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Granularity adjustment for Basel II

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Abstract

The credit value-at-risk model underpinning the Basel II Internal Ratings-Based approach assumes that idiosyncratic risk has been diversified away fully in the portfolio, so that economic capital depends only on systematic risk contributions. We develop a simple methodology for approximating the effect of undiversified idiosyncratic risk on VaR. The supervisory review process (Pillar 2) of the new Basel framework offers a potential venue for application of the proposed granularity adjustment (GA).

Our GA is a revision and extension of the methodology proposed in the Basel II Second Consultative Paper. The revision incorporates some technical advances as well as modifications to the Basel II rules since the Second Consultative Paper of 2001. Most importantly, we introduce an "upper bound" methodology under which banks would be required to aggregate multiple exposures to the same underlying obligor only for a subset of their obligors. This addresses what appears to be the most significant operational burden associated with any rigorous assessment of residual idiosyncratic risk in the portfolio. For many banks, this approach would permit dramatic reductions in data requirements relative to the full GA.

Key words: Basel II, granularity adjustment, value-at-risk, idiosyncratic risk

JEL Codes: G31, G28

Non-Technical Summary

Name concentration in a lending portfolio arises when there are few borrowers in a bank portfolio or when loan amounts are very unequal in distribution. The portfolio credit risk model underpinning the Basel II Internal Ratings-Based (IRB) approach does not account for name concentration. Rather, it assumes that the bank's portfolio is perfectly fine-grained, meaning that every single loan accounts only for a very small share in the total portfolio exposure. Real bank portfolios are, of course, not perfectly fine-grained.

In this paper, we develop a simple granularity adjustment (GA) for quantifying the contribution of name concentrations to portfolio risk. The supervisory review process (Pillar 2) of the new Basel framework offers a potential venue for application of the proposed method. This version of the GA is a revision and extension of earlier methodologies, and is intended in particular to reduce the cost of implementation. In practical application, it is the data inputs (and not the formulae applied to those inputs) that can pose the most serious operational burden. When a bank has multiple exposures to the same underlying obligor, it is necessary that these multiple exposures be aggregated into a single exposure for the purpose of assessing the effect of name concentration (whether by our methodology or any robust potential alternative). This aggregation of information is currently a significant challenge for many banks. Our revised GA proposal permits banks to calculate a conservative "upper bound" on the GA on the basis of the largest exposures in the portfolio, and thereby avoids the need for aggregation of data on each and every obligor. For many banks this approach would permit dramatic reductions in data requirements relative to earlier granularity adjustment proposals.

We apply our GA methodology to a variety of realistic portfolios drawn from the German central credit register (Millionenkreditmeldewesen). The results show that the effect of name concentration can be material, and that our proposed GA is a robust tool for its assessment.

Nicht-Technische Zusammenfassung

Adressenkonzentration in einem Kreditportfolio entsteht, wenn sehr wenige Kreditnehmer in dem Portfolio sind oder wenn die Kreditvolumina sehr ungleich verteilt sind. Das Kreditrisikomodell, welches dem Internen Ratings-Basierten (IRB) Ansatz von Basel II unterliegt, berücksichtigt die Adressenkonzentration nicht. Es wird vielmehr sogar angenommen, dass das Portfolio einer Bank perfekt granular ist, in dem Sinne, dass jeder einzelne Kredit nur einen sehr kleinen Anteil zum Gesamtportfolio beiträgt. Reale Bankportfolios sind selbstverständlich nicht perfekt granular.

In dieser Arbeit stellen wir eine einfache Granularitätsanpassung (GA) als Methode vor, mit der der Beitrag von Adressenkonzentration zum Risiko eines Portfolios quantifiziert werden kann. Das bankenaufsichtliche Überprüfungsverfahren (Säule 2) unter Basel II bietet ein Anwendungsfeld für die vorgeschlagene Methode. Diese Version der GA ist eine Überarbeitung und Erweiterung früherer Methoden und dient insbesondere dazu, die Kosten für eine Umsetzung in der Praxis zu reduzieren. In praktischen Anwendungen stellen meistens die benötigten Daten (und nicht die Formel, die auf diese Daten angewendet wird) das größte Hindernis dar. Wenn eine Bank mehrere Kredite an denselben Kreditnehmer vergeben hat, erfordert die Messung von Adressenkonzentration, dass diese Kredite aggregiert werden. Das ist unabhängig davon, ob die von uns vorgeschlagene Methode oder eine beliebige robuste Alternative verwendet wird. Diese Aggregation von Kreditinformationen stellt momentan eine wesentliche Herausforderung für die Banken dar. Unsere überarbeitete GA bietet den Banken die Möglichkeit, eine obere Schranke für die GA in einem Portfolio zu berechnen, indem sie sich ausschließlich auf Informationen über die größten Kredite stützt. Dadurch entfällt die Notwendigkeit, Daten für jeden einzelnen Kreditnehmer zu aggregieren. Für viele Banken würde dieser Ansatz eine erhebliche Reduktion der Datenanforderungen im Vergleich zu früheren Methoden zur Bestimmung der Granularitätsanpassung darstellen.

Wir wenden unsere GA Methode auf mehrere realistische Portfolios an, die auf dem Millionenkreditmeldewesen basieren. Unsere Ergebnisse zeigen, dass der Effekt der Adressenkonzentration bedeutend sein kann und dass die von uns vorgeschlagene GA eine robuste Methode für ihre Messung darstellt.

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1. Introduction

In the portfolio risk-factor frameworks that underpin both industry models of credit VaR and the Internal Ratings-Based (IRB) risk weights of Basel II, credit risk in a portfolio arises from two sources, systematic and idiosyncratic. Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on the performance of borrowers. Borrowers may differ in their degree of sensitivity to systematic risk, but few firms are completely indifferent to the wider economic conditions in which they operate. Therefore, the systematic component of portfolio risk is unavoidable and only partly diversifiable. Idiosyncratic risk represents the effects of risks that are particular to individual borrowers. As a portfolio becomes more fine-grained, in the sense that the largest individual exposures account for a smaller share of total portfolio exposure, idiosyncratic risk is diversified away at the portfolio level.

Under the Asymptotic Single Risk Factor (ASRF) framework that underpins the IRB approach, it is assumed that bank portfolios are *perfectly* fine-grained, that is, that idiosyncratic risk has been fully diversified away, so that economic capital depends only on systematic risk. Real-world portfolios are not, of course, perfectly fine-grained. The asymptotic assumption might be approximately valid for some of the largest bank portfolios, but clearly would be much less satisfactory for portfolios of smaller or more specialized institutions. When there are material name concentrations of exposure, there will be a residual of undiversified idiosyncratic risk in the portfolio. The IRB formula omits the contribution of this residual to required economic capital.

