

Corporate Taxation and Carbon Emissions

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Research Question

Is there an environmental bias in corporate taxation?

- If so, through which mechanism?
- Does it matter quantitatively for carbon emissions?

This Paper

- Estimates tax advantage for carbon-intensive firms
 - ⇒ large implicit subsidy (5-7 USD/tonne of carbon)
 - ⇒ works indirectly through debt tax shield
- Builds GE multi-sector model (calibrated to the U.S. economy)
(closed economy, taxes, debt/equity choice, input/output intermediate and investment networks)
- Studies alternative policy scenarios
 - In particular, remove tax shield of debt
 - ⇒ GDP falls by 2% and carbon emissions by 5%

Empirical Analysis

Data

- Firms' balance sheet and income statement data
 - Compustat North America Fundamentals
 - Exclude financials
- Carbon emissions at the firm level from Trucost [▶ Coverage](#)
 - covers 70% of publicly listed U.S. firms
 - 90% of their aggregate assets
 - sample period: 2004-2019
- Statutory tax rates on firms' profits [▶ Details on Tax Rate](#)
 - country and state-level corporate tax rates from Tax Foundation
 - location of firms' establishments across US states from Infogroup
 - firms' international sales by country from Factset
- Additional sector-level data for model calibration
 - BEA data for input/output networks and production function parameters

Descriptive Statistics

	Compustat Firms (U.S.) (Obs=13,791)				
	Mean	SD	p1	p50	p99
Carbon Emissions					
Carbon/Sales (tonnes of CO ₂ per k. Sales)	0.220	0.712	0.000	0.019	4.627
Taxes paid by U.S. corporations					
Taxes paid/Sales	0.022	0.026	-0.020	0.015	0.126
Interest × Tax Rate/Sales ("Tax Shield")	0.010	0.015	0.000	0.005	0.082
Firm (Statutory) Tax Rate (in %)	33.737	5.225	22.956	35.000	40.841
Other Variables					
Sales (mn USD)	11,020	31,684	23	2,826	145,224
Firm Age	45.766	30.215	4.000	39.000	128.000
EBITDA/Sales	0.117	0.400	-2.736	0.155	0.622
Share Foreign	0.267	0.274	0.000	0.189	0.944
Debt/Sales	0.511	0.643	0.000	0.300	3.526
PPE/Sales	0.563	0.916	0.010	0.204	4.704

Baseline Specification

Pooled OLS regressions at the firm f -year t level:

$$\text{Taxes/Sales}_{f,t} = \beta \times \text{Carbon/Sales}_{f,t} + \text{controls}_{f,t} + \gamma_{\text{state},t} + \epsilon_{f,t}$$

- controls: profits, size, age, share foreign
- HQ state-year fixed effects $\gamma_{\text{state},t}$
⇒ estimate within firms with the same HQ state-level profit tax rate
- Standard errors clustered at industry (SIC 4) level
- if $\beta > 0$, emission-intensive firms pay more taxes/sales

Note: not interpreted in a causal sense

Carbon Emissions and Corporate Taxes

	Corp. Taxes per k. Sales					
Carbon Intensity (tonnes of CO ₂ per k. Sales)	-4.133*** (0.578)	-4.450*** (0.571)	-4.357*** (0.670)	-4.148*** (1.102)	-6.373*** (1.158)	-6.369*** (1.109)
Year FE	Y	Y		Y	Y	
HQ State x Year FE			Y			Y
Firm Controls		Y	Y		Y	Y
Size Weights				Y	Y	Y
R ²	0.071	0.135	0.190	0.041	0.336	0.416
N	13,791	13,791	13,791	13,791	13,791	13,791

- 1 tonne of carbon emissions associated with \approx 5 USD lower taxes

Carbon Emissions and Debt Tax Shield

$$\text{Taxes} = \underbrace{\text{Taxes} + \text{Interest Payment} \times \text{Tax Rate}}_{\text{Taxes Assuming 100\% Equity}} - \underbrace{\text{Interest Payment} \times \text{Tax Rate}}_{\text{Tax Shield}}$$

