Calendar effects on economic activity

Calendar configurations can have a marked impact on economic activity. In the case of the quarterly rate of change in real gross domestic product (GDP), they account for as much as 1 percentage point. In the monthly movements of industrial output, calendar effects are often on a scale of more than 5 percentage points.

With regard to statistical measurement of calendar effects, a distinction should be made between two aspects. Following European recommendations, patterns that recur annually and are typical of a given month or quarter are assigned to the seasonal component of a time series. In a month with 31 days, more work is performed and more is consumed on average than in a month with 30 days, let alone a month with 28 days. This should be differentiated from effects that result, say, from the shift in the number of working days within the same given month or quarter. In the context of official seasonal adjustment, these are recorded separately as calendar effects.

The forms in which calendar effects appear are manifold and vary depending on the economic sector and the type of economic activity which is measured. For a large number of German economic indicators, the working-day model has proved to be effective in quantifying calendar effects. This model takes account of the fact that a working week of five days is usual in Germany, but that production is sometimes continuous, ie takes place even on public holidays. In the manufacturing sector, for example, one additional working day in the months January to November thus leads, on average, to a 3.4% higher monthly output. The effect is less pronounced in December, since production is cut back anyway in the period around Christmas. The scale of activity in other sectors of the economy, such as transport, likewise follows a working-day pattern. Retail sales, on the other hand, are influenced more by the number of days on which outlets are open for business. However, these effects are not equally strong in every month. For GDP, the calendar effect is derived by aggregation across all sectors of the economy. A 1% increase in the number of working days leads, on average, to a 0.3% rise in overall output.

Such calendar effects prove to be largely stable across time. The increased use of working time accounts and of more flexible working hours has no noticeable impact on this. Furthermore, the estimated relative effects are virtually independent of the cyclical situation.

In principle, the effects on output of "bridge days", the timing of school holidays or of weather conditions can also be estimated by calendar adjustment methods. For example, industrial output on such a bridge day is, on average, about one-third lower than on a normal working day. This effect is not independent of the cyclical situation, however. Difficulties in determining a stable correlation also arise when estimating the effects of school holidays. And, in assessing the effects of the weather using calendar models, subsequent catching-up effects are not clearly quantifiable. Accordingly, in official statistics, such effects are not assigned to the calendar component, but are shown instead in the collective item "irregular effects" of the relevant seasonally adjusted time series.

The importance of calendar effects

Macroeconomic relevance

Calendar configurations have a marked impact on economic activity. For example, real GDP increased at an annual rate of 1.7% in the first quarter of 2012. However, this growth received a boost from the leap year effect (29 February) and from the early Easter business season, which began in March. Calendar influences probably contributed an estimated 0.5 percentage point, so that the calendar-adjusted output rose by "only" 1.2%. A counter-movement took place in the second quarter: at 1.0% up on the previous year, the calendar-adjusted increase was 0.5 percentage point higher than that of the original values. Moreover, since the third quarter of 2012 had one working day less than the corresponding quarter one year previously, the growth rate of the original values in this case, too, is, at 0.4%, below the calendar-adjusted increase of 0.9%. Output in the manufacturing sector and construction output depend to a very large extent on the calendar. It is not uncommon for working-day deviations in a given month to amount to 5%, or even more, of the series level.

