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The imperfect-common-knowledge Phillips curve: Calvo versus Rotemberg

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Non-technical summary

Research Question

Economic agents suffer from information rigidities. For instance, firms are never perfectly informed about the state of the economy. It is impossible for them to acquire all relevant information. The information acquisition also brings a certain degree of imprecision. Economics describes such information rigidities with the concept of imperfect common knowledge. Older literature established what imperfect common knowledge means for the Phillips curve. However, the literature assumed Calvo pricing, under which firms reoptimize prices after receiving a random signal. I am interested in whether I can obtain the same Phillips curve under the competing assumption of Rotemberg, where firms bear price-adjustment costs.

Contribution

The paper specifies a simple dispersed-information environment. Firms in the model pay quadratic price-adjustment costs. Profit maximization delivers the condition for an optimal price. I aggregate and obtain the imperfect-common-knowledge Phillips curve that is implied by Rotemberg pricing.

Results

Under the assumption of imperfect common knowledge, the Phillips curve implied by Calvo differs from the Phillips curve implied by Rotemberg. Inflation in both versions depends on expectations of marginal costs and on expectations of future inflation. Under Rotemberg, inflation additionally depends on expectations of future relative prices.

Nichttechnische Zusammenfassung

Fragestellung

Wirtschaftakteure unterliegen Informationsrigiditäten. Beispielsweise sind Unternehmen niemals vollkommen über die Konjunkturlage unterrichtet, weil es ihnen nicht möglich ist, alle relevanten Informationen zu beschaffen. Die Informationsakquise ist zudem mit einem gewissen Grad an Ungenauigkeit behaftet. In der Volkswirtschaftslehre werden solche Informationsrigiditäten auch als unvollkommenes Wissen bezeichnet. Die bisherige Fachliteratur hat sich vor allem mit der Bedeutung des unvollkommenen Wissens für die Phillips-Kurve beschäftigt. Dies erfolgte allerdings unter Annahme einer Calvo-Preissetzung, wonach Unternehmen ihr Preisniveau nach Erhalt eines willkürlichen Signals anpassen. Die vorliegende Studie untersucht nun, ob sich unter der konkurrierenden Annahme von Rotemberg, wonach Unternehmen bei Preisanpassungen Kosten entstehen, dieselbe Phillips-Kurve ergibt.

Beitrag

Zunächst wird ein Modell mit verstreuter Information spezifiziert. Die Unternehmen im Modell zahlen quadratische Preisanpassungskosten. Die Bedingung für den optimalen Preis ist Gewinnoptimierung. Auf dieser Grundlage wird die aggregierte Phillips-Kurve ermittelt, die sich bei unvollkommener Information und einer Preissetzung nach Rotemberg ergibt.

Ergebnisse

In einem Modell mit verstreuter Information unterscheidet sich die Phillips-Kurve nach Calvo von der nach Rotemberg. In beiden Versionen hängt die Inflation von den Erwartungen hinsichtlich der Grenzkosten sowie der künftigen Inflation ab. Bei Rotemberg hängt die Inflation darüber hinaus von den Erwartungen in Bezug auf die künftigen relativen Preise ab.

BUNDESBANK DISCUSSION PAPER NO 50/2016

The Imperfect-Common-Knowledge Phillips Curve: Calvo versus Rotemberg*

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Abstract

I derive the imperfect-common-knowledge Phillips curve under the assumption of Rotemberg pricing. The curve differs from the Calvo version in one important aspect. Expectations of future relative prices impact inflation.

Keywords: Phillips Curve, Rotemberg, Imperfect Knowledge

JEL classification: D83, E31

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1 Calvo and the Imperfect Common Knowledge

Forecast surveys document that economic agents disagree about the future (e.g., Coibion and Gorodnichenko, 2012; Andrade, Crump, Eusepi, and Moench, 2016). Private information represents a natural explanation for such heterogeneous beliefs. Each economic agent has its specific information set and cannot access all relevant information. The heterogeneous beliefs can be formalized by models with imperfect common knowledge (i.a., Woodford, 2003).