	Hypothetical Taxes Assuming 100% Equity			Tax Shield		
	(Tax+Tax Shield)/Sales (× 1,000)			Interest × Tax Rate/Sales (× 1,000)		
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.089 (1.007)	0.090 (1.036)	0.128 (1.030)	4.355*** (0.565)	4.496*** (0.638)	4.445*** (0.525)
Year FE	Y	Y		Y	Y	
Firm Controls		Y	Y		Y	Y
HQ State × Year FE			Y			Y
R ²	0.046	0.052	0.104	0.050	0.147	0.206
N	13,791	13,791	13,791	13,791	13,791	13,791

- Carbon bias of corporate taxation explained by debt tax shield

Decomposition of the Tax Shield Advantage

$$\text{Tax Shield} = \underbrace{\text{Interest Payment}}_{\text{Debt} \times \text{Interest Rate}} \times \text{Tax Rate}$$

	<u>Tax Shield/Sales</u>	<u>Debt</u>	<u>Interest Rate</u>	<u>Tax Rate</u>
Carbon Intensity	4.445***	0.0218***	-0.018	-0.007
(tonnes of CO ₂ per k. Sales)	(0.525)	(0.022)	(0.091)	(0.037)
Firm Controls	Y	Y	Y	Y
HQ State x Year FE	Y	Y	Y	Y
<i>R</i> ²	0.206	0.158	0.138	0.856
N	13,791	13,791	13,791	13,791

- Tax shield advantage of dirty firms explained by their higher leverage

What Explains Higher Leverage in Dirty Firms?

	PPE/Sales	Debt/Sales		Tax Shield ($\times 1,000$)	
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.511*** (0.035)	0.0218*** (0.022)	-0.014 (0.027)	4.445*** (0.525)	0.076 (0.585)
PPE/Sales			0.454*** (0.040)		8.658*** (0.688)
HQ State \times Year FE	Y	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y	Y
R ²	0.323	0.158	0.439	0.206	0.401
N	13,791	13,791	13,791	13,791	13,791

- Asset tangibility explains carbon bias of corporate taxation

▶ PPE Decomposition

▶ Other leverage determinants

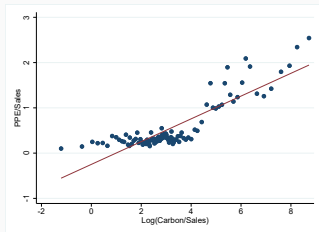
▶ Industry vs firm variation

▶ Energy sector

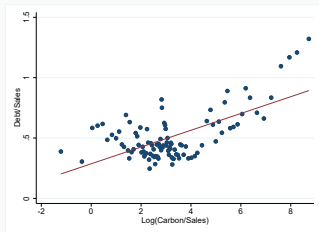
Summing up...

Dirty firms \Rightarrow more tangible assets \Rightarrow higher debt \Rightarrow lower taxes

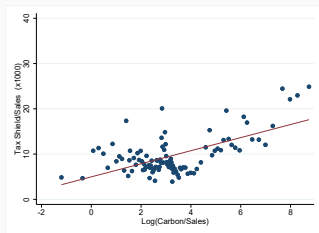
Summing up...



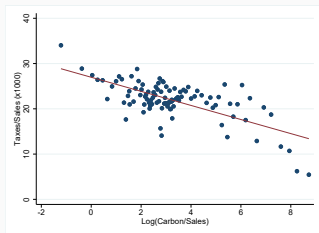
PPE



Debt



Tax Shield



Taxes Paid

The Model

The Model: Households

Representative Household

- consumes $C_t \equiv \prod_{i \in \mathcal{N}} c_{i,t}^{\theta_i}$ with $c_{i,t} \equiv \left(\int_0^1 c_{f,t}^{\frac{\sigma_i-1}{\sigma_i}} dH(f|i) \right)^{\frac{\sigma_i}{\sigma_i-1}}$
→ pays sales tax τ_c
- supplies labor L_t and receives wage w_t
→ pays income tax τ_h
- invests in three types of assets:
 - risk-free government bonds → pays income tax τ_h
 - risky corporate bonds → pays income tax τ_h
 - equity → pays dividend tax τ_d
- preferences: $\frac{1}{1-\varphi} C_t^{1-\varphi} - \frac{\epsilon}{1+\epsilon} L_t^{1+\frac{1}{\epsilon}}$

