarter of and half a year, meaning that the estimates of this horizon are relatively precise.

Several robustness checks are carried out. First, in 2010, there was a major structural break in the accounting rules where the banks’ total assets, especially those of the large banks, increased by roughly 10% (the “Act to Modernize Accounting Law” [‘Bilanzmodernisierungsgesetz’]). This increase in total assets, which was driven purely by changes in accounting rules, led to a corresponding decline in the interest margins. Leaving aside the years from 2010 on does not change the results qualitatively, but increases the statistical significance of the results. Second, an auto-regressive process of order 1 may not be sufficient to adequately describe the time series properties of the interest margins. In order not to lose too many of the yearly observations, we restrict ourselves to including one additional lag. The coefficient of this additional lag turns out to be statistically significant for all samples and margins, but negative, meaning that the pass-through share even becomes smaller (given that the direct effect of the change in the market interest rates mainly remains the same). The qualitative effect on the net interest margin, i.e. negative in the short run and positive in the long run, remains (results are available upon request). Third, the estimations are carried out using ordinary least squares (OLS) and applying the standard errors proposed by Driscoll and Kraay (1998) to better account for correlation across panels and allowing for higher order of autocorrelation in the error terms. The results remain qualitatively unchanged, although there is a tendency to weaker significance (Results are available on request).

5.2. Low interest environment

In Table 6, the results of the optimization (14) concerning the replication strategies are displayed for the six different retail products: daily callable accounts, short-term and long-term savings accounts and term deposits (short-term, medium-term, and long-term).

Number	Product	M1	w1	M2	w2	wP	Explanation
1	Daily callable accounts	6	35.9%			64.1%	94%
2	Savings account (short term)	6	25.5%	54	27.9%	46.6%	89%
3	Savings account (long term)	18	72.4%	30	18.0%	9.6%	98%
4	Term deposits (short term)	6	98.8%			1.2%	91%
5	Term deposits (medium term)	6	83.2%	66	15.2%	1.6%	91%
6	Term deposits (long term)	6	7.2%	120	57.2%	35.7%	60%

Table 6: Solution to the optimization in Equation (14). M1 and M2 are maturities (in months) of the replication strategies; w1 and w2 are the respective weights; wP is the weight of the time-invariant investment strategy. Non-negative constraints on the weights w1, w2 and wP. The column "Explanation" gives the coefficient of determination of the regression in (17) and can be interpreted as the share of the serial variation that is explained by the passive portfolio strategies. Optimization period: January 2003 to September 2008.

The table gives the weights in the two passive trading strategies (w1, w2) and states the maturities of these two trading strategies (M1, M2). In addition, the weight of the passive investment strategy (wP) is given. There are three main results. First, the pass-through of changes in the market interest rates is incomplete for these retail bank deposits, even in the long run. This holds especially true for sight deposits (long-term pass-through of 35.9%) and for short-term saving accounts (53.4%). By contrast, for short-term and medium-term deposits the pass-through is more than 98%. This compares with the estimate of the long-run pass-through to the interest expenses of 70.9% (See Table 4), which can be seen as a weighted average of these figures. Second, trading strategies based on short-term interest rates, mostly 6-month, are always included in the replicating portfolio. This holds especially true for short-term term deposits, where the share of the trading strategy in 6-month-bonds is 98.8%.[7] Third, the replicating portfolios are able to explain around 90% of the serial variation in the product rates (with the exception of long-term deposits, where the share of explanation is only 60% of the serial variation).