Effects on stock and flow variables

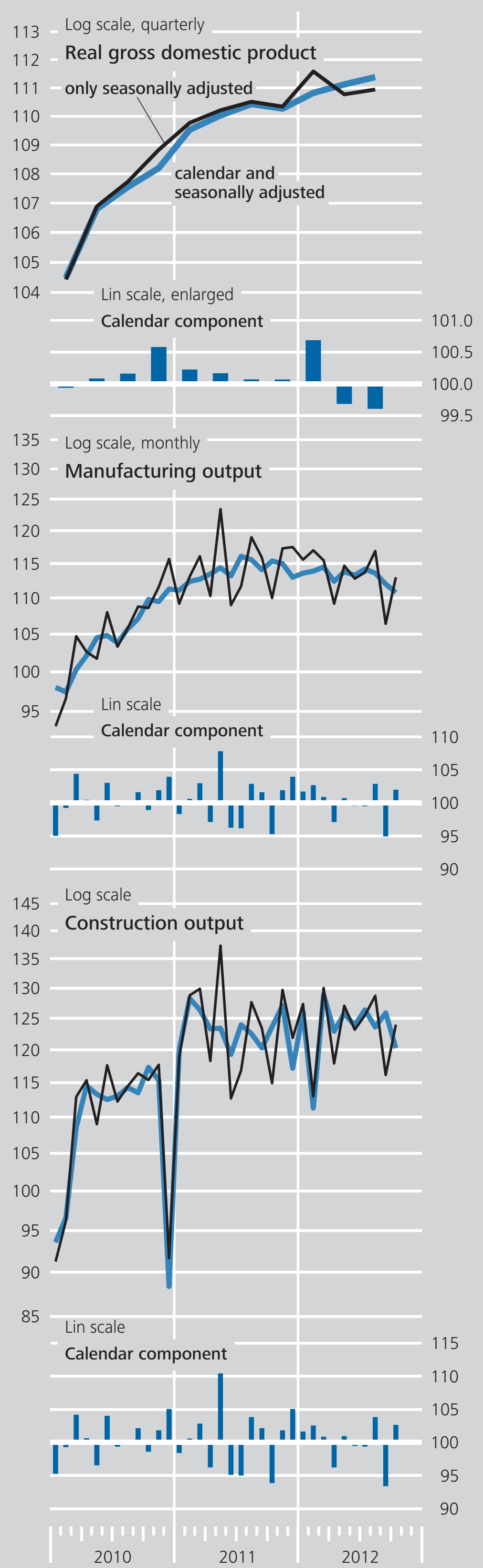
Calendar patterns not only affect flow variables (which are measured over a period of time), however, but also stock variables (which are measured at a point in time). The prices of some services, such as package holidays, depend positively on the timing of movable public holidays such as Easter or Whitsun, around which times demand for travel usually increases. And the volume of overnight deposits held by credit institutions at the end of a month is lower when the day of observation is shortly before the weekend, as this is when many individuals withdraw cash for the weekend ahead.

Seasonal and structural calendar effects

Moreover, the calendar affects seasonal behaviour. In months with 31 days, for instance, more work is performed and more is consumed than in a February with 28 days, and Christmas shoppers push retail sales to seasonal peak levels every December. Following European rec-

Calendar effects in selected economic indicators

2005 = 100



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ommendations,[1] effects that recur annually and can be allocated to a particular month or quarter are assigned to the seasonal component of a time series. Only the other calendar influences that result, for example, from the shift in the number of working days (and therefore in the number of weekends or public holidays) within the same given month or quarter are recorded in the context of official seasonal adjustment as calendar effects. These effects are examined in greater detail below.

Estimating calendar factors

Quantification of calendar effects reflected in daily data …

In order to quantify calendar effects precisely, it would actually be necessary to conduct daily statistical surveys, as then it would be possible to precisely measure production volume on 29 February, for instance, or retail sales on a given day before Easter.

… possible to estimate from monthly indicators, …

But because often only monthly data are available, the relevant calendar effects cannot be calculated directly. Instead, the Easter effect in March or April of each year becomes blurred with all the other effects in the month in question. For this reason, it is only possible to conduct estimates based on comparable calendar configurations that have occurred sufficiently often in the past.

… and difficult to gauge from quarterly data

In the case of quarterly data, for instance, the Easter effect, which sometimes occurs in the first quarter and sometimes in the second, overlaps with all the other effects that arise between 1 January and 30 June. As a result, direct quantification on the basis of quarterly time series leads to greater uncertainties than when using comparable monthly data. The quarterly approach is therefore generally considered difficult. For the same reason, the calendar adjustment of the quarterly national accounts (QNA) in Germany is also carried out using monthly indicators that are closely related to the corresponding QNA variables. These indicators are used to derive monthly calendar factors[2] which are condensed with variable weights into quarterly factors and are consolidated in the national accounts according to the importance of the individual series. The most accurate statistical quantification of calendar effects is possible following this procedure. Moreover, the consistent treatment of monthly and quarterly indicators of sectoral and macroeconomic output is assured; this is very important in analysing and forecasting economic activity.

