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Systematic risk of CDOs and CDO arbitrage

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Abstract

"Arbitrage CDOs" have recorded an explosive growth during the years before the outbreak of the financial crisis. In the present paper we discuss potential sources of such arbitrage opportunities, in particular arbitrage gains due to mispricing. For this purpose we examine the risk profiles of Collateralized Debt Obligations (CDOs) in some detail. The analyses reveal significant differences in the risk profile between CDO tranches and corporate bonds, in particular concerning the considerably increased sensitivity to systematic risks. This has far-reaching consequences for risk management, pricing and regulatory capital requirements. A simple analytical valuation model based on the CAPM and the single-factor Merton model is used in order to keep the model framework simple. Then, the conditional expected loss curve (EL profile) is studied in some detail. In the next step, the asset correlation associated with a CDO tranche is estimated treating the structured instrument as a single-name credit instrument (i.e., a loan equivalent). While tractable, the loan-equivalent approach requires appropriate parameterization to achieve a reasonable approximation of the tranche´s risk profile. We consider the tranche as a "virtual" borrower or bond for which a single-factor model holds. Then, the correlation parameter is calculated via a non-linear optimization. This "bond representation" allows to approximate the risk profile (expressed by the EL profile) using a single-factor model and to express the dependence on the systematic risk factor via the corresponding asset correlation. It turns out that the resulting asset correlation is many times higher than that of straight bonds. Then, the Merton type valuation model for the corresponding bond representations is applied for valuation of the CDO tranches. Using a sample CDO portfolio, some opportunities for "CDO arbitrage" are described where it is assumed that investors are guided solely by the tranches' rating and ignore the increased systematic risk for pricing. In the next section we discuss how tranches with high systematic risk can be generated and how CDO arrangers can exploit this to their advantage. It comes as no surprise that precisely these types of structures featured in many of the CDOs issued prior to the outbreak of the financial crisis.

Keywords: Collateralized debt obligations (CDO), Arbitrage CDOs, credit rating, expected loss profile, bond representation, systematic risk of CDO tranches, CDO pricing

Jel Classifications: C13, G01, G12, G24

Non-technical summary

Due to the information asymmetry between the originators of structured finance instruments and the investors and the lack of reliable data on the collateral pools even sophisticated investors with the necessary knowledge were not able to conduct an extensive valuation and risk analysis of the structured instruments. So they treated them like corporate or sovereign bonds and relied solely on the agencies' ratings. The use of the same rating scale for CDO tranches and corporate bonds by the rating agencies obviously hid that there is a significantly different risk profile also at the same credit rating with regard to the systematic risk. Structured finance instruments appeared very attractive to investors as they could choose between different risk/return profiles. Moreover, CDO tranches tended to offer more yield than corporate bonds or loans with similar ratings.

Among CDOs, "arbitrage CDOs" in particular have recorded enormous growth. Arbitrage CDOs attempt to produce a surplus from the collateral pool's acquired assets on the one hand, and from the tranches sold to the investors after securitization on the other hand. In the present paper we discuss potential CDO arbitrage opportunities due to mispricing. For this purpose the risk profiles of CDO tranches are analysed in some detail. The analyses presented here reveal significant risk profile differences between CDOs and corporate bonds which have far-reaching consequences for risk management, pricing and regulatory capital requirements. Particularly due to the increased sensitivity to systematic risks, CDOs become much riskier investments than comparable corporate bonds. This results *inter alia* in rapid and drastic rating downgrades once the underlying economic environment comes under stress. Another major consequence stems from the price relevance of systematic risks. The considerably increased systematic risk of CDO tranches needs to be compensated for by a significantly higher spread than is usually paid for corporate bonds. If investors do not appreciate the increased systematic risk, this offers arbitrage opportunities for the CDO issuers.

Coval/Jurek/Stafford (2009a, b) and Brennan/Hein/Poon (2009) analyse similar aspects of a possible mispricing of CDO tranches. As in the present paper, the authors argue that investors who solely rely on the rating generally pay a too high price for the CDO tranches. According to Coval/Jurek/Stafford (2009a), this is especially true for senior tranches which default just when the economy is in a very bad state. If a broad equity index is used as a proxy for the market factor, option prices can be used. Using special options on the S&P 500 index with comparable systematic risks, the authors show empirically that the highly rated tranches of the CDX index exhibit market spreads that are significantly too low. By contrast, the results obtained by Brennan/Hein/Poon (2009) indicate that the AAA-rated tranches are only marginally mispriced, and that the highest profits can be gained with junior tranches.

In contrast to these papers we attempt to quantify the size of the CDOs' sensitivity to systematic risks as precisely as possible, while keeping the model framework relatively simple. For this, a single factor model is assumed for the collateral pool assets, and the loss distribution is calculated. Then each of the expected tranche losses are analysed conditional on the market factor realisations. It turns out that expected loss curve conditional on the market factor (EL profile) is much steeper for unfavourable market factor values than for a bond with a comparable rating or for the collateral pool. This expresses the greatly increased sensitivity of the tranches to systematic risk. In the next step, the asset correlation associated with a CDO tranche is estimated treating the structured instrument as a single-name credit instrument (i.e., a loan equivalent). While tractable, the loan-equivalent approach requires appropriate parameterization to achieve a reasonable approximation of the tranche´s risk profile. We consider the tranche as a "virtual" borrower or bond for which a single-factor model holds. Then, the correlation parameter is calculated via a non-linear optimization. This "bond representation" allows to approximate the risk profile (expressed by the EL profile) using a single-factor model and to express the dependence on the systematic risk factor via the corresponding asset correlation. It turns out that the resulting asset correlation is many times higher than that of ordinary bonds. In the next step a pricing model which depends explicitly on the asset correlation as a measure of systematic risk is implemented for the CDO tranches. CDO arbitrage opportunities are then described using a sample CDO portfolio. It is assumed that investors are guided solely by the tranches' rating and ignore the increased systematic risk. We subsequently show how tranches with high systematic risk can be generated and describe how CDO arrangers can exploit this to their advantage. It comes as no surprise that precisely these types of structures featured in many of the CDOs issued prior to the outbreak of the financial crisis.

Nichttechnische Zusammenfassung

Die Informationsasymmetrie zwischen den Originatoren von komplexen strukturierten Kreditprodukten wie CDOs und den Investoren wurde massiv verstärkt durch die mangelnde Verfügbarkeit zeitnaher Daten zu den Collateral Pools. Auch für Investoren mit dem notwendigen Knowhow war es nicht möglich, eine umfassende Bewertung und Risikoanalyse der strukturierten Produkte vorzunehmen. Viele Investoren haben CDO-Tranchen wie Corporate oder Sovereign Bonds betrachtet und sich vollständig auf die Ratings der Produkte verlassen. Die Verwendung derselben Ratingskala für CDO-Tranchen und Corporate Bonds durch die Ratingagenturen hat offensichtlich verschleiert, dass auch bei gleichem Rating ein signifikant unterschiedliches Risikoprofil hinsichtlich des systematischen Risikos vorliegt. Da die Investoren entsprechend ihrem Rendite-/Risikoprofil aus verschiedenen Tranchen auswählen konnten und darüber hinaus die strukturierten Produkte anfänglich höhere Spreads zahlten, erschienen sie sehr attraktiv.

Bei den CDOs haben insbesondere die „Arbitrage-CDOs" ein enormes Wachstum verzeichnet. Arbitrage-CDOs versuchen einen Überschuss aus den erworbenen Assets des Collateral Pools einerseits und den nach der Strukturierung an die Investoren verkauften Tranchen andererseits zu erzielen. Für das Zustandekommen der CDO-Arbitrage gibt es in der Literatur unterschiedliche Begründungen. Im vorliegenden Beitrag wird untersucht, wie durch Fehlbepreisungen Möglichkeiten zur CDO-Arbitrage entstehen. Hierzu werden die Risikoprofile von CDO-Tranchen im Detail untersucht und potentielle Unterschiede zu Corporate Bonds mit vergleichbaren Ratings genauer überprüft. Die vorgestellten Analysen zeigen signifikante Differenzen im Risikoprofil zwischen CDOs und Corporate Bonds, welche weitreichende Konsequenzen in Bezug auf Risikomanagement, Bepreisung und regulatorische Eigenkapitalanforderungen haben. Dies betrifft vor allem die deutlich erhöhte Sensitivität gegenüber systematischen Risiken. Dadurch werden CDOs zu wesentlich riskanteren Investments als vergleichbare Corporate Bonds. Eine Konsequenz sind rapide und drastische Ratingänderungen (Downgrades), wenn die ökonomischen Rahmenbedingungen unter Stress geraten. Eine weitere wichtige Konsequenz, die für eine inadäquate Bepreisung ausschlaggebend ist, ergibt sich aus der Preisrelevanz des systematischen Risikos. Das deutlich erhöhte systematische Risiko der CDO-Tranchen muss durch einen signifikant höheren Spread, als für Corporate Bonds gezahlt wird, kompensiert werden.

Ähnliche Aspekte einer möglichen Fehlbepreisung von CDO-Tranchen untersuchen Coval et al. (2009a,b) und Brennan et al. (2009). Wie im vorliegenden Paper argumentieren die Autoren, dass Investoren, die sich lediglich am Rating orientieren, in der Regel einen zu hohen Preis für die CDO-Tranchen bezahlen. Gemäß Coval et al. (2009a) gilt dies im besonderen Maße für die Senior-Tranchen, die gerade dann ausfallen, wenn sich die

Ökonomie in einem sehr schlechten Zustand befindet. Diese Zustände haben jedoch einen hohen marginalen Nutzen und die zugehörigen Zustandspreise sollten gemäß der Arrow/Debreu-Theorie hoch sein. Anhand spezieller Optionen auf einen Marktindex mit vergleichbaren systematischen Risiken zeigen sie, dass die hoch gerateten Tranchen des CDX Index signifikant zu niedrige Marktspreads aufweisen. Im Gegensatz dazu finden Brennan et al. (2009) heraus, dass die AAA-Tranchen nur geringfügig fehlbepreist sind und dass die höchsten Gewinne bei der Tranchierung mit den riskanteren Junior-Tranchen erzielt werden können.

