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Macroeconomic now- and forecasting based on the factor error correction model using targeted mixed frequency indicators

Jeong-Ryeol Kurz-Kim

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Research question:
Since the influential paper of Stock and Watson (2002), the dynamic factor model (DFM) has been widely used to forecast macroeconomic key variables such as GDP. The DFM is capable of summarizing a huge number of indicators containing macroeconomic, financial and survey data in a small number of factors. This strong advantage of the DFM enables econometricians to overcome the restriction of dimensionality faced by the usual vector autoregressive models.

The DFM also has some weaknesses, however. Three refinements have been considered in recent literature. Because of publication lags regarding macroeconomic key variables, GDP for example, it is necessary to estimate current values for the corresponding quarter (nowcasting) and also to estimate past values (unknown yet) for the preceding quarter. For now-casting, the dynamic factor model is modified by using the mixed data sampling technique. The mixed data sampling technique is able to capture information from high frequency (monthly) indicators in order to estimate low frequency (quarterly) key variables. The second refinement uses pre-selection methods to optimally choose a small number of indicators from a large number of indicators. This kind of pre-selection improves efficiency when extracting factors by applying principal component analysis. This pre-selected set of indicators (called targeted indicators in the literature) is able to avoid arbitrariness by choosing a very large number of initial indicators. The third refinement takes into account the non-stationarity of macroeconomic variables, which was completely ignored by the DFM. The error correction mechanism models the co-integrating relationship between the key variables and factors, and thus captures not only the short-run dynamics, which is the case for the DFM, but also the long-run dynamics of the non-stationary macroeconomic variables.

Contribution:
This paper proposes a forecasting model using targeted mixed-frequency indicators which includes three refinements to the dynamic factor model, namely the mixed data sampling technique, pre-selection methods and the error correction mechanism. These three techniques have already been well considered by many authors, but the novelty of our model is the combination of all three in a single model.

Results:
The empirical results based on euro area data show a superior nowcasting and forecasting performance of our new model compared to that of the subset models, namely the DFM, the DFM plus mixed data sampling, the DFM plus mixed data sampling and pre-selection.

Nichttechnische Zusammenfassung

Fragestellung:
Seit der Publikation von Stock und Watson (2002), fungiert das dynamische Faktormodel (DFM) als eines der am häufigsten genutzten Prognosemodelle für makroökonomische Variablen wie das Bruttoinlandsprodukt. Das DFM ist in der Lage, eine Vielzahl von makroökonomischen, finanzwirtschaftlichen und Umfrage bezogenen Indikatoren in eine kleine Anzahl von Faktoren zusammenzufassen. Dieser Vorteil ermöglicht, die Dimensionsrestriktion der vektorautoregressiven Modelle aufzuheben.

Das DFM hat aber seinerseits auch einige Schwächen. Drei Schwächen und deren Verbesserungsvorschläge werden in der Fachliteratur diskutiert. Wegen der Publikationsverzögerung ist es notwendig, auch eine Schätzung für laufende Quartale (Nowcasting) durchzuführen. Für das Nowcasting lässt sich das DFM mit Verwendung des gemischten Frequenzen-Verfahrens (MIDAS) modifizieren. Dabei nutzt das MIDAS-Verfahren Informationen aus den höher frequentierten (monatlichen) Indikatoren für die Schätzung der niedrig (quartal-) frequentierten Variablen aus. Die zweite Verbesserung besteht darin, eine kleine Anzahl von besonders aussagekräftigen Indikatoren vorab auszuwählen (Vorsortierungsmethode). Diese Art von Vorsortierungen erhöht die Effizienz bei der Bestimmung von Faktoren via Hauptkomponentenanalyse. Die dritte Verbesserung trägt der Nicht-Stationarität der makroökonomischen Variablen Rechnung. Der Fehler-Korrektur-Term berücksichtigt die Kointegrationsbeziehung zwischen der zu prognostizierenden Variable und den zu erklärenden Faktoren. Deshalb kann die Fehler-Korrektur-Modellierungen nicht nur die kurzfristigen Dynamiken, sondern auch die langfristigen Dynamiken erfassen.

Beitrag:
In diesem vorliegenden Aufsatz wird ein Prognosemodell vorgeschlagen, das die drei oben genannten Verbesserungen in sich vereinen kann.

Ergebnisse:
Die empirischen Ergebnisse basierend auf den aggregierten Euroraumdaten, die für die ökonomischen Analysen und Prognosen bei der Bundesbank genutzt werden. Dabei zeigt sich, dass das vorgeschlagene Prognosemodell eine signifikant höhere Güte als konkurrierende Modelle hat.

Macroeconomic now- and forecasting based on the factor error correction model using targeted mixed frequency indicators*

Jeong-Ryeol Kurz-Kim
Deutsche Bundesbank

Abstract

Since the influential paper of Stock and Watson (2002), the dynamic factor model (DFM) has been widely used for forecasting macroeconomic key variables such as GDP. However, the DFM has some weaknesses. For nowcasting, the dynamic factor model is modified by using the mixed data sampling technique. Other improvements are also studied mostly in two directions: a pre-selection is used to optimally choose a small number of indicators from a large number of indicators. The error correction mechanism takes into account the co-integrating relationship between the key variables and factors and, hence, captures the long-run dynamics of the non-stationary macroeconomic variables.

This papers proposes the factor error correction model using targeted mixed-frequency indicators, which combines the three refinements for the dynamic factor model, namely the mixed data sampling technique, pre-selection methods, and the error correction mechanism. The empirical results based on euro-area data show that the now- and forecasting performance of our new model is superior to that of the subset models.

Keywords: Factor model; MIDAS; Lasso; Elastic Net; ECM; Nowcasting; Forecasting.

JEL classification: C18, C23, C51, C52, C53.

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*Contact address: Deutsche Bundesbank, Wilhelm-Epstein-Str. 14, D-60431 Frankfurt am Main, Germany. Phone: +49 69 9566 4576. E-Mail: jeong-ryeol.kurz-kim@bundesbank.de. The author thanks Johannes Hoffmann, Klemens Hauzenberger, Christian Schumacher and Jochen Mankart for their excellent comments and support. This paper represents the author' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or its staff.

1 Introduction

Macroeconomic forecasting is one of the important tasks of both researchers and practitioners. This is because forecasts serve as the basic information for economic policy decisions. Recently, macroeconomic nowcasting[1] has also become an important part of forecasting practice. By performing nowcasting, the mixed data sampling (MIDAS) technique enables us to use information contained in high frequency indicators for forecasting some macroeconomic low frequency key variables. We will come back to discuss this modeling technique, but first review forecasting models in chronological order.

Since the seminal work of Engle and Granger (1987), the error correction model (ECM) was one of the widely used forecasting models in the late 1980s and 1990s. The ECM is useful for *short-run* forecasting,[2] especially in times when the economic environment is strongly above or below the equilibrium and tends to adjust towards it. Another strong advantage of the ECM is that the short-run dynamics and long-run adjustment can be captured separately. This property is useful for forecasters, because it gives them scope for deeper economic interpretations of the development of the economic key variables. This ECM is usually based on a framework of vector autoregression (VAR) which has a drawback of a limited dimensionality, usually denoted the "curse of dimensionality problem" in the literature.

Stock and Watson (2002) popularized the (approximate) dynamic factor model (DFM), which has been widely used in empirical forecasting practice. One of the strongest advantages of the DFM is that it overcomes the dimensionality constraints of the VAR (and VARECM). The DFM is capable of summarizing a large number of indicators in terms of a small number of factors, which can be used for forecasting some economic key variables. However, the DFM in Stock and Watson (2002) is based on stationary data, ie, an integration restriction is imposed by means of a first-difference operation when the underlying macro variables (indicators) are non-stationary. This is one drawback of the DFM, because the DFM cannot use the long-run information which can be only captured by modeling the EC mechanism. Thus, the DFM and the ECM can be regarded as being complementary to each other when they can be combined in a model. One more drawback of the DFM is that it is merely assumed that factors obtained by the principle component (PC) method have some explanatory power for the key variables to be forecast, see Stock and Watson (2002). In other words, the DFM is based on the assumption that *each* of the indicators in a large dataset has some *significant* explanatory power for the key variables. This is, however, not always an adequate assumption in empirical applications.

[1] In the following, the word 'forecasting' is also used as a synonym or generic term for 'nowcasting', unless they need to be distinguished from each other.

[2] Christoffersen and Diebold (1998) studied the implication of co-integration restriction for forecasting and conclude that *this is the opposite of the folk wisdom–it turns out that imposition of co-integrating restrictions helps at short, but not long, horizons.* $\cdots$ *This is because the long-horizon forecast of the error correction term is always 0. (The error correction term, by construction, is covariance stationary with a zero mean.)* [Christoffersen and Diebold, 1988, Journal of Business & Economic Statistics, p. 452] They correctly pointed out that this *folk wisdom*, namely that imposing co-integration would help more in long-horizon forecasting, has arisen from the misinterpretation of the simulation results in Engle and Yoo (1987), who *compared forecasts from a VAR in levels (which impose neither integration nor co-integration) with forecasts from the co-integrated system (which impose both integration and co-integration.) [Christoffersen and Diebold, 1988, Journal of Business & Economic Statistics, p. 455].*

To overcome these drawbacks of the DFM, refinements in two directions have been considered in the literature. Bai (2004) considers the generalized dynamic factor model (GDFM) with non-stationary factors and possible co-integration among them. Recently, Banerjee et al. (2014, 2015) introduced the factor-augmented error correction model (FECM) capturing the co-integrating relationship between factors and the key variables to be forecast and demonstrate empirically the superior forecasting performance of the FECM over the DFM. Their papers used both the dataset in Stock and Watson (2005) for the US and the Euro Area Wide Model dataset of Fagan et al. (2001).

The other refinement focuses on a pre-selection of indicators. The choice of a large dataset is usually based on the economic intuitions or/and the experiences of forecasters, but a degree of arbitrariness cannot be avoided in the selection of a huge number of indicators. In the framework of the DFM, Boivin and Ng (2006) study the relationship between the dimension of the panel data and forecasting performance and conclude that *the factors extracted from as few as 40 series seem to do no worse, and in many cases, better than the ones extracted from 147 series.* Bai and Ng (2008) also argue that $\cdots$ *the dynamic factor model as it stands does not take into account the predictive ability of* X_{it} (indicators) *for* y_t (key variables). Consequently, they proposed *targeted* indicators using some pre-selection methods such as the least absolute shrinkage and selection operator (lasso)[3] and report improvements at all forecast horizons over the DFM using fewer but informative indicators. In the framework of Bayesian shrinkage, De Mol et al. (2008) showed that a wide range of prior choice leads to a forecasting performance that is as good as the principal component (PC) method. This result supports the intuition that a combination of some selection methods and the PC method could improve forecasting performance of the DFM. Li and Chen (2014) also report in a slightly different regard[4] on a significant improvement in forecasting performance based on the lasso regression as compared with the DFM alone. Using the dynamic sparse partial least squares method which selects an informative subset of indicators, Fuentes et al. (2015) demonstrate empirically, based on the same database as Stock and Watson (2005), a better performance in improving efficiency compared with that of the DFM.

One more modification can be achieved by adopting the mixed data sampling (MIDAS) technique. Ghysels et al. (2004) and Ghysels et al. (2007) introduced this method to explain low frequency variables using information contained in high frequency data. It is a useful tool, especially for nowcastings when quarterly GDP, which is usually known some weeks after the end of quarter, should be forecast in current quarters by means of the monthly data available. Marcellino and Schumacher (2010), for example, combine the DFM with the MIDAS technique for nowcasting German GDP. Götz et al. (2014) combine the MIDAS technique with EC modeling and empirically demonstrate the usefulness of the EC term in the MIDAS context.

In this paper, we introduce a factor error correction model using targeted mixed-frequency indicators (which can be synonymously regarded as a lasso-based factor-augmented mixed-frequency error correction model (LFMECM)), which takes into ac-

[3]Tibshirani (1996) introduced this method to obtain greater prediction accuracy and economic interpretability for estimation in linear models. See the subsequent section for some details on the lasso technique.

[4]They compare the forecasting performance of pooling forecasters based on the DFM and based on the lasso regression without pooling

count all three refinements of the DFM, namely the MIDAS technique, the pre-selection method, and EC modeling. As will be shown, the GDFM of Bai (2004) is a basic element of our LFMECM. The linkage between the possible explanatory power of the factors and the key variables to be forecast will be reinforced using the lasso technique (Tibshirani, 1996), or rather, the elastic net (EN) (Zou and Hastie, 2005) at the beginning of modeling our LFMECM. From the pre-selected panel data (targeted indicators), the PC method will extract the long-run and short-run factors separately. The long-run factors will build the EC term, while the short-run factors represent the short-run dynamics as processed in the DFM by Stock and Watson (2002). Here, the long-run factors are estimated at a quarterly frequency. The build-up of the short-run lags, however, is at a monthly frequency, although our key variables to be forecast such as GDP are observed at a quarterly frequency. The two different frequencies will be connected by the MIDAS technique.

The paper is organized as follows. Section 2 describes the LFMECM and its three elements, the pre-selection methods (namely, the lasso and EN techniques), the DFM in the framework of EC modeling and the MIDAS techniques. None of these elements is new, each of them is well documented in the literature. What is novel in this paper is the combination of these three elements in a single model. This combination can be justified, as will be partially shown in the theoretical analysis, in the simulation study, and above all in the empirical application, by the fact that each of the three elements contributes to an improvement in the forecasting performance of the standard DFM. Section 3 presents a simulation study to demonstrate how much the MIDAS and the ECM can be expected to contribute to the forecasting performance. In this regard, the standard DFM by Stock and Watson (2002) will serve as the benchmark model. In the empirical application based on an euro-area dataset used for short-term macroeconomic analysis at the Deutsche Bundesbank presented in section 4, we show how the pre-selection and estimation of the long- and short-run factors work empirically and compare the nowcasting performance of our LFMECM with that of the 'subset' models, namely the DFM plus the MIDAS technique and the DFM plus the MIDAS technique and the lasso/EN. Section 5 provides some more supplementary extensions and discussions on one-step ahead forecasting. It also sketches out some asymptotic distributions of estimated parameters in the LFMECM. Section 6 summarizes the paper.

2 The factor error correction model using targeted mixed-frequency indicators: the lasso-based factor-augmented mixed-frequency error correction model (LFMECM)

The basic model of the LFMECM is the single equation error correction model (SEECM). The SEECM was developed to capture a stable relationship between consumer expenditure and income in the UK economic equilibrium by Davidson et al. (1978). Banerjee et al. (1990) complete the dynamic SEECM for non-stationary variables by using a linear transformation of the autoregressive-distributed lag model. The SEECM is a widely used model in economic analysis, both in structural analysis and in forecasting practice. This is because the SEECM is capable of capturing both the adjustment towards the economic equilibrium (a stable long-run relationship in level) and the short-run dynamics (in difference) and, hence, can reproduce economic equilibrium hypotheses in a statistical model.[5] In terms of forecasting, the ECM is rather useful for short-horizon forecasting exercises—*this is the opposite of the folk wisdom* as noted by Christorfferson and Diebold (1998).[6] The structure of the LFMECM consists of the SEECM plus three further methodological techniques: a pre-selection method, ie, the lasso and the EN method, extracting long- and short-run targeted indicators via the PC method, the dynamic factor EC modeling, and the MIDAS technique. In the next subsections, each of these methods will be briefly summarized in the context of the LFMECM.

2.1 Pre-selection of indicators: the lasso and the elastic net

Lasso: The lasso technique, popularized by Tibshirani (1996), is used to estimate and select variables simultaneously. Pre-selection can be achieved by minimizing the residual sum of squares subject to the sum of the absolute value of the coefficients being less than a threshold parameter. The lasso estimate by Tibshirani (1996) in penalized form is given as:

$$\beta^{lasso} = \text{argmin}_{\beta} \left\{ \frac{1}{2N} \sum_{i=1}^{N} (y_i - x_i^T \beta)^2 + \theta \sum_{j=1}^{n} |\beta_j| \right\}, \tag{1}$$

where y $(T \times 1)$ is a key variable of interest, X $(T \times N)$ a panel of indicators, and n the number of pre-selected indicators. The expression in (1) is the ordinary least squares (OLS) with a L_1-norm penalty multiplied by the lasso parameter, θ. This second term serves as a selection mechanism to set some coefficients for indicators of little relevance with respect to the GDP to zero in the panel data. As documented in many empirical papers, for example Korobilis (2013) and Gefang (2014), this pre-selection improves forecasting accuracy and, at the same time, guarantees greater explanatory power for the factors estimated from the (pre-selected) panel data for the key variables. Regarding the DFM of Stock and Watson (2002), the lasso selects a set of targeted indicators in the sense of Bai and Ng (2008) for forecasting a certain economic variable (GDP in our case). Because of the reinforcement of the correlation between GDP and the panel data, it justifies the assumption of the approximate DFM of Stock and Watson (2002) that the estimated factors have some explanatory power concerning the key variable to be forecast.[7] The lasso estimates in (1) have the oracle property (Fan and Li, 2001) by being given the necessary condition (see Theorem 1, Zou, 2005), meaning that the lasso correctly detects the sparse subset indicators and the lasso estimates follow asymptotic normality. Regarding the determination of the lasso parameter, θ, Tibshirani (1996) proposes three methods via minimizing the prediction errors regarding the response variable, y. Slightly differently, we choose the optimal lasso parameter by minimizing historical forecasting MSE and/or

[5] See Banerjee et al. (2014) for empirical forecasting exercises; Engle and Yoo (1987), and Christorfferson and Diebold (1998) for the ECM and forecasting; Kremers et al. (1992), Ericsson (1994), Banerjee et al. (1999) for theoretical analysis of the SEECM.

[6] For this reason, our LFMECM focuses on now- and one-step ahead forecasting. Kurz-Kim (2008) also concludes based on his simulation that *the SEECM produces superior forecasts for short horizons, but not for long horizons.*

[7] Stock and Watson (2002, p. 148) merely assume existence of a causal relationship between the panel data (without pre-selection) represented by factors and the key variable to be forecast.

MAD in our empirical applications. In practice, the set of optimal indicators chosen by the optimal lasso parameter is not restricted to be the same for every time period. Instead, the indicators are selected at each forecasting point in time, and the forecasting equation is re-estimated after new factors are estimated, as also recommended in Bai and Ng (2008).

Elastic net (EN): Zou and Hastie (2005) propose the EN, which can be regarded as a 'generalized' lasso technique with regard to the penalty term whose extreme is either lasso or ridge regression.[8] The EN estimate is given as:

$$\beta^{en} = \text{argmin}_{\beta} \left\{ \frac{1}{2N} \sum_{i=1}^{N} \left(y_i - x_i^T \beta \right)^2 + \theta \left[\sum_{j=1}^{n} \left(\frac{1-\alpha}{2} \beta_j^2 + \alpha |\beta_j| \right) \right] \right\}. \quad (2)$$

The generalization of the lasso is carried out by the tuning parameter, $\alpha \in [0, 1]$. For $\alpha \in (0, 1)$, the penalty term interpolates between the L^1- and L^2-norm of β.[9] The EN is the same as the lasso in (1) when $\alpha = 1$ and the ridge regression when $\alpha = 0$. This generalization has important advantages in empirical forecasting applications. Macroeconomic panel data often have the $N >> T$ problem and high pairwise correlations of the indicators in a group. For such cases, as pointed out in Zou and Hastie (2005), the length of time dimension T would be the upper limit of the number of selected variables. In the latter case, the lasso takes just one variable of the highly correlated variables instead of a factor of the group. In our empirical application using the euro area panel data, these two advantages of the EN play a useful role. We will come back to this topic later.

