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Time dynamic and hierarchical dependence modelling of an aggregated portfolio of trading books – a multivariate nonparametric approach

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Abstract

From a banking supervisory perspective, this paper analyses aspects of market risk of an aggregated trading portfolio comprised of the trading books of 11 German banks with a regulatory approved internal market risk model. Based on real, clean profit and loss data and Value-at-Risk estimates of the 11 banks, the paper specifically models and analyzes the portfolio's dependence and diversification structure, indispensable for financial stability studies. The high sensitivity of market risk measurements with respect to the dependence structure of the underlying portfolio is nowadays a well-known fact. However, only few techniques for high-dimensional and hierarchical dependence analysis have been proposed and studied in the financial literature so far. One reason is certainly the increasing complexity of the statistical theory, which is commonly referred to as the curse of high-dimensionality. The present paper develops and applies multidimensional (asymptotic) test statistics based on the copula theory with the aim of detecting significant long-term level changes in the supervisory portfolio's dependence over time. Furthermore, a statistical hypothesis test is proposed to identify the distinct contributions of sub-portfolios towards the overall dependence level in a hierarchical manner. The utilized techniques are distribution-free and, in particular, are invariant with respect to the marginal return distributions.

Keywords: Multivariate dependence modelling, multivariate Spearman's rho, time-varying copula, asymptotic test theory, hierarchical testing, control chart theory

JEL classification: C12, C13, C14

Non-technical summary

The dependence structure between the returns of financial assets often changes during market crisis towards a higher degree of co-movement. As a consequence, the profits and losses (P&L) across banks may exhibit stronger co-movement, too.

In this paper, two test procedures are developed in order to analyze the multivariate dependence structure between financial asset returns. In particular, we investigate the dependence structure between the daily P&L's of 11 German banks during the period from 2001 to 2006, while the corresponding daily Value-at-Risk estimates of the banks are also included in the analysis. First, we discuss a test procedure which is designed to detect possible structural change points of the dependence between the banks' P&L's over time, that is, we analyze those points in time where the dependence has changed significantly. Second, another test procedure determines those groups of banks that have distinctively contributed to the change of the dependence structure in the event of a structural change point. In order to quantify the dependence structure, we use the dependence measure Spearman's rho. In contrast to the common correlation coefficient, Spearman's rho represents a nonparametric dependence measure. The test procedures are thus quite general and, for example, no assumptions about the underlying marginal distributions have to be made.

By means of the developed test procedures, we are able to identify three structural change points of the dependence between the banks' P&L's. In April 2002, the high level of dependence observed after the events of September 11 is significantly decreasing. Along with the uncertainty and the sideward trends of the European and US financial markets at the beginning of the year 2004, the dependence significantly increases in February 2004. The clear upward trends of most stock markets in the world from 2005 onwards lead to a decreasing dependence among the banks' P&L's, which turns out to be statistically significant around April 2005. The second test procedure implies that in April 2002, for example, the structural change in dependence is driven more by the large banks.

Nichttechnische Zusammenfassung

In Krisenzeiten beobachtet man häufig eine Änderung der Abhängigkeitsstruktur der Renditen von Finanzanlagen; typischerweise bewegen sich die Renditen verstärkt gemeinsam in eine Richtung. Dies kann dazu führen, dass auch die Eigenhandelsergebnisse (Profits & Losses, kurz P&L) der Banken untereinander sich stärker gemeinsam in eine Richtung bewegen.

In diesem Paper werden zwei statistische Tests entwickelt, welche die multivariate Abhängigkeitsstruktur der Renditen analysieren. Im Speziellen untersuchen wir die Abhängigkeitsstruktur der täglichen P&L elf deutscher Banken über den Zeitraum von 2001 bis 2006. Zusätzlich zur P&L der Handelsbücher beziehen wir auch die täglichen Value-at-Risk Schätzungen der Banken in die Analyse ein. Zum einen stellen wir einen Test auf mögliche Strukturbrüche der Querschnittsabhängigkeit der P&L-Daten im zeitlichen Verlauf vor, d.h. der Test untersucht Zeitpunkte, an denen sich die Abhängigkeitsstruktur der P&L zwischen den Banken signifikant geändert hat. Zum anderen wird in einem weiteren Test diejenige Gruppe von Banken bestimmt, welche zum Zeitpunkt des Strukturbruchs maßgeblich zur Änderung der Abhängigkeit beigetragen haben. Zur Quantifizierung der Querschnittsabhängigkeit wird das Abhängigkeitsmaß Spearman's rho verwendet. Anders als der übliche Korrelationskoeffizient stellt Spearman's rho ein nichtparametrisches Abhängigkeitsmaß dar. Die entwickelten Tests sind somit allgemeiner Natur, und es müssen zum Beispiel keine Annahmen über die zugrunde liegenden Randverteilungen getroffen werden.

Mittels der Testverfahren können drei Strukturbrüche im Beobachtungszeitraum identifiziert werden. Die nach den Ereignissen des 11. Septembers 2001 hohe Querschnittsabhängigkeit der P&L- Daten fällt im April 2002 signifikant. Im Zuge der Seitwärtsbewegungen an den europäischen und amerikanischen Aktienmärkten zu Beginn des Jahres 2004 steigt die Abhängigkeit im Februar 2004 wieder signifikant an. Die Aufwärtsbewegungen der meisten Aktienmärkte in der Welt von 2005 an führen zu einer Verringerung der Querschnittsabhängigkeit, die sich im April 2005 als signifikant erweist. Das zweite Testverfahren zeigt, dass zum Beispiel im April 2002 der Strukturbruch der Querschnittsabhängigkeit hauptsächlich durch die größeren Banken ausgelöst wurde.

Contents

Time dynamic and hierarchical dependence modelling of an aggregated portfolio of trading books – a multivariate nonparametric approach[1]

1 Introduction

The analysis of the dependence structure of a portfolio of risky assets has attracted increasing interest in the scientific literature as well as among practitioners over the last decade. First, the globalizing and interdependence of financial markets require thorough portfolio risk modelling and management that can react quickly to changing market situations. This is particularly important when market conditions deteriorate and the dependence between asset returns increases – which is also known as the 'correlation breakdown', see, for example, Karolyi and Stulz (1996), Longin and Solnik (2001), Campbell et al. (2002), Bae et al. (2003), Patel (2005), Minderhoud (2006), Rodriguez (2007), or Bartram et al. (2007) – and a proper understanding of the portfolio's cross correlation structure and diversification may be essential to preserve the financial stability of a bank or even an entire banking sector. Simultaneously, the rising awareness of modelling portfolio dependence may certainly be put down to the recent market turbulence in the entire financial sector. A number of empirical studies such as Duellmann et al. (2007) show, for example, that the Value-at-Risk (VaR) – the best-established measure for assessing the market risk in a portfolio of risky assets – reacts with particular sensitivity towards any changes of the cross-correlation structure in the portfolio. It is thus essential for every financial or supervisory institution to properly understand, model, and analyze the dependence structure in a given portfolio of interest.

Based on real profit and loss (in short: P&L) data and associated VaR estimates from 11 German banks having a regulatory approved internal market-risk model, the paper is written from the perspective of a supervisor. The supervisor aggregates the respective bank trading portfolios into a supervisory portfolio and analyzes the inherent systemic market risk. Specifically, the paper's main focus lies on modelling and analyzing the portfolio's

[1] The authors would like to thank Thilo Liebig and Friedrich Schmid for their support and stimulating discussions. They gratefully acknowledge the hospitality and support of the Deutsche Bundesbank. The views expressed in this paper represent the authors' personal opinions and does not necessarily reflect the views of the authors' associated institutions.

dependence structure both over time and across banks. It represents one of the few papers that are based on real P&L data and VaR forecasts from banks having a regulatory approved internal market-risk model. Berkowitz and O'Brien (2002) analyze the daily VaR forecasts and corresponding P&L series of six large US banks to evaluate the performance of the banks' VaR models, while Jaschke et al. (2003) provide a comparable analysis of VaR forecasts and P&L series of 13 German banks having a regulatory approved internal market model in the year 2001. The present paper is a sequel to Memmel and Wehn (2006), who focus on the VaR estimation of a supervisory portfolio by using different cross-correlation estimates under the assumption of normally distributed asset returns. By contrast, we utilize copula theory which allows for a more sophisticated analysis of the portfolio's dependence structure.

Copula techniques are frequently applied in the quantitative finance literature. We mention Embrechts et al. (2001), McNeil et al. (2005), O'Kane and Schloegl (2005), Charpentier (2006), Patton (2006), Savu and Trede (2008), and Giacomini et al. (2008). We advocate bivariate or multivariate versions of the dependence measure, Spearman's rho, in order to quantify (the time variation of) the portfolio's dependence structure (cf. Schmid and Schmidt (2007)). Being the best-known copula-based dependence measure in econometrics, Spearman's rho represents the natural alternative to Pearson's correlation coefficient. The latter is known to be less appropriate when dealing with non-normal or, more generally, non-elliptical distributions; cf. Embrechts et al. (2002) for a list of possible pitfalls. Regarding the time-dynamic analysis of the dependence structure, our particular interest from the supervisory perspective lies in detecting long-term level shifts of dependence over time. As a result, possible level shifts should be detected as soon as new information arrives. We develop a two-step test procedures which takes both aspects into account. It is of sequential form and is based on the concept of control charts. For an overview of control chart theory and of existing types of control charts; we refer to Schmid and Knoth (2001) and Wieringa (1999). There is a large literature on detecting structural changes in time series; we refer to Pawlak et al. (2004) and references therein. For example, Steland (2002) and Golosnoy and Schmid (2007) utilize control chart techniques for financial risk and portfolio analysis. Furthermore, we establish an (asymptotic) hypothesis test to analyze the hierarchical dependence structure at some point in time. The proposed statistical test procedures and the dependence measures have the following advantages: They are based solely on the copula and, thus, invariant with respect to the marginal distribution functions. They support a nonparametric approach, i.e. they are free of any distributional assumption. Based on multivariate techniques, they also allow for a dependence analysis in high dimensions.

The rest of the paper is organized as follows. Section 2 describes the modelling approach of the supervisory portfolio and introduces the notion of multivariate Spearman's rho to measure portfolio dependence. In Section 3, the test procedures are developed and the relevant theoretical results on Spearman's rho established. A nonparametric bootstrap method is also discussed, which helps in the estimation and testing process. The theoretical findings are applied to the supervisory portfolio in Section 4.

2 Modelling the portfolio dependence structure

2.1 The data

The empirical analysis is based on P&L data and VaR forecasts of the trading book of these 11 German banks which had a regulatory approved internal market risk model during the period from January 2001 to December 2006. The data, which are available on a daily basis, are maintained by the banks and reported in the Basel II framework to the supervisor; altogether we have $1,435$ observations. Throughout, we consider clean P&L data which, in contrast to the economic P&L of a trading book, do not take commissions and fees, intra-day gains and losses into account. According to regulations, the VaR forecasts are calculated at a confidence level of 99% and for a one-day horizon. Note that the data for the year 2007 onwards have not been disclosed yet.

2.2 The standardized profits and losses

For each bank $i \in \mathbb{N}, i \in \{1, \dots, d\}$, the daily clean P&L of the trading book at discrete time t is modelled by a random variable $G_{t,i}$. Since we do not consider economic P&L in this paper, we shortly refer to the $G_{t,i}$ as the P&L. Suppose $w_{t,i} = (w_{t,i}^1, \dots, w_{t,i}^m)'$ represents the positions of bank i on m financial instruments whose corresponding prices at time t are modelled by the random vector $P_t = (P_t^1, \dots, P_t^m)'$. Then $G_{t,i}$ takes the form

$$G_{t,i} = \sum_{j=1}^{m} w_{t-1,i}^j (P_t^j - P_{t-1}^j), \quad i = 1, \dots, d. \tag{1}$$

A central objective of a bank's internal risk model is to analyze and predict the future P&L distribution of the trading book by taking all past information into account. If the information flow available up to time t is modelled by the σ-algebra $(\mathcal{F}_{t,i})_{t\geq 0}$, $i = 1, \ldots, d$, the interest lies thus in determining the conditional distribution function of $G_{t,i}$, denoted by $F_{t,i}(x) = \mathbb{P}(G_{t,i} \leq x|\ \mathcal{F}_{t-1,i})$. If not stated otherwise, w.l.o.g. we assume that $F_{t,i}(x)$ has infinite support. The Value-at-Risk (VaR) at confidence level α, $V_{t,i}$, is then obtained as the $(1-\alpha)$-quantile of $F_{t,i}$, i.e.

$$V_{t,i} = F_{t,i}^{-1}(1-\alpha), \quad i = 1, \ldots, d. \tag{2}$$

As already mentioned above, we consider data $(G_{t,i}, V_{t,i}), i = 1, \ldots, d$, which arise within a regulatory approved internal market-risk models. Hence, α is set to 0.99 in line with the supervisory requirement for approval of internal market-risk models. Note that in this setting, VaR is a negative number. For background reading on the VaR, we refer, for instance, to Artzner et al. (1999) and Jorion (2006).

Assume that the random vector $\mathbf{G}_t = (G_{t,1}, \ldots, G_{t,d})'$, with $G_{t,i}$ as defined in (1), represents the set of bank P&Ls in the supervisory portfolio at time t. Consider the random vector $\mathbf{S}_t = (S_{t,1}, ..., S_{t,d})'$ defined by

$$\mathbf{S}_t = \Big(-\frac{G_{t,1}}{V_{t,1}}, \ldots, -\frac{G_{t,d}}{V_{t,d}} \Big)' = -\mathbf{V}_t \mathbf{G}_t \tag{3}$$

with diagonal matrix $\mathbf{V}_t = diag\{\frac{1}{V_{t,1}}, \ldots, \frac{1}{V_{t,d}}\}$. The P&Ls are standardized by dividing each bank's P&L by the respective VaR forecast for that day; $\mathbf{S}_t$ is therefore commonly referred to as the standardized P&L or returns at time t. As shown later, for our purposes it suffices to concentrate on the modelling of $\mathbf{S}_t$.

Remark. The standardization in (3) is motivated by the following considerations: If, conditional on the information $\mathcal{F}_{t-1,i}$, the $G_{t,i}$ are normally distributed, i.e. $G_{t,i} \mid \mathcal{F}_{t-1,i} \sim N(0, \sigma^2_{t,i})$, the standardized returns $S_{t,i}$ take the form $S_{t,i} = -G_{t,i}/V_{t,i} = -\{\Phi^{-1}(1-\alpha)\}^{-1} G_{t,i}/\sigma_{t,i}$. That is, the standardization is with respect to the P&L's time-varying volatility in this case and any temporal dependence of the $G_{t,i}$ which is induced by a time-varying volatility (e.g. if asset prices follow a GARCH model) is removed. The standardized returns usually serve as a basis for the validation of a bank's VaR model, see , for example, Jaschke et al. (2003).

Consider the (conditional) joint distribution function $F_{t,\mathbf{S}_t}(\mathbf{x}) = \mathbb{P}(\mathbf{S}_t \leq \mathbf{x}|\ \mathcal{G}_{t-1})$ of $\mathbf{S}_t$ with marginal distribution functions $F_{t,\mathbf{S}_{t,i}}(x) = \mathbb{P}(S_{t,i} \leq x|\ \mathcal{G}_{t-1}),\ i = 1, \ldots, d$. Here, the σ-algebra $(\mathcal{G}_t)_{t\geq 0}$ represents the information flow available up to time t to the supervisor.