Im Unterschied zu den beiden Arbeiten wird im vorliegenden Beitrag versucht, die Größenordnung der Sensitivität von CDOs gegenüber systematischen Risiken möglichst exakt zu quantifizieren. Dabei soll der Modellrahmen möglichst einfach gehalten werden. Hierzu wird für die Assets des Collateral Pools ein Ein-Faktor-Modell unterstellt und die Verlustverteilung ermittelt. Dann werden die erwarteten Verluste der Tranchen jeweils bedingt auf die Realisierungen des Marktfaktors analysiert. Die bedingte Expected-Loss-Kurve (EL-Profil) wird mit Hilfe einer Simulation ermittelt. Im nächsten Schritt wird die Assetkorrelation einer CDO-Tranche geschätzt. Dabei wird die Tranche als „fiktiver" Schuldner bzw. Bond aufgefasst, für den approximativ ebenfalls ein Ein-Faktor-Modell gilt. Die Approximation wird so vorgenommen, dass das simulierte EL-Profil möglichst gut nachgestellt wird, und der Korrelationsparameter wird mit Hilfe einer nichtlinearen Optimierung ermittelt. Durch die „Bond-Repräsentation" gelingt es, das Risikoprofil von CDO-Tranchen mit Hilfe eines Ein-Faktor-Modells zu approximieren und die Abhängigkeit vom systematischen Risikofaktor durch den entsprechenden Korrelationsparameter auszudrücken. Die resultierende Assetkorrelation ist um ein Vielfaches höher als für gewöhnliche Bonds. Dann wird für die CDO-Tranchen ein Preismodell implementiert. Hierzu wird die risikoneutrale Ausfallwahrscheinlichkeit mit Hilfe des strukturellen Firmenwertmodells ermittelt und in Abhängigkeit von der realen Ausfallwahrscheinlichkeit und dem systematischen Risiko spezifiziert, wobei die Gültigkeit des (zeitstetigen) CAPM unterstellt wird. Danach werden anhand eines Beispiel-CDOs Möglichkeiten zur „CDO-Arbitrage" beschrieben, wenn sich die Investoren nur am Rating der Tranchen orientieren und das erhöhte systematische Risiko außer Acht lassen. Schließlich wird im letzten Kapitel die Konstruktion von Tranchen mit hohem systematischen Risiko dargestellt und beschrieben, wie dies von den CDO-Arrangeuren zu ihren Gunsten ausgenützt werden kann. Es ist nicht überraschend, dass gerade derartige Strukturen in vielen der in der Zeit vor Ausbruch der Krise emittierten CDOs zu finden sind.

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Systematic Risk of CDOs and CDO Arbitrage*

1 Introduction

The technique of securitisation allows to transfer a portfolio of illiquid assets or the default risks associated with the assets to the structured credit markets using various credit transfer instruments and to distribute much of the underlying credit risk to end-investors. This led to a dramatic rise in the market for credit transfer instruments, especially Credit Default Swaps (CDS) und Collateralized Debt Obligations (CDOs). For example, the volume of outstanding CDS has increased more than tenfold over the past few years.

Among CDOs, "arbitrage CDOs" in particular have recorded enormous growth. Arbitrage CDOs attempt to produce a surplus from the collateral pool's acquired assets on the one hand, and from the tranches sold to the investors after securitization on the other hand. The Figure below shows the issuance volumes between Q1 2004 and Q4 2008. The issuance volumes of "Balance Sheet CDOs" are given for comparison.

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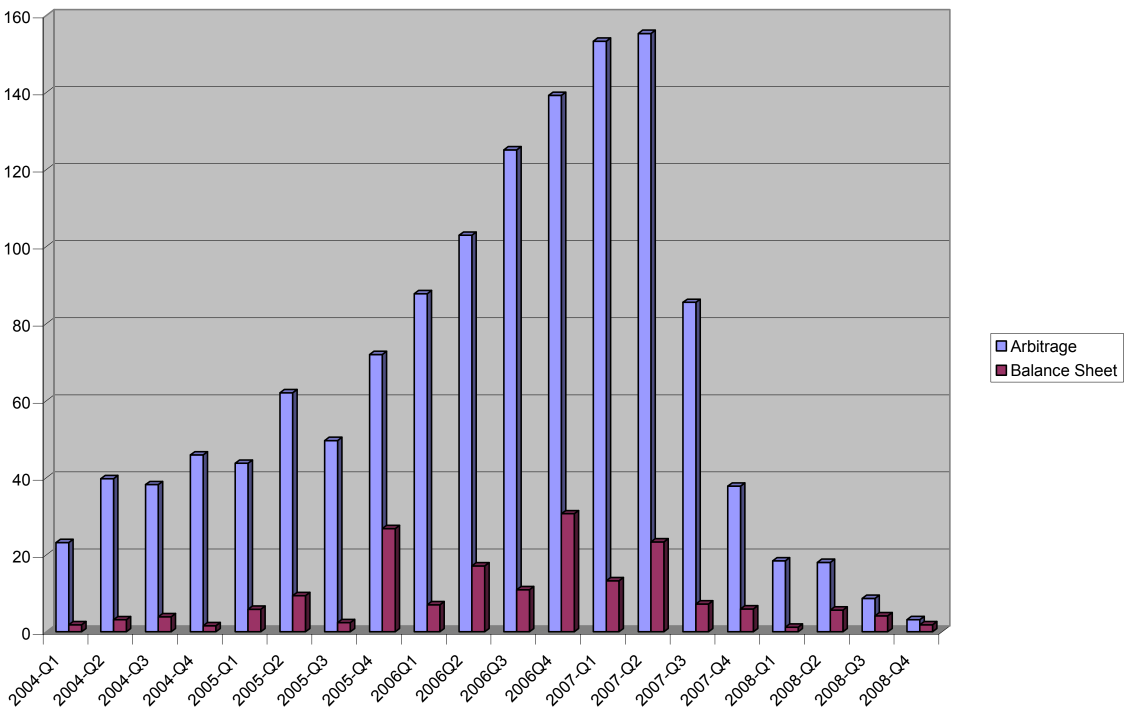

Figure 1: Issuance volumes of arbitrage CDOs and balance sheet CDOs between 2004 and 2008 (source: SIFMA)

In the case of arbitrage CDOs, there is a sharp increase until the beginning of the financial crisis from around US$23 billion in Q1 2004 to over US$155 billion in Q2 2007. A correspondingly large volume of loans had to be granted in order to generate this high volume of CDOs, with loans with higher yields (and greater risks) being preferred. This created a huge leap in the demand for subprime loans. CDO trusts were constantly looking for BBB-rated MBS (mortgage backed securities) bonds, which offered relatively high yields. The CDO trust could finance their purchase by issuing highly rated CDO bonds (tranches) paying lower yields. The business was so lucrative that there were insufficient MBS bonds in circulation to form CDO collateral pools in 2006 and 2007. A solution was found with the introduction of credit default swaps on asset-backed securities (ABS CDS). The result was a large number of CDO collateral pools "topped up" with synthetic MBS bonds (MBS CDS). This environment encouraged questionable practices by some lenders, and lead to a dramatic fall in credit lending standards that was often described in the media.[1]

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[1] For a description of the chronology and causes of the financial crisis, see, for example, Crouhy et al (2008), Hull (2009), van Deventer (2008), Ashcraft and Schürmann (2009), Krinsman (2007), Zimmerman (2007), Borio (2008), Rudoph (2008) or the German Council of Economic Experts (2007), Chapter 3.

The valuation by rating agencies is crucial for complex structured finance instruments such as CDOs. Due to their complexity, however, it is difficult to rate CDOs. The waterfall structures and loss distributions are always different and have to be analysed and simulated on an individual basis. Also, many collateral pools contain synthetic ABS CDS, which increases the difficulty in rating these assets.

A further problem is the information asymmetry between the originators of the structured finance instruments and the investors. Generally, data on the collateral pools are not available on an individual basis, except for synthetic CDOs with corporate CDSs in the collateral pool. As the database had not been disclosed, investors had no reliable basis upon which to quantify the risks of these securities, and even investors with the necessary knowledge were not able to conduct an extensive valuation and risk analysis of the structured instruments. They had to almost completely rely on the agencies' ratings. Even though the regulatory authorities have pointed out that the use of ratings does not excuse investors from conducting their own risk analyses, many of them relied in practice purely on external ratings for valuing structured credit products (see Bank of England, 2007). Many investors regarded CDO tranches like corporate or sovereign bonds with the same rating and, in particular, also equated their risks with those of bonds.

Structured finance instruments appeared very attractive to investors as they could choose between different risk/return profiles. Moreover, CDO tranches tended to offer more yield than corporate bonds or loans with similar ratings. This price advantage did not last, however. Between 2005 and the beginning of the subprime crisis, rising investor confidence in the new asset class led to a rising demand. Thus the spread differences narrowed and CDOs and corporate bonds with comparable ratings were quoted at almost equal prices (see Brennan/Hein/Poon, 2009, Figure 1).