The Model: Firms

Continuum of monopolistic competitive firms in each sector

⇒ Representative Firm (in each sector)

- owned by consumers, maximizes PV of dividends
- issues risky corporate bonds
- hires labor $\ell_{i,t}$
- purchases intermediates $x_{ij,t}$ from sector j
- owns capital $k_{i,t}^s$ of type $s \in \{\text{structures, equipment, intangibles}\}$
 - law of motion: $k_{i,t+1}^s = (1 - \delta_i^s)k_{i,t}^s + I_{i,t}^s$
 - investment network $\rightarrow I_{i,t}^s \equiv \prod_j (i_{ij,t}^s)^{\omega_{ij}^s}$

The Model: Firms

- Cobb-Douglas production function (relaxed in the extensions)

$$y_{i,t} = z_i \left(\prod_{j \in \mathcal{N}} x_{ij,t}^{\alpha_{ij}} \right)^{1-\gamma_i} \left(\ell_{i,t}^{\phi_i^\ell} \cdot \prod_{s \in \mathcal{S}} (k_{i,t}^s)^{\phi_i^s} \right)^{\gamma_i}$$

- pay profit tax τ_p (deductibles: **interest**, inputs, depreciation, R&D)
- **produce carbon emissions** $\Rightarrow e_i \times y_{i,t}$

The Model: Default

Default

In every period, random fraction of firms defaults:

- some firms are restructured (only debt-holders receive payment)
- other firms are liquidated (no creditor receives payment)

⇒ Debt and equity are risky

Leverage

Firms issue debt $b_{i,t+1}$ subject to

$$b_{i,t+1} \leq \frac{1}{1 + r_{i,t+1}^b} \sum_{s \in \mathcal{S}} \psi_{i,s} q_{i,t+1}^s k_{i,t+1}^s$$

⇒ Fraction $\psi_{i,s}$ is capital and sector specific

Mechanism - Rental rate of capital

Rental rate of type- s capital *with* tax shield

$$R_{i,\text{before}}^s \equiv \delta_i^s + r_i^b \frac{\psi_{i,s}}{1 + r_i^b} + \frac{1}{1 - \tau_p} r_i^e \left(1 - \frac{\psi_{i,s}}{1 + r_i^b}\right)$$

Rental rate of type- s capital *without* tax shield

$$R_{i,\text{after}}^s \equiv \delta_i^s + \frac{1}{1 - \tau_p} r_i^b \frac{\psi_{i,s}}{1 + r_i^b} + \frac{1}{1 - \tau_p} r_i^e \left(1 - \frac{\psi_{i,s}}{1 + r_i^b}\right)$$

Therefore,

$$dR_i^s \equiv R_{i,\text{after}}^s - R_{i,\text{before}}^s = \frac{r_i^b}{1 + r_i^b} \cdot \frac{\tau_p}{1 - \tau_p} \psi_{i,s} \geq 0$$

⇒ increasing in capital pledgeability $\psi_{i,s}$

Mechanism - Which sectors are hurt the most?

Partial Equilibrium (fix C & prices)

▶ PE vs. GE

($\mathcal{D}_i \equiv$ demand, $\mathcal{C}_i \equiv$ total cost per unit of y_i)

$$d \log y_i = \frac{d \log \mathcal{D}_i(p_i, C)}{d \log p_i} \times \sum_s \frac{d \log \mathcal{C}_i(\{R_i^s\}_s, w, \{p_j\}_j)}{d R_i^s} \times d R_i^s$$

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proportional
to $\psi_{i,s}$

Mechanism - Which sectors are hurt the most?