Observe that, in general, $\mathcal{G}_t$ does not coincide with $\mathcal{F}_{t,i}$, $i = 1, \ldots, d$. Below, we always consider conditional distribution functions, taking with respect to the σ-algebra $(\mathcal{G}_t)_{t\geq 0}$ since we take the position of the supervisor; however, we will omit the condition for notational reasons. We make the following two assumptions on $\mathbf{S}_t = (S_{t,1}, ..., S_{t,d})'$, which are also supported by empirical analysis (see Section 4):

(A1) The standardized returns $\mathbf{S}_t$ are independent over time. Furthermore, for every $i = 1, \ldots, d$, the marginal distribution functions $F_{t,\mathbf{S}_{t},i}$ are continuous and $S_{t,i}$ is identically distributed for all t; it follows that for every i we have $F_{t,\mathbf{S}_t,i}(x) = F_{\mathbf{S}_t,i}(x)$.

(A2) The hypothesis of identical marginal distribution functions $F_{\mathbf{S}_t,i}, i = 1, \ldots, d$, is rejected, i.e. $F_{\mathbf{S}_t,i} \neq F_{\mathbf{S}_t,j}$ for at least one pair (i, j) with $i \neq j$ and $i, j = 1, \ldots, d$.

Assumption (A2) requires an individual modelling of the marginal distribution functions $F_{\mathbf{S}_t,1}, \ldots, F_{\mathbf{S}_t,d}$ in order to quantify the risk arising from each single bank as accurately as possible. Thus, using the concept of copulas, we assume that $F_{t,\mathbf{S}_t}$ takes the form

$$F_{t,\mathbf{S}_t}(\mathbf{x}) = C_t^S(F_{\mathbf{S}_t,1}(x_1), \ldots, F_{\mathbf{S}_t,d}(x_d)), \quad \text{for all } \mathbf{x} \in \mathbb{R}^d, \tag{4}$$

with time-varying copula C_t^S, which uniquely exists according to Sklar's Theorem (Sklar (1959)) if the $F_{\mathbf{S}_t,i}$ are continuous functions. The copula C_t^S splits the joint distribution function into the univariate marginal distribution functions and the time-varying dependence structure represented by C_t^S. Hence, estimation of $F_{t,\mathbf{S}_t}$ breaks down into the estimation of the margins $F_{\mathbf{S}_t,1}, \ldots, F_{\mathbf{S}_t,d}$ and the copula C_t^S. See Nelsen (2006) for a general overview on copulas. An elaboration of time-varying copulas in the context of Value-at-risk calculations can, for example, be found in Giacomini et al. (2008).

It is important to note that the standardization of the P&L in (3) does not change the dependence structure between the banks' P&Ls, as shown in the next corollary.

Corollary 1 *Suppose* $\mathbf{S}_t$ *has joint distribution function* $F_{t,\mathbf{S}_t}$ *with copula* C_t^S *as in* (4) *and assume that* $\mathbf{G}_t$*, defined in (1), has joint distribution* $F_{t,\mathbf{G}_t}$ *with copula* C_t^G *and continuous marginal distribution functions* $F_{t,i}(x)$ *and infinite support. Then, conditioned on the information up to time* $t-1$, $C_t^S = C_t^G$*, i.e. the standardization of* $\mathbf{G}_t$ *does not change the dependence structure represented by the copula* C_t^G*.*

Proof. Using the notation in (3) and the fact that $V_{t,i}$ is strictly negative, the transformation function $\alpha_{t,i}(x) = -x/V_{t,i}$ is, conditioned on the information up to time $t-1$, strictly increasing. According to Theorem 2.7 in Embrechts et al. (2001), each copula is invariant with respect to strictly increasing transformations of the marginal distributions. □

2.3 The dependence measure Spearman's rho

We use the dependence measure Spearman's rho in order to quantify the dependence structure of the supervisory portfolio, which constitutes a direct functional of the copula. As such, not only does it allow concentrating on the standardized returns $\mathbf{S}_t$ for an analysis of the dependence structure between the banks according to Corollary 1, it also captures the time variation of the portfolio dependence structure represented by the copula. A multivariate generalization of (bivariate) Spearman's rho makes it possible to measure the portfolio's dependence structure by either one single 'key' parameter referring to the entire portfolio or by multiple lower-dimensional parameters referring to sub-portfolios.

We define Spearman's rho in a general setting and a time-static setting first. Consider the d-dimensional random vector $\mathbf{X} = (X_1, ..., X_d)'$ with continuous joint distribution function F and continuous univariate marginal distribution functions $F_i, i = 1, \ldots, d$. Think of $\mathbf{X}$ as representing the returns of d assets in a portfolio. Furthermore, $F(\mathbf{x}) = C(F_1(x_1), ..., F_d(x_d))$ with unique copula C. Bivariate Spearman's rho of the two-dimensional random vector (X, Y) with copula C can be written as

$$\rho = 12 \int_0^1 \int_0^1 C(u, v) du dv - 3. \tag{5}$$

A natural d-dimensional extension of Spearman's rho for the d-dimensional random vector $\mathbf{X}$ is given by

$$\rho_{d,\mathbf{X}} = h(d) \Big\{ 2^d \int_{[0,1]^d} C(\mathbf{u}) \, d\mathbf{u} - 1 \Big\} \tag{6}$$

with $h(d) = \frac{d+1}{2^d - (d+1)}$. This version was originally discussed in Ruymgaart and van Zuijlen (1978); we also refer to Wolff (1980), Joe (1990), and Nelsen (1996). Later we also consider sub-portfolios i.e. we are interested in quantifying the dependence structure of a portfolio consisting of only those X_i where $i \in \mathcal{I}$ with index set $\mathcal{I} \subseteq \{1, \ldots, d\}$ and cardinality $2 \leq |\mathcal{I}| \leq d$. Analogously to (6), we define the $|\mathcal{I}|$-dimensional Spearman's rho as

$$\rho_{|\mathcal{I}|,\mathbf{X}} = h(|\mathcal{I}|) \Big\{ 2^{|\mathcal{I}|} \int_{[0,1]^{|\mathcal{I}|}} C_{i_1, \ldots, i_{|\mathcal{I}|}}(\mathbf{u}) \, d\mathbf{u} - 1 \Big\} \tag{7}$$

with $\mathcal{I} = \{i_1, \ldots, i_{|\mathcal{I}|}\}$. Here, $C_{i_1, \ldots, i_{|\mathcal{I}|}}$ refers to the $|\mathcal{I}|$-dimensional copula which corresponds to the $i_1, \ldots, i_{|\mathcal{I}|}$-margin of C. Obviously, for $\mathcal{I} = \{1, \ldots, d\}$, $\rho_{|\mathcal{I}|,\mathbf{X}} = \rho_{d,\mathbf{X}}$. If it is clear from the context, we will suppress the subindex $\mathbf{X}$.

A nonparametric estimator for $\rho_{|\mathcal{I}|}$ is obtained via the empirical copula, what has been considered in Schmid and Schmidt (2007). We therefore assume that neither F or C nor the marginal distribution functions $F_i, i = 1, ..., d$, of the d-dimensional random vector $\mathbf{X}$ are known. Given a random sample of returns $(\mathbf{X}_j)_{j=1,...,n}$, the empirical copula is defined as

$$\hat{C}_n(\mathbf{u}) = \frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\mathbf{1}_{\{\hat{U}_{ij,n}\leq u_i\}} \quad \text{for} \quad \mathbf{u} = (u_1, \dots, u_d) \in [0,1]^d$$

with $\hat{U}_{ij,n} = \frac{1}{n}$ (rank of X_{ij} in $X_{i1}, \dots, X_{in}$). Empirical copulas were first introduced and studied by Deheuvels (1979) under the name of 'empirical dependence functions'. Note that the estimation is based on the ranks of the observations rather than on the actual observations themselves. This yields the following nonparametric estimator for $\rho_{|\mathcal{I}|}$

$$\widehat{\rho}_{|\mathcal{I}|,n} = h(|\mathcal{I}|)\Big\{\frac{2^{|\mathcal{I}|}}{n}\sum_{j=1}^{n}\Big[\,(1-\hat{U}_{i_1 j,n})\dots(1-\hat{U}_{i_{|\mathcal{I}|}j,n})\Big] - 1\Big\}. \tag{8}$$

Details of the derivation of $\widehat{\rho}_{|\mathcal{I}|,n}$ are provided in Appendix 5; the corresponding estimator for ρ_d follows by setting $\mathcal{I} = \{1, \dots, d\}$.

Observe that another multivariate version of $|\mathcal{I}|$-dimensional Spearman's rho is given by (see, for example, Kendall (1970))

$$\tilde{\rho}_{|\mathcal{I}|} = h(2)\Big\{2^2\sum_{\substack{k<l\\k,l\in\mathcal{I}}}\binom{|\mathcal{I}|}{2}^{-1}\int_{[0,1]^2} C_{kl}(u,v)dudv - 1\Big\} \tag{9}$$

where C_{kl} denotes the bivariate copula which corresponds to the kth and lth margin of C, $k, l \in \mathcal{I}$. $\tilde{\rho}_{|\mathcal{I}|}$ describes the average of all pairwise Spearman's rho coefficients (cf. formula (5)) and a nonparametric estimator can be derived analogously as above via the empirical copula (see Appendix 5 for details). We have

$$\widehat{\tilde{\rho}}_{|\mathcal{I}|,n} = \frac{12}{n}\binom{|\mathcal{I}|}{2}^{-1}\sum_{\substack{k<l\\k,l\in\mathcal{I}}}\sum_{j=1}^{n}(1-\hat{U}_{kj,n})(1-\hat{U}_{lj,n}) - 3. \tag{10}$$

Multivariate Spearman's rho $\widehat{\rho}_{|\mathcal{I}|,n}$ naturally decreases with increasing dimension owing to the increasing dispersion of data points. This is illustrated by Figure 1 as opposed to the average of all pairwise Spearman's rho coefficients $\widehat{\tilde{\rho}}_{|\mathcal{I}|,n}$. At the same time, the variance of both estimators is considerably decreasing with increasing dimensions.

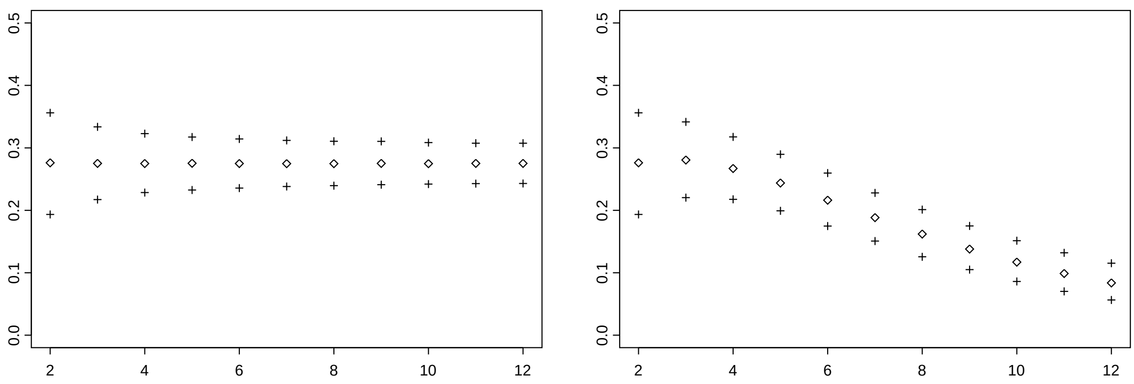


Figure 1: Empirical mean and 95%-confidence interval of the average of all pairwise Spearman's rho coefficients $\widehat{\tilde{\rho}}_{d,n}$ (left) and multivariate Spearman's rho $\widehat{\rho}_{d,n}$ (right) for different dimensions d; the results were obtained by sampling: 10,000 independent random samples of sample size $n = 500$ from a d-dimensional equi-correlated normal distribution with pairwise linear correlation coefficients are all equal to 0.3.

We concentrate below on multivariate Spearman's rho $\rho_{|\mathcal{I}|}$ since it represents a real multivariate measure of concordance according to Joe (1990) and Nelsen (1996)(see also Úbeda-Flores (2005) and Taylor (2007)). In particular, it accounts for multivariate concordance ordering and $\rho_{|\mathcal{I}|} = 0$ if $C = \Pi$ with Π the independence copula. In contrast, $\tilde{\rho}_{|\mathcal{I}|}$ is determined by the two-dimensional marginal copulas only. Above all, $\tilde{\rho}_{|\mathcal{I}|}$ may be zero even if all margins are pairwise stochastically independent. Various tests on independence are, furthermore, often based on such multivariate versions, see, for example, Genest and Rémillard (2004) and references therein. Note that all results established below hold for the version $\tilde{\rho}_{|\mathcal{I}|}$, too.

3 Time-dynamic and hierarchical testing for long-term level shifts of Spearman's rho

The adequate measurement and evaluation represents only a first step in the context of a thorough and comprehensive analysis of a portfolio's dependence structure, allowing for an initial identification of salient features of dependency. However, a second step, verifying whether these effects are really significant, is mandatory.

The aim of this section is therefore twofold. First, we develop a time-dynamic two-step test procedure to detect long-term level changes in portfolio dependence over time, which uses the concept of control charts. Second, an asymptotic hypothesis test is developed to analyze portfolio dependence in a hierarchical manner across all sub-portfolios at a given point in time. We will refer to the latter as hierarchical testing in order to distinguish it from the time-dynamic testing.

Both approaches utilize multivariate Spearman's rho, as introduced in (7), to measure portfolio dependence. Thus, both procedures are free of any distributional assumption and may be applied to general financial portfolios if the assumption of independent *standardized* asset returns over time holds. The next section establishes some basic theoretical results first.

3.1 General concepts

The following two-sample (asymptotic) test distribution for Spearman's rho can be established. It extends results about the asymptotic normality of multivariate Spearman's rho (as defined in (6)) given in Schmid and Schmidt (2007).