In this paper, the risk profiles of CDO tranches are analysed in some detail and potential differences to corporate bonds that have comparable ratings are examined more closely. The analyses presented here reveal significant risk profile differences between CDOs and corporate bonds which have far-reaching consequences for risk management, pricing and regulatory capital requirements. Particularly due to the increased sensitivity to systematic risks CDOs become much riskier investments than comparable corporate bonds. This results *inter alia* in rapid and drastic rating downgrades once the underlying economic environment comes under stress. Another major consequence stems from the price relevance of systematic risks. The considerably increased systematic risk of CDO tranches needs to be compensated for by a significantly higher spread than is usually paid for corporate bonds. If investors do not appreciate the increased systematic risk, this offers arbitrage opportunities for the CDO issuers.

Coval/Jurek/Stafford (2009a,b) and Brennan/Hein/Poon (2009) analyse similar aspects of a possible mispricing of CDO tranches. As in the present paper, the authors argue that investors who solely rely on the rating generally pay a too high price for the CDO tranches. According to Coval/Jurek/Stafford (2009a), this is especially true for senior tranches which default just when the economy is in a very bad state. These states have a high marginal utility, however, and according to the Arrow-Debreu theory, the associated state prices should be high. If a broad equity index is used as a proxy for the market factor, option prices can be used. When the index does exhibit such unfavourable conditions, the "volatility skew" becomes important, and investors should be entitled to an additional spread for bearing this risk. Using special options on the S&P 500 index with comparable systematic risks, the authors show empirically that the highly rated tranches of the CDX index exhibit market spreads that are significantly too low. By contrast, Brennan/Hein/Poon (2009) find that the AAA-rated tranches are only marginally mispriced, and that the highest profits can be gained with junior tranches.

In line with these two articles we also see limited risk assessment of CDO tranches prior to the crisis. Market participants based their investment decisions on a tranche's rating grade and otherwise assumed default behaviour like that of straight bonds, in particular concerning the exposure to systematic risks. As a consequence non-equity CDO tranches were overvalued. In contrast to these articles, our objective is not precisely quantify the degree of mispricing in the CDO market. Instead we concentrate on an illustrative presentation of differences in the default behaviour of corporate bonds and CDO tranches using a single-factor model framework, which can be derived from a simple analytical valuation model based on the CAPM and the Merton model. This allows developing a parsimonious pricing approach. As it relates directly to the mentioned differences between bonds and CDO tranches and because of its less complicated structure it sheds light on the consequences of possible misjudgements concerning the risk properties of CDO tranches. By applying our valuation model on a sample arbitrage CDO transaction, we conclude that there is no CDO arbitrage possible as long as investors take the special default behaviour of CDO tranches into account. However, if investors base their investment decisions solely on the ratings of the CDO tranches, this creates a market environment that allows CDO arrangers to gain some kind of arbitrage profit.

In contrast to the above mentioned papers, here we attempt to quantify the size of the CDOs' sensitivity to systematic risks. For this purpose we analyse the expected tranche loss of a CDO tranche conditional on the market factor realisations (EL profile). In the next step, the asset correlation associated with a CDO tranche is estimated treating the structured instrument as a single-name credit instrument (i.e., a loan equivalent). While tractable, the loan-equivalent approach requires appropriate parameterization to achieve a reasonable approximation of the tranche´s risk profile. We consider the tranche as a "virtual" borrower or bond for which a single-factor model holds. Then, the correlation parameter is estimated via an approximation of the EL profile. This "bond representation" allows to approximate the risk profile using a single-factor model and to express the dependence on the systematic risk factor via the corresponding asset correlation. It turns out that the resulting asset correlation is many times higher than that of ordinary bonds. Then, the CDO tranches are priced by applying the valuation model to their corresponding bond representations. In the following section, opportunities for "CDO arbitrage" are described using a sample CDO portfolio. It is assumed that investors are guided solely by the tranches' rating and ignore the increased systematic risk. We show how tranches with high systematic risk can be generated and describe how CDO arrangers can exploit this to their advantage. It comes as no surprise that precisely these types of structures featured in many of the CDOs issued prior to the outbreak of the financial crisis.

The rest of the paper is organized as follows. Section 2 provides a simple analytic valuation model based on the CAPM and the single-factor Merton model of debt pricing. Section 3 discusses pooling and tranching and reallocating risks. First, rating-based risk measures are presented. Then, the "bond representation" treating a CDO tranche as a single-name credit instrument is described. Section 4 illustrates tranche valuation and CDO arbitrage. In section 5 is analysed how tranches with high systematic risk can be generated, and how CDO issuers can exploit this to their advantage. A discussion and conclusion is given in section 6.

2 A framework for credit modelling and valuation

Collateral pools of structured finance transactions are typically composed of assets like e. g. loans, bonds or CDS contracts. The risk properties of portfolio derivatives such as CDOs are influenced by the default behaviour of the collateral pool's assets. Thus, modelling credit risk of plain vanilla credit instruments forms the basis of modelling credit risk of structured finance instruments. In addition, the valuation model for simple credit instruments, introduced in this chapter, will be developed further to a model for evaluating structured credit products.

Asset-value-model

In this paper the borrowers' default behaviour is described by a parsimonious Vasicek style one-factor-model which is derived from a Merton asset-value model. With the assumption of a CAPM-conform asset pricing on capital markets, we are able to calculate risk-neutral default probabilities within this model framework, which enables us to price credit risky assets.

We assume that the value $V_{i,t}$ of a firm i follows a geometric Brownian motion with drift μ_i and volatility σ_i, ie

$$\frac{dV_{i,t}}{V_{i,t}} = \mu_i \cdot dt + \sigma_i \cdot dW_{i,t} \tag{1}$$

$W_{i,t}$ denotes the Wiener process. It is also assumed that for the firm value at time t=0 $V_{i,0} > 0$ holds.

It follows from the assumption of geometric Brownian motion that:

(1) At maturity T and for a given value $V_{i,0}$, the firm's value $V_{i,T}$ equals

$$V_{i,T} = V_{i,0} \cdot \exp\{(\mu_i - \sigma_i^2 / 2) \cdot T + \sigma_i \cdot W_{i,T}\} \tag{2}$$

(2) The log return $S_{i,T} = \ln \frac{V_{i,T}}{V_{i,0}}$ is normally distributed and can be derived from

$$S_{i,T} = \left(\mu_i - \frac{\sigma_i^2}{2}\right) \cdot T + \sigma_i \cdot \sqrt{T} \cdot W_{i,T} \tag{3}$$

By the introduction of a systematic risk factor M_T in the spirit of the one-period CAPM, that influences the asset returns of all firms, equation (3) can be rewritten as

$$S_{i,T} = \left(\mu_i - \frac{\sigma_i^2}{2}\right) \cdot T + \beta_i \cdot \sigma_m \cdot \sqrt{T} \cdot M_T + \sigma_{u,i} \cdot \sqrt{T} \cdot U_{i,T} \tag{4}$$

(i = 1,…,n). M_T denotes the standard normally distributed market factor and U_{iT} denotes the idiosyncratic risk factor of firm i which is also standard normal. We assume that the random variables $U_{1T},...,U_{nT}$ and M_T are independent. β_i represents the beta factor of firm i. σ_m and $\sigma_{u,i}$ are the volatilities of the market factor and of the idiosyncratic risks, respectively.

Expectation and variance of $S_{i,T}$ are

$$E(S_{i,T}) = \left(\mu_i - \frac{\sigma_i^2}{2}\right) \cdot T$$

and

$$Var(S_{i,T}) = \sigma_i^2 \cdot T = (\beta_i^2 \cdot \sigma_m^2 + \sigma_{u,i}^2) \cdot T \,.$$

A statistical default model (one-factor-model)

An additional assumption is that debt consists of a zero-coupon bond with a nominal value K_i and maturity T, as in Merton's firm value model. A default can only occur at maturity, and only when the firm's value at maturity T is smaller than the debt value. The liabilities therefore represent preferential claims on the firm's value. At maturity, the liabilities match the nominal value of the zero-coupon bond.

Then, the real probability of default (PD) can be calculated:

$$\begin{aligned} p_{i,T} &= P\left(V_{i,T} \le K_i \middle| V_{i,0}\right) = \\ &= P\left(S_{i,T} \le \ln K_i - \ln V_{i,0}\right) = \\ &= P\left(R_{i,T} \le c_{i,T}^P\right) = \\ &= \Phi\left(c_{i,T}^P\right) \end{aligned} \tag{5}$$

where $R_{i,T}$ is the standardised rate of return and the standardised default threshold

$$c_{i,T}^P = \frac{\ln \frac{K_i}{V_{i,0}} - (\mu_i - \sigma_i^2/2) \cdot T}{\sigma_i \sqrt{T}}.$$

For the standardised rate of return $R_{i,T}$, the following holds:

$$\begin{aligned} R_{i,T} &= \frac{S_{i,T} - \left(\mu_i - \sigma_i^2/2\right) \cdot T}{\sigma_i \cdot \sqrt{T}} = \frac{\beta_i \cdot \sigma_m \cdot \sqrt{T} \cdot M_T + \sigma_{u,i} \cdot \sqrt{T} \cdot U_{i,T}}{\sqrt{\beta_i^2 \cdot \sigma_m^2 + \sigma_{u,i}^2} \cdot \sqrt{T}} = \\ &= \sqrt{\rho_i} \cdot M_T + \sqrt{1 - \rho_i} \cdot U_{i,T}. \end{aligned} \tag{6}$$

where $\rho_i = \frac{\beta_i^2 \cdot \sigma_m^2}{\beta_i^2 \cdot \sigma_m^2 + \sigma_{u,i}^2}$

With equation (6) we arrive at the well-known single-factor-model. In particular in homogeneous portfolios ($\rho_i = \rho$) the sensitivity coefficient ρ is often referred to as the asset correlation.

