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The macroeconometric model of the Bundesbank revisited

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Non-technical summary

The macroeconometric model of the Bundesbank is an estimated, quarterly, semi-structural macroeconomic model for the German economy. It is used in the Macroeconomic Analysis and Projection Division of the Bundesbank's Directorate General Economics as a central coordinating tool in the bi-annual forecasts that contribute to the Eurosystem's Broad Macroeconomic Projection Exercises (BMPEs). In between projection rounds, the model elasticities are used to assess the economic impact of updated projection assumptions. The projection baseline is developed in an iterative process to which Bundesbank experts (e.g. for prices, the labour market or fiscal developments) contribute, and the macroeconometric model is used to develop a consistent macro outlook and projection narrative. Naturally, the model is also frequently applied for simulation analyses both in the projection context and beyond. It is designed to strike a balance between the theoretical foundations of its individual equations and model blocks and the empirical fit, while remaining sufficiently flexible to enable its users to integrate new empirical or theoretical findings. Given the model's role as a coordinating device in the projections and its broad use for simulations within the division, it aims to take into account sector or variable-specific modelling approaches developed and used by Bundesbank experts in order to represent them – to the greatest extent feasible – in a unifying macro framework.

While the macroeconometric model can be characterised as a traditional semi-structural model, some of its modelling approaches may not necessarily be shared with other mainstream models of this kind. For instance, its production technology incorporates energy as a third input factor besides capital and labour. The behavioural equation for private consumption disaggregates the effects of disposable income components and takes into account the small impact of households' net financial wealth. The modelling of business investment allows for a role of credit supply effects via indicators derived from survey information on bank lending standards. The macroeconometric model contains a relatively detailed fiscal block that enables the impact of a variety of fiscal policy measures to be assessed. On the nominal side, the price block combines the modelling of demand deflators and the most important HICP components for which simulation output is frequently required.

Simulation results as depicted for isolated shocks in the Basic Model Elasticities (BMEs) illustrate notable effects of external shocks, such as those to foreign demand or the exchange rate, which can be explained by the importance of extra-euro area trade in Germany's exports and overall final demand. Fiscal shocks also show strong demand effects not only if they directly affect GDP components such as government consumption or investment, but also in cases where they are transmitted via households' disposable income, such as wage tax and social security contribution shocks. In the model, prices generally react with a lag, and their reaction to shocks is fairly muted over the four-year horizon considered, unless they are directly affected, such as in case of indirect tax changes.

Nichttechnische Zusammenfassung

Das makroökonometrische Modell der Bundesbank ist ein geschätztes semi-strukturelles makroökonometrisches Quartalsmodell für die deutsche Wirtschaft. Es wird in der Abteilung Konjunktur und Wachstum des Zentralbereichs Volkswirtschaft als zentrales koordinierendes Instrument in den halbjährlichen Prognosen eingesetzt, die in die Broad Macroeconomic Projection Exercises (BMPEs) des Eurosystems einfließen. Zwischen den Projektionsrunden werden die Modellelastizitäten dazu genutzt, die gesamtwirtschaftlichen Auswirkungen aktualisierter Prognoseannahmen abzuschätzen. Die Prognosebasislinie wird in einem iterativen Prozess entwickelt, zu dem die Experten der Bundesbank (z.B. für die Preis- und Lohnentwicklung oder die öffentlichen Finanzen) beitragen. Mit Hilfe des makroökonometrischen Modells werden daraus ein konsistenter gesamtwirtschaftlicher Ausblick und ein Narrativ für die Projektionen erarbeitet. Zudem wird das Modell regelmäßig für Simulationsanalysen eingesetzt, sowohl im Zusammenhang mit den Prognosen als auch außerhalb des Projektionsprozesses. Das Modell soll die theoretische Fundierung seiner Einzelgleichungen und Blöcke mit guten empirischen Eigenschaften kombinieren. Dabei soll es ausreichend flexibel bleiben, sodass auch neue empirische oder theoretische Befunde integriert werden können. Vor dem Hintergrund seiner Rolle als koordinierendes Instrument im Prognoseprozess soll das makroökonometrische Modell einen Rahmen bieten, in dem Modellierungsansätze, die von Bundesbankexperten für einzelne Variablen oder Sektoren entwickelt und genutzt werden, soweit wie möglich berücksichtigt werden können.

Einige der im makroökonometrischen Modell genutzten Modellierungsansätze können sich auch von denen anderer traditioneller semi-struktureller Modelle unterscheiden. So umfasst die Produktionstechnologie mit Energie neben Arbeit und Kapital noch einen dritten Produktionsfaktor. In der Verhaltensgleichung für die privaten Konsumausgaben werden die Auswirkungen verschiedener Einkommenskomponenten unterschieden sowie ein (geringer) Einfluss des Nettogeldvermögens der privaten Haushalte abgebildet. Die Modellierung der gewerblichen Investitionen lässt einen Einfluss von Kreditangebotseffekten zu, die anhand von Umfrageinformationen zum Kreditgeschäft der Banken quantifiziert werden. Das makroökonometrische Modell beinhaltet einen relativ detaillierten Fiskalblock, der es ermöglicht, die gesamtwirtschaftlichen Auswirkungen verschiedener fiskalischer Maßnahmen mit dem Modell abzuschätzen. Der Preisblock kombiniert Modellierungen für die einzelnen Nachfragedeflatoren und die wichtigsten Komponenten des HVPI, für die regelmäßig Simulationsergebnisse bereitgestellt werden müssen.