Theorem 2 *Consider two stochastically independent random samples* $(\mathbf{X}_s)_{s=1,\ldots,n}$ *and* $(\mathbf{Y}_s)_{s=1,\ldots,m}$ *from the d-dimensional random vectors* $\mathbf{X}$ *and* $\mathbf{Y}$ *with d-dimensional distribution functions* $F_{\mathbf{X}}$ *and* $F_{\mathbf{Y}}$*, continuous marginal distribution functions and copulas* $C_{\mathbf{X}}$ *and* $C_{\mathbf{Y}}$*. Assume that the i-th partial derivatives of* $C_{\mathbf{X}}$ *and* $C_{\mathbf{Y}}$ *exist and are continuous for* $i = 1, \ldots, d$*. Let* J *be the set of all subsets* $\mathcal{I}$ *of* $\{1, \ldots, d\}$*. For* $A \subseteq J$ *with cardinality* $|A| = k$*, suppose that* $S_{A,n,\mathbf{X}}$ *and* $S_{A,m,\mathbf{Y}}$*, respectively, denote the k-dimensional random vectors of all sample versions* $\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}}$ *and* $\widehat{\rho}_{|\mathcal{I}|,m,\mathbf{Y}}$ *of Spearman's rho with* $\mathcal{I} \in A$*, as calculated from the above random samples according to (8). Let also* $\boldsymbol{\rho}_{A,\mathbf{X}}$ *and* $\boldsymbol{\rho}_{A,\mathbf{Y}}$ *be the corresponding vectors of the true values* $\rho_{|\mathcal{I}|,\mathbf{X}}$ *and* $\rho_{|\mathcal{I}|,\mathbf{Y}}$ *of Spearman's rho. We denote by* $||\cdot||$ *an arbitrary matrix norm on the space* $[-1,1]^k$*.*
Under the assumption that $\boldsymbol{\rho}_{A,\mathbf{X}} = \boldsymbol{\rho}_{A,\mathbf{Y}}$ *and with* $m := m(n)$ *such that* $\frac{\sqrt{n}}{\sqrt{m(n)}} \to c$ *for* $n \to \infty$*, we have*

(i) for each $A \subseteq J$ *with* A *being a single set* $\mathcal{I}$

$$\sqrt{n}\Big(\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}} - \widehat{\rho}_{|\mathcal{I}|,m(n),\mathbf{Y}}\Big) \xrightarrow{d} Z \sim N(0,\sigma^2) \quad \text{as } n \to \infty, \tag{11}$$

with variance

$$\sigma^2 = 2^{2|\mathcal{I}|}h(|\mathcal{I}|)^2 \int_{[0,1]^d}\int_{[0,1]^d} \Big[I\!\!E\{\mathbb{G}_{C_{\mathbf{X}}}(\boldsymbol{u}^{(\mathcal{I})})\mathbb{G}_{C_{\mathbf{X}}}(\boldsymbol{v}^{(\mathcal{I})})\} + c^2 I\!\!E\{\mathbb{G}_{C_{\mathbf{Y}}}(\boldsymbol{u}^{(\mathcal{I})})\mathbb{G}_{C_{\mathbf{Y}}}(\boldsymbol{v}^{(\mathcal{I})})\}\Big] d\mathbf{u}d\mathbf{v}. \quad (12)$$

Here, $\mathbf{u}^{(\mathcal{I})}$ *and* $\mathbf{v}^{(\mathcal{I})}$ *refer to the d-dimensional vectors* $\mathbf{u}$ *and* $\mathbf{v}$*, respectively, whose components – except the ones with* $i \in \mathcal{I}$ *– are all 1. Moreover,* $\mathbb{G}_C(\mathbf{u}) = \mathbb{B}_C(\mathbf{u}) - \sum_{i=1}^d D_i(\mathbf{u})\mathbb{B}_C(\mathbf{u}^{(i)})$ *and the process* $\mathbb{B}_C$ *is a tight centered Gaussian process on* $[0,1]^d$ *with covariance function*

$$E\{\mathbb{B}_C(\mathbf{u})\mathbb{B}_C(\mathbf{v})\} = C(\mathbf{u} \wedge \mathbf{v}) - C(\mathbf{u})C(\mathbf{v}).$$

(ii) Furthermore, for each $A \subseteq J$*, it follows that*

$$\sqrt{n}||S_{A,n,\mathbf{X}} - S_{A,m(n),\mathbf{Y}}|| \xrightarrow{w} W \quad \text{as } n \to \infty,$$

with non-degenerated random variable W*.*

The *proof* is given in Appendix 5. □

Next, we derive two results which will be relevant for the time-dynamic considerations of Spearman's rho. In these, a moving window estimation is considered to evaluate Spearman's rho over time.

Theorem 3 *Consider the random sample* $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ *from the d-dimensional random vector* $\mathbf{X}$ *with d-dimensional distribution function* F *and copula* C *whose marginal distribution functions are assumed to be continuous. For an index set* $\mathcal{I} \subseteq \{1,\ldots,d\}$*, let* $\widehat{\rho}^{\,t}_{|\mathcal{I}|,n}$ *denote the corresponding estimator for Spearman's rho at time t as defined in (8), calculated based on an (equally weighted) moving window of size n, i.e. Spearman's rho estimator is based on the sample* $\mathbf{X}_{t-n+1},\ldots,\mathbf{X}_t$*. We have*

(i) for any fixed $s \in \mathbb{N}$ *with* $s < n$*,*

$$n\left(\widehat{\rho}^{\,t}_{|\mathcal{I}|,n} - \widehat{\rho}^{\,t-s}_{|\mathcal{I}|,n}\right) \xrightarrow{d} Z^{t,s}_{|\mathcal{I}|} \quad \text{as } n \to \infty, \quad (13)$$

with non-degenerated centered random variable $Z^{t,s}_{|\mathcal{I}|}$ *which is bounded with* $|Z^{t,s}_{|\mathcal{I}|}| \leq s$*.*

(ii) Furthermore,

$$n^2 \; Cov\Big(\widehat{\rho}^{\,t}_{|\mathcal{I}|,n} - \widehat{\rho}^{\,t-s}_{|\mathcal{I}|,n} \,,\, \widehat{\rho}^{\,t-r}_{|\mathcal{I}|,n} - \widehat{\rho}^{\,t-r-s}_{|\mathcal{I}|,n}\Big) \longrightarrow 0 \quad as\; n \to \infty \tag{14}$$

for fixed $r, s \in \mathbb{N}$ and $n > r > s > 0$. Moreover, the limiting variables $Z^{t,s}_{|\mathcal{I}|}$ and $Z^{t-r,s}_{|\mathcal{I}|}$ are stochastically independent for $n > r > s > 0$.

The *proof* is given in Appendix 5. □

Consider a d-dimensional time series $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ fulfilling assumption (A1) and (A2) as elaborated in Section 2.2 with joint distribution function F_t and copula C_t. The dependence structure of $\mathbf{X}_t$ is described by the copula C_t. In general, it is difficult to perform hypothesis tests on the copula when no further structural assumptions are imposed. However, the presumption of a parametric model would be too restrictive for our purposes. Therefore, we assume that Spearman's rho determines the dependence structure and make the following additional assumption:

(A3) At any time t, the dependence structure is completely described by (an adequate set of $|\mathcal{I}|$-dimensional) Spearman's rho (at time t).

Note that for the majority of parametric families of copulas, there exists a bijective relationship between the copula C and a set of $|\mathcal{I}|$-dimensional Spearman's rho coefficients $\rho_{|\mathcal{I}|}$, which can be illustrated by three examples.

1. Let C be a member of the two-dimensional Farlie-Gumbel-Morgenstern family of copulas with parameter $\theta \in [-1, 1]$. Then, bivariate Spearman's rho ρ equals $\theta/3$ (see Nelsen (2006), p.168).
2. Let C be the d-dimensional Gaussian copula with correlation matrix $\Sigma = (r_{i,j})_{1\le i,j\le d}$. The copula can be fully described by considering the $\binom{d}{2}$-dimensional vector of all bivariate Spearman's rho coefficients $\rho_{i,j}, i < j$. In particular, $\rho_{i,j} = 6/\pi \arcsin(r_{i,j}/2)$.
3. Let C be a four-dimensional hierarchical Archimedean copula which is constructed by coupling the two-dimensional Archimedean copulas $C_{(1)}$ and $C_{(2)}$, generated by the (strict) generators $\phi_{(1)}$ and $\phi_{(2)}$, respectively, using a third (strict) generator $\phi_{(3)}$.

Hence,

$$\begin{aligned}
C(u_1, u_2, u_3, u_4) &= C\{C_{(1)}(u_1, u_2), C_{(2)}(u_3, u_4)\} \\
&= \phi_{(3)}{}^{-1}[\phi_{(3)} \circ \phi_{(1)}{}^{-1}\{\phi_{(1)}(u_1) + \phi_{(1)}(u_2)\} \\
&\quad +\phi_{(3)} \circ \phi_{(2)}{}^{-1}\{\phi_{(2)}(u_3) + \phi_{(2)}(u_4)\}]. \qquad (15)
\end{aligned}$$

Joe (1997), Section 4.2, states the conditions which need to be fulfilled so that construction (15) is a copula function. For example, in case the $\phi_{(i)}, i = 1, 2, 3$ are generators of the Gumbel copula with parameters $\theta_i \geq 1$, then C is a copula if $\theta_3 < \theta_1$ and $\theta_3 < \theta_2$. The dependence structure of C is then completely described by the three-dimensional vector consisting of the pairwise Spearman's rho coefficients $\rho_{(1)}$ and $\rho_{(2)}$ corresponding to the marginal copulas $C_{(1)}$ and $C_{(2)}$, respectively, and multivariate Spearman's rho ρ_4 as defined in (6). We refer to Savu and Trede (2006) and Hofert (2008) for further examples of hierarchical copulas and related estimation and simulation techniques.

3.2 Detecting long-term changes in the level of Spearman's rho over time

This section elaborates a procedure to detect level changes in the portfolio dependence over time by using multivariate Spearman's rho. Specifically, our interest lies in detecting long-term level changes of Spearman's rho which, in addition, should be indicated as early as possible, i.e. as soon as new information has arrived. The procedure is thus of sequential form and consists of two (consecutive) steps, which are illustrated in Table 1: Being based on a control chart for Spearman's rho, Phase 1 sequentially monitors the series in order to detect level shifts of Spearman's rho. As far as possible long-term changes of dependence are concerned, it acts as an early indication or warning system. After having been signalled, a shift in dependence in Phase 1, Phase 2 verifies whether a long-term rather than a short-term change is experienced; naturally, further observations need to be awaited for this purpose. The procedure in Phase 2 is therefore of static, retrospective form and can be regarded as a kind of 'dependence backtesting'.

Let $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ denote a d-dimensional time series fulfilling assumption (A1), (A2), and (A3) as elaborated in Sections 2.2 and 3.1 with joint distribution function F_t and copula C_t. For notational reasons, the description of the two phases is based on the $|\mathcal{I}|$-dimensional Spearman's rho coefficient $\rho^t_{|\mathcal{I}|}$ for arbitrary index set $\mathcal{I} \subseteq \{1, \ldots, d\}$. (Note that the follow-

Table 1: **Setup of test procedure**.

Test type	Test procedure
Early indication system of change of dependence	Phase 1: Control chart for Spearman's rho
Detection of sustainable change of dependence	Phase 2: 'Dependence-backtesting'

ing elaborations could be generalized to the case of an adequate vector of Spearman's rho coefficients, which should be considered otherwise, as discussed above; we also refer to the hierarchical testing in Section 3.3). The corresponding series of estimators of Spearman's rho based on an equally weighted moving window of size n is denoted by $(\widehat{\rho}^{\,t}_{|\mathcal{I}|,n})_{t\in\mathbb{Z}}$.

Phase 1. In this phase, a nonparametric control chart for detecting level changes in multivariate Spearman's rho is developed. The concept of control charts represents one of the most important tools of statistical process control and is often used to implement sequential approaches. According to a predefined decision rule, a control chart compares control statistics with given control limits at each time point. The value of the control statistics is calculated based on the observations of the process to be monitored; if it lies outside of the control limits, the control chart gives a signal, indicating that the monitored process is 'out-of-control' in the sense specified beforehand. See also Golosnoy and Schmid (2007) for monitoring the optimal portfolio weights by means of control charts.

Let us therefore assume that, up to time t', there are no changes in portfolio dependence. Formally, $\rho^{t}_{|\mathcal{I}|} = \rho$ for fixed but unknown parameter ρ and for all $t \leq t'$. We fix a lag parameter $s \in \mathbb{N}, s < n$, which allows the choice of the time frequency (e.g. daily or weekly) for monitoring Spearman's rho. At each time $t = t' + ks, k = 1, 2, 3, \ldots$, we then consider the hypothesis

$$\mathrm{H}_{0,t}: \quad \rho^{t}_{|\mathcal{I}|} = \rho \quad \text{versus} \quad \mathrm{H}_{1,t}: \quad \rho^{t}_{|\mathcal{I}|} \neq \rho. \tag{16}$$

We thereby reject the null hypothesis at time t if

$$n\,(\widehat{\rho}^{\,t}_{|\mathcal{I}|,n} - \widehat{\rho}^{\,t-s}_{|\mathcal{I}|,n}) > c_2 \quad \text{or} \quad n\,(\widehat{\rho}^{\,t}_{|\mathcal{I}|,n} - \widehat{\rho}^{\,t-s}_{|\mathcal{I}|,n}) < c_1 \tag{17}$$

with predefined constant c_1 and c_2. Adopting the terminology of control charts, $Y_t :=$

$n(\widehat{\rho}^{t}_{|\mathcal{I}|,n} - \widehat{\rho}^{t-s}_{|\mathcal{I}|,n})$ represents the control statistics and the control limits are given by c_1 and c_2. The process $\widehat{\rho}^{t}_{|\mathcal{I}|,n}$ is 'in control' as long as the null hypothesis is not rejected; if it is rejected (i.e. the control chart gives a signal), it is concluded that the process is out of control. Note that Y_t represents an (asymptotic) unbiased estimator of $n(\rho^{t}_{|\mathcal{I}|} - \rho^{t-s}_{|\mathcal{I}|})$ if the process is in control, i.e. $\mathbb{E}(Y_t)$ is asymptotically zero in this case. We thus concentrate on sequentially monitoring the mean or the location of the process $\widehat{\rho}^{t}_{|\mathcal{I}|,n}$. In particular, we reject the null hypothesis if Y_t exceeds, or is less than, the level c_2 or c_1, respectively. According to Theorem 3, part (i), Y_t has a limiting distribution if the process is in control. For a given significance level, the control limits $c_i, i = 1, 2$, can be chosen as the respective quantiles of this distribution owing to the second part of Theorem 3 (see also the remark at the end of this section).

Two phases must be distinguished when working with control charts. In a first step, the control chart has to be calibrated, i.e. the control limits c_1 and c_2 must be determined adequately. This is generally done based on a 'pre-sample' of the process, i.e. a sample of past observations. Note that, in order to obtain reasonable control limits, the pre-sample must stem from the in-control process. The second step comprises the actual usage of the control chart to analyze and monitor new observations which are drawn sequentially from the process.

Phase 2. The analysis in Phase 1 aims at identifying shifts of Spearman's rho. If, as from the supervisory perspective, the focus lies in detecting long-term, sustaining (level) changes in portfolio dependence, a second phase is added subsequently to the first phase. In this, we understand by a long-term change that, after the shift indicated by Phase 1, Spearman's rho stays at this level throughout a specified period.

Assume, therefore, that Phase 1 gives a signal at time $t^\star > t^{'}$. Based on further $n^\star = n - s$ observations of the process, Phase 2 compares Spearman's rho over distinct time periods before and after $t^\star$. Specifically, we verify whether there is a significant difference between Spearman's rho calculated based on the periods $[t^\star - n + 1, t^\star - s]$ and $[t^\star + 1, t^\star + n^\star]$. We further assume that Spearman's rho does not change throughout the latter period.