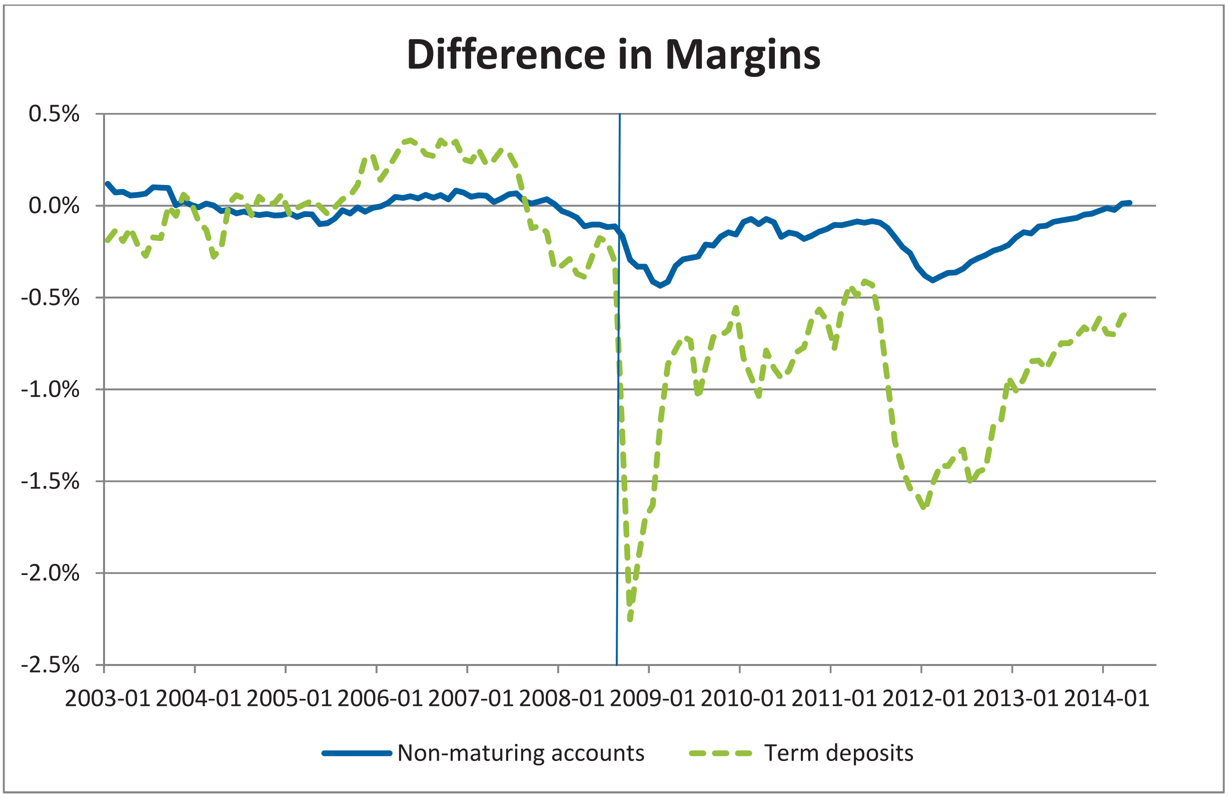

Figure 1: Difference in margins for bank products, relative to the average margin from January 2003 to September 2008. Margins are derived as the difference relative to a portfolio of investment strategies whose composition is determined in the period Jan. 2003 to Sep. 2008. "Non-maturing accounts" comprise "daily callable accounts" and two kinds of savings accounts. "Term deposits" comprise three retail kinds of retail deposits (up to 1 year, 1 up to 2 years, more than 2 years of maturity). The differences in margins are weighted with the volume of the amounts (daily callable and savings accounts) and with the volume of new business (term deposits).

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[7] Deposits like this, i.e. deposits with a very close link to market interest rates, best correspond to the ones in the model of Dell'Ariccia et al. (2014). However, in October 2014, the volume of these short-term deposits in Germany was less than one-tenth of that of sight deposits (see Deutsche Bundesbank, 2014).

In Figure 1, the change in the margin that the banks earn on the deposit products is displayed. As defined in subsection 3.2, the margin is the difference between the product interest rate and the return on a portfolio of passive investment strategies in government bonds. We see that, after the cut-off date of September 2008, the margin has become dramatically smaller. This holds true especially for the term deposits, where the average change in the margin is -0.97% p.a., and not so much for the non-maturing accounts (daily callable accounts and savings accounts), where the average change in the margin is only -0.19% p.a. However, if we assume that the low interest rate environment did not start until August 2012, then the changes in the margins are smaller, namely -0.87% p.a. and -0.11% p.a., respectively.

For the non-maturing accounts (daily callable accounts and savings accounts), the change in margin vanishes in fact at the latest available date. The relatively large reduction in the margin of the term deposits is due to the high weights of the market rates in the replicating portfolio: When the market rates reached zero or even negative values, the replicating portfolio followed this development, but the bank rates stayed significantly positive, which compressed the margins.

6. Conclusion

Our analysis suggests that, in the long run, an increase in the level of interest rates leads to an increase in the banks' net interest margin. This finding adds a further perspective to the common wisdom that banks lose in the event that interest rates rise. The story seems somehow more complicated: While it seems that banks lose in the short run in an environment of rising interest rates, they benefit in the long run from interest rates being at a higher level. This empirical finding seems to be relevant for the question of how banks react to structurally changing interest rates levels, because a bank's net interest margin has a huge impact on its behavior. Our empirical results further show that the turning point, i.e. the horizon where the positive and the negative effects offset each other, is at about one and half years. This finding concerns the design of stress test scenarios, because the stress scenarios are often embedded in an environment of rising interest rates, where the stress test horizon is up to three years, so that scenarios like this are not adverse for the banks.