In den Simulationsergebnissen für isolierte Schocks im Rahmen der Basic Model Elasticities (BMEs) spiegelt sich die große Bedeutung des Handels mit Ländern außerhalb des Euroraums für den deutschen Außenhandel und die deutsche Wirtschaft insgesamt wider. So zeigen externe Schocks, wie solche auf die ausländische Nachfrage oder die Wechselkurse, deutliche gesamtwirtschaftliche Auswirkungen. Auch fiskalische Schocks ziehen starke Effekte auf die gesamtwirtschaftliche Nachfrage nach sich. Dies ist nicht nur der Fall, wenn sie Komponenten des Bruttoinlandsprodukts betreffen, wie den Staatskonsum oder die Staatsinvestitionen, sondern auch, wenn die Transmission über das verfügbare Einkommen der privaten Haushalte läuft, wie z.B. für Schocks auf die Lohnsteuer oder die Sozialversicherungsbeiträge. Die Reaktion der Preise im Modell läuft verzögert ab und fällt über den hier betrachteten Vierjahreshorizont im Allgemeinen gedämpft aus. Stärkere Effekte ergeben sich, wenn der Schock direkt auf die Preise wirkt, wie z.B. bei Veränderungen der indirekten Steuern.

The Macroeconometric Model of the Bundesbank Revisited[1]

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Abstract

The use of a semi-structural model as one of the workhorse tools for macroeconomic analysis and projections at the Bundesbank has a long-standing tradition. In an iterative projection process, the macroeconometric model's main task is to merge projections by Bundesbank experts for various areas of the economy and other information into a unifying framework in order to develop a consistent narrative on the outlook for the German economy. Besides this, the model is frequently used to perform simulation analyses both in the projection context and beyond. Its structure thus has to be broad enough to capture the key interdependencies within the macroeconomy whilst also including a sufficiently detailed setup. This paper summarises the model's main equation blocks, shows growth decompositions of key macro variables and presents selected simulation results in order to illustrate key transmission channels.

The model has a traditional semi-structural character and is set in a national accounts framework. Most of its central behavioural equations follow an error-correction mechanism and are estimated individually based on quarterly data. One particular feature is the constant elasticity of substitution (CES) production technology with a two-level nested structure that integrates energy as a third input factor in addition to capital and labour. In the model's price block, behavioural equations for most deflators and HICP core inflation are jointly specified and estimated via a system approach. Moreover, the model contains a rich fiscal block.

Keywords: Semi-structural model, Germany, macroeconomic forecasting, policy simulations
JEL-Classification: C30, E10, E17, E6

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1 Introduction

For at least the past half century, large-scale macroeconometric models have played a prominent role in policy simulation and forecasting exercises performed by various policy-making institutions worldwide, including central banks. Although this model class takes a back seat in academia nowadays owing to its largely eclectic nature, it remains a firm fixture in model suites required for practical economic policy analysis. For this reason, numerous central banks and international policy-making institutions allocate considerable resources to keeping these models running and upgrading them periodically in line with changing economic conditions and new theoretical developments.[2]

The use of large macroeconometric models for various tasks, including forecasting exercises and policy assessment, has a long-standing tradition at the Bundesbank, too. The Bundesbank's first macroeconometric model was developed in the first half of the 1970s.[3] It was a single-country model consisting of 96 equations based on semi-annual data. Over a period of more than four decades, it has constantly been developed and adapted to the changing economic environment. In 1978, the first version based on quarterly data was published.[4] The reunification of Germany in 1990 posed a serious challenge for macroeconomic modelling, since the economic structure of the eastern part of Germany was decidedly distinct from that of the western region. Consequently, a separate block for the East German economy, consisting of 76 equations, was integrated into the model.[5] The separate modelling of the East German economy was abandoned some years later when market-based economic processes were sufficiently established.

The next step in the development of the model was taken after the introduction of the single currency. In 2000, a new version of the model, known as the *Macro-Econometric Multi-Country Model* (MEMMOD), was published.[6] MEMMOD saw the model reach its largest dimensions: It included nine country blocks with a total number of 690 equations aiming at capturing the growing importance of international trade. Besides a more detailed block for the German economy, the model included blocks for Belgium, France, Italy, the Netherlands, the United Kingdom, the United States, Canada and Japan as well as one block to model the common monetary policy within the euro area and a foreign trade block. MEMMOD's behavioural equations for interest and exchange rates as well as inflation rates incorporated forward-looking expectations.

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2 According to the Work stream on Eurosystem modelling (2021), almost all central banks in the Eurosystem (including the ECB) maintain semi-structural models and a large majority of them conduct their macroeconomic projections using these models.