The impact of undiversified idiosyncratic risk on portfolio VaR can be assessed via a methodology known as *granularity adjustment*. The basic concepts and approximate

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form for the granularity adjustment were first introduced by Gordy in 2000 for application in Basel II (see Gordy, 2003). It was then substantially refined and put on a more rigorous foundation by Wilde (2001b) and Martin and Wilde (2003).[1] In this paper, we propose and evaluate a granularity adjustment (GA) suitable for application under Pillar 2 of Basel II (Basel Committee on Bank Supervision, 2006).

Our proposed methodology is similar in form and spirit to the granularity adjustment that was included in the Second Consultative Paper (CP2) of Basel II (Basel Committee on Bank Supervision, 2001). Like the CP2 version, the revised GA is derived as a first-order asymptotic approximation for the effect of diversification in large portfolios within the CreditRisk$^+$ model of portfolio credit risk. Also in keeping with the CP2 version, the data inputs to the revised GA are drawn from quantities already required for the calculation of IRB capital charges and reserve requirements.

In practical application, it is the data inputs (and not the formulae applied to those inputs) that can pose the most serious obstacles to cost-effective implementation. For this reason, we should elaborate here on an important caveat to our claim that all GA inputs are made available in the course of calculating IRB capital and reserve requirements. When a bank has multiple exposures to the same underlying obligor, it is required that these multiple exposures be aggregated into a single exposure for the purpose of calculating GA inputs. For the purpose of calculating IRB capital requirements, by contrast, the identity of the obligor is immaterial, as capital charges depend only on characteristics of the loan and obligor (e.g., type of loan, default probability, maturity) and not on the name of the borrower per se. This is a great convenience when data on different sorts of exposures are held on different computer systems, as the job of calculating capital may be delegated to those individual systems and reported back as subportfolio aggregates which can then be added up in a straightforward fashion to arrive at the bank-level capital and reserve requirements. When we measure granularity, we cannot ignore borrower identity. From the perspective of single name concentration, ten loans of 1 million Euros each to ten distinct borrowers jointly carry much less idiosyncratic risk than the same ten loans made to a single borrower. It is understood that the need to aggregate

[1] The results of Martin and Wilde (2003) can be viewed as an application of theoretical work by Gouriéroux, Laurent, and Scaillet (2000). Other early contributions to the GA literature include Wilde (2001a) and Pykhtin and Dev (2002). Gordy (2004) presents a survey of these developments and a primer on the mathematical derivation.

information across computer systems on multiple exposures to a single borrower is the most significant challenge for banks in implementing a granularity adjustment. In defense of this aggregation requirement, we note that such aggregation would be necessary in *any* effective measure of granularity, and so is not a drawback peculiar to the GA we propose in this paper. Furthermore, one might ask how a bank can effectively manage its name concentrations without the ability to aggregate exposures across the different activities of the bank.

To reduce the burden associated with exposure aggregation, the revised GA provides for the possibility that banks be allowed to calculate the GA on the basis of the largest exposures in the portfolio, and thereby be spared the need to aggregate data on each and every obligor. To permit such an option, regulators must be able to calculate the largest possible GA that is consistent with the incomplete data provided by the bank. Our approach, therefore, is based on an upper bound formula for the GA as a function of data on the m largest capital contributions out of a portfolio of n loans (with $m \leq n$). As m grows towards n (i.e., as the bank provides data on a larger share of its portfolio), the upper bound formula converges to the "full portfolio" GA. The advantage to this approach is that the bank can be permitted to choose m in accordance with its own trade-off between higher capital charges (for m small) and higher data collection effort (for m large).

Our revised methodology takes advantage of theoretical advances that have been made since the time of CP2. In particular, the GA of CP2 required a first-stage calculation in which the portfolio would be mapped to a homogeneous portfolio of similar characteristics. In the revision GA, the heterogeneous portfolio is used directly in the formula. The resulting algorithm is both simpler and more accurate than the one of CP2.

Last, our revised methodology is adapted to the changes in the definition of regulatory capital. At the time of CP2, capital requirements were expressed in terms of expected loss (EL) plus unexpected loss (UL), whereas the finalized Basel II distinguishes UL capital requirements from EL reserve requirements. The GA is invariant to EL so is unaffected by this definitional issue. However, the *inputs* to the GA do depend on the distinction between EL and UL, and so the formulae have been modified accordingly.

The methodology for the GA is set out in Section 2. In Section 3, we show how to construct an upper bound based on partial information for the portfolio. Section 4 describes the dataset that we have used for our numerical studies. The performance of the GA is assessed in various ways in Section 5. We conclude with some thoughts on the role of model choice in crafting a granularity adjustment and with a list of some tasks left for future work.

2. Methodology

In principle, the granularity adjustment can be applied to *any* risk-factor model of portfolio credit risk, and so we begin with a very general framework. We mainly follow the treatment of Martin and Wilde (2003) in the mathematical presentation, though our parameterization of the GA formula will differ. Let X denote the systematic risk factor. For consistency with the ASRF framework of Basel II and for ease of presentation, we assume that X is unidimensional (i.e., that there is only a single systematic factor). Let n be the number of positions in the portfolio, and assume that exposures have been aggregated so that there is a unique obligor for each position. Let U_i denote the loss rate on position i, let A_i denote its exposure at default (EAD_i), and let L_n be the loss rate on the portfolio of the first n positions, i.e.,

$$L_n = \sum_{i=1}^{n} s_i \cdot U_i, \tag{1}$$

where s_i denotes the portfolio share of each instrument $s_i = A_i / \sum_{j=1}^{n} A_j$.

Let $\alpha_q(Y)$ denote the q^{th} percentile of the distribution of some random variable Y. When economic capital is measured as value-at-risk at the q^{th} percentile, we wish to estimate $\alpha_q(L_n)$. The IRB formula, however, delivers the q^{th} percentile of the conditional expected loss $\alpha_q(E[L_n|X])$. The difference $\alpha_q(L_n) - \alpha_q(E[L_n|X])$ is the "exact" adjustment for the effect of undiversified idiosyncratic risk in the portfolio. Such an exact adjustment cannot be obtained in analytical form, but we can construct a Taylor series approximation in orders of $1/n$. Define the functions $\mu(X) = E[L_n|X]$ and $\sigma^2(X) = V[L_n|X]$ as the conditional mean and variance of the portfolio loss respectively, and let h be the probability density function of X. Wilde (2001b) shows that the first-order granularity adjustment is given by

$$GA = \frac{-1}{2h(\alpha_q(X))} \frac{d}{dx} \left(\frac{\sigma^2(x)h(x)}{\mu'(x)} \right) \Big|_{x=\alpha_q(X)} \tag{2}$$

This general framework can accommodate any definition of "loss." That is, we can measure the U_i on a mark-to-market basis or an actuarial basis, and either inclusive or exclusive of expected loss. The latter point is important in light of the separation of "total capital" (the concept used in CP2) into its EL and UL components in the final Basel II document. Say we measure the U_i and L_n inclusive of expected loss, but wish to define capital on a UL basis. Let UL_n be the "true" UL for the portfolio, and let UL_n^{asympt} be its asymptotic approximation which assumes that the idiosyncratic risk is diversified away. Then

$$\alpha_q(L_n)-\alpha_q(E[L_n|X]) = (UL_n + EL_n)-\left(UL_n^{asympt} + E[E[L_n|X]]\right) = UL_n-UL_n^{asympt}$$

because the unconditional expected loss ($EL_n = E[L_n]$) is equal to the expectation of the conditional loss ($E[E[L_n|X]]$). Put more simply, expected loss "washes out" of the granularity adjustment.