The second part of our analysis shows that banks are negatively affected by a low-interest rate environment. The zero lower bound of deposit products puts some additional stress on banks, especially concerning the margin of term deposits.

Appendices

Appendix 1

The delta method states that, if the standardized vector x is asymptotically normally distributed, i.e. $\sqrt{T}\cdot\left(x-\mu\right)\longrightarrow N\left(0;\Sigma\right)$, and $f(\cdot)$ is a differentiable function, the expression $\sqrt{T}\cdot\left(f(x)-f(\mu)\right)$ is asymptotically normally distributed with expectation zero and variance $\left(\frac{\partial f}{\partial x}\right)'\Sigma\left(\frac{\partial f}{\partial x}\right)$ (See Greene, 2003, pp. 913f). In our paper, $x=\left(\hat{\beta}_{L,1},\hat{\beta}_{L,2},\hat{\beta}_{D,1},\hat{\beta}_{D,2}\right)'$. The following table gives $f(\cdot)$ and $\partial f/\partial x$ for the different cases.

Case i	Elasticity / test statistics	$f_i(x)\big\vert_{x=\mu}$	$\left(\partial f_i \big/ \partial x\right)'\big\vert_{x=\mu}$
1	short-term interest income	$\beta_{L,2}$	$\left(0,1,0,0\right)$
2	long-term interest income	$\frac{\beta_{L,2}}{1-\beta_{L,1}}$	$\left(\frac{\beta_{L,2}}{(1-\beta_{L,1})^2},\frac{1}{1-\beta_{L,1}},0,0\right)$
3	short-term interest expenses	$\beta_{D,2}$	$\left(0,0,0,1\right)$
4	long-term interest expenses	$\frac{\beta_{D,2}}{1-\beta_{D,1}}$	$\left(0,0,\frac{\beta_{D,2}}{(1-\beta_{D,1})^2},\frac{1}{1-\beta_{D,1}}\right)$
5	Short-term NIM	$\beta_{L,2}-\beta_{D,2}$	$\left(0,1,0,-1\right)$
6	Long-term NIM	$\frac{\beta_{L,2}}{1-\beta_{L,1}}-\frac{\beta_{D,2}}{1-\beta_{D,1}}$	$\left(\frac{\beta_{L,2}}{(1-\beta_{L,1})^2},\frac{1}{1-\beta_{L,1}},\frac{-\beta_{D,2}}{(1-\beta_{D,1})^2},\frac{-1}{1-\beta_{D,1}}\right)$
7	LvsS	$f_5\left(\mu\right)\cdot f_6\left(\mu\right)$	$\left(f_5\left(\mu\right)\cdot\frac{\beta_{L,2}}{(1-\beta_{L,1})^2},f_6(\mu)+\frac{f_5\left(\mu\right)}{1-\beta_{L,1}},-f_5\left(\mu\right)\cdot\frac{\beta_{D,2}}{(1-\beta_{D,1})^2},-f_6\left(\mu\right)-\frac{f_5(\mu)}{1-\beta_{D,1}}\right)$

Table A1: Parameters for the calculation of tests statistics using the delta method. "NIM" denotes net interest margin; "LvsS" is defined in Equation (11) as the test statistics to check whether there is a change in the sign of the relationship between interest rates and a bank's net interest margin. Interest income and interest expenses relative to total assets.

Appendix 2

The change in the net interest margin can be written as

$$\Delta NIM(k)=\beta_{L,2}\cdot\frac{1-\beta_{L,1}^{k}}{1-\beta_{L,1}}-\beta_{D,2}\cdot\frac{1-\beta_{D,1}^{k}}{1-\beta_{D,1}}. \tag{19}$$

The variable k^{*} denotes the horizon, for which this change equals zero, i.e.

$$\Delta NIM(k^{*})=0\ . \tag{20}$$

Using the theorem about implicit functions, we get

$$\frac{\partial k^*}{\partial \beta_{i,j}} = -\frac{\dfrac{\partial \Delta NIM}{\partial \beta_{i,j}}}{\dfrac{\partial \Delta NIM}{\partial k^*}} \tag{21}$$

where the numerator and the denominators of (21) for the four different cases $i = L, D$, $j = 1, 2$ can be obtained as follows:

$$\frac{\partial \Delta NIM}{\partial \beta_{L,1}} = \beta_{L,2} \cdot \frac{-k^* \cdot \beta_{L,1}^{k^*-1} \cdot \left(1-\beta_{L,1}\right) + \left(1-\beta_{L,1}^{k^*}\right)}{\left(1-\beta_{L,1}\right)^2} \tag{22}$$

