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Endogenous credit derivatives and bank behavior

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Abstract

Instruments for credit risk transfer arise endogenously from and interact with optimizing behavior of their users. This is particularly true with credit derivatives which are usually OTC contracts between banks as buyers and sellers of credit risk. Recent literature, however, does not account for this fact when analyzing the effects of these instruments on banking. The present paper closes this gap by explicitly modelling the market for credit derivatives and its interaction with banks' loan granting and deposit taking activities.

Keywords: credit risk, credit derivatives, bargaining

JEL classification: D53, D82, G11, G14, G21

Non-technical summary

Owing to their business operations, banks are heavily exposed to credit risk. This has traditionally been managed by passive strategies such as portfolio selection and loan sales. The increasing availability of new instruments such as credit derivatives and synthetic structures in the recent past, however, has allowed a more active management of credit risk, ie credit risk transfer.

While practitioners as well as researchers increasingly take new instruments into account, effects with respect to banks' behaviour and the stability of financial systems are not that clear. In particular, (theoretical) analyses are unable to derive unambiguous results regarding the way credit risk transfer affects exposure to credit risk either on the bank-level or the level of the financial system as a whole. The main reason for this is that theoretical models usually consider the structure of credit derivatives and synthetic instruments given exogenously. However, because of the OTC character of these new instruments, one might expect interdependencies to appear between contract design and banks' behavior.

The model used in the present paper considers two banks which do not compete for depositors or lenders. Banks have the opportunity to transfer credit risk applying a credit derivative in the form of a credit default swap (CDS). In this regard it is assumed that bank 1 only sells credit risk and bank 2 only buys credit risk. In order to take into account the OTC character the process of negotiating credit derivatives is modelled as a cooperative game. In this way, one is able to show that there appear portfolio effects in addition to the ones familiar in the context of banking activities. In particular, we find the probability distributions of banks' returns from loan business to improve in the sense of first-order stochastic dominance. This, in turn, reduces banks' costs when funding loans by deposits. The reason for this is that risk-averse depositors demand a lower risk premium when a bank's outcome from loan business becomes less risky. Furthermore, this allows banks to grant more loans, which reinforces portfolio effects. Moreover, by taking a closer look at the financial system one finds, first, that aggregate credit risk is reduced and, second, that credit risk transfer allocates total credit risk among banks efficiently.

In this regard, the optimal credit derivative just covers parts of the protection buyer's total exposure to credit risk. In particular, the banks agree just to transfer credit risk to the amount which affects the protection buyer's ability to meet his own depositors' claims. In return, the protection seller gets payed a fixed premium which ensures that total profits from credit risk transfer are shared optimally among the banks.

Recent descriptive data regarding CDS transactions basically seem to support the main results of the theoretical model in the present paper. The nominal amount of CDS contracts outstanding steadily increased during the last few years which is particularly true with portfolio CDS contracts. Moreover, it could be observed that banks which participate in credit risk transfer expanded loan granting activities. Unfortunately, currently there is no sufficient data available which allows for a more detailed empirical analysis – especially with respect to banks' effective total exposure to credit risk.

Nicht technische Zusammenfassung

Aufgrund ihrer Geschäftstätigkeit sind Banken in besonderem Maße dem Kreditrisiko ausgesetzt, bei dessen Management sie sich traditionell auf eher passive Strategien wie Portfolioselektion oder Kreditverkäufe beschränken mussten. Die zunehmende Verbreitung von Instrumenten wie Kreditderivaten und synthetischen Verbriefungen erleichtern den Banken jedoch den Handel von Kreditrisiken und ermöglichen so ein aktives Risikomanagment.

Obwohl Praxis wie auch Wissenschaft diesen Entwicklungen zunehmend Beachtung schenken, sind die Auswirkungen der Nutzung von Kreditderivaten und synthetischen Verbriefungen auf Banken und Finanzsystem erst wenig erforscht. Insbesondere gelingt es der einschlägigen theoretischen Literatur bislang nicht, eindeutige Aussagen hinsichtlich der Beeinflussung des Kreditrisikos insgesamt auf Bank- und Finanzsystemebene herauszuarbeiten. Zurückzuführen ist dies in erster Linie darauf, dass üblicherweise in den Analysen von exogen gegebenen Instrumenten für den Kreditrisikohandel ausgegangen wird. In der Praxis werden diese jedoch in der Regel durch die beteiligten Vertragspartner bilateral ausgehandelt, so dass Abhängigkeiten zwischen dem Verhalten von Banken und der Ausgestaltung von Kreditderivaten und synthetischen Verbriefungen zu erwarten sind.

Das Modell der vorliegenden Arbeit bezieht diese Abhängigkeiten explizit in die Analyse ein. Ausgangspunkt der Modellierung sind zwei Banken, die in unterschiedlichen Märkten im Kredit- und Einlagenbereich tätig sind und somit keinem Wettbewerb unterliegen. Durch ein Kreditderivat in Form eines Credit Default Swap (CDS) erhalten die Banken die Möglichkeit zum Kreditrisikohandel, wobei Bank 1 lediglich als Verkäufer und Bank 2 lediglich als Käufer von Kreditrisiko auftritt. Die Modellierung der optimalen Gestaltung des Kreditderivats erfolgt hierbei als kooperatives Verhandlungsspiel unter symmetrischer Information zwischen den Vertragspartnern. In diesem Rahmen gelingt es zu zeigen, dass mit Hilfe des optimalen Kreditrisikohandels Diversifikationseffekte erzeugt und ausgenutzt werden können, die zusätzlich zu den ohnehin im Bankgeschäft existierenden Diversifikationseffekten auftreten. Diese äußern sich in Form einer Verbesserung der Wahrscheinlichkeitsverteilung der riskanten Erträge aus dem Kreditgeschäft der am Risikohandel beteiligten Banken im Sinne stochastischer Dominanz erster Ordnung. In der Folge sind die Banken in der Lage, ihre Kreditvergabe zu geringeren Kosten durch Kundeneinlagen zu refinanzieren, da das gesunkene Ertragsrisiko ein Absenken der an die risikoaversen Einleger zu zahlenden Risikoprämie erlaubt. Als Resultat werden mehr Kredite vergeben und die auftretenden Portfolioeffekte weiter verstärkt. Bei Betrachtung des gesamten Finanzsystems wird zudem deutlich, dass durch den Risikohandel das insgesamt auftretende Kreditrisiko reduziert und dieses geringere Risiko anschließend effizient auf die beteiligten Banken aufgeteilt werden kann.

Darüber hinaus zeigt sich, dass das im Modell bestimmte optimale Kreditderivat nicht das gesamte Kreditrisiko des Risikoverkäufers an den Risikokäufer überträgt. Vielmehr wird Kreditrisiko nur in dem Umfang an den Sicherungsgeber transferiert, soweit es die Ansprüche der Einleger des Risikoverkäufers tangiert. Die gleichzeitig fällige Prämie für den Risikohandel wird so bemessen, dass die durch diesen realisierbaren Gewinne unter Berücksichtigung des Kontrahentenrisikos optimal unter beiden Banken aufgeteilt werden.

Die vorliegenden deskriptiven Daten scheinen die Ergebnisse aus der Modellierung grundstzlich zu stützen. So nimmt nicht nur das Nominalvolumen ausstehender Credit Default Swaps stetig zu, vielmehr gewinnen portfoliobasierte CDS zunehmend an Bedeutung. Zudem ist in den zurückliegenden Jahren eine Ausweitung der Kreditvergabe der am Kreditrisikohandel beteiligten Banken zu beobachten. Aufgrund der unzureichenden Datenlage gestaltet sich eine detailliertere empirische Überprüfung der im Modell abgeleiteten Ergebnisse – insbesondere hinsichtlich der effektiven Gesamtrisikoposition der Banken – derzeit allerdings als schwierig.

Contents

Endogenous Credit Derivatives and Bank Behavior

1 Introduction

The formation and tremendous growth of markets for credit derivatives during the past two decades has significantly extended banks' opportunities to actively manage credit risk. Whether this is beneficial to participating banks or even the financial system at all is still an unresolved question. That is, recent academic analysis is not able to determine unambiguous conclusions regarding the effects arising from banks' use of credit derivatives.

The reason for this is that in the literature credit derivatives do not emerge endogenously from rational behavior of involved institutions. Rather, structures of the instruments and pricing techniques are considered exogenous. Banks' decisions are, hence, only passive in the sense that behavior is simply adjusted to actual market conditions. The possible impact of banks' decisions on the market environment is usually not considered. As a result, it is found that credit derivatives allow one to exploit effects of portfolio diversification on the one hand, and on the other hand, banks may adjust their decisions with respect to granting loans. When they decide to issue more risky loans it is unclear whether this offsets the portfolio effect.

In order to determine the net effect of this tradeoff one needs to consider that credit derivatives are in general over-the-counter (OTC) contracts. That is, buyers and sellers of credit risk exposures bilaterally negotiate the terms of credit derivatives contracts. If one assumes rational behavior of both parties one might expect interdependencies between details of credit derivatives and banks' loan granting and deposit taking activities.

The present paper starts from this consideration and explicitly models the market for credit derivatives.[1] One is able to analyze the optimal structure of the credit derivatives contracts as well as its interactions with banks' decisions in the deposit and loan business, and the model picks up a number of real-world facts recently observed in markets for credit derivatives.