In the GA formula, the expressions for $\mu(x)$, $\sigma^2(x)$ and $h(x)$ are model-dependent. For application of the GA in a supervisory setting, it would be desirable to base the GA on the same model as that which underpins the IRB capital formula. Unfortunately, this is not feasible for two reasons: First, the IRB formula is derived within a single-factor mark-to-market Vasicek model closest in spirit to KMV Portfolio Manager. The expressions for $\mu(x)$ and $\sigma^2(x)$ in such a model would be formidably complex. The effect of granularity on capital is sensitive to maturity, so simplification of the model to its default-mode counterpart (closest in spirit to a two-state CreditMetrics) would entail a substantive loss of fidelity. Furthermore, even with that simplification, the resulting expressions for $\mu(x)$ and $\sigma^2(x)$ remain somewhat more complex than desirable for supervisory application. The second barrier to using this model is that the IRB formula is not fit to the model directly, but rather is linearized with respect to maturity. The "true" term-structure of capital charges in mark-to-market models tends to be strongly concave, so this linearization is not at all a minor adjustment. It is not at all clear how one would alter $\mu(x)$ and $\sigma^2(x)$ to make the GA consistent with the linearized IRB formula.

As fidelity to the IRB model cannot be imposed in a direct manner, we adopt an indirect strategy. We base the GA on a model chosen for the tractability of the resulting expressions, and then reparameterize the inputs in a way that restores consistency as much as possible. Our chosen model is an extended version of the single factor CreditRisk$^+$ model that allows for idiosyncratic recovery risk.[2] As

[2] CreditRisk$^+$ is a widely-used industry model for portfolio credit risk that was proposed by

CreditRisk$^+$ is an actuarial model of loss, we define the loss rate as $U_i = \mathrm{LGD}_i \cdot D_i$, where D_i is a default indicator equal to 1 if the obligor defaults, 0 otherwise. The systematic factor X generates correlation across obligor defaults by shifting the default probabilities. Conditional on $X = x$, the probability of default is

$$\mathrm{PD}_i(x) = \mathrm{PD}_i \cdot (1 - w_i + w_i \cdot x)\,.$$

where PD_i is the unconditional probability of default. The factor loading w_i controls the sensitivity of obligor i to the systematic risk factor. We assume that X is gamma-distributed with mean 1 and variance $1/\xi$ for some positive ξ.[3] Finally, to obtain an analytical solution to the model, in CreditRisk$^+$ one approximates the distribution of the default indicator variable as a Poisson distribution.

In the standard version of CreditRisk$^+$, the recovery rate is assumed to be known with certainty. Our extended model allows LGD_i to be a random loss-given-default with expected value ELGD_i and variance VLGD_i^2. The LGD uncertainty is assumed to be entirely idiosyncratic, and therefore independent of X.

We next obtain the $\mu(x)$ and $\sigma^2(x)$ functions for this model. Let us define at the instrument level the functions $\mu_i(x) = E[U_i|x]$ and $\sigma_i^2(x) = V[U_i|x]$. By the conditional independence assumption, we have

$$\begin{aligned}\mu(x) &= E[L_n|x] = \sum_{i=1}^{n} s_i \mu_i(x) \\ \sigma^2(x) &= V[L_n|x] = \sum_{i=1}^{n} s_i^2 \sigma_i^2(x).\end{aligned}$$

In CreditRisk$^+$, the $\mu_i(x)$ function is simply

$$\mu_i(x) = \mathrm{ELGD}_i \cdot \mathrm{PD}_i(x) = \mathrm{ELGD}_i \cdot \mathrm{PD}_i \cdot (1 - w_i + w_i \cdot x).$$

For the conditional variance, we have

$$\sigma_i^2(x) = E[\mathrm{LGD}_i^2 \cdot D_i^2|x] - \mathrm{ELGD}_i^2 \cdot \mathrm{PD}_i(x)^2 = E[\mathrm{LGD}_i^2] \cdot E[D_i^2|x] - \mu_i(x)^2. \quad (3)$$

As D_i given X is assumed to be Poisson distributed, we have $E[D_i|X] = V[D_i|X] = \mathrm{PD}_i(X)$, which implies

$$E[D_i^2|X] = \mathrm{PD}_i(X) + \mathrm{PD}_i(X)^2.$$

Credit Suisse Financial Products (1997).

[3] Note that we must have $E[X] = 1$ in order that $E[\mathrm{PD}_i(X)] = \mathrm{PD}_i$.

For the term $E[\mathrm{LGD}_i^2]$ in the conditional variance, we can substitute

$$E[\mathrm{LGD}_i^2] = V[\mathrm{LGD}_i] + E[\mathrm{LGD}_i]^2 = \mathrm{VLGD}_i^2 + \mathrm{ELGD}_i^2$$

This leads us to

$$\begin{aligned} \sigma_i^2(x) &= \left(\mathrm{VLGD}_i^2 + \mathrm{ELGD}_i^2\right) \cdot \left(\mathrm{PD}_i(X) + \mathrm{PD}_i(X)^2\right) - \mu_i(x)^2 \\ &= \mathcal{C}_i \mu_i(x_q) + \mu_i(x_q)^2 \cdot \frac{\mathrm{VLGD}_i^2}{\mathrm{ELGD}_i^2} \end{aligned}$$

where $\mathcal{C}_i$ is defined as

$$\mathcal{C}_i \equiv \frac{\mathrm{ELGD}_i^2 + \mathrm{VLGD}_i^2}{\mathrm{ELGD}_i}. \qquad (4)$$

We substitute the gamma pdf $h(x)$ and the expressions for $\mu(x)$ and $\sigma^2(x)$ into equation (2), and then evaluate the derivative in that equation at $x = \alpha_q(X)$. The resulting formula depends on the instrument-level parameters PD_i, w_i, ELGD_i and VLGD_i.