$$\frac{\partial \Delta NIM}{\partial \beta_{L,2}} = \frac{1-\beta_{L,1}^{k^*}}{1-\beta_{L,1}} \tag{23}$$

$$\frac{\partial \Delta NIM}{\partial \beta_{D,1}} = -\beta_{D,2} \cdot \frac{-k^* \cdot \beta_{D,1}^{k^*-1} \cdot \left(1-\beta_{D,1}\right) + \left(1-\beta_{D,1}^{k^*}\right)}{\left(1-\beta_{D,1}\right)^2} \tag{24}$$

$$\frac{\partial \Delta NIM}{\partial \beta_{D,2}} = -\frac{1-\beta_{D,1}^{k^*}}{1-\beta_{D,1}} \tag{25}$$

$$\frac{\partial \Delta NIM}{\partial k^*} = -\beta_{L,2} \cdot \frac{\ln \beta_{L,1} \cdot \beta_{L,1}^{k^*}}{1-\beta_{L,1}} + \beta_{D,2} \cdot \frac{\ln \beta_{D,1} \cdot \beta_{D,1}^{k^*}}{1-\beta_{D,1}} \tag{26}$$

Using the delta method (as outlined in Appendix 1), one can calculate the asymptotic standard deviation of k^* in a closed-form expression.

Appendix 3

In the event that one has to choose exactly one possible passive investment strategy, the minimization (14) reduces to

$$\min_{M_1} \left(\min_{m, w_1, w_P} \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t^2 \right) \tag{27}$$

subject to

$$\varepsilon_t = R_t - \left(m + w_1 \cdot z_t(M_1) + w_P \cdot r_P\right) \tag{28}$$

and

$$w_1 + w_P = 1 \tag{29}$$

Again, this minimization problem can be seen as a linear regression:

$$R_t - r_P = \alpha + \beta_1 \left(z_t(M_1) - r_P\right) + \varepsilon_t \tag{30}$$

where $m = \alpha$, $w_1 = \beta_1$ and $w_P = 1 - \beta_1$. For a linear regressions with only one regressor, we get $R^2 = \hat{\rho}^2_{M_1}$, where R^2 is the coefficient of determination and $\hat{\rho}_{M_1}$ is the empirical correlation coefficient between R_t and $z_t(M_1)$. Using $\min_{m,w_1,w_P} \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t^2 =: \hat{\sigma}^2_\varepsilon = \left(1 - \hat{\rho}^2_{M_1}\right)\hat{\sigma}^2_R$, we can rewrite the minimization in (27) as

$$\min_{M_1} \hat{\sigma}^2_R \left(1 - \hat{\rho}^2_{M_1}\right), \tag{31}$$

where $\hat{\sigma}^2_R$ is the empirical variance of R_t. For positive correlations, the approach (31) is equivalent to the approach by the European Central Bank (2006), which is

$$\max_{M_1} \hat{\rho}_{M_1} \tag{32}$$

Appendix 4

Let $Z_t = \left(z_t(M_1),...,z_t(M_n)\right)'$ be a vector of returns from passive investment strategies, let R_t be the interest rate of the retail product. The vector c includes all the covariances, i.e. $c = \left(\text{cov}(z_t(M_1), R_t),....,\text{cov}(z_t(M_n), R_t)\right)'$, the matrix $\Omega = \text{var}\left(Z_t\right)$ is the covariance matrix of the vector Z_t. As the inner optimization of (14) is equivalent to linear regression, the solution for the vector of coefficients w is (see Kempf and Memmel, 2006)

$$w^* = \Omega^{-1} c \ . \tag{33}$$

Next, we consider the correlation between the return of an arbitrary portfolio $w'Z_t$ of passive investment strategies and the return R_t of the retail product. The squared correlation coefficient for arbitrary portfolio weights w is given by

$$\rho^2 = \frac{\text{cov}\left(w'Z_t, R_t\right)^2}{\text{var}\left(w'Z_t\right) \cdot \sigma^2_R} = \frac{\left(w'c\right)^2}{w'\Omega w \cdot \sigma^2_R} \tag{34}$$

where σ^2_R is the variance of the product interest rate R_t.

From matrix theory, we know that the maximal squared correlation is (See Judge and Bock, 1978, p. 317, Theorem A.3.14.)

$$\rho^2_{\max} = \frac{c'\Omega^{-1}c}{\sigma^2_R} \tag{35}$$

When using the solution (33) as the weights in Equation (34), we get – after some algebraic conversions – the maximal value as given in (35).

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