At $t^\star$, we then consider the hypothesis

$$\mathrm{H}_0: \quad \rho^{t^\star + n^\star}_{|\mathcal{I}|} = \rho^{t^\star - s}_{|\mathcal{I}|} \quad \text{versus} \quad \mathrm{H}_1: \rho^{t^\star + n^\star}_{|\mathcal{I}|} \neq \rho^{t^\star - s}_{|\mathcal{I}|},$$

In this context, the statistics

$$T = \frac{\sqrt{n}(\widehat{\rho}^{t^\star+n^\star}_{|\mathcal{I}|,n^\star} - \widehat{\rho}^{t^\star-s}_{|\mathcal{I}|,n^\star})}{\hat{\sigma}^B_{|\mathcal{I}|}} \tag{18}$$

is – under the null hypothesis – asymptotically standard normally distributed according to Theorem 2. Here, $(\hat{\sigma}^B_{|\mathcal{I}|})^2$ represents the consistent bootstrap estimator for the variance of $\sqrt{n}(\widehat{\rho}^{t^\star+n^\star}_{|\mathcal{I}|,n^\star} - \widehat{\rho}^{t^\star-s}_{|\mathcal{I}|,n^\star})$ which is introduced in Section 3.4. We thus reject H_0 at $t^\star$ at level α if $|T| > z_{1-\alpha/2}$, where $z_{1-\alpha/2}$ denotes the $(1-\alpha/2)$-quantile of the standard normal distribution. Note that, by excluding the time point $t^\star$ from the analysis and choosing the window size $n^\star$, it is guaranteed that the above test is independent of Phase 1.

Finally, it is concluded that $\widehat{\rho}^{t}_{|\mathcal{I}|,n}$ is out-of-control at time t if

(B1) both Phase 1 and Phase 2 give a signal at time t and

(B2) the control statistics of Phase 1 and the test statistics T of Phase 2 have the same sign.

We refer to such events fulfilling (B1) and (B2) simply as signals; an event signalled in Phase 1 is called 'alarm' below. The described procedure is applied to the supervisory portfolio in Section 4.3 for $s = 1$ and $\mathcal{I} = \{1, \dots, d\}$.

Remark. Concentrating in Phase 1 on the differences $\widehat{\rho}^{t}_{|\mathcal{I}|,n} - \widehat{\rho}^{t-s}_{|\mathcal{I}|,n}$ rather than on the actual series $\widehat{\rho}^{t}_{|\mathcal{I}|,n}$ itself offers several advantages. First, the latter approach would yield a control statistics involving the parameter ρ, leaving us with an additional parameter to estimate in order to set up the control chart. Furthermore, note that the series $\widehat{\rho}^{t}_{|\mathcal{I}|,n}$ exhibit a very high serial correlation. In the context of a sequential test, the control limits would therefore have to be adapted and are generally more difficult to determine (see, for instance, Golosnoy and Schmid (2007) for the determination of the control limits in this situation in a parametric context). In contrast, the second part of Theorem 3 guarantees that the differences are (asymptotically) independent, making it possible to obtain the limits c_1 and c_2 by probabilistic considerations. For completeness, we refer to Pawlak et al. (2004) who mention another basic approach that determines the control limits dependent on the size of the jump which one seeks to detect with high probability.

3.3 Hierarchical testing

In contrast to the time-dynamic approach in the previous section, we now consider a static approach. By fixing a particular time point t, the key question is whether there is a significant difference in the dependence level before and after t. The approach differs from Phase 2 above insofar as we now include all sub-portfolios of the overall portfolio in the analysis. In doing so, we may be able to identify the 'dependence-drivers', namely those sub-portfolios which show a change of dependence around time point t.

As usual, let $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ denote a d-dimensional time series fulfilling assumption (A1), (A2), and (A3) as elaborated in Sections 2.2 and 3.1. For $l \in \mathbb{N}$ with $1 \leq l \leq d-1$, let also J be the set of all subsets $\mathcal{I}$ of the index set $\{1,\ldots,d\}$ with cardinality $|\mathcal{I}| > l$. We define by $\boldsymbol{\rho}_J^t$ the vector of all $|\mathcal{I}|$-dimensional Spearman's rho coefficients $\rho_{|\mathcal{I}|}^t$ at time $t \in \mathbb{Z}$ with $\mathcal{I} \in J$. An estimator of the latter is given by $\widehat{\boldsymbol{\rho}}_{J,n}^t$ based on samples $\mathbf{X}_{t-n+1},\ldots,\mathbf{X}_t$ in a moving window with window size n.

In order to detect whether there is a level change of dependence at time t, we compare the values of Spearman's rho of all sub-portfolio combinations in J between the adjacent and non-overlapping windows of fixed size n before and after t. Let us therefore assume that, throughout both periods, Spearman's rho does not change for any $\mathcal{I} \in J$, respectively. We then consider the hypothesis

$$\mathrm{H}_0 : \boldsymbol{\rho}_J^{t-1} = \boldsymbol{\rho}_J^{t+n-1} \quad \text{versus} \quad \mathrm{H}_1 : \boldsymbol{\rho}_J^{t-1} \neq \boldsymbol{\rho}_J^{t+n-1}. \tag{19}$$

For each $\mathcal{I} \in J$, we would reject the null hypothesis of identical Spearman's rho in the respective time periods at level $\alpha_{\mathcal{I}}$ if $|Q_{\mathcal{I},n}^t| > z_{1-\alpha_{\mathcal{I}}/2}$ with

$$Q_{\mathcal{I},n}^t = \frac{\sqrt{n}(\widehat{\rho}_{|\mathcal{I}|,n}^{t+n-1} - \widehat{\rho}_{|\mathcal{I}|,n}^{t-1})}{\hat{\sigma}_{|\mathcal{I}|}^B}.$$

Here, $(\hat{\sigma}_{\mathcal{I}}^B)^2$ represents the consistent bootstrap estimator of the variance of $\sqrt{n}(\widehat{\rho}_{|\mathcal{I}|,n}^{t+n-1} - \widehat{\rho}_{|\mathcal{I}|,n}^{t-1})$ as introduced in Section 3.4. $Q_{\mathcal{I},n}^t$ is, under the null hypothesis, (asymptotically) standard normally distributed according to Theorem 2 (cf. also formula (18)) since $\widehat{\rho}_{|\mathcal{I}|,n}^{t+n-1}$ and $\widehat{\rho}_{|\mathcal{I}|,n}^{t-1}$ are based on independent samples.

The null hypothesis in (19) is rejected at significance level β_l if $|Q_{\mathcal{I},n}^t| > z_{1-\alpha_{\mathcal{I}}/2}$ for some

$\mathcal{I} \in J$, that is

$$\mathbb{P}\Big(\bigcup_{\substack{\mathcal{I}\in J \\ |\mathcal{I}|>l}} \Big\{ |Q^t_{\mathcal{I},n}| > z_{1-\alpha_{\mathcal{I}}/2} \Big\} \Big) = \beta_l. \tag{20}$$

The interrelationship between β_l and α is complicated – however, it may be approximated by Bonferroni's method as carried out in Section 4.4. For convenience, we choose $\alpha_{\mathcal{I}} = \alpha$ for all $\mathcal{I}$ and obtain

$$\mathbb{P}\Big(\bigcup_{\substack{\mathcal{I}\in J \\ |\mathcal{I}|>l}} \Big\{ |Q^t_{\mathcal{I},n}| > z_{1-\alpha/2} \Big\} \Big) = \mathbb{P}(\sup_{\substack{\mathcal{I}\in J \\ |\mathcal{I}|>l}} |Q^t_{\mathcal{I},n}| > z_{1-\alpha/2}) = \beta_l$$

with $\sup_{\{\mathcal{I}\in J\ ,\ |\mathcal{I}|>l\}} |Q^t_{\mathcal{I},n}| = \max_{\{\mathcal{I}\in J\ ,\ |\mathcal{I}|>l\}} |Q^t_{\mathcal{I},n}|$ as J is finite. Hence, a test statistics for H_0 is given by $\max_{\{\mathcal{I}\in J\ ,\ |\mathcal{I}|>l\}} |Q^t_{\mathcal{I},n}|$ which has a limiting distribution according to Theorem 2 with $||.||$ being the maximum norm $||.||_\infty$. By changing the parameter l one can move from one portfolio's hierarchy level to another one. Note that, for small hierarchical level l, the power of the test will decrease owing to the inclusion of a larger set of sub-portfolios.

The hierarchical test procedure is applied to the supervisory portfolio in Section 4.4.

3.4 The Bootstrap

A nonparametric bootstrap is used in order to estimate the distribution of the limiting variable W in Theorem 2 since it generally cannot be derived explicitly and depends on the choice of the matrix norm. We refer to Shao and Tu (1995) and Efron and Tibshirani (1993) for background reading on the nonparametric bootstrap and to Van der Vaart and Wellner (1996) and Fermanian et al. (2004) for details on the bootstrap in the context of empirical processes.

The next theorem first verifies that the nonparametric bootstrap method works.

Theorem 4 *Let $(\mathbf{X}^B_l)_{l=1,\dots,n}$ and $(\mathbf{Y}^B_l)_{l=1,\dots,m}$ denote the bootstrap sample which is obtained by sampling from the independent random samples $(\mathbf{X}_l)_{l=1,\dots,n}$ and $(\mathbf{Y}_l)_{l=1,\dots,m}$ with replacement, respectively. For $A \subseteq J$, let further $S_{A,n,\mathbf{X}}$, $S_{A,m,\mathbf{Y}}$ be the vector of sample versions of Spearman's rho as given in Theorem 2 and let $S^B_{A,n,\mathbf{X}}$, $S^B_{A,m,\mathbf{Y}}$ denote the corresponding estimators for the bootstrap samples $(\mathbf{X}^B_l)_{l=1,\dots,n}$ and $(\mathbf{Y}^B_l)_{l=1,\dots,m}$. Then, under the assumptions of Theorems 2 and with $m := m(n)$ such that $\frac{\sqrt{n}}{\sqrt{m(n)}} \to c$ for $n \to \infty$, the* *sequences* $\sqrt{n}||S^B_{A,n,\mathbf{X}} - S^B_{A,m(n),\mathbf{Y}} - (S_{A,n,\mathbf{X}} - S_{A,m(n),\mathbf{Y}})||$ *converge weakly to the same limit as* $\sqrt{n}||S_{A,n,\mathbf{X}} - S_{A,m(n),\mathbf{Y}}||$ *with probability one.*

Even if, in case $|A| = 1$, asymptotic normality of $\sqrt{n}\Big(\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}} - \widehat{\rho}_{|\mathcal{I}|,n,\mathbf{Y}}\Big)$ can be shown in Theorem 2, the asymptotic variance of the limiting variable is usually of complicated form (cf. Schmid and Schmidt (2006) for the explicit calculation of the asymptotic variance of multivariate Spearman's rho for some copulas). Therefore, the variance is determined by the above-described bootstrap method in this case, and we denote the corresponding consistent bootstrap variance estimator by $(\hat{\sigma}^B_{|\mathcal{I}|})^2$.

Our assumption of independent standardized sample returns justifies the application of the bootstrap method with replacement; otherwise, different methods must be applied (see, for instance, Davison and Hinkley (1997)). Regarding the number of bootstrap replications, $B = 500$ and $B = 2500$ are generally considered to be large enough to give good estimates of the variance and the empirical distribution, respectively (cf. Efron and Tibshirani (1993)). Schmid and Schmidt (2006) also show that the bootstrap variance estimator for multivariate Spearman's rho performs well in finite samples.

4 Empirical results

Before analyzing the dependence structure of the supervisory portfolio by means of the presented methods above, we briefly examine the standardized returns $\mathbf{S}_t$ of the supervisory portfolio introduced in Section 2.2.

4.1 Standardized returns

We start with the verification of the assumptions (A1) and (A2) in Section 2.2; further empirical analysis of the banks' individual standardized returns can be found in Memmel and Wehn (2006) for the years 2001 to 2004 and Jaschke et al. (2003) for the year 2001.

The squared standardized returns contain only minor serial correlation; for illustration, Figure 2 gives the autocorrelation function across all banks; the same holds for the standardized returns themselves, too. Thus, assumption (A1) of serial independence of the standardized returns can be justified. Note that this is also consistent with the literature on volatility modelling; see Andersen et al. (2006) for an overview. According to the rea-

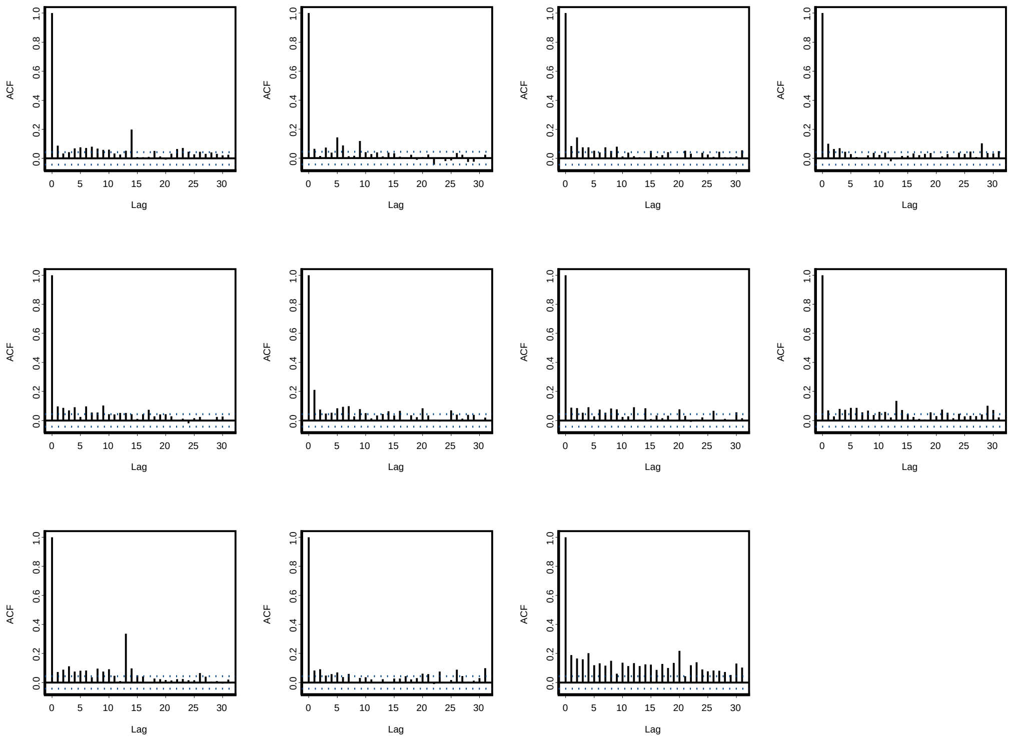


Figure 2: Autocorrelation function of the squared standardized returns of each bank of the supervisory portfolio.

soning in Section 2.2, this confirms that the VaR models used by the banks work quite accurately over the considered data time horizon (cf. Jaschke et al. (2003)).

Table 2 reports the first four moments of the distribution of the standardized returns across all 11 banks. The results show that the kurtosis varies from bank to bank, ranging from 3.79 to 13.39. In order to verify whether those differences are statistically significant, we perform several pairwise tests on equal kurtosis, which are based on a bootstrap procedure. More specifically, we draw (with replacement) a bootstrap sample from $\mathbf{S}_t, t = 1, \ldots, T$, and determine for all pairs of banks the empirical confidence interval for the difference in kurtosis; an extract is given in Table 3. The table shows that the difference in kurtosis of 32 out of 55 possible pairs of banks is significantly different from zero, which justifies assumption (A2) of differing marginal distribution functions of the standardized returns.