2.2 Long-run and short-run targeted indicators

Suppose that we have a set of non-stationary monthly panel data, $X_{i,t}^M$, with the cross-section dimension $i = 1, \ldots, N$ and the time domain dimension $t = 1, \ldots, T^M$. Moreover, we have a non-stationary quarterly GDP, Y_t^Q with $t = 1, \ldots, T^Q$ which has to be forecast. It is assumed that both of them ($X_{i,t}^M$, Y_t^Q) are non-stationary, where the symbols ^{Q} and ^{M} stand for a quarterly and monthly frequency, respectively. Using the EN technique, we now try to obtain a set of long-run targeted indicators and short-run targeted indicators.

In order to apply the EN technique to the (non-stationary) quarterly GDP series, we transform the (non-stationary) monthly panel into quarterly panel. As usually recommended in literature, we also take values of every last month in a quarter and regard them as quarterly data, as $X_{it}^Q := X_{i,1:3:T}$.[10] Using the EN technique in (2), where Y_t^Q (X_{it}^Q) is used as a variable on the left (right) hand side, we choose a subset of X_{it}^Q as our (long-run) targeted indicators for the (non-stationary) level of GDP. In order to obtain a set of short-run targeted indicators, we simply use a difference operator and build quarterly growth rates of endogenous and exogenous variables in (2) as: $y_t^Q := \Delta Y_t^Q$ and $x_{it}^Q := \Delta X_{it}^Q$. Using the EN technique in (2), where y_t^Q (x_{it}^Q) is used as a variable on the left

[8] See Hoerl and Kennard (1970) for the ridge regression.

[9] As will be shown in our empirical application, the best forecasting models in the sense of the mean-squared forecasting error have usually an $\alpha \in (0, 1)$.

[10] The empirical reason for this choice shows a better forecasting performance of our model than other possibilities such as mean value of three months in a quarter. Furthermore, from econometric point of view this choice ensures the asymptotic nominal size of tests for co-integration in the presence of mixed sampling frequencies and temporal aggregation. See also subsection 5.3 for more discussions on this topic.

(right) hand side, we first choose a subset of x_{it}^Q (quarterly difference) and determine x_{it}^M (monthly difference) as our targeted (short-run) indicators for the (stationary) difference of GDP.

This means that the stationary regressors in our model are *not* the differenced non-stationary factors, but factors derived from the differenced non-stationary (and, hence, stationary) panel data. The theoretical reason for this is that the sum of a short-run factor is not equal to the corresponding long-run factor, when the innovation process as a linear combination of the panel data and their factors is I(1)-process. See Bai and Ng (2004) for more details on this topic. The empirical reason for this is that, as will be shown in our empirical applications, the subset of targeted long-run indicators is usually different to that of targeted short-run indicators. Furthermore, it is also economic intuition that some economic variables relate more to long-run dynamics, others more to short-run dynamics of GDP.[11]

2.3 Dynamic factor EC modeling

Our starting point is the GDFM of Bai (2004):

$$\begin{aligned} X_{i,t}^M &= A_i(L)F_t^M + e_{i,t} \\ F_t^M &= F_{t-1}^M + \epsilon_t, \end{aligned}$$

where $A_i(L)$ is a vector of polynomials of the lag operator; each of $e_{i,t}$ and ϵ_t is a stationary zero-mean vector process. In this model, the partial sum of errors drives the factors and, therefore, factors are non-stationary. These non-stationary factors as common trend again drive (ie, are causal to) each of the variables in the panel data. Furthermore, the relationship between $X_{i,t}^M$ and F_t^M is also dynamic.

$$\begin{aligned} Y_t^Q &= A_j(L)F_{jt}^Q + e_t \\ F_{jt}^Q &= F_{j,t-1}^Q + \epsilon_{jt}. \end{aligned}$$

Now, consider a factor augmented autoregressive model with exogenous variables (FARX) which contains the generalized dynamic component of factors in the sense of Bai (2004), and the dynamic component of endogenous variable in the sense of Stock and Watson (2002), as:

$$Y_t^Q = \sum_{p=1}^{P+1} b_p Y_{t-p}^Q + \sum_{q=0}^{Q_j+1} a_{q_j} F_{j,t-q_j}^Q + u_t \tag{3}$$

The equation in (3) can be now transformed in the SEECM without any change in the residual structures of the FARX.[12] Using $y_t^Q := \Delta Y_t^Q$ and $f_{jt}^Q := \Delta F_{jt}^Q$, the SEECM can be given as:

$$y_t^Q = b[Y_{t-1}^Q - \beta F_{t-1}] - \sum_{p=1}^{P} b_p y_{t-1}^Q + \sum_{q_j=0}^{Q_j} a_{q_j} f_{jt-q_j} + u_t \tag{4}$$

[11] In the context of the co-integration analysis, they are usually defined as permanent and transitory components. See, for more details, Cochrane (1994).

[12] For this transformation, see Davisdson et al. (1978) and Banerjee et al. (1990).

where $b = \sum_{p=1}^{P} b_p - 1$ the loading parameter for the error correction term; $\beta = \sum_{j=1}^{J} \sum_{q=0}^{Q_j} a_{q_j}/b$ the co-integrating parameter for the estimated regressors (long-run factors); P is the order of lagged endogenous variables; Q_j is the order of j-th lagged exogenous variables; J is the number of exogenous variables (usually symbolized by r in the frame of factor models.)[13]

In the next step, we substitute $f_{jt}^{Q} = \Delta F_{jt}^{Q}$ with f_{jt}^{M} in the following factor expression:

$$\begin{aligned} y_t^M &= a_j(L) f_{jt}^M + z_t \\ f_{jt}^M &= f_{j,t-1}^M + \varepsilon_{jt}, \end{aligned}$$

where y_t^M are the unobservable monthly changes in GDP and f_{jt}^M are the short-run factors derived from the monthly short-run indicators x_{it}^M.
Remark (a): The error correction term can be regarded as the static factor model (Bai, 2004, p. 139):

$$\begin{aligned} x_{it}^l &= \sum_{j=1}^{r(l)} \lambda_{ij} f_{jt}^l + e_{it}^l \\ f_t^l &= f_{t-1}^l + \epsilon_t^l, \end{aligned}$$

where e_{it}^l is a stationary error process and ϵ_t^l is a $(r \times 1)$-dimensional stationary zero-mean vector process.

Substituting x_{it}^l with y_t, we get a factor augmented ECM system that is exactly the same (up to the factor augmentation) as the triangular ECM, introduced by Phillips (1991, Econometrica, p. 286) as

$$y_t = \sum_{j=1}^{r} \beta_j \hat{f}_{jt}^l + e_t \tag{5}$$

$$\hat{f}_t^l = \hat{f}_{t-1}^l + \varepsilon_t^l, \tag{6}$$

where β_j is a $(1 \times r)$ matrix of co-integrating coefficients.[14] Phillips (1991) interprets (5) as a stochastic version of the linear equilibrium relationship, where e_t represents the stationary deviation from equilibrium. We will use this triangular ECM in our empirical application in order to calculate departures from the economic equilibrium for our EC term in (4). We will come back to this issue later.

[13] An example of this ECM transformation is given by setting $P = 1; Q = 0; J = 1$ as follows:

$$\begin{aligned} Y_t^Q &= b_1 Y_{t-1}^Q + b_2 Y_{t-2}^Q + a_0 F_t^Q + a_1 F_{t-1}^Q + u_t \\ Y_t^Q - Y_{t-1}^Q &= -Y_{t-1}^Q + b_1 Y_{t-1}^Q + b_2 Y_{t-1}^Q - b_2 Y_{t-1}^Q + b_2 y_{t-2} + a_0 F_{t-1}^Q - a_0 F_{t-1}^Q + a_0 F_{t-1}^Q - a_1 F_{t-1}^Q + u_t \\ y_t^Q &= (b_1 + b_2 - 1) Y_{t-1}^Q - b_2 y_{t-1}^Q + a_0 f_t^Q + (a_0 + a_1) f_{t-1}^Q + u_t \\ y_t^Q &= b[Y_{t-1}^Q - \beta F_{t-1}] - b_1 y_{t-1}^Q + a_0 f_t + u_t \end{aligned}$$

Analogously, arbitrarily higher lag orders for both endogenous and exogenous variables of the FARX and the number of exogenous variables as well can be transformed in SEECM.

[14] We set the dimension of y_t to one. This is because we can forecast only a single variable with one dataset which was chosen in relation to y_t by the lasso technique.

Remark (b): According to Corollary 1 of Theorem 3 in Bai and Ng (2006) and the analytical results of Banerjee et al. (2014), the forecasting error of models with a co-integrating term is smaller than that of models without a co-integrating term (the difference amount to the product of the squared loading parameter plus the squared error correction term), when there exists a stable long-run relationship among the variables involved.

2.4 Mixed frequency in the framework of ECM

In order to be able to nowcast, we introduce the MIDAS technique regarding the short-run dynamics of exogenous variables (the short-run factors). In order to incorporate EC modeling into the MIDAS technique in a slightly different way from the usual EC modeling, we derive the (stationary) short-run factors from the differenced non-stationary panel data, instead of using the differenced (non-stationary) long-run factors as discussed above. The static error correction term still remains as a static, quarterly relationship between GDP and the long-run factors, but the short-run dynamics of exogenous variables are now disaggregated, or rather, used at a monthly frequency as derived from the monthly indicators.[15] Combining this MIDAS technique, the final form of our LFMECM is given as:

$$y_t^Q = b\left[Y_{t-1}^Q - \sum_{k=1}^{r^l} \beta_k F_{k,t-1}^Q\right] + \sum_{i=1}^{p} b_i y_{t-i}^Q + \sum_{k=1}^{r^s}\sum_{j=0}^{q_k}\sum_{m=0}^{2} a_{kjm} f_{k,t-j-m/3}^M + u_t^Q, \tag{7}$$

where y^Q are changes in quarterly GDP; r^l the optimal number of long-run factors–in the sense of Bai (2004); F_k^Q the k-th long-run factor; r^l the optimal number of short-run factors–in the sense of Bai and Ng (2002); $f_{k,t-j-m/3}^M$ the j-th lag with the m-th month of the k-th short-run factor. Using the LFMECM in (7), we now- and forecast quarterly growth rates of GDP by means of i) a quarterly EC term taken from the quarterly non-stationary factors $(y_{t-1}^Q - \sum_{k=1}^{r^l} \beta_k F_{k,t-1}^Q)$ multiplied by its loading parameter, b; ii) p times of quarterly lagged endogenous variables $(\sum_{i=1}^{p} b_i y_{t-i}^Q)$; iii) $\sum_{k=1}^{r^s}\sum_{j=0}^{q_k} 1 \times 3$ times of each monthly lagged exogenous variables $(\sum_{k=1}^{r^s}\sum_{j=0}^{q_k}\sum_{m=0}^{2} a_{kjm} f_{k,t-j-m/3}^M)$; and iv) a disturbance term (u_t^Q) with the usual zero-mean normality assumption. In our forecasting practice, we use unrestricted MIDAS, ie, every lag of monthly data has to be estimated. In general, this unrestricted MIDAS suffers from parameter proliferation for samples of a relatively small size. The reason for using the unrestricted MIDAS are, however, two-fold: as will be shown, the optimal number of short-run factors for our stationary panel as determined by the criteria in Bai and Ng (2002) is one. Therefore, the number of terms having to be estimated is not large, so that the advantage of efficiency offered by unrestricted MIDAS outweighs the disadvantage of parameter proliferation. The second and perhaps more important reason is a practical one. An estimated unrestricted model gives forecasters a clearer understanding of the changes and contributions of each monthly lag with respect to the now-/forecasting values. This will, therefore, enable us to better interpret our now-/forecasts.

[15] Götz et al. (2014) propose an error correction model using the MIDAS technique. Despite the unconventional concept of a *dynamic* co-integrating relationship, they show that their model produces a better forecasting performance than models without a error correction term.

3 A simulation study

To check the possible improvement in forecasting performance brought about by MIDAS and EC modeling, we conduct a Monte-Carlo simulation. For this purpose, we generate a co-integrating relationship directly from a set of three (non-stationary) factors which are assumed to be given. This means that the pre-selection is not considered in the simulation study.[16] Consequently, our benchmark model is the DFM for this simulation study which is compared with the MDFM (DFM with the MIDAS technique only), the FECM (DFM with the EC technique only) and the FMECM (DFM with both the MIDAS and the EC technique).

Data generating process: The data generating process is the triangular system introduced by Phillips (1991), which has a very similar structure to the GDFM given by Bai (2004). Under the assumption of a stable long-run relationship among the non-stationary variables (GDP and the factors in our case), the data generating process is given as:[17]

$$\begin{aligned} Y_t &= \sum_{i=1}^{r} \beta_i F_{it} + u_{0t}, \quad t = 1, \dots, T^M \\ F_{it} &= F_{i,t-1} + u_{it}, \end{aligned}$$

with $\begin{bmatrix} u_{0t} \\ u_{1t} \\ \vdots \\ u_{rt} \end{bmatrix} := u_t = e_t + \sum_{j=1}^{\infty} m_j e_{t-j}, \quad e_t \sim iid\ N(0, \Sigma)$. Furthermore, we construct a pseudo quarterly time series as $Y_t^Q = Y_{\{t,1:3:T^M\}}, \quad t = 1, \dots, T^Q$.

Simulation design: To generate samples based on the SEECM in (4), we set a number of factors, $r = 3$, length of factors, $T^M = 259$ (hence, $T^Q = 87$) for the sake of empirical relevance, co-integrating parameters, $\beta_1 = \beta_2 = \beta_3 = 1$ without any loss of generality, $\Sigma = \begin{bmatrix} 1 & 0.75 & 0.5 & -0.25 \\ 0.75 & 1 & 0 & 0 \\ 0.5 & 0 & 1 & 0 \\ -0.25 & 0 & 0 & 1 \end{bmatrix}$, $j = 1$; $m_1 = -0.25 \times \mathrm{I}_4$. The four forecasting models, DFM MDFM and FECM, are given as:

$$y_t^Q = \sum_{i=1}^{p} b_i y_{t-i}^Q + \sum_{k=1}^{r^s} \sum_{j=0}^{q_k} a_{kj} f_{k,t-j}^M + u_t^Q; \tag{8}$$

$$y_t^Q = \sum_{i=1}^{p} b_i y_{t-i}^Q + \sum_{k=1}^{r^s} \sum_{j=0}^{q_k} \sum_{m=0}^{2} a_{kjm} f_{k,t-j-m/3}^M + u_t^Q; \tag{9}$$

[16] One reason for excluding a pre-selection in this way is the intractability of constructing a data generating process which reflects our empirical data appropriately. A more important reason is the indifference of a possible pre-selection regarding the comparison of forecasting performance among the four models examined in our simulation.

[17] See also Phillips and Hansen (1990) for a comparison of the data generating process.

$$y_t^Q = b\left[Y_{t-1}^Q - \sum_{k=1}^{r^l}\beta_k F_{k,t-1}^Q\right] + \sum_{i=1}^{p} b_i y_{t-i}^Q + \sum_{k=1}^{r^s}\sum_{j=0}^{q_k} a_{kj} f_{k,t-j}^M + u_t^Q, \quad (10)$$

$$y_t^Q = b\left[Y_{t-1}^Q - \sum_{k=1}^{r^l}\beta_k F_{k,t-1}^Q\right] + \sum_{i=1}^{p} b_i y_{t-i}^Q + \sum_{k=1}^{r^s}\sum_{j=0}^{q_k}\sum_{m=0}^{2} a_{kjm} f_{k,t-j-m/3}^M + u_t^Q, \quad (11)$$

respectively. The model in (8) is the popular model proposed in Stock and Watson. The model in (9) is used in Marcellino and Schumacher (2010). The model in (10) is introduced in Banerjee et al. (2014, 2015). The model in (11) seems to be exactly the same as that in (7), but each of the long- and short-run factors in (7) is taken from a set of targeted indicators (via a pre-selection), while those in (11) are not. For specification of the four forecasting models, we set the lag order of the three exogenous variables (pseudo factors) to $q_1 = q_2 = q_3 = 4$. We perform a recursive one-step ahead out of sample forecasting with $t_{start} = 51$. Consequently, we collect 27 forecasters from each of the four forecasting models. 10,000 replications were made.

Simulation results: The simulation results are summarized in Table 1 and in Figure 1. The numbers in Table 1 are the average of the 10,000 root mean-squared forecasting errors (RMSFEs or MSEs for short) and mean absolute deviation (MAD). Each statistic (MSE and MAD) is calculated from the 27 one-step ahead forecasters which are generated by the four forecasting models.

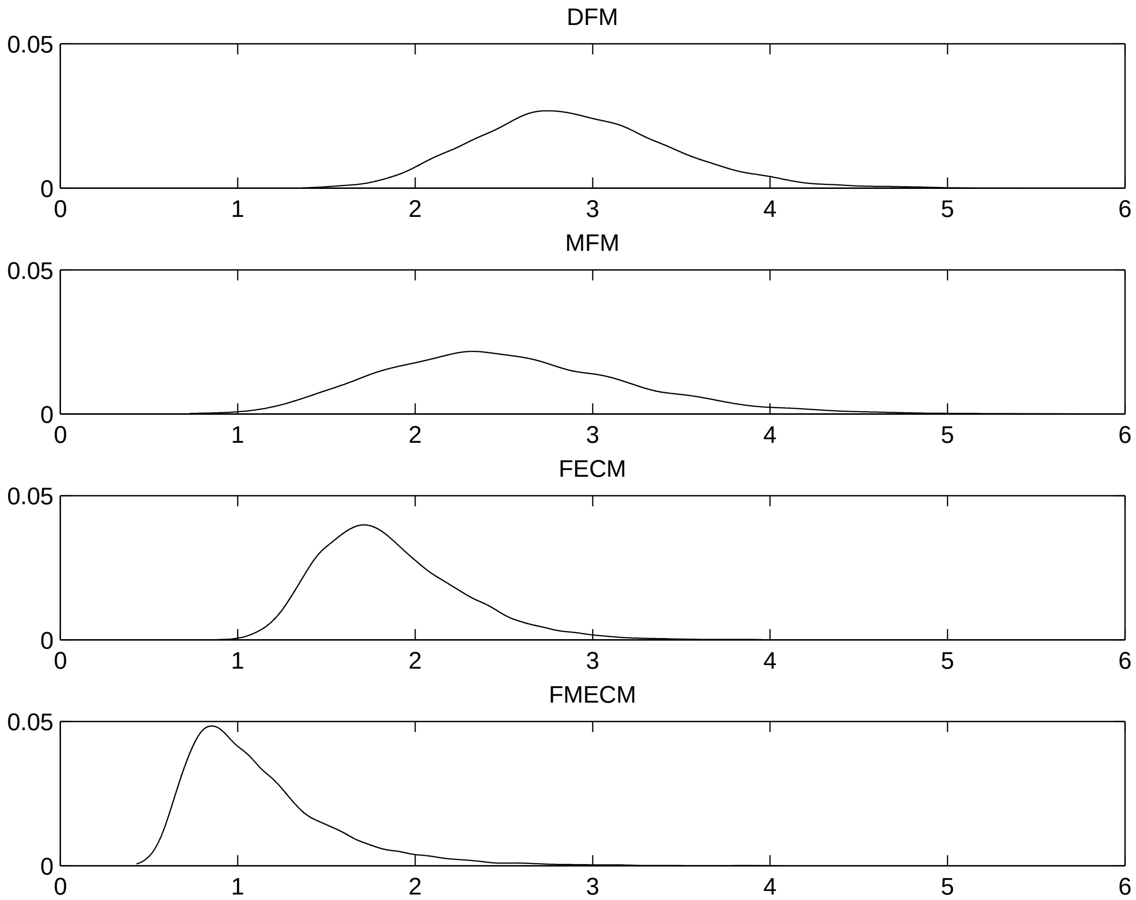
Table 1. Forecasting performance[a]

Model / Statistics	DFM	MDFM	FECM	MFECM
MSE	3.59 (2.49)	3.12 (2.17)	2.31 (1.60)	1.44
MAD	2.84 (2.58)	2.46 (2.24)	1.82 (1.65)	1.10

[a]The numbers in parentheses are ratios in comparison with the MFECM.