The conditional probability of default with given realisation of the market factor results in

$$p_{i,T}(M_T) = \Phi\left(\frac{c_{i,T}^P - \sqrt{\rho_i} \cdot M_T}{\sqrt{1 - \rho_i}}\right). \tag{7}$$

In practice, the probability of default $p_{i,T}$ may be given externally by the obligor`s rating. In this case the (standardised) default threshold can be calculated from $c_{i,T}^P = \Phi^{-1}(p_{i,T})$.

In addition, a default time τ_i is defined for every borrower, and $\mathbf{1}_{\{\tau_i \le T\}}$ denotes the default indicator, which takes on the value 1 if the expression in brackets holds (i.e. a default occurs until maturity) and is otherwise 0.

If we denote the recovery rate conditional on the default of borrower i by RR_i (which is assumed fixed) and his exposure at default by EAD_i, the loss contribution of the borrower is given by

$$L_i(T) = EAD_i \cdot (1 - RR_i) \cdot \mathbf{1}_{\{\tau_i \leq T\}},$$

and the expected loss is

$$EL_i(T) = EAD_i \cdot (1 - RR_i) \cdot p_{i,T} \tag{8}$$

Valuation

In the following, a flat term structure with a risk-free interest rate *r* is assumed. For simplicity, zero bonds with a nominal value *K = 1* and maturity *T* are considered. The current price of a zero bond (with default risk) is denoted by $B^d(0,T)$.

In general, the price of a derivative financial instrument is the present value of the expected cash flows under the risk-neutral measure *Q*.

For a zero bond subject to default risk (with *K* = 1), we obtain the following (for simplicity the index *i* has been omitted):

$$B^d(0,T) = \exp(-rT) \cdot E^Q\left(1_{\{\tau > T\}} + RR(\tau) \cdot 1_{\{\tau \leq T\}}\right)$$

where τ denotes the stochastic default time and $RR(\tau)$ the recovery rate.

Assuming a constant recovery rate $RR(\tau) = RR$, we get

$$B^d(0,T) = B(0,T) \cdot \left(Q(\tau > T) + RR \cdot \left(1 - Q(\tau > T)\right)\right). \tag{9}$$

$Q(\tau > T) = 1 - q_T$ denotes the risk-neutral survival probability, q_T denotes the risk-neutral default probability, and $B(0,T)$ is the price of the risk-free bond.

Using the bond price, the credit spread *s* can also be calculated[2]:

$$s = -\frac{1}{T}\ln B^d(0,T) - r$$

Risk-neutral default probabilities

Just like the real PDs, the risk-neutral default probabilities can be derived from the asset-value model. For the log asset return of firm *i* under the risk-neutral measure follows

$$S_{i,T} = \left(r - \frac{\sigma_i^{\,2}}{2}\right) \cdot T + \beta_i \cdot \sigma_m \cdot \sqrt{T} \cdot M_T + \sigma_{u,i} \cdot \sqrt{T} \cdot U_{i,T} \tag{10}$$

[2] However, often higher spreads are observed on the market than the values calculated using the Merton model, see, for example, Jones/Mason/Rosenfeld (1984) and Kim/Ramaswamy/Sundaresan (1993).

The probability under the measure Q, the risk-neutral default probability is:

$$\begin{aligned} q_{i,T} &= Q\left(V_{i,T} \leq K_i \middle| V_{i,0}\right) \\ &= Q\left(S_{i,T} \leq \ln K_i - \ln V_{i,0}\right) \\ &= Q\left(R_{i,T} \leq c_{i,T}^{Q}\right) \\ &= \Phi\left(c_{i,T}^{Q}\right) \end{aligned}$$

Because the asset-value model's drift parameter (the expected asset return) is smaller under the measure Q, the risk-neutral default probability is higher than the real default probability. The same holds for the default threshold, as we again model default event via standardized asset returns.

The default threshold for modelling risk-neutral probabilities is given by

$$c_{i,T}^{Q} = \frac{\ln \frac{K_i}{V_{i,0}} - \left(r - \sigma_i^2 / 2\right) \cdot T}{\sigma_i \sqrt{T}} = c_{i,T}^{P} + \frac{\left(\mu_i - r\right)}{\sigma_i} \cdot \sqrt{T}$$

With the CAPM relationship $\mu_i = r + \beta_i \cdot \left(\mu_m - r\right)$, this formula can be rewritten as

$$c_{i,T}^{Q} = c_{i,T}^{P} + \frac{\left(\mu_m - r\right)}{\sigma_m} \cdot \sqrt{\rho_i} \cdot \sqrt{T} = c_{i,T}^{P} + \delta \cdot \sqrt{\rho_i} \cdot \sqrt{T}$$

where δ denotes the Sharpe ratio of the market.

The connection between real and risk-neutral default probabilities can be expressed by the following equation:

$$q_{i,T} = \Phi\left(\Phi^{-1}\left(p_{i,T}\right) + \sqrt{\rho_i} \cdot \delta \cdot \sqrt{T}\right) \tag{11}$$

The difference between real and risk neutral default probability represents a risk premium for systematic risk. The magnitude of the risk premium follows the logic of the CAPM. Systematic uncertainties concerning future cash-flows of an investment cannot be eliminated through diversification, and thus represent risks, that have to be compensated by higher returns of that investment.

The market factor M_T determines via the asset-value process the conditional default probability and therefore the conditional expected cash-flows to investors. The higher ρ_i, the higher the sensitivity of credit instruments to systematic risks and the higher the risk premium asked by investors.

Finally, let us consider the special case of the asset with the maximum systematic risk for a given real PD. In this case, $\rho_i = 1$. According to (11), we have

$$
\begin{aligned}
q_{i,T} &= \Phi(\Phi^{-1}(p_{i,T}) + \delta \cdot \sqrt{T}) \\
&= \Phi(-\Phi^{-1}(1 - p_{i,T}) + \delta \cdot \sqrt{T}) \\
&= 1 - \Phi(\Phi^{-1}(1 - p_{i,T}) - \delta \cdot \sqrt{T})
\end{aligned}
$$

Then, the bond price is given by

$$
B^d(0,T) = e^{-rT} \cdot \Phi(\Phi^{-1}(1 - p_{i,T}) - \delta \cdot \sqrt{T}) .
$$

This is the price of a digital call option on the market factor M_T, which pays \$1 if $M_T > S$ and otherwise 0. The strike price S is set so that the option is out of the money with a probability of $p_{i,T}$. Coval/Jurek/Stafford (2009a) call this option the cheapest-to-supply asset with a given rating (expressed by the PD or hitting probability $p_{i,T}$). If the recovery rate is greater than zero, the bond price can be derived from (9).

EL profile and valuation

A connection between the expected losses and therefore the expected cash-flows and the realisation of the market factor M_T can easily be established. Using equations (7) and (8) it follows for the expected loss given the value of the market factor expressed as percentage of exposure at default:

$$
EL_i(T \mid M_T) = (1 - RR_i) \cdot p_{i,T}(M_T) \tag{12}
$$

We refer to this relationship in the following as the "EL profile" considering it as a function of the realisations m_T. It represents in our framework all systematic uncertainty concerning future losses and thus contains all information relevant for pricing considerations.

Further uncertainty, i. e. uncertainty concerning the actual realisations of losses given a value of M_T, is of idiosyncratic nature and can be eliminated through diversification in a sufficiently large portfolio and thus is not relevant for pricing.

Two credit products, showing identical EL-profiles, are exposed to identical systematic risks and therefore should realise the same market price in the absence of arbitrage. This conclusion will be the basis for the evaluation of structured credit products.

3 Tranching and reallocating risks

3.1 Tranching and rating-based risk measures

CDOs are securities which transform the credit risk of the underlying collateral pool into a set of securities with different credit and loss profiles. Here we only consider a simplified version, and do not take into account the waterfall describing the priority of payments which is often quite complex. Instead we assume a strict subordination principle. If there are defaults in the collateral, losses first impact the equity tranche (first-loss piece). If the number of defaults is so great that the losses exceed the equity tranche's volume, the mezzanine tranches incur losses and the mezzanine holders start to see a reduced coupon and principal. The tranche with the highest subordination is the senior tranche. It only has to absorb losses if all tranches with lower subordination are wiped out.

The loss distribution of the underlying collateral pool is crucial for determining each tranche's risk profile. The collateral pool's loss on a percentage basis is an aggregation of individual losses. Using $EAD = \sum_i EAD_i$ we obtain:

$$L(T) = \frac{1}{EAD}\sum_{i=1}^{n} L_i(T) = \frac{1}{EAD}\sum_{i=1}^{n} EAD_i \cdot (1 - RR_i) \cdot \mathbf{1}_{\{\tau_i \leq T\}} \tag{13}$$

The loss distribution of a tranche relates to the loss distribution of the collateral pool. Generally the tranche's status is determined by its upper and lower limits – its attachment and detachment point.

Let us consider a tranche with attachment point $a \in [0;100\%)$ and detachment point $b \in (a;100\%]$. The tranche only suffers losses when the losses in the collateral pool until maturity T exceed the attachment point a. If the accumulated losses in the reference portfolio remain below a, then the tranche does not sustain any losses. On the other hand, if the accumulated losses exceed the detachment point b during the period, this results in a total loss for the tranche.