Statistical model framework

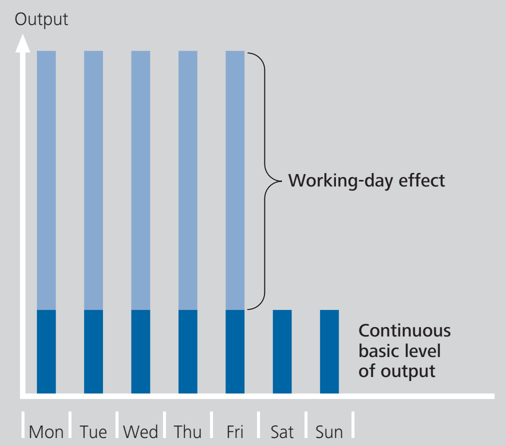
RegARIMA models have prevailed internationally for estimating calendar influences (for details, see annex with methodological notes on pages 59-60). These models can be used to determine semi-elasticities which, for example, indicate the average percentage effect of an additional working day (compared with the month-specific average) on monthly output.

Working-day approach

In Germany the working-day model has proved to be effective for a large number of economic indicators. It is based on a five-day working week and takes into account that, in some cases, production is carried out continuously – meaning even on public holidays. Since an additional working day in a month with a fixed duration always means a weekend day or public holiday day less in that month, the estimated working-day effect reflects exactly the difference, for example, between production on a normal working day and production on a weekend day. A distinction needs to be made between two extreme cases. In an economic sector with purely working-day production (ie without continuous production), the working-day effect is proportional. Assuming a month to have 20 working days on average, one additional working day would lead to an increase in production of 5%. On the other hand, if production is continuous and consistent across all the days of the week, there is no difference be-

1 Eurostat, ESS Guidelines on Seasonal Adjustment, 2009, Methodologies and Working Papers, available at http://epp.eurostat.ec.europa.eu/cache/ITY_OFFPUB/KS-RA-09-006/EN/KS-RA-09-006-EN.PDF.
2 Task force on Seasonal Adjustment of Quarterly National Accounts (2008), Final Report, available at http://www.cmfb.org/pdf/TF-SA%20QNA%20-%20Final%20Report.pdf.

Diagram of the working-day model



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tween the level of production on a working day and a weekend day. In such a case, therefore, no working-day effect is measured. The calendar effect is then fully included in the seasonal component.[3]

Working-day effects in the manufacturing sector

In the manufacturing sector an additional working day usually leads to 3.4% higher production in a month. However, the effect is lower in December (2.6%) because many firms cut back production in the time around Christmas in any case, irrespective of the number of working days.[4]

Weighting regional public holidays

In calculating the number of working days, both national and regional public holidays have to be taken into account. The latter are considered on a pro rata basis according to the share of employees in the federal states affected in each case. For example, Ascension Day, which is not a statutory public holiday in all states, counts for Germany as a whole as 0.3 of a working day. In sectors in which production is highly region-specific, only the regional calendar patterns are relevant, of course. That is why only the working days in North Rhine-Westphalia are used for the adjustment of mining and quarrying production. In this case, an additional working day results in an average monthly increase in production of 2.2%.

Working-day effects in energy supply lower

The working-day effect is lower in the energy supply sector, which largely produces on a continuous basis. An additional working day (meaning one weekend day less in a month of fixed duration) raises output by only 0.8% on average. Thus, the continuous production base is relatively high. This is reflected in the estimate of the leap year effect. Regardless of whether 29 February is a working day or a weekend day, continuous production on that day leads to an average increase of 2.8% for the month; the pure working-day effect described above applies in addition if 29 February is a working day.

Effects strong in construction, but less so in the cold season

In the construction sector the working days have an almost proportional effect in the warm season. Generally speaking, little work is performed at weekends. By contrast, the calendar effects on output are less pronounced in the months November to March, when output is primarily driven by weather conditions. When it is very cold for a prolonged period of time, and ice or snow hinders construction, one additional working day has less of an impact than in the rest of the year. For the quarterly sales figures in specialised construction activities, such month-specific considerations cannot be taken into account when calculating the calendar effects. Converted to a monthly basis, an additional working day leads to an increase in turnover of 1.9% on average.

