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Solving RE models with discontinuous policy rules – an application to minimum wage setting in Germany

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Non-technical summary

Research Question

The legal regulations governing the adjustment of the minimum wage in Germany require the appointed minimum wage commission to present biennial adjustment proposals to the Federal Government. Changes to the minimum wage in line with the commission's meeting schedule, however, produce a policy discontinuity as the minimum wage is held constant in a certain year and adjusted over-proportionately in the subsequent year.

Solving rational expectations models with such policy features is not straightforward as the discontinuities render standard local approximation techniques infeasible.

Contribution

Using the example of minimum wage setting in Germany, the paper illustrates how models with discontinuous policy rules can be solved using the method of undetermined coefficients.

In the stylised model of wage and profit setting employee and employer representatives (the "social partners") aim to minimise the volatility of negotiated wages and corporate profits. Workers in the economy are either negotiated wage earners or minimum wage recipients. While – in line with the legal framework in Germany – adjustments to the minimum wage are based on a discontinuous policy rule, negotiated wages are the outcome of an optimisation problem.

Results

Based on selected model simulations of the outlined model it can be inferred that – conditional on the assumed loss function of the social partners – annual adjustments to the minimum wage may be advantageous *ex ante* despite the minimum wage commission's biennial meeting schedule.

Nichttechnische Zusammenfassung

Fragestellung

Die gesetzlichen Vorgaben zur Anpassung des Mindestlohns in Deutschland sehen vor, dass die hierfür eingesetzte Mindestlohnkommission in zweijährigem Rhythmus der Bundesregierung Anpassungsvorschläge unterbreitet. Bei einem zweijährigen Anpassungsrhythmus ist ein gewisser „Stufeneffekt“ im Zeitablauf unvermeidbar, weil der Mindestlohn in gewissen Jahren konstant gehalten und demgegenüber in den Folgejahren überproportional stark angepasst wird.

Eine Lösung von Modellen mit Unstetigkeiten dieser Art gestaltet sich allerdings als schwierig, da unstete Politikregeln die Anwendung häufig angewandter lokaler Approximationsverfahren erschwert.

Beitrag

Basierend auf dem Beispiel der Mindestlohnsetzung in Deutschland zeigt das Papier, wie Modelle mit unsteten Politikregeln unter Zuhilfenahme der Methode unbestimmter Koeffizienten gelöst werden können.

In dem stilisierten Modell der Lohn- und Gewinnsetzung sind Arbeitnehmer- und Arbeitgebervertreter (die „Sozialpartner“) darum bemüht, die Volatilität von Tariflöhnen und Unternehmensgewinnen zu minimieren. Die Arbeitnehmer der Ökonomie sind entweder Tariflohn- oder Mindestlohnempfänger. Während Anpassungen des Mindestlohns – im Einklang mit den rechtlichen Rahmenbedingungen in Deutschland – auf einer unsteten Politikregel basieren, sind die Tariflöhne das Ergebnis eines Optimierungskalküls.

Ergebnisse

Ausgewählte Modellsimulationen für das dargelegte Modell legen auf Basis der Verlustfunktion der Sozialpartner nahe, dass trotz des zweijährigen Entscheidungsrhythmus der Mindestlohnkommission die *Ex-ante*-Festlegung auf jährliche Anpassungen des Mindestlohns vorteilhaft sein kann.

BUNDESBANK DISCUSSION PAPER NO 35/2016

Solving RE models with discontinuous policy rules – an application to minimum wage setting in Germany*

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Abstract

The legal regulations require the minimum wage in Germany to be adjusted biennially which gives rise to a policy discontinuity. From the perspective of rational expectations models, such policy features render standard local approximation techniques infeasible. The paper presents a stylised model in which negotiated wages and corporate profits are the outcome of an optimisation problem, while changes to the minimum wage are modelled by a discontinuous policy rule. Using the simple example of minimum wage setting in Germany, the paper illustrates how such models can be solved using the method of undetermined coefficients and presents selected simulation results.

Keywords: rational expectations model, discontinuous policy rule, method of undetermined coefficients.

JEL-Classification: E 1, E 6.