Table 1 shows that both the MF and the EC technique improve forecasting performance and that the improvement brought about by the EC mechanism is greater than that of the MF technique. The clear superiority of forecasting performance via the EC technique was expected, because a co-integrating relationship is assumed in the data generating process. In other words, if there exists a stable long-run relationship between the macroeconomic key variables (GDP in our case) and the factors in the real economy, ignoring a co-integration restriction in the building of forecasting models would lead to systematic larger forecasting errors.[18] More detailed information on the forecasting performance of the four models can be found in a distribution of the entire MSEs. Figure 1 shows the distributions of the 10,000 MSEs generated by the four forecasting models.

Figure 1. Forecasting performance: MSE distributions



As is clearly shown in Figure 1, the mean value of the distributions produced by the four models becomes smaller when MIDAS and especially the EC technique is used (also shown in Table 1). At the same time, the variance also becomes smaller, with the reduction in variance brought about by the EC mechanism being larger (3rd panel) than that of the MIDAS technique (2nd panel). Consequently, the MFECM has the smallest mean and variance. The third moment for all four models is right-skewed because of the lower limitation, and this is also the reason why distributions with a small mean seem to be more right-skewed, which is the case for the LFMECM.

[18] Via Corollary 1 of Theorem 3 in Bai and Ng (2006, p. 1139), the h-step forecasting error is given as $N(0, \sigma^2 + var(\hat{y}_{T+h}))$. When the (correct) co-integration restriction is ignored, the h-step forecasting error would be $N(0, \sigma^2 + var(\hat{y}_{T+h})) + b^2 u_{EC}^2$, where b is the loading parameter for the EC term; and u_{EC} is the (ignored) EC term.

4 An empirical application

As discussed in the previous section, the basic element of our LFMECM is the DFM of Stock and Watson (2002) with three added refinements, namely the MIDAS, the lasso/EN technique and the EC mechanism. Because of the complementary property of the added elements, it is of course possible to construct a DFM by using one or some of them as is already shown in our simulation study. In our empirical application, we include two models for the purpose of comparing the forecasting performance which can be regarded as *subset* models of the LFMECM. The three models (two subset models plus the LFMECM) are listed as:

- Mixed-frequency dynamic factor model (MFM = DFM + MIDAS)
- Lasso-based mixed-frequency dynamic factor model (LMDFM = DFM + MIDAS + EN)
- Lasso-based mixed-frequency (dynamic factor) error correction model (LFMECM = DFM + MIDAS + EN + ECM)

The DFM is not included as a competing model in our empirical comparison. This is because the superiority of the MFM and the FECM is already well documented in the literature and also in our simulation study. In line with the results of some recent papers on this topic, each of the three elements, as will be shown in our empirical part, can contribute to improving forecasting performance.[19]

4.1 Data

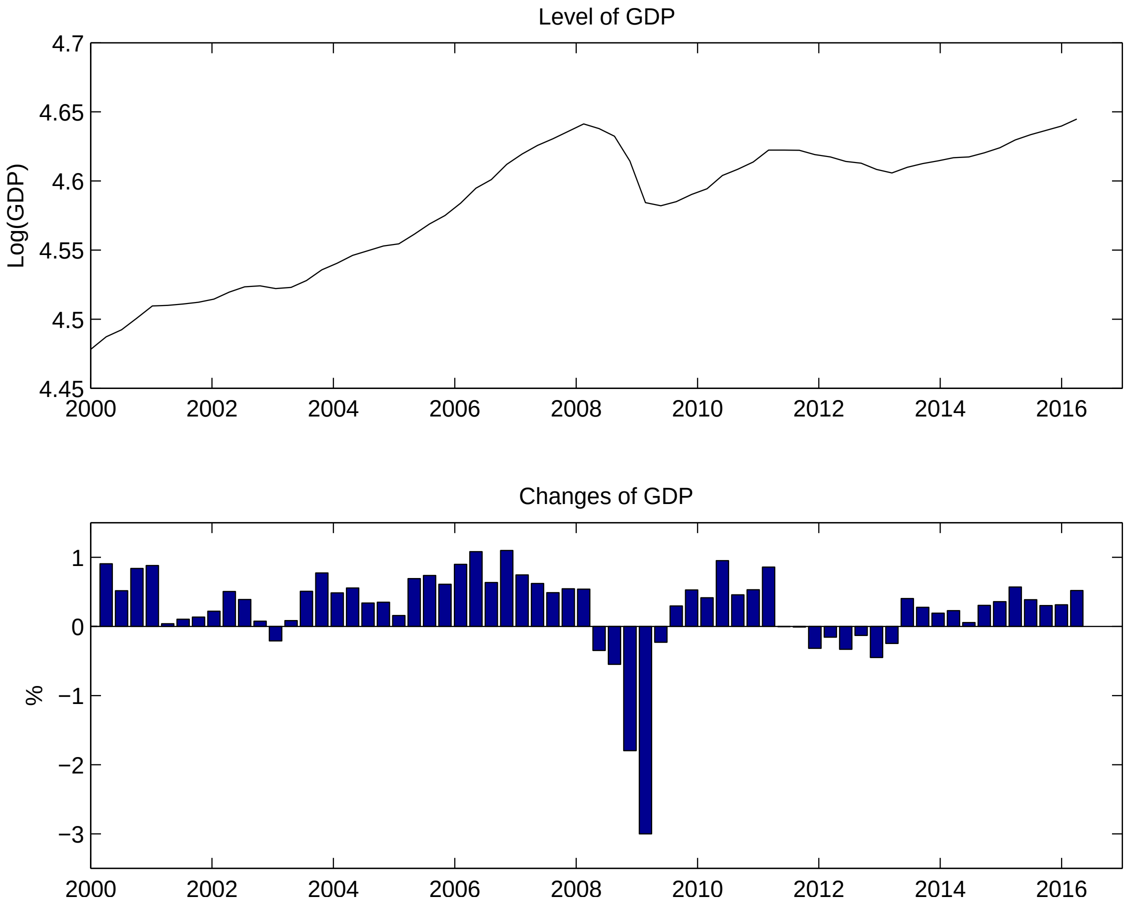
For our empirical application we use the euro area dataset used at the Deutsche Bundesbank for macroeconomic analysis and forecasting. The key variable of interest to be forecast, the euro-area GDP, is aggregated, seasonally and calendar adjusted quarterly data from 2000QI to 2016QI ($T^Q = 65$, number of quarterly observations). The panel data serving as a set of high frequency indicators consists of the 115 monthly time series ($N = 115$, number of time series before pre-selection) and spans 2000M01 to 2016M03 ($T^M = 195$, number of monthly observations).[20] A large part of the panel data is euro-area aggregated data, some of which are disaggregated national data such as industrial production in Germany, France, Italy and Spain, for example. In the face of the data

[19]See Marcellino and Schumacher (2010) for the contribution of the MIDAS technique; Banerjee et al. (2014, 2015) for that of the EC mechanism; and Bai and Ng (2008) for that of the lasso/EN technique.

[20]The dataset used for our nowcasting exercise is a final one, not a real-time data, ie, all revisions made before 31 May 2016 have already been taken into account. Measurement of the influence of revisions on relative forecasting accuracy is another issue and is presumably of little relevance in comparison to forecasting performance.

structure, namely $T^Q < N$ and high pairwise correlations of the indicators in a group, the EN technique has more advantages than the lasso with respect to a pre-selection as discussed in subsection 2.1. The whole dataset used in our empirical application is listed in detail in Appendix A. Figure 1 shows the logarithm of euro-area GDP (upper panel) and its quarterly growth rates (lower panel), where the growth rate for 2000QI is not determined.

Figure 2. Euro-area GDP and its quarterly growth rates, 2000QI - 2016QI



Quarterly GDP in the euro area grew at a steady rate of almost 0.5% up to 2008QII, which was when the recent worldwide economic recession was triggered by the financial crisis. The recovery was disturbed again by what was called the euro crisis. After that (since 2013QII), the average monthly growth rate to date has been a little more than 0.3%. This tells us that a trend break occurred around the financial crisis. We will come back to this issue later, when we estimate the EC term as departures from the long-run equilibrium.

4.2 Determination of model structure

4.2.1 Choice of lasso and tuning parameter

In empirical applications, we will usually have two sets of data. One of them contains only non-stationary indicators, the other only stationary ones. The indicators in the former data are allowed to be included in the latter data in difference. So far, we will have to apply the lasso and the EN technique twice, namely to the *long-run targeted indicators* and the *short-run targeted indicators*.[21] The justification for those separate sets of non- and stationary indicators is based on more economic intuition (than on statistical necessity). It is usually assumed that some economic variables, such as industrial production, foreign trade and number of employees, drive the long-run trend of GDP, while other economic variables, such as financial variables and survey data are more a reflection of the short-run dynamics of GDP.[22] In line with Bai and Ng (2008), the determination of the two kind of predictor sets should be replicated at each forecasting point in time to ensure the sparsistency (selecting the correct sparsity pattern) of the lasso estimates in empirical practice. Furthermore, this replication can also be motivated by the economic intuition that the optimal indicators can vary along the business cycle.

In our empirical application, we use a grid searching method to find an optimal lasso and tuning parameter. To do this, we set $\theta \in [0.01 : 0.01 : 0.5]$ and $\alpha \in [0.01 : 0.01 : 1]$. By construction, the larger θ is, the smaller is the optimal number of pre-selected indicators.[23] If $\alpha = 1$, the pre-selection regression will be the same as the OLS regression; and if $\alpha = 0$, the pre-selection regression would be the same as the ridge regression.[24] By construction of our LFMECM, we need two types of θ: one for the non-stationary data (henceforth called the long-run lasso parameter, denoted as θ^l), and another for the stationary data (henceforth called the short-run lasso parameter, denoted as θ^s). For each of the 101 α, we will have 50^2 combinations of θ^l and θ^s, ie, there are 252,500 combinations of α, θ^l and θ^s in our grid search. From these $252,500$ combinations we pick out the combination as a set of the optimal model parameters, $\hat{\alpha}$, $\hat{\theta}^l$ and $\hat{\theta}^s$ at which the MSE (or MAD, alternatively) of the forecasts is smallest.

Figure 3a shows the minimum MSE (and MAD) values from each of the 50^2 combinations of θ^l and θ^s for the corresponding α, where the upper panel results from monthly type

[21] Analogously to the OLS for non-stationary indicators, the lasso and EN estimates are also consistent and oracle efficient, meaning that they select the correct sparsity pattern. See Kock (2012) on this topic, for example.

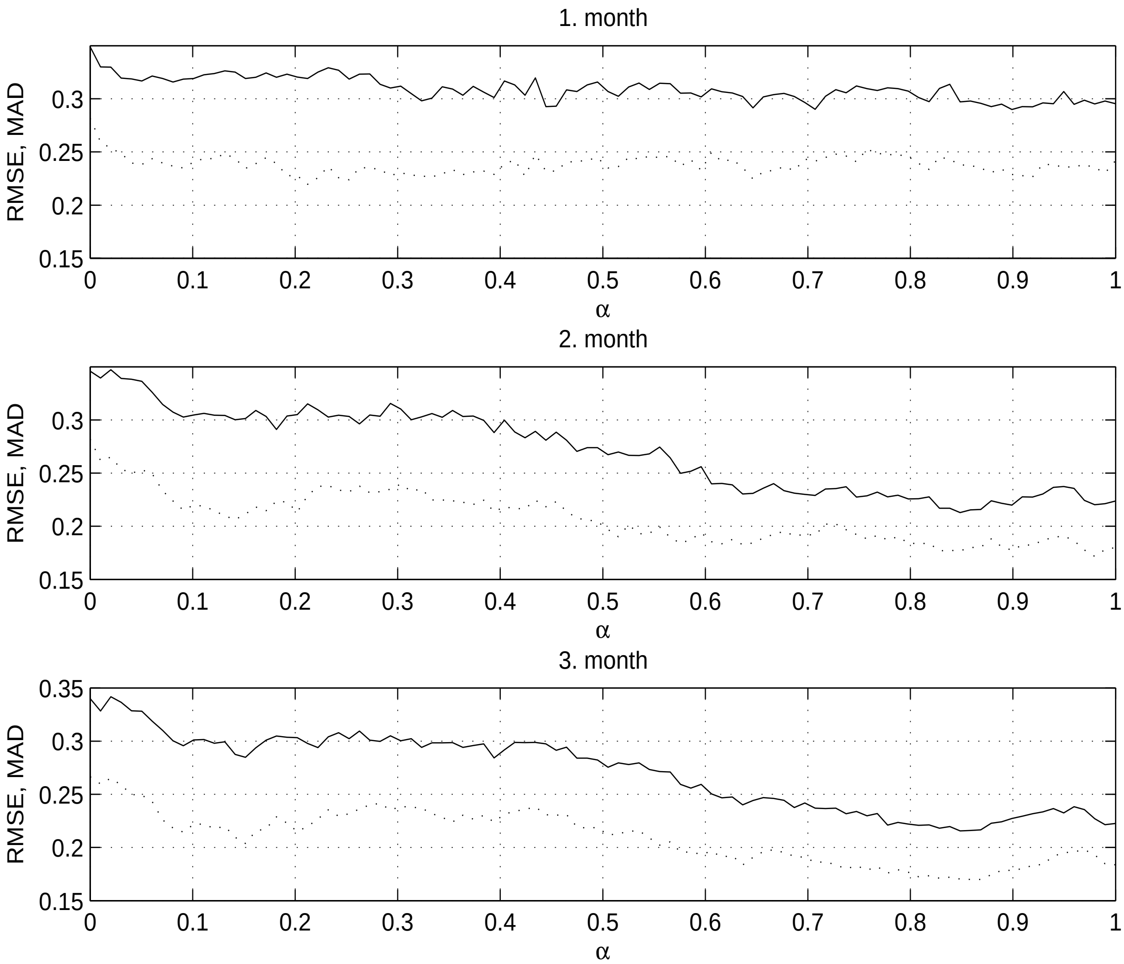
[22] This categorization, especially regarding the short-run dynamics, does not necessarily mean that variables that are more responsible for short-run dynamics are stationary and *vice versa*.

[23] In the empirical applications based on our data, we observed a very small (but still positive) number of pre-selected indicators when $\theta = 0.5$.

[24] Usually, the empirically optimal tuning parameter has been determined $\alpha \in (0.50\ 1)$.

1 (ie, every third month from 2000M01 until 2016M01); the middle panel from monthly type 2 (ie, every third month from 2000M02 until 2016M02); and the lower panel from monthly type 3 (ie, every third month from 2000M03 until 2016M03). In each of the panels, the solid (dotted) line shows the MSE (MAD).

Figure 3a. Model parameters $(\alpha, \theta^l, \theta^s)$ according to MSE and MAD



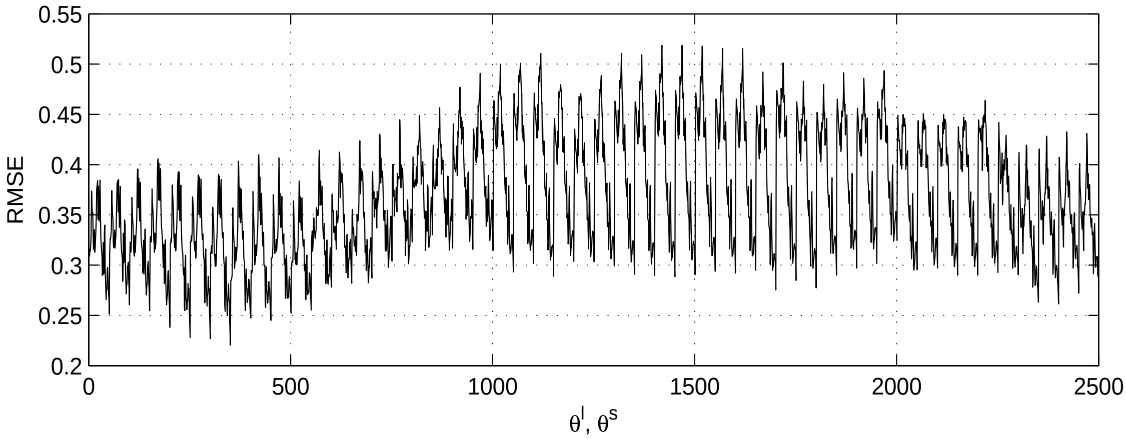
In this example, based on a dataset from 2000QI - 2016QI for quarterly GDP; and 2000M1 - 2016M05 for the monthly panel, the optimal model parameters shown in Figure 3a are $\alpha = 0.90, 0.85, 0.85$ based on MSE, and $\alpha = 0.22, 0.98, 0.86$ based on MAD. For the lasso parameter, $\theta^l = 0.22, 0.06, 0.06$ based on MSE, and $\theta^l = 0.17, 0.06, 0.05$ based on MAD; $\theta^s = 0.50, 0.50, 0.50$ based on MSE, and $\theta^s = 0.49, 0.43, 0.50$ based on MAD.[25]

The 101 points (corresponding to the 101 α values) on the solid (dotted) line in all panels of Figure 3a are the minimums of 2500 MSE (MAD) values determined from the 2500 combinations of θ^l and θ^s. In order to show how the 2500 MSE or MAD values are

[25]For this sample, the long-run lasso parameters (θ^l) are mostly small, while the short-run lasso parameters (θ^s) are mostly large, or rather, equal to the upper bound. Usually, the optimal model parameters do not often take the extreme value of their definition area $\theta^l, \theta^s \in [0.01,\ 0.50]$.

distributed for a given α value, Figure 3b shows the case of the monthly type 2 with a given value of $\alpha = 0.85$. In this case $\theta^l = 0.06$ and $\theta^s = 0.43$ are determined.

Figure 3b. Lasso parameters (θ^l, θ^s) for $\alpha = 0.85$



The optimal model parameters usually change from month to month (but do not do so necessarily).[26] Using the given tuning parameter ($\alpha = 0.85$) and long- and short-run lasso parameter ($\theta^l = 0.06$ $\theta^s = 0.43$) in our example, we would select 27 (*long-run*) targeted indicators for estimating our long-run factors and 15 (*short-run*) targeted indicators for estimating our short-run factors for the time period from 2000QI to 2016QI. Some indicators are included in both of the targeted indicators.[27]

4.2.2 Optimal number of long- and short-run factors

In the framework of the asymptotic principal components for large panels, Bai and Ng (2002) provide some useful criteria to determine the optimal number of factors for stationary panel data, and Bai (2004) considers similar criteria for non-stationary panel data. According to the panel criteria of Bai and Ng (2002), the optimal number of factors can be estimated by minimizing two quantities which stand in a tradeoff relationship by changing the number of factors: the first quantity is a usual sum of the squared residuals from regressions of X_{it} on the k factors for all i and t, given as:

$$V(k, \hat{F}^k) = min_{\Lambda, F^k} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (X_{it} - \lambda_i^{k'} \hat{F}_t^k)^2,$$

[26]The parameter fluctuation (uncertainty) is also observed from quarter to quarter. The reason for this is that the unknown model parameters were determined using a relatively small size of sample with maximal 22 observations.