Nimark (2008) and Melosi (2013) impose imperfect common knowledge and Calvo (1983) pricing on firms. The authors then derive the following Phillips curve:

$$\begin{aligned}\pi_t = {} & (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1} mc_{t|t}^{(k)} \\ & + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\pi_{t+1|t}^{(k)}.\end{aligned} \tag{1}$$

The variables in the Phillips curve have the following definitions.

$$\begin{aligned} mc_{t|t}^{(1)} &= \int_0^1 E_t^j mc_t(j)\,\mathrm{d}j \\ mc_{t|t}^{(k+1)} &= \int_0^1 E_t^j mc_{t|t}^{(k)}\,\mathrm{d}j \quad \forall k=1,2,\ldots \\ \pi_{t+1|t}^{(1)} &= \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j \\ \pi_{t+1|t}^{(k+1)} &= \int_0^1 E_t^j \pi_{t+1|t}^{(k)}\,\mathrm{d}j \quad \forall k=1,2,\ldots \end{aligned}$$

The Calvo pricing and the imperfect common knowledge cause inflation π_t to depend on two sums. The first adds up average higher-order expectations of marginal costs $mc_{t|t}^{(k)}$. The second adds up average higher-order expectations of future inflation $\pi_{t+1|t}^{(k)}$. The parameter β is the discount factor, $(1-\theta)$ the probability of price reoptimization. The operator E_t^j denotes expectations of firm j; $mc_t(j)$ represents marginal costs of firm j.

This is the first paper addressing the question whether the Phillips curve, under the imperfect common knowledge, looks similar if one assumes the Rotemberg (1982) pricing. I derive such a curve and show it differs from Equation 1.

2 Profit Maximization

A continuum of firms $[0;1]$ populates the economy. Each firm produces a differentiated good. A bundler, which uses the Dixit-Stiglitz aggregator, combines the differentiated goods and supplies final good Y_t. A firm $j \in [0;1]$ faces demand

$$Y_t(j) = (Z_t(j))^{-\nu}\, Y_t, \tag{2}$$

where $Z_t(j) = P_t(j)/P_t$ denotes the relative price of firm j. The definition of the aggregate price level P_t implies

$$1 = \int_0^1 (Z_t(j))^{1-\nu} \, \mathrm{d}j. \tag{3}$$

The firm j knows all parameters and completely understands how the economy works. But it doesn't know realizations of all variables. Instead, the firm has the following information set:

$$\mathcal{I}_t^j = \{P_\tau(j), P_{\tau-1}, a_\tau(j) | \tau = t, t-1, t-2, \ldots\}.$$

The firm knows the price it sets today and remembers its past prices. The firm learns the aggregate price level with one period lag. The firm additionally receives a noisy signal $a_t(j)$ about a process a_t which is common to all firms:

$$a_t(j) = a_t + \epsilon_t(j).$$

For instance, $a_t(j)$ could be firm-specific productivity which consists of productivity a_t shared by all firms and an idiosyncratic component $\epsilon_t(j)$. Over all other variables the firm has to form expectations E_t^j. Richer information sets like in Nimark (2008) and Melosi (2013) would change nothing in the derivation of the Phillips curve.