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1 Introduction

The general minimum wage was implemented in Germany in 2015.[1] In the German institutional set-up a minimum wage commission presents proposals on biennial adjustments which are subsequently implemented by the Federal Government.[2] Adjustments in line with the commission's recommendations, however, produce a discontinuity as the minimum wage is held constant in a certain year and adjusted over-proportionately in the subsequent year. This policy discontinuity renders standard local approximation techniques infeasible.[3] The paper therefore illustrates how models featuring discontinuous policy rules can be solved using the method of undetermined coefficients (see, for instance, Christiano, 2002, 1991, and McCallum, 1983, on the general method) using a stylised model of wage and profit setting.

Despite focusing on minimum wage setting in Germany the presented method has, *mutatis mutandis*, a broader applicability to models in which a decision/policy rule is non-linear and either depends on the specific time period (e. g. odd-even rationing) or on certain threshold values (e. g. bank lending decisions depending on required minimum levels of equity).

In the outlined model workers are either negotiated wage earners or minimum wage recipients. While – in line with the legal framework – adjustments to the minimum wage are based on an *ad hoc* and *ex ante* specified rule taking into account the discontinuous minimum wage setting[4], negotiated wages are the outcome of an optimisation problem. Two different policy rules are evaluated on the basis of a (joint) loss function assuming that the employee and employer representatives (the "social partners") aim to minimise the volatility of negotiated wages and corporate profits.

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[1] For work on the impact of sectoral minimum wages in Germany see, for instance, Bachmann et al. (2012) and Müller (2010). A comprehensive review on the general impact of minimum wages on employment is given by Neumark and Wascher (2007).

[2] The minimum wage commission comprises nine members: Three representatives each from the representative organisations of the employers and employees, one chairperson, and two consulting (non-voting) members from academia.

[3] As a shortcut, continuous transition functions have frequently been used to model such non-linearities (see Rieth, 2014, Baldini and Ribeiro, 2008, and Bayoumi et al., 1995, among others). Although easy to implement, important information may be lost when non-linearities are approximated in such a way. Consider, for instance, a model (as the one presented below) where some policy differs depending on the prevailing time period (even versus uneven). Applying a transition function to model this policy feature would then imply that the policy (in the steady state) refers to some "average period", and neither to an even nor to an uneven period. While technically correct, this may be hard to justify from an economic point of view.

[4] Subsequently referred to as "minimum wage rule". Policy makers committing to policy rules are frequently encountered in economic modelling, in particular in the sphere of monetary and fiscal policy.

The remainder of the paper is organised as follows. Section 2 presents the stylised model of wage and profit setting as well as its solution under rational expectations. Section 3 briefly discusses the simulation setup and selected results. Section 4 concludes.

2 The stylised model

2.1 Budget constraint

The stylised economy comprises N_F firms, N_A employees, a negotiated wage setter[5], and a minimum wage setter. Employees are heterogeneous in terms of the wages paid to them. While $n_{L,t} \in (0,\ N_A)$ employees receive the wages negotiated for them by the social partners, $n_{M,t} = N_A - n_{L,t}$ employees are paid the minimum wage.[6] The economy's total income, Y_t, is the sum of the total negotiated wage bill, $n_L W_t$, the total minimum wage bill, $n_M M_t$, and corporate profits, Π_t, with W_t representing the negotiated wage and M_t the minimum wage. The budget constraint at time t is therefore given by

$$Y_t = n_L W_t + n_M M_t + \Pi_t. \tag{1}$$

Let $\bar{X}$ denote the time average of variable X_t and $x_t \equiv \ln(\mathrm{X}_t/\mathrm{X}_{t-1})$. It can then be shown that a first-order Taylor approximation of (1) around $\bar{X}$ produces an equivalent formulation which relates relative changes in total income to changes in wages and corporate profits:

$$y_t = \alpha_1 w_t + \alpha_2 m_t + \alpha_3 \pi_t,$$

with $\alpha_1 \equiv \frac{n_L \bar{W}}{\bar{Y}} = (1-\alpha_3)(1-\beta)$, $\alpha_2 \equiv \frac{n_M \bar{M}}{\bar{Y}} = (1-\alpha_3)\beta$, and $\alpha_3 \equiv \frac{\bar{\Pi}}{\bar{Y}}$. α_3 is the firms' profit share and $\beta \equiv \frac{n_M \bar{M}}{n_M \bar{M} + n_L \bar{W}}$ the share of total minimum wages in the total wage bill.