Estimates for specialised construction activities less exact

National account aggregate for the construction sector

Amongst other things, the weighting of both indicators for the construction sector in the national accounts (on the supply side, gross value added in the construction sector; on the demand side, construction investment) takes into consideration the fact that the results from the first quarter are more strongly affected by specialised construction activities owing to the

3 However, leap years have to be modelled separately, as may be seen below in the treatment of the energy supply sector.
4 Statistically, it makes no sense to divide the calendar effects into the impact of the working days from the beginning of December to the weekend before Christmas and for the time after Christmas owing to the small number of relevant observation values.

Selected calendar effects in the national accounts*

Economic sector	Indicator	Estimation period	Calendar variable	Semi-elasticity[1]	t-value
Production sector (excl construction)					
Manufacturing	Production index	01.1991 – 05.2012	Working days January-November	3.4	46.7
			Working days December	2.6	14.4
Energy supply	Production index	01.1991 – 05.2012	Working days January-November	0.8	5.3
			29 February	2.8	2.7
Mining and quarrying	Production index	01.1991 – 05.2012	Working days North Rhine-Westphalia	2.2	12.8
Construction sector					
Construction of buildings and civil engineering	Production index	01.1991 – 05.2012	Working days April-October	4.5	16.8
			Working days November-March	3.2	9.1
Specialised construction activities	Turnover	1991 Q1 – 2012 Q1	Working days	1.9	3.6
Capital goods excl motor vehicles	Domestic turnover	01.1991 – 05.2012	Working days January-November	3.7	24.4
			Working days December	2.8	8.2
Wholesale	Turnover	01.1994 – 05.2012	Working days	2.9	28.4
Retail	Turnover	01.1994 – 05.2012	Sundays January-November	– 3.1	– 19.5
			29 February	4.2	8.2
			Public holidays March/April	– 1.8	– 6.0
			Public holidays May/June	– 1.7	– 8.5
			German Unity Day (if not a Sunday)	– 1.6	– 2.5
			Easter shopping days	0.4	6.7
			Easter late March/early April	0.6	1.9
Sale of motor vehicles	Turnover	01.1994 – 05.2012	Working days January-November	3.5	22.7
			Working days December	2.4	6.3
Hotels and restaurants	Turnover	01.1994 – 05.2012	29 February	2.4	3.9
Transport					
Railways	Net tonne kilometres in transport of goods	01.1991 – 04.2012	Working days	3.3	13.5
Buses and trains	Person kilometres in regular transport services	1999 Q1 – 2011 Q4	Working days	1.5	3.4
Trucks	Net tonne kilometres	01.1991 – 02.2012	Working days	4.1	29.2
Permitted vehicle type	Owner group				
Passenger vehicles	Private	01.1991 – 06.2012	Working days	4.1	13.4
	Commercial	01.1991 – 06.2012	Working days	3.6	12.6
Commercial vehicles		01.1991 – 06.2012	Working days	3.9	14.7
External trade					
Goods					
Intra trade	Exports	01.1993 – 05.2012	Working days	2.0	10.1
	Imports	01.1993 – 05.2012	Working days	1.6	6.5
Extra trade	Exports	05.2004[2] – 05.2012	Working days	3.0	9.6
	Imports	05.2004[2] – 05.2012	Working days	1.6	4.3
Capital goods excl motor vehicles	Imports	01.2003 – 05.2012	Working days	1.6	3.7
Services	Revenue	01.2003 – 05.2012	Working days	1.4	3.0
	Expenditure	01.1991 – 05.2012	Working days	1.3	4.8

* Calculated using RegARIMA models. **1** A working-day regression coefficient of x means that an additional working day in the month leads on average to an increase of x%. **2** Change in the recording of external trade with the 10 new EU member states.

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weather-related output losses in outside production. Moreover, specialised construction activities have risen steadily in importance compared with construction over the last few decades given the growing volume of renovation and modernisation work carried out on existing buildings.

