Uncertainty, freedom of action and investment behaviour – empirical findings for Germany

Investment in the buildup and modernisation of the capital stock is a key variable in the economic development and prosperity of a country. Private investment requires entrepreneurs to deal with a basic phenomenon: uncertainty. One might say that investment is like "betting on the future".

There is no clear theoretical relationship between investment and uncertainty. Since uncertainty also involves entrepreneurial opportunity, it may not only produce dampening effects but might also promote investment. The extent of the entrepreneurial freedom of action is important in determining which of the two types of effects will prevail.

Drawing on the more recent literature on investment, this article uses corporate balance-sheet data of the Deutsche Bundesbank to examine the investment behaviour of 6,745 enterprises. The main result is that the effect of uncertainty on investment behaviour is definitely negative. For economic policy makers, the maxim that follows from this is to ensure certainty of planning wherever possible and to open up – and keep open – the freedom of entrepreneurs to take action.

The role of uncertainty in investment decisions

An understanding of factors that motivate entrepreneurial fixed investment is extremely important to economic policy makers in general and to central banks in particular. Experience has shown that investment in fixed assets is a very volatile component of macroeconomic demand. This makes it an exceptionally dynamic factor in the entire business cycle, and thus difficult to forecast. However, its second feature is more important. Positive net investment is tantamount to growth of the macroeconomic capital stock and – in line with its factor productivity – of potential output.[1] Investment is therefore a key foundation of material prosperity. By either investing or not investing, burdens and benefits can be shifted back and forth between generations and over time.[2]

In principle, an additional investment project is advantageous if and only if the expected market value of the project exceeds the amount spent on purchasing and installing the capital good. If investment is made, this leads to additional revenue. This "marginal revenue" must be weighed by the investor against the costs being incurred because the funds are tied up in the enterprise for a lengthy period of time and cannot be put to another use.[3]

Betting on the future

Knowledge of future developments is always sketchy. Therefore, investment is essentially akin to "betting on the future". Those who undertake an investment project under market conditions, such as purchasing a warehouse, building a multi-family house or establishing a new production line, will not be able to avoid accountability to themselves or others regarding the expected revenue. But important data on the cost side such as input prices, wages and interest, are hidden in the future.

Entrepreneurial risk

If the investors' expectations turn out to be erroneous, they must bear the economic consequences of unfavourable sales or cost developments. On the other hand, they also reap the benefits of unexpectedly favourable market developments. Hence investors tie their economic fate to factors which they have only incomplete knowledge of and control over. This is the essence of their entrepreneurial risk, and without the willingness to incur such a risk, i.e. to "bet on the future", all long-term economic activity would cease. Uncertainty is what makes entrepreneurship, in its strict sense, necessary in the first place. Within society's division of labour, one might say that entrepreneurs are experts in dealing with uncertainty.

In light of the mediocre success that traditional models have had in attempting to satisfactorily explain observed patterns of investment behaviour, research on the effects of uncer-

1 For details on this see: Deutsche Bundesbank, Trends in and structure of the overall capital stock, Monthly Report, November 1998, page 25ff.
2 See: Deutsche Bundesbank, Development of public sector investment, and its financing, Monthly Report, April 1999, page 29ff.
3 For more on the theory and empirical experience of investment demand, see: Chirinko, R.S., Business fixed investment spending: modeling strategies, empirical results and policy implications, Journal of Economic Literature, 31, pages 1875 to 1911, 1993, and Caballero, R., Aggregate investment, in: Taylor, J.B. and M. Woodford (eds.), Handbook of Macroeconomics, pages 813 to 862, 1999.

tainty has undergone a renaissance. What may come as a surprise, though, is that increased risk does not necessarily harm investment activity per se.

Risk aversion: uncertainty as a "bad"

However, there are certainly a number of valid reasons for such a negative effect. Most economic agents tend to be risk-averse. Presented with a choice, they prefer certain returns over uncertain returns if the expected level is the same in both cases. Entrepreneurial risk is thus subjectively perceived as a "bad" which is only accepted in return for the prospect of higher returns. Generally this extra return will have to be higher if the existing information on the future is less precise. Such a correlation between the non-diversifiable, systematic part of risk and the risk premium demanded by investors continues to hold when all options for diversifying risk have been exhausted.

Asymmetrical information: uncertainty makes financing more difficult

The financing of risky projects can turn out to be difficult owing to what is called the principal-agent problem. If a company's management is better informed than the external providers of capital (lenders or shareholders), management will have an incentive to exploit this knowledge advantage. For their part, providers of capital must live with the consequences of such selfish behaviour – which can range anywhere from lower returns to total loss of invested capital – and will do all they can to hedge against such losses. For good reason, therefore, providers of capital are more careful with commitments if the decision makers are largely uncontrolled. Information asymmetry is especially great if the profitability of an investment project is difficult to assess from the outside – meaning that a higher level of uncertainty exists. Besides additional risk premiums or limits in the amount of funds lent by banks, a further consequence may be that promising projects will not find anyone willing to finance them in the market for equity, either. That has nothing to do with the risk propensity of providers of capital: asymmetrical information also makes financing more difficult in those cases where all parties involved are indifferent to risk.