[5] The negotiated wage setter can be thought of as an "aggregate" of the employee and employer representatives.

[6] The model shall not be regarded as fully reflecting the wage setting behaviour but rather serves as an example how models of such a kind can be solved using the proposed method. For the sake of simplification, it is assumed that the number of negotiated wage and minimum wage recipients does not change over time such that $n_{L,t} = n_L$ and $n_{M,t} = n_M \forall t$ applies. Differences in wages could be motivated by heterogeneity of employees with respect to their labour productivity. The analyses abstract from the precise mechanisms that divide employees into recipients of the negotiated wage and minimum wage, respectively. Unemployment is also abstracted from.

2.2 Setting of negotiated and minimum wages

It is assumed that there is a fundamental difference in the wage-setting behaviour of the economy's two wage-setters. The stylised minimum wage setter is able to commit to an *ad hoc* and *ex ante* specified minimum wage rule $\Omega(\cdot)$. Since changes in the minimum wage are supposed to track realised changes in negotiated wages, and the minimum wage commission presents biennial adjustment proposals, it is appealing to model the adjustment to the minimum wage in period t as a function of past changes to the negotiated wage. In line with the legal regulations in Germany, two possible rules m_t^a and m_t^b are considered. Specifically, these are

$$m_t^a = \begin{cases} w_{t-1} + w_{t-2} & , t = 1,3,5, \dots \\ 0 & , t = 0,2,4, \dots \end{cases}$$

and

$$m_t^b = w_{t-1}.$$

The discontinuous rule m_t^a takes into account the biennial meeting schedule of the minimum wage commission as changes to minimum wages are based on the cumulative change to negotiated wages observed over the last two periods.[7] Without loss of generality, it is assumed that the minimum wage setter can only decide upon changes to the minimum wage in uneven periods t. In even periods the minimum wage would accordingly remain unchanged. As an alternative, it is also reasonable to assume annual adjustments in line with m_t^b despite the commission's biennial meeting schedule such that adjustments are made more continuously.[8]

Unlike the setting of the minimum wage, both negotiated wages and corporate profits are the outcome of an optimisation problem. Assume that there exists a negotiated wage setter who minimises the expected discounted total loss

$$\mathcal{L} = E_t \textstyle\sum_{t=0}^{\infty} \delta^t \frac{\Lambda_t}{2} \qquad (2)$$

subject to

[7] The expression for m_t^a results from solving $1 + m_t = (1 + w_{t-1})(1 + w_{t-2})$ and defining $m_t \equiv m_t^a$. It is assumed for simplicity that the "compound interest effect", $w_{t-1}w_{t-2}$, is equal to zero.

[8] The legal regulations on adjusting the minimum wage in Germany require the minimum wage commission to present biennial adjustment proposals to the Federal Government, which are subsequently implemented by the Federal Government. The negotiated pay rates index of the Federal Statistical Office (*Tarifverdienstindex*) is particularly important to the commission's recommendations. However, the Federal Government is not bound to the commission's biennial meeting schedule and implementations can in principle also occur on an annual basis. For adjustments based on m_t^b, the stylised model assumes that the commission can condition its recommendations for the adjustment of the minimum wage in the year following its meeting on the development of the negotiated pay rates index.

$$y_t = \alpha_1 w_t + \alpha_2 m_t^i + \alpha_3 \pi_t, \tag{3}$$

where $i \in \{a, b\}$ with respect to w_t and π_t.[9] $\Lambda_t \equiv w_t^2 + \phi\pi_t^2$ is the per-period loss function[10], E_t the expectations operator conditional on information available at time t, $\delta \in (0,\ 1)$ the discount factor, and ϕ a constant welfare weight capturing the importance of stabilising π_t relative to w_t. Hence, volatile negotiated wage and profit paths tend to result in losses for the negotiated wage setter according to Λ_t. The resulting first-order conditions differ according to the assumed rule. In the case of m_t^a, the state-dependent first-order condition is given by

