Who cares about Inflation?
Endogenous Expectation Formation of Heterogeneous Households

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Household level inflation expectations are highly dispersed.

- 4pp interquartile range for US households.
- Households make substantial errors.

Today: Focus on co-movement of these errors with wealth.

Key results:
1. Empirically: Wealthier households make less errors.
2. Rationalized by endogenous (costly) expectation formation.
3. Theoretical response of HHs’ consumption to expected inflation depends on wealth.

⇒ Aggregate response determined by co-movement of expectations and wealth.
Empirical Observations
Empirical Setup

DNB Household Survey (CentERpanel)
Annual data on asset holdings and inflation expectations. Individual level (household heads), waves 2010-2018. \( \sim 11,500 \) observations in total.

Empirical Methodology:
- Construct net financial wealth (NFW): Net wealth ex housing, vehicles and businesses.
- Sort HHs into wealth decile groups: Decile of NFW distribution in year of observation.
- Compute expectation errors: Ex-ante point forecast – ex-post realized inflation rate.
- Compute “aggregate” statistics: Mean absolute errors and standard deviation of errors within each decile group.
Expectation errors co-move with wealth.

- Drop of 0.5 pp in standard deviation and mean absolute error from peak to bottom.
- Highest standard deviation and absolute errors around zero NFW (2nd decile).
- Robust to age / education controls.

Expectation Errors by Wealth Decile Groups
95% confidence bands. Data from DNB Household Survey waves 2010-2018.
A Theory of Expectation Formation and Wealth
A Model of Inflation Expectations

Realized inflation (exogenous):

\[ \pi_{t+1} = (1 - \rho)\mu + \rho \pi_t + \epsilon_{t+1} \quad \epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2_{\epsilon}) \]

Households:

- Know underlying process, persistence \( \rho \), long-run mean \( \mu \).
- Informed about current inflation \( \pi_t \).
- Signal \( \hat{e} \) with noise \( s \), noise reduced by effort \( n \).

\[ \hat{e}^i_{t+1} = e_{t+1} + s^i_{t+1} \quad s^i_{t+1} \sim \mathcal{N}(0, \sigma^2_s(n^i_t)) \]

- Assume pure noise:

\[ s^i_t \perp s^j_t \forall i, j, t, \quad e_t \perp s^i_t \forall i, t \quad \text{and} \quad s^i_t \perp s^i_{t+1} \forall i, t \]
A Model of Inflation Expectations - Errors

Bayesian updating with weight $\omega_{t+1}^i(n_t^i) = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_s^2(n_t^i)}$.

Posterior belief:

$$e_{t+1}|\hat{e}_{t+1}^i, n_t^i \sim \mathcal{N}(\omega_{t+1}^i(n_t^i)\hat{e}_{t+1}^i, \omega_{t+1}^i(n_t^i)\sigma_s^2(n_t^i))$$

Expectation error:

$$err_{t+1}^i = \omega_{t+1}^i(n_t^i)s_{t+1}^i - (1 - \omega_{t+1}^i(n_t^i))e_{t+1}$$

Implications:

- Error = overreaction to noise + underreaction to news.
- Standard deviation and mean absolute error increase in subjective uncertainty ($\Rightarrow$ increase in noise).
HH Problem with Endogenous Expectations

- Infinitely lived households, Epstein-Zin preferences.
- Receive risky **real** income \( y \).
- Save/borrow between periods at constant **nominal** rate \( r^n \).
  \[ \Rightarrow \text{Inflation is a risk to the real interest rate.} \]
- Expectations as before.