3 See Deutsche Bundesbank (1975).

4 See Deutsche Bundesbank (1978). An interim update of the model followed four years later in Deutsche Bundesbank (1982). A description of the use of the model for forecasting the West German macroeconomic development at that time is given in Deutsche Bundesbank (1989).

5 See Deutsche Bundesbank (1994).

6 See Deutsche Bundesbank (2000a).

However, over the years, it became clear that the operational reality of the newly established European System of Central Banks was diminishing the benefits of maintaining a large multi-country model within a particular national central bank (NCB). Macroeconomic projections for the euro area were established and are still undertaken as a joint exercise by the participating NCBs and the ECB in a bottom-up approach. Common technical assumptions are derived from agreed-upon methodologies and ECB experts present an outlook for the world economy, which is then discussed in the Working Group of Forecasting of Eurosystem experts. In this framework, the approach best suited for regularly developing projection baselines and undertaking related simulation exercises for the German economy appeared to be a national model that could easily incorporate the common projection assumptions. Thus, in 2004, the German block of the multi-country model was extracted and became the de facto starting point for the Bundesbank's current macroeconometric model for the German economy.[7]

Apart from substituting seasonally unadjusted data by seasonally and working-day adjusted time series, the German block generally preserved its structure, with the pre-unification West German data still included in the estimation sample of major equations. The forward-looking expectation formation was eliminated, mainly because the financial variables that previously incorporated forward-looking expectations became exogenous in the national model. Although the general framework of definition equations proved its usefulness and applicability hereafter, the structure of the German economy changed substantially over the years. In particular, shortly after the beginning of the new millennium, Germany underwent a longer phase of subdued economic growth. An extended period of wage moderation had already begun in the second half of the 1990s. Following labour market reforms introduced in the first half of the 2000s, the unemployment rate, which had been ratcheting upwards since the 1970s, started a downward trend. Against the background of labour market reforms and demographic changes, private consumption dynamics altered noticeably and the saving rates of households rose. The financing structure of German enterprises also underwent a distinct transformation. Equity ratios increased constantly and the role of bank credit diminished in the first decade of the new millennium. In addition, a striking transition in the current account position occurred.[8] In 2002, the current account deficit switched to a surplus that increased considerably over the following years. The outbreak of the financial crisis in 2008 and the subsequent worldwide recession triggered new changes in Germany's economic dynamics. In spite of a deep recession, Germany recovered quickly from the crisis. There was no substantial increase in unemployment, and domestic demand strengthened considerably over the subsequent protracted recovery period that began in 2010, associated with robust employment growth and

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[7] In order to complete the class of semi-structural models for simulation analyses focusing on international settings with a key role for endogenous international feedback loops, the commercially available large multi-country model NiGEM of the National Institute of Economic and Social Research was acquired. It is still frequently applied in the Directorate General Economics of the Bundesbank. See Deutsche Bundesbank (2021c) for a recent example, where NiGEM simulations are used to analyse the potential impact of a sharp downturn on the Chinese real estate market on economic activity in a number of countries. See information on NiGEM on the NIESR website.

[8] This development can be traced back to several factors as highlighted in Bursian et al. (2020).

considerably higher wage growth compared to the preceding decade. All these gradual developments made the use of the macroeconometric model, as it stood then, increasingly challenging, despite regular re-estimations with up-to-date data. The need for a profound revision of the Bundesbank's macroeconometric model became apparent.

Besides aiming to take into account new empirical regularities and theoretical developments by re-assessing and re-estimating every behavioural equation using the data for the unified Germany, starting from 1991, the focus was also on tackling a number of more general issues. The production function was augmented with energy as an additional input, not least in light of the growing importance of climate-related aspects. In order to allow for elasticities of substitution below one, a CES-production technology was employed. The behavioural equation for private consumption expenditure, the largest component of final demand, was thoroughly revised, taking into account disaggregate income and wealth effects. Given the experiences gained during the financial crisis, a number of channels for capturing possible credit supply restrictions were integrated into the model. Finally, a more adequate representation of the German housing market had to be included, since house prices and construction activities in Germany started to surge after the Global Financial Crisis and policymakers began to focus more closely on their development. A larger revision of the model's price block, abstracting from the approach of a central price variable and moving towards individual price Phillips curves estimated in a system approach, was undertaken. Closely linked to this was a reassessment of the modelling of negotiated wages, which represents the key wage variable in the projection exercises. Additions to the labour market modelling sought to better reflect the increasing tightness that characterised the German labour market. Finally, a more detailed modelling of indirect taxation was implemented in order to capture the transmission to individual price variables more adequately and to separately take into account energy taxation. However, abstracting from the various revisions and extensions, the model's overall structure and the framework of definition equations were left in place to the extent possible, as they had proven valuable over the course of the last decade and a half.