Irreversibility and the "wait-and-see attitude"

A third chain of causality has only recently come to the forefront of academic debate and has also entered the literature of business management as the real option model of investment.[4] Traditional models of investment take account of uncertainty by converting the expected returns into what are called certainty equivalents or include a risk premium in the discount factor. However, this does not account for the fact that in most cases investment is difficult or impossible to reverse. This engenders asymmetrical adjustment costs. Whereas in principle it is relatively easy to raise the capital stock, it is often only possible to get rid of it by either selling it far below its original price or scrapping it. If the investor has the option of postponing the implementation of such a project, tying up capital irreversibly becomes more expensive if the degree of uncertainty is high. The less investors know about future developments, the more

4 The flood of relatively recent studies on uncertainty and irreversibility was unleashed by: McDonald, R.L. and D.R. Siegel, The value of waiting to invest, Quarterly Journal of Economic, 101, pages 706 to 727, 1986. The monograph by Dixit, A.K and R.S. Pindyck, Investment under uncertainty, Princeton, 1994 is the first summary of this new literature.

difficult it is for them to make an irreversible commitment in the present.

The more uncertain the future, ...

Where investment is made immediately, the company not only gives up the monetary equivalent of the capital good it has purchased, but also sacrifices the option of being able to decide again at a later time based on improved information. From the company's point of view such an investment opportunity is a "real option", the value of which can, in principle, be calculated just like that of a call option on a dividend-bearing security. If investment is made now, the option on a future decision is sacrificed. In a rational investment decision, the value of this option must be taken into account as an additional cost component. This raises the opportunity cost of the decision to invest, which, in turn, raises the profitability threshold. This yield premium, as is typical of option prices, is dependent on the level of uncertainty regarding the course of future profitability. In Appendix 1 this subject will be discussed more thoroughly for the two-period case.

... the more valuable freedom of action is

Where uncertainty is great, it often appears advisable for entrepreneurs to hold on to their option, thereby maintaining their ability to adapt to economic developments which are still murky. Therefore, a relatively long period of heightened uncertainty can easily lead to a "wait-and-see" attitude towards investment. This general principle can also be applied to the demand for durable consumer goods, efforts to explore sources of raw materials and decisions to enter or exit the market. The effect of job protection rules on the demand for labour can also be studied using this versatile analytical instrument.[5]

Uncertainty as an entrepreneurial opportunity

The influence channels outlined so far tend to suggest that uncertainty has a dampening impact on investment activity. However, increasing uncertainty heightens not only the risks but also the opportunities of entrepreneurial activity. Those investing in their capital stock today are at the same time purchasing new opportunities to take action which remain closed to the hesitant. If an entrepreneur is able to adjust in a sufficiently flexible manner to the situation, uncertainty can even add economic value to the investment project. This mechanism, also known as the Hartman-Abel effect,[6] applies where, at the time of the investment, uncertainty about the output price exists, yet the amounts of variable factors of production being deployed, such as labour, energy or raw materials, can optimally be adjusted to different demand situations. In Appendix 2 this will be explained in depth using a two-period example.

Flexibility and freedom of action

If investors are able to adapt production optimally to changing circumstances, they thereby systematically increase the weight of favourable developments relative to less favourable events. Thus, uncertainty increases the expected value of the return and can thus be an independent incentive to invest. To a certain degree this is a complement to earlier

5 See: Bentolila, S. and G. Bertola, Firing costs and labour demand: How bad is Eurosclerosis? Review of Economic Studies, 57, pages 381 to 402, 1990.
6 See: Hartman, R., The effect of price and cost uncertainty on investment, Journal of Economic Theory, 5, pages 258 to 266, 1972, and Abel, A.B., Optimal investment under uncertainty, American Economic Review, 73, pages 228 to 233, 1983.

statements in this article that in an irreversible investment situation the cost of tying up capital rises with growing uncertainty. This pattern once again underlines the fact that flexibility and freedom of action are key variables in investment behaviour.

Effect depends on circumstances

On the other hand, though, the above considerations make it clear that the impact of uncertainty on the investment behaviour of profit-oriented entrepreneurs depends on the specific circumstances. Depending on the type of project, the enterprise's technology, its market position and the nature of uncertainty, the underlying phenomenon of uncertainty can either prevent or attract entrepreneurial investment.[7]

How and to what extent does uncertainty affect the "bet on the future" when looking at an aggregate of projects and enterprises? This can only be answered empirically, yet the findings do not permit any definite conclusions to be made. There are only a very few results available for Germany,[8] and only one study uses individual data.[9]

Microeconomic data are fundamentally better suited to explain the role of uncertainty than macroeconomic data. A research project run by the Bundesbank therefore attempted to use a large and in many respects unique set of individual balance sheets to gather more exact information about the relationship between investment and uncertainty.[10]

Size distribution of firms in the sample, by average number of employees

Average number of employees (n)	No. of enterprises	%
$n < 20$	675	10.01
$20 < n \leq 100$	2,622	38.87
$100 < n \leq 500$	2,547	37.76
$n > 500$	901	13.36