[27]The numbers of the pre-selected 27 long-run targeted indicators are 9, 11, 13, 14, 15, 16, 22, 31, 34, 40, 42, 44, 48, 59, 62, 75, 79, 83, 85, 86, 92, 99, 101, 103, 104, 113, 114, and the numbers of the pre-selected 15 short-run targeted indicators are 2, 9, 13, 14, 16, 18, 22, 36, 59, 60, 69, 96, 109, 113, 115. See the description of data in Appendix A for matching the numbers to the corresponding indicators.

where $\hat{F}_t^k$ are k-dimensional estimated factors which will decrease as k increases. The second quantity is a function of k, N and T as a penalty for overfitting, given as

$$kg(N,T),$$

which will increase as k increases. The optimal number of factors can be obtained by minimizing the sum of the two quantities, namely panel criteria (PC) depending on k as:

$$PC(k) := V(k, \hat{F}^k) + kg(N,T).$$

Then, the optimal number of factors is determined in an interval between 1 and a certain upper limit, say k_{max}, as:

$$\hat{k} = argmin_{k\in[1,k_{max}]} PC(k).$$

Bai and Ng (2002) consider three kinds of penalty terms ($kg(N,T)$) and provide three panel criteria as

$$\begin{aligned}
PC_1(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right) \\
PC_2(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{N+T}{NT}\right)\ln C_{NT}^2 \\
PC_3(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{\ln C_{NT}^2}{C_{NT}^2}\right),
\end{aligned}$$

where $C_{NT} = \min\{\sqrt{N}, \sqrt{T}\}$ and $\hat{\sigma}^2$ provides a proper scaling of the penalty term, and can be replaced by $\hat{V}(k_{max}, \hat{F}^{k_{max}})$ in empirical applications. All three criteria are asymptotically equivalent, but have different properties in finite samples (as will also be shown in our empirical applications).

Bai (2004) also provides three criteria to determine the optimal number of factors for non-stationary panel data. They are very similar to those of the panel criteria for stationary panel above, and are called integrated panel criteria (IPC), as

$$\begin{aligned}
IPC_1(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\alpha_T\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right) \\
IPC_2(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\alpha_T\left(\frac{N+T}{NT}\right)\ln C_{NT}^2 \\
IPC_3(k) &= V(k, \hat{F}^k) + k\hat{\sigma}^2\alpha_T\left(\frac{N+T-k}{\ln(NT)}\right),
\end{aligned}$$

where $\alpha_T = T/(4\ln\ln T)$.

Using the PC and IPC, we estimate the optimal number of long-run and short-run factors for our empirical data. We do this by first applying the EN technique to select a subset from both our (whole) non-stationary and stationary panel data, where, as discussed in previous subsection, the non-stationary data are observed (constructed) at a quarterly frequency and the stationary panel data at a monthly frequency. The results from our empirical data are summarized in Table 2.

Table 2. Optimal number of factors [a]

Data / Criteria / k	$\tilde{X}_{it}$ $IPC_1(k)$	$IPC_2(k)$	$IPC_3(k)$	ΔX_{it} $PC_1(k)$	$PC_2(k)$	$PC_3(k)$
1	0.7268	0.7343	**1.0385**	**0.8716**	**0.8769**	**0.8630**
2	**0.7065**	**0.7216**	1.3254	0.9444	0.9551	0.9273
3	0.7872	0.8098	1.7088	1.0184	1.0343	0.9927
4	0.9162	0.9463	2.1359	1.1205	1.1418	1.0863
5	1.0465	1.0842	2.5600	1.2458	1.2725	1.2030
6	1.1897	1.2349	2.9925	1.3826	1.4146	1.3312
7	1.3496	1.4024	3.4371	1.5244	1.5616	1.4644
8	1.5187	1.5790	3.8864	1.6680	1.7106	1.5995

[a] $\tilde{X}_{it}$ means the non-stationary panel at a quarterly frequency after a pre-selection via the EN method, and ΔX_{it} means the stationary panel at a monthly frequency after a pre-selection via the EN method.

Table 2 shows that the optimal number of indicators for the stationary factors is 1 according to all three criteria, and 2 for the non-stationary factors according to the $IPC_1(k)$ and $IPC_2(k)$; 1 according to the $IPC_3(k)$. Consequently, we choose 1 factor from the stationary panel for the short-run dynamics in our model. The results for the non-stationary panel are not unique. But we choose 2 factors from the non-stationary panel.[28] The third criterion $IPC_3(k)$ is rather conservative and is no more strongly consistent when N is large relative to T. More results to determine the optimal number of factors for the quarterly panel data and the monthly panel without applying the EN (ie, for the whole set of our panel data) are given in Table C1 and Table C2 in Appendix C.

4.2.3 Other specifications

For empirical applications, the lag order of the endogenous variable p and the k exogenous variable $q_1, \cdots, q_k$ have to be determined in (7), where $k = 1$ according to Table 1. The usual lag criteria can be used; we use an empirical one. More precisely, we compare the MSE of models with different lag lengths for some given α, θ^l and θ^s. From this method, a lag length of 4 for both the endogenous and the exogenous variable turns out to be an optimal one. For quarterly variables like GDP, the lag length of 4 is usually used in empirical works and makes sense economically, even if the data are seasonally adjusted. Consequently, we set $p = q_1 = 4$.

[28] In a historical evaluation, it turned out that the now-/forecasting performance measured by the MSE based on a two long-run factors is superior to that based a one long-run factor.

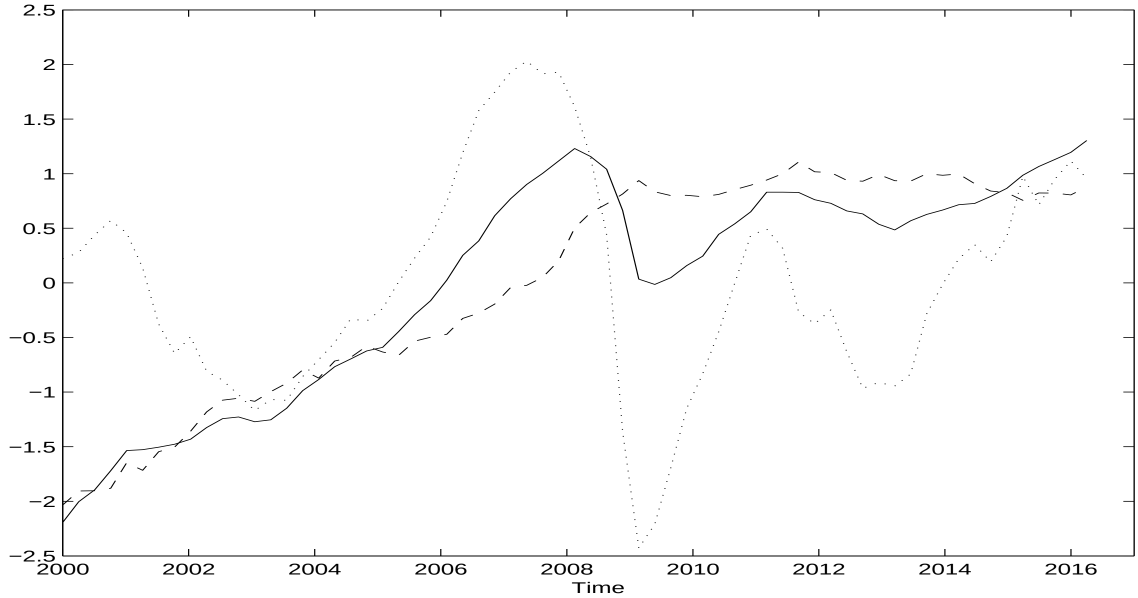
4.3 An example: the latest nowcasting

Before we present the nowcasting performance of the models based on our 22 historical data samples (in the next subsection), this subsection presents one example for the (longest) time period from 2001QI to 2016QI, and hence, the latest nowcasting. We will nowcast three times, in January 2016, February 2016 and March 2016, for 2016QI. At the beginning of building a nowcasting model, we have to determine optimal model parameters and the optimal number of long- and short-run factors. For the latest nowcasting, we use the longest sample for which the choice of tuning and lasso parameters, and the number of the long- and short-run factors have been already done in previous subsections.

4.3.1 Estimated LFMECM

To estimate our LFMECM, we first need to estimate the long- and short-run factors. According to the PC and IPC criteria we choose 2 long-run factors and 1 short-run factor. Figure 4 shows the two estimated long-run factors together with GDP, where the solid line signifies GDP, the dashed line the first long-run factor, and the dotted line the second long-run factor. All three time series are standardized to aid visual comparison.

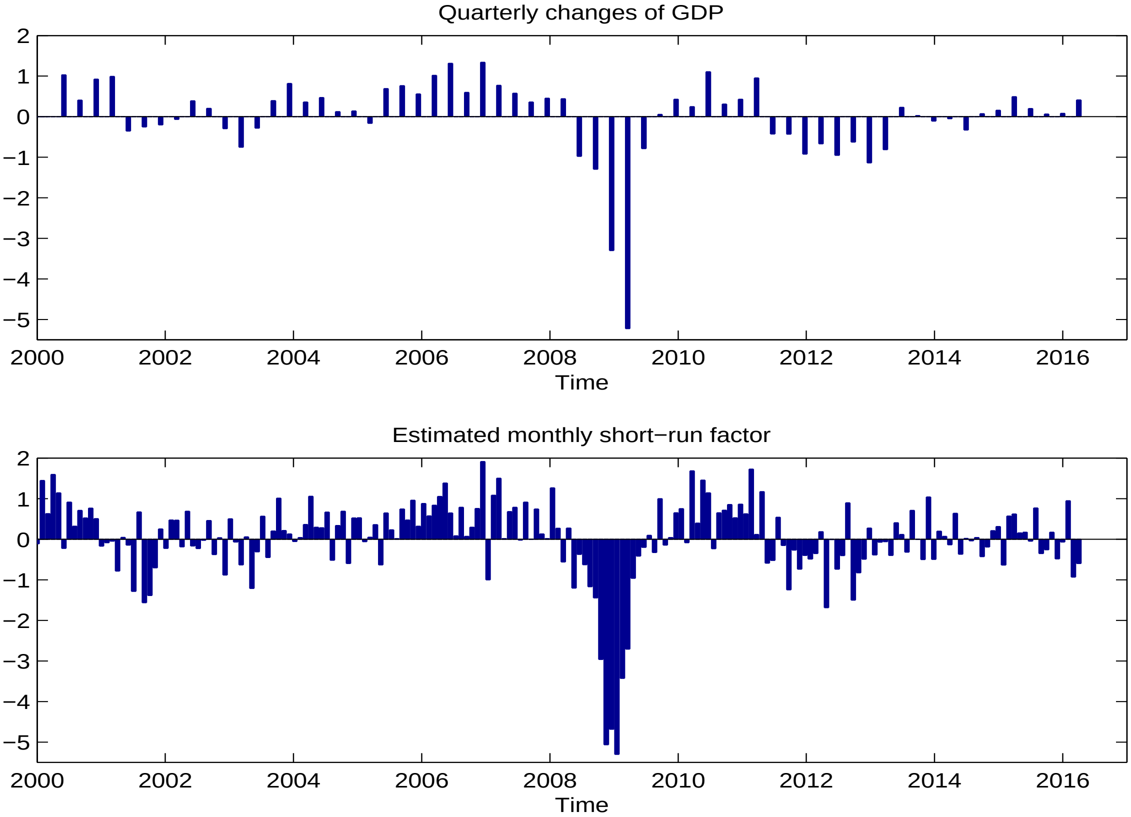
Figure 4. Estimated long-run factors



It seems that the first long-run factor carries the (linear) GDP trend (where the possible trend break can again be seen here), the second long-run factor the cyclical part of the GDP.

We now add the one estimated short-run factor to estimate the LFMECM in the second step. Figure 5 shows quarterly changes of the GDP (upper panel) and the one estimated monthly short-run factor (lower panel).

Figure 5. Estimated short-run factor



As Figure 5 clearly shows, the estimated monthly short-run factor is very similar to the changes in GDP. This means that short-run factors capture changes in the variable of interest, while the long-run factors explain trends and/or the cyclical component. As already discussed in Figure 1 for GDP, Figure 5 also reflects two aspects in the development of GDP. During the two crises (financial and euro crisis) GDP was below the equilibrium. The growth rates before the financial crisis lie above the equilibrium regarding the whole sample period.

To estimate our LFMECM, we use the standard two-step method of Engle and Granger (1987). For the estimation of the EC term, we use a static regression containing the level of GDP and the two estimated long-run factors plus a constant term. In order to match the observational frequency of quarterly GDP and the long-run monthly factors we use every third one (last month of a quarter) of the long-run monthly factors.[29] The static long-run relationship is given as:

$$Y_t^Q = c + \beta_1 F_{1t}^Q + \beta_2 F_{2t}^Q + u_t^{EC}. \tag{12}$$

[29] The use of factors observed in the last month of a quarter also has consequences in the asymptotic distributions of estimated co-integrating parameters. We will discuss on this topic again later.

In the spirit of Bai (2004) and Phillips (1991) the long-run factors are given as:

$$\begin{aligned} F^Q_{1t} &= F^Q_{1t-1} + u_{1t} \\ F^Q_{2t} &= F^Q_{2t-1} + u_{2t}. \end{aligned}$$

This static long-run relationship in (12) is usually estimated based on the ordinary least squares (OLS) estimation. Alternatively, one can also use the fully-modified OLS (FMOLS) of Phillips and Hansen (1990). The FMOLS takes the contemporaneous dynamic relationship between GDP and the factors, namely the endogeneity problem, in a static EC regression into account. Furthermore, the FMOLS also corrects the serial correlation of the residuals from the static EC regression. The two corrections by the FMOLS are performed via the long-run covariance (as referred to in the literature) between the innovation processes u^{EC}_t, u_{1t}, u_{2t}. Define two quantities using $u_t = [u^{EC}_t \; u_{1t} \; u_{2t}]$ as:

$$\begin{aligned} \Omega &= \lim_{T^Q \to \infty} \frac{1}{T^Q} \sum_{t=1}^{T^Q} \sum_{s=1}^{T^Q} \mathrm{E}[u_s u_t'] = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \\ \Lambda &= \lim_{T^Q \to \infty} \frac{1}{T^Q} \sum_{t=1}^{T^Q} \sum_{s=1}^{t} \mathrm{E}[u_s u_t'] = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}, \end{aligned}$$

where Ω_{11} and Λ_{11} are a scala, and Ω_{22} and Λ_{22} are a (2×2)-dimensional matrix.[30] Under the assumption of co-integration among the non-stationary variables involved, Ω is proportional to the spectral density evaluated at frequency 0 (free from autocorrelation and endogeneity). Calculate the bias due to endogeneity as:

$$\Lambda^+_{21} = \Lambda_{21} - \Lambda_{22}\Omega_{22}^{-1}\Omega_{21}.$$

Estimate the error correction term as:

$$\hat{u}^{EC}_t = (y_t - \hat{\Omega}_{12}\hat{\Omega}_{22}^{-1}\hat{u}_{2t}) - \left(\sum_{t=1}^{T^Q} (y_t - \hat{\Omega}_{12}\hat{\Omega}_{22}^{-1}\hat{u}_{2t}) F_t' - (0 \; \hat{\Lambda}^{+'}_{21}) \right) \left(\sum_{t=1}^{T^Q} F_t F_t' \right) \tag{13}$$

To calculate the long-run variance and covariance matrix for finite samples, we use kernel estimations containing a bandwidth parameter. To meet a significant degree of serial correlation from the residuals u_t, Hansen (1992) suggests a pre-whitening of the estimated residuals $\hat{u}_t$ via a vector autoregression with order 1 as $\hat{u}_t = \hat{\phi}\hat{u}_{t-1} + \hat{e}_t$. The kernel estimator takes the form as given in Hansen (1992):

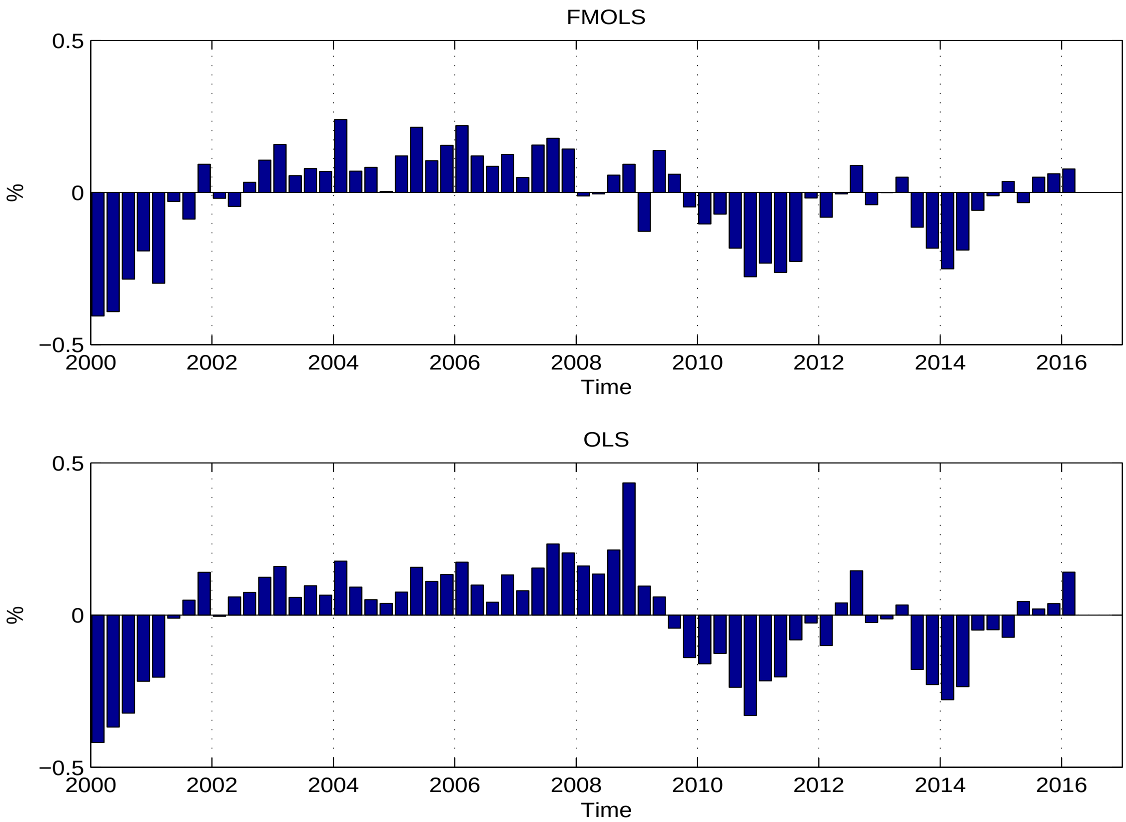
$$\begin{aligned} \hat{\Omega} &= \sum_{j=-T^Q}^{T^Q} w(j/M) \frac{1}{T^Q} \sum_{t=j+1}^{T^Q} \hat{e}_{t-j}\hat{e}_t' \\ \hat{\Lambda} &= \sum_{j=0}^{T^Q} w(j/M) \frac{1}{T^Q} \sum_{t=j+1}^{T^Q} \hat{e}_{t-j}\hat{e}_t', \end{aligned}$$

[30] See Phillips and Hansen (1990) and Hansen (1992).

where $w(\cdot)$ is a weight function containing a bandwidth parameter, M. Some kernel estimations are considered in the literature. To avoid arbitrariness and also to improve the MSE of the semi-parametric FMOLS estimate, based on the minimization of asymptotic truncated MSEs, Andrews (1991) proposes a plug-in bandwidth estimator, $\hat{M}$, for the Bartlett, the quadratic spectral and the Parzen kernel estimation. In our empirical application, we adopt the Parzen kernel estimation whose bandwidth parameter is usually larger than those of the Bartlett, the quadratic spectral and/or the Turkey and Hanning kernel.[31]

The estimated co-integrating parameters in (12) are 0.26 and 0.16, with corresponding highly significant t-values of 47.73 and 19.82 for β_1 and β_2, respectively.[32] Figure 6 shows the estimated co-integration residuals ($\hat{u}_t^{EC}$), where the upper panel shows residuals via the FMOLS estimation, and the lower panel those via the OLS estimation.

Figure 6. Estimated co-integration residuals



[31]The bandwidth parameter of the Parzen kernel for our empirical data is ≈3, while those of the quadratic spectral and the Turkey and Hanning kernel are ≈2 and that of the Bartlett kernel is ≈1. However, the difference in the estimated co-integration residuals among the kernel estimations is, for our empirical data, not very significant.

