

Discussion Paper

Deutsche Bundesbank
No 55/2021

The hockey stick Phillips curve and the effective lower bound

Gregor Boehl

(University of Bonn)

Philipp Lieberknecht

(Deutsche Bundesbank)

Editorial Board:

Daniel Foos
Stephan Jank
Thomas Kick
Martin Kliem
Malte Knüppel
Christoph Memmel
Panagiota Tzamourani

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet <http://www.bundesbank.de>

Reproduction permitted only if source is stated.

ISBN 978-3-95729-865-2

ISSN 2749-2958

Non-technical summary

Research Question

After the Global Financial Crisis of 2007/2008, inflation seemed disconnected from economic activity. This paper analyzes whether a lower bound on nominal interest rates may have contributed to this observed inflation puzzle.

Contribution

This paper considers a theoretical model featuring financial frictions regarding firms' external financing. We analyze macroeconomic dynamics following financial shocks, both analytically and numerically, and discuss the implications for monetary policy.

Results

If nominal rates are at their lower bound, inflation declines less after contractionary financial shocks compared to normal times. This happens because credit spreads, which are part of firms' marginal costs, rise after financial shocks. A similar effect occurs for forward guidance, such that this channel may dampen the potency of this instrument.

Nichttechnische Zusammenfassung

Fragestellung

Im Zuge der globalen Finanzkrise von 2007/2008 bestand scheinbar kein Zusammenhang zwischen Inflation und ökonomischer Aktivität. Dieses Forschungspapier untersucht, ob eine bindende Zinsuntergrenze zu dieser empirischen Beobachtung beigetragen haben könnte.

Beitrag

Dieses Forschungspapier präsentiert ein theoretisches Modell mit Finanzmarktfriktionen in Bezug auf die externe Finanzierung von Unternehmen. Wir analysieren die makroökonomischen Dynamiken nach finanziellen Schocks, sowohl analytisch als auch numerisch, und diskutieren die daraus resultierenden Implikationen für die Geldpolitik.

Ergebnisse

An der Zinsuntergrenze sinkt die Inflation weniger stark nach adversen finanziellen Schocks als in normalen Zeiten. Dies geschieht aufgrund von Risikoaufschlägen in den Kreditzinsen; diese sind Teil der Produktionskosten der Unternehmen und steigen nach finanziellen Schocks. Ein ähnlicher Effekt ist bei Ankündigungen der Zentralbank über zukünftige Aktionen (“forward guidance”) zu beobachten, so dass dieser Kanal die Wirksamkeit dieses Instrumentes einschränkt.

The Hockey Stick Phillips Curve and the Effective Lower Bound*

Gregor Boehl
University of Bonn

Philipp Lieberknecht
Deutsche Bundesbank

Abstract

We show that if business cycles are driven by financial shocks, the interplay between the effective lower bound (ELB) and the costs of external financing can generate an additional supply-side channel, which causes a disconnect between inflation and output. In normal times, factor costs dominate firms' marginal costs and hence inflation; credit spreads and the nominal interest rate, which together constitute external financing costs, balance out in response to a financial shock. When nominal rates are constrained by the ELB, larger spreads can partly offset the effect of lower factor costs on firms' price setting. The Phillips curve is hence flat at the ELB, but features a positive slope in normal times and thus an overall hockey stick shape. This mechanism also weakens the effects of forward guidance on inflation, since such policy reduces spreads and thereby financing costs.

Keywords: Phillips curve, financial frictions, effective lower bound, disinflation, forward guidance

JEL classification: C62, C63, E31, E32, E44, E52, E58, E63.

*Contact address: Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main, Germany. E-Mail: philipp.lieberknecht@bundesbank.de. We are grateful to Joachim Keller, Keith Kuester, Vivien Lewis, Taisuke Nakata, Johannes Wieland, and participants of several conferences and a seminar at the Bundesbank for helpful discussions and comments. Parts of the research leading to the results in this paper has received financial support from the Alfred P. Sloan Foundation under the grant agreement G-2016-7176 for the Macroeconomic Model Comparison Initiative (MMCI) at the Institute for Monetary and Financial Stability. Gregor Boehl gratefully acknowledges financial support by the DFG under CRC-TR 224 (project C01) and under project number 441540692. The views expressed in this paper are those of the authors and do not necessarily coincide with the views of the Deutsche Bundesbank or the Eurosystem.

1 Introduction

What is the relationship between inflation and economic activity? Given the fundamental role of these two concepts, it is troubling that this question is currently (still) puzzling the economic profession. After the Global Financial Crisis of 2007/2008 and the associated financial turmoil, inflation seemed disconnected from economic activity, leading to puzzles of both “missing disinflation” and “missing inflation” (Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2015; Lindé and Trabandt, 2019). These observations raised considerable interest in analyzing the seemingly flat Phillips curve. While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the effective lower bound (ELB) on nominal interest rates, which was reached by several central banks around the globe in recent years, coincidental with the observed inflation puzzles.

In this paper, we show how the interplay of the ELB and financial frictions reshapes the relation between inflation and output if financial shocks are the main driver of business cycles. From recent research, it is understood that financial distortions can be crucial for firms’ price setting behavior and, thereby, for inflation dynamics (e.g. Gilchrist et al., 2017). We argue that during normal times, firms’ marginal costs are dominated by the procyclical costs of employing production factors, which hence determine their price setting. In the presence of financial frictions, marginal costs further contain the costs of external financing. These consist of the safe real interest rate and a countercyclical credit spread reflecting financial frictions. While these two components roughly balance out in normal times over the business cycle, larger credit spreads can substantially offset lower production factor costs if the nominal rate is constrained by the ELB. In this case, the costs of external financing considerably weaken the supply-side link between output and prices. As a result, financial shocks at the ELB induce only moderate disinflationary responses, and may in extreme cases even be inflationary. The *observational Phillips curve*¹ is thus shaped like a hockey stick: it features the usual positive slope in normal times, but is flat for large negative output gaps when the ELB binds.

We show these results using a tractable New Keynesian DSGE model featuring financial frictions. In the model, workers need to be paid before production (as in Ravenna and Walsh, 2006), generating external financing needs for the entrepreneurs operating the firms. Due to a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999), the costs of external finance comprise a risk premium, which depends positively on entrepreneurs’ (countercyclical) leverage. We focus on the effects of financial shocks in the form of risk premium shocks in the spirit of Smets and Wouters (2007). These shocks are known to have a large explanatory power for the joint movement of consumption and investment following the 2007/2008 recession (Gust et al., 2017; Kulish et al., 2017; Boehl et al., 2021).

Our first contribution is to show that the expectation of a longer ELB duration can be associated with weaker disinflationary effects of financial shocks. We provide analytical solutions both for normal times and for a binding ELB, which reveal that this case occurs for large financial shocks if the elasticity of the credit spread with respect to entrepreneur

¹We use this term to refer to the reduced-form relationship between realized (equilibrium) values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms’ price setting behavior.

leverage is sufficiently large.² The analytical solutions furthermore highlight that even an overall increase of inflation following contractionary financial shocks is possible, and may in particular occur if the ELB is expected to bind for an extended period of time. In this case, financial shocks move output and inflation in opposite directions, thus appearing as supply-type disturbances, in sharp contrast to their usual appearance as demand shocks.

Simulating the model numerically, we find that the observational Phillips curve, i.e. the relationship between realized inflation and the output gap, features a striking hockey stick shape: For normal times with positive or mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space. In contrast, the slope is considerably flat for significantly negative output gaps when the ELB is binding. This provides an explanation for the puzzles of missing disinflation consistent with the observed timing of events. As seen in Figure 1, in 2008:Q4, corporate spreads with rating BAA peaked at almost 6%, while the Federal funds rate reached its lower bound of almost 0% in 2009:Q1. Both elevated corporate spreads and the binding lower bound persisted until the end of 2015, coincidental with the observed inflation puzzles.

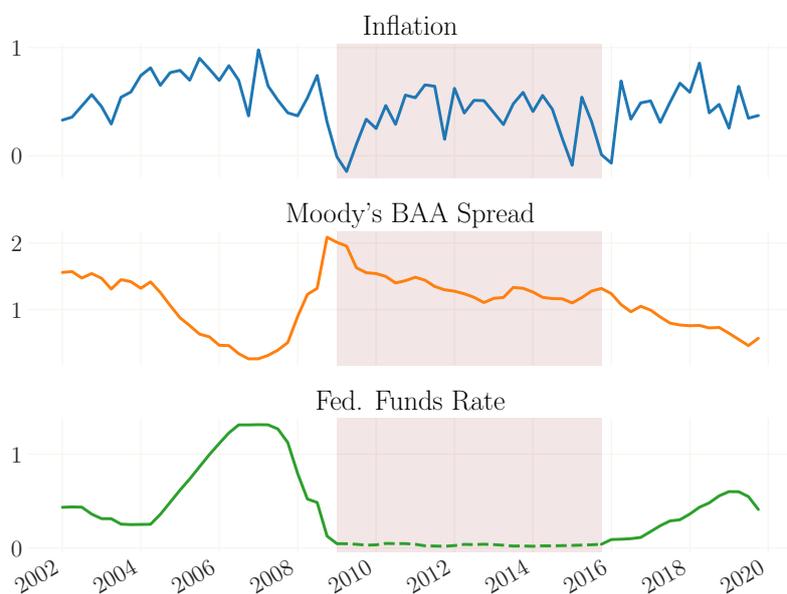


Figure 1: Time series of inflation, BAA spread and US interest rates from 2002 to 2020. Quarterly data in percentage points. The shaded area depicts the episode during which the ELB was binding.

As our second contribution, we discuss the associated implications for monetary policy. We show that monetary policy shocks generate macroeconomic dynamics that are highly similar to financial shocks. As a consequence, forward guidance shocks with relatively low persistence can even be disinflationary: the effect of keeping expected refinancing costs lower in the future may dominate the long-run effect of increasing the price level by stimulating consumption. This observation provides an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggests

²Accordingly, this effect depends crucially on the presence of financial frictions and is hence absent in the standard New Keynesian model.

that any forward guidance must be undertaken with vigor; it is only effective if private agents assess the announcement as credible and persistent. Furthermore, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses guaranteeing a determinate equilibrium are rather smaller.