We now reparameterize the inputs. Let $\mathcal{R}_i$ be the EL reserve requirement as a share of EAD for instrument i. In the default-mode setting of CreditRisk$^+$, this is simply

$$\mathcal{R}_i = \mathrm{ELGD}_i \cdot \mathrm{PD}_i\,.$$

Let $\mathcal{K}_i$ be the UL capital requirement as a share of EAD. In CreditRisk$^+$, this is

$$\mathcal{K}_i = E[U_i | X = \alpha_q(X)] = \mathrm{ELGD}_i \cdot \mathrm{PD}_i \cdot w_i \cdot (\alpha_q(X) - 1) \qquad (5)$$

When we substitute $\mathcal{R}_i$ and $\mathcal{K}_i$ into the CreditRisk$^+$ GA, we find that the PD_i and w_i inputs can be eliminated. We arrive at the formula

$$\begin{aligned} GA_n = \frac{1}{2\mathcal{K}^*} \sum_{i=1}^n s_i^2 \Bigg[&\left(\delta \mathcal{C}_i (\mathcal{K}_i + \mathcal{R}_i) + \delta(\mathcal{K}_i + \mathcal{R}_i)^2 \cdot \frac{\mathrm{VLGD}_i^2}{\mathrm{ELGD}_i^2} \right) \\ &- \mathcal{K}_i \left(\mathcal{C}_i + 2(\mathcal{K}_i + \mathcal{R}_i) \cdot \frac{\mathrm{VLGD}_i^2}{\mathrm{ELGD}_i^2} \right) \Bigg], \end{aligned} \qquad (6)$$

where $\mathcal{K}^* = \sum_{i=1}^n s_i \mathcal{K}_i$ is the required capital per unit exposure for the portfolio as a whole and where

$$\delta \equiv (\alpha_q(X) - 1) \cdot \left(\xi + \frac{1 - \xi}{\alpha_q(X)} \right).$$

Note that the expression for δ depends only on model parameters, not data inputs, so δ is a regulatory parameter. It is through δ that the variance parameter ξ influences the GA. In the CP2 version, we set $\xi = 0.25$. Assuming that the target solvency probability is $q = 0.999$, this setting implies $\delta = 4.83$. This is the value used in the numerical exercises of Section 5, but we also examine the sensitivity of the GA to the choice of ξ. Alternative calibrations of ξ are explored in the Appendix. For policy purposes, it is worthwhile to note that setting $\xi = 0.31$ would be well within any reasonable empirical bounds on this parameter, and would yield the parsimonious integer value $\delta = 5$.

The volatility of LGD (VLGD) neither is an input to the IRB formula, nor is it restricted in any way within the IRB model. Banks could, in principle, be permitted or required to supply this parameter for each loan. Given the scant data currently available on recoveries, it seems preferable to impose a regulatory assumption on VLGD in order to avoid the burden of a new data requirement. We impose the relationship as found in the CP2 version of the GA:

$$\mathrm{VLGD}_i^2 = \gamma\,\mathrm{ELGD}_i(1 - \mathrm{ELGD}_i) \tag{7}$$

where the regulatory parameter γ is between 0 and 1. When this specification is used in industry models such as CreditMetrics and KMV Portfolio Manager, a typical setting is $\gamma = 0.25$.

The GA formula can be simplified somewhat. The quantities $\mathcal{R}_i$ and $\mathcal{K}_i$ are typically small, and so terms that are products of these quantities can be expected to contribute little to the GA. If these second-order terms are dropped, we arrive at the simplified formula:

$$\widetilde{GA}_n = \frac{1}{2\mathcal{K}^*} \sum_{i=1}^{n} s_i^2 \mathcal{C}_i \left(\delta(\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i\right). \tag{8}$$

Here and henceforth, we use the tilde to indicate this simplified GA formula. The accuracy of this approximation to equation (6) is evaluated in Section 5.

Before proceeding, we pause to mention some alternative methodologies. Perhaps the very simplest approach would be based on a Herfindahl-Hirschman Index (HHI), which is defined as the sum of the squares of the portfolio shares of the individual exposures. Holding all else equal, the closer the HHI of a portfolio is to 1 the more concentrated the portfolio is, so the higher the appropriate granularity add-on charge. As with any ad hoc approach, it is difficult to say what the "appropriate" add-on for a given HHI should be. Furthermore, as we will see in Section 5, the effect of granularity on economic capital is quite sensitive to the credit quality of the portfolio, so the HHI approach would need to somehow take this into account. One suspects that an appropriately modified HHI-based approach would be no less complex than a model-based approach and certainly would be less robust. Finally, an HHI-based approach does not avoid in any way the operational burden associated with aggregation of multiple exposures to a single exposure per obligor.

Another approach, due to Vasicek (2002), lies somewhere between ad hoc and model-based. In this method, one augments systematic risk (by increasing the factor loading) in order to compensate for ignoring the idiosyncratic risk. The trouble is that systematic and idiosyncratic risk have very different distribution shapes. This method is known to perform quite poorly in practice.

Much closer to our proposal in spirit and methodology is the approach of Emmer and Tasche (2005). Emmer and Tasche (2005) offer a granularity adjustment based on a one-factor default-mode CreditMetrics model, which has the advantage of relative proximity of the model underpinning the IRB formula. As discussed earlier, however, we believe this advantage to be more in appearance than in substance because of the importance of maturity considerations in the IRB model. As a mark-to-market extension of the Emmer and Tasche (2005) GA appears to be intractable, maturity considerations would need to be introduced indirectly (as in our proposal) through the inputs. Reparameterization along these lines is feasible in principle, but would lead to a rather more complicated formula with more inputs than our CreditRisk$^+$-based GA.

Finally, an alternative that has not been much studied is the saddlepoint based method of Martin and Wilde (2003). Results in that paper suggest that it would be quite similar to the GA in performance and pose a similar tradeoff between fidelity to the IRB model and analytical tractability. Indeed, it is not at all likely that the saddlepoint GA would yield a closed-form solution for any industry credit risk model other than CreditRisk$^+$.

3. An upper bound based on incomplete data

As discussed in the introduction, aggregation of multiple exposures into a single exposure per obligor is very likely to be the only substantive challenge in implementing the granularity adjustment. To reduce this burden on the banks, we propose that banks be permitted to calculate the GA based on a *subset* consisting of the largest exposures. An upper bound can be calculated for the influence of exposures that are left out of the computation. This approach is conservative from a supervisory point of view because the upper bound is always at least as large as the "true" GA. The bank can therefore be given the flexibility to find the best trade-off between the cost of data collection and the cost of the additional capital associated with the upper bound.

In order to convey most clearly the intuition behind our approach, we first present the upper bound in the special case of a portfolio that is homogeneous in PD and ELGD. We then present the upper bound for the more realistic case of a heterogeneous portfolio.

3.1. Homogeneous case

The simplest upper bound is for the case in which exposures are homogeneous in PD and ELGD, but heterogeneous in exposure size. Assume that the bank has determined the m largest aggregate exposures in the portfolio of n obligors ($m \leq n$), and that we have sorted these aggregated EAD values as $A_1 \geq A_2 \geq \ldots \geq A_m$. The shares $s_1 \geq s_2 \geq \ldots \geq s_m$ are, as in Section 2, calculated with respect to the *total* portfolio EAD in the denominator. This latter quantity certainly will be available in the bank's balance sheet.