The production of the firm obeys constant returns to scale. Real marginal costs $MC_t(j)$ are firm-specific (e.g., because of firm-specific productivity). The firm has to pay price-adjustment costs, which depend on the steady-state gross inflation rate Π and the aggregate demand Y_t. In every period the firm maximizes expected nominal profits discounted by $\Lambda_{\tau|t}$.

$$\max_{P_t(j)} E_t^j \sum_{\tau=t}^{\infty} \Lambda_{\tau|t} \left[P_\tau(j) Y_\tau(j) - P_\tau MC_\tau(j) Y_\tau(j) - P_\tau \frac{\Xi}{2} \left(\frac{P_\tau(j)}{P_{\tau-1}(j)} - \Pi \right)^2 Y_\tau \right]$$
$$\text{s.t. } (2)$$

I can write the first-order condition as

$$\begin{aligned} 0 = E_t^j \big\{ & \left[1 - \nu + \nu MC_t(j) \left(Z_t(j)\right)^{-1}\right] Y_t(j) \\ & - \Xi \Pi_t \left(\Pi_t(j) - \Pi\right) \left(Z_{t-1}(j)\right)^{-1} Y_t \\ & + \Lambda_{t+1|t} \Xi \Pi_{t+1} \left(\Pi_{t+1}(j) - \Pi\right) \Pi_{t+1}(j) \left(Z_t(j)\right)^{-1} Y_{t+1} \big\}, \end{aligned} \tag{4}$$

where $\Pi_t = P_t/P_{t-1}$ and $\Pi_t(j) = P_t(j)/P_{t-1}(j)$.

3 The Phillips Curve

I linearize the first-order condition (4). For convenience, I define $\pi_t(j) = \Pi_t(j) - \Pi$, $mc_t(j) = (MC_t(j) - MC(j))/MC(j)$, $z_t(j) = (Z_t(j) - Z(j))/Z(j)$, and $\kappa = (\nu - 1)/(\Xi\Pi)$. Given the information set $\mathcal{I}_t^j$ the firm knows its today's inflation ($E_t^j \pi_t(j) = \pi_t(j)$). I can express the first-order condition as (see Appendix A)

$$\pi_t(j) = \kappa E_t^j mc_t(j) + \beta E_t^j \pi_{t+1}(j) - \kappa E_t^j z_t(j). \tag{5}$$

The standard logic of price setting holds. If the firm expects higher marginal costs, it requires a higher price. The price-adjustment costs generate forward-looking firm-specific inflation. The firm doesn't want to set its price too high because the demand would be too low. Therefore, the expected relative price negatively affects the firm-specific inflation.

The definition of the relative price leads to

$$z_t(j) = \frac{1}{\Pi}\pi_t(j) - \frac{1}{\Pi}\pi_t + z_{t-1}(j),$$

where $\pi_t = \Pi_t - \Pi$. I use this relation and substitute for $\pi_{t+1}(j)$ and $z_t(j)$ in Equation 5:

$$\begin{aligned}\pi_t(j) = &\frac{\kappa\Pi}{(1+\beta)\,\Pi+\kappa} E_t^j mc_t(j) + \frac{\beta\Pi}{(1+\beta)\,\Pi+\kappa} E_t^j \pi_{t+1} \\ &+ \frac{\beta\Pi^2}{(1+\beta)\,\Pi+\kappa} E_t^j z_{t+1}(j) + \frac{\beta\Pi+\kappa}{(1+\beta)\,\Pi+\kappa} E_t^j \pi_t \\ &- \frac{(\beta\Pi+\kappa)\,\Pi}{(1+\beta)\,\Pi+\kappa} E_t^j z_{t-1}(j).\end{aligned}$$

The information set $\mathcal{I}_t^j$ ensures the firm knows its past relative price ($E_t^j z_{t-1}(j) = z_{t-1}(j)$). I integrate over all firms and apply two results which originate from Equation 3. The relative prices sum to zero ($\int_0^1 z_{t-1}(j)\,\mathrm{d}j = 0$); the aggregate inflation is the average of firm-specific inflations ($\pi_t = \int_0^1 \pi_t(j)\,\mathrm{d}j$). I obtain (see Appendix B)