The probability that the accumulated losses in the collateral pool exceed the attachment point a by the end of the contract duration T,

$$p_T^{tr} = P(L(T) > a) = E(\mathbf{1}_{\{L(T) > a\}}), \tag{14}$$

denotes the "hitting probability" or PD of the tranche. It indicates the probability that the tranche will be hit during the period, i.e. whether it will suffer any loss. It depends solely on the subordination, i.e. the attachment point a, and not from the tranche "thickness" $b - a$.

If the tranche is hit, then the size of the loss is of course also relevant. The accumulated percentage tranche loss can be derived from

$$L_{tr}(T) = \frac{1}{b-a}(L(T)-a)\cdot \mathbf{1}_{\{a\le L(T)\le b\}} + \mathbf{1}_{\{L(T)>b\}}\,. \tag{15}$$

An important parameter, which is needed for the pricing in particular, is the expected tranche loss. This is given by

$$E(L_{tr}(T)) = P(L(T) > b) + \frac{1}{b-a}\int_a^b (x-a)dF_{L(T)}(x). \tag{16}$$

Using partial integration, this can be expressed by (see Jobst, 2007, p 298)

$$E(L_{tr}(T)) = \frac{1}{b-a}\int_a^b (1-F_{L(T)}(x))dx. \tag{17}$$

$1-F_{L(T)}(x)$ represent the "hitting probabilities" as a function of x. In special cases the expected tranche loss can be approximated analytically or asymptotically, but generally it has to be calculated based on simulation.

In general, the rating of a tranche is based on the hitting probability or the expected loss. If the rating is based on the hitting probability, it represents a "first dollar of loss" rating and the likelihood of an investor facing any loss at all until maturity is estimated. The "thickness" or width $b-a$ of the tranche is taken into consideration for a rating based on expected tranche loss.

Other important risk measures include the variance or volatility of the tranche loss, the tranche's unexpected loss, or value at risk and expected shortfall parameters. Generally these measures can only be determined by simulation.

3.2 The systematic risk of CDO tranches

In this section the sensitivity of CDO tranches to systematic risks is studied in some detail. This involves analysing the default behaviour of CDO tranches as a function of the market factor M (for simplicity subscript T is omitted) and comparing it with that of the collateral pool or a corporate bond with a comparable rating. To illustrate the differences, a sample portfolio is considered for the collateral pool, and a sample CDO is generated. The collateral pool loss, the tranche losses (see (15)), and the corresponding risk measures are calculated using stochastic simulation.

Sample collateral pool and CDO

We consider the default behaviour and potential losses of a CDO with a maturity of five years (T=5). A homogeneous portfolio of 100 equally weighted corporate bonds with a BBB rating is used as the collateral pool. The empirical five-year S&P default rate of $p_T = 3.25\%$ is used to estimate the unconditional probabilities of default. In addition, a homogeneous asset correlation of $\rho = 0.10$ and a constant recovery rate of $RR_i = 0.4$ is assumed for all collateral pool assets. The following tranching scheme with the associated attachment and detachment points is used:

Tranche	Attachment Point	Detachment Point	Hitting Probability
Senior	11.5%	100%	0.0014
Mezzanine	6.5%	11.5%	0.0316
Junior	4%	6.5%	0.1282
Equity	0	4%	0.8545

Table 1: Definition of CDO tranches

The hitting probabilities are calculated via simulation according to (14). If the tranche rating is based on the hitting probability, the mezzanine tranche has roughly a BBB rating which matches the rating of the bonds in the collateral pool. With a five-year hitting probability of 14 bp the senior tranche has a AAA rating.

Expected loss profile

To analyse the increased sensitivity of CDO tranches to systematic risk, we consider the EL-profile, i.e. the expected tranche loss conditional on the realisation m of the market factor M

$$E(L_{tr}(T) \mid m)$$

as a function of m.

The slope of the EL profile for poor (i.e. negative) market factor values is a measure of sensitivity to systematic risk.

First, the collateral pool itself is analysed. If the pool is regarded as an index or considered as a single tranche, the homogeneous portfolio, based on the assumptions made and the single factor model (6), gives us

$$E(L(T)\,|\,m) = (1 - RR) \cdot p_T(m)$$

where $$p_T(m) = \Phi\left(\frac{\Phi^{-1}(p_T) - \sqrt{\rho} \cdot m}{\sqrt{1-\rho}}\right).$$

However, this is exactly the same EL profile as for any bond contained in the collateral pool. This means that, in absolute value, the collateral pool and the bond contain exactly the same systematic risk. The diversification is expressed in the fact that the idiosyncratic risks are reduced and therefore the share of systematic risk increases. In the case at hand of a homogeneous portfolio, conditional on the market factor the losses are binomially distributed and the variance of the conditional losses is reduced as the number of bonds in the pool increases.

The situation changes dramatically if the loss distribution is split into several tranches and a single tranche is analysed. For example, the mezzanine tranche of our sample CDO shows the following EL profile (calculated via simulation):

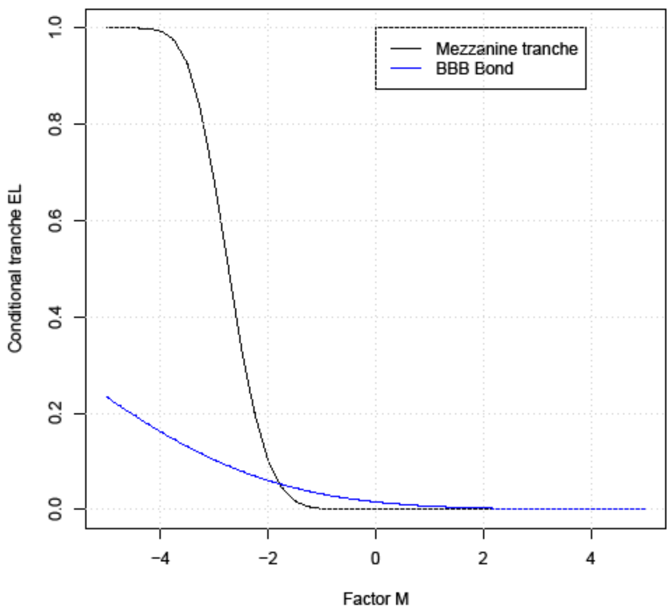

Figure 2: EL profiles of the BBB mezzanine tranche and BBB bond

The EL profile for one of the collateral pool's bonds (BBB rated) is also shown for comparison. Notice that the hitting probability of the mezzanine tranche approximately matches the bonds default probability. This suggests that the tranche's rating based on the hitting probability would also be BBB. Figure 2 graphically illustrates that the default behaviour of CDO tranches and bonds differs drastically with regard to the influence of systematic risk. Given unfavourable values for the systematic risk factor, the curve of the tranche rises much more steeply than for a corporate bond with similar rating.

A consequence of the sharply increased systematic risk is rapid and drastic rating downgrades in bad economic times. In a good macroeconomic environment (high values of M), the tranche is hardly ever hit, while the corresponding conditional expected losses for the collateral pool or for a single corporate bond are always slightly higher. In such a favourable scenario, changes to the systematic risk factor have almost no effect on the hitting probability or expected loss of the tranche, and CDO ratings appear very stable with regard to macroeconomic changes. This impression is deceptive, however. If the systematic risk factor comes under stress, CDOs (unlike corporate bonds) prove extremely sensitive to an economic downturn. In this critical region, even a small change of the market factor causes a dramatic deterioration in the tranche's credit quality. It is precisely this phenomenon that can be observed in the US subprime crisis. Falling house prices and rising interest rates triggered a downward spiral resulting from rising insolvencies, house sales and plummeting prices, and the macroeconomic situation of the US housing market entered a critical phase. This led to drastic downgrades of hundreds of CDO tranches which contained these loans in their reference portfolios.

Another important consequence stems from the price relevance of systematic risk, which was possibly not adequately appreciated by investors. The considerably increased systematic risk of the tranches needs to be compensated for by a significantly higher spread compared with corporate bond spreads. If the prices of CDO tranches and bonds are the same or do not differ sufficiently, CDO issuers can exploit this to their advantage. This may provide a further explanation for the emergence of "CDO arbitrage". The price-relevant aspects are discussed in detail in section 4.

Loan equivalent "bond representation" for quantifying the systematic risk of a CDO tranche

The sensitivity of a borrower or bond, respectively, to systematic risk is expressed in the single factor model by the correlation parameter ρ (assumed to be homogeneous). In this section we estimate the asset correlation associated with a CDO tranche treating the structured instrument as a single-name credit instrument. Thus we treat the CDO tranche as a "virtual" borrower or bond, and approximate its default behaviour likewise using a single factor model. Our aim is to estimate the "virtual" asset correlation ρ^{tr} of the CDO tranche.

Considering a CDO tranche as a virtual bond, the conditional hitting probability is expressed in the single-factor model as a function of M by

$$p_T^{tr}(m) = \Phi\left(\frac{c^{tr} - \sqrt{\rho^{tr}} \cdot m}{\sqrt{1-\rho^{tr}}}\right). \tag{18}$$

The default threshold c^{tr} can be calculated using the unconditional hitting probability given in (14). In this approach (see, for example, Yahalom/Levy/Kaplan, 2008) the loss given default (LGD) of the tranche has to be specified in a separate model.

Here we introduce another approach. A key observation is that the LGD of a CDO tranche is always a random variable that depends on the market factor. This is because the tranche loss depends on the defaults in the collateral pool, and the defaults, in turn, depend on the market factor. Note that this is usually not the case with a straight bond where default and LGD may be independent. A CDO tranche's sensitivity to systematic risk can be increased further by the dependence of tranche loss on the market factor. This additional systematic risk is contained in the expected loss profile. Therefore we consider the EL profile as the tranche`s risk profile and not the tranche's conditional hitting probability. Moreover, as outlined in the previous section, the EL profile contains all the information necessary for pricing. The EL profile is obtained via stochastic simulation. For the "bond representation", however, we assume a constant *LGD** for the tranche (instead of the random loss given default depending on the market factor).