The issue of missing (dis-)inflation in the recent years was first brought up by [Ball and Mazumder \(2011\)](#) and subsequently confirmed for many advanced economies by [Friedrich \(2016\)](#). Manifold of explanations were put forward, encompassing anchored expectations ([Ball and Mazumder, 2018](#); [Coibion and Gorodnichenko, 2015](#)), various measures of economic slack ([Gordon, 2013](#); [Watson, 2014](#)), supply shocks and wage rigidities ([Daly and Hobijn, 2014](#); [Lindé and Trabandt, 2019](#)), optimal monetary policy, potentially in combination with financial frictions ([Lieberknecht, 2019](#); [Sims and Wu, 2019](#); [McLeay and Tenreyro, 2020](#)) or global factors ([Bobeica and Jarociński, 2019](#); [Forbes, 2019](#)). Compared to this literature, we provide a complementary explanation for inflation dynamics that also matches the particular timing of the observed missing (dis-)inflation: the ELB affects the cyclicity of marginal costs via the costs of external financing, thereby leading to an observational disconnect between inflation and output.

A related strand of the literature investigates these recent inflation dynamics through the lens of New Keynesian DSGE models, notably [Christiano et al. \(2015\)](#), [Del Negro et al. \(2015b\)](#) and [Gilchrist et al. \(2017\)](#). In line with our paper, these contributions show that adding financial frictions to DSGE models helps to explain the missing disinflation puzzle in the US. Closely related to our work, [Gilchrist et al. \(2017\)](#) explain inflation dynamics via financial distortions, i.e. larger credit spreads in recessions. While our paper shares this argument, we provide additional insights that a binding ELB strongly amplifies the effects of financial frictions, such that credit spreads may even dominate inflation dynamics. This is in line with [Bianchi and Melosi \(2017\)](#) and [Boehl and Strobel \(2020\)](#), who find that accounting for the ELB substantially improves the empirical fit of estimated DSGE models.

Our hockey-stick Phillips curve is also well-supported by recent empirical work showing that financial shocks can be disinflationary if supply-side effects dominate demand effects. Various contributions find empirical evidence in favor of such a (financial) cost channel ([Barth III and Ramey, 2001](#); [Chowdhury et al., 2006](#); [Tillmann, 2008](#); [Abbate et al., 2016](#)). Similarly, [Gaiotti and Secchi \(2006\)](#) find this cost channel to be proportional to working capital, using Italian firm-level data. Our paper provides a theoretical foundation for these papers and highlights that both the degree of financial frictions and a binding ELB are particularly relevant.

Lastly, our paper is related to the literature on Neo-Fisherianism, which argues that the causality between the policy rate and inflation is positive even in the short run ([Gabaix, 2016](#); [Cochrane, 2011, 2016, 2017](#)). We show that such effects may arise at the ELB, and even in normal times if the elasticity of marginal costs to the risk spread is large enough. This is in contrast to [García-Schmidt and Woodford \(2019\)](#), who argue that Neo-Fisherian effects arise only after credible changes in long-run monetary policy targets.

We continue in Section 2 by outlining the model and discussing the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ELB. Section 6 concludes.

2 Model

Our analysis is based on a tractable New Keynesian DSGE model featuring financial frictions. The setup is based on [Boehl \(2020\)](#) and [Lieberknecht \(2019\)](#), to which we refer for further details. Production is subject to a working capital channel as in [Ravenna and Walsh \(2006\)](#). A distinct role for external finance is motivated via a costly state verification problem in the spirit of [Townsend \(1979\)](#) and [Bernanke et al. \(1999\)](#). Entrepreneurs operating wholesale firms borrow from financial intermediaries to finance production, and their shares are traded on financial markets. Their (homogeneous) good is sold to a monopolistic retail sector where diversification takes place. The resulting final goods are sold to a representative household, who consumes and supplies labor in a perfectly competitive labor market. A central bank sets the nominal interest rate subject to an effective lower bound.

2.1 Households

Households maximize the expected present value of lifetime utility by choosing consumption of a composite good C_t and hours devoted to the labor market H_t . They can deposit monetary savings D_t at financial intermediaries (banks in the following), for which they receive the gross nominal interest rate R_t in the next period. The household's optimization problem is completely standard and yields the usual Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \quad (1)$$

$$H_t^\eta = W_t C_t^{-\sigma}, \quad (2)$$

where Π_t is gross inflation and W_t is the real wage. U_t is a *financial shock*, i.e. a premium on the risk-free interest rate reflecting the state of the financial system (as in [Smets and Wouters, 2007](#)). This type of shock features the highest explanatory power regarding the post-2000 macroeconomic dynamics across all standard shocks, and can explain a large share of the joint dynamics of consumption, investment and inflation following the 2007/2008 financial crisis ([Gust et al., 2017](#); [Kulish et al., 2017](#); [Boehl et al., 2021](#)). The parameters σ, η and β are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

2.2 Wholesale and retail firms

The wholesale sector consists of a continuum of firms indexed by j . Each firm is operated by a risk-neutral entrepreneur and produces a homogeneous good using a production function that is linear in labor (the only production factor) subject to a firm-specific idiosyncratic productivity shock. Workers have to be paid before production takes place, while returns are realized at the end of the period. This working capital channel (also labeled the *cost channel*) motivates a positive role for external finance. The loan volume demanded by each entrepreneur is the difference between her desired working capital $W_t H_{j,t}$ and her equity $N_{j,t}$. The realization of the idiosyncratic productivity shock is

private information of the entrepreneur; banks can only observe produced output when paying monitoring costs.

The contract that solves this costly state verification problem specifies that the interest rate on a loan obtained by an entrepreneur from the intermediary $R_{j,t}^L$ contains an endogenous risk premium on the prevailing real interest rate. The risk premium is a credit spread that depends positively on the individual firm's leverage $LEV_{j,t} = \frac{W_t H_{j,t}}{N_{j,t}}$. Intuitively, when the leverage ratio decreases, the premium on external finance falls because more collateral is provided such that the loan becomes less risky.

It can be shown that all entrepreneurs make identical choices in equilibrium, such that the aggregate loan rate is given by

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t, \quad (3)$$

where $z'(\cdot) > 0$. Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. In the aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding. It follows that firms' marginal costs are given by

$$MC_t = W_t R_t^L = W_t z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t. \quad (4)$$

With respect to equity financing, we assume that entrepreneurs can issue equity in the stock market, which is bought by risk-neutral financial traders associated with the financial intermediaries. Imposing no arbitrage on financial markets and noting that entrepreneurs must be indifferent between external finance and equity finance in equilibrium implies that the expected return on equity equals the loan rate. Invoking rule-of-thumb behavior from financial traders, it can be shown that the evolution of equity is given by

$$N_t = \Psi(Y_t), \quad (5)$$

with $\Psi'(\cdot) > 0$, such that equity financing is procyclical with respect to output, as in standard financial accelerator models à la [Bernanke et al. \(1999\)](#).

After wholesale goods have been produced, retailers buy the homogeneous good $Y_{j,t}$ on the wholesale market. After differentiation, they sell it in the monopolistically competitive good market. Firms' price setting decisions are subject to nominal rigidities à la [Calvo \(1983\)](#), i.e. a fraction ζ firms cannot adjust prices each period. This gives rise to a classic New Keynesian Phillips curve, whose properties are studied in detail below.

2.3 The central bank

The central bank follows a standard rule for the notional gross nominal interest rate R_t^n ,

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \exp(v_t), \quad (6)$$

where v_t is a monetary policy shock following an AR(1) process. The interest rate on deposits R_t is subject to a ELB constraint and cannot fall below \bar{R} ,

$$R_t = \max \{ \bar{R}, R_t^n \}. \quad (7)$$

Note that when the ELB constraint in Equation (7) binds, a negative realization of v_t can be understood as a *forward guidance shock* as it prolongs the expected duration of the ELB.

2.4 Understanding the components of marginal costs

In our framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. The role of financial frictions for marginal costs and inflation dynamics is thus best understood by studying the New Keynesian Phillips curve. Linearizing around an efficient steady state³, and denoting log-deviations from steady state via lower-case letters, the New Keynesian Phillips curve may be represented in the familiar textbook form

$$\pi_t = \kappa mc_t + \beta E_t[\pi_{t+1}], \quad (8)$$

with slope $\kappa = \frac{(1-\zeta\beta)(1-\zeta)}{\zeta}$. Hence, financial frictions do not alter the price setting behavior of firms per se; prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of marginal costs, which are given by

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \quad (9)$$

where s_t denotes the linearized credit spread $s_t = z(lev_t) + u_t$. This highlights that marginal costs consist of three components: a) the real wage (also called *factor costs* in the following), as in the standard NK model, b) the risk-free real interest rate and c) the credit spread. The latter two components jointly constitute the costs of external finance.

In the following, we take a closer look at the cyclicity of these components. In a financial accelerator economy, the credit spread (also known as the external finance premium) is countercyclical (Bernanke et al., 1999), which is also supported empirically (Gilchrist and Zakrajšek, 2012). The elasticity of the credit spread with respect to entrepreneur leverage $\nu = z'(\cdot)$ can be shown to be positive (Lieberknecht, 2019). In turn, this implies that a countercyclical spread requires entrepreneur leverage to be countercyclical as well. Using the household's intra-temporal optimality condition, leverage is given by

$$lev_t = -(\psi - 1 - \sigma - \eta)y_t, \quad (10)$$

where $\psi = \Psi'(\cdot)$ denotes the elasticity of equity with respect to output. The necessary and sufficient condition for leverage to be countercyclical is thus that the term in brackets is larger than zero. This implies the following parameter restriction:

³Steady state subsidies from the government (financed by lump-sum taxes) can correct for the two inefficiencies arising from monopolistic competition and the presence of financial frictions (Lieberknecht, 2019).