$$w_t - \frac{\phi\alpha_1}{\alpha_3}\pi_t = \begin{cases} \delta^2 \dfrac{\phi\alpha_2}{\alpha_3} E_t\pi_{t+2} & , t = 1,3,5, \dots \\ \delta \dfrac{\phi\alpha_2}{\alpha_3} E_t\pi_{t+1} & , t = 0,2,4, \dots \end{cases} \tag{4}$$

and in case of m_t^b by

$$w_t - \frac{\phi\alpha_1}{\alpha_3}\pi_t = \delta \frac{\phi\alpha_2}{\alpha_3} E_t\pi_{t+1}. \tag{5}$$

2.3 Model solution under rational expectations

The model is to be solved such that the resulting paths $\{w_{t,} m_{t,} \pi_t\}_{t=0}^{\infty}$ are in line with rational expectations. Combining the first-order condition (4) with the budget constraint (3) results in a new optimality condition of the form

$$y_t = \begin{cases} b_1\pi_t + b_2\pi_{t-1} + b_3\pi_{t-2} + b_4 E_t\pi_{t+1} + b_5 E_t\pi_{t+2} & , t = 1,3,5, \dots \\ b_6\pi_t + b_7 E_t\pi_{t+1} & , t = 0,2,4, \dots \end{cases} \tag{6}$$

where

$$b_1 \equiv \frac{\phi({\alpha_1}^2 + \delta^2{\alpha_2}^2) + {\alpha_3}^2}{\alpha_3}, b_2 \equiv \frac{\phi\alpha_1\alpha_2}{\alpha_3}, b_3 \equiv b_2,$$

$$b_4 \equiv \frac{\delta^2\phi{\alpha_2}^2}{\alpha_3}, b_5 \equiv \delta^2 b_2, b_6 \equiv \frac{\phi{\alpha_1}^2 + {\alpha_3}^2}{\alpha_3},$$

and

$$b_7 \equiv \delta b_2.$$

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[9] As the minimum wage rule has been decided upon *ex ante*, the negotiated wage setter takes its functional form $\Omega(\cdot)$ into account. It is assumed here that the minimum wage rule has already been substituted into the budget constraint (3).

[10] Analyses based on quadratic loss functions are widespread in macroeconomics, in particular in the analysis of optimal monetary policy. See, for instance, Walsh (2010); and for applications Beck and Wieland (2007), Galí and Monacelli (2005), Rudebusch (2002), and Bursian and Roth (2014), among many others.

It is reasonable to conjecture that the state dependency in (6) carries over to the model solution. Therefore, the functional form for the solution

$$\pi_t = \begin{cases} \theta_3^a y_t + \theta_4^a \pi_{t-1} + \theta_5^a \pi_{t-2} + c_2 & ,t = 1,3,5,\ldots \\ \theta_1^a y_t + \theta_2^a \pi_{t-1} + c_1 & ,t = 0,2,4,\ldots \end{cases} \tag{7}$$

that no longer depends on expectations is assumed, where $\theta_1^a, \theta_2^a, \theta_3^a, \theta_4^a, \theta_5^a, c_1$, and c_2 are constant and yet to be determined parameters. Under the method of undetermined coefficients, the expectations $E_t\pi_{t+1}$ and $E_t\pi_{t+2}$ in (6) are now eliminated using (7). A comparison of the resulting condition with (7) then allows the parameters of the proposed model solution to be determined. A mathematical contradiction in the process of solving the model, however, would indicate that the functional form for the model solution was not specified correctly.