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The data for empirical review

The Bundesbank's corporate balance-sheet statistics are by far the most comprehensive statistically evaluated collection of German non-financial enterprises' annual accounts. The data date back to when the Bundesbank was still conducting bill-based business. The discounting of a trade bill of exchange made it necessary to check enterprises' credit rat-

7 See: Dixit, A.K. and R.S. Pindyck, op. cit., chapters 6 and 11, and Darby, J., A.J. Hughes Hallet, J. Ireland and L. Piscitelli, The impact of exchange level uncertainty on the level of investment, Economic Journal, 109, pages C55 to C67, 1999.
8 See, among others: Seppelfricke, P., *Investitionen unter Unsicherheit. Eine theoretische und empirische Untersuchung für die Bundesrepublik Deutschland,* Frankfurt am Main, 1996, and Werner, T., *Die Wirkung von Wechselkursvolatilitäten auf das Investitionsverhalten. Eine theoretische und empirische Analyse aus der Perspektive der Realoptionstheorie,* Kredit und Kapital, 34, pages 1 to 27, 2001 (available only in German).
9 This study, on the basis of data from 70 German firms, identifies a positive correlation between investment and uncertainty. See: Böhm, H., M. Funke and N.A. Siegfried, Discovering the link between uncertainty and investment – Microeconometric evidence from Germany, in Deutsche Bundesbank (ed.), Investing today for the world of tomorrow. Berlin, Heidelberg, New York, 2001.
10 See: v. Kalckreuth, U., Exploring the role of uncertainty for corporate investment decisions in Germany, Economic Research Centre of the Deutsche Bundesbank, Discussion Paper 5/00, 2000.

Composition of the sample, by sector

Sector (according to SYPRO)	No. of enter-prises	No. of obser-vations
Mineral oil processing	16	132
Quarrying and manufacture of mineral products	222	1,645
Iron and steel industry	118	859
Non-ferrous metal industry	64	495
Foundries	100	724
Drawing plants, cold rolling mills, secondary transformation of metals, etc.	284	2,087
Manufacture of structural metal products, rolling stock	236	1,680
Mechanical engineering	1,169	8,726
Manufacture of road vehicles and repair of motor vehicles	166	1,255
Shipbuilding	8	63
Manufacture of aircraft and spacecraft	4	32
Electrical engineering, repair of electrical household goods, etc.	385	2,921
Manufacture of precision and optical instruments, clocks and watches	285	2,119
Manufacture of tools and finished metal goods	526	3,967
Manufacture of musical instru-ments, toys and games, fountain pens, etc.	134	944
Chemical industry	349	2,629
Data processing equipment	19	130
Manufacture of ceramic goods	70	523
Manufacture and processing of glass	75	546
Wood-working	257	1,813
Manufacture of wood products	196	1,406
Manufacture of pulp, paper and board	193	1,444
Processing of paper and board	50	391
Printing and duplicating	268	1,998
Manufacture of plastic products	444	3,282
Manufacture of rubber products	59	455
Manufacture of leather and leather products	56	453
Textile industry	327	2,410
Clothing industry	208	1,528
Food and drink industry, tobacco products	448	3,302
Total	6,745	49,959

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ing. In that context, up until the end of the nineties an average of nearly 70,000 annual accounts were sent to the Bundesbank's branch offices every year for processing. Once they have been thoroughly reviewed and checked, these accounts form the data base for corporate balance-sheet statistics. With respect to manufacturing companies in western Germany, the data base covers around 75 % of the aggregate turnover, but only around 8 % of the total number of enterprises.

Technically speaking, the set of data underlying this study is called a panel. It contains not only information on a large number of individual enterprises at a fixed point in time but can also follow the development of those enterprises over time. A panel contains a considerably larger volume of information than purely cross-sectional data or time series.

However, it is not possible to use the whole panel of data from corporate balance-sheet statistics for econometric evaluation. For comparability reasons, the study is limited to western German manufacturing corporations and to the period between 1987 and 1997. Furthermore, only part of the annual accounts contains sufficiently exact information on the capital stock and investment. For statistical reasons, uncertainty can only be measured in those enterprises where annual accounts exist for at least eight consecutive financial years. After excluding statistical outliers, the panel is left with 6,745 firms and nearly 50,000 observations. Though this is not a representative sample in the strict statistical sense, it still mirrors the structure of the western German manufacturing industry relatively well. The median number of persons employed is 118, and a rather large share of small and medium-sized enterprises, which form the backbone of western German industry, is taken into account.

The estimation approach

In order to be econometrically useful, a measure of uncertainty must vary not only over time but also among firms. Uncertainty is a characteristic of the subjective image that decision-makers form of their environment and cannot be recorded with the same precision as the prices or quantity of output.

Profit uncertainty ...

The reference variable is profit, defined as the difference between earnings and costs. For both variables, uncertainty indicators are constructed from their respective volatility. The underlying premise is as follows: the more sharply revenue and costs have fluctuated in the past few years, the less certain the entrepreneur in question is regarding their future development.

... is sales uncertainty ...