In particular, a Nash-bargaining situation between two monopolistic banks is considered in which the terms of Credit Default Swap (CDS) contracts are negotiated. That is, the assumption of Nash-bargaining captures the OTC character of credit derivatives. Furthermore, when focusing on CDS contracts one acknowledges that this is the most common type of present credit derivatives. Moreover, since banks make up the largest group of sellers as well as buyers of credit risk the effects of

[1] The model is based on chapter 5 of my dissertation completed at the University of Augsburg.

credit risk transfer within the banking system are considered. The assumption of monopolistic banks permits one to focus on the pure effects of credit risk transfer and prevents distorting effects which may arise from competition among banks. The results, which are derived within this setup, undoubtedly help to better understand the effects of credit risk transfer ongoing at present.

The main results may be summarized as follows. The application of a CDS contract positively affects participating banks in the sense that they are less exposed to credit risk than before entering into a CDS contract. In fact, the CDS contract synthetically merges the loan portfolios of both banks and in this way reinforces the effect of portfolio diversification. In this regard, the optimal structure of the CDS contract allows a bank to exploit corresponding benefits which are then shared among banks. For this purpose, the optimal CDS contract exhibits a number of interesting features.

In particular, it is found that credit risk is only transferred to the extent that the risk seller is not able to meet their own depositors' claims. Credit risk exposure which only affects the risk seller's positive profits above this threshold is not handed over to the risk buyer. The risk buyer, in turn, agrees to cover losses of the protection buyer only if the claims of the risk buyer's depositors are not put at risk. A CDS contract, thus, shifts credit risk to the bank which is better off and hence better able to absorb risk. In other words, CDS contracts ensure that credit risk is allocated efficiently among banks.

A more efficient allocation of credit risk, then, allows banks to expand granting loans and taking deposits. This enhances portfolio diversification even more and reinforces risk reducing effects of credit risk transfer. As a result, one observes a complementary interrelationship between credit risk transfer on the one hand and banks' loan and deposit decisions on the other hand.

In this way the present paper contributes to the relevant literature in a number of aspects. First, explicitly modelling the market for credit derivatives considerably extends the usual view of the literature. For example Broll and Welzel (2002), Broll et al. (2002) and Broll et al. (2003) analyze a bank's decisions on loans, deposits and risk management applying an industrial organization approach. That is, the bank's technology of producing loans and deposits is given exogenously. In addition, credit risk may be traded with some standardized derivatives instruments at fixed price. In this context a bank has an incentive to only sell credit risk. The extend of risk sold, then depends on the correlation between the risk exposure and the risk of the underlying of the derivative. In contrast, the present paper argues that endogenous credit risk transfer and OTC markets for credit derivatives is beneficial for banks regardless if they are sellers or buyers of credit risk. Moreover, credit derivatives will be designed such that the risk of the underlying perfectly matches the risk seller's exposure to credit risk. Hence, there will always be a perfect hedge.

Another important aspect addressed in the present paper considers the interaction between diversification effects and bank's decision making process. Recent papers of Wagner and Marsh (2004), Wagner (2006a), Wagner (2006b) and Wagner (2007) start from exogenously given credit derivatives. With this assumption credit derivatives might be used to utilize diversification effects, but the consequences regarding total exposures to credit risk cannot be derived. In contrast, in the present paper it is argued that credit risk transfer allows banks to exploit diversification effects without increasing the overall risk in the market. Hence, credit risk transfer is not only a matter of allocating risk in the financial markets efficiently, but may also help to reduce the market's exposure to risk. In this way, the present paper is able to show that a large number of arguments are parallel to findings in the literature on reinsurance agreements. The seminal papers of Borch (1960), Borch (1961), Borch (1962) and Blazenko (1986) derive similar results in the case of insurers reinsuring some of their risk. An important difference, however, is that in these papers the structure of reinsurance agreements is not derived endogenously. Rather, a number of plausible exogenous reinsurance designs are considered. Therefore, the results of the present paper are more general.

The analysis to follow is also related to papers of Hart (1975), Santomero and Trester (1998) and Duffee and Zhou (2001) which examine the effects of innovations in financial markets in general. A common argument in these papers is that there may occur situations when financial innovations might reduce welfare. However, this result is largely driven by exogenous creation of instruments and asymmetric information. While informational problems in financial markets are not explicitly addressed in the present paper the endogenous character of credit derivatives helps to tradeoff a number of effects figured out in these seminal papers and find unambiguous net effects.

The remainder of the paper is structured as follows: Section 2 sets up the model. Section 3 considers portfolio aspects in detail which are used to derive the main results of the paper in section 4. Section 5 concludes. Mathematical proofs can be found in the appendix.

2 The Model

Assumptions

For the analysis of endogenous credit risk transfer consider the following situation. Two monopolistic banks, B1 and B2, may bilaterally negotiate a CDS contract and in this way set up a market for credit risk. In this regard, it is assumed that B1 just sells credit risk (buys protection) and B2 just buys credit risk (sells protection).[2] The purpose of the CDS contract is to enable trading credit risk without affecting the banks' balance sheets. Therefore, the contract needs to define the (part of) B1's exposure to credit risk to be transferred to B2 and the price being payed in exchange by the protection seller (B1).

In particular, the CDS contract transfers credit risk by defining a payment from the protection seller to compensate the protection buyer for a loss of at least a single loan. This implies that B1 first needs to specify a number s of loans out of B1's loan portfolio for which credit risk is transferred. Below, s will be referred to as the underlying of the CDS contract. Furthermore, denote R the payment of B2 to B1 which depends, first, on the underlying's return Y_s and, second, on B2's outcome from the loan business X_l. Besides endogenously determining the levels of Y_s which trigger a payment from B2 to B1 (default events) the functional definition of $R(Y_s, X_l)$ allows for considering counterparty risk. That is, for a given Y_s B2 might be allowed to reduce the payment R depending on B2's disposable funds. In this regard it needs to be mentioned that from B2's point of view R is subordinated to its depositors' claims. In addition, let Θ denote the fixed premium B1 has to pay to B2 in exchange for credit risk transfer. Finally, it is assumed that there does not appear any asymmetric information between the banks. In this way, one is able to focus on effects of credit risk transfer without being distorted by incentive aspects.

This opportunity for credit risk transfer needs to be incorporated into a financial system in order to analyze interdependencies between credit risk transfer and banks' behavior. For this purpose, consider two (geographically) separated financial markets. Each of the aforementioned banks is a monopolist in one of these markets. Moreover, in B1's (B2's) region there is a finite number n_1 (n_2) of borrowers. Each borrower has an investment project with positive net present value available which generates an ex ante random outcome and requires one unit of external funds. Denote $y_i \in [0, \overline{y}]$ and $x_i \in [0, \overline{x}]$ the outcome of borrower i's investment project and $f(y_i)$ and $a(x_i)$ the corresponding probability distribution functions (pdf) in B1's and B2's region, respectively. Investment projects in a certain region are assumed identical and stochastically independent. Since banks as well as borrowers are assumed to lack any own funds, one needs to add lenders to the setup.

The assumptions regarding lending below truly extend the standard models in the literature.[3] In particular, there is a finite number m_1 (m_2) of risk-averse lenders in B1's (B2's) region. Let $U(\cdot)$ and $V(\cdot)$ denote lenders' strictly increasing von-

[2]While this assumption seems to be quite restrictive at first glance, one might argue that it is the bank's net position in credit risk transfer which is captured by the assumption. Hence, there appears no loss of generality in this way.

[3]Note that so far assumptions simply represented an application of the standard models of Townsend (1979), Diamond (1984), and Williamson (1986) to a situation of multiple financial markets.

Neumann-Morgenstern utility function in the region of B1 and B2, respectively.[4] Each lender is endowed with one unit of funds which they wish to profitably invest. Moreover, the number of lenders – and hence the amount of funds – in each region are assumed to be sufficient in order to run all investment projects. Furthermore, lenders have different levels of reservation utilities which they can realize by choosing some opportunities outside the model. However, while each lender exactly knows his level of reservation utility, banks as well as borrowers only know the range of possible utility levels and some kind of probability distribution thereof. In this regard, let $U_{Rj} \in [\underline{U_R}, \overline{U_R}]$ ($V_{Rj} \in [\underline{V_R}, \overline{V_R}]$) with $\underline{U_R} > 0$ ($\underline{V_R} > 0$) denote the domain and $g(U_{Rj})$ ($b(V_{Rj})$) the corresponding pdf of lender j's reservation utility in B1's (B2's) region. In this way it becomes possible to consider some kind of supply function for financial funds in the model. Note, if a bank, for example, wants to take more deposits, the price of deposits will increase, too. While this relationship is standard in economics, the standard literature on financial intermediation so far did not account for this fact. Rather, the focus of this strand of literature was on asymmetric information within financial contracting situations.

In the context of the present model, there is also asymmetric information between banks and borrowers, on the one hand, and lenders and banks, on the other hand. In particular, assume that banks (lenders) need to incur some fixed cost c in order to observe the actual outcome of a borrower's investment project (a bank's loan business). That is, the costly state verification problem which is well known from the literature may also appear in the present model. In this context, it is noteworthy to mention that the verification cost c is the same regardless of the region or the relationship considered. In this way, one prevents that results of the decision making process are driven by such differences.