Retail trade shows different calendar patterns

Whereas monthly production in most sectors is mainly affected by the working days from Monday to Friday, retail turnover is influenced more by the number of days on which outlets are open or closed for business. Calendar effects are not equally strong in each month in the retail trade. One Sunday more (and therefore one shop opening day less) in the months January to November means, on average, a 3.1% drop in sales. In December, the number of shop opening days shows no significant impact, however, because Christmas trade, the predominant factor, takes place regardless of the number of days on which retail outlets are open for business. An additional shop opening day as a result of a leap year, on the other hand, leads to an increase in February retail sales by 4.2%. In the case of retail sales of food, beverages and tobacco products, consumers do more of their shopping on Thursdays and, in particular, on Fridays compared with the other weekdays.

Effect of the timing of Easter …

Public holidays also affect aggregate retail trade turnover. On the one hand, Good Friday and Easter Monday reduce turnover in the respective March or April. On the other hand, Easter stimulates sales on the days prior to the Easter period. The bulk of Easter purchases are made during the last two weeks before Easter. Depending on when Easter falls, this results in additional sales proceeds in March and/or April.

… and other public holidays

The first of May (if it does not fall on a Sunday), Ascension Day or Whit Monday have a dampening effect on turnover in the months May and June by an average of 1.7% in each case. German Unity Day has to be taken into consideration in October. If this national public holiday does not fall on a Sunday, the result is a 1.6% drop in retail sales.

Aggregate calendar effects in trade

In the national accounts, trade includes retail and wholesale trade as well as the sale of motor vehicles. For the latter two components, only the number of working days in the five-day week is of significance according to empirical studies. Although many motor vehicle traders are open for business on Saturdays as well, it is possible that the sales generated on a Saturday are not entered into the accounts until the following Monday, so that they are statistically allocated to the respective month under that day. Another factor could be that new vehicles are mainly picked up from the dealer on the day they are registered – which can only be on a normal working day. With regard to the aggregation of the calendar factors for gross value added in the trade sector as a whole, it may be seen that the importance of wholesale trade is greatest, accounting for almost 50%. Retail trade follows (over 35%), with the sale of motor vehicles accounting for around 15%. Only in the fourth quarter is the retail trade's share roughly two percentage points higher than usual due to the Christmas effect, while the weights of the other segments are correspondingly lower.

Transport, storage, machinery and equipment as well as external sector

Activities in other economic sectors such as transport also follow a working-day pattern (transport production is measured by person kilometres and net tonne kilometres recorded by the railways, buses and trains as well as by net tonne kilometres for trucks). The exports and imports recorded in the national accounts are based, respectively, on the indicators of the exports and imports of extra trade and intra trade in goods as well as revenue and expenditure from cross-border trade in services. The quarterly series currently show an average working-day elasticity of around 0.4 for exports and around 0.3 for imports.

Working-day elasticity of GDP

The calendar factor for real GDP is ultimately derived from all these components. Macroeconomic working-day elasticity is currently around

0.3. This means that one percent more working days leads to an average increase in overall economic output by 0.3%, whereby the calendar effect is usually somewhat lower in the fourth quarter because of the Christmas effect.

Calendar factors affected neither by working time accounts …

The introduction of working time accounts and of more flexible working hours has no noticeable impact on the calendar effects described above. As stipulated in the collective wage agreements, credit hours are mainly to be accumulated and used up during the normal working week from Monday to Friday. This provision therefore does not affect the average gap between the economic activity on a working day and a weekend day, which is the only factor responsible for the effects within the working-day model.

… nor by the business cycle

The estimated relative calendar effects (semi-elasticities), too, are virtually independent of the cyclical situation. Output per working day is greater during boom periods than recessions. However, as long as the continuously produced goods rise by the same extent, the relative working-day effects do not change (compared with the weekend day effects). This makes it possible to predict them accurately.

Changes resulting from shift in economic structure

Nevertheless, the relative importance of calendar effects does change over time. This can result in aggregates from shifts in the importance of economic sectors with strong or weak working-day effects. For example, the working-day elasticity of gross value added in the construction sector is decreasing in the long term because specialised construction services, which show relatively little calendar impact, are gaining in importance.

Special aspects: bridge days, school holidays and weather conditions

In addition to the effects described above and taken into account in the official seasonal adjustment, there are other calendar effects whose impact is verifiable as an average. However, quantifying this impact causes problems.