Table 2: Descriptive statistics of the standardized returns for each bank of the supervisory portfolio.

Bank	Mean	St. deviation	Skewness	Kurtosis
1	0.0419	0.3225	-0.2422	13.3883
2	0.084	0.3721	0.1502	4.3238
3	0.0115	0.3805	-0.1155	4.5387
4	0.0905	0.3796	0.0284	4.2795
5	0.0046	0.2728	-0.0657	4.5791
6	0.0141	0.3523	-0.0659	4.2136
7	-0.0012	0.2741	-0.1607	5.6778
8	0.0125	0.4344	-0.2966	6.638
9	-0.011	0.4363	-0.3023	8.3636
10	0.0256	0.2786	0.0639	3.7949
11	-0.0532	0.3199	-0.323	5.7466

Table 3: 90% bootstrap confidence $(\hat{c}_l^B, \hat{c}_u^B)$ intervals for the difference in kurtosis for those pairs of banks where the difference is statistically different from 0, based on 10,000 bootstrap replications.

Bank	Bank	$\hat{c}_l^B$	$\hat{c}_u^B$	Bank	Bank	$\hat{c}_l^B$	$\hat{c}_u^B$
1	2	3.2529	13.8552	4	8	-5.1587	-0.1354
1	3	2.9961	13.6637	4	9	-6.2071	-1.6245
1	4	3.2552	13.9064	4	11	-2.4966	-0.3446
1	5	2.9149	13.6022	5	9	-5.771	-1.4653
1	6	3.3427	13.9024	5	10	0.0943	1.4968
1	7	1.8224	12.6234	5	11	-2.149	-0.1508
1	8	0.4997	12.1646	6	7	-2.6159	-0.3265
1	10	3.7619	14.3313	6	8	-5.1781	-0.2711
1	11	1.7493	12.4315	6	9	-6.1313	-1.7594
2	7	-2.5906	-0.1189	6	11	-2.3826	-0.6456
2	8	-5.1116	-0.1101	7	9	-4.9096	-0.0857
2	9	-6.1154	-1.5992	7	10	0.7575	3.0069
2	11	-2.4091	-0.335	8	10	0.7036	5.5623
3	9	-5.9475	-1.3387	9	10	2.2056	6.5528
3	11	-2.3295	-0.0193	9	11	0.1447	4.7268
4	7	-2.668	-0.1946	10	11	-2.8096	-1.0732

4.2 Multivariate Spearman's rho of the supervisory portfolio

Figure 3 (left panel) illustrates the evolution of multivariate Spearman's of the standardized returns of the supervisory portfolio; the estimation is based on a window size of $n = 150$. In addition, the horizontal line illustrates the average (multivariate) Spearman's rho over the entire time horizon. The figure shows that Spearman's rho fluctuates over time: The situation of deteriorating financial markets after the events of September 11 is accompanied by a steady increase in Spearman's rho. This is consistent with the observations of high asset volatilities and correlations during this time period and an increase in medium-term interest rates from October 2001 onwards (Jaschke et al. (2003)). After its first peak at the beginning of 2002, Spearman's rho falls sharply. Thereafter, a period of relatively low portfolio dependence is observable, which was characterized by medium-term interest rates at an all-time low and stabilizing markets. The year 2004 reveals an initially steady, then sudden rise in Spearman's rho to its second peak in December 2004. During 2005, which proved to be a financial year of rising markets, Spearman's rho peaks off and remains relatively low for the rest of the observation period.

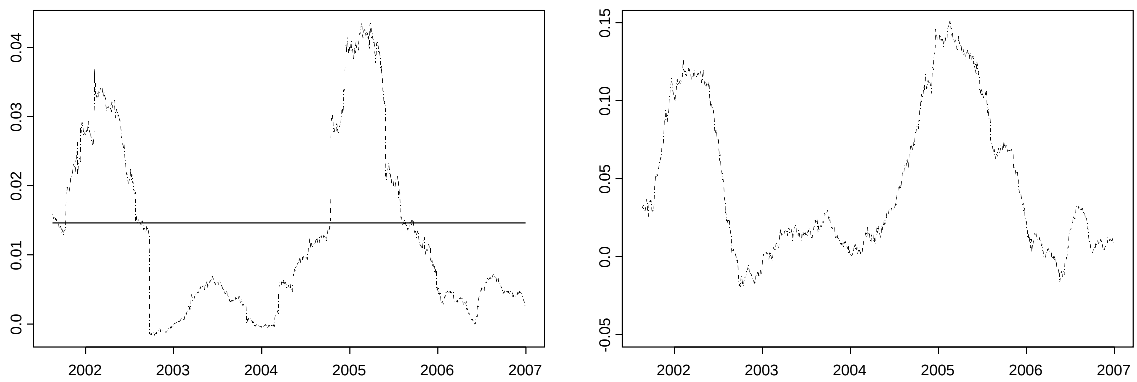


Figure 3: Time-varying (dashed line) and average (solid line) multivariate Spearman's rho $\widehat{\rho}_{11,\mathbf{S}}$ (left panel) and average of all pairwise Spearman's rho coefficients $\widehat{\tilde{\rho}}_{11,\mathbf{S}}$ (right panel) of the standardized returns of the supervisory portfolio, based on a moving window of size $n = 150$.

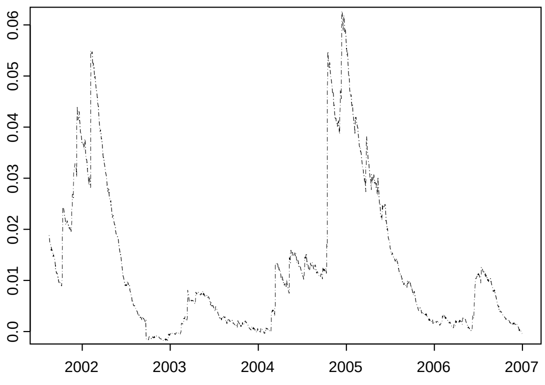


Figure 4: Exponentially weighted multivariate Spearman's rho with decay factor $\lambda = 0.985$, based on a moving window of size $n = 150$.

Some developments are particularly noticeable, such as the sudden upward movement of

Spearman's rho on 6 February 2002 and on 13 October 2004. On these two days, the P&L or standardized returns, respectively, of all banks in the portfolio proved to be negative, i.e. all banks simultaneously realized losses. For comparison, we provide in Figure 3 (right panel) the average of all pairwise Spearman's rho coefficients (see also Memmel and Wehn (2006) for an analysis of the supervisory portfolio's VaR based on the average linear correlation coefficient). While the multivariate Spearman's rho reacts sensitive to these days of simultaneous negative movements, the average of all pairwise Spearman's rho shows a more gradual and steady increase and does not emphasize those extreme events. However, those events may be of particular interest to the supervisor in general, especially as the simultaneous realization of losses across all banks happened only on four days altogether during the observation period and these days revealed the by far highest losses.

The sudden decrease in Spearman's rho in September 2002 must be put down to the nature of the moving window approach. For benchmarking, we calculate multivariate Spearman's rho anew, but this time, allocate different, exponentially decreasing weights to the observations (see Figure 4). This approach is analogous to the exponentially weighted moving window approach (EWMA) by RiskMetrics. As the figure implies, it emphasizes those days where the banks' P&L simultaneously move in the same direction, like in October 2004. Though the exponentially weighted version of multivariate Spearman's rho represents an alternative estimator, its statistical properties are more difficult to establish. All further analysis is based on an equally weighted moving window of size $n = 150$.

4.3 Level changes of Spearman's rho of the supervisory portfolio over time

The time-dynamic test procedure proposed in Section 3.2 is applied to the supervisory portfolio. We set $s = 1$ and thus focus on monitoring Spearman's rho of the supervisory portfolio daily.

The main motivation of the control chart design in Phase 1 is the fact that the first differences of Spearman's rho estimates $\widehat{\rho}_{d,n}^{t}$ are (asymptotically) serially uncorrelated. The sample autocorrelation functions of the original time series and the first difference are given in Figure 5. For calibrating the control chart in Phase 1, we use the first 150 observations (denoted as pre-sample) of the series $\{n(\widehat{\rho}^{t}_{d,n} - \widehat{\rho}^{t-1}_{d,n})\}$ to determine the control limits c_1 and c_2, according to the procedure described in Section 3.2 and in line with the window size $n = 150$.

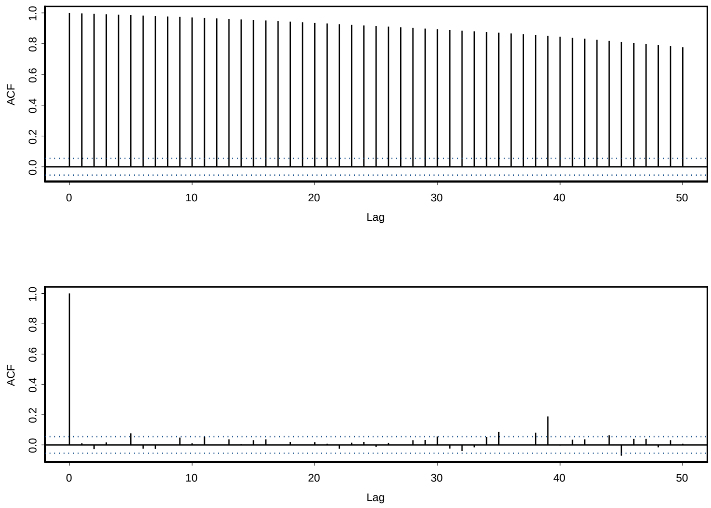
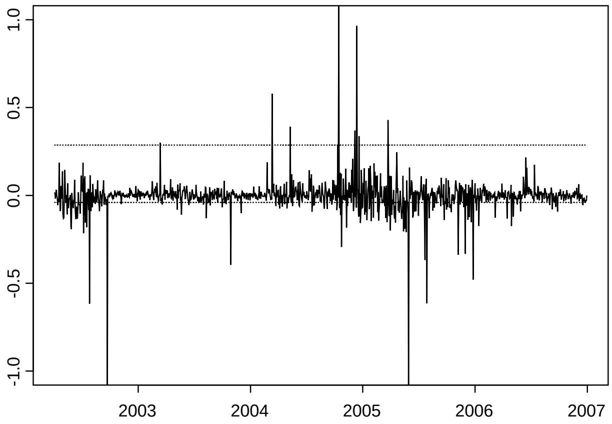


Figure 5: Sample autocorrelation function of the time series of Spearman's rho estimates $\widehat{\rho}^{t}_{d,n}$ (upper panel) and of the first differences $\widehat{\rho}^{t}_{d,n} - \widehat{\rho}^{t-1}_{d,n}$ (lower panel) of the supervisory portfolio with window size $n = 150$.

Estimates $\hat{c}_1$ and $\hat{c}_2$ of the control limits c_1 and c_2 are given as the empirical $\alpha/2$- and $(1-\alpha/2)$-quantiles of the pre-sample; the confidence level α is set to 0.05 in both Phase 1 and Phase 2. The control chart of Phase 1 as well as the results of the test procedure are given in Figure 6. Here, $\hat{c}_1$ and $\hat{c}_2$ are -0.03962 and 0.28709, respectively; altogether, we observe 241 alarms in Phase 1. Proceeding with Phase 2, where the estimation of the bootstrap variance is based on 500 bootstrap replications, we obtain two signals at time $t = 308$ and $t = 1038$. The corresponding values of the test statistics are provided in the table of Figure 6. It becomes clear from Figure 3 that both signals occur at the respective global downward movements of Spearman's rho at the beginning of the years 2002 and 2005.

After an alarm has been triggered in Phase 1 and, thus, an early warning of level change in dependence has occurred, market-relevant factors and events should be analyzed around the time the alarm occurred in order to find a possible economic interpretation for the shift in dependency. As elaborated above, the next observations (in our case, 149 observations)

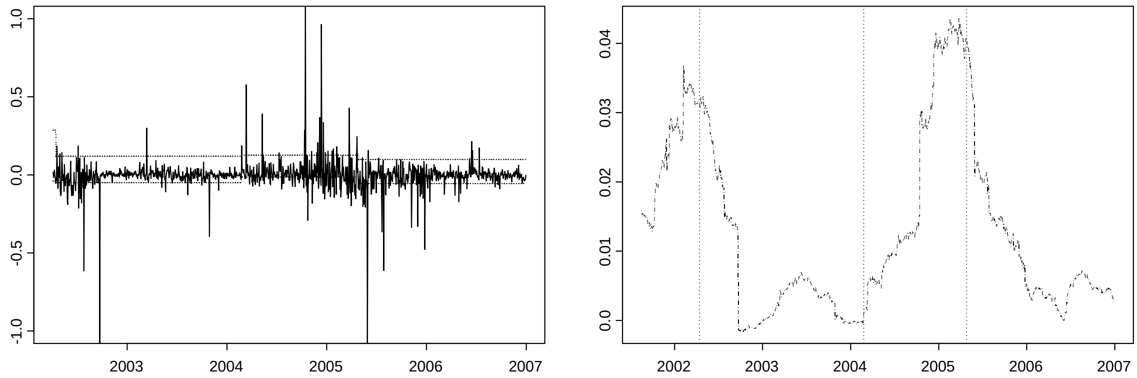


t	date	Phase 1 $[\hat{c}_1, \hat{c}_2]$	$n(\widehat{\rho}^{t}_{d,n} - \widehat{\rho}^{t-1}_{d,n})$	Phase 2 T	p-value
308	12.04.2002	$[-0.03962, 0.28709]$	-0.05490001	-2.73087	0.00632
1038	26.04.2005	$[-0.03962, 0.28709]$	-0.08792838	-2.06398	0.03902

Figure 6: Upper panel: Control chart in Phase 1 of differences $n(\widehat{\rho}^{t}_{d,n} - \widehat{\rho}^{t-1}_{d,n})$ with the estimated control limits $\hat{c}_1$ and $\hat{c}_2$ (horizontal lines); lower panel: Summary of the test statistics including those dates with significant signals. Here, T refers to the test statistics given in (18). The results are based on $\alpha = 0.05$, 500 bootstrap replications, and window size $n = 150$.

are then to be awaited in Phase 2 in order to test for a significant long-term level change of dependence. Note, however, that by leaving the control limits unchanged throughout the whole period, we would not use the information provided by the test procedure, i.e. that a signal has occurred.