Some general explanations should also be dedicated to the explicit role of the macroeconometric model in regular work. The model acts as a workhorse in the periodic construction of the macroeconomic outlook for the German economy. It is important to note that the projections are not the result of a purely model-based forecast. Instead, they are the outcome of a comprehensive and iterative process that combines forecasts for individual parts of the overall economy, also taking into account specific judgements by Bundesbank experts, e.g. on inflation, the labour market, the real estate market or fiscal policy. The macroeconometric model provides a consistent framework for these forecasts, allowing an assessment from a macro perspective and the development of a common, cohesive and consistent narrative. In addition, the model also has to be sufficiently flexible in order to allow a timely incorporation of off-model information based either on expert judgement or on higher frequency short-term indicators, for instance. Furthermore, the model needs to be sufficiently flexible in order to permit a fairly swift adjustment of its equations between the projection exercises in case findings from the preceding exercise need to be incorporated or sensitivity analyses are conducted within a projection exercise.[9]

Given this particular role of the macroeconometric model, its development and maintenance take into account the approaches used by Bundesbank experts for the various parts of the economy and integrate them into the model framework to the extent possible and appropriate. Since the experts' approaches can be much more detailed, disaggregated or even less model-based, this often entails a certain trade-off between theoretical and empirical considerations as well as practical issues. The outcome of this challenge is presented in the subsequent sections.

This paper documents the status quo of the Bundesbank's macroeconometric model as of autumn 2021. Given the fact that the COVID-19 pandemic has brought the long-lasting boom phase the German economy has enjoyed since 2010 to an abrupt end, this may be an appropriate point in time to record the status quo. It also seems particularly reasonable because potential shifts in structural relationships induced by the pandemic and massive data outliers in corresponding time series might require model adjustments that cannot be fully anticipated, let alone implemented yet.

The paper summarises the results of the major revisions to the Bundesbank's macroeconometric model undertaken over the past years. In doing so, it takes stock and gives an account of the model that has been employed for projection exercises and related policy simulations in recent years. Furthermore, this documentation forms the basis for future projects as well as ongoing work involving adapting the model to the changing environment and augmenting it with new features – for instance incorporating alternative forms of expectation formation, adding to the purely adaptive processes that are in place now.

The remainder of the paper is set up as follows: The next section outlines the structure of the model, with various subsections describing the relevant equation blocks. Section 3 presents selected growth decompositions of key model variables and a recent model application dealing with the amount of precautionary savings of German households in the COVID-19 pandemic. The fourth section displays and discusses the results of a number of simulations that are undertaken frequently with the macroeconometric model both for diagnostic purposes as well as for policy analyses, where several of the isolated shocks considered here are usually combined. The final section concludes.

[9] When conducting the macroeconomic projection exercise, the Bundesbank's suite of short-term forecasting models is used to transfer the information embedded in available economic data and forward-looking indicators into the macroeconometric model. See Deutsche Bundesbank (2018b) for an overview of the short-term economic forecasting tools at the Bundesbank. Recently, this model suite has been augmented with a weekly economic activity index for Germany developed by Eraslan and Götz (2021).

2 Model structure

This section outlines the structure of the macroeconometric model. For the purpose of this paper, we use the acronym BbkM-DE for the model. The first subsection deals with the general framework. The subsequent subsections describe the particular model blocks, starting with the production technology and corresponding factor demands, and continuing with the main domestic demand components (private consumption, business investment and residential investment). Wage and price setting mechanisms follow. International trade, government and financial market blocks are explained at the end of the section.

2.1 General framework

BbkM-DE's main fields of application are, and will continue to be, the medium-run projection exercises of the Bundesbank, which are part of the Eurosystem's *Broad Macroeconomic Projection Exercise* (BMPE), both for developing the projection baseline and carrying out related simulation analyses. This requires a high level of practicability in the model structure, since all results have to be expressed in terms of national accounts (NA) figures in a most direct way. The need to represent a broad picture of the key interdependencies in the macroeconomy and to maintain technical flexibility and practicability somewhat limits the depth of the model's theoretical concepts as well as the complexity of the mathematical structures. Instead, these characteristics feature in a set of DSGE models that the Bundesbank has available in its model portfolio. Amongst others, this model suite contains a comprehensive three-region DSGE model developed by Hoffmann et al. (2021) and a medium-scale two-agent New Keynesian (TANK) model introduced by Gerke et al. (2020). Further macro models that are part of the Bundesbank's model portfolio can be found in Bursian et al. (2020).

In general, BbkM-DE preserves the framework of its predecessors. The long-run properties of the model are determined by a neoclassical production function, postulating optimising behaviour of economic agents. In general, the demand for particular production factors (capital, labour and energy) is derived theoretically from the production function. However, in the short run, the model possesses Keynesian features since a gradual adjustment to the long-run neoclassical steady state is assumed. Estimated error correction (EC) models ensure the gradual adjustment in factor demand equations. Besides its neoclassical core, the model is set up in an eclectic fashion, driven by a mix of theoretical considerations and empirical regularities. The expectation formation is entirely backward-looking. This reduces model complexity. The model consists of 71 behavioural (including auxiliary) equations and 187 definitions, yielding a total of 258 equations. Behavioural equations are regularly re-estimated, usually twice per year.

The system of national accounts for Germany constitutes the main statistical framework for the model and for definition equations in particular. While the main income and expenditure components for the accounts of households, the government and the foreign account are directly modelled in BbkM-DE, the corporate account constitutes a largely residual item in order to close the system. For an overview of sector accounts in BbkM-DE see Annex I.