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proportional to $q_i^s k_i^s / p_i y_i$

proportional to $\psi_{i,s}$

Mechanism - Which sectors are hurt the most?

Partial Equilibrium (fix C & prices)

▶ PE vs. GE

($\mathcal{D}_i \equiv$ demand, $\mathcal{C}_i \equiv$ total cost per unit of y_i)

$$d \log y_i = \underbrace{\frac{d \log \mathcal{D}_i(p_i, C)}{d \log p_i}}_{\text{demand elasticity}} \times \sum_s \underbrace{\frac{d \log \mathcal{C}_i(\{R_i^s\}_s, w, \{p_j\}_j)}{d R_i^s}}_{\text{proportional to } q_i^s k_i^s / p_i y_i} \times \underbrace{d R_i^s}_{\text{proportional to } \psi_{i,s}}$$

Mechanism - Which sectors are hurt the most?

Partial Equilibrium (fix C & prices)

► PE vs. GE

($\mathcal{D}_i \equiv$ demand, $\mathcal{C}_i \equiv$ total cost per unit of y_i)

$$d \log y_i = \underbrace{\frac{d \log \mathcal{D}_i(p_i, C)}{d \log p_i}}_{\text{demand elasticity}} \times \sum_s \underbrace{\frac{d \log \mathcal{C}_i(\{R_i^s\}_s, w, \{p_j\}_j)}{d R_i^s}}_{\text{proportional to } q_i^s k_i^s / p_i y_i} \times \underbrace{d R_i^s}_{\text{proportional to } \psi_{i,s}}$$

Which sectors are hurt the most?

- Those using types of capital which are easier to collateralize
- Those using more capital in their production function