$$\begin{aligned}\pi_t = &\frac{\kappa\Pi}{(1+\beta)\,\Pi+\kappa} mc_{t|t}^{(1)} + \frac{\beta\Pi}{(1+\beta)\,\Pi+\kappa} \pi_{t+1|t}^{(1)} \\ &+ \frac{\beta\Pi^2}{(1+\beta)\,\Pi+\kappa} z_{t+1|t}^{(1)} + \frac{\beta\Pi+\kappa}{(1+\beta)\,\Pi+\kappa} \pi_{t|t}^{(1)},\end{aligned} \tag{6}$$

where $mc_{t|t}^{(1)} = \int_0^1 E_t^j mc_t(j)\,\mathrm{d}j$, $\pi_{t+1|t}^{(1)} = \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j$, $z_{t+1|t}^{(1)} = \int_0^1 E_t^j z_{t+1}(j)\,\mathrm{d}j$, and $\pi_{t|t}^{(1)} = \int_0^1 E_t^j \pi_t\,\mathrm{d}j$. I repeatedly take expectations E_t^j and integrate over Equation 6 to get expressions for $\pi_{t|t}^{(1)}$, $\pi_{t|t}^{(2)}$, etc. Then in Equation 6 I repeatedly substitute for $\pi_{t|t}^{(1)}$, $\pi_{t|t}^{(2)}$, etc. Finally, I obtain the imperfect-common-knowledge Phillips curve under Rotemberg:

$$\begin{aligned}\pi_t = &\frac{(\nu-1)\,\Pi}{(1+\beta)\,\Xi\Pi^2+\nu-1} \sum_{k=1}^{\infty} \left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\,\Xi\Pi^2+\nu-1}\right)^{k-1} mc_{t|t}^{(k)} \\ &+ \frac{\beta\Xi\Pi^2}{(1+\beta)\,\Xi\Pi^2+\nu-1} \sum_{k=1}^{\infty} \left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\,\Xi\Pi^2+\nu-1}\right)^{k-1} \pi_{t+1|t}^{(k)} \\ &+ \frac{\beta\Xi\Pi^3}{(1+\beta)\,\Xi\Pi^2+\nu-1} \sum_{k=1}^{\infty} \left(\frac{\beta\Xi\Pi^2+\nu-1}{(1+\beta)\,\Xi\Pi^2+\nu-1}\right)^{k-1} z_{t+1|t}^{(k)},\end{aligned} \tag{7}$$

where $mc_{t|t}^{(k+1)} = \int_0^1 E_t^j mc_{t|t}^{(k)}\,\mathrm{d}j$, $\pi_{t+1|t}^{(k+1)} = \int_0^1 E_t^j \pi_{t+1|t}^{(k)}\,\mathrm{d}j$, and $z_{t+1|t}^{(k+1)} = \int_0^1 E_t^j z_{t+1|t}^{(k)}\,\mathrm{d}j$ for $k = 1, 2, \ldots$.

4 The Difference between Calvo and Rotemberg

Now it becomes clear how the Phillips curves in Equation 1 and Equation 7 differ. Under the Rotemberg pricing, inflation still depends on expectations of marginal costs and on expectations of future inflation. But a new term appears. Inflation, in addition, depends on average higher-order expectations of future relative prices $z^{(k)}_{t+1|t}$.

In models with perfect information the Phillips curves under Calvo and Rotemberg easily resemble each other up to the first order. One just needs to assume zero trend inflation or full price indexation (i.a., Ascari and Rossi, 2012).

Here, under the imperfect common knowledge, the Phillips curves differ despite of the full indexation. The difference results from the heterogeneous beliefs. Each firm has private information in its information set and forms its own beliefs. Therefore, the average higher-order expectations of future relative prices don't equal zero, and the Phillips curves differ.

If the firms were still imperfectly informed, but information wasn't private, the average higher-order expectations of future relative prices would equal zero. In consequence, the Phillips curves under Calvo and Rotemberg would become identical.