Assumption 1. *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta. \quad (11)$$

Using these insights about entrepreneur leverage, marginal costs can be written as

$$mc_t = \gamma y_t + (r_t - E_t[\pi_{t+1}]) + u_t, \quad (12)$$

with

$$\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \quad (13)$$

capturing the elasticity of marginal costs with respect to output. The factor cost component, i.e. term $\sigma + \eta$ in γ , is procyclical. As output increases, expanding production requires firms to offer a larger real wage in order to increase labor supply. The financing cost component, the term $-\nu(\psi - 1 - \sigma - \eta)$, is countercyclical given Assumption 1 as the spread is dominated by the procyclicality of net worth. The financial shock u_t is also countercyclical, as output falls for positive realizations of u_t . The cyclicity of the cost channel is ambiguous and depends on the source of aggregate fluctuations, as this determines the endogenous nominal interest rate reaction by the central bank. For demand-side shocks – like the financial shock we consider – that reduce inflation, the nominal interest rate is procyclical. The three components of marginal costs are thus characterized by opposing cyclicity over the business cycle: factor costs and the pure interest rate channel are procyclical, whereas the external finance premium is countercyclical. Since firms' price setting is tightly connected to marginal costs, the relative dynamics of these components over the business cycle are thus crucial for inflation dynamics.

3 Financial shocks at the effective lower bound

In this section, we analyze how a binding ELB affects the transmission of financial shocks in the economy. To this end, we derive closed-form general equilibrium solutions for normal times and for when the economy is at the ELB. Contrasting these two cases highlights that macroeconomic dynamics at the ELB may be fundamentally different.

3.1 The propagation of financial shocks in normal times

We first analyze the macroeconomic effects of financial shocks in normal times, i.e. when the ELB is non-binding. The model can be represented in three equations:⁴

$$\pi_t = \kappa\gamma y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa(r_t + u_t), \quad (14)$$

$$y_t = -\sigma^{-1}(r_t - E_t[\pi_{t+1}] + u_t) + E_t[y_{t+1}], \quad (15)$$

$$r_t = \max\{\phi_\pi\pi_t + \phi_y y_t + v_t, \bar{r}\}. \quad (16)$$

⁴See the Appendix for more details on this particular representation.

The exogenous processes for the financial shock u_t and the monetary policy shock v_t are

$$u_t = \rho u_{t-1} + \epsilon_t, \quad (17)$$

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}. \quad (18)$$

Equation (14) again represents the New Keynesian Phillips curve, where the third and fourth term reflect the financing cost channel and the purely exogenous markup that arises from financial shocks increasing the credit spread.⁵ Equation (15) is the Euler equation, and Equation (16) is the monetary policy rule setting the (notional) interest rate.⁶ In normal times, the ELB constraint does not bind, such that (15) and (16) are identical to the textbook New Keynesian model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, highlighting that the financial accelerator is a supply-side friction that directly affects inflation dynamics.

We solve the model via the method of undetermined coefficients and guess that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock.⁷

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times are given by*

$$\pi_t = a_0 u_t, \quad (19)$$

$$y_t = b_0 u_t, \quad (20)$$

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1 - \rho)}{(1 - \beta\rho)(\sigma(1 - \rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1 - \rho)(\phi_\pi - \rho)}, \quad (21)$$

$$b_0 = -\frac{1 + (\phi_\pi - \rho)a_0}{\sigma(1 - \rho) + \phi_y}. \quad (22)$$

Proof. See Appendix. ■

In combination with Proposition 1, the following Lemma 1 shows that financial shocks are (usually) a particular form of demand shocks. A positive financial shock increases the wedge between the interest rate controlled by the central bank and the return on bonds held by households, thereby reducing current consumption. Thus, a positive financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation declines as factor costs dominate over external financing costs.

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times are negative, i.e.*

$$a_0 < 0, \quad (23)$$

$$b_0 < 0, \quad (24)$$

⁵This exogenous effect differentiates financial shocks from pure demand shocks (e.g. natural rate shocks) that appear solely in the Euler equation.

⁶Note that for financial shocks, the responses of output and the output gap are identical: an efficient economy without nominal rigidities and financial frictions does not respond to financial shocks.

⁷We assume that determinacy conditions hold. See Section 5.3 and Footnote 8 for a closer analysis.

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \quad (25)$$

Proof. See Appendix. ■

The analytical solutions from Proposition 1 display precisely the different channels through which the financial shock operates. In a_0 , the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following a positive financial shock, factor costs decrease, because labor demand falls given the decline in demand (the first part of $\kappa\gamma$). This in turn reduces inflation. At the same time, the financial shock increases the costs of production via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in factor costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator in a_0 . This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output, the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, lower nominal interest rates in reaction to the overall decline in inflation also decrease marginal costs directly. This amplifies the disinflationary response and the cost channel thus weakens the overall stabilizing property of the central bank’s interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas factor costs and the pure cost channel amplify the disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively large.⁸ In this case, factor costs dominate the price setting of firms, whereas the interest rates and credit spreads approximately balance out.

Nevertheless, as summarized in Lemma 2 below, the analytical solutions reveal that an overall increase of inflation following positive financial shocks is in principle possible. This situation may occur if the credit spread channel dominates both factor costs and the pure cost channel because the credit spread sensitivity to leverage is large:

Lemma 2. *The impact response of inflation to a financial shock in normal times is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \quad (26)$$

Proof. See Appendix. ■

⁸Lemma 1 guarantees that the denominator in a_0 is positive, which is required for determinacy (as shown in the Appendix). Intuitively, the model is only determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. The combination of a positive numerator from Lemma 1 and determinacy thus yields $a_0 < 0$ (note the minus in front of the fraction).

Note that this results directly from the presence of financial frictions linking credit spreads to marginal costs: in the absence of financial frictions, the policy functions in Proposition 1 are unambiguously negative. In the following, we want to focus on the case in which our financial shock is a classic demand shock to maintain the analogy to the Global Financial Crisis. We hence generally assume that Equation (26) is not satisfied such that a_0 remains negative:

Assumption 2. *The elasticity of the credit spread to entrepreneur leverage satisfies Condition (25) from Lemma 1.*

This implies the natural case of an upward sloping Phillips curve for financial shocks in normal times, i.e. a positive relationship between inflation and output.

3.2 The propagation of financial shocks at the ELB

We now turn to the case of a binding ELB. To this end, we assume that a financial shock endogenously brought the economy to the ELB and makes private agents expect the ELB to bind for a certain number of periods (often called the *ELB spell duration*, e.g. Holden, 2019). In this section, we take this ELB spell duration as given and do *not* adjust agents' expectations on the spell duration to any *additional* shocks, which we discuss in Section 4. This scenario hence focuses on *marginal* effects of (further) financial shocks at the ELB. While this perspective abstracts from the mapping between shocks and the expected duration of the ELB, it allows for a straightforward analytical comparison to the case of normal times.

The equilibrium responses of inflation and output can be characterized by recursive policy functions which are conditionally linear given the expected ELB spell:

Proposition 2. *Suppose that the ELB on nominal interest rate is expected to bind for $k > 0$ periods. Then, the impact responses of inflation and output to a financial shock are given by*

$$\pi_t = a_k u_t, \quad (27)$$

$$y_t = b_k u_t, \quad (28)$$

where

$$a_k = \kappa (1 - \gamma\sigma^{-1}) \left(1 + \frac{\bar{r}}{u_t}\right) + \rho (\beta - \kappa + \kappa\gamma\sigma^{-1}) a_{k-1} + \rho\kappa\gamma b_{k-1}, \quad (29)$$

$$b_k = -\sigma^{-1} \left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1} a_{k-1} + \rho b_{k-1}, \quad (30)$$

and $\{a_0, b_0\}$ as in Proposition 1.

Proof. See Appendix. ■

To interpret Proposition 2, consider the inflation response for an expected ELB duration of one quarter ($k = 1$), i.e. a_1 , and recall that Assumption 2 guarantees negative policy functions a_0 and b_0 . This implies that both the second and third term in Equation (29) are negative. The term in front of a_0 is close to unity for persistent shocks and

shows the ELB’s amplification property: the impact response of inflation increases (ceteris paribus) in the expected length of the ELB. This reflects the inability of the central bank at the ELB to counteract further contractionary shocks via additional (conventional) monetary stimulus. At the same time, the resulting upward pressure on real interest rates depresses consumption, and accordingly output.

However, there is an opposing effect on the overall inflation response, captured by the first term in Equation (29). This term can be positive, such that there is potential for a policy function for inflation that is concave in the expected ELB spell duration. In other words, it is possible that the disinflationary effect following positive financial shocks is *lower* if the ELB is expected to bind for a longer period of time. A necessary condition for a concave inflation policy function is that $1 > \gamma\sigma^{-1}$, which is equivalent to the following Lemma 3.

Lemma 3. *The inflation policy function is concave if the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta}. \quad (31)$$

It thus follows from Lemma 3 that the overall response of inflation following inflationary shocks depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage. An alternative way to see this is to note that

$$\frac{\partial a_k}{\partial \nu} = \frac{\partial a_k}{\partial \gamma} \frac{\partial \gamma}{\partial \nu} > 0. \quad (32)$$

The first term in Equation (29) depends negatively on γ . The second depends positively on γ , but following the recursion brings up a_0 , which is negative following Assumption 2. The last term is positive in γ as well, while $b_{k-1} < 0$ for all reasonable calibrations. The effect of an increase in ν is hence unambiguous: a larger elasticity of the credit spread with respect to entrepreneur leverage ceteris paribus increases the inflationary effect of financial shocks.

Intuitively, a concave policy function for inflation requires that the credit spread channel (the left-hand side in Equation (31)) dominates both the factor cost channel and the financial cost channel (the right-hand side in Equation (31)). If financial frictions are sufficiently pronounced such that ν is large, credit spreads may dominate the price setting of firms, thereby increasing inflation ceteris paribus. This result also naturally implies that the corresponding effect is absent in the standard NK model (in which $\nu = 0$).⁹ For the following analysis, we capture this scenario via the following assumption:

Assumption 3. *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition (31) from Lemma 3.*

Note that Assumption 3 is weaker than the counterpart in Assumption 2. A further requirement for a concave policy function is that financial shocks are sufficiently large.

⁹Given financial frictions, a concave policy function is also possible for natural rate shocks. This requires a larger elasticity of the credit spread to compensate for the missing purely exogenous markup effect. It holds that $a_k^{\text{nrs}} = a_k^{\text{fs}} - \kappa$, where “nrs” stands for natural rate shock, and “fs” for financial shock.