Owing to the assumptions above the decision making process can be represented as a three-stage game. It starts (stage 1) with banks' decisions on optimal contracts, ie loan, deposit, and CDS contracts are determined. In particular, there is a (natural) sequence of decisions at this stage which starts with determining optimal volumes of loan and deposit contracts by both banks. In addition, B1 optimizes the underlying s of the CDS contract. If one refers to this set of decisions as stage 1a, at stage 1b payment $R(Y_s, X_l)$ and premium Θ of the CDS contract are negotiated. At stage 1c both banks decide on the optimal structure of loan and deposit contracts. At stage 2 banks offer loan and deposit contracts to lenders and borrowers who then

[4]Note, the assumption that lenders behave risk-averse while banks and borrowers are risk-neutral captures the consequences of limited liability of the latter. While Froot et al. (1993) and Froot and Stein (1998) present arguments for risk averse behavior of borrowers and banks, Jensen and Meckling (1976) explain that because of limited liability their level of risk aversion is less than the one of lenders. This is particularly true if one considers lenders to be private households which are not able to well diversify their claims. Hence, the assumptions with respect to preferences towards risk of the present model are the most simple way to consider these arguments.

decide on accepting or not. If deposit and loan contracts have been accepted by lenders and borrowers, investment projects will be carried out and outcomes from investment projects and the banks' loan business will be realized. Since outcomes are observed only by borrowers and banks, they would be apt to exploit their informational advantage opportunistically and renege on contracts (stage 3). Therefore, rational behavior by participating actors will take this into account when designing contracts.

Fundamentals

For the analysis of the three-stage-game presented above, the concept of subgame-perfect Nash equilibrium will be applied. For this purpose one considers stage 3 first. That is, provided that contracts had already been written and accepted, incentive compatibility of deposit and loan contracts is examined in a first step. The analysis, then, continues with stage 2. In this regard, conditions will be figured out which ensure that lenders and borrowers are willing to accept deposit and loan contracts. When so doing results derived for stage 3 are taken into account. At the end, stage 1 is examined. For this purpose analysis starts with determining optimal structures of deposit and loan contracts (stage 1c), and then proceeds with analyzing optimal CDS contracts and the optimal volumes of deposit and loan contracts. In either case results derived for subsequent stages are taken into account. Moreover, there are some aspects to be noted which might simplify the analysis.

In particular, because of the assumption of separate financial markets interdependencies between both banks' decisions may arise only from credit risk transfer. This makes it possible to examine optimal deposit and loan contracts of a bank independently from the decisions of the other bank as long as CDS contracts are taken as given. Furthermore, let k (l) and d (e) denote the volume of loan and deposit contracts of B1 (B2). Then B1 is able to grant k loans only if it takes enough deposits to cover the premium of the CDS contract in addition: $d = k + \Theta$. B2, however, can reduce the volume of deposits taken in order to grant l loans due to Θ: $e = l - \Theta$. This argument also considers the fact that deposit taking is costly to the banks. Therefore, there is no reason for any bank to take more deposits than necessary to run loan business and credit risk transfer. In the following analysis this is considered as given.

As explained above, stage 3 of the earlier outlined game is examined first. In this regard, it should be noted that there are two sources of incentive problems. First, in the context of the loan contract a borrower might misreport the project outcome to the bank if he will be able to reduce payment in this way. Second, a bank might claim that proceeds from the loan business do not suffice to meet prespecified payments to depositors. Hence, loan and deposit contracts need to specify payments and situations in which the better informed party is monitored in order to ensure incentive compatibility.

As can be easily verified, optimal loan and deposit contracts which solve this problem show the structure of standard debt contracts. Denote $t_K(y_i)$ ($p_K(x_i)$) a borrower's payment to B1 (B2) and $T_D(\mathcal{Y}_k)$ ($P_D(X_l)$) B1's (B2's) average payment per unit of loans to depositors. The structure of optimal incentive compatible loan and deposit contracts, then, is

$$T_D(\mathcal{Y}_k) = \begin{cases} T_{D0} & ; \mathcal{Y}_k \geq T_{D0} \\ \mathcal{Y}_k - \frac{d}{k}c & ; \mathcal{Y}_k < T_{D0} \end{cases} \quad \text{and} \quad P_D(X_l) = \begin{cases} P_{D0} & ; X_l \geq P_{D0} \\ X_l - \frac{e}{l}c & ; X_l < P_{D0} \end{cases} \tag{1}$$

$$t_K(y_i) = \begin{cases} t_{K0} & ; y_i \geq t_{K0} \\ y_i - c & ; y_i < t_{K0} \end{cases} \quad \text{and} \quad p_K(x_i) = \begin{cases} p_{K0} & ; x_i \geq p_{K0} \\ x_i - c & ; x_i < p_{K0} \end{cases} . \tag{2}$$

That is, the solution of the costly state verification problem in the literature still applies in the context of the present model.[5] Of course, the interpretation of the optimal contracts is also straightforward. The contracting parties of either contract agree on a fixed payment which is denoted T_{D0}, P_{D0}, t_{K0}, and p_{K0} in the respective cases. As long as this fixed payment is made, there is no verification. If, however, the payment falls short of this fixed amount, there will be verification and the payment will be as large as possible. In the latter situation, verification costs need to be payed in addition. Compared to the literature, there is, however, one point in which the present model is different.

Taking a closer look at optimal deposit contracts, a banks' payment depends on the total outcome from its loan business. In this regard, $\mathcal{Y}_k$ and X_l denote B1's and B2's total average outcome per unit of loans from the loan business. Furthermore, define

$$\mathcal{Y}_k = \frac{1}{k}\left(Y + R(Y_s, X_l)\right) \text{ with } Y = \sum_{i=1}^{k} t_K(y_i) \text{ and } X_l = \frac{1}{l}X \text{ with } X = \sum_{i=1}^{l} p_K(x_i).$$

From these definitions it is obvious that B1's outcome from the loan business includes repayments from loan contracts as well as CDS payments. In addition, B2's payments to depositors are not affected from credit risk transfer since CDS payments are subordinated to depositors' claims. The question is, however, whether these optimal deposit and loan contracts are accepted by lenders and borrowers.

This latter question is exactly the focus of stage 2 of the present three-stage game which will be analyzed next. Note, borrowers and lenders are willing to accept offered loan and deposit contracts only if it is beneficial to them. And this means that a certain borrower's/lender's expected utility when a loan or deposit contract

[5]For a detailed proof of this result see Pausch (2006, p. 128ff) and the arguments of Gale and Hellwig (1985) and Townsend (1979).

is accepted needs to be at least as large as the expected utility from loan/deposit contracts without credit risk transfer.[6] In formal terms this may be written from the borrowers' point of view as

$$\frac{k}{n_1}\left[\int_0^{t_{K0}}\left(y_i-\overline{t_K(y_i)}-c\right)dF(y_i)+\int_{t_{K0}}^{\overline{y}}(y_i-t_{K0})\,dF(y_i)\right]\geq \mathrm{E}\left(\Pi_{1-nCDS}\right) \quad (3)$$

$$\frac{l}{n_2}\left[\int_0^{p_{K0}}\left(x_i-\overline{p_K(x_i)}-c\right)dA(x_i)+\int_{p_{K0}}^{\overline{x}}(x_i-p_{K0})\,dA(x_i)\right]\geq \mathrm{E}\left(\Pi_{2-nCDS}\right) \quad (4)$$

where (3) represents the participation constraint of a borrower in B1's region and (4) is the participation constraint of a borrower in B2's region. Lenders' participation constraint can be written in a similar way:

$$G\left(\int_0^{T_{D0}}U\left(\frac{k}{d}\overline{T_D(\mathcal{Y}_k)}\right)dH(\mathcal{Y}_k)+\int_{T_{D0}}^{\overline{\mathcal{Y}_k}}U\left(\frac{k}{d}T_{D0}\right)dH(\mathcal{Y}_k)\right)=\frac{d}{m_1}$$
$$\geq G\left(\mathrm{E}U_{1-nCDS}\right) \quad (5)$$

$$B\left(\int_0^{P_{D0}}V\left(\frac{l}{e}\overline{P_D(X_l)}\right)dW(X_l)+\int_{P_{D0}}^{p_{K0}}V\left(\frac{l}{e}P_{D0}\right)dW(X_l)\right)=\frac{e}{m_2}$$
$$\geq B\left(\mathrm{E}V_{2-nCDS}\right) \quad (6)$$

where (5) denotes the participation constraint of a lender in B1's region and (6) represents the participation constraint of a lender in B2's region. In the lenders' case $H(\mathcal{Y}_k)$ and $W(X_l)$ denote the cumulative probability distribution functions of $\mathcal{Y}_k$ and X_l, respectively. Furthermore, due to the definitions of both variables it must be true that

$$\mathcal{Y}_k \in [0,\overline{\mathcal{Y}_k}] \text{ and } X_l \in [0,p_{K0}]$$

where $\overline{\mathcal{Y}_k}$ represents some upper bound of $\mathcal{Y}_k$ depending on $R(Y_s, X_l)$.

With respect to lenders' participation constraint it should be noted that neither bank can observe a certain lenders reservation utility. Therefore, deposit contracts do not differ among lenders. As a result, a bank which wishes to take a certain amount of deposits needs to write the uniform deposit contract such that the probability that a lender's reservation utility is less than the expected utility from a deposit contract equals the desired share of lenders who accept deposit contracts.