When PD and ELGD are homogeneous, we have $\mathcal{K}_i = \mathcal{K}^* = \mathcal{K}$ and $\mathcal{R}_i = \mathcal{R}$ for all i, and similarly $\mathcal{C}_i = \mathcal{C}$ is also independent of i. Hence the simplified GA reads

$$\widetilde{GA}_n = \frac{1}{2\mathcal{K}}\mathcal{C}(\delta(\mathcal{K}+\mathcal{R})-\mathcal{K})\cdot HHI,$$

where HHI is the Herfindahl-Hirschman Index

$$HHI = \sum_{i=1}^{n} s_i^2.$$

Using only the first $m \leq n$ exposures, and defining S_m as the cumulative share of these exposures, $S_m = \sum_{i=1}^{m} s_i$, we know that HHI is bounded by

$$HHI = \sum_{i=1}^{m} s_i^2 + \sum_{i=m+1}^{n} s_i^2 \leq \sum_{i=1}^{m} s_i^2 + s_m \cdot \sum_{i=m+1}^{n} s_i = \sum_{i=1}^{m} s_i^2 + s_m \cdot (1 - S_m).$$

This leads to the following upper bound for the simplified granularity adjustment

$$\widetilde{GA}_n^{upper} = \frac{1}{2\mathcal{K}}\mathcal{C}(\delta(\mathcal{K}+\mathcal{R})-\mathcal{K}) \cdot \left(\sum_{i=1}^{m} s_i^2 + s_m \cdot (1-S_m)\right). \tag{9}$$

3.2. Heterogeneous case

In the general case of a heterogeneous portfolio, the upper bound becomes more complicated because the meaning of "largest exposures" is no longer unambiguous. Do we mean largest by EAD, by capital contribution, or by some other measure? It turns out that we require information on both the distribution of aggregated positions by EAD and by capital contribution. Specifically, we assume:

1. The bank has identified the m obligors to whom it has the largest aggregated exposures measured in capital contribution, i.e., $A_i \cdot \mathcal{K}_i$. Denote this set of obligors as Ω. For each obligor $i \in \Omega$, the bank knows $(s_i, \mathcal{K}_i, \mathcal{R}_i)$.
2. For the $n-m$ exposures that are unreported (that is, exposures for which the obligor is not in Ω), the bank determines an upper bound on share (denoted $\bar{s}$) such that $s_i \leq \bar{s}$ for all i in the unreported set.
3. The bank knows $\mathcal{K}^*$ and $\mathcal{R}^*$ for the portfolio as a whole.

The first assumption is straightforward and unavoidable, as this is where the need arises to aggregate multiple exposures for a subset of obligors in the portfolio. Internal risk management reporting typically includes a list of the "tallest trees" in capital usage by customer, and therefore it is reasonable to assume that aggregated capital contribution data for the largest customers are internally available. If such data are unavailable, we might question whether the bank is making any substantive business use of its internal economic capital models.

The second assumption is perhaps more difficult, but is necessary in order to obtain a bound on unreported exposure shares. A bank can easily identify $\bar{s}$ if, for example, internal risk management systems report on the obligors to which the bank has the greatest exposure in EAD.[4] Denote this set by Λ, and let λ be the smallest s_i in this

[4]For example, there may be a lending rule that requires the director of the bank to sign off on all loans above a certain threshold.

set. Then $\bar{s}$ is either the largest of the s_i which is in Λ but not in Ω or (if this set is empty) simply λ, i.e.,

$$\bar{s} = \max\{s_i : s_i \in \Lambda \backslash \Omega \cup \{\lambda\}\}$$

The third assumption hardly needs justification, as these porfolio-level quantities are calculated in the course of determining IRB capital requirements. In particular, $\mathcal{K}^*$ and $\mathcal{R}^*$ can be obtained in the usual manner without aggregation of exposures by obligor.

We generalize the notation $\mathcal{K}^*$ and $\mathcal{R}^*$ so that

$$\mathcal{K}^*_k = \sum_{i=1}^{k} s_i \mathcal{K}_i \qquad \text{and} \qquad \mathcal{R}^*_k = \sum_{i=1}^{k} s_i \mathcal{R}_i,$$

i.e., $\mathcal{K}^*_k$ and $\mathcal{R}^*_k$ are partial weighted sums of the $\mathcal{K}_i$ and $\mathcal{R}_i$ sequences, respectively. Finally, for notational convenience define

$$Q_i \equiv \delta(\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i.$$

Using the above notation, the GA can be reformulated as follows

$$\begin{aligned} \widetilde{GA}_n &= \frac{1}{2\mathcal{K}^*} \sum_{i=1}^{n} s_i^2 \mathcal{C}_i (\delta(\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i) \\ &= \frac{1}{2\mathcal{K}^*} \cdot \left(\sum_{i=1}^{m} s_i^2 Q_i \mathcal{C}_i + \sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \right). \end{aligned} \tag{10}$$

The summation over 1 to m is known by Assumption 1. By Assumption 2, we know that $\bar{s} \geq s_i$ for $i = m+1, \ldots, n$. Our assumption on VLGD in equation (7) is sufficient to guarantee that $\mathcal{C}_i \leq 1$. Therefore,

$$\sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \leq \bar{s} \sum_{i=m+1}^{n} s_i Q_i = \bar{s} \left(\delta \sum_{i=m+1}^{n} s_i (\mathcal{K}_i + \mathcal{R}_i) - \sum_{i=m+1}^{n} s_i \mathcal{K}_i \right).$$

Next observe that

$$\begin{aligned} \sum_{i=m+1}^{n} s_i \mathcal{K}_i &= \mathcal{K}^* - \mathcal{K}^*_m \\ \sum_{i=m+1}^{n} s_i \mathcal{R}_i &= \mathcal{R}^* - \mathcal{R}^*_m. \end{aligned}$$

Assumption 1 implies that $\mathcal{K}_m^*$ and $\mathcal{R}_m^*$ are known to the bank. Thus we arrive at

$$\sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \leq \bar{s}((\delta-1)(\mathcal{K}^*-\mathcal{K}_m^*)+\delta(\mathcal{R}^*-\mathcal{R}_m^*)). \tag{11}$$

Finally we obtain the following upper bound for the heterogeneous case

$$\widetilde{GA}_m^{upper} = \frac{1}{2\mathcal{K}^*}\left(\sum_{i=1}^{m} s_i^2 Q_i \mathcal{C}_i + \bar{s}\left((\delta-1)(\mathcal{K}^*-\mathcal{K}_m^*)+\delta(\mathcal{R}^*-\mathcal{R}_m^*)\right)\right). \tag{12}$$

4. Data on German bank portfolios

To show the impact of the granularity adjustment on economic capital we need to apply the GA to realistic bank portfolios. We use data from the German credit register, which includes all bank loans greater or equal to 1.5 Million Euro. This data set has been matched to the firms' balance sheet data to obtain obligor specific PDs. More specifically, a logistic regression model based on balance sheet data between 12 and 24 months before default classified as default balance sheets has been used.[5] The resulting portfolios are much smaller than the portfolios reported in the German credit register, however, there are still a number of banks with more than 300 exposures in this matched data set which we consider as an appropriate size for calculating the GA. We grouped the banks in large, medium, small and very small banks where large refers to a bank with more than 4000 exposures, medium refers to one with $1000-4000$ exposures, small refers to a bank with $600-1000$ exposures and very small to a bank with $300-600$ exposures.