For real sales, initially a first-order autoregressive equation with a firm-specific constant was estimated for 78 groups of firms. The groups were formed using the characteristics of sector and size. The residues of the estimation for a given company and a given year can be interpreted as a firm-specific "sales shock". As an indicator of sales uncertainty, the square of the sum of residual squares was formed for the past three years.

... plus cost uncertainty

An analogous approach was used to construct an indicator for cost uncertainty. A real cost variable was formed by calculating the difference between sales and the operating result and deflating it with output prices. Since costs are to a great degree determined by the amount sold, in a second step for each company a least-squares regression of costs on real sales was conducted. This filtered out all direct and indirect linear effects of earnings on costs. The residues left over are interpreted as time-specific and firm-specific "cost shocks" and taken to form uncertainty indicators similarly to sales shocks.

An accelerator equation

The decision on the investment equation to be estimated is also important. If a significant factor is left out, something might be attributed to uncertainty although its cause is completely different. The study uses the frequently adopted "accelerator equation", in which changes in real sales play a key role. Moreover, the company's cash flow was entered into the equation to take account of enterprises' profit expectations and financing constraints. The influence of interest and taxes on the user costs of capital, just like turbulence following German unification and other cyclical disruptions, were filtered out by adjusting for all macroeconomic movements using time dummies. Finally, the two uncertainty variables were included and tested both in isolation and in combination with one another.

The main results of the study

As the estimated results show, uncertainty may be classified as clearly disruptive to demand for investment. The strength of the negative effect is considerable. This becomes evident if one looks at an average deviation of the indicators from their mean value, also known as a standard deviation. If both indicators deviate upwards by one standard deviation, companies' investment will decline by an average of around 6½%. A downward deviation would, ceteris paribus, cause investment activity to go up accordingly. In addition, sales and cost uncertainty have roughly the same importance, i.e. each is responsible for roughly half of the overall effect.

Conversely, this means that the potentially positive impact, the Hartman-Abel effect, is apparently not dominant in Germany. As explained above, this transmission mechanism presupposes that the use of important variable factors can be adjusted to changing market conditions rapidly and at minor cost. In many cases the interplay of various labour-market-policy rules and high recruitment costs make such rapid adjustment to changing operational conditions more difficult. In such an institutional environment, though, the Hartman-Abel effect can only prevail in certain market niches, if at all.[11]

However, diagnosing a net negative effect does not permit one to conclude which of the three possible channels – risk aversion, asymmetrical information or irreversibility – is the chief culprit. Particularly the role of the financial structure in investment behaviour will need to be clarified in further studies by the Bundesbank and the research network of the Eurosystem's central banks.

In economic policy terms, it becomes clear that it will be much less difficult for German entrepreneurs to "bet on the future" if they know where they stand within their economic environment. However, this certainly cannot mean that the government could or should relieve them of their entrepreneurial risk. Yet the government, through reliable economic policies, can provide a stable foundation for private-sector planning. This would also enhance companies' willingness to invest.

However, an even more important element in regulatory policy terms is to create and maintain entrepreneurial freedom of action. The more the hands of tomorrow's investors are tied, the more hesitant they will be today and the less real capital they will provide to the site in question. By contrast, more flexibility in operations management, such as in regulating working hours, will create incentives to invest. The additional capacity, once it has been created, also benefits labour as a factor of production, through increased employment and, over the longer term, higher wages.

11 If the capital intensity chosen at one point in time must be maintained until the end of a capital good's life-cycle, this does not mean that the corporate sector could not adjust to changes in the business environment over the medium and long term. Investment in rationalisation is just as possible as is non-reinvestment or the dropping of expansion plans or even terminating operations at certain sites entirely. The medium-term consequences of the attempt to use the short-term immobility of capital as a factor of production for high wage demands are covered in: Deutsche Bundesbank, Factor prices, employment and capital stock in Germany: results of a simulation study, Monthly Report, July 2001, page 49ff.

Estimation of an accelerator equation with uncertainty

There is a long-run equilibrium between the capital stock and output, which is determined by relative factor prices. Investment determines the growth of the capital stock and is therefore in a fixed relationship to output growth. The influence of the user cost of capital is included by using time dummies under the assumption that capital use costs are the same for all firms at a given point in time. The cash flow per unit of capital has a dual function in the equation: as a measure of free liquidity it determines the scope for the enterprise's internal finance, and as a measure of profitability, it plays a significant role in expectations formation. Last but not least, measures for sales uncertainty and cost uncertainty are entered. The estimated behaviour equation for firm i is:

$$\frac{I_{i,t}}{K_{i,t-1}} = \sum_{m=0}^{M} \beta^{S}_{t=m} \hat{S}_{i,t-m} + \sum_{n=0}^{N} \beta^{F}_{t-n} \frac{F_{i,t-n}}{K_{i,t-n-1}} + \beta^{U} U_{i,t} + v_{i,t},$$

where $v_{i,t} = \alpha_i + \lambda_t + \xi_{i,t}$.