Counterfactual: No Debt Tax Shield

Remove tax shield of debt \Rightarrow interest no longer deductible

- **Aggregate effects**

GDP: -2.12%, consumption: -1.66%

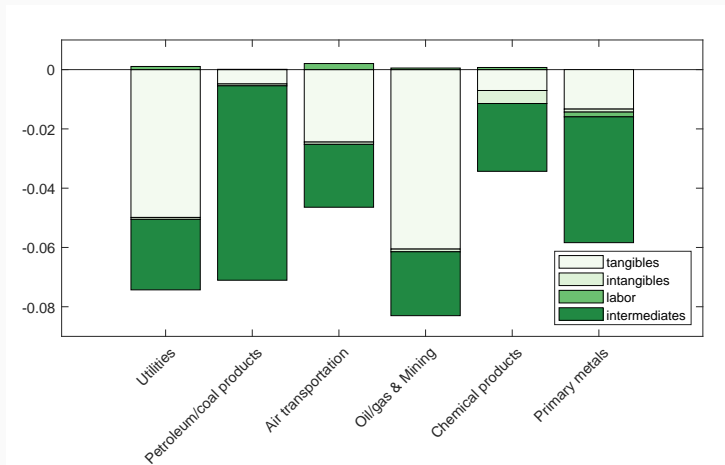
total emissions: -5.37%

▶ Energy elasticity

▶ Sensitivity

Counterfactual: No Debt Tax Shield

Key result: the most polluting sectors are more affected

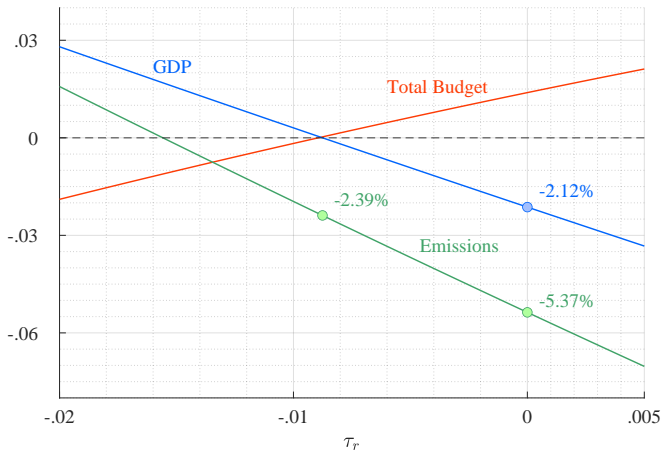


56 BEA sectors in calibration

6 sectors above generate more than 85% of aggregate emissions

Offsetting removal of tax shield with revenue subsidy

Output neutral counterfactual: -2% emissions



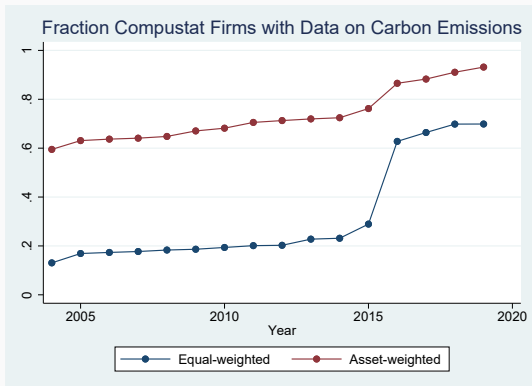
Conclusion

- Large environmental bias in corporate taxation

Debt tax shields subsidize firms with more tangible assets

- A policy that removes the tax advantage of debt
 - ⇒ has disproportionate effect on polluting sectors
 - ⇒ has large impact on total emissions

Coverage of Compustat firms with data on carbon emissions in Trucost



This figure reports the fraction of Compustat firms for which we observe information on carbon emissions in Trucost.

Details on Tax Rate

Construction of Tax Rate:

- ✓ for domestic sales,
 - use Infogroup data for location of establishments
 - compute state-level tax rate (weighted by sales/employment)
 - sum to federal tax rate

- ✓ for foreign sales,
 - use Factset data for sales in different countries
 - compute weighted average of country-level tax rate (includes regional/state tax)

Robustness

Panel A:		Taxes per k. Sales						
Carbon Intensity	-3.435***	-3.735***	-4.183***	-6.669***	-4.093***	-4.516***	-4.188***	-3.932***
(tonnes of CO ₂ per k. Sales)	(0.879)	(0.716)	(1.521)	(1.306)	(0.875)	(0.603)	(0.676)	(0.639)
R ²	0.270	0.272	0.219	0.359	0.202	0.172	0.189	0.190
N	2,686	4,079	9,547	2,321	11,141	11,576	13,791	13,791
Panel B:		Tax Shield per k. Sales						
Carbon Intensity	4.659***	4.214***	4.975***	3.631***	4.808***	4.500***	4.331***	3.617***
(tonnes of CO ₂ per k. Sales)	(0.725)	(0.594)	(0.962)	(0.500)	(0.741)	(0.778)	(0.517)	(0.520)
R ²	0.334	0.263	0.156	0.511	0.200	0.216	0.206	0.199
N	2,686	4,079	9,547	2,321	11,141	11,576	13,791	13,791
HQ State x Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y	Y	Y	Y	Y
Robustness Test	Private	Domestic	International	Reported	Estimated	EPA	Scope 1+2	Scope 1+2+3