The "implied" hitting probability conditional on the market factor is determined from

$$p_T^{tr}(m) = \frac{E(L_{tr}(m))}{LGD*} \tag{19}$$

The tranche's unconditional hitting probability is given by

$$p_T^{tr} = \frac{E(L_{tr})}{LGD*}.$$

The unconditional expected tranche loss is also calculated in the simulation, and the default threshold c^{tr} is given by

$$c^{tr} = \Phi^{-1}(p_T{}^{tr}).$$

In the next step, the tranche LGD* is determined as the maximum loss of the tranche.

The maximum loss of the collateral pool is given by

$$L_{\max} = \frac{1}{EAD}\sum_{i=1}^{n} EAD_i(1 - RR_i)$$

which reduces to $L_{max} = 1 - RR = LGD$ for a homogeneous portfolio.

Based on this, the maximum loss for a tranche with attachment point a and detachment point b is

$$LGD^* = \frac{\min(L_{\max}, b) - a}{b - a}$$

if $L_{\max} > a$. In the unrealistic case $L_{\max} < a$, the tranche cannot be hit at all. Its LGD is then always zero. In general, $LGD^* = 1$ for all tranches except senior or super-senior tranches. Thus $LGD^* < 1$ only for the tranche with the highest seniority (with detachment point 1). For the non-senior tranches, the “implied” hitting probability profile of the "virtual" approximating bond is set equal to the EL profile, while the EL profile is scaled up for the senior tranche with $LGD^* < 1$.

Finally, the “virtual” asset correlation ρ^{tr} is calculated by means of optimization

$$arg\,min_{\rho^{tr}} \left\{ \sum_{k=1}^{K} \left[p_T^{\;tr}(m_k) - \tilde{p}_T^{\;tr}(m_k) \right]^2 \middle| \rho^{tr} \in [0,1] \right\} \tag{20}$$

where

$$\tilde{p}_T^{tr}(m_k) = \Phi\left(\frac{\Phi^{-1}(p_T^{\;tr}) - \sqrt{\rho^{tr}} \cdot m_k}{\sqrt{1-\rho^{tr}}} \right) \tag{21}$$

and $p_T^{\;tr}(m_k)$ is given in (19) evaluated at m_k. $(m_k)_{k=1}^{K}$ is a sufficiently accurate discretization of the support of *M*.

In summary, a CDO tranche is approximated by a “virtual” bond in a single-factor model according to (6) with unconditional (“implied”) hitting probability $p_T^{\;tr}$, asset correlation ρ^{tr} and LGD* as the loss given default. This “bond representation” is a more appropriate approximation of the default behaviour and the loss profile than an approximation based on the conditional hitting probability of the tranche.

The estimation results for all tranches of the sample CDO are summarised in Table 2.

Tranche	p_T^{tr} (%)	ρ^{tr}	LGD*
Senior (11.5 - 100%)	0.00615	0.3196	0.548
Mezzanine (6.5 - 11.5%)	0.93444	0.7572	1
Junior (4 - 6.5%)	6.45767	0.7518	1
Equity (0 - 4%)	43.57131	0.4214	1
Collateral Pool	3.25	0.1	0.6

Table 2: Risk parameters for the CDO tranches and the collateral pool

From Table 2 it can be seen that in our approach p_T^{tr} is rather an artificial parameter for fitting the EL profile than a hitting probability. p_T^{tr} corresponds to the unconditional expected loss for the non-senior tranches of the sample transaction. The parameter ρ^{tr} can be interpreted as the tranche's "virtual" asset correlation. It can be seen that the values of ρ^{tr} are much higher than those for comparable corporate bonds (e.g. $\rho = 0.1$ for the collateral pool's assets) reflecting the drastically increased sensitivity of CDO tranches to systematic risks.

The following Figure shows the bond representation's fit for the mezzanine tranche. It indicates a very good fit of the EL profile obtained from the simulation and function $\tilde{p}_T^{tr}(m_k)$ in (21) derived from the optimization procedure with given default threshold and estimated "virtual" asset correlation.[3]

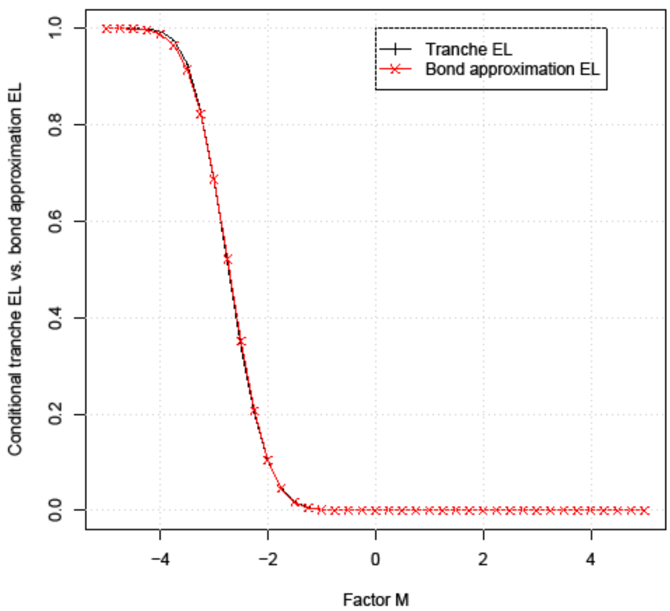

Figure 3: Comparison of the mezzanine tranche's EL profile with the bond representation's profile

Impact of subordination

For the rating process of a structured finance transaction a tranche`s subordination plays a key role. The higher the subordination the more losses in the collateral pool are absorbed by tranches with lower seniority. Thus, higher seniority lowers the hitting probability and allows higher rating grades. The EL profiles for the tranches are shown in Figure 4 below.

[3] In the case of equity tranches and broad senior tranches, the bond approximation reveals somewhat larger deviations from the EL profile of each CDO tranche. However, the approximation errors are negligible for the issues discussed in the present article.

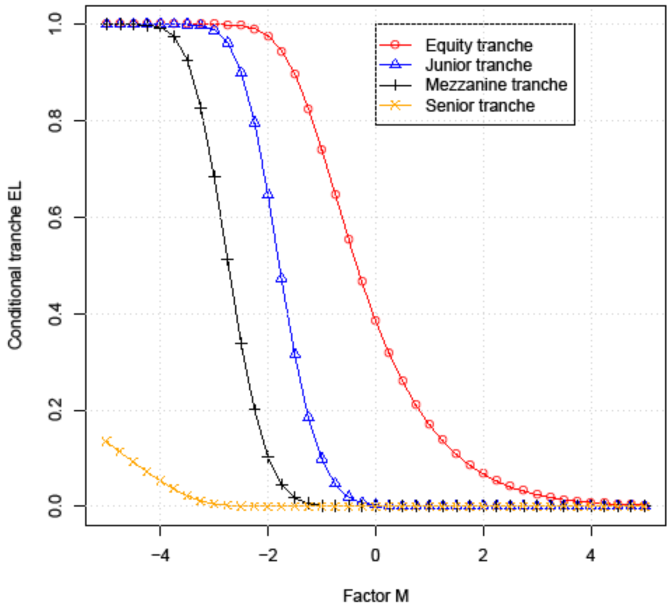

Figure 4: EL profiles of all tranches of the sample CDO

The position of the critical regions, in which the market factor has a strong effect on the tranche`s losses (the EL profile has a steep slope), depends on the degree of subordination. This area can be shifted towards the left side of the diagram (i.e. a scenario of economic depression) by increasing the subordination. It is evident that the macroeconomic environment must worsen greatly before a senior tranche is hit. However, if the critical area is reached, the credit assessment of even highly rated tranches may suffer drastic downgrades as we have seen in the current subprime crisis.

Investment in CDO tranches versus investment in the collateral pool

At this point the key difference between an investment in the collateral pool ("vertical" tranching) and investment in a tranche ("horizontal" tranching) can be made clear. Let us consider the sample collateral pool with an average BBB rating and the mezzanine tranche (6.5 - 11.5%), which also has a BBB rating in accordance with its hitting probability. If the transaction's total nominal amount is 1 billion Dollar, an investor can, for example, purchase the mezzanine tranche for $50 million or he can invest this amount in a portfolio which is comparable with or equivalent to the collateral pool. By construction, both investments have the same implied PD. However, the two investments have significant differences regarding their other risk characteristics. The main difference stems from the fact that, in the latter case, the investment is made in the entire loss distribution of the pool, whereas in the case of a tranche, the investment is made in a specific part of the loss distribution. The collateral pool's risks are completely reallocated by the tranching process. On the one hand, a large senior tranche is created which has a much smaller default risk than that of the collateral pool's assets. On the other hand, the systematic risk (sensitivity to the market factor) of all tranches rises massively. It is true that building a large collateral pool diversifies idiosyncratic risk, so that only systematic risk remains. Such an investment in a well diversified pool is seen as an advantage of CDO tranches. However, the systematic risk remains unchanged in absolute terms in the collateral pool ($\rho = 0.1$; see Table 2), while, by contrast, the tranches' systematic risk rises dramatically (e.g. $\rho^{tr} = 0.75$ for junior and mezzanine tranches; see Table 2). For this reason, a portfolio of BBB tranches from different securitisations (of the same type), for example, is much riskier than a portfolio of comparable bonds or investments in different collateral pools. In particular, the addition of CDO tranches to bond or loan portfolios raises the probability of large portfolio losses, i.e. CDO tranches affect the tail of the loss distribution. A consequence is considerably higher risk contributions measured by value at risk or expected shortfall contributions, respectively.