For illustration that the new term doesn't generally equal zero, recall the definition

$$z^{(1)}_{t+1|t} = \int_0^1 E^j_t z_{t+1}(j) \, \mathrm{d}j$$

and imagine the following scenario. A large negative productivity shock a_t hits the economy. The firms observe their firm-specific $a_t(j)$ and cannot distinguish a_t from $\epsilon_t(j)$. The firms misinterpret the situation and think they experience a large negative idiosyncratic shock $\epsilon_t(j)$. The firms expect high marginal costs and consequently require high prices. Because each firm interprets the low productivity as idiosyncratic, each firm expects to have a high relative price today. Due to the price stickiness each firm also expects to have a high relative price tomorrow ($E^j_t z_{t+1}(j) > 0$). So the new term in the Phillips curve easily becomes non-zero.

The expectations of relative prices are more than a surprising algebraic feature of the Rotemberg pricing. They demonstrate how communication of a central bank can change the behavior of inflation. Suppose the central bank provides the public with model-consistent expectations of relative prices ($E^j_t z_{t+1}(j) = E_t z_{t+1}(j)$). Because then

$$\begin{aligned} z^{(1)}_{t+1|t} = \int_0^1 E^j_t z_{t+1}(j) \, \mathrm{d}j = \int_0^1 E_t z_{t+1}(j) \, \mathrm{d}j = \\ = E_t \int_0^1 z_{t+1}(j) \, \mathrm{d}j = 0, \end{aligned}$$

expectations of relative prices $z^{(k)}_{t+1|t}$ for any order k would equal zero. Under such communication inflation would again only depend on expectations of marginal costs and on expectations of future inflation.

What the difference between Calvo and Rotemberg exactly means for model dynamics, estimation, and welfare analysis, I leave for future research.

A How to Obtain Equation 5

I take the first-order condition (4). I apply the first-order Taylor expansion around the steady state.

$$
\begin{aligned}
0 = E_t^j \Bigg\{ \Bigg\{ & \left[1-\nu+\nu MC(j)(Z(j))^{-1}\right] Y(j) + \nu(Z(j))^{-1} Y(j)\left(MC_t(j)-MC(j)\right) \\
& - \nu MC(j)(Z(j))^{-2} Y(j)\left(Z_t(j)-Z(j)\right) \\
& + \left[1-\nu+\nu MC(j)(Z(j))^{-1}\right]\left(Y_t(j)-Y(j)\right) \Bigg\} \\
& - \Bigg\{ \Xi\Pi\left(\Pi(j)-\Pi\right)(Z(j))^{-1}Y + \Xi\left(\Pi(j)-\Pi\right)(Z(j))^{-1}Y\left(\Pi_t-\Pi\right) \\
& + \Xi\Pi(Z(j))^{-1}Y\left(\Pi_t(j)-\Pi(j)\right) \\
& - \Xi\Pi\left(\Pi(j)-\Pi\right)(Z(j))^{-2}Y\left(Z_{t-1}(j)-Z(j)\right) \\
& + \Xi\Pi\left(\Pi(j)-\Pi\right)(Z(j))^{-1}\left(Y_t-Y\right) \Bigg\} \\
& + \Bigg\{ \overline{\Lambda_{t+1|t}}\Xi\Pi\left(\Pi(j)-\Pi\right)\Pi(j)(Z(j))^{-1}Y \\
& + \Xi\Pi\left(\Pi(j)-\Pi\right)\Pi(j)(Z(j))^{-1}Y\left(\Lambda_{t+1|t}-\overline{\Lambda_{t+1|t}}\right) \\
& + \overline{\Lambda_{t+1|t}}\Xi\left(\Pi(j)-\Pi\right)\Pi(j)(Z(j))^{-1}Y\left(\Pi_{t+1}-\Pi\right) \\
& + \Big[\overline{\Lambda_{t+1|t}}\Xi\Pi\Pi(j)(Z(j))^{-1}Y \\
& + \overline{\Lambda_{t+1|t}}\Xi\Pi\left(\Pi(j)-\Pi\right)(Z(j))^{-1}Y\Big]\left(\Pi_{t+1}(j)-\Pi(j)\right) \\
& - \overline{\Lambda_{t+1|t}}\Xi\Pi\left(\Pi(j)-\Pi\right)\Pi(j)(Z(j))^{-2}Y\left(Z_t(j)-Z(j)\right) \\
& + \overline{\Lambda_{t+1|t}}\Xi\Pi\left(\Pi(j)-\Pi\right)\Pi(j)(Z(j))^{-1}\left(Y_{t+1}-Y\right) \Bigg\} \Bigg\}
\end{aligned} \tag{8}
$$