This chapter relies on certain notational conventions in order to describe the individual model blocks: Model equations are presented in a more condensed manner in order to streamline the notation. Variables' names are based on acronyms that more or less follow German terminology. Most of the variables' labels mentioned in the main text already correspond to their original names as used in the model environment (apart from Section 2.2, which gives only a very stylised representation of the production technology). There are some exceptions where the names of variables are only used in the main text. These are constructed variables and are marked by a tilde. In a few cases (e.g. interest rates, unemployment rate, output gap), the variables in the model database are measured in percent, while their notation in the main text refers to their decimal equivalent. Lower case letters represent a log of the respective variable. All the actual model variables and the exact specifications of the equations can be found in Annex II and III. In the model equations shown in the main text, coefficients are only distinguished with regard to the short term and the long term, with a particular label for the error correction (EC) coefficient. Estimated values are not explicitly documented in the main text, but only if deemed necessary in selected cases. Only the sign of the coefficients is indicated (except for constants and autoregressive terms). A full overview of the estimated values is given in the appendix. In general, dummy variables are not documented in the main text, but can be retraced in the aforementioned list of equations, together with the equation-specific estimation sample. If not explained otherwise, the model equations are estimated by ordinary least squares (OLS).[10] Hyperlinks (marked in blue) can be used for the key model equations in order to switch between the main text and the appendix.

2.2 Production technology

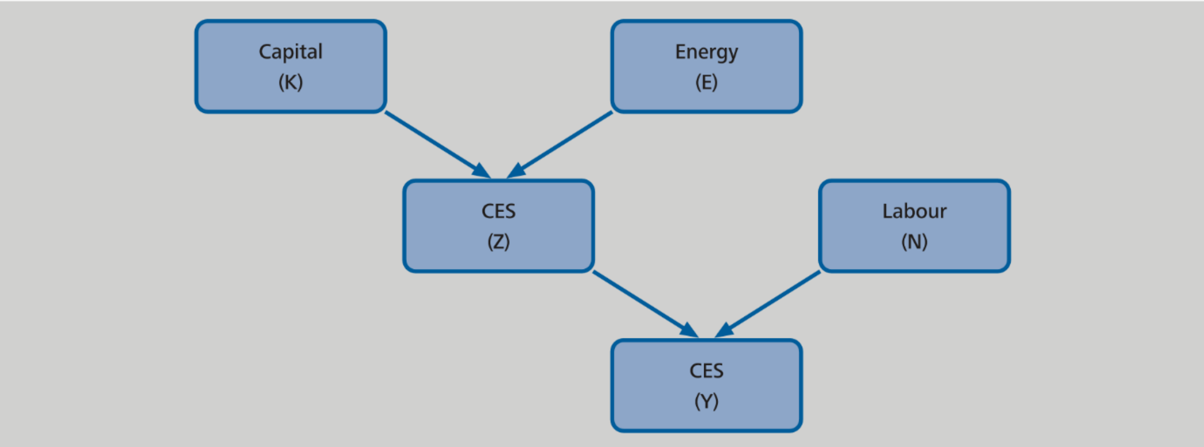
The production function is one of the model's key elements. It determines the core of the supply side in the economy and explains how input factors are transformed into the final good. A specification typically used in macroeconomic models (e.g. in the former version of the Bundesbank model) would be the Cobb-Douglas-type production function that takes two input factors, labour and capital, into account. BbkM-DE refrains from this approach and allows for more flexibility by specifying a constant-elasticity-of-substitution (CES)-type production function. In addition, the production technology is extended to also comprise energy as a third input factor. Increasing the empirical flexibility comes at the cost of higher complexity since this functional form is highly nonlinear.

As opposed to its Cobb-Douglas counterpart, a CES production function allows for an elasticity of substitution between the input factors that can deviate from unity. This implies that the resulting labour share is not restricted to be constant but can vary over time. However, adding

[10] When error-correction mechanisms are modelled in two steps, the fitted residual from the estimation of the long-run equation is used as an explanatory variable (lagged by one period, i.e. ec_{t-1}) in the short-run equation, thus representing the error-correction term.

a third input factor also increases the complexity of the function and requires further assumptions on the particular specification. For example, the specific way in which the input factors are bundled has to be explicitly stated in the functional form. Consequently, the specification that was ultimately chosen represents the outcome of different theoretical, empirical and practical considerations. The following graph illustrates the particular structure of the production function used in BbkM-DE.

Figure 1: Production function in BbkM-DE



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The **production function** has a two-level nested structure.[11] The inner bundle combines the two input factors capital and energy in a CES-type manner. On the upper level, a CES composite connects the inner nest to the third input factor, i.e. labour. Thus, the functional form allows for elasticities of substitution between the components not equal to unity at both levels.