To accommodate privacy restrictions on these data, we aggregate portfolios for three different banks into a single data set. We then sort the loans by exposure size and remove every third exposure. The resulting portfolio of 5289 obligors is still realistic in terms of exposure and PD distribution and is similar in size to some of the larger portfolios in the matched data set of the German credit register and the firm's balance sheet data. The mean of the loan size distribution is 3973 thousand Euros and the standard deviation is 9435 thousand Euros. Quantiles are reported in Table 1. Henceforth, we refer to this portfolio as "portfolio A."

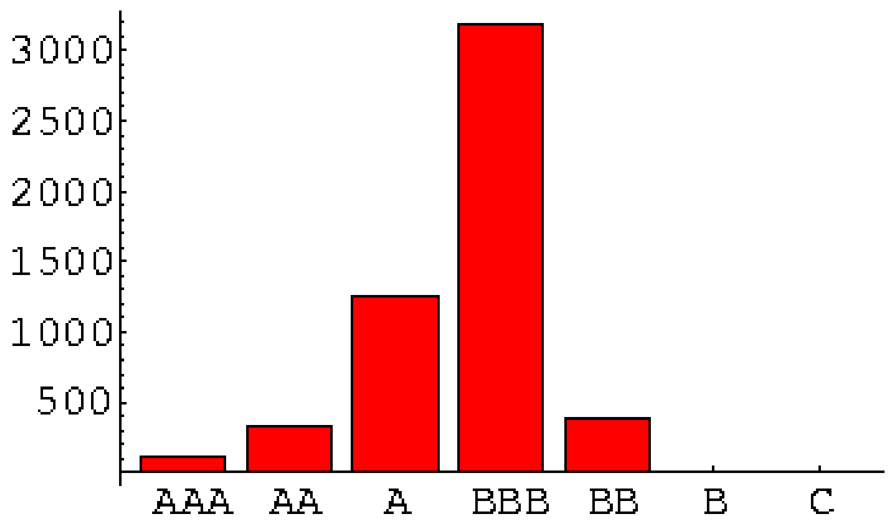
[5]The model has been found to provide a high accuracy in terms of an area under the ROC curve of more than 0.8. See Gerke et al. (2006).

Table 1
Exposure distribution in Portfolio A

Level	Quantile
5%	50.92
25%	828.80
50%	1811.75
75%	3705.50
95%	13637.36

Figure 1 shows the PD distribution for the aggregated portfolio A for different PD categories which we denote here by S&P's common rating grades. The PD ranges for the different rating grades are listed in Table 2 below.

Figure 1. Borrower Distribution by Rating Grade



The average PD of the data set is 0.43% and hence lower than the average PD of a portfolio of a smaller or medium sized bank in Germany, which is approximately 0.8% (Kocagil et al., 2001, p. 8). Moody's, for example, understates average net

Table 2
PD ranges associated with rating buckets

Rating Grade	PD Ranges in %
AAA	PD ≤ 0.02
AA	$0.02 \leq$ PD ≤ 0.06
A	$0.06 \leq$ PD ≤ 0.18
BBB	$0.18 \leq$ PD ≤ 1.06
BB	$1.06 \leq$ PD ≤ 4.94
B	$4.94 \leq$ PD ≤ 19.14
C	$19.14 \leq$ PD

loan provisions of 0.77% for German banks during the period 1989 – 1999 (Kocagil et al., 2001, p. 7), which is more than two times the average loss of the firms in our sample during the same period. Approximately 70% of the portfolio in our data set belongs to the investment grade domain (i.e., rated BBB or better) and the remaining 30% to the subinvestment grade. In smaller or medium sized banks in Germany the percentage of investment grade exposures in a portfolio is approximately 37% (Taistra et al., 2001, p. 2). As a consequence the value of the GA in our aggregated portfolio A will be smaller than the GA in a true bank portfolio of similar exposure distribution.

The data set does not contain information on LGD, so we impose the Foundation IRB assumption of ELGD = 0.45.

5. Numerical results

In Table 3, we present granularity adjustments calculated on real bank portfolios varying in size and degree of heterogeneity. As we would expect, the GA is invariably small (12 to 14 basis points) for the largest portfolios, but can be substantial (up to 161 basis points) for the smallest. The table demonstrates the strong correlation between Herfindahl index and GA across these portfolios, though of course the correspondence is not exact as the GA is sensitive to credit quality as well. As a reference portfolio, we included a portfolio with 6000 loans each of PD = 0.01 and ELGD = 0.45 and of homogeneous EAD. The GA for the largest real portfolio is roughly six times as large as the GA for the homogeneous reference portfolio, which demonstrates the importance of portfolio heterogeneity in credit concentrations.

Table 3
Granularity Adjustment for real bank portfolios

Portfolio	Number of Exposures	HHI	GA (in %)
Reference	6000	0.00017	0.018
Large	> 4000	< 0.001	$0.12 - 0.14$
Medium	$1000 - 4000$	$0.001 - 0.004$	$0.14 - 0.36$
Small	$600 - 1000$	$0.004 - 0.011$	$0.37 - 1.17$
Very Small	$250 - 600$	$0.005 - 0.015$	$0.49 - 1.61$

We have also computed the VaR in the CreditRisk$^+$ model and the relative add-on for the GA on the VaR. For a large portfolio this add-on is 3% to 4% of VaR. For a medium sized bank the add-on lies between 5% and 8% of VaR. In a study based on applying a default-mode multi-factor CreditMetrics model to US portfolio data, Heitfield et al. (2006) find that name concentration accounts for between 1% and 8% of VaR depending on the portfolio size. These results are quite close to our own for the GA, despite the difference in model and data.