For the steady states it holds: $Z(j) = 1$, $Y(j) = Y$, $\Pi(j) = \Pi$, $\overline{\Lambda_{t+1|t}} = \beta/\Pi$, and $MC(j) = (\nu-1)/\nu$. I simplify Equation 8.

$$
\begin{aligned}
0 = E_t^j \Bigg\{ \Bigg\{ & \nu Y\left(MC_t(j)-MC(j)\right) - (\nu-1)Y\left(Z_t(j)-Z(j)\right) \Bigg\} \\
& - \Bigg\{ \Xi\Pi Y\left(\Pi_t(j)-\Pi(j)\right) \Bigg\} \\
& + \Bigg\{ \beta\Xi\Pi Y\left(\Pi_{t+1}(j)-\Pi(j)\right) \Bigg\} \Bigg\}
\end{aligned} \tag{9}
$$

I introduce variables expressed in deviations from the steady state.

$$\begin{aligned} mc_t(j) &= (MC_t(j) - MC(j))\,/MC(j) \\ z_t(j) &= (Z_t(j) - Z(j))\,/Z(j) \\ \pi_t(j) &= \Pi_t(j) - \Pi(j) \end{aligned}$$

I then rewrite Equation 9:

$$E_t^j \pi_t(j) = \frac{\nu - 1}{\Xi\Pi} E_t^j mc_t(j) + \beta E_t^j \pi_{t+1}(j) - \frac{\nu - 1}{\Xi\Pi} E_t^j z_t(j). \tag{10}$$

The information set $\mathcal{I}_t^j$ implies $E_t^j \pi_t(j) = \pi_t(j)$. Let's define $\kappa = (\nu - 1)\,/\,(\Xi\Pi)$. From Equation 10 I obtain Equation 5

$$\pi_t(j) = \kappa E_t^j mc_t(j) + \beta E_t^j \pi_{t+1}(j) - \kappa E_t^j z_t(j). \tag{5}$$

B How to Obtain Equation 6

First, notice that I can express the relative price of the firm j as follows:

$$Z_t(j) = \frac{P_t(j)}{P_t} = \frac{P_t(j)}{P_t}\frac{P_{t-1}(j)}{P_{t-1}(j)}\frac{P_{t-1}}{P_{t-1}} = \Pi_t(j)\frac{1}{\Pi_t} Z_{t-1}(j).$$

This means up to the first order:

$$z_t(j) = \frac{1}{\Pi}\pi_t(j) - \frac{1}{\Pi}\pi_t + z_{t-1}(j), \tag{11}$$

where $\pi_t = \Pi_t - \Pi$. I can rewrite Equation 11 for period $t+1$:

$$\pi_{t+1}(j) = \pi_{t+1} + \Pi z_{t+1}(j) - \Pi z_t(j). \tag{12}$$

I substitute for $\pi_{t+1}(j)$ in Equation 5 by Equation 12:

$$\pi_t(j) = \kappa E_t^j mc_t(j) + \beta E_t^j \pi_{t+1} + \beta\Pi E_t^j z_{t+1}(j) - (\beta\Pi + \kappa)\, E_t^j z_t(j). \tag{13}$$