[32]In section 5 we will discuss critical values of estimated parameters in non-stationary regressions.

The two residuals seem rather similar at first glance. The differences seem to be clearer in periods of major economic fluctuations, excessive booms or crises. Consequently, the largest difference is found in 2009QI, which is the quarter in which the largest economic contraction in the euro area (as part of the recent world-wide economic recession caused by the 2008 financial crisis) was observed (see also the upper panel in Figure 5). The EC term (departure from the economic equilibrium) from the OLS estimation for this quarter shows a positive value, while that from the FMOLS estimation shows a negative value. This means that the EC mechanism for the next period, as a product of a negative significant loading parameter (which is, by construction, always negative if significant) and the corresponding EC residual, will be a negative value via the OLS estimation, and a positive value via the FMOLS estimation. It is generally expected that the long-run trend will move towards the economic equilibrium, ie, the EC mechanism should be positive in recession, and negative in an excessive boom.

Together with the estimated co-integration residuals and one short-run factor, we now estimate our LFMECM for nowcasting in a historical setting from 2010QIV to 2016QI based on our monthly panel from 2000M01 to 2016M03. For each quarter we need three estimated nowcasting models for monthly types 1, 2 and 3. More precisely, we estimate our LFMECM for nowcasting the quarter 2010QIV based on our monthly panel from 2000M01 to 2010M10 as the first historical model for monthly type 1, and replicate the same estimation for nowcasting the quarter 2010QII using our monthly panel from 2000M01 to 2010M04 as the second historical model for monthly type 1, and so on. Consequently, the last (22th) historical model for monthly type 1 for nowcasting the quarter 2016QI will then be the estimation using our monthly panel from 2000M01 to 2016M01. The same procedure was performed for monthly types 2 and 3.

Before we present the historical nowcasting performance of the models, we will show the estimated LFMECM for the (longest) time period from 2000QI to 2016QI as an example of nowcasting in practice. Table 3a presents the estimated coefficients of our LFMECM in (7) for all three monthly types 1, 2 and 3.

Table 3a. Estimated coefficients based on MSE[a]

Type of month	Lag(i,j) Coefficients		1	2	3	4
1	$\hat{c}$	1.61(3.93)				
	$\hat{b}$	-0.20(-1.49)				
	$\hat{b}_i$		0.28(1.42)	0.07(0.34)	0.08(0.41)	-0.07(-0.42)
	$\hat{a}_{1j0}$		-1.59(-5.49)	0.50(1.57)	0.23(0.69)	0.16(0.48)
	$\hat{a}_{1j1}$		-1.08(-3.16)	0.45(1.19)	-0.35(-0.95)	0.33(0.98)
	$\hat{a}_{1j2}$		-0.05(-0.14)	0.52(1.43)	0.09(0.25)	-0.33(-1.10)
2	$\hat{c}$	1.11(3.49)				
	$\hat{b}$	-0.00(-0.06)				
	$\hat{b}_i$		0.11(0.70)	0.02(0.15)	0.39(2.42)	0.08(0.97)
	$\hat{a}_{1j0}$		1.24(6.06)	0.25(1.08)	-0.53(-2.29)	-0.37(-1.67)
	$\hat{a}_{1j1}$		0.99(4.80)	0.01(0.05)	0.26(1.10)	-0.31(-1.46)
	$\hat{a}_{1j2}$		0.74(2.95)	-0.41(-1.56)	-0.17(-0.65)	-0.59(-2.73)
3	$\hat{c}$	0.68(5.60)				
	$\hat{b}$	0.03(0.37)				
	$\hat{b}_i$		0.02(0.11)	0.12(0.81)	0.20(1.32)	0.03(0.51)
	$\hat{a}_{1j0}$		0.71(3.53)	0.73(3.01)	-0.18(-0.74)	-0.25(-1.05)
	$\hat{a}_{1j1}$		0.86(4.06)	0.27(1.21)	-0.55(-2.51)	-0.25(-1.27)
	$\hat{a}_{1j2}$		0.80(3.97)	0.10(0.47)	0.12(0.52)	-0.18(-1.00)

[a]The corresponding t-values are given in parentheses. All estimated coefficients for high frequency terms including constant terms are multiplied by 10^3.

The three estimated models for the three different monthly types differ from each other. The monthly frequencies in the first lag seem to usually be significant for all three monthly types. The estimated t-statistics for the loading parameters are -1.49, -0.06 and 0.37 and, hence, not significant for all three monthly types according to the critical values of the t_{ECM} test given in Banerjee et al. Some of the coefficients for high frequency terms for all three monthly types are significant in different lags, and some are not. For our 2016QII nowcasting, we take all lags (both lag endogenous and lag exogenous variables including the EC term) ie, we do not delete insignificant lags.[33] The estimated model chosen based on MAD is given in Table 3b, where models for monthly types 2 and 3 are the same as those based on MSEs (and therefore not reported).

[33]We tried various significance levels to eliminate insignificant lags from our nowcasting model, but little improvement can be achieved by any significance levels.

Table 3b. Estimated coefficients based on MAD[a]

Type of Month	Lag(i,j) Coefficients		1	2	3	4
1	$\hat{c}$	1.48(3.54)				
	$\hat{b}$	0.09(0.68)				
	$\hat{b}_i$		0.25(1.26)	0.09(0.40)	0.14(0.70)	-0.05(-0.28)
	$\hat{a}_{1j0}$		-1.50(-5.09)	0.48(1.48)	0.06(0.18)	0.05(0.14)
	$\hat{a}_{1j1}$		-1.42(-4.08)	0.38(1.00)	-0.15(-0.41)	0.31(0.88)
	$\hat{a}_{1j2}$		-0.16(-0.45)	0.71(1.93)	0.18(0.52)	-0.21(-0.69)

[a]The corresponding t-values are given in parentheses. All estimated coefficients for high frequency including constant terms are multiplied by 10^3.

4.4 Nowcasting results

From our quarterly data from 2000QI - 2016QI, we use the first 43 observations as our starting estimation periods, ie, our first nowcasting begins with 2010QIV and ends in 2016QI. For each of the 22 quarters, we will nowcast three times, at the first month, at the second month and at the last month of the corresponding quarter.[34] Therefore, we will collect 22 nowcasters for each of the three models (MFM, LMFM and LFMECM) and for each of the monthly types.

4.4.1 Historical comparison

Figure 7a shows the results of our empirical historical nowcasting performance for the three models whose optimal model parameters were measured by the MSEs, where the upper panel shows results nowcasted by the MFM, the middle panel by the LMFM and the bottom panel by the LFMECM. In each panel, monthly type 1 is marked with an 'x'; monthly type 2 with a '+'; the monthly type 3 with 'o'; and the dotted line shows the growth rates of aggregate GDP for the euro area.

[34]The difference between the three monthly types lies in the availability of the latest observations from the whole monthly time series. In practical forecasting exercises based on unbalanced data, this means that this forecasting timing will be called "monthly type 1" if at least one of the longest time series in the panel reaches the first month of a quarter, namely January, April, August and October. Monthly types 2 and 3 can be determined in a similar fashion. The longest time series is/are usually the soft data, such as survey data, because of their short lag in availability.

Figure 7a. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

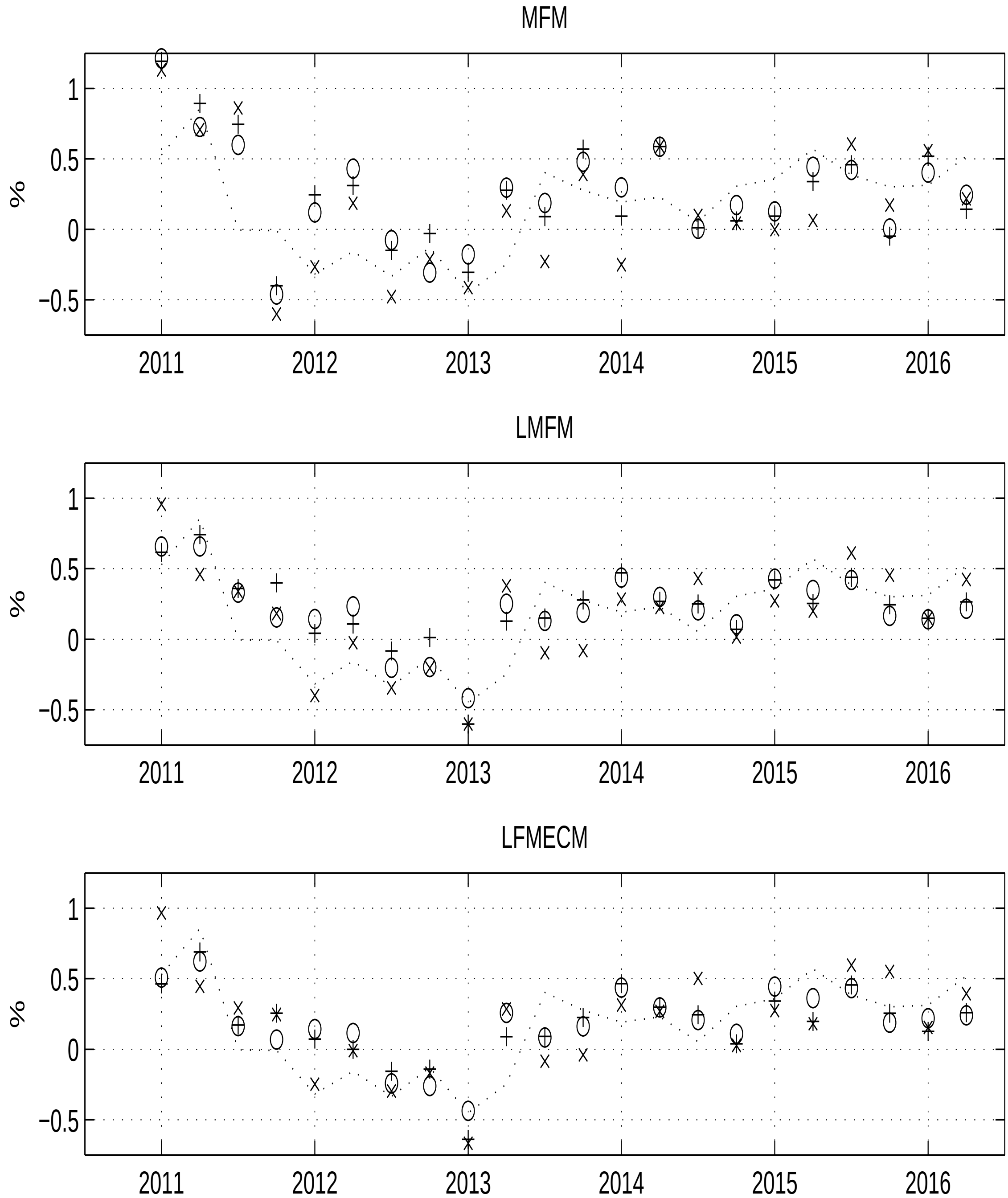


Figure 7a shows the consistency of each model, ie, the improvement in nowcasting performance by adding new information month by month in a quarter. For all three models, we can observe the consistency characteristic, ie, in general (but not for every quarter), the 'o' symbol (signifies the third month) is located most closely around GDP; the 'x' symbol (signifies the first month) furtherst away from GDP (in both signs); and the '+' symbol (signifies the second month) is located somewhere between the two. The results with the same setting, but based on the MAD, are shown in Figure D1 in Appendix D.

Figure 7b shows the same results of our empirical historical nowcasting performance for the three models as presented in Figure 7a. The only difference is that the nowcasters are re-scaled according to the growth rates of realized GDP. The results with the same setting, but based on the MAD, are shown in Figure D2 in Appendix D.

Figure 7b. Historical growth rates of GDP and nowcasters 2010QIV - 2016Q

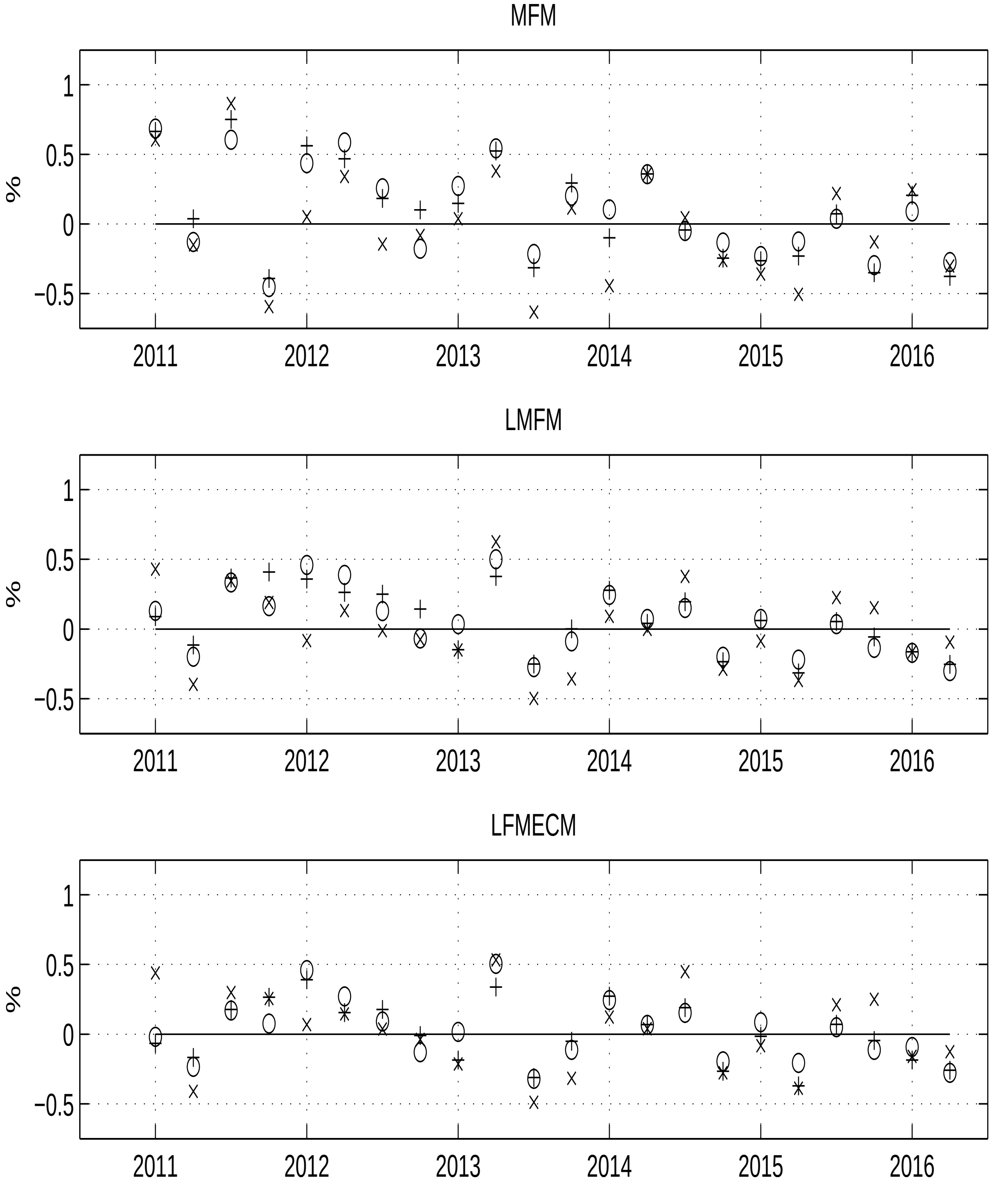


Figure 8a shows a comparison of the empirical historical nowcasting performance for the three models in each panel for the three different monthly types. For this nowcasting exercise, the optimal model parameters were measured by the MSE and all the other settings are the same as those used in Figure 7a.

Figure 8a. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

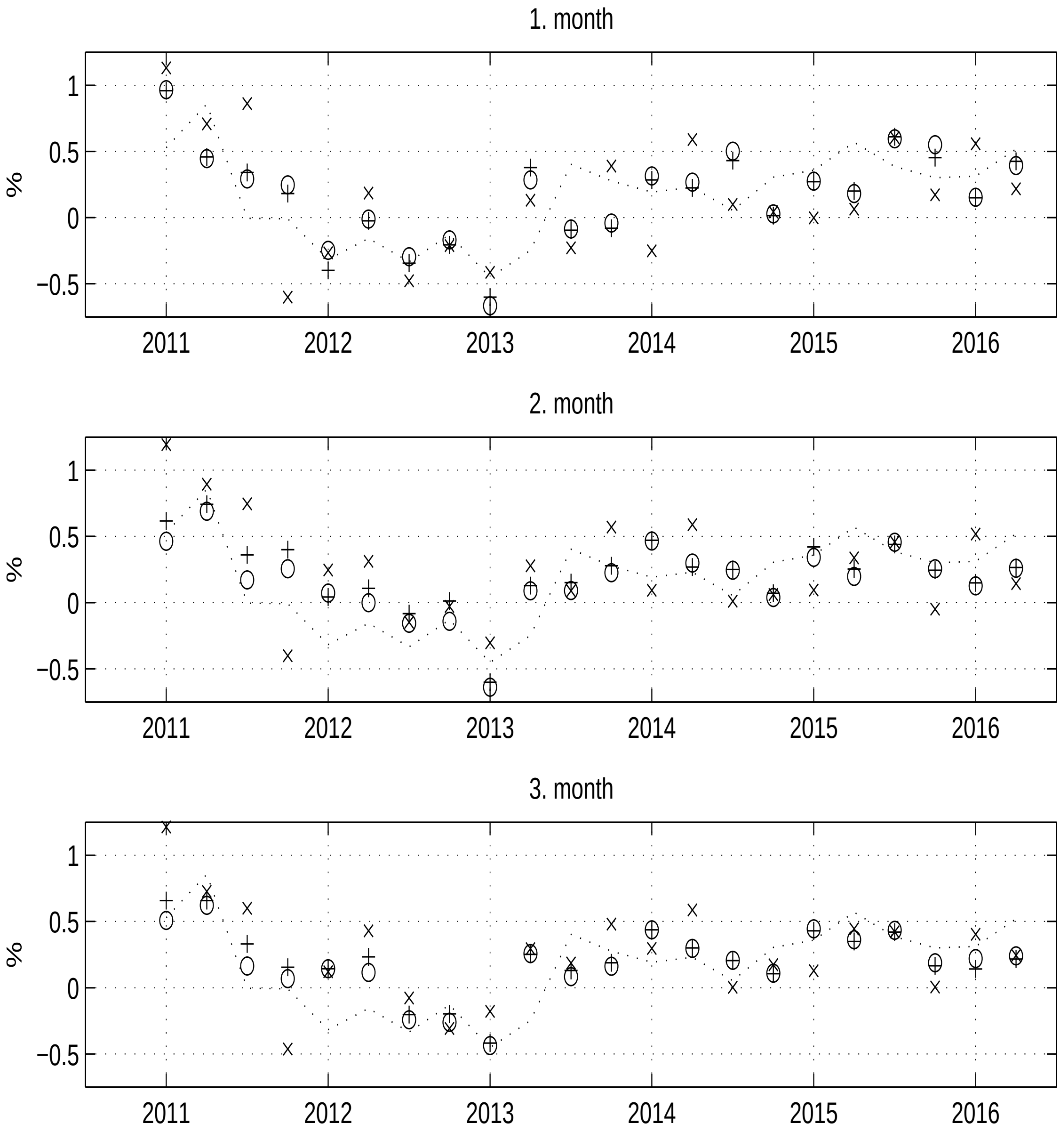


Figure 8a shows that, as expected, in all three monthly types the nowcasters from the LFMECM (denoted as 'o') are often closer to realized GDP than those from the MFM (denoted as 'x'); while those from the LMFM (denoted as '+') are located somewhere between the two. The results with the same setting, but based on the MAD, are shown in Figure D3 in Appendix D.