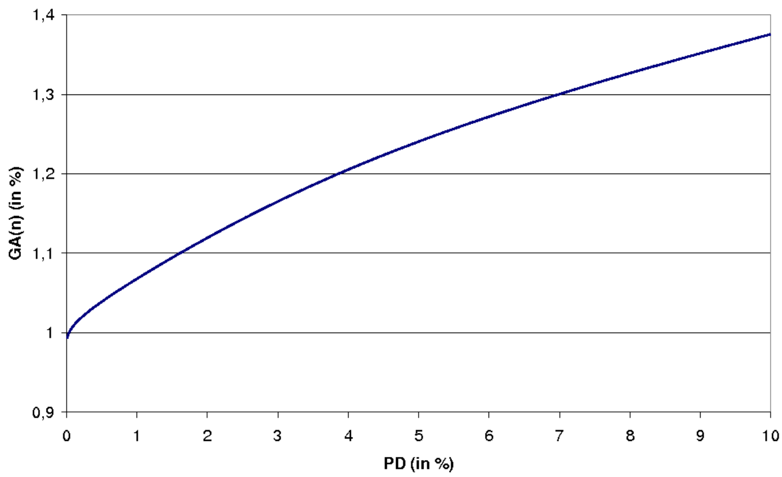
Table 4 shows the relative add-on for the granularity adjustment on the Risk Weighted Assets (RWA) of Basel II for small, medium and large portfolios as well as for the reference portfolio with 6000 exposures of unit size. The reference portfolio is used to point out the influence of the GA even for large portfolios that would be seen as very fine-grained. For the reference portfolio of 6000 exposures of unit size with homogeneous PD = 1% and ELGD = 45% the GA is approximately 0.018% and the IRB capital charge is 5.86%. Thus the add-on due to granularity is approximately 0.3% and the economic capital to capture both systematic risk and risk from single name concentration is 5.878% of the total portfolio exposure. For the real bank portfolios of our data set the add-on for the GA is higher than for the reference portfolio, although it is still quite small for large and even for some of the medium sized bank portfolios. For smaller portfolios with 300 to 1000 exposures the add-on for the GA is more significant.

Table 4
GA as percentage add-on to RWA

Portfolio	Number of Exposures	Relative Add-On for RWA
Reference	6000	0.003
Large	> 4000	0.04
Medium	$1000 - 4000$	$0.04 - 0.10$
Small	$300 - 1000$	$0.17 - 0.32$

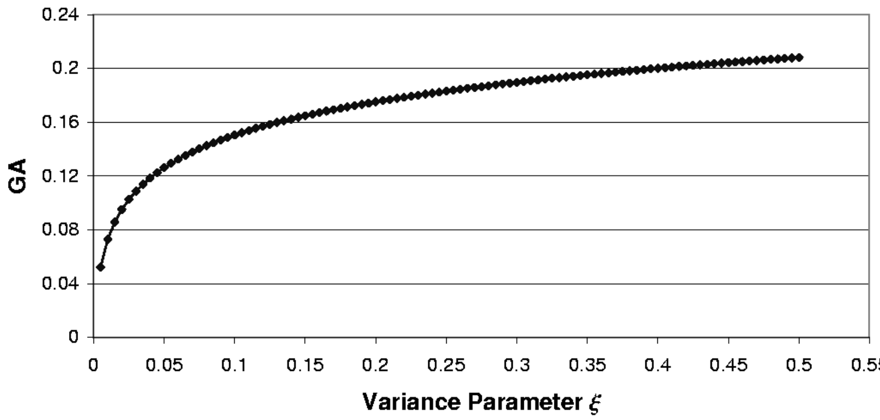
Figure 2 shows the dependence of the simplified GA on the default probability. Each point on the curve represents a homogeneous portfolio of $n = 100$ borrowers of the given PD. Dependence on portfolio quality is non-negligible, particularly for lower-quality portfolios. Such dependence cannot be accommodated naturally and accurately in ad hoc methods of granularity adjustment based on exposure HHI.

Figure 2. Effect of Credit Quality on Simplified GA



The sensitivity of the GA to the variance parameter ξ of the systematic factor X is explored in Figure 3. We see that the granularity adjustment is strictly increasing in ξ, and that the degree of sensitivity is not negligible. Increasing ξ from 0.2 to 0.3 causes a 10% increase in the GA for Portfolio A. Uncertainty in dependence parameters of this sort is a perennial challenge in portfolio credit risk modeling. A guiding principle in the design of Basel II has been to impose regulatory values on parameters (e.g., the asset correlation parameter ρ) that cannot be estimated to reasonable precision with extant data. Similar judgmental treatment is required here. While the absolute magnitude of the GA is sensitive to ξ, its relative magnitude across bank portfolios is much less so. In this sense, the proper functioning of the GA as a supervisory tool does not materially depend on the precision with which ξ is calibrated.

Figure 3. Effect of the Variance of the Systematic Factor on Simplifed GA



Our next task is to verify the accuracy of the simplified granularity adjustment $\widetilde{\mathrm{GA}}$ as an approximation to the "full" GA of equation (6). We construct six stylized portfolios of different degrees of exposure concentrations. Each portfolio consists of $n = 1000$ exposures and has constant PD and ELGD fixed at 45%. Portfolio P0 is completely homogeneous whereas portfolio P50 is highly concentrated since the largest exposure $A_{1000} = 1000^{50}$ accounts for 5% of the total exposure of the portfolio. The values for both the simplified $\widetilde{GA}_n$ and the full GA for each of these portfolios are listed in Table 5. We see that the approximation error increases with concentration and with PD. For realistic portfolios, the error is trivial. Even for the case of portfolio P10 and PD = 4%, the error is only 3 basis points. The error

Table 5
Approximation error of the simplified $\widetilde{GA}_n$

Portfolio	P0	P1	P2	P10	P50
PD = 1%					
Exposure A_i	1	i	i^2	i^{10}	i^{50}
$\widetilde{\text{GA}}$ in %	0.107	0.142	0.192	0.615	2.749
GA in %	0.109	0.146	0.197	0.630	2.814
PD = 4%					
Exposure A_i	1	i	i^2	i^{10}	i^{50}
$\widetilde{\text{GA}}$ in %	0.121	0.161	0.217	0.694	3.102
GA in %	0.126	0.168	0.227	0.726	3.243

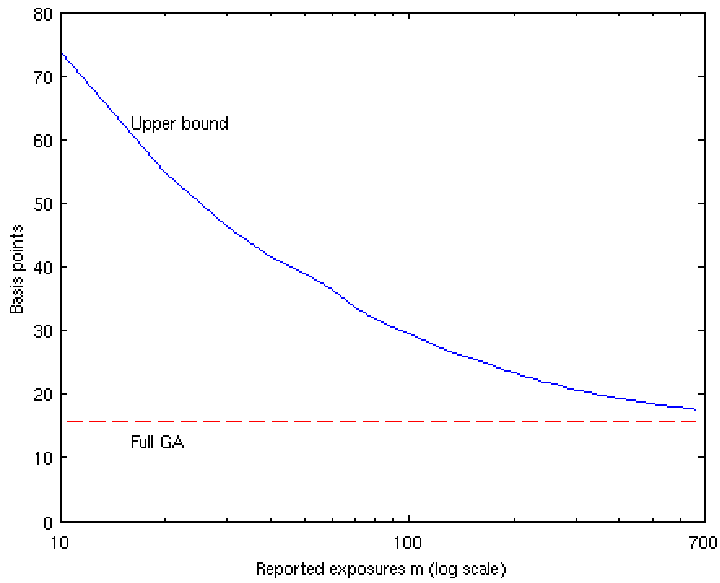
grows to 12 basis points in the extreme example of P50 and PD = 4%, but even this remains small relative to the size of the GA.