To put it into a formal stylised representation:

$$Y_t = Y_0 \cdot \left(\alpha_0 \cdot \left(e^{g_N (t-t_0)} \cdot \frac{N_t}{N_0} \right)^{\frac{\sigma_{NZ}-1}{\sigma_{NZ}}} + (1-\alpha_0) \cdot \left(\frac{Z_t}{Z_0} \right)^{\frac{\sigma_{NZ}-1}{\sigma_{NZ}}} \right)^{\frac{\sigma_{NZ}}{\sigma_{NZ}-1}}$$

with

$$Z_t = Z_0 \cdot \left(\gamma_0 \cdot \left(\frac{K_t}{K_0} \right)^{\frac{\sigma_{KE}-1}{\sigma_{KE}}} + (1-\gamma_0) \cdot \left(\frac{E_t}{E_0} \right)^{\frac{\sigma_{KE}-1}{\sigma_{KE}}} \right)^{\frac{\sigma_{KE}}{\sigma_{KE}-1}}.$$

Total output (Y_t) is a function of labour (N_t), capital (K_t) and energy input (E_t), where g_N represents a labour-specific technology trend, which is modelled as a quadratic trend for the observation sample and as a linear trend in the forecasting period. Its corresponding parameters are estimated together with the elasticities of substitution between the respective

[11] De Nederlandsche Bank applies a similar approach in its macroeconometric model of the Dutch economy DELFI (see Berben et al. (2018)) but, uses a three-factor two-level CES production function with a (K,N)E structure.

inputs, i.e. σ_{NZ} and σ_{KE}. The production function is specified in a normalised form, as suggested by the literature (see, e.g., Klump et al. (2012)). This necessitates calibrating the parameters that reflect the relative cost share of labour and capital in production (α_0, γ_0) as well as scaling input and output measures by particular values $(N_0, K_0, E_0, Z_0, Y_0)$ before estimation. In practice, the respective sample means are used to determine these parameters and scaling values.

In order to estimate the elasticities of substitution and the parameters governing labour-specific technology, a system approach is used. The system comprises the production function and the first order conditions of the representative optimising firm as proposed by León-Ledesma et al. (2010). To control for cross-correlation of the equation-specific residuals, a seemingly unrelated regression (SUR) estimator is applied.

The production function in BbkM-DE describes the evolution of potential output.[12] For that purpose, labour input represents the potential labour force accompanied by a respective wage measure (W_t). Capital input and user costs of capital are modelled according to Knetsch (2013), who refers to Jorgenson and Griliches (1967) and uses a Törnqvist approximation scheme to build an aggregate series of capital services. First, asset-specific user costs are calculated for five types of fixed assets: dwellings, other buildings and structures, transport equipment, other machinery and equipment, and other fixed assets. In order to compute an aggregate measure of capital services, each asset type is weighted by its share in total user costs.[13] In addition, asset-specific user costs are aggregated over the individual assets to obtain a user costs measure $(P_{K,t})$ as average over all capital goods.[14] Note that capital services and the associated user costs in BbkM-DE encompass both private and government entities. Energy input series are computed using official energy consumption data provided by Arbeitsgemeinschaft Energiebilanzen e.V. (AGEB).[15] The AGEB data are merged with information from input-output tables in line with Knetsch and Molzahn (2012) to calculate the volume measure and a corresponding deflator series $(P_{E,t})$, whereby different energy carriers are taken into account to capture shifts in the underlying energy mix.

Factor demands are derived from the CES production structure as the first-order conditions of the representative firm's optimising behaviour. For **labour input** (scaled and in logs), the condition reads as follows:[16]

$$n_t = y_t - \sigma_{NZ} \cdot \left(w_t - p_{Y,t}\right) + (\sigma_{NZ} - 1) \cdot g_{N,t} \,.$$

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12 The total output series used for the estimation is computed as the sum of gross value added and energy consumption as suggested by Knetsch and Molzahn (2012).

13 See also Deutsche Bundesbank (2012). Since the outcome of this approach is a series expressed as a Divisia index, it has to be rescaled using an appropriate factor in order to convert the index series into volumes chained at previous year prices. This scaling factor is calibrated such that the resulting series matches the historical average of the relative factor share between labour and capital.

14 See also Bursian and Nagengast (2020), who compute country-specific aggregate measures of the user costs of capital based on the most disaggregated level for the ten asset classes available in the EU KLEMS database.

15 See the energy consumption data on the AGEB website.

16 Small letters here indicate the log deviations of the variable from its normalised value, i.e. $x_t = \log(X_t/X_0)$.

For **capital and energy**, optimal demands (scaled and in logs) are given by the following equations:

$$k_t = y_t - \sigma_{NZ} \cdot (p_{Z,t} - p_{Y,t}) - \sigma_{KE} \cdot (p_{K,t} - p_{Z,t}) \,,$$

$$e_t = y_t - \sigma_{NZ} \cdot (p_{Z,t} - p_{Y,t}) - \sigma_{KE} \cdot (p_{E,t} - p_{Z,t}),$$

where $P_{Z,t}$ and $P_{Y,t}$ represent the price measure of the inner and the upper nest, respectively. These equations build the stylised basis of the long-run relationships in the corresponding behavioural equations for the input factors in the model. More details on these aspects can be found in the sections on business investment, the labour market and energy imports, respectively.