4 Valuation of tranches and CDO arbitrage

The simple valuation model introduced in section 2 is used to identify potential possibilities of mispricing of the securities generated by the tranching. We do not compare expliciltly model and market spreads (see, for example, Coval/Jurek/Stafford, 2009a, and Brennan/Hein/Poon, 2009). Instead we analyze which opportunities for CDO arbitrage are available if investors ignore the tranches' systematic risk and base their pricing solely on the rating in the form of hitting probability or expected losses.

The results presented in the preceding sections allow us to achieve the above mentioned objective of an approximate tranche pricing with the following three steps:

(1) Starting point is the aggregation of all pricing relevant information and results in generating the tranche's EL-profile (by simulation, see sections 2 and 3.2).

(2) The risk parameters of a "virtual" bond are calculated in a way, that the EL-profile of this bond replicates the tranche's profile. We call this the "bond representation". (see section 3.2)

(3) This bond can be priced with equation (9) presented in section 2; sharing the same sensitivity towards systematic risks, the tranche and the virtual bond are equally priced. As a result, we arrive at the price for the tranche. Again, we assume a zero-coupon structure.

It can be seen from (9) that the bond price falls as the risk-neutral PD increases. In line with (11), the risk-neutral PD increases – *ceteris paribus* – as the systematic risk rises. This means that bonds with the same real PD p_T should have very different prices depending on their exposure to systematic risk.

Application to the sample CDO

In the following the present values of the tranches and the collateral pool are calculated using the results from Table 2. This involves determining the risk-neutral PD using equation (11), incorporating each of the "virtual" asset correlations of the bond representation. For the Sharpe ratio of the market we assume $\delta = 0.4$ Then, the values of the tranches are calculated from (9), assuming a constant risk-free interest rate $r = 0.04$. The valuations are always based on a nominal of 100. The results are shown in Table 3.

Tranche	Price
Senior (11,5 % - 100,0 %)	81.843
Mezzanine (6,5 % - 11,5 %)	77.140
Junior (4,0 % - 6,5 %)	63.100
Equity (0,0 % - 4,0 %)	27.650
Collateral Pool	78.960

Table 3: Tranche prices (based on bond representation)

The sum of all tranche values weighted by their respective share in the capital structure equals the value of the collateral pool (up to rounding errors).

For comparison, another valuation of the non-equity tranches is conducted, assuming a uniform asset correlation of $\rho = 0.1$ for these tranches according to the assets in the collateral pool. This is designed to replicate the valuation approach of an investor, who has the rating of a tranche (i.e. hitting probability or expected loss) at hand, but ignores the increased sensitivity to systematic risks and instead assumes a "bond-typical" behaviour. The value of the equity tranche is then calculated such that the sum-total matches the value of the collateral pool. Table 4 shows the results:

Tranche	Price
Senior (11,5 % - 100,0 %)	81.868
Mezzanine (6,5 % - 11,5 %)	80.293
Junior (4,0 % - 6,5 %)	72.980
Equity (0,0 % - 4,0 %)	16.812
Collateral Pool	78.960

Table 4: Tranche prices (based on an asset correlation of $\rho = 0.1$)

A comparison of the results in Tables 3 and 4 shows how a CDO arranger who retains the equity tranche can exploit the situation to his advantage. If he succeeds in selling the non-equity tranches in line with their rating (according to the real PD or expected losses) like corporate bonds, investors will pay an excessive price and will receive inadequate spreads for the tranches' systematic risk. For the equity tranche, by contrast, the situation is reversed. The value in Table 4 shows that the price of the equity tranche as the "residual tranche" is significantly too low. If the equity tranche spread is an "excess spread", this creates the possibility to generate a positive net capital value from the overall transaction. Put another way, the holders of the equity tranche charge a very high premium for the apparently high risk.[4]

The results indicate that investors should receive much higher spreads for their investments in CDOs as a risk premium than that which they are paid on comparable corporate bonds. This was obviously not the case in the years prior to the start of the subprime crisis (see Brennan et al, 2008). During this period there were virtually no differences between the prices of CDO tranches and those of similarly rated corporate bonds. In some cases the spreads of corporate bonds were even lower than the spreads of tranches with similar ratings.

In summary, there is a general way to CDO arbitrage when the non-equity tranches of a CDO can be sold at the prices of corporate bonds with comparable ratings. In this case, it is

[4] This underscores a remark by Tavakoli (2006), which points out that investors in equity tranches should never accept an upfront payment and a fixed spread, which is usual with credit indices. Instead, they should always demand the excess spread.

advantageous for the CDO arranger to put together the collateral pool in such a way that the tranching creates securities with a very high systematic risk whose true prices should be close to those of the respective cheapest-to-supply assets. This provides the maximum potential profit for the arranger from the transaction.

In the next section we describe some possibilities of generating tranches with high systematic risk. It comes as no surprise that precisely these types of transactions can be found in many CDOs issued prior to the outbreak of the financial crisis.

5 Generating tranches with high systematic risk

5.1 Increasing the number of assets in the collateral pool

It is often claimed that a key advantage of investing in a CDO tranche is that it offers the possibility to invest in an already well diversified portfolio. Additionally, if the systematic risk of the portfolio is low, the unexpected loss decreases and realised losses are usually close to expected loss. While this is basically correct, it is often forgotten that this applies only when an investment is made in the entire collateral pool, in other words when just one single tranche is formed. When tranches are created, however, a reallocation of risks occurs whereby the systematic risk of each tranche increases drastically and is much higher than the systematic risk in the collateral pool. This sensitivity to systematic risk is strengthened if the number of assets in the collateral pool is increased. To illustrate this, let us consider the case in which the number of assets in the homogeneous collateral pool is increased from 100 to 500, while the individual risk characteristics of the assets remain unchanged. A tranching scheme is used which produces approximately the same ratings for the non-equity tranches as the structure shown in Table 1 in line with their hitting probability. The results are shown in Table 5.

Tranche	AP	DP	Hitting Probability
Senior	10.25%	100%	0.0017
Mezzanine	5.75%	10.25%	0.0317
Junior	3.5%	5.75%	0.1335
Equity	0	3.5%	

Table 5: Definition of the CDO tranches (n=500)

The following results are obtained for the bond representation:

Tranche	Implied HP (%)	ρ^{tr}	LGD*
Senior (10.25 - 100%)	0.0055	0.5464	0.554
Mezzanine (5.75 - 10.25%)	0.9947	0.8990	1
Junior (3.5 - 5.75%)	7.091	0.9038	1
Equity (0 - 3.5%)	49.809	0.4114	1
Collateral Pool	3.25	0.1	0.6

Table 6: Parameter estimations for the CDO tranches (n=500)

First it can be seen that the collateral pool's risk characteristics – as far as they are relevant for the CDOs' risk analysis – have not changed. Although the relative share of idiosyncratic risk has dropped due to further diversification, this is not price-relevant. In absolute terms (expressed by the covariation with the market factor), the systematic risk is unchanged. The price of the collateral pool thus also remains the same.

However, for the CDO tranches matters have changed. Apart from the equity tranche, all tranches show a further increase in systematic risk. As described above, this strengthens the CDO arbitrage. The following Table shows the tranche prices according to bond approximation including the actual systematic risk, as well as the prices of comparable corporate bonds $(\rho = 0.1)$. Additionally, the prices for the respective cheapest-to-supply assets are given for the non-equity tranches.

Tranche	Price	Price	Price
	(actual systematic risk)	(Corporate Bonds)	(Cheapest-to-Supply)
Senior	81.842	81.865	81.806
Mezzanine	76.207	80.209	75.683
Junior	59.923	72.228	58.727
Equity	23.309	7.339	

Table 7: Tranche prices including the actual systematic risk, valued like comparable corporate bonds, and cheapest-to-supply assets

It can be seen that the valuation of the non-equity tranches on the basis of corporate bonds produces roughly the same prices as before because of the comparable ratings of the tranches. However, taking the actual systematic risk into account, the prices ought to be distinctly lower – especially for the junior and mezzanine tranches – to compensate for the increased risk. On the other hand, the equity tranche offers extremely high spreads and thus drastically higher potential returns because of its very low price.

Finally it should be noted that a sufficiently large number of assets in the collateral pool always produces tranches with high systematic risk. This also holds if the asset correlation between the assets in the reference portfolio is very low, some of them may even be uncorrelated. Thus the process of bundling a large number of assets with a low systematic risk via securitisation with subsequent tranching generates securities with a very high systematic risk. Among others, this applies to US residential mortgage backed securities (RMBS).

A further example illustrates the point. If an (extremely low) asset correlation of 0.01 is assumed for the assets in the collateral pool (other risk parameters remain unchanged) and the number of assets is increased to 2000 (which is the case with many ABS and RMBS), the tranche from 3.5 to 5.75%, which now has approximately an A rating, has an estimated asset correlation of 0.6.