It should be noted that results sketched so far represent optimal decisions at stages 3 through 1c of the three-stage game. The volumes of deposit and loan contracts as well as terms of CDS are taken as given in this context. Therefore, these results need to be considered when examining stages 1b and 1a of the game in the following section.

[6] In this regard see Pausch (2006) for a basic version of the present model without credit risk transfer. There it is argued that banks which grant loans and take deposits provide strictly positive profits to borrowers and lenders. Moreover, they earn a strictly positive expected profit for themselves.(p. 88ff)

3 Portfolio effects and optimal deposit and loan contracts

From taking a closer look at borrowers' and lenders' participation constraints (3), (4) and (5), (6) it is obvious that a bank's volume of loans is very important for the level of expected profit and expected utility in the respective cases. A more detailed examination of this matter, therefore, seems to be necessary.[7]

Considering borrowers' participation constraints first, one observes that a borrower's repayment t_{K0} (p_{K0}) increases when a bank decides to grant more loans. More formally, applying the implicit function theorem to B1's and B2's borrowers' participation constraints – which are binding in the optimum – yields

$$\frac{dt_{K0}}{dk} > 0 \text{ and } \frac{dp_{K0}}{dl} > 0. \tag{7}$$

At a first glance this result seems to be counterintuitive. However, if one considers that with increasing its volume of loans a bank rises a borrower's probability of successfully entering into a loan contract. It should be clear that in this way the expected profit of a borrower grows – all other things being equal. The monopolistic bank is then able to skim the borrower's additional expected profit by increasing the repayment written down in the loan contract. At the end, a borrower's expected profit remains unchanged and corresponding participation constraints are still binding.

At the deposit side of banks' business things are more complicated. As can be easily seen from lenders' participation constraints (5) and (6), there are two ways in which banks' decisions on the volume of loans affects lenders' expected utility. First, a direkt effect appears, ie k and l enter (5) and (6) directly. Second, there is also an indirect effect which affects a lender's expected utility via probability distribution functions.

This latter indirect effect is worth to be examined in more detail. In this regard, it proves useful to consider the cumulative distribution function of X_l first. Note, earlier it was argued in this context that by the assumption of CDS payment being subordinated B2's depositors' claims are not affected by the CDS contract. Therefore, one can apply the convolution theorem[8] to determine the value of the cdf W of X_l at an arbitrary realization γ_l and given volume of loans l as

$$W(\gamma_l|l) = \int\limits_{\{(p_{K1},\ldots,p_{Kl}):\sum_{i=1}^{l} p_{Ki} \leq l\gamma_l\}} a(p_{K1}) \ldots a(p_{Kl}) dp_{Kl} \ldots dp_{K1}.$$

[7]The arguments in the present section are more intuitive in order to economize on space. For a strictly formal treatment of the aspects see Pausch (2006, p. 132ff).

[8]See Larsen and Marx (1986, p. 142f).

In this latter equation we have, moreover, used the facts that $p_{Ki} = p_K(x_i)\ \forall\ i$ and x_i being independently identically distributed with density function $a(\cdot)$. Now, adding a further loan to B2's portfolio yields

$$\begin{aligned} W(\gamma_l|l+1) &= \int\limits_{\{(p_{K1},\ldots,p_{Kl}):\sum_{i=1}^{l} p_{Ki}\le(l+1)\gamma_l\}} a(p_{K1})\ldots a(p_{Kl})\cdot \\ &\cdot \int\limits_{\{p_{Kl+1}:p_{Kl+1}\le(l+1)\gamma_l-\sum_{i=1}^{l} p_{Ki}\}} a(p_{Kl+1})dp_{Kl+1}dp_{Kl}\ldots dp_{K1}. \end{aligned}$$

Obviously, both equations only differ in the term

$$\int\limits_{\{p_{Kl+1}:p_{Kl+1}\le(l+1)\gamma_l-\sum_{i=1} lp_{Ki}\}} a(p_{Kl+1})dp_{Kl+1} = A_{Kl+1}\left((l+1)\gamma_l - \sum_{i=1}^{l} p_{Ki}\right) \le 1,$$

that is the value of the cdf of $p_K(x_{l+1})$ at $(l+1)\gamma_l - \sum_{i=1}^{l} p_{Ki}$.

Since the same line of arguments may be applied to the cdf $H(\cdot)$ of $\mathcal{Y}_k$ at any realization γ_k one can conclude

$$\frac{\partial H(\gamma_k)}{\partial k} \le 0\ ;\ \frac{\partial W(\gamma_l)}{\partial l} \le 0\ ;\ \forall\ \gamma_k\ ,\ \gamma_l \tag{8}$$

given that k and l are sufficiently large to be considered continuous. In words, what has been figured out is that there appears a portfolio effect in the sense of first-order stochastic dominance as a bank increases the number of issued loans. That means that the probability of low realizations of outcome from the loan business decreases when the loan portfolio of a bank grows. This may also be interpreted as a reduction of a bank's total exposure to credit risk due to pooling of a large number of independent risky loans.

It is worth noting at this point that the portfolio effects which appear in the present paper differ from similar effects in the standard literature. In particular, the arguments of Diamond (1984), Williamson (1986), or Krasa and Villamil (1992a) and Krasa and Villamil (1992b) apply portfolio effects based on the law of large numbers or the large deviation principle, respectively, to figure out the value of banks. For this reasoning, however, banks need to be very large since portfolio effects in these papers only arise in the limit. This is not the case in the context of the present paper. A portfolio effect in the sense of first-order stochastic dominance occurs even when a bank is small. Hence, the value of a bank depends on the cost of taking deposits. Therefore, the optimal size of a bank in the present model may be determined by explicitly trading off benefits and costs of banking which was not possible in the literature so far.

In order to determine the cost of deposit taking the relationship between the size of the loan portfolios k and l and the optimal payments T_{D0} and P_{D0} in the banks' deposit contracts, respectively, need to be examined. Unfortunately, by applying the implicit function theorem to lenders' participation constraints (5) and (6) one is not able to find an unambiguous relationship. Rather, there appears a tradeoff of two effects. In this regard the portfolio effect, in general, reduces banks' credit risk exposure which is also beneficial from lenders' point of view. Since the probabilities of low outcomes from the banks' loan business decrease due to the portfolio effect, lenders' risk of low repayments when entering into deposit contracts also decreases. Hence, banks are able to reduce risk premia – note that lenders behave risk-averse – and therefore T_{D0} and P_{D0} might decrease when k and l are increased, respectively. However, because of $k = d - \Theta$ and $l = e + \Theta$, for given Θ k and l can increase only if the banks take more deposits d and e. For this purpose, lenders' expected utility from entering into deposit contracts needs to increase which means that banks need to raise T_{D0} and P_{D0}. Hence, the net effect depends on the relative strength of both effects just explained.

However, since the present paper aims at an examination of an interior optimum for k and l it is obvious that the total effect of increasing the volume of loans cannot be negative. If this would be the case, increasing k and l would allow to reduce T_{D0} and P_{D0}, respectively, which reduces banks' costs of deposit taking. At the same time, due to (7) expected profits of both banks would increase. In this situation further increasing the volumes of loans is beneficial. Therefore, an optimum is arrived only if

$$\frac{\partial T_{D0}}{\partial k} \geq 0 \; ; \; \frac{\partial P_{D0}}{\partial l} \geq 0. \tag{9}$$

The cost of deposit taking of both banks are, in addition, affected by the CDS contract. In particular, the premium Θ might shift T_{D0} and P_{D0} whereas the payments $R(Y_s, X_l)$ $\forall$ (Y_s, X_l) might furthermore shift the optimal level T_{D0}.[9] In this regard, applying the implicit function theorem to lenders' participation constraints (5) and (6) yields

$$\frac{\partial T_{D0}}{\partial \Theta} > 0 \text{ and } \frac{\partial P_{D0}}{\partial \Theta} < 0. \tag{10}$$

The reason for this is that a higher Θ means to increase (decrease) the volume of deposits (loans) of B1 in order to hold constant the volume of loans (deposits). However, taking more deposits forces the B1 to increase lenders' expected utility from accepting deposit contracts. In contrast, reducing the volume of loans and leaving constant the amount of deposit contracts hurts the portfolio effect and increases B1's exposure to risk. Risk averse lenders will demand a higher risk premium in this latter situation. Hence, in either case T_{D0} needs to increase when Θ increases. With respect to B2 the converse argument is true: for B2 Θ represents an additional source of income. As a result, B2 may reduce deposit taking in order to issue

[9]Note, P_{D0} of B2 is independent from $R(\cdot)$ due to the subordinated character of the CDS contract.

a certain volume of loans. Hence, P_{D0} may decrease and deposit taking becomes less expensive to B2 when Θ increases.