This can be seen by inspecting the term $(1 + \frac{\bar{r}}{u_t})$, which is only positive if the following Assumption holds:

Assumption 4. *The financial shock size satisfies*

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \quad (33)$$

Figure 2 displays the policy functions a_k and b_k under two illustrative calibrations. In the first case, the parameters satisfy Assumptions 1, 2 and 4: the spread is countercyclical, financial shocks have conventional effects in normal times and the shock is relatively large. In the second case, the calibration additionally satisfies Assumption 3. In the first case, the policy functions for inflation and output are strictly decreasing in the expected ELB spell duration; a longer expected ELB duration implies a stronger macroeconomic effect of additional financial shocks. In the second case, however, the policy function for inflation is concave, peaking at an expected ELB duration of six quarters in positive territory. In other words, if the ELB is expected to bind for a longer period of time, the overall inflation response may even turn positive. In this case, financial shocks appear as supply-type disturbances, i.e. moving output and inflation in opposite directions, whereas they usually belong to the class of demand shocks. This illustrates that inflation dynamics following financial shocks may be fundamentally different at the ELB compared to normal times.

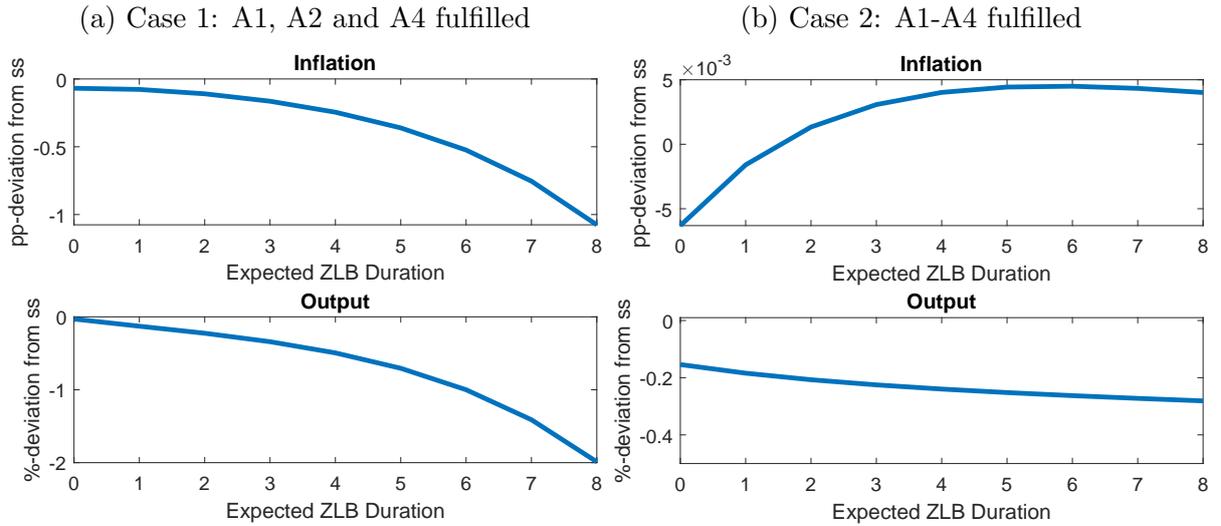


Figure 2: Expected ELB duration and impact response following financial shocks.

4 Numerical results and the hockey stick Phillips curve

In this section, we supplement our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. We employ numerical solution methods to treat the expected ELB spell as endogenous to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

4.1 Calibration and solution method

Throughout this section, we fix the model’s structural parameters to standard values taken from [Woodford \(2003\)](#), and adjust them to the most recent estimates (up until 2019) from [Boehl and Strobel \(2020, BS20\)](#) henceforth). We set $\beta = 0.99$, representing the standard view of a quarterly model. We calibrate $\sigma = 1$, which is a common assumption in line with a balanced growth path and also backed by BS20. Following the same line of reasoning, we set $\eta = 0.5$. We calibrate the fraction of non-adjusting price setters ζ to the commonly found textbook value of 0.66. This is conspicuously lower than the larger estimates from [Smets and Wouters \(2007\)](#) and BS20, as we want to avoid assuming a flat New Keynesian Phillips curve ex-ante.

For the parameters pertaining to the financial frictions, we fix $\psi = 8$ such that the output effects of financial shocks are amplified by approx. 20% relative to the standard NK model, which is roughly in line with the amplification degree documented by [Bernanke et al. \(1999\)](#). For our baseline szenaro we assume that $\nu = 0.25$, which implies an elasticity of firms’ marginal cost w.r.t. the risk spread of one quarter. We conduct comparative exercises with regard to this parameter further below and in section 5.

Regarding monetary policy parameters, we set ϕ_π to 1.5 (a commonly used standard prior), and ϕ_y to 0.2. In line with the estimates of BS20, the latter value is large relative to the standard prior mean of 0.125. As the authors argue, this reflects the strong reaction of the Fed to output during the ELB episode from 2009–2015, during which inflation was close to its target value while the level of output remained persistently depressed. We set $\rho = 0.9$, reflecting a lasting, quite persistent financial shock which resembles a post-2009 scenario.

The analytical solutions shown in the previous section hold for the impact period when the shock occurs, under the assumption that the expected duration of the ELB k is given. However, in general and in the absence of special policy measures such as forward guidance, k is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by [Boehl \(2021\)](#). A brief description of the solution method is outlined in C.

4.2 Impulse responses to financial shocks

The left panel of Figure 3 displays impulse responses following contractionary financial shocks of differing size. For the impact responses, these correspond to the analytical policy functions in Proposition 1 and Proposition 2. A one-percent shock (yellow line) is not strong enough to cause the ELB to be binding. As a result, the dynamics look conventional (for demand-side shocks), with inflation (and marginal costs) falling in response to the shock. As the shock size increases, the ELB spell duration increases. Respectively, the initial response of inflation shifts upwards, in line with the analytical insight from Assumption 4. For a large value of u_t , the initial response of inflation becomes positive.¹⁰

In the right panel of Figure 3, we consider a large financial shock, pushing the economy to the ELB, for different values of ν . As highlighted by the graphs, marginal costs

¹⁰Note that the lines are simply shifted outwards in case of a larger initial shock, since the responses of endogenous variables are a simple linear map of u_t and u_t decreases each period by $(1 - \rho)$.

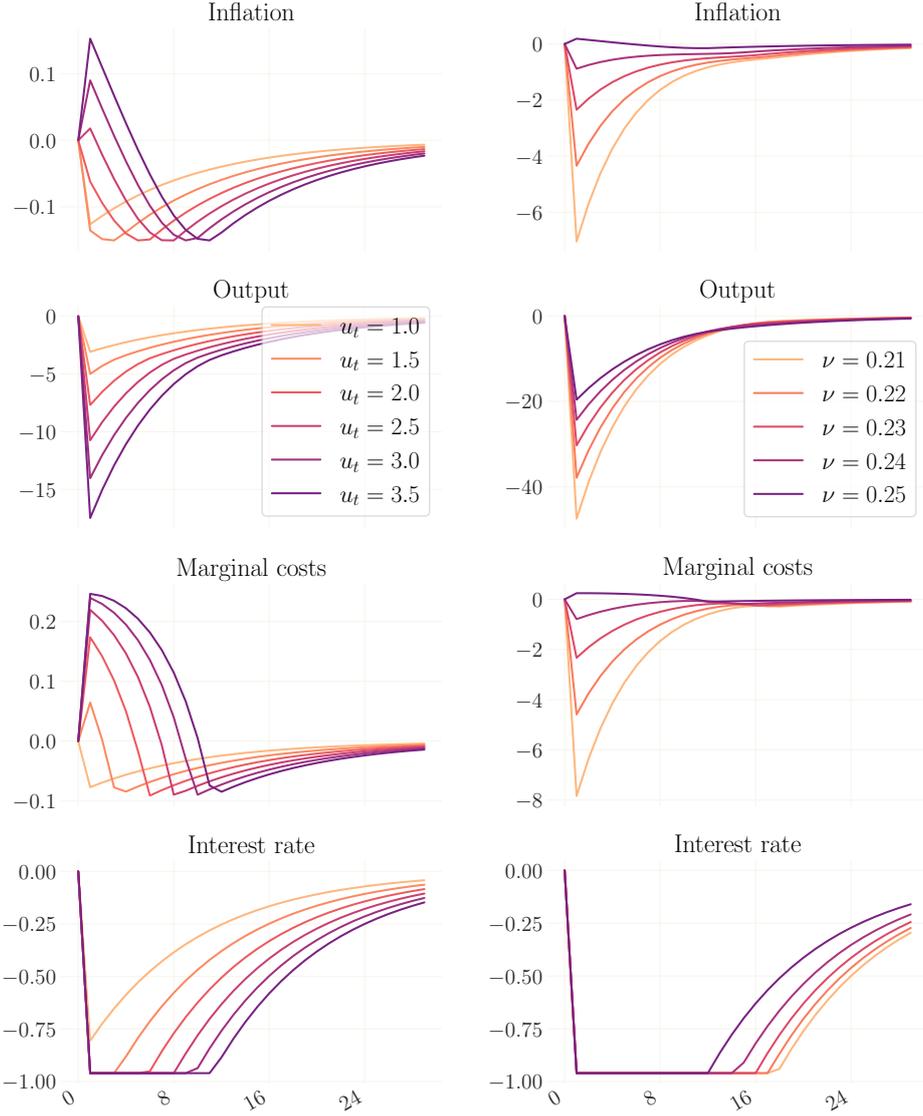


Figure 3: Impulse response functions. *Left:* to different financial shocks for $\nu = 0.25$. *Right:* to a 3% financial shock for different values of ν causing the ELB to be binding.

decrease less if financial frictions are stronger, which also translates to inflation. For $\nu = 0.25$, inflation actually increases, whereas the same calibration yields regular dynamics in the absence of the ELB (left side of Figure 3). This corresponds to the standard case outlined in the previous section: the elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not so excessively large such that a positive response emerges in normal times.¹¹

¹¹As Proposition 2 suggests, the persistence of financial shocks ρ is another central parameter for inflation dynamics, both at the ELB and for the rather extreme Neo-Fisherian case. A lower value of ρ yields a more concave inflation policy function (c.f. Equation 29). A lower ρ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock. We illustrate this in Figure A3 in the Appendix. We discuss the role of persistence in more detail in Section 5.

4.3 The observational hockey stick Phillips curve

Figure 4 plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$ -space. We interpret this as the *observational Phillips curve*, i.e. the realized values of inflation and output (gap) that would be observed in general equilibrium.¹² This is in contrast to the theoretical New Keynesian Phillips curve – as shown in Equations (8) and (14) – which represents firms’ price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 4 is the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is positive, in line with standard theory. However, for substantially negative values of output (caused by large financial shocks) the observational Phillips curve flattens out at the ELB.