- Each period:
  0. Wealth known, learn about \( \pi \) and \( y \).
  1. Chose noise of signal (costly effort \( n \)).
  2. Receive signal and update beliefs.
  3. Decide on consumption and saving/borrowing.
HH Problem with Endogenous Expectations

Consumption-savings choice, conditional on $n$ and $\hat{e}$:

$$V(a, y, \pi, n, \hat{e}) = \max_{c, a'} \left( c^{1-\gamma} + \beta \left( E_{\pi'} [\tilde{V}(a', y', \pi')^{1-\alpha} | \hat{e}, n, \pi, y] \right)^{\frac{1-\gamma}{1-\alpha}} \right)^{\frac{1}{1-\gamma}}$$

s.t. $c + a' = \frac{1 + r^n}{1 + \pi} a + y - F(n)$

Information choice:

$$\tilde{V}(a, y, \pi) = \max_{n} E_{\hat{e}} [V(a, y, \pi, n, \hat{e}) | n]$$

Functional Forms:

$$\sigma_s(n) = \frac{\chi}{1 + n} \quad F(n) = (\theta n)^\phi$$
Expectation Errors along the Wealth Distribution Model

Model matches decline of errors in wealth.

- Drop of 0.5 pp in standard deviation from peak to bottom.
- Highest standard deviation and absolute errors around zero wealth (2nd decile).

Expectation formation is driven by exposure to real interest rate risk.

Expectation Errors by Wealth Decile Groups
Simulated model implied statistics.
Co-movement of expectations and wealth important for aggregate consumption response.

- Noise reduces consumption response to signal.
- Inflationary signal has heterogeneous income effects.
  \[ \Rightarrow \text{Consumption response declining in wealth.} \]

Attentive low wealth HHs most responsive to expected inflation.

Marginal Propensity to Consume on Signal
Change in consumption policy if \( \pi_t = 2 \) and \( \hat{e}_{t+1} \) changes from 0 to 1pp.
Conclusion

Findings:
1. Dispersion and mean absolute error vary with wealth.
2. Rationalized in a model with endogenous expectations.
3. Response to expectations depends on noise and wealth.

⇒ Attentive low wealth HHs most responsive, but low wealth HHs most likely inattentive.

Co-movement of wealth and expectations could impact...
... effectiveness of forward guidance.
... intertemporal substitution channel in general.
Appendix
Dispersion of Inflation Expectations (US Data)

Time Series of Inflation Expectations
Data from Michigan Consumer Survey and FRED.
Related Literature

Expectation formation

▶ Models: Coibion and Gorodnichenko (2012), Mankiw et al. (2003), Patton and Timmermann (2010)...

⇒ Here: Precision/uncertainty and influence of wealth.

Inflation expectations and household choice:

▶ Armantier et al. (2015), Crump et al. (2015), Dräger and Nghiem (2018), Vellekoop and Wiederholt (2018), Coibion et al. (2019)...

⇒ Here: Reverse impact of choices on expectations.

Heterogeneous agent models with information frictions:

▶ Auclert et al. (2019), Broer et al. (2018), Carroll et al. (forthcoming), Lei (2019), Yin (2018)...

⇒ Here: Endogenous, disciplined by (expectation) data.
### Net Financial Wealth Decile Groups - Summary

<table>
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<tr>
<th>Decile</th>
<th>N</th>
<th>Min</th>
<th>Max</th>
<th>1pct</th>
<th>99pct</th>
<th>N err</th>
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<td>111,379</td>
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<td><strong>Total</strong></td>
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<td>-1,791,382</td>
<td>3,940,520</td>
<td>-48,500</td>
<td>529,396</td>
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</table>

Data from DNB Household Survey waves 2010-2018. Net financial wealth refers to net wealth ex housing, mortgages, businesses and vehicles. Decile groups overlap due to differences in cut-offs across waves.
# Expectation Error - Summary

## Expectation Error by Decile Groups - Summary

<table>
<thead>
<tr>
<th>Decile</th>
<th>Mean</th>
<th>Sd</th>
<th>Mean (abs)</th>
<th>N</th>
</tr>
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<td>1.75</td>
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<td>2</td>
<td>1.71</td>
<td>2.00</td>
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<td>1.24</td>
<td>1.52</td>
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<td>6</td>
<td>1.19</td>
<td>1.39</td>
<td>1.42</td>
<td>1,150</td>
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<td>7</td>
<td>1.25</td>
<td>1.43</td>
<td>1.46</td>
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<td>8</td>
<td>1.22</td>
<td>1.46</td>
<td>1.44</td>
<td>1,174</td>
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<td>9</td>
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<td>10</td>
<td>1.14</td>
<td>1.37</td>
<td>1.36</td>
<td>1,201</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>1.32</strong></td>
<td><strong>1.60</strong></td>
<td><strong>1.55</strong></td>
<td><strong>11,536</strong></td>
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</tbody>
</table>