Bridge-day effects can be proved, …

One such example is the bridge day. This term refers to a day that falls between a statutory public holiday and the weekend. Bridge days are counted as normal working days for purposes of the working-day model. However, employees increasingly take advantage of them to use up flexitime credit hours or for a long weekend. For this reason, output in the manufacturing sector is, on average, around one-third lower on a bridge day than on a normal working day. At the turn of the year, however, when many firms reduce production in any case, the bridge-day effect is lower. Of course, a series that is additionally adjusted for bridge days is smoother than a series that is only calendar and seasonally adjusted because some variance is assigned to the bridge-day effects and filtered out.

… but they give rise to estimation problems

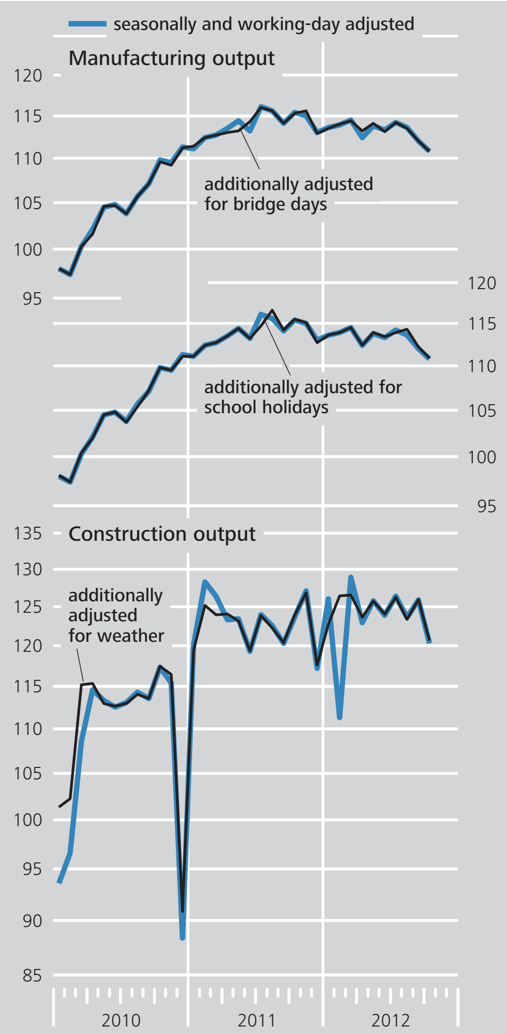
Quantification difficulties arise, however, due to the fact that the effect of bridge days is not proportional to their number. The effect is smaller in months with two bridge days, because in many cases only one of the two days is used for a long weekend. Thus, automatically taking the one-bridge-day effect into consideration twice would lead to an over-adjustment. On the other hand, an estimate that took all of these special factors into consideration would not be statistically secured due to the small number of observation values, nor could it be carried out on a disaggregated level. Moreover, it has to be borne in mind that an additional day of holiday taken on a bridge day implies a day of holiday less over the rest of the year, which means that a counter-entry is necessary in order to avoid distortions that affect economic analysis in the seasonally and calendar-adjusted series. However, indirect effects and their temporal distribution cannot be reliably ascertained.

Growing importance and cyclicality of bridge-day effects

In addition, bridge-day effects are not stable over time. On the one hand, they have been growing in importance for decades; on the

Adjustment for bridge days, school holidays and weather

2005 = 100, log scale



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other, the extent to which they are used depends on the economic situation. In phases of weak economic activity or crisis, bridge days tend to be used more as a means of quickly bringing production into line with an unfavourable orders situation. Conversely, somewhat less use seems to be made of bridge days, on average, during boom periods.

School holidays restrain output, …

Whether and how much work is performed on a given day does not depend solely on whether that day is a normal working day, a national or regional public holiday or a bridge day, however. The timing of school holidays is also of relevance. Many employees have school-age children, and so interrupt their work during the school holidays. Businesses, too, temporarily stop production. Since school holidays do not fall at the same time throughout Germany and their timing varies from year to year in the individual federal states, the effect is not completely captured by seasonal adjustment. For this reason, it could be modelled within the scope of calendar adjustment in principle, as it is also treated in this context by European recommendations.[5]

… but are not securely quantifiable …

It becomes evident that the strength of the impact caused by a shift of holidays greatly depends on the individual months. The effects of the movement of holidays are strongest in July and August. There is little or no evidence of such effects in other months. The month-specific estimate is based on a very small number of observation values, meaning that the result is not stable over time: further individual observations may have a strong impact on the estimated result.