5.2 Inclusion of assets with a high systematic risk in the collateral pool

Another way of generating tranches with a very high systematic risk is to include assets which already have a high systematic risk in the collateral pool, while leaving the number of assets at roughly 100 to 150. Examples are ABS CDOs (SF CDOs), in particular ABS CDOs with subprime exposure, whose collateral pools contain RMBS mezzanine tranches, for instance. These assets already have a high systematic risk, and second-layer securitisation further significantly raises the systematic risk of the resulting tranches. The situation is similar for many transactions of the CDO-squared type. The collateral pools of these structured finance instruments are frequently composed of tranches which are in turn the product of securitisations from the same or similar segments (eg car loans, credit card receivables, leasing contracts), or include tranches of synthetic CDOs with overlapping names. In all these cases, the tranches of the first-layer securitisations are already highly correlated. The second-layer securitisation further increases the sensitivity of the resulting tranches to systematic risk.

Credit default swaps (CDSs) on ABS or MBS additionally increase sensitivity to systematic risk. This was evident, in particular, on the market for MBS CDOs with subprime exposure. If an underlying MBS tranche suffers a loss, this directly leads to a credit event for the associated synthetic MBS CDS. Correlation is perfect in this case. This leveraging of real MBS bonds multiplies the effect of the defaults on subprime loans and may result in an abrupt loss of value if economic conditions deteriorate.

For synthetic CDOs, the credit risks in the collateral pool are transferred via credit default swaps (CDSs). For these transactions, it is useful to look for CDSs that offer as high a spread as possible for a given rating. This is the case where the underlying enterprises exhibit additional risks, in particular systematic risks. For example, according to estimates (see Bruyere, 2006, p 126), the collateral pools of the vast majority of synthetic arbitrage CDOs in 2002 and 2003 included the tobacco company Philip Morris. This was because the potential risk of further legal sanctions against the tobacco industry was reflected in a high spread, but the rating was unchanged at BBB. As the agency models only took into account the default risk (in the form of the BBB rating) in their tranche rating, this offers additional opportunities for CDO arbitrage for the issuers.

5.3 Creating as many tranchelets as possible

Finally, it is possible to slice the collateral portfolio up into a large number of very thin tranches (tranchelets). These tranchelets are extremely sensitive to systematic risk. Thus it is always advantageous for CDO issuers to generate as many tranches as possible in a securitisation.

Similarly, virtually only tranches with a width of just a few percentage points were generated in the construction of single-tranche CDO swaps. This also applies structured instruments where several tranches were created but the capital structure was not completely placed with investors. Where the tranches created in this manner are included in another securitisation's collateral pool, the new tranches' sensitivity to the market factor is further raised. If investors ignore the higher systematic risk in such a tranche and focus only on the rating, the seller can exploit this to his advantage, as outlined above.

A particularly instructive example of splitting a tranche into two new tranches with the possibility of CDO arbitrage is where a senior tranche is divided into a super-senior tranche and a lower AAA tranche. This approach is customary for CDO structuring and is discussed very critically, particularly in Tavakoli (2006).

By way of illustration, we will look at the capital structure shown in Table 5. The senior tranche (10.25%-100%) has a hitting probability of 17 basis points (bp) (for a period of 5 years), which corresponds to an AAA rating. If it is assumed, for example, that LIBOR + 30bp is payable for an AAA-rated bond, investors would receive 30bp for almost 90% of the total nominal amount for a synthetic (unfunded) CDO.

Now, a super-senior tranche is set up with an attachment point of 12.25%. This tranche has a hitting probability of about 4 bp and is claimed "virtually without default risk" according to the CDO arranger. It should therefore be possible (and this was generally the case) to insure its risk with a monoliner at low cost of, say, 6 bp or, alternatively, to retain the risk on the books. In this way, a saving of 24 bp is achieved for 87.75% of the total nominal amount.

The hitting probability for the next lower tranche of 10.25%-12.25% remains unchanged. If the rating is based on the hitting probability, the tranche retains its AAA rating and therefore also its price if investors focus only on the rating. However, the other risk characteristics have changed dramatically, in particular in terms of expected loss and sensitivity to systematic risk. As a consequence, the price, too, must change, and investors should demand a significantly higher spread to compensate them for the higher risks. Table 8 shows the results of bond representation and valuation.

Tranche	Implied HP	ρ^{tr}	LGD*	Price
Senior (10.25 - 100%)	0.000055	0.5465	0.554	81.842
Senior(10.25 - 12.25%)	0.000978	0.9547	1	80.792

Table 8: Bond approximation and valuation of the senior tranche of the original structure and the senior tranche with the same attachment point, which is below the super senior tranche

6 Discussion and conclusion

In line with Brennan/Hein/Poon (2009) and Coval/Jurek/Stafford (2009a), this paper presents mispricing of CDO tranches as an explanation for the exceptional growth of arbitrage CDOs prior to the onset of the financial crisis. Although CDO tranches initially offered more yield than cash assets (bonds or loans) with similar ratings, the spread differences, which declined continuously from 2005 onwards, were far too low to compensate for the additional systematic risk. Securitization of the collateral pool and tranching means risks are completely reallocated. While the probability of default and expected loss of a large portion of the capital structure are distinctly lower than for an average collateral pool asset, the systematic risk of all tranches rises dramatically. If investors focus exclusively on the tranches' rating, they are very likely to pay too high a price for the non-equity tranches. As the prices for the tranches, which are weighted according to the capital structure, add up to the value of the collateral pool, the fact that prices for the non-equity tranches are too high inevitably results in the price for the equity tranche being too low.

The "bond representation" described in section 3 allows quantifying systematic risk treating a CDO tranche as a single-name credit instrument. The tranche is considered as a "virtual" borrower or bond for which a single-factor model holds. Then, the exposure to systematic risk is measured by the asset correlation. In addition, we derive risk neutral default probabilities using a simple valuation based on the CAPM and the single-factor Merton model. The risk neutral default probabilities also depend explicitly on the asset correlation. This allows pricing of CDO tranches as an explicit function of asset correlation. Thus we obtain different prices for bonds and tranches with the same rating depending on their exposure to systematic risk. The valuation of tranches and CDO arbitrage opportunities are discussed in section 4.

The desire to exploit CDO arbitrage opportunities usually entails the construction of tranches with a high systematic risk. How this can be achieved was described in section 5. The collateral pool's high degree of diversification was always regarded as a particular advantage (see, for example, Amato and Remolona, 2003). In the authors' view, less collateral is needed to cover unexpected default losses if pool diversification is high, thus raising "arbitrage profits". However, in section 5 it is shown that increasing the number of assets in the collateral pool leads to higher sensitivity to systematic risk of the tranches. Therefore, it is the investors who should be entitled to the "arbitrage profits" to compensate them for the tranches' much higher systematic risk. In addition, a large number of assets is not necessary if the assets in the collateral pool already exhibit a high systematic risk. Prominent examples are

ABS CDOs, in particular MBS CDOs. The collateral pools of these CDO-squared instruments include a large proportion (roughly 80%, see BCBS, 2008) of BBB-rated RMBS mezzanine tranches. The RMBS tranches have a high hitting correlation. This means that the second-layer tranches are much more sensitive to systematic risks. If the degree of hitting correlation of the tranches in the collateral pool is underestimated, the second-layer capital structure is such that the senior tranche with a very good rating is much too large. Another potential danger for MBS CDOs with subprime exposure results from the relatively small width of the mezzanine tranches in the collateral pool (2.5% to 4%) as well as the relatively low original level of subordination (between 3% and 5%). Prepayments during the contract's life can temporarily increase subordination, but overall it remains low. Rising default rates for subprime loans combined with declining recovery rates may result in the BBB-rated mezzanine tranches in the original RMBS suffering losses. The extreme correlation then means a high likelihood that all BBB-rated tranches of subprime RMBS will be hit at the same time. In addition, their small width means that there is a high likelihood that they will all experience a total loss at the same time. This scenario means that the senior tranches of subprime MBS CDOs will likewise default. These subprime CDOs therefore exhibit an "all or nothing" risk profile[5]: either the subprime default rates remain below the thresholds and the senior tranches of the subprime CDOs do not suffer any loss at all, or the thresholds are breached, and the senior tranches could all be wiped out.

The opportunity to increase the value of the equity tranche by sophisticated tranching also undermines the importance investors in senior tranches attach to the fact, that the arranger of a transaction keeps the first-loss piece. Usually it is assumed, that this prevents the arranger which is mostly also the designer of the collateral pool, from overinvesting and thus ensures high collateral quality which also strengthens the position of senior tranche investors. However, if the arranger can gain profit solely from tranching, these profits offset losses from poor investment decisions, in other words, the credit quality of the collateral pool's assets becomes less important as long as there are enough assets to form collateral pools. Thus, our findings can to some extent explain the surge in demand for MBS bonds before the financial crisis as well as the deteriorating credit quality in collateral pools.

Another aspect concerns the role of through-the-cycle ratings for structured credit instruments. One of the key conclusions of this paper is that both the (conditional) hitting probabilities and the expected loss profiles for CDO tranches are highly dependent on systematic risks. The transition probabilities are similarly influenced by systematic risks. When the economy is doing well, CDO ratings are very stable. They can, however, become

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[5] See also BCBS (2008), p. 6 and appendix C

very volatile in an economic downturn, and there may be significant rating downgrades within a short space of time. This specific behaviour under stress makes a through-the-cycle rating for CDOs as well as the use of average transition matrices, which take no account of the current position in the credit cycle, questionable. In general, through-the-cycle ratings favour CDO arbitrage opportunities, in particular during an economic downturn. Consider, for instance, a synthetic CDO's collateral pool consisting of CDS. Then, the higher default risks mean the CDS spreads are very likely to widen during an economic downturn. On the other hand, the through-the-cycle ratings remain largely unchanged. If a new CDO is structured with the same collateral pool, the resulting tranching scheme and the tranche prices are the same if one focuses exclusively on the rating, while the originator's revenues from the collateral pool have risen.

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