The negotiated wage setter takes the policy discontinuity resulting from the biennial minimum wage adjustment explicitly into account when forming its expectations which therefore differ depending on the period when they are formed (even versus uneven). Based on (7), they are given by

$$E_t\pi_{t+1} = \begin{cases} \theta_1^a E_t y_{t+1} + \theta_2^a \pi_t + c_1 & ,t = 1,3,5,\ldots \\ \theta_3^a E_t y_{t+1} + \theta_4^a \pi_t + \theta_5^a \pi_{t-1} + c_2 & ,t = 0,2,4,\ldots \end{cases} \tag{8}$$

and

$$E_t\pi_{t+2} = \begin{cases} \theta_3^a E_t y_{t+2} + \theta_4^a E_t\pi_{t+1} + \theta_5^a \pi_t + c_2 & ,t = 1,3,5\ldots \\ \theta_1^a E_t y_{t+2} + \theta_2^a E_t\pi_{t+1} + c_1 & ,t = 0,2,4\ldots \end{cases} \tag{9}$$

(8) and (9), in turn, also depend on expected income changes which makes it necessary to specify the exact nature of the income process.[11] It is assumed that income changes evolve according to a first-order autoregressive process of the form

$$y_{t+1} = \rho y_t + d + \varepsilon_{t+1},$$

where d refers to the drift parameter and ρ to the autocorrelation coefficient. ε_{t+1} is an i. i. d. shock process with $E_t\varepsilon_{t+1} = 0$ and $Var(\varepsilon_{t+1}) = \sigma^2$. Accordingly, (8) and (9) can be rewritten such that

$$E_t\pi_{t+1} = \begin{cases} \theta_1^a[\rho y_t + d] + \theta_2^a \pi_t + c_1 & ,t = 1,3,5,\ldots \\ \theta_3^a[\rho y_t + d] + \theta_4^a \pi_t + \theta_5^a \pi_{t-1} + c_2 & ,t = 0,2,4,\ldots \end{cases} \tag{10}$$

and

[11] From an efficiency wage perspective, for instance, the wage setting behaviour can also increase productivity and income. Such and similar general equilibrium effects are abstracted from.

$$E_t\pi_{t+2} = \begin{cases} \theta_3^a[\rho^2 y_t + d(1+\rho)] + \theta_4^a E_t\pi_{t+1} + \theta_5^a\pi_t + c_2 & ,t = 1,3,5\ldots \\ \theta_1^a[\rho^2 y_t + d(1+\rho)] + \theta_2^a E_t\pi_{t+1} + c_1 & ,t = 0,2,4\ldots \end{cases} \quad (11)$$

$E_t\pi_{t+1}$ and $E_t\pi_{t+2}$ in (6) are eliminated using (10) and (11) such that

$$\pi_t = \begin{cases} \frac{b_8}{b_{10}} y_t - \frac{b_2}{b_{10}}\pi_{t-1} - \frac{b_3}{b_{10}}\pi_{t-2} - \frac{b_9}{b_{10}} & ,t = 1,3,5,\ldots \\ \frac{1-\rho b_7\theta_3^a}{b_6 + b_7\theta_4^a} y_t - \frac{b_7\theta_5^a}{b_6 + b_7\theta_4^a}\pi_{t-1} - \frac{b_7(\theta_3^a d + c_2)}{b_6 + b_7\theta_4^a} & ,t = 0,2,4,\ldots \end{cases} \quad (12)$$

where

$$b_8 \equiv 1 - \rho^2 b_5\theta_3^a - \rho\theta_1^a(b_5\theta_4^a + b_4),$$

$$b_9 \equiv b_4(\theta_1^a d + c_1) + b_5\theta_4^a(\theta_1^a d + c_1) + b_5(\theta_3^a d(1-\rho) + c_2),$$

and

$$b_{10} \equiv b_1 + b_4\theta_2^a + b_5(\theta_2^a\theta_4^a + \theta_5^a).$$

A comparison of the coefficients in (7) and (12) gives rise to a non-linear system of seven equations in seven unknowns which can be solved numerically for the parameters $\theta_1^a, \theta_2^a, \theta_3^a, \theta_4^a, \theta_5^a, c_1$, and c_2:[12]

$$\theta_1^a(b_6 + b_7\theta_4^a) = 1 - \rho b_7\theta_3^a$$

$$\theta_2^a(b_6 + b_7\theta_4^a) = -b_7\theta_5^a$$

$$b_{10}\theta_3^a = b_8$$

$$b_{10}\theta_4^a = -b_2$$

$$b_{10}\theta_5^a = -b_3$$

$$c_1(b_6 + b_7\theta_4^a) = -b_7(\theta_3^a d + c_2)$$

$$b_{10}c_2 = -b_9.$$

After substituting the model solution for profit setting (7) in the budget constraint (3), a state-dependent model solution for negotiated wage setting can also be determined. Solving the model under m_t^b using (5) is standard and skipped for brevity.