Year-by-Year

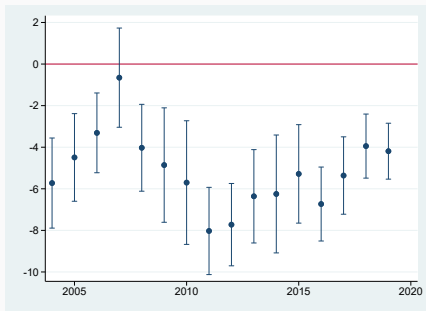


Figure 2: Taxes

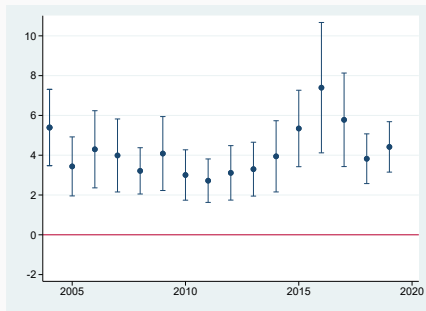


Figure 3: Tax Shield

Leave-one-out

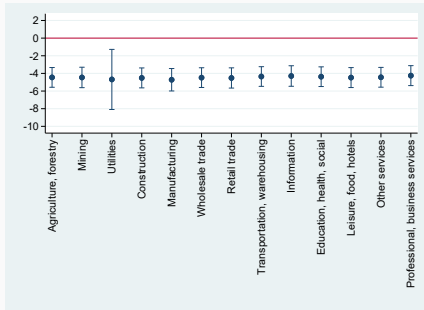


Figure 4: Taxes

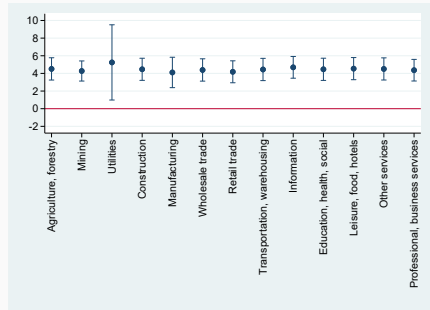


Figure 5: Tax Shield

Log Specifications

	Taxes per k. Sales			Tax Shield per k. Sales		
Log(Carbon Intensity)	-1.568*** (0.438)	-1.744*** (0.414)	-1.600*** (0.466)	1.441*** (0.467)	1.505*** (0.470)	1.398*** (0.443)
Year FE	Y	Y		Y	Y	
HQ State x Year FE			Y			Y
Firm Controls		Y	Y		Y	Y
R ²	0.071	0.136	0.188	0.039	0.136	0.192
N	13,791	13,791	13,791	13,791	13,791	13,791
Implied Subsidy (USD/tonnes CO2e)	7					

100% increase in carbon intensity (given a mean of 0.22 tonnes CO₂ per 1,000 USD of sales) is associated with 1.6 USD lower taxes (per 1,000 USD of sales).

- 1 tonne of carbon emissions associated with ≈ 7 USD lower taxes

PPE Decomposition

Panel A: Without Firm Controls	GrossPPE/Sales	Machinery/Sales	Buildings/Sales	Leases/Sales	Land/Sales	ConstrlnProg/Sales	Other/Sales
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.529*** (0.110)	0.482*** (0.097)	-0.009 (0.021)	-0.022*** (0.007)	0.009 (0.009)	0.015** (0.007)	0.003 (0.007)
HQ State x Year FE	Y	Y	Y	Y	Y	Y	Y
R ²	0.228	0.196	0.316	0.172	0.202	0.140	0.085
N	8,132	8,132	8,132	8,132	8,132	8,132	8,132
Panel B: With Firm Controls	GrossPPE/Sales	Machinery/Sales	Buildings/Sales	Leases/Sales	Land/Sales	ConstrlnProg/Sales	Other/Sales
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.530*** (0.107)	0.479*** (0.093)	-0.012 (0.022)	-0.018*** (0.006)	0.009 (0.009)	0.015** (0.007)	0.004 (0.007)
HQ State x Year FE	Y	Y	Y	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y	Y	Y	Y
R ²	0.237	0.208	0.323	0.297	0.213	0.147	0.092
N	8,132	8,132	8,132	8,132	8,132	8,132	8,132
Dep Var Mean	0.534	0.309	0.105	0.038	0.022	0.016	0.020

- Correlation driven entirely by Machines & Equipment

Other Leverage Determinants?