In this equation $I_{i,t}$ is the real expenditure of firm i in year t on investment in fixed capital. $\hat{S}_{i,t}$ is the growth rate of real sales, expressed as the first difference of the logarithm. $F_{i,t}$ is real cash flow, $K_{i,t-1}$ the real capital stock at the end of the previous period, and $U_{i,t}$ one of the two measures of uncertainty. The disturbance term $v_{i,t}$ is composed of a firm-specific constant α_i, a time-specific disturbance λ_t which is the same for all enterprises, and an idiosyncratic disturbance $\xi_{i,t}$.

Fixed-effect estimations of an accelerator equation with uncertainty
Dependent variable: real investment per unit of capital

Variables	(1)	(2)	(3)	(4)
Sum of coefficients for real sales growth of the years (t–3) to t	0.2448** (0.0249) P<0.0005	0.2478** (0.0249) P<0.0005	0.2407** (0.0250) P<0.0005	0.2437** (0.0250) P<0.0005
Sum of coefficients for real cash flow per unit of capital of the years (t–3) to t	0.1353** (0.0130) P<0.0005	0.1350** (0.0130) P<0.0005	0.1358** (0.0129) P<0.0005	0.1356** (0.0130) P<0.0005
Sales uncertainty from shocks of the years (t–3) to (t–1)		– 0.0457** (0.0172) P=0.008		– 0.0425* (0.0173) P=0.014
Cost uncertainty from shocks of the years (t–3) to (t–1)			– 0.1693** (0.0464) P<0.0005	– 0.1612** (0.0465) P=0.001
Number of observations	29,724			
Number of firms	6,745			

Additional regressors: time dummies and a constant. In brackets: standard deviations of the estimated coefficients, robust against general heteroscedasticity and autocorrelation within the observation unit. P-values: significance level. — * Significant at the 5 % level. — ** Significant at the 1 % level.

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Annex 1:

The option pricing model as an explanatory approach

In the following text, a closer look will be taken at investment projects which are not only irreversible but where there is also discretionary scope regarding the choice of when to invest. Those two characteristics are crucially important under uncertainty. The key statement is: in order for an irreversible investment to be made under uncertainty, the expected return must be greater than if the investment were freely reversible or if certainty were to exist. There are two equivalent methods of justifying this in economic terms:

- Irreversibility constitutes a restriction in future entrepreneurial activity. There is a certain probability that this restriction will be binding in future periods, in which the capital stock will prove to be too big. Yet since the latter cannot be reduced, the capacity overhang causes losses which could have been avoided if the investor would have waited. If the investment is to be made, the expected return must cover this possibility of a loss.

- Making an irreversible investment robs the investor of the opportunity to make the investment decision at a later point in time. This possibility – given imperfect competition, at any rate – generally has a positive value. From the entrepreneur's point of view the investment opportunity represents a real option, the value of which can be calculated like that of an option to purchase a dividend-bearing security. If the investment is made, the option is gone. Its value must therefore be taken into account in the investment decision as an additional opportunity cost.

1. A simple two-period model

This basic statement can be explained in somewhat more depth using a two-period model.[12] To start with, the structure of two-period irreversible investment decisions is characterised by maximising expected returns. Then, the principles of option valuation are explained. Building upon that, it will be shown that an irreversible investment decision can just as easily be described as a decision on whether to exercise a real option.

In the example, it shall be assumed that a potential investor wants to build a factory and has a choice between two possible dates for investment. There are irreversible costs of $I = 1{,}600$ money units (MU), i.e. these costs must be borne in full even if the project ends up losing money. In every period, the factory produces one unit of output, the price of which moves at random. Let the price at time $t = 0$ be 200 MU. In the next period it will rise with a probability of $q = 0.5$ to 300 MU. With the reverse probability $1 - q = 0.5$ it will fall to 100 MU. Then the price remains at its new level forever.

For simplicity, this model does not include ongoing production costs; the development of prices is therefore identical with the development of returns. Finally, it shall also be assumed for simplicity that either the investor is risk-neutral or the price development risk can be eliminated by means of a suitable method of diversification. In this case, the expected payment flows are relevant to the investment decision, and they can be discounted at

12 For a similar explanation see, among others: Pindyck, Robert S. (1991), Irreversibility, Uncertainty and Investment. Journal of Economic Literature, 29, pages 1110 to 1148. The formulas and calculations in this section were borrowed from Pindyck.

interest rate R for risk-free investment. In this example, let R = 10 %.

If the investor is unable to delay the decision, it is necessary to check whether the investment will lead to a positive net present value (NPV) if it is made in t = 0. The result for the (expected) net present value is:

$$(1)\ E(NPV) = -1{,}600 + \sum_{t=0}^{\infty} \frac{E(P_t)}{(1.1)^t} = 600,$$

since $E(P_t)$ = 200 for all values of t. Since the present value of the expected net payments exceeds the one-off purchase costs, the investment should be made according to the net present value criterion.

Now, if the investor is able to postpone the decision to a later point in time, this creates the opportunity to track price developments. If the price goes down to $P_1 = 100$, the investment no longer pays off: the present value of the returns in t = 1 would then only be 1,100 MU, which would not cover the initial outlays. For $P_1 = 100$ no investment is made and the losses associated with this environment are avoided. By contrast, in the best-case scenario, where $P_1 = 300$, investment is profitable. Further hesitation would be senseless, only leading to the loss of current income.