[12] It is ensured that only meaningful parameter values are used for the simulations. A solution to the non-linear system is considered meaningful in an economic sense if positive income changes *ceteris paribus* also lead to positive changes in corporate profits (i. e. $\theta_1^a > 0$ and $\theta_3^a > 0$).

3 Simulation of wage and profit setting

3.1 Simulation setup and calibration

This section illustrates how the obtained model solutions can be used in order to generate simulated paths $\{w_{t,}m_{t,}\pi_t\}_{t=0}^N$ with N referring to the total number of periods.

Table 1: Calibration

Parameter	Value	Description	Source
δ	1/(1.03)	Discount factor	Christiano et al. (2005)
α_3	0.500	Firms' profit share	$1 -$ mean of the nominal ratio of employee compensation in Germany over GDP (time period: 2000 to 2014)
β	0.050	Share of total minimum wages in the total wage bill	Estimate based on SOEP data[13]
d	0.014	Drift parameter of the income process	Estimate of an *AR(1)* model with drift based on annual data for the "real value" of GDP (time period: 1991 to 2014)
σ	0.010	Standard deviation of the shock process	Standard value
ϕ	1.000	Welfare weight	Standard value[14]

The simulation of wage and profit setting requires a calibration of the key model parameters which can be found in Table 1. Conditional on m_t^a and given an income process $\{y_t\}_{t=0}^N$, the paths $\{w_{t,}m_{t,}\pi_t\}_{t=0}^N$ can be determined together with the budget constraint (3) and equation (7). The scenarios for both rules are simulated independently and evaluated based on the discounted total loss (2).[15]

[13] See Wagner et al. (2007) for an introduction to the German Socio-Economic Panel Study (SOEP) or for further details http://www.diw.de/en/soep.

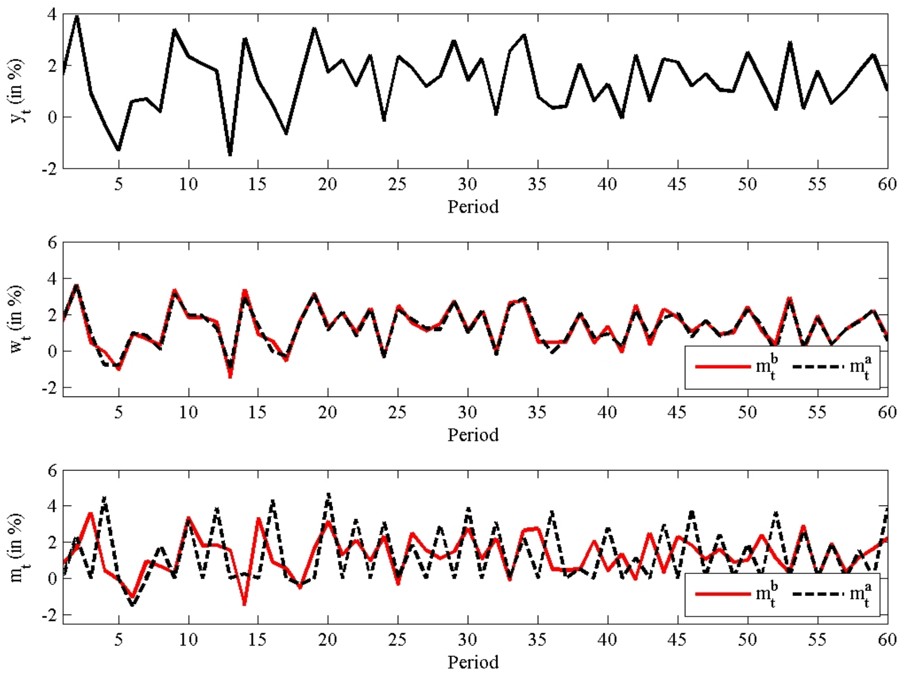
[14] The calibration has been chosen for expositional purposes: For $\phi = 1.0$ the negotiated wage setter endeavours to equally stabilise wages and corporate profits. Based on the assumption of risk-neutral employers and risk-averse employees, a negotiated wage setter representing both employees and employer representatives is likely to stabilise wages more strongly at the expense of corporate profits. This is the case for $0 \leq \phi < 1$. Results are robust.