Finally, we use Portfolio A to demonstrate the effectiveness of the upper bound provided in Section 3. In Figure 4, we show how the gap between the upper bound and the "whole portfolio" GA shrinks as m (the number of positions included in the calculation) increases. With only 150 exposures included out of 5289 in the whole portfolio, this gap is only 10 basis points. With 300 exposures included, the gap shrinks to 5 basis points. The tightness of the upper bound is undoubtedly somewhat sensitive to the characteristics of the portfolio, but from these results we can tentatively conclude that the upper bound approach performs quite well.

6. Discussion

This paper sets forth a granularity adjustment for portfolio credit VaR that accounts for a risk that is not captured by the Pillar 1 capital requirement of the Basel II IRB approach. Our GA is a revision and extension of the methodology first introduced in the Basel II Second Consultative Paper. The revision incorporates some technical advances as well as modifications to the Basel II rules since CP2. Most importantly,

Figure 4. Tightness of the Upper Bound



we introduce an "upper bound" methodology that addresses the most significant source of operational burden associated with the assessment of residual idiosyncratic risk in the portfolio (whether through the proposed GA or by any other rigorous methodology). For many banks, this approach would permit dramatic reductions in data requirements at modest cost in additional capital requirement.

We have examined the numerical behavior of the GA across a range of portfolio types and studied its robustness to model parameters. Two further potential sources of inaccuracy should be considered. First, the GA formula is itself an asymptotic approximation, and so might not work well on very small portfolios. We do not see this issue as a material concern. In general, the GA errs on the conservative (i.e., it overstates the effect of granularity), but is quite accurate for modest-sized portfolios of as few as 200 obligors (for a low-quality portfolio) or 500 obligors (for an investment-grade portfolio). Second, the IRB formulae are based on a rather different model of credit risk, so we have a form of "basis risk" (or "model mismatch"). This is potentially a more serious issue. However, the great advantage to the particular model we use to underpin the GA is its analytical tractability. This tractability permits us to reparameterize the GA formula in terms of the IRB reserve requirement and capital charge, the latter of which includes a maturity adjustment. In effect, we obtain an indirect form of maturity adjustment in the GA through maturity-adjustment of the inputs, rather than maturity adjustment in the formula itself. Furthermore, without the analytical tractability of our approach, it would not have been possible to derive a useful upper bound methodology.

For application in practice, a more important limitation of our methodology is that we assume each position is an unhedged loan to a single borrower. How should we incorporate credit default swaps (CDS) and loan guarantees in a granularity adjustment? Credit risk mitigation activities can decrease name concentration (say, through purchase of CDS on the largest exposures in the portfolio) or actually indirectly give rise to name concentration in exposure to providers of credit protection. We will address this problem in future work.

Appendix: Calibration of variance parameter ξ

In models such as CreditMetrics that assume Gaussian systematic factors, the shape of the distribution for X does not depend on the variance. For this reason, one can normalize the variance to one without any loss of generality. By contrast, when X is gamma-distributed as in CreditRisk$^+$, skewness and kurtosis and other shape measures for X are not invariant to the variance, and so this parameter must be calibrated. In principle, the parameter ξ presents an extra degree of freedom for better fitting the model to data, and so is welcome. In practice, however, extremely long time-series would be required to get a reasonably precise fit. One sees users impose a fairly wide range of values for ξ, say between 0.2 and 2. Lower values of ξ imply greater systematic risk, which generally leads to higher economic capital requires, but which minimizes the GA as a share of economic capital.

Recall that ξ influences the GA through the δ parameter. In Table 6, we report δ for representative values of ξ (holding fixed $q = 0.999$). From this, we conclude that a range of values $4.5 < \delta < 6.5$ would not be out of line with common practice.

Another way to calibrate ξ is to match the variance of the default probability when portfolio maturity is one year. When $M = 1$, the IRB model collapses to the default-mode CreditMetrics model, and this variance has tractable form Gordy (2000)

$$V_i^{\mathrm{CM}} = Var[PD_i(X)] = \Phi_2(\Phi^{-1}(\mathrm{PD}_i), \Phi^{-1}(\mathrm{PD}_i), \rho_i) - PD_i^2. \tag{13}$$

Table 6
δ as a function of ξ ($q = 0.999$)

ξ	0.20	0.25	0.35	0.50	0.75	1.00	1.50	2.00
δ	4.66	4.83	5.09	5.37	5.68	5.91	6.23	6.45

where ρ_i is the Basel II asset correlation parameter and Φ_2 denotes the bivariate normal cdf. The corresponding variance for CreditRisk$^+$ is

$$V_i^{CR+} = Var[PD_i(X)] = (PD_i \cdot w_i)^2/\xi. \tag{14}$$

Equating the two variance expressions gives

$$\xi = \frac{\Phi(\Phi^{-1}(\mathrm{PD}_i), \Phi^{-1}(\mathrm{PD}_i), \rho_i) - PD_i^2}{PD_i^2 \cdot w_i^2}. \tag{15}$$

Next, we obtain an expression for the factor loading w_i by matching asymptotic UL capital charges across the same two models:

$$\begin{aligned} \mathcal{K}_i^{CR+} &= \mathrm{ELGD}_i \cdot \mathrm{PD}_i \cdot w_i \cdot (\alpha_q(X) - 1) \\ \mathcal{K}_i^{CM} &= \Phi\left(\sqrt{\frac{1}{1-\rho_i}}\Phi^{-1}(\mathrm{PD}_i) + \Phi^{-1}(q)\sqrt{\frac{\rho_i}{1-\rho_i}}\right) \end{aligned}$$

and so

$$w_i = \frac{\Phi\left(\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(\mathrm{PD}_i) + \Phi^{-1}(q)\sqrt{\frac{\rho}{1-\rho}}\right) - \mathrm{PD}_i}{\mathrm{PD}_i \cdot (\alpha_q(X) - 1)}. \tag{16}$$

We substitute this expression for w_i into equation (15) to get an implicit formula for ξ that depends only on PD, the corresponding ρ in the IRB formula, and $\alpha_q(X)$. This last quantity depends on ξ, so we must solve using a nonlinear root-finding algorithm.

An obvious drawback to this method is that the estimated value of ξ depends on the chosen PD, whereas ξ ought to be independent of portfolio characteristics. When PD is set to 1%, we obtain the value $\xi = 0.206$, which is roughly consistent with the our baseline parameterization of $\xi = 0.25$.

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