	Debt/Sales			Tax Shield (x 1,000)		
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.219*** (0.021)	0.173*** (0.022)	-0.021 (0.025)	4.362*** (0.449)	3.611*** (0.517)	0.053 (0.570)
PPE/Sales			0.432*** (0.036)			8.018*** (0.689)
Rated		0.384*** (0.052)	0.232*** (0.025)	7.961*** (1.211)	5.179*** (0.582)	
Dividend Payer		0.020 (0.034)	-0.044** (0.022)	0.061 (0.788)	-1.197** (0.575)	
M/B		-0.052*** (0.016)	-0.021* (0.012)	-0.892** (0.375)	-0.261 (0.300)	
Cash-Flow Volatility		0.105 (0.095)	0.094** (0.046)	7.721** (3.013)	7.474*** (1.871)	
Depreciation/Assets		0.076 (0.807)	-2.184*** (0.534)	38.134** (18.923)	-4.203 (12.970)	
RD/Sales		0.743** (0.304)	0.420* (0.223)	12.772* (6.962)	6.007 (5.657)	
Advertising/Sales		-0.123 (0.396)	0.715** (0.359)	-12.063 (7.337)	4.417 (6.161)	
EBITDA/Sales		0.296 (0.234)	0.021 (0.148)	-0.570 (4.909)	-6.321* (3.304)	
Log(Sales)		-0.068*** (0.016)	-0.012 (0.009)	-1.975*** (0.353)	-0.940*** (0.207)	
Log(Firm Age)		-0.065** (0.031)	-0.061*** (0.021)	-1.092 (0.790)	-0.932 (0.574)	
Share Foreign		-0.209*** (0.066)	-0.010 (0.047)	-6.995*** (1.419)	-3.275*** (1.104)	
HQ State x Year FE	Y	Y	Y	Y	Y	Y
R ²	0.138	0.255	0.482	0.118	0.286	0.434
N	13,791	13,520	13,520	13,791	13,520	13,520

Industry vs. Firm-level Variation

	PPE/Sales	Debt/Sales	Tax Shield per k. Sales	Taxes per k. Sales
Carbon Intensity Industry	0.819*** (0.081)	0.327*** (0.050)	6.861*** (1.066)	-6.978*** (1.010)
Firm Residual Carbon Intensity	0.241*** (0.078)	0.119*** (0.028)	2.075*** (0.640)	-2.041** (0.853)
HQ State x Year FE	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y
R^2	0.359	0.164	0.213	0.193
N	13,791	13,791	13,791	13,791

- Industry (SIC 4) main driver, but carbon bias also within industry

Energy Sector

	Carbon Intensity	PPE/Sales	Debt/Sales	Tax Shield per k. Sales	Taxes per k. Sales
Panel A:	Carbon intensity				
Carbon Intensity (tonnes of CO ₂ per k. Sales)	0.278*** (0.076)	0.118*** (0.041)	2.648*** (0.942)	-2.898** (1.408)	
Year FE	Y	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y	Y
R ²	0.559	0.294	0.335	0.236	
N	969	969	969	969	969
Panel B:	Fossil fuel energy production capacity				
Fossil Fuel Capacity (gigawatts per k. Sales)	0.609*** (0.058)	0.190*** (0.065)	0.090*** (0.027)	2.262*** (0.646)	-2.748** (1.056)
Year FE	Y	Y	Y	Y	Y
Firm Controls	Y	Y	Y	Y	Y
R ²	0.637	0.448	0.217	0.263	0.246
N	969	1,296	1,296	1,296	1,296

- Carbon bias also within energy production sector

Calibration

⇒ Use “exact hat algebra” for counterfactuals, compare steady states

Main parameters:

- ✓ intermediates network & input shares from BEA data
- ✓ investment networks as in Lehn & Winberry (2020)
- ✓ leverage and interest rates from Compustat data
- ✓ equity returns from CRSP
- ✓ estimate leverage constraint from Compustat and BEA data
 - $\hat{\psi}_{\text{struct.}} = \hat{\psi}_{\text{equip.}} = 0.43$
- ✓ profit taxes: $\tau_p = 0.25$ (average tax)
- ✓ time discount $\beta = 0.99$, Frisch elast. $\epsilon = 0.5$, income elast. $\varphi = 1.7$