If the expected value of the net present value NPV* is calculated for the delay of investment for one period, the result is:

$$E(NPV^*) = 0.5 \cdot 0 + 0.5 \cdot \left[-\frac{1{,}600}{1.1} + \sum_{t=1}^{\infty} \frac{300}{(1.1)^t} \right] = 772.73\ MU$$

It is apparent that the project's expected present value is greater if the decision is put off. A rational investor will therefore not make the investment right away although the expected net present value is positive. Instead, it is better to wait and invest only when the price movements are advantageous. The difference between the two net present values, 172.73 MU, denotes the value of the option of waiting one period before deciding whether or not to invest.

2. The binomial model of option pricing

Treating an investment opportunity as an option in the analytical sense, too, is possible and revealing. The decision maker has the right to make an investment outlay within a certain time period and to acquire a project whose value fluctuates stochastically over time. This decision resembles that of the owner of a call option on a dividend-bearing security, such as a share. The decision on the optimum time of the investment can therefore be modelled just like the decision on exercising a call option.

In the following, the binomial option pricing model will be explained, with the help of which such options can be evaluated. It was developed by Cox, Ross and Rubinstein (1979).[13] In its simplest version, with two points in time, it may be applied directly to the above case.

A security will be examined, the price of which (prior to dividend payment) is subject to a simple stochastic process:

$$V_t = w_t V_{t-1}$$

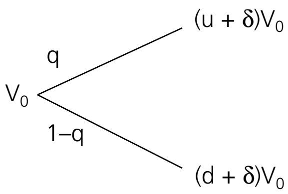
The factor w_t is a random variable. With a probability of q it assumes the value u and, with the reverse probability 1 – q, a lower value d. The values

13 See: Cox, J.C., S.A. Ross and M. Rubinstein, Option pricing: A simplified approach, Journal of Financial Economics, 7, pages 229 to 263, 1979.

u and d therefore give the price changes in the best-case and worst-case scenarios, respectively. At the end of period t, or at the beginning of t + 1, the holder of the security receives a dividend of

$D_t = \delta V_t$.

Holding a security during a period, i.e. between t = 0 and t = 1, will lead in the second period, with a probability of 1, to the value $(u+\delta)V_0$ and with the probability of 1–q to a lower value $(d+\delta)V_0$.



A call option gives its holder the right to buy the security at a fixed price K, the strike price, from the option seller. There are two basic types. A European option grants the right to purchase the paper at only one single point in time, at maturity. The holder of an American option, by contrast, can exercise the right to purchase at any single point in time prior to maturity. Here, a European option with a maturity of one period will be examined; the purchaser therefore has the right to purchase the underlying security in the following period at price K. The value of the option at the time of maturity, F_1, can be determined for all states of nature. It is calculated using the returns under optimum behaviour:

(2) $F_1 = \max[V_1 - K, 0]$

This value is a function of $V_1 = w_1 V_0$ and thus itself a random variable. F_u is the value for favourable price developments, F_d that given unfavourable price developments. Both values are non-negative. It is in the option's nature that it does not need to be exercised and therefore never leads to net out-payments. Option price theory looks at the prices of other marketable assets and calculates the value of the option according to the best arbitrage possibilities.

To value the option an "equivalent portfolio" is built with a pattern of yields that exactly tracks developments in the value of the option. It has two components: a number of units of the underlying security, and credit liabilities. The composition of the equivalent portfolio is selected in such a way as to generate the same payments as the option itself under all possible states of nature. Then, in equilibrium, the market price of the option must equal the value of its equivalent portfolio. Otherwise, by issuing new options while simultaneously purchasing the equivalent portfolio or by buying the option and selling the portfolio, one can reap virtually unlimited risk-free arbitrage profits.

In general, the value of a financial investment is measured in terms of the cash flows it generates. If a financial investment leads to the same payments as another financial investment for all states of nature with a positive probability, those investments are perfect substitutes and have the same value.

Let it be assumed that the investor, by issuing or purchasing zero-coupon bonds, could invest any amount of money at the market rate R or borrow money. Let the associated interest factor be $r = (1+R)$. If R and δ are equilibrium rates of return, the following must hold:

$$u + \delta > r > d + \delta$$

The interest factor belonging to the risk-free security must be higher than the percentage value development of the risky security given an unfavourable price development. Otherwise, even given an unfavourable price development, the total return on the share would be higher than that on the risk-free investment, and unlimited arbitrage profits would be possible. For the same reason, given a favourable development, the return on the security must be higher than r.