Data from DNB Household Survey waves 2010-2018. Summary statistics for expectation error by net financial wealth decile groups, computed as ex-ante point forecast for inflation minus ex-post realized inflation rate.
Mean Error by Wealth Deciles

Mean Expectation Error by Wealth Quintiles
95% confidence bands. Data from DNB Household Survey waves 2010-2018.
Empirical Observations - Robustness

Controls - Age and Education:
- Findings are robust by age / education groups.

Michigan Survey of Consumers:
- No comprehensive wealth data, proxy by stock investment.
- Errors declining in stock investment.
- Robust to controlling for age / education.
Error Standard Deviation - Controls

(a) by age

(b) by education

Standard Deviation of Errors by Wealth Quintile Groups

95% confidence bands. Data from DNB Household Survey waves 2010-2018. Combination of youngest age and highest wealth quintile omitted due to lack of observations.
Mean Absolute Error - Controls

(a) by age

(b) by education

Mean Absolute Expectation Error by Wealth Quintile Groups

95% confidence bands. Data from DNB Household Survey waves 2010-2018. Combination of youngest age and highest wealth quintile omitted due to lack of observations.
Mean Error - Controls

(a) by age

(b) by education

Mean Expectation Error by Wealth Quintile groups

95% confidence bands. Data from DNB Household Survey waves 2010-2018. Combination of youngest age and highest wealth quintile omitted due to lack of observations.
Expectation Errors along the Wealth Distribution
Michigan Data

Expectation Errors by Investment Quintile Groups
Error Standard Deviation - Controls

Michigan Data

(a) by age

(b) by education

Standard Deviation of Errors by Investment Quintile Groups
Mean Absolute Error by Investment Quintile Groups

(a) by age

(b) by education

Mean Error by Wealth Quintiles
Michigan Data

Mean Error by Investment Quintile Groups
Mean Error - Controls
Michigan Data

(a) by age
(b) by education

Mean Error by Investment Quintile Groups
Allow disagreement about long-run mean \( \mu_i \sim \mathcal{N}(\bar{\mu}, \sigma_{\mu}^2) \), \( \bar{\mu} \neq \mu \).

For a group \( g \) assuming that \( n_i^t = \bar{n}_g^t \ \forall \ i \in g \) we get:

Variance of expectation errors:

\[
\text{Var}(\text{err}_{t+1}^i | g) = (1 - \rho)^2 \sigma_{\mu,g}^2 + \omega_{t+1}(\bar{n}_t^g)\sigma_s^2(\bar{n}_t^g)
\]

Mean absolute error:

\[
\mathbb{E}^g[|\text{err}^i|] = \sqrt{\text{Var}^g(\text{err}_{t+1}^i) \frac{2}{\pi} e^{-\frac{\text{err}^g_{t+1}^2}{2\text{Var}^g(\text{err}_{t+1}^i)}} - \text{err}^g \left(1 - 2\Phi\left(\frac{-\text{err}^g}{\sqrt{\text{Var}^g(\text{err}_{t+1}^i)}}\right)\right)}
\]
Covariance of expectation errors:

$$\text{Cov}(\text{err}_{t+1}^i, \text{err}_t^i | g) = (1 - \rho)^2 \sigma_{\mu,g}^2$$

⇒ Variance = covariance + subjective uncertainty.
## Two Period Model - Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$\gamma$</td>
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</tr>
<tr>
<td>$\alpha$</td>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$\rho_y$</td>
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<tr>
<td>$\sigma_y$</td>
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<tr>
<td>$r^h$</td>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$\mu$</td>
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<tr>
<td>$\sigma_e$</td>
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<tr>
<td>$\chi$</td>
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</table>
Relative response is up to 1% of current consumption.

Relative Marginal Propensity to Consume on Signal
Percentage change in consumption policy if $\pi_t = 2$ and $\hat{e}_{t+1}$ changes from 0 to 1pp.
References


References