Therefore, we apply the test procedure anew – only, this time, we recalibrate the control chart of Phase 1 each time after a signal has been observed: The control limits are re-estimated based on the 150 observations following (and including) the signal, and the chart is restarted. The corresponding output is given in Figure 7. We provide the corresponding control chart of Phase 1 together with the re-estimated control limits whose values are explicitly given in the table. This time, 230 alarms are obtained in Phase 1, leaving us with three signals in Phase 2. Hence, in addition to the signals obtained from the control chart without re-calibration, we observe a signal at $t = 755$. As Figure 7 shows, this new signal

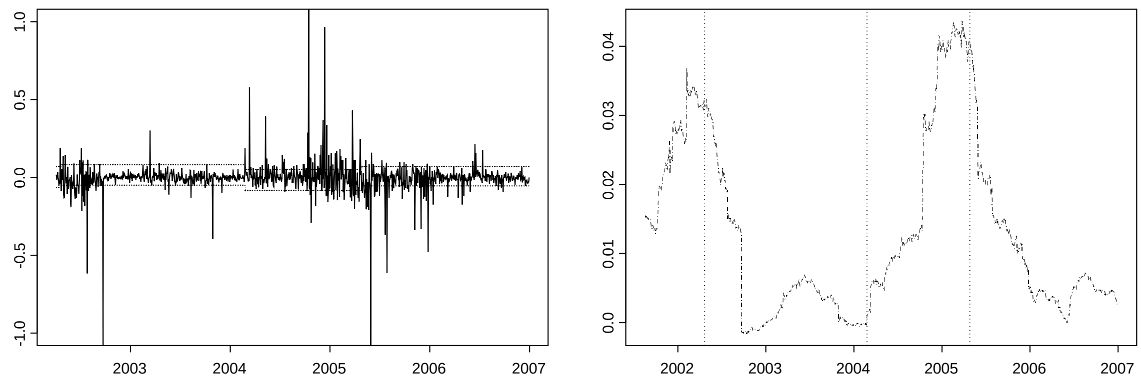


		Phase 1		Phase 2	
t	date	$[\hat{c}_1, \hat{c}_2]$	$n(\widehat{\rho}^t_{d,n} - \widehat{\rho}^{t-1}_{d,n})$	T	p-value
308	12.04.2002	$[-0.03962, 0.28709]$	-0.05490001	-2.73087	0.00632
755	23.02.2004	$[-0.04952, 0.11975]$	0.18811689	2.74026	0.00614
1038	26.04.2005	$[-0.01370, 0.12654]$	-0.08792838	-2.06398	0.03902
	>26.04.2005	$[-0.05506, 0.09861]$			

Figure 7: Upper left panel: Control chart in Phase 1 of differences $n(\widehat{\rho}^t_{d,n} - \widehat{\rho}^{t-1}_{d,n})$ with the estimated control limits $\hat{c}_1$ and $\hat{c}_2$ (horizontal lines); upper right panel: multivariate Spearman's rho with signals (vertical lines); lower panel: Summary of the test statistics including those dates with significant signals. Here, T refers to the test statistics given in (18). The results are based on $\alpha = 0.05$, 500 bootstrap replications, and window size $n = 150$.

occurs at the global increase of Spearman's rho at the beginning of the year 2004.

Below we discuss an alternative approach concerning the calibration of the control chart in Phase 1. To reduce the volatility of the estimation of the control limits, we now adopt the following procedure: By means of the nonparametric bootstrap method introduced in Section 3.4, we determine for each t the $\alpha/2$- and $(1-\alpha/2)$-quantiles of the bootstrap distribution of the control process Y_t as introduced in Section 3.2. The control limits are then estimated as the median of the respective quantile estimates of the 150 observation in the pre-sample. We only provide the results of the test procedure including re-calibration of the control limits since this seems the natural approach according to our discussion above; the number of bootstrap replications to estimate the control limits is set to 2500.



		Phase 1		Phase 2	
t	date	$[\hat{c}_1, \hat{c}_2]$	$n(\widehat{\rho}_{d,n}^t - \widehat{\rho}_{d,n}^{t-1})$	T	p-value
313	19.04.2002	$[-0.06369, 0.06914]$	-0.088354	-2.69338	0.00707
755	23.02.2004	$[-0.04931, 0.08124]$	0.18811689	2.74026	0.00614
1038	26.04.2005	$[-0.08226, 0.05006]$	-0.08792838	-2.06398	0.03902
	>26.04.2005	$[-0.05463, 0.06977]$			

Figure 8: Upper left panel: Control chart in Phase 1 of differences $n(\widehat{\rho}_{d,n}^t - \widehat{\rho}_{d,n}^{t-1})$ with the estimated control limits $\hat{c}_1$ and $\hat{c}_2$ (horizontal lines); upper right panel: multivariate Spearman's rho with signals (vertical lines); lower panel: Summary of the test statistics including those dates with significant signals. Here, T refers to the test statistics given in (18). The results are based on $\alpha = 0.05$, 2500 (calibration of control limits in Phase 1) and 500 (variance estimation in Phase 2) bootstrap replications, and window size $n = 150$.

The estimates of the control limits are shown in the table of Figure 8. At the beginning, $\hat{c}_1 = -0.06369$ and $\hat{c}_2 = 0.06914$; observe that, in most cases, the bootstrap yields smaller values for $\hat{c}_2$ than the previous calibration method. The alternative method yields signals at almost the same time points. Only the first signal now occurs slightly later at time $t = 313$ instead of $t = 308$, but still identifies the global decrease in Spearman's rho. Since the alternative calibration method yields more robust estimates of the control limits, it might be even possible to reduce the number of required observations in the pre-sample. The impact of reducing the sample size is illustrated in Figure 9, where the estimates of the control limits are calculated depending on the size of the pre-sample. The solid vertical line represents the actual choice of $n = 150$, implying that a smaller sample size does not seem to be reasonable.

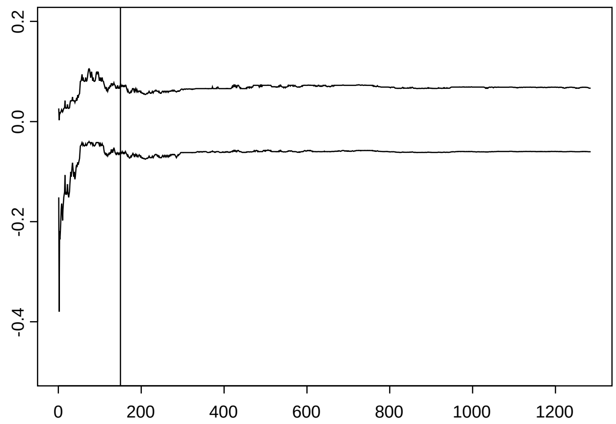


Figure 9: Estimated control limits (at $\alpha = 0.05$) as a function of the size of the pre-sample. The solid vertical line represents the sample size of $n = 150$.

Summarizing the above findings, the test procedure based on control chart theory (discussed in Section 3.2) helps to identify the global level changes of Spearman's rho. The high level of dependence between the banks' returns, as observed after the events of September 11 (cf. Section 4.2), is significantly decreasing after April 2002, as detected by our statistical tests. Along with the uncertainty and the sideward trends of the European and US financial markets at the beginning of the year 2004, the dependence significantly increases after February 2004. Finally, the clear upward trends of most stock markets in the world from 2005 onwards lead to a decreasing dependence among the banks' returns, which is statistically significant around April 2005. A detailed analysis of the banks' trading portfolios would now enable a further explanation of the evolution of the dependence.

4.4 Hierarchical considerations for the supervisory portfolio

The hierarchical testing described in Section 3.3 answers the question as to which groups of banks of the supervisory portfolio show a significant change of dependence around a predefined time point when applied to its standardized returns.

According to Bonferroni's inequality, we have

$$\mathbb{P}\Big(\bigcup_{\substack{\mathcal{I}\in J \\ |\mathcal{I}|>l}} \Big\{ |Q^t_{\mathcal{I},n}| > z_{1-\alpha/2} \Big\} \Big) \leq \sum_{\substack{\mathcal{I}\in J \\ |\mathcal{I}|>l}} \mathbb{P}(|Q^t_{\mathcal{I},n}| > z_{1-\alpha/2}) = \beta_l,$$

and we may choose α in such a way that

$$\alpha \sum_{\substack{k=l \\ |\mathcal{I}|=k}}^{d} \binom{d}{k} = \beta_l,$$

and thus, $\alpha = \beta_l / \{2^d - \sum_{k=0}^{l-1} \binom{d}{k}\}$.

Table 4: Output of the hierarchical testing for the supervisory portfolio at the time points $t = 313$ (19.04.2002), $t = 755$ (23.02.2004), and $t = 1038$ (26.04.2005) for $l = 8$. Value of the test statistics $\max_{\substack{I \in J \\ |\mathcal{I}|>8}} |Q^t_{\mathcal{I},n}|$ for testing the overall null hypothesis (19). Calculations are based on 500 bootstrap replications and $\beta_8 = 0.1$.

t	date	$\max_{\substack{I \in J \\ \|\mathcal{I}\|>8}} \|Q^t_{\mathcal{I},n}\|$	$z_{1-\alpha/2}$
313	19.04.2002	4.53089	3.176131
755	23.02.2004	3.46422	3.176131
1038	26.04.2005	2.71896	3.176131

For the supervisory portfolio, $d = 11$ and we set the overall test level β_l to 0.1 and $l = 8$. Hence, $\alpha = 0.00149$ and $z_{1-\alpha/2} = 3.176131$, in this case. Furthermore, we concentrate on the three time points identified as level changes of portfolio dependence in the previous section, though any other time point might be possible, too. The value of the test statistics $\max_{\{I \in J\, ,\, |\mathcal{I}|>8\}} |Q^t_{\mathcal{I},n}|$ at those three time points is given in Table 4. It follows that the null hypothesis (19) has to be rejected at level β_8 at the time points $t = 313$ and $t = 755$, implying that dependency has significantly changed in a period before and after the time points among the portfolios with dimension greater than 8. By contrast, we cannot reject the null hypothesis at the time point $t = 1038$.

For the time points $t = 313$ and $t = 755$, Table 5 further provides all sub-portfolios with dimension greater than 8, showing a significant change of Spearman's rho at level α at the respective time points. Altogether, there are 23 sub-portfolios with dimension 9 or 10 at $t = 313$ with the 9-dimensional sub-portfolio consisting of the banks $2, 3, 4, 5, 6, 7, 8, 10$, and 11 having the smallest p-value. At $t = 755$, only sub-portfolios with dimension 9 turn out to have a significant change in Spearman's rho at level α at this time point. Here, the sub-portfolio consisting of the banks $1, 2, 3, 4, 6, 8, 9, 10$, and 11 has the smallest p-value. For

Table 5: Output of the hierarchical testing for the supervisory portfolio at the time points $t = 313$ (19.04.2002) and $t = 755$ (23.02.2004) for $l = 8$. Significant sub-portfolio combinations $\mathcal{I}$, corresponding value of the statistics $Q^t_{\mathcal{I},n}$, and p-value. Calculations are based on 500 bootstrap replications and $\beta_l = 0.1$.

t=313			t=755		
$\mathcal{I}$	$Q^t_{\mathcal{I},n}$	p-value	$\mathcal{I}$	$Q^t_{\mathcal{I},n}$	p-value
1 2 3 4 5 6 7 8 10 11	-3.39774	0.00068	1 2 3 4 5 6 8 9 10	3.35484	0.00079
2 3 4 5 6 7 8 9 10 11	-3.82334	0.00013	1 2 3 4 5 6 9 10 11	3.36225	0.00077
1 2 3 4 5 6 7 8 9	-3.23098	0.00123	1 2 3 4 6 7 8 9 11	3.23002	0.00124
1 2 3 4 5 6 7 8 10	-4.25733	0.00002	1 2 3 4 6 8 9 10 11	3.46422	0.00053
1 2 3 4 5 6 7 10 11	-3.54487	0.00039	1 2 3 4 7 8 9 10 11	3.2428	0.00118
1 2 3 4 5 6 8 10 11	-3.60147	0.00032	2 3 4 5 6 8 9 10 11	3.28402	0.00102
1 2 3 4 5 7 8 10 11	-3.33302	0.00086	2 3 4 6 7 8 9 10 11	3.37724	0.00073
1 2 3 4 6 7 8 10 11	-4.00944	0.00006			
1 2 3 5 6 7 8 10 11	-3.99211	0.00007			
1 2 4 5 6 7 8 9 10	-3.29191	0.001			
1 2 4 5 6 7 8 10 11	-4.25264	0.00002			
1 2 5 6 7 8 9 10 11	-3.19147	0.00142			
1 3 4 5 6 7 8 10 11	-3.67182	0.00024			
2 3 4 5 6 7 8 9 10	-3.35125	0.0008			
2 3 4 5 6 7 8 9 11	-3.77839	0.00016			
2 3 4 5 6 7 8 10 11	-4.53089	0.00001			
2 3 4 5 6 7 9 10 11	-3.66836	0.00024			
2 3 4 5 6 8 9 10 11	-3.58624	0.00034			
2 3 4 5 7 8 9 10 11	-3.29662	0.00098			
2 3 4 6 7 8 9 10 11	-3.92365	0.00009			
2 3 5 6 7 8 9 10 11	-3.9123	0.00009			
2 4 5 6 7 8 9 10 11	-4.13888	0.00003			
3 4 5 6 7 8 9 10 11	-3.6275	0.00029			

illustration, Figure 10 illustrates, at the two time points, the evolution of Spearman's rho of the sub-portfolio showing the highest significant change in Spearman's rho in contrast to the sub-portfolio of the same dimension having the largest p-value. In general, considerably converse developments are observable around the respective time points. Note that, at $t = 313$, the two smallest banks, bank 1 and 9 (as measured in terms of average VaR), appear considerably less often than any other bank. This may imply that dependence around $t = 313$ is driven more by the larger banks. If more information about the banks were available, possible common factors driving dependency might be identified in the respective time period.

The result of the hierarchical testing does not imply that there is no significant change of dependence at time point $t = 1038$ at all; however, there is none among the sub-portfolios with dimension greater than 8. Furthermore, the fact that more than three times as many sub-portfolios with dimension greater than 8 showing a significant change in dependence could be observed at $t = 313$ than at $t = 755$ might provide an indication of the stability of the financial markets around that time – the more so as financial markets indeed displayed a more volatile behaviour in 2002 than at the beginning of 2004.

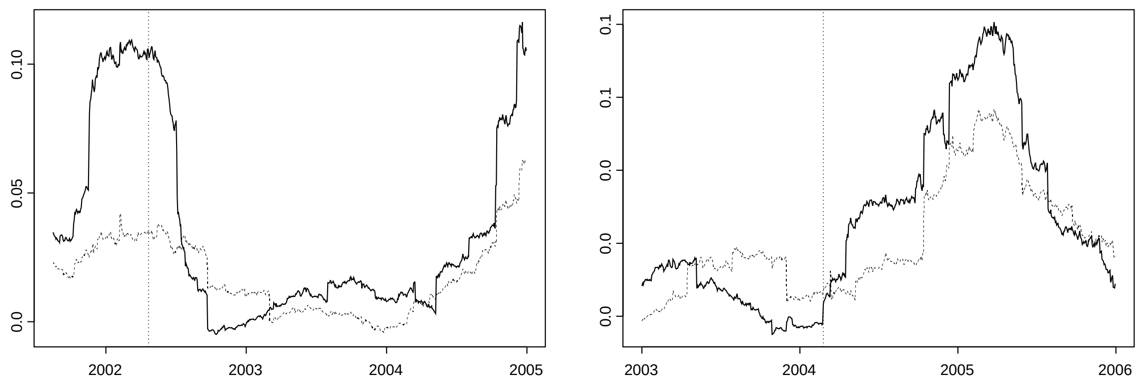


Figure 10: Development of Spearman's rho of sub-portfolios. Left panel: Sub-portfolio of banks $2, 3, 4, 5, 6, 7, 8, 10$, and 11 (solid line) against sub-portfolio of banks $1, 2, 3, 4, 6, 7, 8, 9$, and 11 (dotted line) with the vertical line representing the time point $t = 313$ (19.04.2002); Right panel: Sub-portfolio of banks $1, 2, 3, 4, 6, 8, 9, 10$, and 11 (solid line) against sub-portfolio of banks $1, 2, 3, 4, 5, 6, 7, 8$, and 10 (dotted line) with the vertical line representing the time point $t = 755$ (23.02.2004).