Figure 8b shows a comparison of the empirical historical nowcasting performance for the three models in each panel for the three different monthly types as presented in Figure 8a. The only difference is that the nowcasters are re-scaled according to the growth rates of realized GDP.

Figure 8b. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

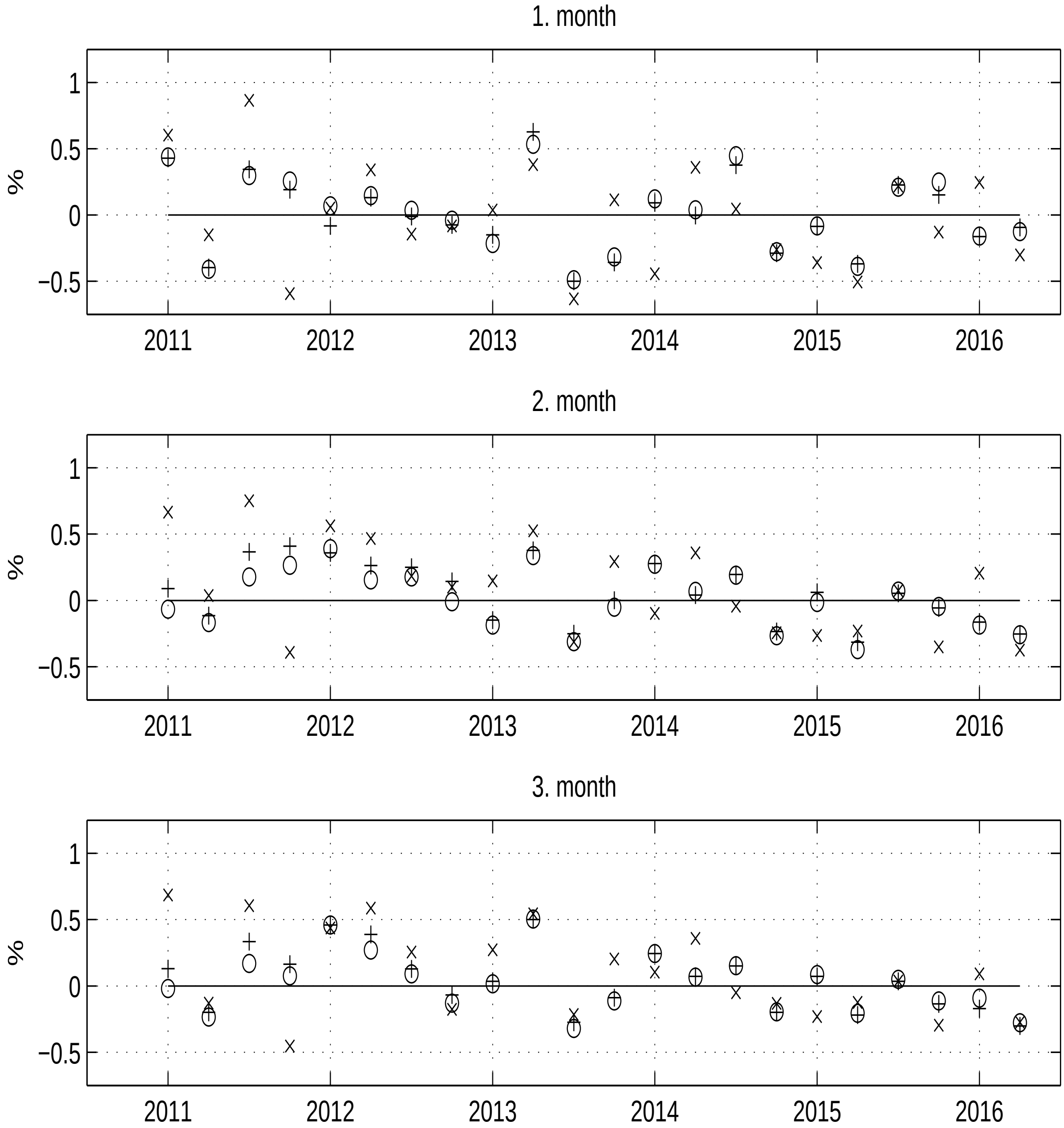


Figure 8b gives a clear picture of the forecasting consistency and forecasting accuracy of the three models as demonstrated in Figures 7a and 8a. The results with the same setting, but based on the MAD, are shown in Figure D4 in Appendix D.

Table 4 summarizes the results measured by the MSE and the MAD shown in the previous Figures (Figure 7a-b and Figure 8a-b), where the numbers in parentheses are ratios compared with those of the LFMECM.

Table 4. Empirical nowcasting performance (in %) [a]

Month	Statistic (Model)	MFM	LMFM[b]	LFMECM[c]
1.	MSE	0.39 (1.34)	0.29 (1.00)	0.29
	MAD	0.31 (1.35)	0.23 (1.00)	0.23
2.	MSE	0.37 (1.68)	0.24 (1.09)	0.22
	MAD	0.30 (1.67)	0.20 (1.11)	0.18
3.	MSE	0.35 (1.59)	0.24 (1.09)	0.22
	MAD	0.28 (1.56)	0.20 (1.11)	0.18

[a]The numbers in parentheses are the ratios based on the FLMECM. [b]For the LMFM, we set $\alpha = 0.90, 0.85, 0.85$ and $\theta = 0.50, 0.50, 0.50$ based on both criteria, namely MSE and MAD, as optimal model parameters for month types 1, 2 and 3 (ie, first, middle and last months in a quarter), respectively. [c]For the LFMECM, we set $\alpha = 0.90, 0.85$, $\theta^l = 0.22, 0.06, 0.06$ and $\theta^s = 0.50, 0.50, 0.50$ for both criteria, namely MSE and MAD, as optimal model parameters for month types 1, 2 and 3 (ie, first, middle and last months in a quarter), respectively. The optimal model parameters based on the two criteria are often the same ones, but that is not necessarily always the case.

In general, the use of the lasso technique substantially improved the nowcasting performance of the MFM, and a further improvement was achieved by using the EC mechanism. This is true of all types of months. The largest difference for both MSE and MAD can be seen in the 2nd month: the lasso technique improves the nowcasting performance of the MFM by more than 50% measured by both statistics; and the EC technique subsequently improves the nowcasting performance of the LMFM by more than 10% again. The relative improvements in the 3rd month are similar to (albeit slightly smaller than) those in the 2nd month.

When evaluating forecasting models, the measure of bias can also be of interest, although it is a part of the MSE. Table 5 shows the empirical results regarding bias.

Table 5. Bias of empirical nowcasts (in %)

	Model	MFM	LMFM	LFMECM
Month	Statistic			
1.	Bias	-0.02	-0.00	0.03
	$\max^{+}$	0.87	0.63	0.53
	$\max^{-}$	-0.63	-0.50	-0.48
2.	Bias	0.09	0.06	0.01
	$\max^{+}$	0.75	0.41	0.40
	$\max^{-}$	-0.39	-0.32	-0.37
3.	Bias	0.10	0.05	0.02
	$\max^{+}$	0.69	0.50	0.50
	$\max^{-}$	-0.45	-0.30	-0.32

Table 5 indicates that the improvement in nowcasting performance by the LMFM and the LFMECM as compared to the MFM is achieved not only by reducing the variance of the nowcasters but also by diminishing their bias. The reduction of bias behaves from 0.1% in the MFM to 0.05% in the LFM and to 0.02% in the LFMECM in the 3rd month, for example. The positive (over-estimation) maximum nowcasting errors decline from 0.69% inthe MFM to 0.50% in the LFM and to 0.50% in the LFMECM in the 3rd month; and the negative (under-estimation) maximum nowcasting errors decline from -0.45% in the MFM to 0.30% in the LFM and to 0.32% in the LFMECM in the 3rd month. Improvement from the LMFM to the LFMECM cannot be observed in every criterion.

4.4.2 Several tests regarding forecast accuracy

***F*-test**: Using the standard F-test we test the equality of two variances of residuals estimated by the three nowcasting models. The sum of squared residuals of an estimated forecasting model in comparison with that of another forecasting model shows the relative forecasting performance. This is because forecasting errors are a function of the sum of squared estimated residuals. The F-statistic is given as

$$F = \left[(n_1 - 1)^{-1} \sum_{i=1}^{n_1} (X_{1i} - \bar{X}_1) \right] \Bigg/ \left[(n_2 - 1)^{-1} \sum_{i=1}^{n_2} (X_{2i} - \bar{X}_2) \right], \tag{14}$$

where n_1 and n_2 are sample sizes of the two random variables X_1 and X_2, respectively. In our case, X_1 and X_2 are the residuals of two paired nowcasting models, and $n_1 = n_2 = 1089$ for the whole nowcasting periods resulting from 22 nowcasting models for each of the paired models (ie, 39 estimated residuals from each of the nowcasting models for 2010QIV, 40 estimated residuals for 2011QI, $\cdots$, 60 estimated residuals for 2016QI). The results of the F-test are summarized in Table 6.

Table 6. F-test for equality of two variances of empirical nowcasters (in %) [a]

Model	MFM/LMFM	MFM/LFMECM	LMFM/LFMECM
F-statistic	1.1286	1.1612	1.0289

[a]The corresponding critical values at the 95% and 99% significance levels are 1.1039 and 1.1519, respectively, which were calculated by a simulation.

Table 6 shows a large improvement of nowcasting performance through the LMFM and especially through the LFMECM. The improvement through the LMFM and the LFMECM in comparison with the MFM is statistically highly significant at the 95% and 99% levels. For a detailed comparison for every nowcasting period, see Table 1 in Appendix E.

Theil's U-statistic: Theil (1966) introduced a U-statistic as a measure of forecasting accuracy given as:

$$U = \left[\sum_{h=T+1}^{T+H} (\hat{y}_h - y_{T+h})^2 \right]^{1/2} \Bigg/ \left[\sum_{h=T+1}^{T+H} y_{T+h}^2 \right]^{1/2}, \tag{15}$$

where $\hat{y}$ and y stand for a pair of predicted and observed quantity. Because the benchmark is the realized quantity in this statistic, it can be interpreted as a performance measure of forecasting models of interest in comparison with a constant model. Therefore, U-statistic values that are lower than 1 signify an improvement over the constant forecast. The forecasters of the constant model would be the mean of the realizations in forecast periods *a posteriori* or the mean of the observations in estimation period *a priori* for *all* forecast periods. Under the assumption of no structural breaks, the two means will be the same. The empirical result of the Theil's U-test is summarized in Table 7.

Table 7. Theil's U-statistic of empirical nowcasters (in %) [a]

Model / Month	MFM	LMFM	LFMECM
1.	1.15	0.87	0.87
2.	1.05	0.68	0.65
3.	0.99	0.70	0.65

As shown in Table 7, the more information is available, the better every model works, (as the MIDAS technique should do). In other words, the forecasting performance in month 2 is better than that in month 1; and the forecasting performance in month 3 is better than that in month 2. However, the improvement of the MFM can be achieved only marginally in the 3rd month. For month types 1 and 3, the MFM works even worse than the constant model. The values of the U-statistic for the LMFM and the LFMECM are clearly lower than 1, where the values for the LFMECM are smaller than those for the LMFM, except in month 1.

5 Discussion

5.1 Balancing the data

In panel data with a monthly frequency, the lengths of the individual indicators are usually unequal because of the different publication lags. Soft indicators, like survey and financial data, usually have very short publication lags, while hard indicators, like industrial productions and labor markets, have relatively longer publications lags, usually of two or three months. For now- and forecasting, one should, therefore, first fill the missing current observations in comparison to the longest indicator(s). The topic of balancing data is, however, not part of this paper because we concentrate on comparing the forecasting performance of a set of competing models. Nevertheless, we show one alternative balancing method based on the generalized autoregression (GAR) method introduced in Kurz-Kim (2008) and compare its efficiency with that of the EM algorithm widely used in the literature. A brief description of the GAR and results of the comparison are given in Appendix B.

5.2 One-step-ahead forecasting

The main focus of the paper is nowcasting, but the models can be also compared in terms of their short-run forecasting performance. In general, the main arguments for using the LFMECM and the main results of the LFMECM remain unchanged for forecasting exercises which were discussed and shown by the nowcasting exercises above. We merely summarize the results of the one-step-ahead forecasting exercise which are presented analogously to Table 3a-b in Table 8a-b.

Table 8a. Estimated coefficients[a]

Month	Coefficients Lag(i,j)		1	2	3	4
1^b	$\hat{c}$	0.66(1.15)				
	$\hat{b}$	-0.06(-0.21)				
	$\hat{b}_i$		0.57(2.85)	0.34(1.63)	0.02(0.11)	-0.28(-1.27)
	$\hat{a}_{1j0}$		0.75(3.11)	-0.35(-1.16)	-0.67(-2.17)	-0.08(-0.26)
	$\hat{a}_{1j1}$		-0.42(-1.46)	0.44(1.40)	0.74(2.52)	0.22(0.83)
	$\hat{a}_{1j2}$		-0.37(-1.19)	-0.34(-1.07)	-0.39(-1.25)	0.08(0.31)
2^c	$\hat{c}$	2.15(4.31)				
	$\hat{b}$	-0.36(-3.23)				
	$\hat{b}_i$		0.14(0.80)	0.01(0.07)	-0.03(-0.16)	-0.14(-0.78)
	$\hat{a}_{1j0}$		0.80(2.76)	-0.60(-0.90)	-0.30(-0.90)	-0.65(-1.97)
	$\hat{a}_{1j1}$		0.97(3.23)	0.04(-1.19)	-0.38(-1.19)	-0.13(-0.42)
	$\hat{a}_{1j2}$		-0.42(-1.38)	0.89(2.69)	0.83(2.69)	0.43(1.80)
3^d	$\hat{c}$	1.82(3.93)				
	$\hat{b}$	-0.23(-1.68)				
	$\hat{b}_i$		-0.02(-0.10)	0.07(0.45)	0.15(0.91)	0.02(0.18)
	$\hat{a}_{1j0}$		1.05(5.01)	-0.18(-0.73)	0.61(2.43)	0.48(1.96)
	$\hat{a}_{1j1}$		0.29(1.16)	-0.66(-2.57)	-0.12(-0.46)	-0.29(-1.17)
	$\hat{a}_{1j2}$		0.61(2.67)	0.23(1.02)	-0.49(-2.20)	-0.28(-1.27)

[a]The corresponding t-values are given in parentheses. All estimated coefficients for high frequency terms including constant terms are multiplied by 10^3. t-statistics are given in absolute values with the exception of $\hat{t}_{ECM}$. $^b\alpha = 0.37, \theta^l = 0.16, \theta^s = 0.20.$ $^c\alpha = 0.97, \theta^l = 0.43, \theta^s = 0.25.$ $^d\alpha = 0.76, \theta^l = 0.50, \theta^s = 0.01.$

Table 8b. Estimated coefficients[a]

Month	Coefficients Lag(i,j)		1	2	3	4
1^b	$\hat{c}$	1.90(3.46)				
	$\hat{b}$	-0.21(-2.20)				
	$\hat{b}_i$		0.55(2.88)	0.22(1.05)	-0.12(-0.57)	-0.42(-1.84)
	$\hat{a}_{1j0}$		0.43(1.75)	-0.33(-1.13)	-0.45(-1.54)	-0.41(-1.33)
	$\hat{a}_{1j1}$		-0.22(-0.76)	0.24(0.75)	1.21(4.05)	0.38(1.38)
	$\hat{a}_{1j2}$		-0.42(-1.32)	-0.32(-1.01)	-0.15(-0.47)	0.21(0.79)
2^c	$\hat{c}$	2.15(4.31)				
	$\hat{b}$	-0.36(-3.23)				
	$\hat{b}_i$		0.14(0.80)	0.01(0.07)	-0.03(-0.16)	-0.14(-0.78)
	$\hat{a}_{1j0}$		0.80(2.76)	-0.60(-1.85)	-0.30(-0.90)	-0.65(-1.97)
	$\hat{a}_{1j1}$		0.97(3.23)	0.04(0.13)	-0.38(-1.19)	-0.13(-0.42)
	$\hat{a}_{1j2}$		-0.42(-1.38)	0.89(2.74)	0.83(2.69)	0.43(1.80)
3^d	$\hat{c}$	0.59(1.19)				
	$\hat{b}$	-0.04(-0.22)				
	$\hat{b}_i$		0.09(0.34)	0.34(1.31)	0.22(0.88)	0.03(0.13)
	$\hat{a}_{1j0}$		1.87(5.99)	-0.97(-2.61)	0.32(0.86)	0.04(0.11)
	$\hat{a}_{1j1}$		0.89(2.65)	-0.96(-2.72)	-0.61(-1.75)	0.19(0.60)
	$\hat{a}_{1j2}$		0.11(0.33)	-0.22(-0.61)	-0.38(-1.06)	-0.42(-1.39)

[a]The same as in Table 8a. $^b\alpha = 0.97, \theta^l = 0.43, \theta^s = 0.25.$ $^c\alpha = 0.97, \theta^l = 0.44, \theta^s = 0.19.$ $^d\alpha = 0.74, \theta^l = 0.18, \theta^s = 0.49.$

Table 9 summarizes the results measured by the MSE analogously to Table 4, where the numbers in parentheses are ratios compared with those of the LFMECM.

Table 9. Empirical forecasting performance (in %) [a]

	Model	MFM	LMFM[b]	LFMECM[c]
Month	Statistic			
1.	MSE	0.48 (1.23)	0.42 (1.08)	0.39
2.	MSE	0.49 (1.36)	0.44 (1.22)	0.36
3.	MSE	0.35 (1.59)	0.24 (1.09)	0.22

[a]The numbers in parentheses are the ratios based on the FLMECM. [b]For the LMFM, we set $\alpha = 0.97, 0.97, 0.85$ and $\theta = 0.19, 0.25, 0.50$ based on MAD, as optimal model parameters for month types 1, 2 and 3 (ie, first, middle and last months in a quarter), respectively. [c]For the LFMECM, we set $\alpha = 0.90, 0.85$, $\theta^l = 0.44, 0.43, 0.06$ and $\theta^s = 0.19, 0.25, 0.50$ based on MAD, as optimal model parameters for month types 1, 2 and 3 (ie, first, middle and last months in a quarter), respectively.

The forecasting accuracy of all three models is, as expected, generally worse than that of the nowcasting exercise (shown in Table 4). The relative ratios between the MFM and LFMECM became smaller, while those between the LMFM and LFMECM became larger. The consistency of forecast timing is not given for the LFM and the LMFM (from monthly type 1 to 2), whereas it is still given for the LFMECM. The results based on the MAD are very similar to those based on the MSE, and hence not reported.

To sum up, the one-step-ahead forecasting exercise confirms that the modeling of the two refinements to the MFM, namely the lasso and EC techniques, improve not only nowcasting, but also forecasting performance (at least one-step-ahead forecasting).

5.3 Asymptotic distributions of the estimated LFMECM parameters

This subsection roughly sketches the asymptotic distributions of the estimated LFMECM parameters. Most of the relevant results are already documented in the literature. We merely reproduce them in the context of the LFMECM.[35]

Suppose that we have monthly panel data X_{it} $i = 1, \ldots, N$ $t = 1, \ldots, T^M$; and a key variable to be forecast y_t $t = 1, \ldots, T^Q$. Before we analyze asymptotics of our LFMECM,

[35]Because we use the estimated LFMECM with all terms (including possibly insignificant terms) as our forecasting model, the asymptotic analysis of the estimated parameters is of theoretical interest.

it is necessary to mention the lasso and the EN technique influence on the asymptotic distribution of the estimated parameters of our LFMECM. In the first stage of building our LFMECM, a pre-selection method like lasso or/and EN technique was already applied. This kind of pre-selection method obviously reduces the cross-section dimension. Consequently, the factors used in our LFMECM as regressors are estimated from this pre-selected (reduced) panel. As analyzed in Bai and Ng (2006), the least squares estimates from the factor augmented regressions are $\sqrt{T}$-consistent and asymptotically normal if $\sqrt{T}/N \to 0$. Therefore, the reduction of the cross-section dimension via a pre-selection method has (only) an effect on the rate of convergence and no influence on the asymptotic distributions of the estimated parameters of the LFMECM.