… and are dependent on the cyclical situation

As with the adjustment for bridge days, the economic situation here, too, has an impact on the size of the effect. In the years 2011 and 2012, many firms in the car manufacturing industry refrained from cutting back production during the summer holidays to be able to meet the growing demand.

Weather effects in calendar models …

In connection with calendar adjustment, European statistical recommendations[6] also address the question of weather effects on construction output such as those that result from differing numbers of ice or snow days, as the same method can be used to measure all these effects. Because weather conditions can vary greatly from one region to the next, local weather data have to be weighted according to their importance in explaining construction

5 Eurostat, ESS Guidelines on Seasonal Adjustment (2009), Methodologies and Working Papers, Item 1.1.
6 See Task Force on Seasonal Adjustment of Quarterly National Accounts, 2008, Final Report, and Eurostat, ESS Guidelines on Seasonal Adjustment (2009), Methodologies and Working Papers.

output and aggregated. It would actually be necessary to use data that cover the location of each building site. But since such data are not available, use is made of a breakdown of employees in the construction sector by company domicile as an approximation for weighting data from selected meteorological offices.

… attest to significant relevance, …

According to the above method, construction output falls considerably as a result of one additional ice day.[7] The effects are strongest in February and March. The series which is additionally adjusted for weather conditions is, of course, also smoother than the series which is only adjusted for calendar and seasonal effects, because the additional variable explains part of the variance. Nevertheless, a number of months still show remarkably high residual weather effects in series that are actually weather-adjusted. In other months there is more of an over-adjustment.

… but catching-up effects not clearly quantifiable

But the list of problems in quantifying the weather effect does not stop there. The days on which ice and snow days fall have a varying impact not only between, but also within, individual months. For example, work is discontinued at many building sites between Christmas and New Year in any case, so that the weather conditions are only of minor importance on those days. Moreover, sustained periods of cold and the weather-related production backlogs are made up for in subsequent periods; statistically, however, these catching-up effects cannot be satisfactorily quantified. For these reasons, the official seasonal and calendar adjustment does not give special consideration to non-seasonal weather effects.

Conclusion

Calendar configurations affect economic activity in many different ways. A varying number of working days has a particularly strong impact on sectoral and macroeconomic output, and the effect is greater the less continuous production is. The estimated calendar effects are not dependent on the economic situation; nor has the introduction of working time accounts had any impact. Retail sales are primarily affected by the number of days on which outlets are open for business. Easter plays a particular role in this context. On the other hand, special problems arise with respect to the quantification of bridge days, school holidays or the effects of weather conditions. Official statistics are therefore not adjusted for such effects. Official calendar adjustment filters out only those influences that are statistically significant, can be explained in economic terms and are sufficiently stable over time, which is to say they have a high level of predictability.

Annex

Methodological notes

Estimating calendar effects using a RegARIMA model

From an estimation point of view, calendar effects cannot be determined using simple regression analysis models in which, for example, only the number of working days explains economic activity. Such an approach would be misspecified, as it would not take the systematic trend-cyclical and seasonal influences into account.[8] In order to avoid false conclusions, these time series components therefore have to be integrated. This is done with the help of RegARIMA models by using regular and seasonal differences as well as ARMA parameters. As the abbreviation "RegARIMA" suggests, this model class combines a regression model with ARIMA techniques.

$$(1)\quad (1 - B)^d (1 - B^S)^D \ln y_{i,j} = (1 - B)^d (1 - B^S)^D \sum_{k=1}^{n} \beta_k (x_{k_{i,j}} - \bar{x}_{k_i}) + w_{i,j},$$

7 On an ice day the maximum air temperature is below 0°C, which means permanent frost.
8 For statistical purposes, the flawed modelling would be reflected in a significant autocorrelation of the residuals.

with

$$\phi_p\,(\mathrm{B})\Phi_P\,(\mathrm{B}^S)w_{i,j} = \theta_q\,(\mathrm{B})\Theta_Q\,(\mathrm{B}^S)a_{i,j}\,.$$

Here, i=1,...,S (with S=12 representing monthly data and S=4 representing quarterly data) and j stands for the year. The expression $(1-\mathrm{B})^d$ defines a regular difference operator of order d and $(1-\mathrm{B}^S)^D$ denotes a seasonal difference operator of order D using the lag operator B, where $\mathrm{B}^m z_t := z_{t-m}$.