Now, let Q_V be the number of dividend-yielding securities contained in the equivalent portfolio and Q_B the number of zero bonds with a price normalised to unity. Q_B is negative if the investor borrows money on the capital market. The market value of the portfolio, in $t=0$, is:

(3) $F_0 = V_0 Q_V + Q_B$

At the point in time $t=1$, after the dividends have been paid and the interest has matured, the value of the portfolio is:

$F_1 = (w + \delta) V_0 Q_V + r Q_B$

The portfolio, Q_V and Q_B, must be chosen such that its value corresponds to that of the option in every state of nature:

$$uV_0Q_V + \delta V_0 Q_V + rQ_B = F_u$$
$$dV_0Q_V + \delta V_0 Q_V + rQ_B = F_d$$

Solving this system leads to:

$$Q_B = \frac{F_u (d + \delta) - F_d (u + \delta)}{(d - u)r}, \text{ and } V_0 Q_V = \frac{F_d - F_u}{d - u}$$

This determines the equivalent portfolio. The value of the European option is equal to the value of the portfolio in $t=0$ according to equation (3):

$$(4)\ F_0 = V_0 Q_V + Q_B = \frac{F_u (r - d - \delta) - F_d (r - u - \delta)}{r(u - d)}$$

Let it now be assumed that $uV_0 > K > dV_0$. In this case, the optimum behaviour for the investor is, where the price pattern is favourable, to exercise the option, and to let it elapse where prices are unfavourable. This means that for (2):

$F_u = uV_0 - K$ and $F_d = 0$

By inserting it into (4) one obtains

$$(5)\ F_0 = \frac{uV_0 - K}{r} \frac{r - d - \delta}{u - d} > 0$$

for the value of the European option at the starting time $t=0$. The holder of an otherwise identical American option can choose whether to exercise it right away, thus obtaining in $t=0$ the payment $V_0 - K$, or to hold the option for another period. If the option is not exercised, its value then corresponds to that of a European option. That means its value is Φ_0:

(6) $\Phi_0 = \max[V_0 - K, F_0]$

The option is exercised if the realised yield is higher than the value of the option when held, i.e. if the following holds:

$$V_0 - K \geq \frac{F_u (r - d - \delta) - F_d (r - u - \delta)}{r(u - d)}$$

3. The investment opportunity as a "real option"

Following these preliminary considerations, the decision problem described in section 1 can be described as a problem of evaluating an option to buy. The strike price K corresponds to irreversible investment outlays, with I=1,600 MU. The interest factor is r = 1.1. The expected net value of immediate investment corresponds to the market value of the security in the option pricing problem, so that:

$$V_0 = E \sum_{t=0}^{\infty} \frac{P_t}{(1.1)^t} = 2{,}200$$

Accordingly, the present value of future returns in the next period is $uV_0 = 3{,}300$ MU given favourable price movements and $dV_0 = 1{,}100$ MU given unfavourable price movements. The return of 200 MU from the sale of the product for immediate investment corresponds in economic terms to a dividend payment of $(1+R)\cdot 200$, resulting in $D_0 = 220$. For the value of a European option containing these features, by insertion into (5) one obtains:

$$F_0 = \frac{3{,}300 - 1{,}600}{1.1} \cdot \frac{1.1 - 0.5 - 0.1}{1} = 772.73 \text{ MU}$$

That would be the amount paid in the financial markets for the right to make an investment in the following period. Instead, though, investment can also be made immediately. The investment opportunity is therefore not a European option but an American option and must be valued according to (6) at $\Phi_0 = \max[V_0 - I, F_0]$. Immediate investment is preferable for $V_0 - I \geq F_0$. Since

$$V_0 - I < \frac{uV_0 - I}{r} \frac{r - d - \delta}{u - d}, \text{ or } V_0 - I = 600 < 772.73$$

immediate investment would be unwise. Therefore, $\Phi_0 = F_0$.

This illuminates from another angle the problem of choosing the right time to invest. The net present value criterion does not accurately evaluate the investment opportunity and fails as a decision-making rule since it does not completely describe the decision problem – it does not take into account all the opportunity costs. Valuation in terms of the net present value implies that the opportunity costs consist of nothing but the forfeited returns discounted at the risk-free interest rate R. In reality, though, when investment is made in t=0 the possibility of investing at a later time is lost, and with it the ability to decide on the basis of new information.

In order to make the investment seem profitable, in the event of immediate investment the expected total earnings less interest must not only cover sunk costs but also the value of the investment opportunity in the next period under optimum behaviour influenced by new information. The value of that opportunity under optimum behaviour shows up as the value of an option with the appropriate characteristics.

Annex 2:

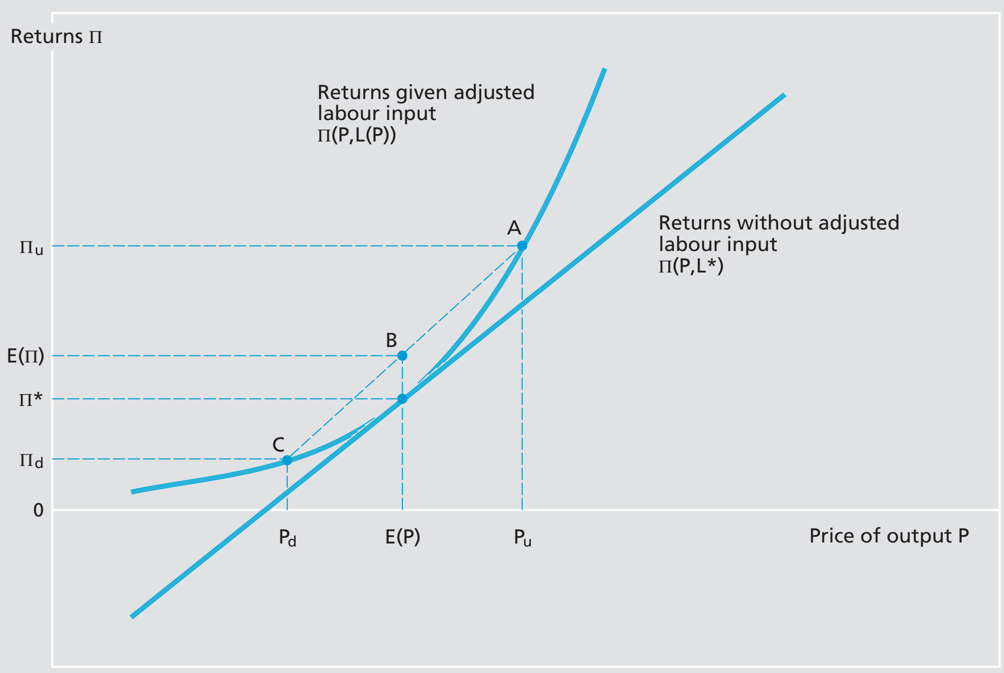
Uncertainty as an entrepreneurial opportunity

Wherever an enterprise is able to adapt optimally to a changing environment, uncertainty may also add value to an investment project. An example will serve in the following to explain this principle, called the Hartman-Abel effect.

Unlike the option pricing model introduced in Appendix 1, there is no delaying possibility here: investment is made either immediately or not at all. Besides capital, another variable factor is entered into the equation, L, which will be called labour here. The nominal wage rate is w MU, the lifespan of the project is infinite, and the production function is

$$Y = F(K,L) = K^{1/2}L^{1/2}$$

Returns as a function of the price of output



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The purchase of a capital unit $K=1$ is considered. The optimal labour input maximises the periodically occurring returns:

$$\Pi(P,L) = PY - wL = PL^{1/2} - wL \rightarrow \text{max!}$$

Depending on the sales price P, this leads to a demand for labour L(P) and returns $\Pi(P,L(P))$ where

$$L(P) = \frac{1}{4}\left(\frac{P}{w}\right)^2 \text{ and } \Pi(P,L(P)) = \frac{1}{4}P\left(\frac{P}{w}\right).$$

Let w = 100 MU. In a case of certainty, if the sales price in this period and all subsequent periods is $P=200$ MU, we then obtain $L^*=1$ and $\Pi^*=100$ MU. Now let us assume uncertainty: let the price, with a probability of q = 0.5, take on a value of $P=P_u=300$ MU for this and all subsequent periods, otherwise a value of $P=P_d=100$ MU is assumed. The expected value E(P) thus remains unchanged at 200 MU.

Now it is of crucial importance to know whether the investor, to match the demand situation, can freely determine the input of factors. If the labour input must be definitively set at the time of the investment,[14] the decision-maker maximises expected returns: the result is an optimum labour input of $L^*=1$ as before and expected returns of

$$E(\Pi) = q\Pi(P_u,L^*) + (1-q)\Pi(P_d,L^*) = 100 \text{ MU}$$

just as in the certainty case. However, if the investor only needs to determine the labour input once it

14 This is the case for what is called putty-clay technology, where factors can be substituted for one another prior to installation but inputs are fixed following installation.

is known what the sales situation is like, then it is optimal to use more labour if demand is high and less labour if demand is low. In both cases the investor can increase returns compared with the fixed labour input situation. For $P = P_u = 300$ the investor then demands $L(P_u) = 2.25$, and for $P = P_d = 100$ labour demand is confined to $L(P_d) = 0.25$. This corresponds to annual returns of

$$E(\Pi) = q\Pi(P_u, L(P_u)) + (1-q)\Pi(P_d, L(P_d)) = 125 \text{ MU}.$$

At an interest rate of 10 % the present value of returns is now 1,375 MU compared with only 1,100 MU under either certainty or uncertainty without the possibility of adjustment. If the investor is able to adjust, uncertainty adds value to the project. At the same time, in a situation of uncertainty the value of the project is increased by the possibility of adjustment. The project is more profitable and will also be implemented at a higher price. This relationship can be easily demonstrated in graph form.

The straight line $\Pi(P,L^*)$ shows the returns as a function of the price of output P for a given labour input L*. The relationship is linear because no adjustment is made. However, if labour input is adjusted, the returns in the best-case scenario, point A, as well as that in the worst-case scenario, point B, are both higher than in the non-adjustment case. The graph of the returns function is a convex curve, and the expected value, point C, is higher than in the case of certainty.

There are many ways to generalise this principle. However, if it is possible not only to adjust but also to wait, combined with irreversibility, the impact of uncertainty is ambiguous: it increases the present value of returns yet at the same time creates an option value for the possibility of waiting. [15]

15 See: Dixit, A.K. and R.S. Pindyck, op. cit., chapters 6 and 11, and Darby, J., A.J. Hughes Hallet, J. Ireland and L. Piscitelli, op. cit.