[15] The model was simulated for $S = 100$ independent realisations of the income process each with dimension $(N \times 1)$. The results presented are based on the average discounted total loss resulting from the S simulations. The number of periods N was selected such that all subsequent periods $N + i$ with

3.2 Selected simulation results

For expositional purposes, Figure 1 shows the evolution of w_t and m_t based on one particular realisation of the income process y_t. The figure contains the simulated income process as well as the paths for the negotiated wage and the minimum wage under m_t^a and m_t^b.[16]

Figure 1
Simulated data



Under m_t^a, adjustments to the minimum wage take place on a biennial schedule and its growth rates thus show an alternating profile. In uneven periods, the minimum wage is not adjusted, and growth rates are zero. Conversely, in even periods, an adjustment by the cumulative change to negotiated wages observed in the two preceding periods is implemented. Compared with m_t^b, adjustments therefore tend to be stronger in even periods. Under m_t^b, however, the minimum wage is always adjusted based on the

$i > 0$ are negligible for loss evaluation. In addition, the first 25 % of the periods of each simulated process have been disregarded as "burn-in" in order to ensure that the starting values required for the simulation of the paths $\{w_t, m_t, \pi_t\}_{t=0}^{N}$ do not have a significant impact on the qualitative results. The autocorrelation coefficient turned out not to be statistically significant for an *AR(1)* model with drift based on annual data. It is therefore not depicted in Table 1. Results are, however, similar when setting $\rho > 0$.

[16] The share of total minimum wages in the total wage bill will probably be rather small in Germany. Based on own estimates using SOEP data, the upper bound is calibrated at 5 %. In order to better visualise the impact of different minimum wage rules on the negotiated wage setting, β was set to 0.3 only in the simulations underlying Figure 1.

change of the negotiated wage observed in the preceding period. As a result, the dynamics of both the negotiated wage and the minimum wage are the same except for a one period phase shift.

As the negotiated wage setter takes into account the effects of its current decisions on future developments, the choice of the minimum wage rule has an indirect impact on the dynamics of negotiated wages. Hence, w_t in Figure 1 differs conditional on the respective minimum wage rule. The impact of each rule on the variability of negotiated wages, however, is not immediately obvious. Therefore, Table 2 shows standard deviations of key model variables and resulting total losses based on the S independent simulations.

Table 2: Standard deviations (SD) of key model variables in % and discounted total loss $\mathcal{L}$

$\phi = 1.0$	$SD(w_t)$	$SD(m_t)$	$SD(\pi_t)$	$\mathcal{L}$
m_t^a	1.0034	1.6775	1.0561	1.9768
m_t^b	1.0035	1.0036	1.0548	1.9741

Income volatility under m_t^b translates almost one-for-one into the volatility of the minimum wage. Conversely, under m_t^a, its volatility is significantly higher as expected owing to the biennial adjustment schedule. The discounted total loss is smaller under m_t^b suggesting that annual adjustments to the minimum wage may be superior compared to biennial adjustments in line with the minimum wage commission's meeting schedule.

4 Conclusion

This paper outlined a simple model of minimum wage setting in Germany in which the social partners minimise a (joint) loss function that penalises volatility in negotiated wages and in corporate profits. In line with the legal framework, adjustments to the minimum wage were modelled by an *ad hoc* and *ex ante* specified discontinuous policy rule: In even periods, the minimum wage was not adjusted whereas in uneven periods, adjustments by past changes to negotiated wages were implemented. Solving stylised rational expectations models featuring such discontinuities is not straightforward and using continuous transition functions as a shortcut to model those non-linearities may be hard to justify from an economic point of view.

The paper therefore illustrated how to solve the presented model using the method of undetermined coefficients. Based on selected model simulations it could be inferred that – conditional on the assumed loss function of the social partners – annual adjustments to the minimum wage may be advantageous despite the minimum wage commission's biennial meeting schedule.

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