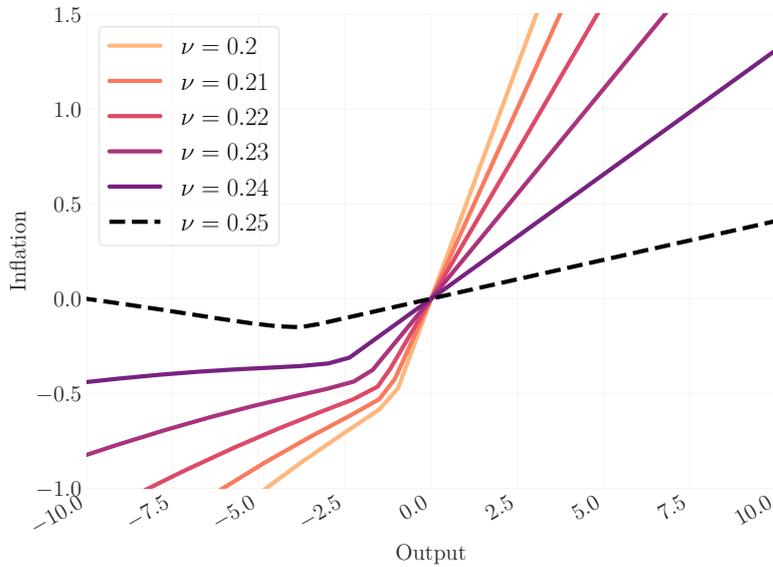


Figure 4: Observed Phillips Curve for an economy facing financial shocks. For each value of ν , we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t .

As the elasticity of the credit spread to leverage, ν , increases, the Phillips Curve becomes flatter for both regimes: the hockey stick not only rotates in the origin, but also the ratio of the two slopes decreases. For $\nu = 0.24$, the observed slope in the region of -3% output is almost zero, while having a conventional slope in the origin. For a value of $\nu = 0.25$, we observe that the credit spread effect at the ELB is strong enough that inflation actually increases with output, while the Phillips curve is still upwards sloping in normal times.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ELB and financial frictions would inherently conclude that the Phillips curve is “dead”. This observation emerges even though the New Keynesian Phillips curve

¹²Note again that the output response following financial shocks is identical to the output gap response, see Footnote 6. As such, the figure can equivalently be interpreted as showing the output gap - inflation space.

is alive and well: the relationship between firms' prices and marginal costs, governed by the Calvo parameter, is intact.¹³ However, the credit spread channel dominates firms' price setting at the ELB and thus blurs the supply-side link between output and prices. The flat observational Phillips curve at the ELB implies that a proper identification of the relationship between inflation and output is challenging if business cycles are driven by financial shocks.

5 Monetary policy at the effective lower bound

We now turn to the implications for monetary policy. From the viewpoint of central banks, the hockey stick Phillips curve raises difficulties in interpreting the observational Phillips curve. To make matters worse, the effects of monetary policy itself are also affected by financial frictions and the ELB. We analyze this aspect by considering both monetary policy shocks – in particular forward guidance shocks at the ELB – and the systematic behavior of central banks, governed by the monetary policy rule.

5.1 Interpreting the observational Phillips curve

Our result about the flat observational Phillips curve at the ELB means that the correct identification of the relationship between inflation and output is challenging. This is because policymakers need to infer the structural relationship between inflation and output (i.e. the structural New Keynesian Phillips slope and determinants of firms' price setting) using only observed equilibrium values. This requires estimates of contemporaneous macroeconomic shocks, private sector expectation of the ELB length and the currently prevailing degree of financial frictions. Acquiring this level of information in real time seems hardly possible in practice.

Abstracting from this challenge, the flat observational Phillips curve does not necessarily constitute a threat to policymakers per se. At the ELB, (further) contractionary risk shocks do not lead to a substantial decline of inflation. Given a strong mandate to stabilize inflation, a lower deflationary pressure is equivalent to a lower sense of urgency for monetary policy to act. This also means that central banks might not necessarily be forced to resort to unconventional monetary policy instruments at the ELB.

However, one could argue that the source of inflation is important as well, raising further difficulties for monetary policy. In our theoretical framework, the lower deflationary pressure at the ELB following financial shocks stems from higher credit spreads. If credit spreads are major determinants of inflation at the ELB, this also implies that central banks should be predominantly concerned with financial conditions. In such a situation, reducing financial distress directly via appropriate monetary policy operations on money markets might be the most efficient way to steer inflation. Unfortunately, central banks might find themselves in a catch-22 situation. On the one hand, high credit spreads might reflect substantial distress in the financial sector, thereby constituting a concern from a financial stability perspective. On the other hand, lower credit spreads induced by looser monetary and financial conditions increase the deflationary pressure. Therefore, a finan-

¹³Note again that our calibration avoids pre-assuming a flat New Keynesian Phillips curve, with the Calvo parameter $\zeta = 0.66$ being considerably lower than the estimate of $\zeta = 0.85$ in BS20.

cial recovery might not necessarily be associated with (a revival of) inflation. As such, disentangling the role of real marginal costs and credit spreads for overall inflation seems important to design appropriate monetary (and macroprudential) policies.

5.2 Reversal effects of forward guidance

A crucial insight regarding monetary policy shocks v_t is that in normal times, they generate *identical* macroeconomic dynamics as financial shocks. The three-equation representation from Section 3.1 reveals that monetary policy shocks appear in the same places as financial shocks. Therefore, in this framework and away from the ELB, monetary policy shocks and financial shocks are observationally equivalent in terms of inflation and output; they are only distinguishable via the response of the interest rate. As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions, which also implies that the possibility of *Neo-Fisherian* effects of monetary policy shocks in normal times (an increase in inflation after rising interest rates) for extreme calibrations. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly in normal times.

The insight that both shocks appear in the same places features major implications for forward guidance monetary policy at the ELB, which is the second important contribution of this paper. At the ELB, through their shock persistence, monetary policy shocks govern the expectations regarding the future interest rate path, acting like explicit forward guidance by the central bank. Forward guidance hence generates the same macroeconomic dynamics at the ELB as financial shocks.¹⁴ However, unfortunately for monetary policy, our previous results thus imply that forward guidance at the ELB might not be particularly effective and may even be associated with unintended effects on inflation. Notably, this includes the possibility that forward guidance at the ELB may be *disflationary*, i.e. inducing Neo-Fisherian effects by *decreasing* inflation, while raising output.

Intuitively, forward guidance shocks induce three different and partially opposing effects on inflation. First, expected rates are lower, which transmits to the economy via the standard (demand-side) Euler channel. Second, lower expected rates decrease expected marginal costs via the financing cost channel. Third, agents expect that the inversion of the policy function will remain active for more periods. The first effect leads to an unambiguous increase in output. The second effect clearly depresses inflation. The third effect prolongs the reversal of the inflation response that is induced by the ELB via the credit channel. As forward guidance raises output, this could also trigger a drop in inflation. Which of these effects dominates depends crucially on the forward guidance persistence and the degree of financial frictions.

As an illustration, Figure 5 shows impulse responses following forward guidance shocks at the ELB given different values for ν . In the left panel, the stronger internal propagation of the forward guidance shock caused by a higher ν leads to a longer ELB period than in the right panel. This means that, given the same financial shock, during the extended ELB period the interest rate is much lower than in the absence of forward guidance, which causes marginal costs to fall. Since expected lower marginal cost are anticipated by firms

¹⁴At the ELB, monetary policy shocks v_t and financial shocks u_t are hence not distinguishable, given the same persistence.

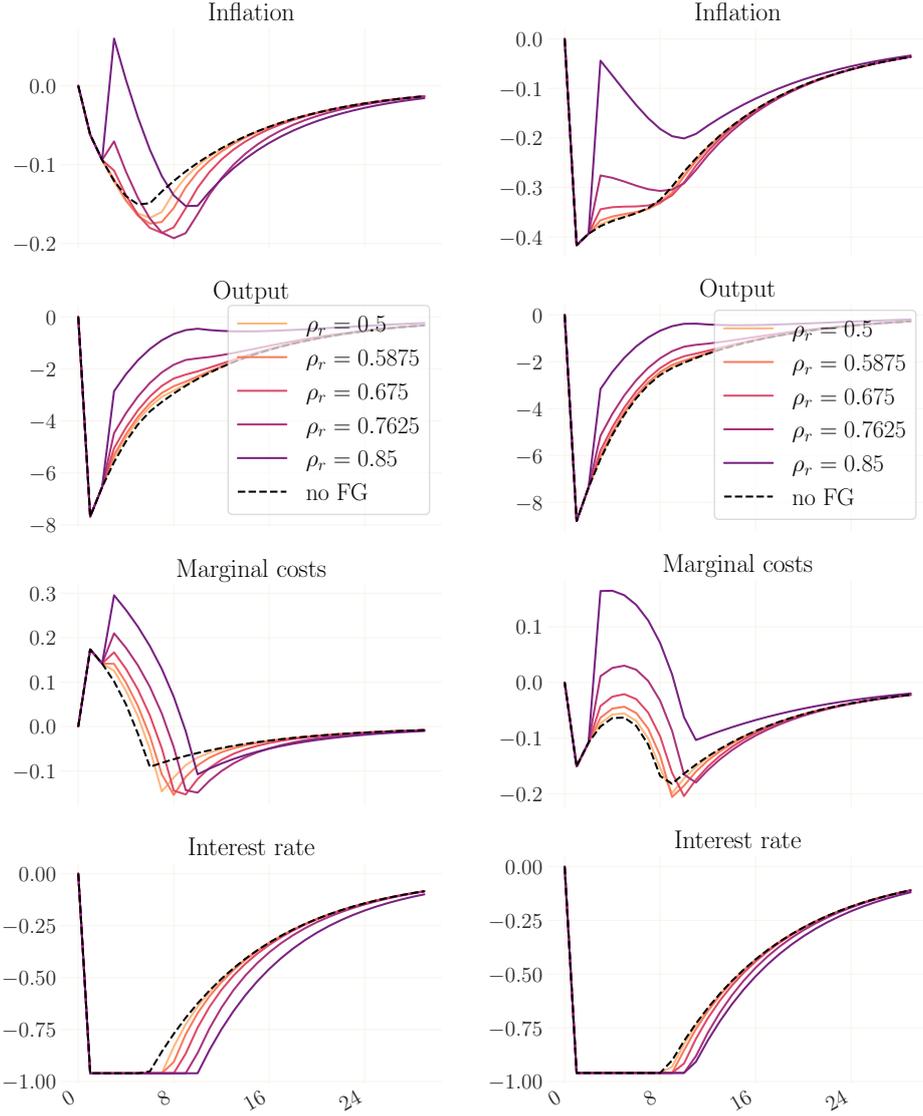


Figure 5: Impulse response functions for a 2% financial shock. Colored lines are the financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. *Left*: for $\nu = 0.25$. For many values of ρ_r , the forward guidance shock is disinflationary. *Right*: for $\nu = 0.24$. For this value of ν the forward guidance shock is not disinflationary.