Based on German data, the calibrated values of the factor shares α_0 and γ_0 are set to historical averages of about 0.65 and 0.90, respectively. The system estimation procedure yields values for the substitution elasticities significantly below unity. The estimates are approximately 0.15 for σ_{NZ} and around 0.34 for σ_{KE}. However, there is only scarce and mixed evidence in the literature to which this finding can be directly compared.[17] Besides, differences in the estimates have to be assessed with caution as they rely on studies that differ in methodology, specification, data set and the period under consideration.[18]

2.3 Private consumption behaviour

Private consumption represents the most important demand component in Germany. Almost 40% of final demand can be attributed to household consumption expenditure. While it is not surprising that there is a close connection between private consumption and disposable income of households at the aggregate level, the empirical literature considers various additional determinants of private consumption expenditure. Some of them are reflected in the modelling approach of consumption dynamics in Germany presented in this section.

The basic idea of the consumption equation in BbkM-DE draws on Lettau and Ludvigson (2001, 2004), whose approach is also used in the analysis of German data by Hamburg et al. (2008). The specification in BbkM-DE generally follows these studies but also reflects some refinement. The subsequent paragraphs deal with several of these issues.

From a theoretical point of view, the core of the equation is derived from the intertemporal budget constraint of the representative household. According to Campbell and Mankiw (1989), the optimal (log of) consumption-wealth ratio depends positively on the difference between

[17] Kemfert (1998) obtains estimates of 0.85 for σ_{NZ} and 0.65 for σ_{KE} based on West German industry data for the period 1960 to 1993, whereas Van der Werf (2008) finds values close to unity for σ_{NZ} or slightly above one (1.2) for σ_{KE} in case of West Germany within a panel analysis of OECD data covering the years 1978-1996. Knetsch and Molzahn (2012) only consider the substitution elasticity between capital and energy in two specific sectors of the German economy, which they estimate at around values of 0.87 for industry and 0.26 in case of the transportation sector.

[18] Gechert et al. (2021) show, for instance, that the average substitution elasticity between labour and capital found in the literature shrinks from 0.9 to 0.3 once corrected for publication bias, use of aggregated data, and the omission of the first-order condition for capital.

future rates of return on wealth and future consumption growth. Following Lettau and Ludvigson (2001, 2004), by decomposing aggregate wealth into asset holdings and human capital, a corresponding cointegrating relationship can be formulated. Given that human capital is unobservable but represents the present value of future income, they suggest using current income as a proxy. This allows a long-run relationship between consumption, (asset) wealth and (disposable) income to be specified.

The specification takes into account that both wealth and income comprehend components that can yield different marginal propensities to consume (i.e. the additional amount spent on consumption due to a unit increase of the respective determinant, MPCs). Disaggregate wealth effects are well studied in the literature, whereas de Bondt et al. (2019, 2020) point out that this has not been the case for income.[19] Hansen (1996) already noted that consumption behaviour might differ between households that receive income mainly from entrepreneurial activities and assets as compared to other households.[20] The decomposition of disposable income in the consumption equation of BbkM-DE originates from this approach and had already been implemented in MEMMOD. Accordingly, disposable income is split into two components: labour income including monetary transfer payments ($YV_{1,t}$) and other income ($YV_{2,t}$).[21] Disaggregation of wealth in the model follows the literature and differentiates between financial (NGV_t) and housing wealth.[22] In fact, estimation results obtained over the past years indicate that housing wealth does not have a significant impact on private consumption. This may reflect an ambiguous role of housing wealth at the aggregate level, which is also indicated by other empirical studies using German data.[23] A related interpretation is given at the end of this section.

The consumption equation contains additional explanatory variables. A measure ($\tilde{\pi}_t$) accounts for adjustment effects for capital losses/gains on financial wealth due to inflation in the spirit of von Ungern-Sternberg (1981) and Hansen (1996) and similar to effects that can be found in the Bank of Italy quarterly econometric model (BIQM).[24] The real long-term interest rate (RLR_t)

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[19] Wealth effects depend on the households' asset portfolio and therefore can differ due to the households' liquidity and risk preferences or the collateral value of the assets (see also Altissimo et al (2005) or Case et al. (2005)).

[20] Although Hansen (1996) gauges this aspect to be more relevant for short-term consumption behaviour, he estimates cointegrating relationships between consumption and income in Germany, where the latter is decomposed based on the argument mentioned in the main text.

[21] Other income is computed as a residual on the income side of households as disposable income less the sum of net wages and salaries and monetary transfer payments. This measure mainly contains profit and property income from households but also includes net current transfers from abroad and changes in net equity of households in pension funds reserves. De Bondt et al. (2019) distinguish between labour, transfer and property income, for instance.

[22] Housing wealth is computed as the (smoothed) private residential capital stock valued at the housing price described in Section 2.5.

[23] See, for instance, the overview on empirical findings provided by de Bondt et al. (2019).