To figure out the relationship between T_{D0} and $R(\cdot)$ it should, first, be noted that $R(\cdot)$ does not enter constraint (5) directly. Rather, $R(\cdot)$ may be interpreted as an additional source of income of B1. Hence, it affects T_{D0} via the probability distribution function $H(\cdot)$ of $\mathcal{Y}_k = \frac{1}{k}(Y + R(Y_s, X_l))$. It is, therefore, necessary to examine $H(\cdot)$ in more detail. Basically, for this purpose the well known convolution theorem will be applied. However, the approach needs to be modified because Y and $R(Y_s, X_l)$ are stochastically dependent due to Y_s. One, therefore, derives

$$\begin{aligned} H(\gamma_k) \quad = \quad & \int\limits_{\{t_K(y_1),\ldots,t_K(y_{k-s}):\sum_{i=1}^{k-s} t_K(y_i)\leq k\gamma_k\}} f(t_K(y_1))\ldots f(t_K(y_{k-s}))\cdot \\ & \cdot \int\limits_{\{t_K(y_{k-s+1}),\ldots,t_K(y_k):Y_s\leq k\gamma_k-\sum_{i=1}^{k-s} t_K(y_i)\}} f(t_K(y_{k-s+1}))\ldots f(t_K(y_k))\cdot \\ & \cdot F_{R|Y_s}\left(k\gamma_k - \sum_{i=1}^{k} t_K(y_i)\right) dt_K(y_k)\ldots d_K(y_1). \end{aligned} \tag{11}$$

In equation (11) loans $i \in \{s-k+1;\ldots;k\}$ form the underlying of the CDS contract with corresponding total outcome of $Y_s = \sum_{i=k-s+1}^{k} t_K(y_i)$. Moreover, $F_{R|Y_s}$ denotes the conditional cumulative distribution function of $R(\cdot)$ given a certain realization of Y_s. If one, now, considers some realization (y_s, x_l) of (Y_s, X_l) from (11) it is obvious that

$$\frac{\partial H(\gamma_k)}{\partial R(Y_s, X_l)} \leq 0 \ ; \ \forall \ \gamma_k \ ; \ (Y_s, X_l). \tag{12}$$

In this regard, increasing $R(\cdot)$ for any given realization of (Y_s, X_l) apparently reduces the probability that $\mathcal{Y}_k$ falls below some arbitrary threshold γ_k. That is, the payment of the CDS contract reinforces the portfolio effect in B1's loan business. As a result, B1's loan business becomes less risky which is beneficial to lenders and allows to reduce the risk premium implied in B1's deposit contracts. Hence, it must be true that

$$\frac{\partial H(\gamma_k)}{\partial R(Y_s, X_l)} \leq 0 \text{ and } \frac{\partial T_{D0}}{\partial R(Y_s, X_l)} \leq 0 \ ; \ \forall \gamma_k \ ; \ (Y_s, X_l). \tag{13}$$

With these findings in mind, one can proceed with the determination of the optimal CDS contract and optimal decisions on volumes of deposit and loan contracts in the next section.

4 The optimum

The fundamental results outlined in the previous sections allow for an examination of optimal endogenous CDS contracts and optimal volumes of deposits and loans in the following. More formally, analysis of the three-stage game proceeds with an examination of stage 1b and, thereafter, stage 1a. That is, one continues to apply the technique of backward induction to stages 1b and 1a taking into account results at stages 3 through 1c.

4.1 The optimal CDS contract

In order to consider the OTC character of a CDS contract banks' negotiation process is modelled as a cooperative game. The idea is that banks' objective to maximize expected profit also implies maximizing benefits from credit risk transfer. However, banks are free to enter into a CDS contract or not. Hence, the negotiation process will be successful only if either bank is better off with credit risk transfer. Therefore, rational behavior of banks makes for maximizing joint profit from credit risk transfer and sharing proceeds between banks. Given the assumptions of the present model this cooperative game can be solved applying the bargaining solution of Nash (1953).[10]

For this purpose banks determine the premium Θ and payments $R(Y_s, X_l) \; \forall \; (Y_s, X_l)$ of the CDS contract such that the increase of joint profit due to credit risk transfer is maximized. This, however, ist constrained by the fact that any payment $R(Y_s, X_l)$ is bounded above by B2's remaining funds after having payed depositors. Furthermore, payments $R(Y_s, X_l)$ are assumed to be non-negative and premium Θ is strictly positive in an effective CDS contract. Formally this maximization problem can be written as

$$\max_{\Theta, \{R(Y_s,X_l)\}_{(Y_s,X_l)}} [\mathrm{E}(\pi_{1CDS}) - \mathrm{E}(\pi_1)][\mathrm{E}(\pi_{2CDS}) - \mathrm{E}(\pi_2)] \tag{14}$$

$$\text{s.t. } R(Y_s, X_l) \leq l(X_l - P_{D0}(\Theta)) \; \forall \; X_l > P_{D0} \tag{15}$$

$$R(Y_s, X_l) \geq 0 \; \forall \; X_l > P_{D0} \tag{16}$$

$$\Theta > 0. \tag{17}$$

In order to solve the optimization problem optimal volumes of deposit and loans contracts of both banks as well as the optimal underlying of the CDS are considered as given. As a result, one can state and prove

[10] See Pausch (2006, S. 176ff) for a more detailed discussion regarding the application of the Nash bargaining solution.

Proposition 1 *For any given k, l, and s the optimal CDS contract is characterized by the following payment function:*

$$R(Y_s, X_l) = \begin{cases} \min\left\{\min\{st_{K0}; kT_{D0}\} - Y_s; l(X_l - P_{D0}) - \epsilon(\overline{Y_s}(X_l), X_l)\right\} & if() \\ 0 & else \end{cases}$$

with $() = \{X_l > P_{D0};\ Y_s < \min\{st_{K0}; kT_{D0}\}\}$ *and* $\epsilon(\overline{Y_s}(X_l), X_l) \to 0$. *The optimal premium* Θ *ensures an optimal allocation of credit risk to both banks.*

Proof: See the appendix. □

At first glance, the structure of the optimal CDS contract looks quite complex and, hence, some comments seem to be appropriate. In this regard, note first that both banks agree on a kind of default event which triggers a payment of B2 to B1. This is the interpretation of the term $\min\{st_{k0}; kT_{D0}\} - Y_s$. In particular, the total outcome from the underlying of the CDS contract is compared to a threshold which is the minimum of the highest realization of Y_s and B1's depositors' total claims. If Y_s falls short of this threshold, B2 will have to cover exactly the difference. This, of course, requires that B2's funds are enough after paying depositors' claims. Otherwise, B2 will pay a lower amount to B1 in case of default. In this latter situation, note that B2 will never pay all remaining funds to B1. Else, it is obvious from constraint (15) that B2 would end up without any own funds and expected profit of $\mathrm{E}(\pi_{2CDS}) = 0 < \mathrm{E}(\pi_2)$. That is, whenever B2 has to pay some strictly positive $R(\cdot)$ to B1 an arbitrary low amount $\epsilon(\cdot)$ will be retained. Therefore, constraint (15) will never bind for any $X_l > P_{D0}$. At the end, there will be, of course, no strictly positive payment $R(\cdot)$ as long as there is no default at B1 and/or B2's earnings from granting loans do not exceed depositors' claims. As a result, with determining optimal payments $R(\cdot)$ the banks optimize the allocation of credit risk in the present model of a financial system.

But this is only part of the story. The arguments above made clear that the CDS contract allows B1 to get rid of some part of credit risk which is, instead, borne by B2. Bearing this in mind, it is obvious that B1 gains from credit risk transfer whereas B2 might suffer a loss. Therefore, there needs to be a compensation to B2 which optimally allocates benefits from credit risk transfer. This is what determines the optimal premium Θ. Unfortunately, the optimal allocation of benefits from writing a CDS contract depends on a number of parameters just like the probability distributions of investment projects' outcomes and lenders' utility functions. One is, thus, not able to derive more detailed results regarding the optimal level of Θ.

Moreover, results have been derived for given underlying and volumes of deposit and loan contracts of both banks. Therefore, changing one of these parameters might affect $R(\cdot)$ and Θ which makes it necessary to do some comparative statics. However, in order to economize on space results in this regard will be derived intuitively.[11] In this way, it is found out that

$$\frac{\partial R(Y_s, X_l)}{\partial s} \geq 0 \ ; \ \frac{\partial \Theta}{\partial s} \leq 0 \ ; \ \forall (Y_s, X_l). \tag{18}$$

That is, as long as $s < k$ holds increasing s means that there are more loans of B1 included in the underlying of the CDS contract. This causes a portfolio effect in the sense of first order stochastic dominance: the probability of low realizations and, hence, the probability of high payments of B2 decreases. In other words, B2's risk of covering losses from B1's credit risk decreases. This makes the CDS contract more beneficial for B2 but less valuable for B1. As a result, B1 is less willing to enter into a CDS contract and adjustments need to occur to ensure both banks to enter into the contract. For this purpose $R(\cdot)$ needs to increase and / or Θ has to decrease. Furthermore, when $s \geq k$ then there is no change in the underlying and no portfolio effect appears. Therefore, in this situation both banks' benefits remain unchanged when s increases and no adjustments of $R(\cdot)$ and Θ will occur.