(via the Phillips curve), the fall in inflation is *larger* than without forward guidance. This effect is absent in the right panel of Figure 5 because the shock does not prolong the ELB period significantly.

Lemma 4. *At the ELB, forward guidance shocks v_t may be associated with Neo-Fisherian effects such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \quad (34)$$

Note that the condition in Lemma 4 is a necessary, but not a sufficient condition. To

see this, assume a combination (ρ, ν) for which a given shock u_t is disinflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from Equation (29) in Proposition 2 that a smaller ρ (or here: ρ_r) can reduce the weight on the (negative terminal) second and third term. In that sense, a decrease in ρ has a similar effect as an increase in ν . We show this effect in Figure A3 in the Appendix.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is to some extent a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its communication strategies. As illustrated in Figure 5, a monetary policy shock with low persistence (i.e. low credibility) can hence trigger negative inflation responses because the short-run effect of decreasing financial costs dominates the longer-term effect that works through the household Euler Equation. As such, non-credible forward guidance may be associated with undesirable macroeconomic dynamics.

5.3 Monetary policy rules at the ELB

We now turn to the systematic behavior of central banks. At first glance, it may seem that these rules are irrelevant at the ELB. However, they are in fact crucial for macroeconomic dynamics because rational private agents take the monetary policy rule into account when forming expectations about future variables and the remaining ELB duration. As such, choosing an appropriate monetary policy rule is of central importance for central banks at the ELB as well. From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium.

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1 - \beta}{\kappa\gamma} \phi_y > 1, \quad (35)$$

$$\kappa(\sigma^{-1}\gamma - 1) \phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \quad (36)$$

Proof. See Appendix. ■

Equation (A74) may be interpreted as a modified Taylor principle for a financial accelerator economy. If the central bank decides to react to inflation only ($\phi_y = 0$), a necessary condition is that the associated coefficient ϕ_π needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that financial frictions affect the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation in the presence of financial frictions.

However, Equation (A75) may constitute additional complications for the design of monetary policy rules. To see this, suppose that $(\sigma^{-1}\gamma - 1) < 0$, which is exactly the condition for a concave policy function of inflation at the ELB, i.e. Assumption 3. In this

case, Equation (A75) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower (upper) bound restriction for the response to output (inflation). In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 6 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage ν increases, a higher value for ϕ_y is necessary to keep the model determined for high values of ϕ_π . For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

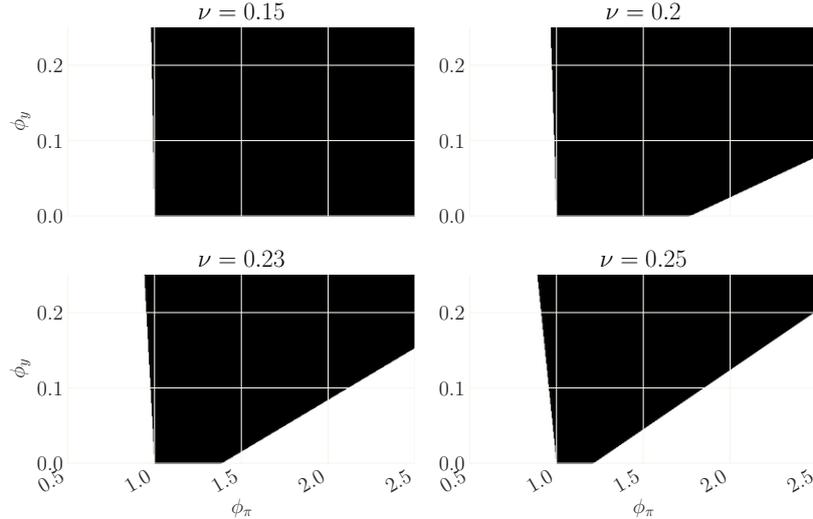


Figure 6: Determinacy regions for different values of ν .

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ELB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ELB, conventional monetary policy wisdoms are abolished: short-lived forward guidance shocks may be associated with Neo-Fisherian inflation effects, and determinacy considerations may place rather tight restrictions on appropriate monetary policy rules.

6 Conclusion

This paper argues that a binding effective lower bound (ELB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity if financial shocks are the main driver of business cycles. At the ELB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. Via this supply-side mechanism, the Phillips curve features a considerably flatter slope when the ELB binds compared to normal times. As a consequence, the resulting observational Phillips curve is shaped like a hockey stick. These findings constitute a complementary explanation for the recently observed inflation puzzles.

Our results translate into strong implications on the conduct of forward guidance, and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run disinflationary effects via the firms' refinancing cost channel, and a longer-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first disinflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only forward guidance with a high expected persistence succeeds in fostering inflation and growth.

References

- Abbate, A., Eickmeier, S., and Prieto, E. (2016). Financial shocks and inflation dynamics. Discussion Papers 41/2016, Deutsche Bundesbank.
- Ball, L. and Mazumder, S. (2011). Inflation Dynamics and the Great Recession. *Brookings Papers on Economic Activity*, 42(1 (Spring)):337–405.
- Ball, L. and Mazumder, S. (2018). A Phillips Curve with Anchored Expectations and Short-Term Unemployment. *Journal of Money, Credit and Banking*, 51(1):111–137.
- Barth III, M. J. and Ramey, V. A. (2001). The cost channel of monetary transmission. *NBER Macroeconomics Annual*, 16:199–240.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics*, 1:1341–1393.
- Bianchi, F. and Melosi, L. (2017). Escaping the Great Recession. *American Economic Review*, 107(4):1030–1058.
- Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311.
- Bobeica, E. and Jarociński, M. (2019). Missing Disinflation and Missing Inflation: A VAR Perspective. *International Journal of Central Banking*, 15(1):199–232.
- Boehl, G. (2020). Monetary Policy and Speculative Asset Markets. Technical report, University of Bonn and University of Mannheim, Germany.
- Boehl, G. (2021). Efficient solution and computation of models with occasionally binding constraints. IMFS Working Paper Series 148, Goethe University Frankfurt, Institute for Monetary and Financial Stability (IMFS).
- Boehl, G., Goy, G., and Strobel, F. (2021). A structural investigation of quantitative easing. Technical report.
- Boehl, G. and Strobel, F. (2020). US business cycle dynamics at the zero lower bound. Technical Report 65/2020.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.
- Carlstrom, C. T., Fuerst, T. S., and Paustian, M. (2015). Inflation and output in New Keynesian models with a transient interest rate peg. *Journal of Monetary Economics*, 76:230 – 243.
- Chowdhury, I., Hoffmann, M., and Schabert, A. (2006). Inflation dynamics and the cost channel of monetary transmission. *European Economic Review*, 50(4):995–1016.
- Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the Great Recession. *American Economic Journal: Macroeconomics*, 7(1):110–67.

- Cochrane, J. H. (2011). Determinacy and identification with Taylor rules. *Journal of Political economy*, 119(3):565–615.
- Cochrane, J. H. (2016). Do higher interest rates raise or lower inflation? *Unpublished paper, February*.
- Cochrane, J. H. (2017). The new-Keynesian liquidity trap. *Journal of Monetary Economics*, 92:47–63.
- Coibion, O. and Gorodnichenko, Y. (2015). Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation. *American Economic Journal: Macroeconomics*, 7(1):197–232.
- Daly, M. C. and Hobijn, B. (2014). Downward Nominal Wage Rigidities Bend the Phillips Curve. *Journal of Money, Credit and Banking*, 46(S2):51–93.
- Del Negro, M., Giannoni, M., and Patterson, C. (2015a). The forward guidance puzzle. Staff Reports 574, Federal Reserve Bank of New York.
- Del Negro, M., Giannoni, M. P., and Schorfheide, F. (2015b). Inflation in the Great Recession and New Keynesian Models. *American Economic Journal: Macroeconomics*, 7(1):168–196.
- Forbes, K. (2019). Inflation dynamics: Dead, dormant, or determined abroad? Working Paper 26496, National Bureau of Economic Research.
- Friedrich, C. (2016). Global inflation dynamics in the post-crisis period: What explains the puzzles? *Economics Letters*, 142(C):31–34.
- Gabaix, X. (2016). A behavioral new keynesian model. Technical report, National Bureau of Economic Research.
- Gaiotti, E. and Secchi, A. (2006). Is there a cost channel of monetary policy transmission? An investigation into the pricing behavior of 2,000 firms. *Journal of Money, Credit and Banking*, (525):2013–2037.
- García-Schmidt, M. and Woodford, M. (2019). Are low interest rates deflationary? A paradox of perfect-foresight analysis. *American Economic Review*, 109(1):86–120.
- Gilchrist, S., Schoenle, R., Sim, J., and Zakrajšek, E. (2017). Inflation Dynamics during the Financial Crisis. *American Economic Review*, 107(3):785–823.
- Gilchrist, S. and Zakrajšek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102(4):1692–1720.
- Gordon, R. J. (2013). The Phillips Curve is Alive and Well: Inflation and the NAIRU During the Slow Recovery. NBER Working Papers 19390, National Bureau of Economic Research, Inc.
- Gust, C., Herbst, E., López-Salido, D., and Smith, M. E. (2017). The empirical implications of the interest-rate lower bound. *American Economic Review*, 107(7):1971–2006.