$x_{k_{i,j}} - \bar{x}_{k_i}$ is the k-th regressor, which is defined as the deviation of the monthly value from the long-term month-specific average, which as a part of the seasonal component is not assigned to the calendar factors. β_k is the corresponding regression coefficient, which can be interpreted as semi-elasticity. ϕ_p, Φ_P, θ_q and Θ_Q represent polynomials of grade p, P, q or Q. $a_{i,j}$ is assumed white noise.

This yields, as the estimated calendar component,

(2) $\hat{c}_{i,j} = \exp\left(\sum_{k=1}^{n} \hat{\beta}_k (x_{k_{i,j}} - \bar{x}_{k_i})\right).$

Deriving quarterly calendar factors from monthly time series

To derive quarterly calendar factors from monthly time series, the following model for decomposing time series is assumed for both the monthly and the corresponding quarterly time series.

(3) $y_t = t_t \cdot s_t \cdot c_t \cdot i_t\,.$

The original values y can be expressed as a product of the unobservable components trend cycles (t), season (s), calendar (c) and irregular influences (i). The index t beside the original values and the components stands for the time. The corresponding seasonally and calendar adjusted series is defined as

(4) $\frac{y_t}{s_t \cdot c_t} = t_t \cdot i_t\,.$

As a simple example, it is assumed for flows that the quarterly original value (Y) results from the sum of the relevant monthly values ($y^{(1)}$, $y^{(2)}$ and $y^{(3)}$):[9]

(5) $Y_t = y_t^{(1)} + y_t^{(2)} + y_t^{(3)}.$

Using (3), it follows that

(6) $$\begin{aligned} Y_t &= T_t \cdot S_t \cdot C_t \cdot I_t \\ &= t_t^{(1)} \cdot s_t^{(1)} \cdot c_t^{(1)} \cdot i_t^{(1)} + t_t^{(2)} \cdot s_t^{(2)} \cdot c_t^{(2)} \cdot i_t^{(2)} \\ &\quad + t_t^{(3)} \cdot s_t^{(3)} \cdot c_t^{(3)} \cdot i_t^{(3)}. \end{aligned}$$

Thus, the required quarterly calendar factor can be represented as

(7) $$\begin{aligned} C_t &= \frac{t_t^{(1)} \cdot s_t^{(1)} \cdot i_t^{(1)}}{T_t \cdot S_t \cdot I_t} \cdot c_t^{(1)} + \frac{t_t^{(2)} \cdot s_t^{(2)} \cdot i_t^{(2)}}{T_t \cdot S_t \cdot I_t} \cdot c_t^{(2)} \\ &\quad + \frac{t_t^{(3)} \cdot s_t^{(3)} \cdot i_t^{(3)}}{T_t \cdot S_t \cdot I_t} \cdot c_t^{(3)}. \end{aligned}$$

The calendar factor of the quarterly series is therefore a weighted arithmetic average of the monthly calendar factors, where the monthly share of the quarterly calendar-adjusted data serves as the weight. If it is postulated that relationship (5) is satisfied even when there is no calendar influence, then

(8) $$\begin{aligned} T_t \cdot S_t \cdot I_t &= t_t^{(1)} \cdot s_t^{(1)} \cdot i_t^{(1)} + t_t^{(2)} \cdot s_t^{(2)} \cdot i_t^{(2)} \\ &\quad + t_t^{(3)} \cdot s_t^{(3)} \cdot i_t^{(3)}. \end{aligned}$$

C_t can therefore be determined on the basis of monthly data by inserting equation (8) into equation (7). This takes the varying month-specific importance of the calendar effect in the quarterly series into account. Thus, the share of a monthly calendar factor in the quarterly factor generally does not correspond to one-third.

9 Depending on the index type, in some cases considerably more complex mathematical relationships can result between the monthly and quarterly values. When this is the case, the following remarks are to be adjusted accordingly.