5 Conclusion

A multivariate version of the copula-based dependence measure Spearman's rho is proposed for modelling the (time-dynamic) dependence structure of the supervisory portfolio. Several important theoretical results on the asymptotic behaviour of Spearman's rho thus allow a thorough analysis of the portfolio's dependence structure in two ways. A time-dynamic two-step test procedure – partly based on a nonparametric control chart for Spearman's rho – detects (sustaining) level shifts of the portfolio's dependence over time, while the hierarchical testing serves to identify sub-portfolios causing a significant change in dependence around some specific time point. The proposed methods are general and can be applied to any series of multivariate asset returns in finance where the assumption of independent standardized returns holds.

Our empirical study of the supervisory portfolio identifies significant changes in the dependence level at three time points during the period from 2001 to 2006. At two of those time points the hierarchical analysis reveals a significant change in dependence for all sub-portfolios comprising more than eight banks.

Appendix

Details on the derivation of the estimators $\widehat{\rho}_{|\mathcal{I}|,n}$ and $\widehat{\widetilde{\rho}}_{|\mathcal{I}|,n}$

The estimation of the $|\mathcal{I}|$-dimensional versions of Spearman's rho $\rho_{|\mathcal{I}|}$ and $\tilde{\rho}_{|\mathcal{I}|}$ as defined in formula (7) and (9), respectively, is based on the empirical copula, which is derived as follows.

Consider a random sample $(\mathbf{X}_j)_{j=1,\dots,n}$ from the d-dimensional random vector $\mathbf{X}$ with joint distribution function F, continuous marginal distribution functions $F_{X_i}, i = 1, \dots, d$, and copula C which are assumed to be unknown. The univariate marginal distribution functions F_{X_i} are estimated by their empirical distribution functions

$$\hat{F}_{i,n}(x) = \frac{1}{n} \sum_{j=1}^{n} \mathbf{1}_{\{X_{ij} \leq x\}}, \quad \text{for } i = 1, \dots, d \text{ and } x \in \mathbb{R}.$$

Furthermore, let $\hat{F}_n(\mathbf{x}) = \frac{1}{n} \sum_{j=1}^{n} \Pi_{i=1}^{d} \mathbf{1}_{\{X_{ij} \leq x_i\}}, \mathbf{x} \in \mathbb{R}^d$, denote the multivariate empirical distribution function. Standardizing the margins with $\hat{U}_{ij,n} := \hat{F}_{i,n}(X_{ij})$ for $i =$

$1,\dots,d,\ j=1,\dots,n$, and $\hat{\mathbf{U}}_{j,n}=(\hat{U}_{1j,n},\dots,\hat{U}_{dj,n})$ yields the empirical copula of C

$$\hat{C}_n(\mathbf{u})=\frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\mathbf{1}_{\{\hat{U}_{ij,n}\le u_i\}}\quad\text{for}\quad\mathbf{u}=(u_1,\dots,u_d)\in[0,1]^d. \tag{21}$$

By replacing the copulas in the definitions (7) and (9) by their empirical counterparts, we obtain the following estimators

$$\begin{aligned}\widehat{\rho}_{|\mathcal{I}|,n} &= h(|\mathcal{I}|)\Big\{2^{|\mathcal{I}|}\int_{[0,1]^{|\mathcal{I}|}}\hat{C}_{i_1,\dots,i_{|\mathcal{I}|},n}(\mathbf{u})\,d\mathbf{u}-1\Big\}\\ &= h(|\mathcal{I}|)\Big\{\frac{2^{|\mathcal{I}|}}{n}\sum_{j=1}^{n}\Big[\,(1-\hat{U}_{i_1j,n})\dots(1-\hat{U}_{i_{|\mathcal{I}|}j,n})\Big]-1\Big\}.\end{aligned}$$

and

$$\begin{aligned}\widehat{\widehat{\rho}}_{|\mathcal{I}|,n} &= h(2)\Big\{2^2\sum_{\substack{k<l\\k,l\in\mathcal{I}}}\binom{|\mathcal{I}|}{2}^{-1}\int_{[0,1]^2}\hat{C}_{kl,n}(u,v)dudv-1\Big\}\\ &= \frac{12}{n}\binom{|\mathcal{I}|}{2}^{-1}\sum_{\substack{k<l\\k,l\in\mathcal{I}}}\sum_{j=1}^{n}(1-\hat{U}_{kj,n})(1-\hat{U}_{lj,n})-3.\end{aligned}$$

Proofs of Theorems

The proof of Theorem 2 utilizes the asymptotic properties of the empirical copula process $\sqrt{n}\{\hat{C}_n(\mathbf{u})-C(\mathbf{u})\}$, which has been studied in different settings; we refer, for example, to Rüschendorf (1976), Stute (1984), Van der Vaart and Wellner (1996), Fermanian et al. (2004), and Tsukahara (2005). The asymptotic result on which our derivations are based is given in the next theorem; a proof is given in Fermanian et al. (2004), Theorem 3.

Theorem 5 *Let F be a continuous d-dimensional distribution function with copula C. Under the additional assumption that the i-th partial derivatives $D_iC(\mathbf{u})$ exist and are continuous for $i=1,\dots,d$, we have*

$$\sqrt{n}\{\hat{C}_n(\mathbf{u})-C(\mathbf{u})\}\xrightarrow{w}\mathbb{G}_C(\mathbf{u}).$$

Weak convergence takes place in $\ell^\infty([0,1]^d)$ and

$$\mathbb{G}_C(\mathbf{u})=\mathbb{B}_C(\mathbf{u})-\sum_{i=1}^{d}D_iC(\mathbf{u})\mathbb{B}_C(\mathbf{u}^{(i)}).$$

The vector $\mathbf{u}^{(i)}$ denotes the vector where all coordinates, except the i-th coordinate of $\mathbf{u}$, are replaced by 1. The process $\mathbb{B}_C$ is a tight centered Gaussian process on $[0,1]^d$ with covariance *function*

$$E\{\mathbb{B}_C(\mathbf{u})\mathbb{B}_C(\mathbf{v})\} = C(\mathbf{u}\wedge\mathbf{v}) - C(\mathbf{u})C(\mathbf{v}),$$

i.e. $\mathbb{B}_C$ *is a d-dimensional Brownian Bridge.*

Proof of Theorem 2.

(i) Given the theorem's prerequisites, the combination of Theorem 5 above and Theorem 3 in Schmid and Schmidt (2007) yields that, for a single index set $\mathcal{I}$,

$$\sqrt{n}(\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}} - \rho_{|\mathcal{I}|,\mathbf{X}}) \xrightarrow{d} W_{\mathbf{X}} \sim N(0,\sigma^2_{\mathbf{X}})$$

and

$$\sqrt{m(n)}(\widehat{\rho}_{|\mathcal{I}|,m(n),\mathbf{Y}} - \rho_{|\mathcal{I}|,\mathbf{Y}}) \xrightarrow{d} W_{\mathbf{Y}} \sim N(0,\sigma^2_{\mathbf{Y}})$$

for $n\to\infty$, with

$$\sigma^2_{\mathbf{X}} = 2^{2|\mathcal{I}|}h(|\mathcal{I}|)^2\int_{[0,1]^d}\int_{[0,1]^d} I\!E\{\mathbb{G}_{C_{\mathbf{X}}}(\mathbf{u}^{(\mathcal{I})})\mathbb{G}_{C_{\mathbf{X}}}(\mathbf{v}^{(\mathcal{I})})\}d\mathbf{u}d\mathbf{v}.$$

and

$$\sigma^2_{\mathbf{Y}} = 2^{2|\mathcal{I}|}h(|\mathcal{I}|)^2\int_{[0,1]^d}\int_{[0,1]^d} I\!E\{\mathbb{G}_{C_{\mathbf{Y}}}(\mathbf{u}^{(\mathcal{I})})\mathbb{G}_{C_{\mathbf{Y}}}(\mathbf{v}^{(\mathcal{I})})\}d\mathbf{u}d\mathbf{v}.$$

respectively. Under the assumption that $\rho_{|\mathcal{I}|,\mathbf{X}} = \rho_{|\mathcal{I}|,\mathbf{Y}}$, we have

$$\sqrt{n}\Big(\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}} - \widehat{\rho}_{|\mathcal{I}|,m(n),\mathbf{Y}}\Big) = \sqrt{n}\Big(\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}} - \rho_{|\mathcal{I}|,\mathbf{X}}\Big) - \frac{\sqrt{n}}{\sqrt{m(n)}}\sqrt{m(n)}\Big(\widehat{\rho}_{|\mathcal{I}|,m(n),\mathbf{Y}} - \rho_{|\mathcal{I}|,\mathbf{Y}}\Big)$$

and the assertion follows owing to Slutsky's theorem (see, for instance, Shao (1999), Theorem 1.11) and the fact that $\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{X}}$ and $\widehat{\rho}_{|\mathcal{I}|,n,\mathbf{Y}}$ are based on stochastically independent samples.

(ii) Let $\hat{C}_{n,\mathbf{X}}(\mathbf{u})$ and $\hat{C}_{m,\mathbf{Y}}, \mathbf{u}\in[0,1]^d$, denote the empirical copula of the random sample $(\mathbf{X}_l)_{l=1,\dots,n}$ and $(\mathbf{Y}_l)_{l=1,\dots,m}$, respectively. For $A\subseteq J$ with $|A|=k$ and $m=m(n)$, consider the k-dimensional random vectors $S_{A,n,\mathbf{X}}$ and $S_{A,m(n),\mathbf{Y}}$ which linearly map the empirical copulas $\hat{C}_{n,\mathbf{X}}$ and $\hat{C}_{m(n),\mathbf{Y}}$ into the k-dimensional Euclidean space $\mathbb{R}^k$, respectively. Thus, an application of the generalized continuous-mapping theorem (see , for example, Theorem 1.3.6 in Van der Vaart and Wellner (1996)) together with Theorem 5 yields the weak convergence of $\sqrt{n}\{S_{A,n,\mathbf{X}} - \boldsymbol{\rho}_{A,\mathbf{X}}\}$ and $\sqrt{m(n)}\{S_{A,m(n),\mathbf{Y}} - \boldsymbol{\rho}_{A,\mathbf{Y}}\}$ on the space $\mathbb{R}^k$. Since, in addition, $S_{A,n,\mathbf{X}}$ and $S_{A,m(n),\mathbf{Y}}$ are based on independent samples, joint weak convergence of

$$\Big(\sqrt{n}\{S_{A,n,\mathbf{X}} - \boldsymbol{\rho}_{A,\mathbf{X}}\}, \sqrt{m(n)}\{S_{A,m(n),\mathbf{Y}} - \boldsymbol{\rho}_{A,\mathbf{Y}}\}\Big)'$$

on the product space $\mathbb{R}^{2k}$ is obtained for $n \to \infty$. Finally, the fact that the matrix-norm $||\cdot||$ is also a continuous mapping from the space $\mathbb{R}^{2k}$ into $\mathbb{R}$ and another application of the continuous mapping theorem yields the asserted result. $\square$

Proof of Theorem 3.

(i) Let $\hat{C}_n^t(\mathbf{u}), \mathbf{u} \in [0,1]^d$, denote the empirical copula of the random sample $\mathbf{X}_{t-n+1}, \ldots, \mathbf{X}_t$, $t \in \mathbb{Z}$, and $\hat{F}_n^t(\mathbf{x})$ the corresponding d-dimensional empirical distribution function. Then, the difference $\widehat{\rho}_{|\mathcal{I}|,n}^t - \widehat{\rho}_{|\mathcal{I}|,n}^{t-s} (s < n)$ can be written as

$$\widehat{\rho}_{|\mathcal{I}|,n}^t - \widehat{\rho}_{|\mathcal{I}|,n}^{t-s} = h(|\mathcal{I}|)2^{|\mathcal{I}|} \int_{[0,1]^d} \Big\{ \hat{C}_n^t(\mathbf{u}^{(\mathcal{I})}) - \hat{C}_n^{t-s}(\mathbf{u}^{(\mathcal{I})}) \Big\} d\mathbf{u}.$$

Since $C = \phi(F)$ and $\hat{C}_n^t = \phi(\hat{F}_n^t)$ with Hadamard-differentiable map ϕ (see Fermanian et al. (2004) and Van der Vaart and Wellner (1996) for the relevant definitions and background reading), this yields

$$\widehat{\rho}_{|\mathcal{I}|,n}^t - \widehat{\rho}_{|\mathcal{I}|,n}^{t-s} = h(|\mathcal{I}|)2^{|\mathcal{I}|} \int_{[0,1]^d} \Big\{ \phi(\hat{F}_n^t)(\mathbf{u}^{(\mathcal{I})}) - \phi(\hat{F}_n^{t-s})(\mathbf{u}^{(\mathcal{I})}) \Big\} d\mathbf{u}.$$

Furthermore, observe that

$$n\Big\{ \hat{F}_n^t(\mathbf{x}^{(\mathcal{I})}) - \hat{F}_n^{t-s}(\mathbf{x}^{(\mathcal{I})}) \Big\} = \sum_{j=t-s+1}^{t} \prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{X_{ij} \le x_i\}} - \sum_{j=t-n-s+1}^{t-n} \prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{X_{ij} \le x_i\}} \stackrel{d}{=} Y^{t,s}(\mathbf{x}^{(\mathcal{I})}), \quad (22)$$

the latter distribution being independent of n. An application of the Delta-method given in Theorem 3.9.4 in Van der Vaart and Wellner (1996) leads to

$$n\Big\{ \hat{C}_n^t(\mathbf{u}^{(\mathcal{I})}) - \hat{C}_n^{t-s}(\mathbf{u}^{(\mathcal{I})}) \Big\} = n\Big\{ \phi(\hat{F}_n^t)(\mathbf{u}^{(\mathcal{I})}) - \phi(\hat{F}_n^{t-s})(\mathbf{u}^{(\mathcal{I})}) \Big\} \stackrel{d}{\longrightarrow} \phi'(Y^{t,s})(\mathbf{u}^{(\mathcal{I})}). \quad (23)$$

Finally, the continuous mapping theorem yields

$$n\Big\{ \widehat{\rho}_{|\mathcal{I}|,n}^t - \widehat{\rho}_{|\mathcal{I}|,n}^{t-s} \Big\} \stackrel{d}{\longrightarrow} h(|\mathcal{I}|)2^{|\mathcal{I}|} \int_{[0,1]^d} \phi'(Y^{t,s})(\mathbf{u}^{(\mathcal{I})}) d\mathbf{u} = Z_{|\mathcal{I}|}^{t,s}. \quad (24)$$

(ii) We start by proving that $n\{\hat{C}_n^t(\mathbf{u}^{(\mathcal{I})}) - \hat{C}_n^{t-s}(\mathbf{u}^{(\mathcal{I})})\}$ is uniformly bounded in $\mathbf{u} \in [0,1]^d$ for fixed $s < n$. Observe that

$$\begin{aligned} n\{\hat{C}_n^t(\mathbf{u}^{(\mathcal{I})}) - \hat{C}_n^{t-s}(\mathbf{u}^{(\mathcal{I})})\} &= \sum_{j=t-s+1}^{t} \prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{\hat{U}_{ij,n}^t \le u_i\}} - \\ - \sum_{j=t-n+1}^{t-s} \Big(\prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{\hat{U}_{ij,n}^t \le u_i\}} &- \prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{\hat{U}_{ij,n}^{t-s} \le u_i\}} \Big) - \sum_{j=t-s-n+1}^{t-n} \prod_{\substack{i=1 \\ i \in \mathcal{I}}}^{d} \mathbf{1}_{\{\hat{U}_{ij,n}^{t-s} \le u_i\}}, \end{aligned} \quad (25)$$

where $\hat{U}^t_{ij,n} = \frac{1}{n}$ (rank of X_{ij} in $X_{i(t-n+1)}, \ldots, X_{it}$), based on the sample $\mathbf{X}_{t-n+1}, \ldots, \mathbf{X}_t$, and $\hat{U}^{t-s}_{ij,n} = \frac{1}{n}$ (rank of X_{ij} in $X_{i(t-s-n+1)}, \ldots, X_{i(t-s)}$), which are based on the sample $\mathbf{X}_{t-s-n+1}, \ldots, \mathbf{X}_{t-s}$, respectively.