- Holden, T. D. (2019). Existence and uniqueness of solutions to dynamic models with occasionally binding constraints. EconStor Preprints 144570, ZBW - Leibniz Information Centre for Economics.
- Kiley, M. (2016). Policy Paradoxes in the New-Keynesian Model. *Review of Economic Dynamics*, 21:1–15.
- Kulish, M., Morley, J., and Robinson, T. (2017). Estimating DSGE models with zero interest rate policy. *Journal of Monetary Economics*, 88:35 – 49.
- Lieberknecht, P. (2019). Financial frictions, the Phillips curve and monetary policy. Discussion Papers 47/2019, Deutsche Bundesbank.
- Lindé, J. and Trabandt, M. (2019). Resolving the Missing Deflation Puzzle. CEPR Discussion Papers 13690, C.E.P.R. Discussion Papers.
- McLeay, M. and Tenreyro, S. (2020). Optimal inflation and the identification of the Phillips curve. *NBER Macroeconomics Annual*, 34(1):199–255.
- Ravenna, F. and Walsh, C. E. (2006). Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53(2):199–216.
- Sims, E. R. and Wu, J. C. (2019). The Four Equation New Keynesian Model. NBER Working Papers 26067, National Bureau of Economic Research, Inc.
- Smets, F. and Wouters, R. (2007). Shocks and Frictions in US business cycles: A Bayesian DSGE approach. *The American Economic Review*, 97(3):586–606.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214.
- Tillmann, P. (2008). Do interest rates drive inflation dynamics? an analysis of the cost channel of monetary transmission. *Journal of Economic dynamics and Control*, 32(9):2723–2744.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.
- Watson, M. W. (2014). Inflation Persistence, the NAIRU, and the Great Recession. *American Economic Review*, 104(5):31–36.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Appendix

A Equilibrium equations

This section lists the full set of the equations defining the equilibrium. On the household side, we have the inter-temporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \quad (\text{A1})$$

$$H_t^\eta = W_t C_t^{-\sigma}. \quad (\text{A2})$$

On the firm side, we have the aggregate production function, which is obtained by aggregating over the individual linear production functions:

$$Y_t = \frac{H_t}{v_t^p} \quad (\text{A3})$$

where v_t^p is a measure of price dispersion defined below. Marginal costs are given by Equation (4):

$$MC_t = W_t R_t^L \quad (\text{A4})$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing and make use of two auxiliary variables f_t^1 and f_t^2 :

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \quad (\text{A5})$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta \zeta E_t [\Pi_{t+1}^\varepsilon f_{t+1}^1] \quad (\text{A6})$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta \zeta E_t \left[\left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2 \right] \quad (\text{A7})$$

$$1 = \zeta \left(\frac{1}{\Pi_t} \right)^{1-\varepsilon} + (1 - \zeta) (\Pi_t^*)^{1-\varepsilon} \quad (\text{A8})$$

$$v_t^p = \zeta \Pi_t^\varepsilon v_{t-1}^p + (1 - \zeta) (\Pi_t^*)^{-\varepsilon} \quad (\text{A9})$$

The interest rate specified in the credit contract is defined by Equation (3):

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \quad (\text{A10})$$

Entrepreneur net worth evolves according to Equation (5):

$$N_t = \Psi(Y_t), \quad (\text{A11})$$

The central bank operates according to a monetary policy rule shown in Equation (6)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \exp(v_t), \quad (\text{A12})$$

The effective lower bound (ELB) constraint is given by Equation (7):

$$R_t = \max \{ \bar{R}, R_t^n \} \quad (\text{A13})$$

Finally, the aggregate resource constraint is

$$Y_t = C_t \quad (\text{A14})$$

These 14 conditions define the equilibrium for the 14 endogenous variables

$$(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p), \quad (\text{A15})$$

together with the evolution of the two exogenous shocks:

$$\ln(U_t) = \rho \ln(U_{t-1}) + \epsilon_t \quad (\text{A16})$$

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}. \quad (\text{A17})$$

The linearized equilibrium conditions are as follows:

$$c_t = -\sigma^{-1} (r_t + u_t - E_t \pi_{t+1}) + E_t [c_{t+1}], \quad (\text{A18})$$

$$w_t = \eta h_t + \sigma c_t, \quad (\text{A19})$$

$$y_t = h_t, \quad (\text{A20})$$

$$mc_t = w_t + r_t^L, \quad (\text{A21})$$

$$\pi_t = \kappa mc_t + \beta E_t [\pi_{t+1}], \quad (\text{A22})$$

$$r_t^L = r_t - E_t [\pi_{t+1}] + \nu (w_t + h_t - n_t) + u_t, \quad (\text{A23})$$

$$n_t = \psi y_t, \quad (\text{A24})$$

$$r_t^n = \phi_\pi \pi_t + \phi_y y_t + v_t, \quad (\text{A25})$$

$$r_t = \max \{ \bar{r}, r_t^n \}, \quad (\text{A26})$$

$$y_t = c_t, \quad (\text{A27})$$

$$u_t = \rho u_{t-1} + \epsilon_t, \quad (\text{A28})$$

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}, \quad (\text{A29})$$

$$(\text{A30})$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (A19)-(A24) into one single Phillips curve and using the resource constraint Equation (A27) to eliminate c_t .

B Proofs

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are given by:*

$$\pi_t = a_0 u_t, \quad (\text{A31})$$

$$y_t = b_0 u_t, \quad (\text{A32})$$

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1 - \rho)}{(1 - \beta\rho)(\sigma(1 - \rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1 - \rho)(\phi_\pi - \rho)}, \quad (\text{A33})$$

$$b_0 = -\frac{1 + (\phi_\pi - \rho)a_0}{\sigma(1 - \rho) + \phi_y}. \quad (\text{A34})$$

Proof. The proof relies on the method of undetermined coefficients. We guess that the solution is given by $\pi_t = a_0 u_t$ and $y_t = b_0 u_t$. Using this guess, the system of equation can be written as

$$(1 - \kappa\phi_\pi - \rho(\beta - \kappa))a_0 u_t = \kappa u_t + \kappa(\gamma + \phi_y)b_0 u_t, \quad (\text{A35})$$

$$(1 + \phi_y\sigma^{-1} - \rho)b_0 u_t = -\sigma^{-1}(\phi_\pi - \rho)a_0 u_t - \sigma^{-1}u_t, \quad (\text{A36})$$

where we replaced the nominal interest rate using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by u_t , substituting for b_0 in the first equation using the second equation and rearranging. ■

Lemma 5. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \quad (\text{A37})$$

$$b_0 < 0, \quad (\text{A38})$$

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \quad (\text{A39})$$

Proof. The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of a_0 is positive. Second, the sign of a_0 then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of b_0 follows from a_0 .

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$M\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \quad (\text{A40})$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (15) and (16)

(ignoring exogenous innovations and the ELB) as

$$(1 + \sigma^{-1}\phi_y)y_t = -\sigma^{-1}(\phi_\pi\pi_t - E_t[\pi_{t+1}]) + E_t[y_{t+1}], \quad (\text{A41})$$

$$(1 - \kappa\phi_\pi)\pi_t = \kappa(\gamma + \phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \quad (\text{A42})$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$A\mathbf{x}_t = B\mathbf{x}_{t+1}, \quad (\text{A43})$$

$$\begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \quad (\text{A44})$$

It is straightforward that

$$B^{-1} = \frac{1}{\beta_\kappa} \begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \quad (\text{A45})$$

and hence

$$M = AB^{-1} = \begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \quad (\text{A46})$$

$$= \begin{bmatrix} 1 + \sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1 + \sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma + \phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) \end{bmatrix}, \quad (\text{A47})$$

$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \quad (\text{A48})$$

The eigenvalues of the system are given by $|M - \lambda I| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1 + m_4) = -(1 + \sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi)) \quad (\text{A49})$$

is the negative of the trace and

$$q = m_1m_4 - m_2m_3 = \beta_\kappa^{-1}(1 + \sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \quad (\text{A50})$$

is the determinant. As there are no endogenous states, determinacy under the conditions by [Blanchard and Kahn \(1980\)](#) requires the modulus of both eigenvalues of M to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of M as

$$|\lambda_{1,2}^r| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \quad \text{if } p^2/4 \geq q, \quad (\text{A51})$$

$$|\lambda_{1,2}^i| = \sqrt{p^2/2 - q} > 1 \quad \text{if } p^2/4 < q. \quad (\text{A52})$$

$|\lambda_{1,2}^r|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda_{1,2}^i|$ are corresponding imaginary eigenvalues otherwise. Using the condition in Equation (A51) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \quad (\text{A53})$$

Rearranging the second case in Equation (A51) also implies

$$1 + p + q > 0. \quad (\text{A54})$$

Together with Equation (A53), this implies

$$q > 1. \quad (\text{A55})$$

Equation (A55) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (A53) and Equation (A54) imply that Equation (A52) holds. Therefore, Equations (A53)-(A55) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1 + p + q > 0$, $p < -2$ and $q > 1$ thus read

$$\phi_\pi + \frac{1 - \beta}{\kappa\gamma} \phi_y > 1, \quad (\text{A56})$$

$$\sigma^{-1} \phi_y + \beta_\kappa^{-1} \sigma^{-1} (\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1} (1 - \kappa\phi_\pi) > 1, \quad (\text{A57})$$

$$1 + \sigma^{-1} (\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \quad (\text{A58})$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of a_0 . Let us suppose that the denominator is positive, i.e.

$$(1 - \beta\rho)(\sigma(1 - \rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1 - \rho)(\phi_\pi - \rho) > 0. \quad (\text{A59})$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1 - \beta}{\kappa\gamma} \phi_y - 1 \right) + \frac{1 - \rho}{\kappa\gamma} \left(\kappa\gamma + \beta\phi_y + \sigma(1 - \beta\rho - \kappa(\phi_\pi - \rho)) \right) > 0. \quad (\text{A60})$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (A56). After some algebraic manipulations, one can show that Equation (A57) implies that the second term in brackets is also positive. This shows that the denominator of a_0 is indeed positive.