Recall now our LFMECM

$$y_t^Q = bu_{t-1}^{EC} - \sum_{p=1}^{P} b_p y_{t-1}^Q + \sum_{q_j=0}^{Q_j} a_{q_j} \hat{f}_{jt-q_j} + u_t^Q, \tag{16}$$

where

$$u_{t-1}^{EC} = Y_{t-1}^Q - \hat{\beta}\hat{F}_{t-1}. \tag{17}$$

In this two-step equation system, there are four types of parameters:

Type 1: $\hat{b}_p$ – parameters for the observable stationary regressor, y_t^Q;

Type 2: $\hat{a}_{q_j}$ – parameter for estimated stationary MIDAS regressor, f_t^M;

Type 3: $\hat{\beta}$ – co-integrating parameter for estimated non-stationary regressor, $\hat{F}_{t-1}^Q$;

Type 4: $\hat{b}$ – loading parameter for the estimated error correction term, $\hat{u}_t^{EC}$ via the two-step estimation by Engle and Granger (1987).

- The parameters of the type 1 ($\hat{b}_p$) are standard. They are asymptotically standard normal distributed even if they are combined in a regression with estimated regressors, see Theorem 1 in Bai and Ng (2006).
- Under certain assumptions and $\sqrt{T}/N \to 0$, asymptotic distributions of the parameters for the estimated stationary MIDAS regressors, $\hat{a}_{q_j}$, are also normally distributed according to Theorem 1 in Bai and Ng (2006) and Theorem 3.2 in Ghysels et al. (2004).
- According to Theorem 1(a) in Phillips (1986), asymptotic distributions of the parameters for the estimated non-stationary regressor, β the co-integrating parameter, is a functional form of two Brownian motions (partial sums of Y^Q and F^Q). Ghysels and Miller (2015) examine the effects of mixed sampling frequencies and temporal

aggregation on the size of commonly used tests for co-integration. One of their conclusions is that the nominal size is obtained asymptotically when all series are skip sampled in the same way, for example, end-of-period sampling. This is just what we are doing: we take factors every third month to match the quarterly GDP so as to build the error correction dynamics.

- The asymptotic distribution of the loading parameter for the error correction term, b, is again non-standard and a functional form of two Brownian motions (partial sums of u_t^{EC} and u_t^{Q}) as shown in Kremers et al. (1992) and Banerjee et al. (1999). Consequently, the critical values of the t_{ECM} test are tabulated in Banerjee et al. (1999).

6 Concluding remarks

This paper introduced a novel now- and forecasting model which contains three refinements to the dynamic factor model of Stock and Watson (2002). Our lasso-based factor-augmented error correction model combines the advantages of three refinements: the pre-section methods (the lasso and the EN technique) reinforce the explanatory power of the estimated factors (extracted from a small number of targeted indicators) for the key variables to be forecast; the MIDAS technique enables us to nowcast some key variables using high frequency indicators; EC modeling is capable of capturing not only the short-run dynamics, but also long-run ones and, therefore, provides forecasters with a deeper insight into the forecasts as described in section 2. A simulation study presented in section 3 confirms an improvement in forecasting performance by each of the elements embedded in our model. An empirical application in section 4 and partly in section 5 clearly demonstrates an improvement in now- and forecasting performance of the standard factor model through our LFMECM.

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A Appendix A: Data description

GDP : euro area 19, quarterly, working day and seasonally adjusted, index

Industrial production (total 13)

1. Euro area 19 (fixed composition) - IP index, total industry
2. Euro area 19 (fixed composition) - IP index, total industry (excluding construction)
3. Euro area 19 (fixed composition) - IP index, manufacturing
4. Euro area 19 (fixed composition) - IP index, construction
5. Euro area 19 (fixed composition) - IP index, all buildings
6. Euro area 19 (fixed composition) - IP index, all civil engineering works
7. Euro area 19 (fixed composition) - IP index, total industry excluding construction and MIG Energy
8. Euro area 19 (fixed composition) - IP index, electricity, gas, steam and air conditioning supply
9. Euro area 19 (fixed composition) - IP index, MIG capital goods industry
10. Euro area 19 (fixed composition) - IP index, MIG durable consumer goods industry
11. Euro area 19 (fixed composition) - IP index, MIG energy
12. Euro area 19 (fixed composition) - IP index, MIG intermediate goods industry
13. Euro area 19 (fixed composition) - IP index, MIG non-durable consumer goods industry

Retail (total 4)

14. Euro area 19 (fixed composition) - total turnover index, retail trade including fuel, except of motor vehicles and motorcycles
15. Euro area 19 (fixed composition) - total turnover index, manufacture of food products; manufacture of beverages
16. Euro area 19 (fixed composition) - total turnover index, retail sale of non-food products including fuel
17. Euro area 19 (fixed composition) - car registration, new passenger car, absolute value

Labor market (total 1)

18. Euro area 19 (fixed composition) - standardized unemployment rate, total (all ages), Eurostat

Industry survey (total 7)

19. Industrial confidence Indicator (Q2 + Q4 + Q5) / 3
20. Production trend observed in recent months
21. Assessment of order-book levels
22. Assessment of export order-book levels
23. Assessment of stocks of finished products
24. Production expectations for the months ahead
25. Selling price expectations for the months ahead
26. Employment expectations for the months ahead

Consumer survey (total 8)

27. Confidence indicator (Q2 + Q4 + Q7 +Q11) / 4
28. General economic situation over last 12 months
29. General economic situation over next 12 months
30. Price trends over last 12 months
31. Price trends over next 12 months

32. Unemployment expectations over next 12 months
33. Major purchases at present
34. Major purchases over next 12 months

Construction survey (total 5)

35. Confidence indicator (Q3 + Q4) / 2
36. Building activity development over the past 3 months
37. Evolution of current overall order books
38. Employment expectations over the next 3 months
39. Prices expectations over the next 3 months

Retail trade survey (total 5)

40. Confidence indicator (Q1 - Q2 + Q4) / 3
41. Business activity (sales) development over the past 3 months
42. Volume of stock currently hold
43. Business activity expectations over the next 3 months
44. Employment expectations over the next 3 months

Services survey (total 11)

45. Confidence indicator (Q1 + Q2 + Q3) / 3
46. Business situation development over the past 3 months
47. Evolution of demand over the past 3 months
48. Expectation of demand over the next 3 months
49. Evolution of employment over the past 3 months
50. Expectations of employment over the next 3 months
51. Markit Surveys, euro area manufacturing PMI headline adjusted
52. Markit Surveys, euro area services PMI headline adjusted
53. Markit Surveys, euro area composite (M+S) PMI headline adjusted
54. Markit Surveys, euro area composite (M+S) PMI output index adjusted
55. Markit Surveys, euro area composite (M+S) PMI new orders index adjusted
56. Markit Surveys, euro area composite (M+S) PMI employment index adjusted

Prices (total 6)

57. Euro area 19 (fixed composition) - producer price index, domestic sales, total industry (excluding construction)
58. Euro area 19 (fixed composition) - producer price index, domestic sales, MIG energy
59. Euro area 19 (fixed composition) - producer price index, domestic sales, MIG intermediate Goods industry
60. Euro area 19 (fixed composition) - producer price index, domestic sales, MIG non-durable consumer goods industry
61. Euro area 19 (fixed composition) - HICP - overall index, monthly index
62. Euro area 19 (fixed composition) - HICP - all-items excluding energy and unprocessed food, monthly index, Eurostat

International trade (total 4)

63. Total trade - intra euro area 19 (fixed composition) trade, export ECU/Euro, Eurostat
64. Total trade - extra euro area 19 (fixed composition) trade, export ECU/Euro, Eurostat
65. Total trade - intra euro area 19 (fixed composition) trade, import ECU/Euro, Eurostat

66. Total trade - extra euro area 19 (fixed composition) trade, import ECU/Euro, Eurostat

Foreign countries (USA) (total 5)

67. US, PMI manufacturing index
68. US, unemployment rate
69. US, industrial output, industrial production index
70. US, employment, civilian
71. US, total retail trade

Commodities (total 7)

72. World price index (2010) - raw materials - total (euro based)
73. World price index (2010) - raw materials - excl. energy-based products (euro based)
74. HWWA comodity price index - raw materials - crude oil (USD based)
75. Gold price, US Dollar, fine ounce (fixing in London)
76. Crude oil future price - 1 month ahead
77. World price index (2010) - raw materials - coal (USD based)
78. World price index (2010) - raw materials - copper (USD based)

Financial market (total 3)

79. ECB nominal effective exchange rate
80. ECB real effective exchange rate, CPI deflated
81. ECB nominal effective exchange rate, producer price deflated

Exchange rate (total 3)

82. Euro/USD exchange rate
83. Euro/British pound exchange rate
84. Euro/Yen exchange rate

stock markets (total 4)

85. Euro Stoxx 50 index
86. Euro Stoxx 50 volatility index
87. Standard & Poors 500 Index
88. Dow Jones Index (price-weighted average of 30 blue chips)

Bonds, treasury notes, interest rates (total 13)

89. Government bond rate 10-year, GDP-weighted composition
90. Interest rate, loans
91. Interest rate, housing loans
92. Spread corporate AA and government bond maturities 7-10 years
93. Spread corporate BBB and government bond maturities 7-10 years
94. Eonia
95. 1-month interest rate, Euribor
96. 3-month interest rate, Euribor
97. 6-month interest rate, Euribor
98. 1-year interest rate, Euribor
99. 10-year government bond yield
100. Spread Euribor 1 year 1 month
101. Spread 10 year 3 month

Money (total 3)

102. Money M1
103. Money M2
104. Money M3

Euro countries (total 11)

105. Germany - IP index, total industry (excluding construction)
106. Germany - IP index, construction
107. France - IP index, total industry (excluding construction)
108. France - IP index, construction
109. Italy - IP index, total industry (excluding construction)
110. Italy - IP index, construction
111. Spain - IP index, total industry (excluding construction)
112. Spain - IP index, construction
113. Spain - total turnover index, accommodation and food service activities
114. Spain - total turnover index, information and communication
115. Spain - total turnover index, total of other services and retail trade as covered by the STS Regulation

B Appendix B: Generalized autoregression for data balancing

The missing months are usually the latest two or three months. In order to meet this problem, Stock and Watson (2002) use the EM algorithm. Alternatively, this paper uses the generalized autoregression (GAR) method proposed by Kurz-Kim (2008). As will be shown, this GAR technique turns out to be a useful alternative to the EM algorithm. A brief summary of the GAR is as follows.

Let Δ^d be the *difference operator of order* d, i.e., $\Delta^d y_t = (1-L)^d y_t$, where L is the lag operator as $Ly_t = y_{t-1}$; and Δ_δ be the *δth lag difference operator*, i.e., $\Delta_\delta y_t = y_t - y_{t-\delta}$. Then, a d-order difference AR process of order p (with respect to the usual AR part) and q (with respect to the lag difference) based on a δth lag difference is given as:

$$-\sum_{i=0}^{p}\sum_{j=0}^{q} a_{ij}\Delta^d_{\delta+j} y_{t-i} = \mu + u_t, \quad \delta \in \mathbb{N},\ q < \delta.$$

As is usual for empirical macroeconomic applications, let $d = 1$ and $\delta = 4$; we then have

$$\begin{aligned}\Delta_4 y_t \;=\;& \mu \qquad\qquad + a_{10}\Delta_4 y_{t-1} \;+\cdots+ a_{p0}\Delta_4 y_{t-p} + \\ & a_{01}\Delta_{4+1} y_t + a_{11}\Delta_{4+1} y_{t-1} + \cdots + a_{p1}\Delta_{4+1} y_{t-p} + \\ & \vdots \\ & a_{0q}\Delta_{4+q} y_t + a_{1q}\Delta_{4+q} y_{t-1} + \cdots + a_{pq}\Delta_{4+q} y_{t-p} + u_t\end{aligned}$$

The usual difference AR(p) is a special case with $q = 0$ as:

$$-\sum_{i=0}^{p} a_{i0}\Delta_\delta y_{t-i} = \mu + u_t,$$

with $a_{00} = -1$. The *quasi* difference autoregression (QAR) of order q (QAR(q)) is another special case with $p = 0$ as:

$$-\sum_{j=0}^{q} a_{0j}\Delta_{\delta+j} y_t = \mu + u_t,$$

with $a_{00} = -1$.

Now, a sum of the two special cases AR(p) and QAR(q) results in the generalized linear (first-)difference autoregression of order p and q (GAR(p, q)) based on the δth lag difference (see Kurz-Kim, 2008, JoF):

$$\Delta_\delta y_t = \mu + \sum_{i=1}^{p} a_{i0}\Delta_\delta y_{t-i} + \sum_{j=1}^{q} a_{0j}\Delta_{\delta+j} y_t + u_t \quad \delta \in \mathbb{N},\ q < \delta. \tag{18}$$

The factor induced balancing via the EM-algorithm recommended by Stock and Watson (2002) is an iterative method with the following steps. Suppose we have panel data with a cross section dimension of N and time dimension of T, where every observation in this panel data is known. However, at time $T+1$ only some of the time series, say n, are available, but the rest of the N time series, say $m := N - n$ is still not available.

Step 1. Estimates the factors and loading parameters based on the panel data $(N \times T)$ as:

$$x_{it} = \sum_{k=1}^{r} \hat{\lambda}_{ki}^{(T)} \quad \hat{f}_{ki,t}^{(N)} + \hat{u}_{it} \quad i \in [1, N]; \; t \in [1, T].$$

Step 2. Estimates the factors and loading parameters based on the panel data $(n \times T + 1)$ as:

$$x_{i,t+1} = \sum_{k=1}^{r} \hat{\lambda}_{ki}^{(T+1)} \quad \hat{f}_{ki,t+1}^{(n)} + \hat{u}_{i,t+1} \quad i \in [1, n]; \; t \in [1, T+1].$$

Step 3. Estimates the unknown variables at $T+1$ using the estimated loading parameters from Step 1. and the estimated factors from Step 2. as:

$$\hat{x}_{i,T+1}^{F} = \sum_{k=1}^{r} \hat{\lambda}_{ki}^{(T)} \quad \hat{f}_{ki,t+1}^{(n)} + \hat{u}_{i,T+1} \quad i \in [n+1, N].$$

Analogously, the missing observations $T+2, \cdots$ can be estimated when the missing observations a month ago become available.

Figure A1 shows a comparison of the MSEs of the 71 estimated variables in the period from 2003M1 until 2014M12 which, because of publication lags, are not usually available at the end of each month. The length of the bars in Figure A1 is the difference of the MSE between the GAR and the EM method.

Figure A1. Comparison of the empirical MSEs of missing variables:
MSE(GAR)-MSE(FM): 2003M1 – 2014M12

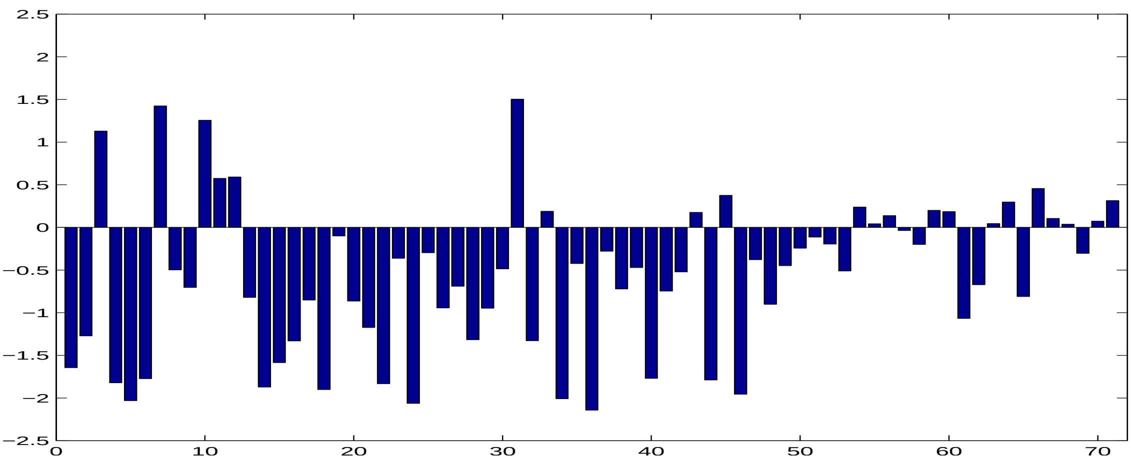


Figure A1 encourages the application of the GAR method in empirical practice. The estimated MSEs of 50 out of 71 variables of the GAR method are smaller than those of the EM method. The estimated MSEs of 21 out of 71 variables of the EM method are smaller than those of the GAR method, but most of the differences are not very large.

C Appendix C

For more information Appendix C provides optimal numbers of long- and short-run factors depending on data frequency and without any use of pre-selection. In comparison with Table 1 in subsection 3.3.2, Table C1 based on quarterly data (constructed by just a picking method) instead shows 3 as an optimal number of long-run factors (whereas that of the short-run factors remains unchanged at 1). But we use the monthly data containing more information as a basis for determining an optimal number of factors. Tables C2 and C3 indicate that an optimal number of factors is a little larger than those based on a pre-selected dataset.

Table C1. Optimal number of factors[a]

	Data	$\tilde{X}_{it}$			ΔX_{it}		
k	Criteria	$IPC_1(k)$	$IPC_2(k)$	$IPC_3(k)$	$PC_1(k)$	$PC_2(k)$	$PC_3(k)$
1		0.6083	0.6168	0.7163	**0.8743**	**0.8914**	**0.8497**
2		0.4687	0.4856	**0.6807**	0.9445	0.9786	0.8953
3		**0.4381**	**0.4635**	0.7503	1.0422	1.0933	0.9683
4		0.4479	0.4817	0.8562	1.1622	1.2303	1.0637
5		0.4705	0.5128	0.9710	1.2945	1.3797	1.1714
6		0.5069	0.5575	1.0956	1.4432	1.5454	1.2954
7		0.5567	0.6159	1.2298	1.5984	1.7177	1.4261
8		0.6145	0.6821	1.3679	1.7668	1.9031	1.5698

[a] $\tilde{X}_{it}$ means the non-stationary panel at a quarterly frequency after a pre-selection via the EN method, and ΔX_{it} means the stationary panel at a quarterly frequency after a pre-selection via the EN method.

Table C2. Optimal number of factors[a]

	Data	$\tilde{X}_{it}$			ΔX_{it}		
k	Criteria	$IPC_1(k)$	$IPC_2(k)$	$IPC_3(k)$	$PC_1(k)$	$PC_2(k)$	$PC_3(k)$
1		0.6773	0.6868	0.7939	0.8637	0.8701	0.8458
2		0.5180	0.5370	**0.7499**	0.8428	**0.8556**	**0.8069**
3		**0.4677**	**0.4961**	0.8136	**0.8380**	0.8572	0.7842
4		0.4972	0.5351	0.9557	0.8446	0.8702	0.7729
5		0.5387	0.5861	1.1085	0.8586	0.8907	0.7690
6		0.5969	0.6538	1.2768	0.8820	0.9204	0.7744
7		0.6725	0.7388	1.4610	0.9125	0.9574	0.7871
8		0.7506	0.8264	1.6465	0.9453	0.9966	0.8020

[a] $\tilde{X}_{it}$ means the non-stationary panel at a monthly frequency without a pre-selection, and ΔX_{it} means the stationary panel at a monthly frequency without a pre-selection.