Taking a look at the effect of changing B1's volume of loans k on the details of the CDS contract $R(\cdot)$ and Θ, in general, one is not able to derive unambiguous results. The reason for this is that there appears a tradeoff. On the one hand, increasing k reinforces B1's portfolio effect in the sense that low realizations of B1's outcome from granting loans become less likely. On the other hand, in order to increase k B1 needs to take more deposits. For this purpose payments to depositors have to increase which makes B1's costs of deposit taking to increase, too. The net effect, however, depends on the relative strength of both partial effects and general unambiguous conclusions are not available. But if one considers a situation of optimal decisions it is obvious that reducing $R(\cdot)$ and / or rising Θ when k is increased makes the CDS contract less beneficial for B1. That is, when $R(\cdot)$ decreases and / or Θ increases low realizations in B1's earnings become more likely and / or credit risk transfer becomes more expensive. As a result, B1's depositors demand even higher repayment due to higher risk premia which leaves B1 worse off and less willing to enter into a CDS contract. Therefore, in the optimum it must be true that

$$\frac{\partial R(Y_s, X_l)}{\partial k} \geq 0 \ ; \ \frac{\partial \Theta}{\partial k} \leq 0 \ ; \ \forall \ (Y_s, X_l). \tag{19}$$

4.2 Optimizing banking activities and hedging

As explained earlier, the final step in the analysis of banks' behavior when credit derivatives are endogenous regards the examination of stage 1a of the three-stage game. That is, in the following optimal amounts of deposit and loan contracts as

[11] For a formal treatment, the reader may be referred to Pausch (2006, ch. 5.2.5).

well as the optimal underlying of the CDS contract will be determined. For this purpose the previous results need to be considered.

Note first that there is a well-defined relationship between a bank's volumes of deposit and loan contracts. In section 2 above, it was explained that $d = k + \Theta$ and $e = l - \Theta$ holds for B1 and B2, respectively. As a result, given the optimal premium Θ it is sufficient to examine optimal volumes of loans explicitly. The optimal numbers of deposit contracts can, then, be calculated as a residual. Second, from the optimal structure of the CDS contract of Proposition 1 one can conclude

$$\overline{\mathcal{Y}_k} = t_{K0}.$$

In words: B1's maximum average outcome per unit of loans is exactly the repayment obligation of a loan contract which does not default. It will prove useful to keep in mind these observations when, in the following, optimal volumes of loan and deposit contracts and the optimal underlying of the CDS contract is determined.

For this purpose, both banks decide on the optimal number of loan contracts in order to maximize expected profits. In addition, B1 determines the optimal level of s which defines the underlying of the CDS contract. Moreover, in either case lenders' participation constraints (5) and (6) need to be considered. Formally, B1's optimization problem may be written as

$$\begin{aligned} &\max_{k,s} \mathrm{E}(\pi_{1CDS}) \\ \text{s.t.} \quad \frac{k + \Theta(k,s)}{m_1} &\geq G(\mathrm{E}U_{1-nCDS}) \\ k, s &> 0. \end{aligned} \tag{20}$$

From the examination of the optimization problems of both banks one can conclude

Proposition 2 *The optimal volume of loans of the protection buyer (B1) does not decrease due to credit risk transfer. B1's optimal volume of deposits strictly increases due to credit risk transfer.*

Proof: see the appendix.

Proposition 1 states that there is an interrelationship between B1's loan granting and deposit taking activities and credit risk transfer. As can be observed from the proof of Proposition 2 B1's optimal volumes of deposits and loans are driven by a tradeoff. On the one hand, offering more loans allows B1 to increase payment t_{K0} which, in turn increases expected profit – see arguments in section 3. On the other hand, in order to grant more loans B1 needs to take more deposits. Since lenders, however, are willing to buy additional deposits only if their expected utility increases, B1 has to pay more T_{D0} – see arguments of section 3. Optimality, then, appears when marginal profit from extending loan business just outweighs marginal costs from taking more deposits.

This tradeoff is affected by the portfolio effect which occurs because of credit risk transfer. In section 3 it has been shown that due to the payment $R(\cdot)$ of the CDS the pdf of B1's earnings improves in the sense of first-order stochastic dominance. From the lenders' point of view this means that B1's loan business becomes less risky. With risk aversion of lenders this portfolio effect reduces B1's cost of deposit taking since less risk in granting loans allows to reduce risk premium for lenders. In other words, credit risk transfer reduces marginal cost of deposit taking. Hence, the tradeoff explained above results in more deposits and loans with credit risk transfer.

This, however, raises the question of the optimal level of credit risk transfer, ie the optimal underlying of the CDS contract needs to be determined. The examination of B1's optimization problem yields

Proposition 3 *The optimal underlying of the CDS contract is the complete loan portfolio of B1.*

Proof: See the appendix. □

The reasons for this result are obvious if one considers the arguments presented in section 4.1 when the optimal CDS contract was examined. In particular, it has been argued that as long as $s < k$ increasing s allows to increase $R(\cdot)$ and reduce Θ. Both effects are beneficial to B1: increasing $R(\cdot)$ reinforces the risk reducing effect of credit risk transfer and reducing Θ makes using the CDS contract less costly. Therefore, it is optimal to choose the largest underlying possible which is the complete loan portfolio of B1.

At the end, we take a closer look at B2's optimal volumes of deposits and loans. In this regard it is easy to see that due to lenders' participation constraint (6) B2 cannot reduce the volume of deposits. If it did, lenders would suffer a loss of expected utility and would, hence, not longer be willing to accept deposit contracts. Due to $l = e + \Theta$ it is obvious that the premium from the CDS contract will be used to increase loan granting. Therefore, with respect to B2 one can summarize:

Proposition 4 *The protection seller's (B2) optimal volumes of deposits and loans increase due to credit risk transfer.*

4.3 Results and Interpretations

The results presented in the previous sections allow for a number of very interesting conclusions and interpretations. In this regard it should, first, be noted that with Proposition 3 one is able to refine the optimal CDS contract.

Proposition 5 *In the optimum the payment function of the CDS contract is*

$$R(Y_k, X_l) = \begin{cases} \min\{k(T_{D0} - Y_k); l(X_l - P_{D0}) - \epsilon(\overline{Y_k}, X_l)\} & \text{if } (*) \\ 0 & \text{else} \end{cases}$$

with $(*) = \{Y_k < T_{D0}; X_l > P_{D0}\}$ *and* $\epsilon(\overline{Y_k}, X_l) \to 0$.

That is, result 5 generalises and extends the main findings known from the literature. In particular, the industrial organisation approach to banking states that a full hedge is optimal only in a situation when there is no basis risk included in a CDS contract. However, owing to the OTC-character of credit risk transfer the present analysis could show that banks' rational behaviour prevents the appearance of basis risk. The optimal CDS contract always exhibits a perfect correlation with B1's exposure to credit risk. Therefore, a full hedge of B1's credit risk is optimal.

Furthermore, from the inspection of Proposition 5 one observes that the default event in the optimum is kT_{D0}. That is, there is a CDS payment only in the event of B1 not being able to meet its depositors' claims. All variations in B1's profit exceeding this threshold are not covered by the CDS contract. Moreover, the CDS payment just covers the claims of B1's depositors. The effects of this kind of contract design are twofold. On the one hand it insures B1's depositors against losses, which allows a reduction of risk premia due to risk aversion, which, in turn, generates additional profits for B1. On the other hand, the CDS contract generates no adverse incentives to B1. This is true since any positive level of profit still exposes B1 to credit risk. Hence – while not explicitly modelled – B1 will not conduct any hazardous action such as reducing monitoring borrowers.

Keeping in mind the arguments lined out in the previous sections, one is now able to derive a number of conclusions about the role and the effects of credit derivatives. The optimal CDS contract implies the following relationship between both banks' average outcomes from granting loans:

$$\mathcal{X}_l = X_l + \frac{k}{l} Y_k - \frac{k}{l} \mathcal{Y}_k \text{ or } \mathcal{X} = (X + Y) - \mathcal{Y}.$$

That is, the CDS contract in fact merges loan portfolios of both banks synthetically and defines how to share total outcome $(X + Y)$ among both banks. Thus, B1 (B2) actually receives $\mathcal{Y}$ $(\mathcal{X})$. Moreover, realizations of $X + Y$ increase compared with the situation without credit risk transfer. The reason is that both banks increase their optimal volume of loans, which causes t_{K0} and p_{K0} to increase. However, this does not mean that, at the same time, banks are more exposed to credit risk than before they started to use CDS contracts. This can be seen from the cumulative probability distribution function of actual outcomes from both banks' loan business.

The cumulative probability distribution function of B1's actual outcomes was shown to be

$$H(\mathcal{Y}_k) = \int_0^{\mathcal{Y}_k} h_{Y_k}(Y_k) F_{R|Y_k}(k(\mathcal{Y}_k - Y_k)) dY_k$$

above.[12] Moreover, the conditional cumulative distribution function $F_{R|Y_k}(\cdot)$ may be rewritten using the previous results as

$$F_{R|Y_k}(k(\mathcal{Y}_k - Y_k)) = W\left(P_{D0} + \frac{k}{l}(\mathcal{Y}_k - Y_k)\right) = \int\limits_0^{P_{D0}+\frac{k}{l}(\mathcal{Y}_k - Y_k)} w(X_l) dX_l.$$

In other words, the probability of R falling short of $k(\mathcal{Y}_k - Y_k)$ for given Y_k corresponds to the probability of B2's outcome from loan business being less than $P_{D0} + \frac{k}{l}(\mathcal{Y}_k - Y_k)$. Hence, the probability distribution of B1's actual outcome $\mathcal{Y}_k$ is derived from a convolution of random variables Y_k and X_l, which causes a portfolio effect in the sense of first-order stochastic dominance. That means the aggregate level of credit risk decreases due to credit risk transfer. Moreover, since the CDS contract then allocates this reduced credit risk among banks, each bank faces a lower level of risk from granting loans. As a result, granting more loans does not increase credit risk at all. This argument is further supported if one considers B2's earnings' cumulative probability distribution function:

$$\mathcal{W}(\mathcal{X}_l) = 1 - H\left(Y_k - \frac{k}{l}(\mathcal{X}_l - X_l)\right).$$

As is obvious from this latter equation, the cumulative probability distribution function of B2's outcome from granting loans is derived by calculating the convolution of probability distributions of Y_k and X_l.