I substitute for $z_t(j)$ in Equation 13 by Equation 11:

$$\begin{aligned} \pi_t(j) = {} & \frac{\kappa\Pi}{(1+\beta)\Pi + \kappa} E_t^j mc_t(j) + \frac{\beta\Pi}{(1+\beta)\Pi + \kappa} E_t^j \pi_{t+1} + \frac{\beta\Pi^2}{(1+\beta)\Pi + \kappa} E_t^j z_{t+1}(j) \\ & + \frac{\beta\Pi + \kappa}{(1+\beta)\Pi + \kappa} E_t^j \pi_t - \frac{(\beta\Pi + \kappa)\,\Pi}{(1+\beta)\Pi + \kappa} E_t^j z_{t-1}(j). \end{aligned} \tag{14}$$

To describe the behavior of the aggregate inflation, I integrate over firm-specific inflations and use Equation 14.

$$
\begin{aligned}
\pi_t &= \int_0^1 \pi_t(j)\,\mathrm{d}j = \int_0^1 \frac{\kappa\Pi}{(1+\beta)\Pi+\kappa} E_t^j mc_t(j) + \frac{\beta\Pi}{(1+\beta)\Pi+\kappa} E_t^j \pi_{t+1} \\
&\quad + \frac{\beta\Pi^2}{(1+\beta)\Pi+\kappa} E_t^j z_{t+1}(j) + \frac{\beta\Pi+\kappa}{(1+\beta)\Pi+\kappa} E_t^j \pi_t - \frac{(\beta\Pi+\kappa)\,\Pi}{(1+\beta)\Pi+\kappa} E_t^j z_{t-1}(j)\,\mathrm{d}j = \\
&= \frac{\kappa\Pi}{(1+\beta)\Pi+\kappa} \int_0^1 E_t^j mc_t(j)\,\mathrm{d}j + \frac{\beta\Pi}{(1+\beta)\Pi+\kappa} \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j \\
&\quad + \frac{\beta\Pi^2}{(1+\beta)\Pi+\kappa} \int_0^1 E_t^j z_{t+1}(j)\,\mathrm{d}j + \frac{\beta\Pi+\kappa}{(1+\beta)\Pi+\kappa} \int_0^1 E_t^j \pi_t\,\mathrm{d}j \\
&\quad - \frac{(\beta\Pi+\kappa)\,\Pi}{(1+\beta)\Pi+\kappa} \int_0^1 E_t^j z_{t-1}(j)\,\mathrm{d}j
\end{aligned} \tag{15}
$$

The information set $\mathcal{I}_t^j$ and the definition of the price level imply:

$$
\int_0^1 E_t^j z_{t-1}(j)\,\mathrm{d}j = \int_0^1 z_{t-1}(j)\,\mathrm{d}j = 0. \tag{16}
$$

Let's define $mc_{t|t}^{(1)} = \int_0^1 E_t^j mc_t(j)\,\mathrm{d}j$, $\pi_{t+1|t}^{(1)} = \int_0^1 E_t^j \pi_{t+1}\,\mathrm{d}j$, $z_{t+1|t}^{(1)} = \int_0^1 E_t^j z_{t+1}(j)\,\mathrm{d}j$, and $\pi_{t|t}^{(1)} = \int_0^1 E_t^j \pi_t\,\mathrm{d}j$. Knowing Equation 16, I rewrite Equation 15 and obtain Equation 6:

$$
\begin{aligned}
\pi_t &= \frac{\kappa\Pi}{(1+\beta)\Pi+\kappa} mc_{t|t}^{(1)} + \frac{\beta\Pi}{(1+\beta)\Pi+\kappa} \pi_{t+1|t}^{(1)} \\
&\quad + \frac{\beta\Pi^2}{(1+\beta)\Pi+\kappa} z_{t+1|t}^{(1)} + \frac{\beta\Pi+\kappa}{(1+\beta)\Pi+\kappa} \pi_{t|t}^{(1)}.
\end{aligned} \tag{6}
$$

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