With the denominator being positive, the sign of a_0 depends on the numerator, including the minus in front of the fraction. The condition for $a_0 < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1 - \rho) > 0. \quad (\text{A61})$$

Using the definition of γ , this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1 - \rho). \quad (\text{A62})$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of b_0 can be determined given the solution for a_0 . The denominator of b_0 is positive for conventional parameters, such that the sign is determined by

the numerator, including the minus. Inserting a_0 , this is given by

$$-1 + (\phi_\pi - \rho) \frac{\kappa\gamma - \kappa\sigma(1 - \rho)}{Z}, \quad (\text{A63})$$

where Z denotes the denominator of a_0 . After some algebraic manipulations, this is equivalent to

$$-Z^{-1} \left((1 - \beta\rho)(\sigma(1 - \rho) + \phi_y) + (1 - \rho)^2 \kappa\sigma \right), \quad (\text{A64})$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ■

Lemma 6. *The impact response of inflation to a financial shock in normal times (without a binding ELB on nominal interest rates) is positive whenever a_0 is positive, i.e. if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \quad (\text{A65})$$

Proof. This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of a_0 to be positive. The condition for $a_0 > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1 - \rho) > \gamma. \quad (\text{A66})$$

Using the definition of γ to obtain

$$\sigma(1 - \rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \quad (\text{A67})$$

and rearranging yields the desired result. ■

Proposition 2. *Suppose that the ELB on nominal interest rate is expected to bind for $k \geq 1$ periods. Then, the impact responses of inflation and output to a financial shock are given by:*

$$\pi_t = a_k u_t, \quad (\text{A68})$$

$$y_t = b_k u_t, \quad (\text{A69})$$

where

$$a_k = \kappa (1 - \gamma\sigma^{-1}) \left(1 + \frac{\bar{r}}{u_t} \right) + \rho (\beta - \kappa + \kappa\gamma\sigma^{-1}) a_{k-1} + \rho\kappa\gamma b_{k-1}, \quad (\text{A70})$$

$$b_k = -\sigma^{-1} \left(1 + \frac{\bar{r}}{u_t} \right) + \rho\sigma^{-1} a_{k-1} + \rho b_{k-1}. \quad (\text{A71})$$

Proof. Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ELB on nominal interest is expected to bind for $k \geq 1$ periods. Denoting the corresponding policy functions for by a_k and b_k , respectively, we can rewrite

the system of equations as

$$a_k u_t = \kappa\gamma b_k u_t + \kappa(\bar{r} + u_t) + (\beta - \kappa)\rho a_{k-1} u_t, \quad (\text{A72})$$

$$b_k u_t = -\sigma^{-1}(\bar{r} + u_t) + \rho\sigma^{-1}a_{k-1}u_t + \rho b_{k-1}u_t, \quad (\text{A73})$$

where the central bank interest rate is replaced by the ELB value. Note that expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ELB duration of $k-1$ under rational expectations, using the law of motion for the financial shocks. The solution is obtained by dividing both equations by u_t , substituting for b_k in the first equation using the second equation and rearranging. ■

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1 - \beta}{\kappa\gamma} \phi_y > 1, \quad (\text{A74})$$

$$\kappa(\sigma^{-1}\gamma - 1) \phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \quad (\text{A75})$$

Proof. The first equation follows directly from the condition $1 + p + q > 0$, which is required to satisfy the [Blanchard and Kahn \(1980\)](#) conditions. This is Equation (A56) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q > 1$, which is Equation (A58) above. ■

C Numerical solution method

For the sake of clarity, we use a different representation of the policy functions to outline the solution procedure. The analytic solutions in Section 3 are expressed in terms recursive policy functions of u_t . A different, non-recursive way of presenting these policy functions is suggested in Boehl (2021). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c} \max \{d\mathbf{x}_t, \bar{r}\} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \quad (\text{A76})$$

where \mathbf{N} is the system matrix of the constrained system, \mathbf{c} contains the coefficients that determine how \mathbf{x}_t is affected by r_t (and thereby also by u_t) and \mathbf{d} contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of r_t .

Assume again that the economy is at the ELB for k periods. Then

$$\mathbf{x}_t + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \quad (\text{A77})$$

$$E_t\mathbf{x}_{t+1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+2} + \mathbf{c}u_{t+1}, \quad (\text{A78})$$

...

$$E_t\mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+k} + \mathbf{c}u_{t+k-1}, \quad (\text{A79})$$

$$E_t\mathbf{x}_{t+k} = \mathbf{A}(0)u_{t+k}. \quad (\text{A80})$$

Recursively inserting (A80) into (A79) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\mathbf{x}_t = \mathbf{N}^k \mathbf{A}(0) \rho^k u_t + \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{c} \rho^i u_t - \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{b} \bar{r}, \quad (\text{A81})$$

$$= \mathbf{A}(k)u_t + \mathbf{a}(k)\bar{r}. \quad (\text{A82})$$

Rewriting (A81) yields

$$\pi_t = A_\pi(k)u_t + a_\pi(k)\bar{r}, \quad (\text{A83})$$

$$y_t = A_y(k)u_t + a_y(k)\bar{r}. \quad (\text{A84})$$

In verbal terms, this implies that depending on the expected number of periods at the ELB k , we can express the vector of controls \mathbf{x}_t as a linear map $A_j(k)$ of u_t and the (constant) vector $a_j(k)$. Both terms are nonlinear functions of k defined on \mathbb{N}_0 . In other words: given k , the policy function is simply a two dimensional linear projection of the scalar u_t .

Definition 1 recapitulates the conditions for k to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

Definition 1 (equilibrium k). For each period t , an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations *exactly* until period $t + k$. Hence,

$$d\mathbf{x}_t > \bar{r} \implies k = 0, \quad (\text{A85})$$

while for $k > 0$ it must hold that

$$\mathbf{d}E_t \mathbf{x}_{t+k} > \bar{r}, \quad (\text{A86})$$

and

$$\mathbf{d}E_t \mathbf{x}_{t+k-1} \leq \bar{r}. \quad (\text{A87})$$

The parsimonious nature of our model allows that, for each u_t , a k can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, k is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in [Boehl \(2021\)](#).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods.

In [Figure A1](#), we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

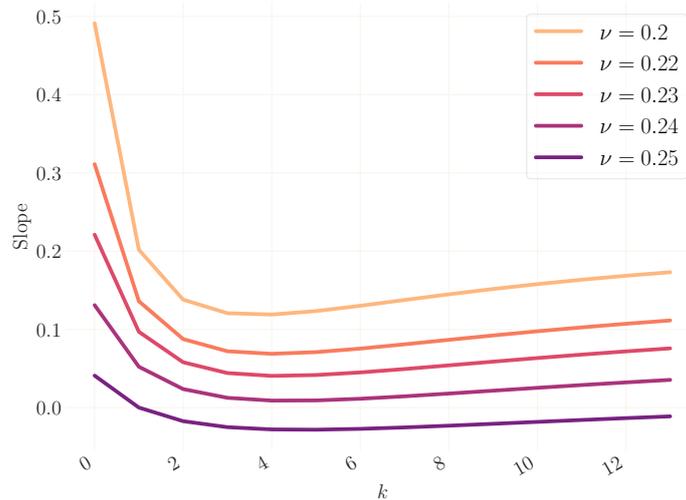


Figure A1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

[Figure A2](#) plots the non-recursive policy functions for π_t . For a more moderate value of ν of 0.2, the mapping $A_\pi(k)$ from $u_t \rightarrow \pi_t$ decreases with k while the linear part $a_\pi(k)$ increases in about the same fashion. As larger shocks are necessary to cause a higher k , the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positive for values of k larger than two. As the static effect is again too weak to counteract, this leads to an increase of inflation on impact, as it is captured in [Figure 3](#).

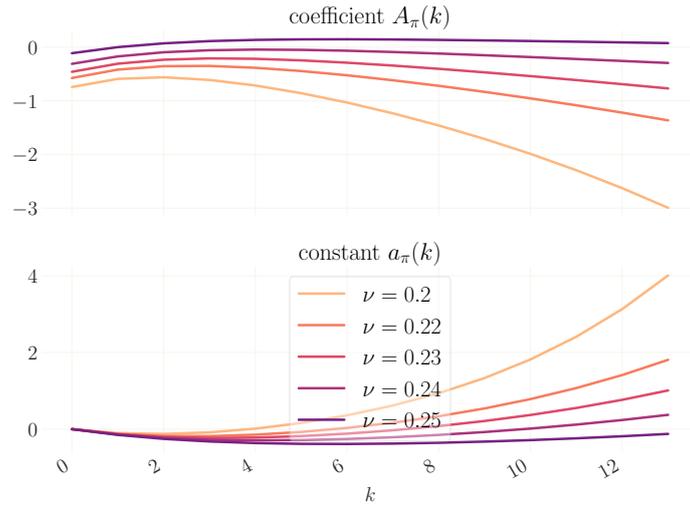


Figure A2: Expected ELB Duration and Impact Response

D Additional figure

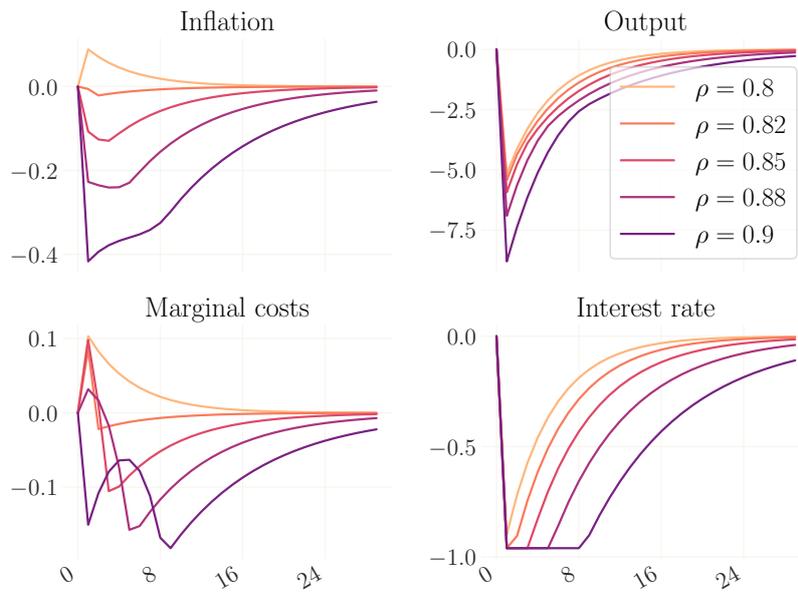


Figure A3: Impulse responses to 2% risk premium shocks for different values of ρ , given $\nu = 0.24$.