A notable aspect in this regard is that, with the optimal CDS contract, credit risk transfer becomes effective only in a well-defined set of situations. In particular, a CDS payment $R(Y_s, X_l) > 0$ appears if, and only if, $Y_k < T_{D0}$ and, at the same time, $X_l > P_{D0}$ holds. That is, at first a positive CDS payment requires B1 to be in financial difficulty and, second, B2 simultaneously has excess funds. Therefore, one might conclude that credit risk transfer from B1 to B2 only appears in a situation when B2 is better able to absorb risk than B1, which undoubtedly enhances financial stability.

5 Conclusion

The present paper analyzed the role of Credit Default Swaps (CDS) in banking in a model in which these instruments arise endogenously from optimizing behavior of banks. For this purpose a cooperative Nash bargaining situation between

[12] Note that we have used the result 3 when formulating this equation.

two monopolistic banks was considered in order to account for the over-the-counter character of CDS contracts. Both banks were, moreover, active in loan granting and deposit taking in a market environment with a costly state verification problem. With these assumptions the model also captures interdependencies between writing a CDS contract on the one hand and decision making in banks' deposit and loan business on the other.

The examination of the resulting three-stage game determined a very interesting optimal structure of the CDS contract. It has been shown that the risk buyer only pays a pre-specified amount to the risk seller, when the latter's funds are insufficient to meet own depositors' claims. Moreover, the payment of the risk buyer exactly covers the difference between the risk seller's funds and his depositors' claims. However, counterparty risk – that is, the risk that the risk buyer is not able to make this payment – prevents that this optimal payment is made at any time. Therefore, the optimal up-front premium of the CDS contract needs to be adjusted adequately.

Furthermore, in the model it has been derived that by using CDS contracts both banks' loan granting and deposit taking activities are extended. The reason for this is, that with an optimal CDS contract both banks synthetically merge their loan portfolios. This creates a portfolio effect which reduces a bank's probability to realize low outcomes from the loan business. As a result, the probability of both banks' depositors claims being met increases. This reduces funding costs of both banks and allows them to expand loan granting and deposit taking activities. Hence, credit risk transfer with credit derivatives is valuable.

However, while these results are quite encouraging it should be noted that the model of the present paper abstracts from important aspects of the real market environment. The two banks in the model, for example, are local monopolists. Hence, there are no forces of competition which might affect banks' behavior in both loan and deposit business and credit risk transfer. But Pausch and Schweimayer (2004) argue in their industrial organization approach with exogenous credit derivatives that banks might use derivatives as strategic device in competition for borrowers and lenders. Nevertheless, against the background of the results in the present paper a more realistic model with endogenous markets for credit risk transfer might help better understand issues in this regard.

Another useful extension of the present model would be to consider asymmetric information between participants in credit risk transfer. The recent literature suggests that this might be a very important issue. For example Morrison (2005) and Parlour and Plantin (2005) examine questions in this regard. However, in their papers credit derivatives are, again, exogenous which captures just part of the relevant effects.

Nevertheless, from an empirical point of view recent descriptive data from CDS markets seem to support the main results of the present paper. That is, besides the steadily increasing nominal amount of CDS contracts outstanding one can observe, in particular, growing relevance of portfolio credit default swaps. In addition, banks which participate in credit risk transfer expanded their loan granting activities. Unfortunately, due to reasons of data availability one is not able at present to perform a more detailed empirical analysis. This is particularly true with respect to an examination of banks' total exposure to credit risk after participating in CDS markets.

To sum up, the considerations of the present paper provide a number of important insights into the nature and the role of credit derivatives. The conclusions are unambiguous since by endogenizing the formation of credit derivatives one is able to trade off diversification effects on the one hand and banks' risk taking behavior on the other. In this way, the present paper makes a first attempt to close a gap in the recent literature on credit risk transfer and credit derivatives. However, as is obvious from the discussion above, there is still a number of important aspects which are not analyzed in sufficient detail. Hence, further research on these issues is necessary.

Appendix

Proof of Proposition 1

In order to derive the optimal CDS contract, at first, expected profits of both banks need to be formulated appropriately:

$$\mathrm{E}(\pi_{1CDS}) = \overline{\mathcal{Y}_k}(\Theta, R(\cdot)) - T_{D0}(\Theta, R(\cdot)) - \int_{T_{D0}(\Theta,R(\cdot))}^{\overline{\mathcal{Y}_k}(\Theta,R(\cdot))} H(\mathcal{Y}_k)d\mathcal{Y}_k \quad (21)$$

$$\mathrm{E}(\pi_{2CDS}) = p_{K0} - P_{D0}(\Theta) - \int_{P_{D0}(\Theta)}^{p_{K0}} \mathcal{W}(\mathcal{X}_l)d\mathcal{X}_l \quad (22)$$

where $\mathcal{X}_l = \frac{1}{l}(X - R(Y_s, X_l))$ with corresponding cdf

$$\mathcal{W}(\delta_l) = \int_{Y_s} h_s(Y_s)\left(1 - F_{R|Y_s}(l(X_l - \delta_l))\right) dY_s.$$

Moreover, $h_s(\cdot)$ denotes the pdf of the realizations of the outcome of the CDS's underlying. In addition, in the following $H_{\overline{s}}(\cdot)$ represents the cdf of the returns from the part of B1's loan portfolio which is not included in the underlying of the CDS.

The following arguments consider the fact that constraint (15) of the maximization problem for the optimal CDS is not binding. It is easily verified that otherwise

B2 were not able to earn any positive profit and were, hence, worse off compared to non-participating in CDS. Using this observation, first-order necessary conditions for an optimal CDS are

$$\frac{\frac{\partial T_{D0}}{\partial \Theta}(1 - H(T_{D0}))}{-\frac{\partial P_{D0}}{\partial \Theta}(1 - W(P_{D0}))} = \frac{[\mathrm{E}(\pi_{1CDS}) - \mathrm{E}(\pi_1)]}{[\mathrm{E}(\pi_{2CDS}) - \mathrm{E}(\pi_2)]} \tag{23}$$

$$\frac{\frac{\partial T_{D0}}{\partial R(Y_s,X_l)}(1 - H(T_{D0}))}{h_s(Y_s)\frac{\partial}{\partial R(Y_s,X_l)}F_{R|Y_s}(R(Y_s, X_l))} +$$

$$+ \int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s + R(Y_s, X_l)))d\mathcal{Y}_k \leq \frac{[\mathrm{E}(\pi_{1CDS}) - \mathrm{E}(\pi_1)]}{[\mathrm{E}(\pi_{2CDS}) - \mathrm{E}(\pi_2)]}$$

$$\forall\ R(Y_s, X_l) \geq 0\ ;\ X_l > P_{D0}. \tag{24}$$

The first step of the proof shows $Y_s + R(Y_s, X_l) \leq kT_{D0}\ \forall\ R(Y_s, X_l) > 0$. For this purpose consider realizations (Y_s^b, X_l^b) and (Y_s^m, X_l^m) with $R(Y_s^b, X_l^b) > 0$ and $(Y_s^m, X_l^m) = \arg\max Y_s + R(Y_s, X_l)\ \forall\ R(Y_s, X_l) > 0$. If in this situation

$$kT_{D0} < Y_s^b + R(Y_s^b, X_l^b) < Y_s^m + R(Y_s^m, X_l^m)$$

would hold, it is obvious from condition (24) that

$$\int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^m + R(Y_s^m, X_l^m)))d\mathcal{Y}_k = \int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^b + R(Y_s^b, X_l^b)))d\mathcal{Y}_k$$

needs to be true. This, however, is impossible since

$$k\overline{\mathcal{Y}_k} - (k - s)t_{K0} - (Y_s^b + R(Y_s^b, X_l^b)) > 0$$

implies

$$\int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^m + R(Y_s^m, X_l^m)))d\mathcal{Y}_k < \int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^b + R(Y_s^b, X_l^b)))d\mathcal{Y}_k$$

– a contradiction. Therefore, in the optimum it must be true that $Y_s + R(Y_s, X_l) \leq kT_{D0}\ \forall\ R(Y_s, X_l) > 0$ holds.