Discussion: elasticity of energy demand

- So far, Cobb Douglas
- Suppose, instead, firms cannot easily substitute away
 - ⇒ elasticity of energy input (“utilities”) < 1
- Set elasticity to 0.8:
 - ⇒ emissions: -4.69%
 - ⇒ emissions decrease by a smaller amount, but still large
- In the long run, Cobb-Douglas reasonable assumption
 - price elasticity of energy increases with time horizon (Labandeira et al., 2017)
 - stable energy share in the long run (Hassler et al., 2021)
 - directed technical change makes firms more energy-efficient

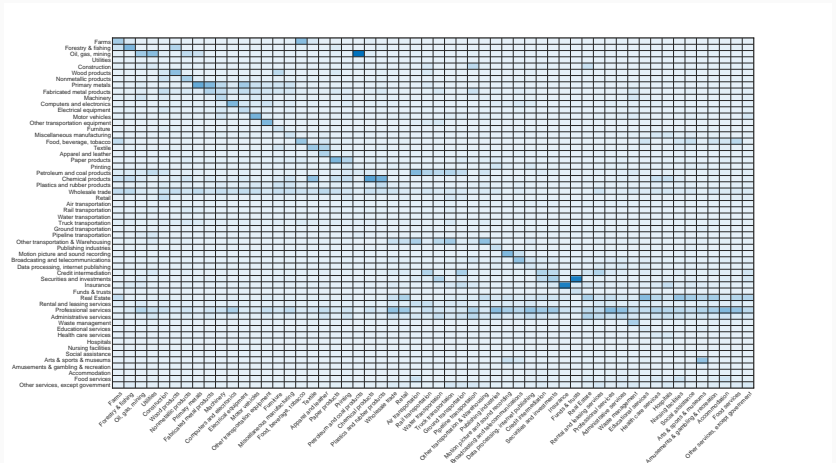
Sensitivity analysis

		ρ		σ		b^ψ	
	baseline	0.8	0.6	1	2	0.30	0.50
emissions	-5.37%	-4.69%	-4.01%	-5.73%	-5.25%	-4.45%	-5.88%
GDP	-2.12%	-2.04%	-2.03%	-2.51%	-2.00%	-1.78%	-2.32%

▶ Back

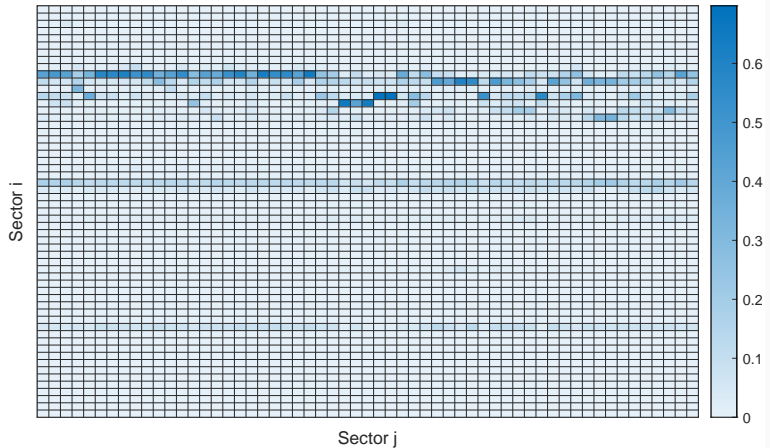
Input-Output Networks

● The intermediates network



Input-Output Networks

- The **investment** (equipment) network



PE versus GE

~~Partial~~ **General** Equilibrium (~~fix C & prices~~)

$$d \log y_i = \frac{d \log \mathcal{D}_i(p_i, \mathbf{C})}{d \log p_i} \times \sum_s \frac{d \log \mathcal{C}_i(\{R_i^s\}_s, \mathbf{w}, \{p_j\}_j)}{d R_i^s} \times d R_i^s$$

($\mathcal{D}_i \equiv$ demand, $\mathcal{C}_i \equiv$ total cost per unit of y_i)