Note that the random variables $\hat{U}^t_{ij,n}$ and $\hat{U}^{t-s}_{ij,n}$ in the middle term of (25) deviate by a maximum of s/n only since the underlying rank order statistics are based on the $(n-s-1)$ common random variables $\mathbf{X}_{t-n+1}, \ldots, \mathbf{X}_{t-s}$ for all $i \in \mathcal{I}$. For each fixed $\mathbf{u} \in [0,1]^d$, there exists at most s index value $j_1, \ldots, j_s \in \{t-n+1, \ldots, t-s\}$ for which the middle term does not equal zero owing to the bijective mapping of $\hat{U}^t_{ij,n}$ and $\hat{U}^{t-s}_{ij,n}$ onto $\{\frac{1}{n}, \ldots, \frac{n}{n}\}$. Thus,

$$\Big|\sum_{j=t-n+1}^{t-s}\Big(\prod_{\substack{i=1\\ i\in\mathcal{I}}}^{d}\mathbf{1}_{\{\hat{U}^t_{ij,n}\le u_i\}} - \prod_{\substack{i=1\\ i\in\mathcal{I}}}^{d}\mathbf{1}_{\{\hat{U}^{t-s}_{ij,n}\le u_i\}}\Big)\Big| \le s \tag{26}$$

for each $\mathbf{u} \in [0,1]^d$. Including the other terms of formula (25) yields

$$|n\{\hat{C}^t_n(\mathbf{u}^{(\mathcal{I})}) - \hat{C}^{t-s}_n(\mathbf{u}^{(\mathcal{I})})\}| \le 3s$$

and, consequently,

$$|n\{\widehat{\rho}^t_{|\mathcal{I}|,n} - \widehat{\rho}^{t-s}_{|\mathcal{I}|,n}\}| \le 3s2^{|\mathcal{I}|}h(|\mathcal{I}|)$$

Thus, the bounded convergence theorem (see, for example, Theorem 10.32 in Wheeden and Zygmund (1977)) together with part (i) of the theorem yields

$$n^2\, Cov\Big(\widehat{\rho}^t_{|\mathcal{I}|,n} - \widehat{\rho}^{t-s}_{|\mathcal{I}|,n}\ ,\ \widehat{\rho}^{t-r}_{|\mathcal{I}|,n} - \widehat{\rho}^{t-r-s}_{|\mathcal{I}|,n}\Big) \longrightarrow Cov\Big(Z^{t,s}_{|\mathcal{I}|}, Z^{t-r,s}_{|\mathcal{I}|}\Big).$$

Finally, formula (22) together with formula (23) shows that the limiting variables $Z^{t,s}_{|\mathcal{I}|}$ and $Z^{t-r,s}_{|\mathcal{I}|}$ are stochastically independent and are thus uncorrelated for $n > r > s > 0$. □

Proof of Theorem 4. Let $\hat{C}^B_{n,\mathbf{X}}(\mathbf{u})$ and $\hat{C}^B_{m,\mathbf{Y}}(\mathbf{u}), \mathbf{u} \in [0,1]^d$, denote the empirical copula of the independent random sample $(\mathbf{X}^B_l)_{l=1,\ldots,n}$ and $(\mathbf{Y}^B_l)_{l=1,\ldots,m}$, obtained by sampling from the independent random samples $(\mathbf{X}_l)_{l=1,\ldots,n}$ and $(\mathbf{Y}_l)_{l=1,\ldots,m}$ with replacement, respectively.

With $m := m(n)$ such that $\frac{\sqrt{n}}{\sqrt{m(n)}} \to c$ for $n \to \infty$, Theorem 5 in Fermanian et al. (2004) implies that $\sqrt{n}(\hat{C}^B_{n,\mathbf{X}} - \hat{C}_{n,\mathbf{X}})$ and $\sqrt{n}(\hat{C}_{n,\mathbf{X}} - C_{\mathbf{X}})$ as well as $\sqrt{m(n)}(\hat{C}^B_{m(n),\mathbf{Y}} - \hat{C}_{m(n),\mathbf{Y}})$ and $\sqrt{m(n)}(\hat{C}_{m(n),\mathbf{Y}} - C_{\mathbf{Y}})$ converge weakly to the same Gaussian limit with probability 1, respectively, for $n \to \infty$. The application of the continuous mapping theorem yields that $\sqrt{n}\{S^B_{A,n,\mathbf{X}} - S_{A,n,\mathbf{X}}\}$ and $\sqrt{n}\{S_{A,n,\mathbf{X}} - \boldsymbol{\rho}_{A,\mathbf{X}}\}$, and $\sqrt{m(n)}\{S^B_{A,m(n),\mathbf{Y}} - S_{A,m(n),\mathbf{Y}}\}$ and $\sqrt{m(n)}\{S_{A,m(n),\mathbf{Y}} - \boldsymbol{\rho}_{A,\mathbf{Y}}\}$ converge to the same limit, respectively, with probability

1. With the assumption that $\boldsymbol{\rho}_{A,\mathbf{X}} = \boldsymbol{\rho}_{A,\mathbf{Y}}$, the assertion follows according to the same reasoning as in the proof for Theorem 2, part (ii). □

References

Andersen, T.G., T. Bollerslev, P.F. Christoffersen, and F.X Diebold, 2005, Practical Volatility and Correlation Modeling for Financial Market Risk Management, in: *Risks of Financial Institutions*, eds. M. Carey and R. Stultz , University of Chicago Press for NBER, 513–548.

Artzner, P.F., F. Delbaen, J.-M. Eber, and D. Heath, 1999, Coherent Measures of Risk, *Mathematical Finance* 9, 203–228.

Bae, K.H., G.A. Karolyi, and R.M. Stulz, 2003, A new approach to measure financial contagion, *Review of Financial Studies* 16, 717–763.

Berkowitz, J., and J. O'Brien, 2002, How accurate are Value-at-Risk Models at Commercial Banks?, *Journal of Finance* 57, 1093–1111.

Bartram, S.M., S.J. Taylor, and Y.H. Wang, 2007, The Euro and European financial market dependence, *Journal of Banking and Finance* 31 (5), 1461–1481.

Campbell, R., K. Koedijk, and P. Kofman, 2002, Increased Correlation in Bear Markets, *Financial Analysts Journal* Jan-Feb, 87–94.

Charpentier, A., 2006, *Dependence structure and limiting results: some applications in finance and insurance*, PhD thesis Universiteit Leuven, Retrievable from: http://www.crest.fr/pageperso/charpent/phd.htm.

Davison, A.C., and D.V. Hinkley, 1997, *Bootstrap Methods and their Applications*, Cambridge University Press.

Deheuvels, P., 1979, La fonction de dépendance empirique et ses propriétés. Un test non paramétrique d'indépendence, *Acad. Roy. Belg., Bull. Cl. Sci.* 65(5), 274–292.

Duellmann, K., M. Scheicher, and C. Schmieder, 2007, Asset correlations and credit portfolio risk - An empirical analysis, *Discussion Paper, Series 2: Banking and Fiancial Studies*, 2007/13, Deutsche Bundesbank.

Embrechts, P., F. Lindskog, and A. McNeil, 2001, Modelling Dependence with Copulas and Applications to Risk Management, in: *Handbook of Heavy Tailed Distributions in Finance*, ed. S. Rachev, Elsevier, Chapter 8, 329-384.

Embrechts, P., A. McNeil, and D. Straumann, 2002, Correlation and Dependency in Risk Management: Properties and Pitfalls, in: *Risk Management: Value at Risk and Beyond*, M.A.H. Dempster (ed.), Cambridge University Press, Cambridge, 176–223.

Efron, B., and R. J. Tibshirani, 1993, *An Introduction to Bootstrap*, Chapman & Hall, New York, London.

Fermanian, J-D., D. Radulovic, and M. Wegkamp, 2004, Weak convergence of empirical copula processes, *Bernoulli* 10 (5), 847–860.

Genest, C., and B. Rémillard, 2004, Tests of Independence and Randomness Based on the Empirical Copula Process, *Test* 13(2), 335–369.

Giacomini, E., W. Härdle, and V. Spokoiny, 2008, Inhomogeneous Dependency Modelling with Time Varying Copulae, *J. Business and Economic Statistics*, forthcoming.

Golosnoy, V., and W. Schmid, 2007, EWMA Control Charts for Monitoring Optimal Portfolio Weights, *Sequential Analysis* 26, 195–224.

Hofert, M., 2008, Sampling Archimedean copulas, *Computational Statistics & Data Analysis* 52(12), 5163–5174.

Jaschke, S., G. Stahl, and R. Stehle, 2003, Evaluating VaR Forecasts under Stress - The German Experience, *CFS Working Paper Series*, 2003/32, Center for Financial Studies.

Joe, H., 1990, Multivariate Concordance, *Journal of Multivariate Analysis* 35, 12–30.

Joe, H., 1997, *Multivariate Models and Dependence Concepts*, Chapman & Hall.

Jorion, P., 2006, *Value at Risk*, 3rd. edn, McGraw-Hill, New York.

Karolyi, G.A. and R.M. Stulz, 1996, Why Do Markets Move Together? An Investigation of U.S.-Japan Stock Return Co-movements, *Journal of Finance* 51, 951–989.

Kendall, M.G., 1970, *Rank Correlation methods*, Griffin, London.

Longin, F., and B. Solnik, 2001, Extreme Correlation of International Equity Markets, *Journal of Finance* LVI, 649–676.

McNeil, A.J., Frey, R. and Embrechts, P., 2005, *Quantitative risk management: concepts, techniques and tools*, Princeton Series in Finance, Princeton University Press.

Memmel, C., and C. Wehn, 2006, Supervisor's portfolio: The market price risk of German banks from 2001 to 2004: Analysis and models for risk aggregation, *Journal of Banking Regulation* 7, 3/4, 310 – 325 .

Minderhoud, K., 2006, Systemic Risk in the Dutch Financial Sector, *De Economist* 154(2), 177–195.

Nelsen, Roger B., 1996, Nonparametric measures of multivariate association, *in: Distribution with Fixed Marginals and Related Topics, IMS Lecture Notes - Monograph Series* 28, 223–232.

Nelsen, R.B., 2006, *An Introduction to Copulas*, 2nd edn., Springer Verlag New York, Inc.

O'Kane, D., and L. Schloegl, 2005, The large homogeneous portfolio approximation with the Student-t copula, *Finance and Stochastics* 4, 577–584.

Patel, N., 2005, Crisis of correlation, *Risk Magazine*, June.

Patton, A., 2006, Modelling asymmetric exchange rate dependence, *International Economic Review* 47(2), 527–556.

Pawlak, M., E. Rafajlowicz, and A. Steland, 2004, On detecting jumps in time series: nonparametric setting, *Journal of Nonparametric Statistic* 16, Numbers 3-4, 329–347.

Rodriguez, J.C., 2007, Measuring Financial Contagion: A Copula approach, *Journal of Empirical Finance* 14 (3), 401–423.

Rüschendorf, L., 1976, Asymptotic normality of multivariate rank order statistics, *Annals of Statistics* 4, 912–923.

Ruymgaart, F.H., and M.C.A. van Zuijlen, 1978, Asymptotic normality of multivariate linear rank statistics in the non-i.i.d. case, *Annals of Statistics* 6 (3), 588–602.

Savu, C., and M. Trede 2006, Hierarchical Archimedean Copulas, working paper, Institute of Econometrics, University of Münster.

Savu, C., and M. Trede 2008, Goodness-of-fit Tests for Parametric Families of Archimedean Copulas, *Quantitative Finance* 8, 109–116.

Schmid, W., and S. Knoth, 2001, Control Charts for Time Series: A Review, *Discussion paper*, No 173, Europe-University Frankfurt (Oder).

Schmid, F., and R. Schmidt, 2007, Multivariate Extensions of Spearman's Rho and Related Statistics, *Statistics and Probability Letters* 77 (4), 407–416.

Schmid, F., and R. Schmidt, 2006, Bootstrapping Spearman's Multivariate Rho, *COMPSTAT, Proceedings in Computational Statistics*, edited by Alfredo Rizzi and Maurizio Vichi.

Shao, Jun, 1999, *Mathematical Statistics*, Springer Verlag New York, Inc.

Shao, J., and D. Tu, 1993, *The Jacknife and Bootstrap*, Springer Verlag, New York, Inc.

Sklar, A., 1959, Fonctions de rèpartition à n dimensions et leur marges, *Publ. Inst. Statist. Univ. Paris* 8, 229–231.

Steland, A., 2002, Sequential credit risk: Can we benefit from sequential nonparametric control?, *Selected Papers Operation Research Proceedings* 43, 337–344.

Stute, W., 1984, The oscillation behavior of empirical processes: The multivariate case, *Annals of Probability* 12, 361–379.

Taylor, M.D., 2007, Multivariate measures of concordance, *Annals of the Institute of Statistical Mathematics* 59(4), 789–806.

Tsukahara, H., 2005, Semiparametric estimation in copula models,*Canadian Journal of Statistics* 33(3), 357–375.

Úbeda-Flores, M., 2005, Multivariate versions of Blomqvist's beta and Spearman's footrule, *Annals of the Institute of Statistical Mathematics* 57(4), 781–788.

Van der Vaart, A.W., and J.A. Wellner, 1996, *Weak Convergence and Empirical Processes*, Springer Verlag, New York.

Wheeden, R.L., and A. Zygmund, 1977, *Measure and Integral: An Introduction to Real Analysis*, Marcel Dekker, Inc., New York.

Wieringa, J. E., 1999, *Statistical process control for serially correlated data*, PhD thesis University of Groningen, retrievable from http://dissertations.ub.rug.nl/faculties/eco/1999/j.e.wieringa/.

Wolff, E.F., 1980, N-dimensional measures of dependence, *Stochastica* 4 (3), 175–188.

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