This result may be extended to $Y_s + R(Y_s, X_l) \leq st_{K0}\ \forall\ R(Y_s, X_l) > 0$ since there may be a difference between k and s. And, indeed, for the situation $st_{K0} \geq kT_{D0}$ this follows immediately from arguments above. In the situation $st_{K0} < kT_{D0}$, however, arguments need to be modified. If one defines (Y_s^m, X_l^m) and (Y_s^b, X_l^b) just like in the previous case it might be asked whether a situation

$$st_{K0} < Y_s^b + R(Y_s^b, X_l^b) < Y_s^m + R(Y_s^m, X_l^m) \leq kT_{D0}$$

with $R(Y_s^b, X_l^b) > 0$ and $R(Y_s^m, X_l^m) > 0$ may occur. If this was the case, one finds for the second-line terms of (24)

$$\int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^m + R(Y_s^m, X_l^m)))d\mathcal{Y}_k < \int_{T_{D0}}^{\overline{\mathcal{Y}_k}} H_{\overline{s}}(k\mathcal{Y}_k - (Y_s^b + R(Y_s^b, X_l^b)))d\mathcal{Y}_k$$

which is the same as in the previous situation. Furthermore, with replacing $\frac{\partial T_{D0}}{\partial R(\cdot)}$ in first-line terms of (24) by expressions derived from applying the implicit function theorem to lenders' participation constraints (5), one finds

$$\frac{\frac{\partial T_{D0}}{\partial R(Y_s^m, X_l^m)}(1 - H(T_{D0}))}{h_s(Y_s^m)\frac{\partial}{\partial R(Y_s^m, X_l^m)} F_{R|Y_s}(R(Y_s^m, X_l^m))} \leq \frac{\frac{\partial T_{D0}}{\partial R(Y_s^b, X_l^b)}(1 - H(T_{D0}))}{h_s(Y_s^b)\frac{\partial}{\partial R(Y_s^b, X_l^b)} F_{R|Y_s}(R(Y_s^b, X_l^b))}.$$

Since it is obvious from above relationships that in the present situation $R(Y_s^m, X_l^m)$ and $R(Y_s^b, X_l^b)$ cannot satisfy first-order necessary conditions (24) simultaneously, it must be true that – considering the earlier result – in an optimal CDS

$$Y_s + R(Y_s, X_l) \leq \min\{st_{K0}; kT_{D0}\} \; \forall \; R(Y_s, X_l) > 0.$$

Moreover, from the arguments presented above it becomes immediately clear that

$$Y_s + R(Y_s, X_l) = \min\{st_{K0}; kT_{D0}\} \; \forall \; R(Y_s, X_l) > 0 \text{ and } (Y_s, X_l)$$

must hold. Otherwise, first order necessary conditions (24) were not satisfied by all optimal payments $R(Y_s, X_l) > 0$ simultaneously. However, in the case that counterparty risk is striking, ie $R(Y_s, X_l) > l(X_l - P_{D0})$, B2 is not able to pay the optimal payment. But from above arguments it is obvious that B2 will pay as much as possible in this situation. That is, B2's retained funds approach zero in this case: $\epsilon(\overline{Y_s}(X_l), X_l) \to 0$. □

Proof of Proposition 2

To prove Proposition 2 maximization problem (20) is solved using the definition of $\mathrm{E}(\pi_{1CDS})$ in the proof of Proposition 1.

The first-order necessary condition for optimal k, then, is

$$\begin{aligned}
-\left[\frac{\partial T_{D0}}{\partial k} + \frac{\partial T_{D0}}{\partial \Theta}\frac{\partial \Theta}{\partial k} + \sum_{(Y_s, X_l)} \frac{\partial T_{D0}}{\partial R(Y_s, X_l)} \frac{\partial R(Y_s, X_l)}{\partial k}\right](1 - H(T_{D0})) - \\
-\int_{T_{D0}}^{t_{K0}} \left(\frac{\partial H(\mathcal{Y}_k)}{\partial k} + \sum_{(Y_s, X_l)} \frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s, X_l)} \frac{\partial R(Y_s, X_l)}{\partial k}\right) d\mathcal{Y}_k + \\
+\lambda_{KG1}\frac{1}{m_1}\left(1 + \frac{\partial \Theta}{\partial k}\right) &= 0.
\end{aligned}$$

Now, let k denote the optimal volume of loans when B1 enters the CDS contract and k' the optimal volume of loans when there is no credit risk transfer. Suppose

$k < k'$, then for the first-order necessary condition one yields

$$-\left[\frac{\partial T_{D0}}{\partial k}|_{k'} + \frac{\partial T_{D0}}{\partial \Theta}\frac{\partial \Theta}{\partial k}|_{k'} + \sum_{(Y_s,X_l)} \frac{\partial T_{D0}}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k'}\right](1 - H(T_{D0}))-$$

$$-\int_{T_{D0}}^{t_{K0}} \left(\frac{\partial H(\mathcal{Y}_k)}{\partial k}|_{k'} + \sum_{(Y_s,X_l)} \frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k'}\right) d\mathcal{Y}_k +$$

$$+\lambda_{KG1}\frac{1}{m_1}\left(1 + \frac{\partial \Theta}{\partial k}|_{k'}\right) \;<\; 0.$$

In this latter equation replace $\frac{\partial T_{D0}}{\partial k}$ and $\frac{\partial T_{D0}}{\partial \Theta}$ by the terms which can be derived from applying the implicit function theorem to lenders' participation constraint (5). Rearranging terms, then, yields

$$g(\cdot)\lambda_{KG1}\left(\frac{\partial \mathrm{E}U([T_D(\mathcal{Y}_k)])}{\partial k}|_{k'} + \frac{\partial \mathrm{E}U([T_D(\mathcal{Y}_k)])}{\partial \Theta}\frac{\partial \Theta}{\partial k}|_{k'}\right) -$$

$$-\sum_{(Y_s,X_l)} \frac{\partial T_{D0}}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k'}(1 - H(T_{D0}))-$$

$$-\int_{T_{D0}}^{t_{K0}} \left(\frac{\partial H(\mathcal{Y}_k)}{\partial k}|_{k'} + \sum_{(Y_s,X_l)} \frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k'}\right) d\mathcal{Y}_k +$$

$$+\frac{1}{m_1}(\lambda_{KG1} - \lambda_{KG1})\left(1 - \frac{\partial \Theta}{\partial k}|_{k'}\right) \;<\; 0.$$

In this latter inequation the last line diminishes since λ_{KG1} and λ_{KG1} are always the same. Moreover, from the arguments presented in section 3 above it is well known that in the optimum $\frac{\partial R(Y_s,X_l)}{\partial k}|_k \geq 0 \; \forall \; (Y_s, X_l)$, $\frac{\partial \Theta}{\partial k}|_k \leq 0$, $g(\cdot), \lambda_{KG1} > 0$, $\frac{\partial}{\partial k}\mathrm{E}U([T_D(\mathcal{Y}_k)]) > 0$, $\frac{\partial}{\partial \Theta}\mathrm{E}U([T_D(\mathcal{Y}_k)]) < 0$, $\frac{\partial T_{D0}}{\partial R(Y_s,X_l)} \leq 0 \; \forall \; (Y_s, X_l)$, $\frac{\partial}{\partial R(Y_s,X_l)}H(\mathcal{Y}_k) \leq 0$, and $\frac{\partial}{\partial k}H(\mathcal{Y}_k)|_k \leq 0$. Therefore, one derives

$$g(\cdot)\lambda_{KG1}\left(\frac{\partial \mathrm{E}U([T_D(\mathcal{Y}_k)])}{\partial k}|_{k} + \frac{\partial \mathrm{E}U([T_D(\mathcal{Y}_k)])}{\partial \Theta}\frac{\partial \Theta}{\partial k}|_{k}\right) -$$

$$-\sum_{(Y_s,X_l)} \frac{\partial T_{D0}}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k}-$$

$$-\int_{T_{D0}}^{t_{K0}} \left(\frac{\partial H(\mathcal{Y}_k)}{\partial k}|_{k} + \sum_{(Y_s,X_l)} \frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial k}|_{k}\right) \;>\; 0$$

which contradicts first-order necessary condition above. Hence in the optimum $k \geq k'$ must be true. Furthermore, from this result and $d = k + \Theta$ it is obvious that $\Theta > 0$ implies a strictly increasing optimal volume of deposits due to credit risk transfer. □

Proof of Proposition 3

From optimization problem (20) one can derive the first-order necessary condition for the optimal s

$$
\begin{aligned}
-\left[\frac{\partial T_{D0}}{\partial \Theta}\frac{\partial \Theta}{\partial s} + \sum_{(Y_s,X_l)} \frac{\partial T_{D0}}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial s}\right] - & \\
-\int_{T_{D0}}^{t_{K0}} \sum_{(Y_s,X_l)} \frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s,X_l)}\frac{\partial R(Y_s,X_l)}{\partial s} d\mathcal{Y}_k \quad &= \quad 0
\end{aligned}
$$

where we have used the fact that Proposition 2 implies that the participation constraint is not binding.

From this latter equation, it can be easily shown that $s < k$ cannot be an optimal solution. The reason is that, owing to $\frac{\partial T_{D0}}{\partial \Theta} > 0$, $\frac{\partial T_{D0}}{\partial R(Y_s,X_l)} \leq 0$, $\frac{\partial H(\mathcal{Y}_k)}{\partial R(Y_s,X_l)} \leq 0$, and $1 - H(T_{D0}) > 0$ both terms on the left hand side of the equation are positive with $s < k$. Moreover, since any $s > k$ does not create a further effect of first-order stochastic dominance, any CDS contract with $s > k$ needs to include the same payments $R(\cdot)$ and premium Θ as does a contract with $s = k$. Therefore, the optimal underlying covers the complete loan portfolio of B1. $\square$

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