Table C3. Optimal number of factors[a]

	Data	$\tilde{X}_{it}$			ΔX_{it}		
k	Criteria	$IPC_1(k)$	$IPC_2(k)$	$IPC_3(k)$	$PC_1(k)$	$PC_2(k)$	$PC_3(k)$
1		0.6352	0.6410	0.7027	0.8574	**0.8682**	0.8319
2		0.4333	0.4451	0.5672	**0.8502**	0.8718	**0.7992**
3		0.3480	0.3657	**0.5468**	0.8804	0.9128	0.8039
4		**0.3388**	**0.3623**	0.6013	0.9133	0.9565	0.8113
5		0.3421	0.3715	0.6670	0.9512	1.0052	0.8237
6		0.3634	0.3987	0.7493	1.0020	1.0667	0.8489
7		0.4001	0.4413	0.8458	1.0562	1.1317	0.8777
8		0.4397	0.4868	0.9439	1.1126	1.1989	0.9085

[a] $\tilde{X}_{it}$ means the non-stationary panel at a quarterly frequency without a pre-selection, and ΔX_{it} means the stationary panel at a quarterly frequency without a pre-selection.

D Appendix D

Appendix D shows the results of the nowcasting performance of the three models chosen by MAD (while Figures 3a-b and 4a-b show those based on MSEs). As Table 1 has already shown, the main conclusions remain unchanged.

Figure D1 shows the results of our empirical historical nowcasting performance of the three models whose optimal model parameters were measured by the MAD, where the upper panel shows results nowcasted by the MFM, the middle panel those by the LMFM and the bottom panel those by the LFMECM. In each panel, Monthly type 1 is marked by an 'x'; monthly type 2 by a '+'; Monthly type 3 by a 'o'; and the dotted line shows the growth rates of aggregate GDP for the euro area.

Figure D1. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

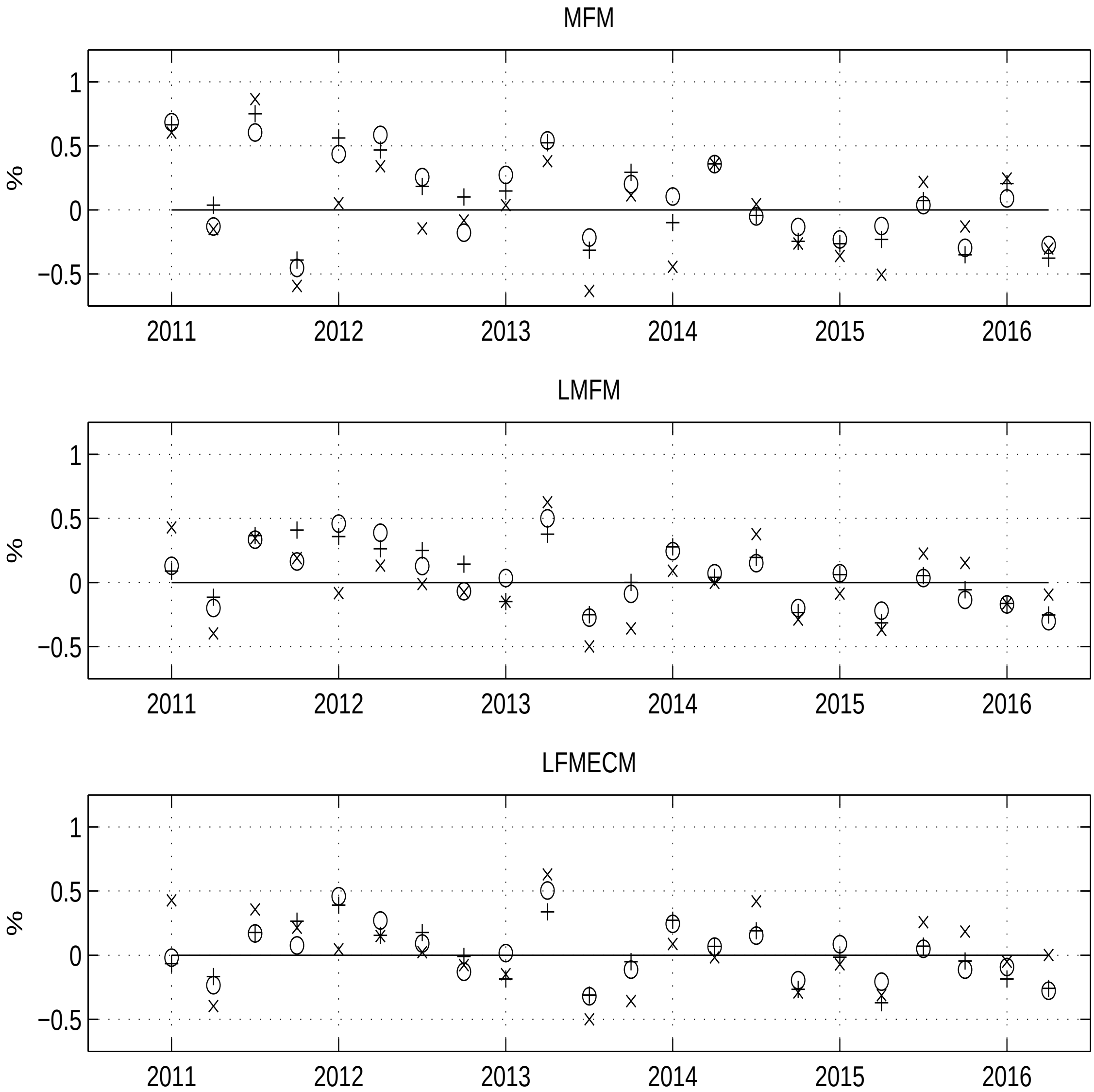


Figure D2 shows the same results of our empirical historical nowcasting performance for the three models as presented in Figure 3a. The only difference is that the nowcasters are re-scaled according to the growth rates of realized GDP.

Figure D2. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

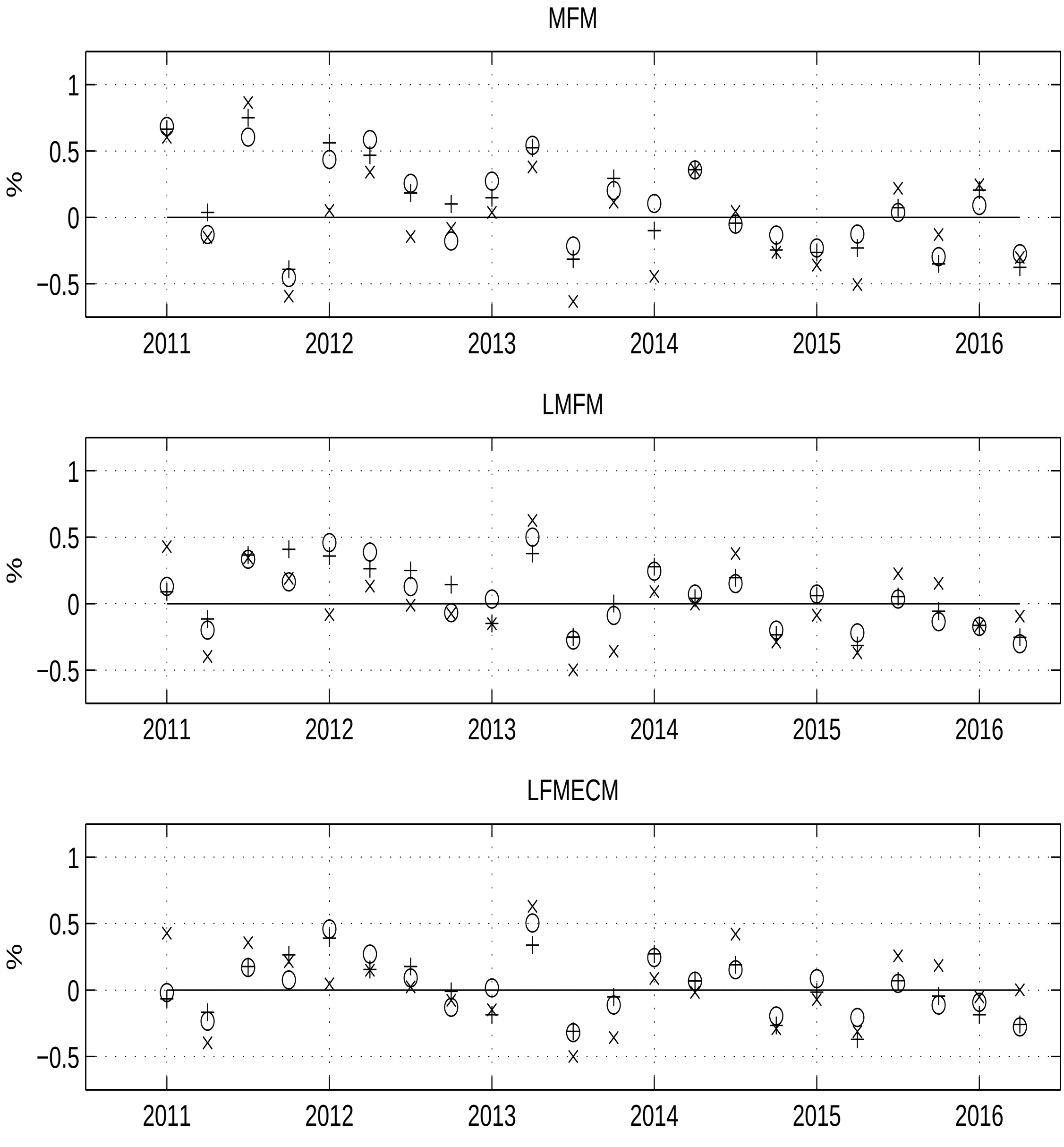


Figure D3 shows the results of our empirical historical nowcasting performance for the three models whose optimal model parameters were measured by the MSE, where the upper panel shows results for Monthly type 1, the middle panel for Monthly type 2, and the bottom panel for Monthly type 3. In each panel, the MFM is marked with an 'x'; the LMFM with a '+'; and the LFMECM with an 'o'.

Figure D3. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI

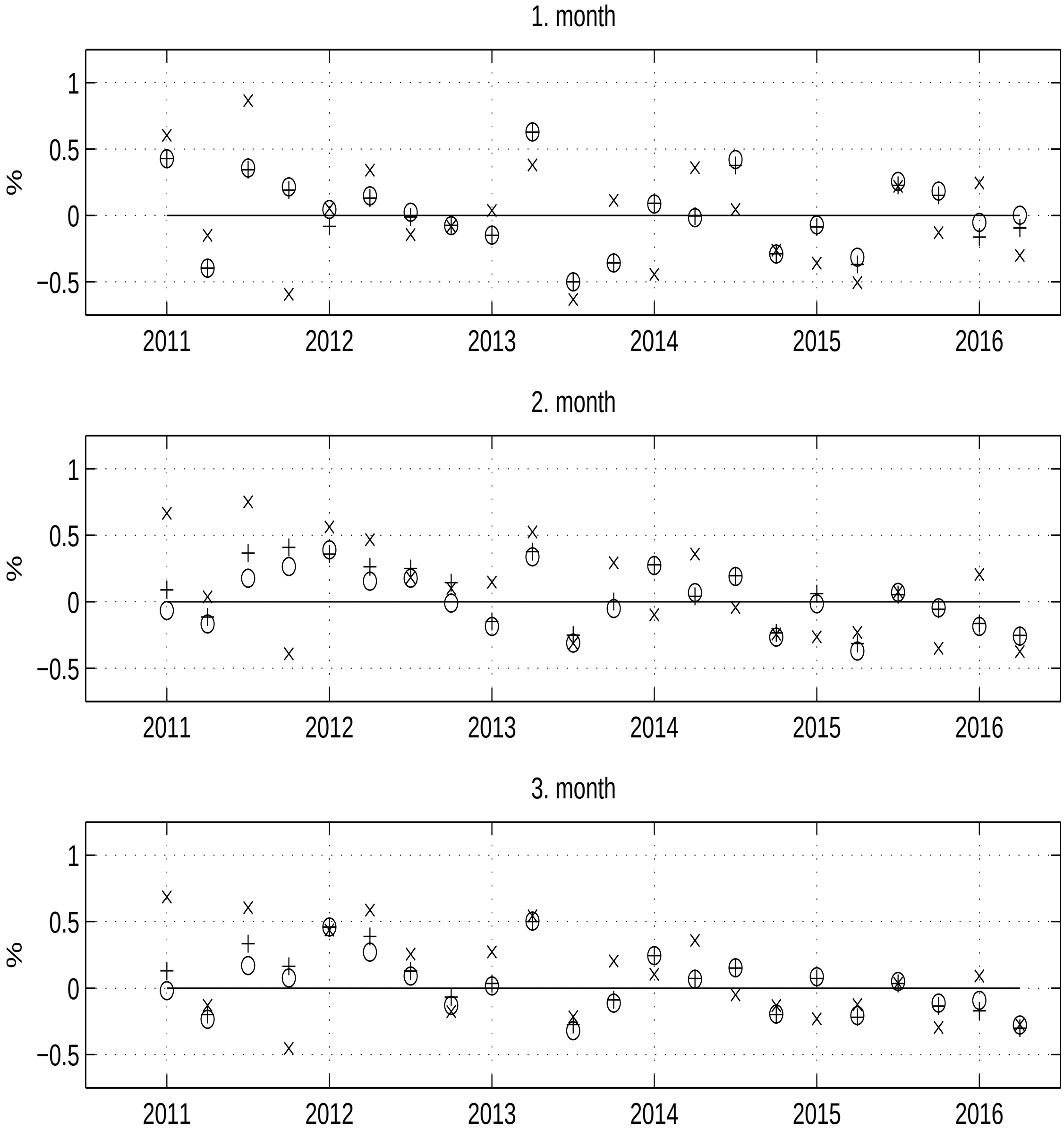
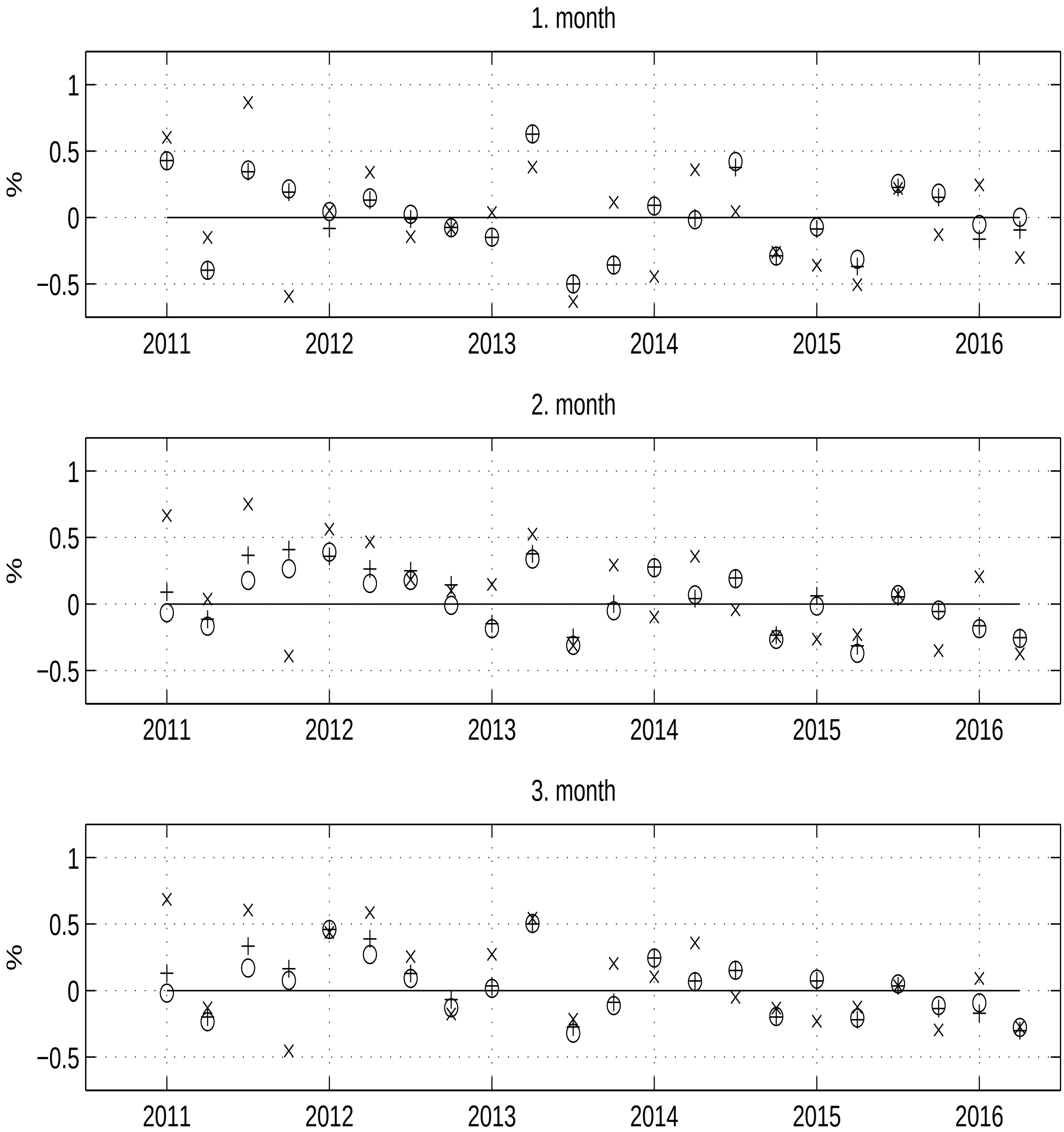


Figure D4 shows the same results of our empirical historical nowcasting performance for the three models as presented in Figure 4a. The only difference is that the nowcasters are re-scaled according to the growth rates of realized GDP.

Figure D4. Historical growth rates of GDP and nowcasters 2010QIV - 2016QI



E Appendix E

Table E1. F-test for equality of two variances of empirical nowcasters (in %)[a]

	Nowcast	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Model	Month																						
MFM	1.	1.04	1.35	1.31	1.25	1.35	1.20	1.21	1.11	1.14	1.06	1.10	1.13	1.11	1.14	1.14	1.25	1.24	1.02	1.04	1.04	1.06	0.96
vs.	2.	1.40	1.64	1.64	1.71	1.65	1.65	1.69	1.66	1.66	1.57	1.71	1.72	1.75	1.78	1.44	1.38	1.37	1.44	1.40	1.40	1.43	1.44
LMFM	3.	1.22	1.46	1.59	1.68	1.75	1.72	1.77	1.80	1.81	1.87	1.88	1.79	1.80	1.81	1.40	1.42	1.40	1.35	1.33	1.33	1.41	1.39
MFM	1.	1.04	1.40	1.37	1.32	1.44	1.20	1.28	1.15	1.20	1.13	1.10	1.14	1.11	1.15	1.16	1.27	1.26	1.03	1.05	1.10	1.10	1.01
vs.	2.	1.46	1.72	1.79	2.03	1.88	1.75	1.88	1.74	1.75	1.64	1.75	1.73	1.82	1.79	1.49	1.41	1.39	1.45	1.40	1.40	1.43	1.44
LFMECM	3.	1.26	1.51	1.73	1.92	1.84	1.87	1.79	1.82	1.81	1.90	1.91	1.80	1.81	1.81	1.40	1.42	1.40	1.35	1.33	1.33	1.42	1.40
LMFM	1.	1.00	1.04	1.05	1.05	1.07	1.01	1.06	1.04	1.05	1.07	1.00	1.01	1.01	1.01	1.02	1.02	1.02	1.01	1.01	1.06	1.04	1.05
vs.	2.	1.04	1.05	1.09	1.19	1.14	1.06	1.11	1.05	1.06	1.04	1.02	1.01	1.04	1.01	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00
LFMECM	3.	1.04	1.03	1.09	1.15	1.05	1.09	1.01	1.01	1.00	1.01	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00

[a] According to the usual cumulative F-distribution, the corresponding critical value at 90% for $n_1 = n_2 = 60$ is 1.40.