[24] See Bulligan et al. (2017) for details on BIQM. The German consumption function in MEMMOD also accounted for this effect. The BbkM-DE specification is based on the idea that an adjusted measure of real disposable income (X) should be applied in empirical analyses: $X_t^* = X_t - \kappa \cdot \pi_t \cdot A_{t-1}$, where π_t denotes the perceived inflation rate, A_t corresponds to the real net financial assets and κ is a parameter to be estimated. This expression can also be approximately written as $log(X_t) - \kappa \cdot \pi_t \cdot A_{t-1}/X_t$. For the estimation of the BbkM-DE equation, this expression is split into two parts and the mean of the ratio between assets and income is used. In line with von Ungern-Sternberg (1981), perceived inflation is replaced by a smoothed inflation rate measure.

also enters the equation with an estimated negative semi-elasticity.[25] Thus, the substitution effect dominates the income effect regarding the direct influence of interest rates on consumption, although the net effect is rather limited.[26] Finally, consumption dynamics depend on the unemployment gap ($\widetilde{U}_t$) as measure for expected income losses due to a higher probability of becoming (or remaining) unemployed.[27] The higher the probability of being unemployed, the stronger the dampening effect on consumption expenditures.

Private consumption is modelled in real and per capita terms. Private consumption and its regressors are deflated by the corresponding price index and population (if necessary). This leads to the following equation for **real private consumption** (CPR) per capita in logs:[28]

$$\Delta \widetilde{cpr}_t = \beta_0 - \gamma \cdot \left(\widetilde{cpr}_{t-1} - \left(c_1 \cdot \widetilde{yv}_{1,t-1} + c_2 \cdot \widetilde{yv}_{2,t-1} - c_3 \cdot \tilde{\pi}_{t-1} + c_4 \cdot \widetilde{ngv}_{t-1} \right) \right) - \beta_1 \cdot \widetilde{RLR}_{t-1}$$

$$- \beta_2 \cdot \widetilde{U}_{t-1} + \beta_3 \cdot \Delta \widetilde{yv}_{1,t} + \beta_4 \cdot \Delta \widetilde{yv}_{2,t} + zz_t^{CPR} \,.$$

Estimation results imply a relatively high speed of adjustment to the long-run equilibrium (that is 0.47 as an absolute value). The long-run MPCs are estimated to lie at around 61 cents for labour including monetary transfer income, 51 cents for other income and 1.8 cents for net financial wealth.[29] The estimated values for the short-run MPCs are 64 cents (labour including monetary transfer income) and 63 cents (other income), whereas no wealth effect turned out to be significant in the short run.

When comparing with other empirical studies using German data, de Bondt et al. (2019) detect somewhat higher average MPCs out of income in the long-run (between 79 and 89 cents) and lower values in the short run (averages between 46 and 54 cents) based on a thick modelling approach. The authors do not find significant impacts of either financial or housing wealth on private consumption expenditure. The same holds for the financial and housing wealth effects on consumption in the analysis conducted by Guerrieri and Mendicino (2018). Omitting

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25 The real interest rate is modelled as the long-term government bond yield less inflation expectations proxied by the smoothed growth rate of the private consumption deflator.

26 Survey-based evidence reveals that households' saving and investment behaviour in Germany is notably influenced by preferences in terms of liquidity and risk content of assets (see Deutsche Bundesbank (2015)). It also suggests that, given the persistently high risk aversion, the level of returns on assets will not strongly affect savings decisions even in the current low-interest-rate environment (In an update of the survey the share of households stating that the low interest rates had led them to save less or not save at all increased from 16% in 2014 to 26% in 2016 (see Marek (2017)).

27 The unemployment gap is measured as the difference between the actual unemployment rate and its exponentially smoothed counterpart, where the latter serves as a proxy for the non-accelerating inflation rate of unemployment (NAIRU) in the model.

28 Note that the specification does not restrict the sum of the income and wealth-related coefficients to one in the long run, which allows for consumption, income and wealth not sharing a common steady state growth path. Hence, this is an example of empirical aspects being given a higher weight compared to theoretical considerations in BbkM-DE. In the case of private consumption, this is justified by three arguments: the ambiguous role of housing wealth described at the end of the section, the restriction being rejected by the data when estimating the equation as presented in the main text and the relatively short projection horizon for which the model is used. According to Jansen (2013) the rejection of long-run homogeneity in the data can be explained by revaluation effects in the wealth components such that the standard accumulation equation (i.e. change in wealth equals the difference between income and consumption) does not hold.

29 The MPCs are computed as the estimated long-run elasticities multiplied by 100 and the average ratio of private consumption expenditures to the respective income or wealth measure, over the estimation period.

housing wealth in the German consumption equation therefore seems to be in line with these empirical studies. Note that Geiger et al. (2016) find a significant, but marginally negative influence of house prices on consumer spending in Germany.

The negligible effect of housing wealth on German consumption behaviour could be down to institutional factors such as the structure of the housing market and the credit market architecture as well as the degree of credit and liquidity constraints of households.[30] Only homeowners can use the higher value of a property as collateral. However, Germany has the second lowest homeownership rate of all OECD countries.[31] With a relatively high share of tenants, the higher expected rents due to rising house prices could lead to consumer restraint on aggregate if the reduction in consumption expenditure by tenants exceeds the increase in consumption by homeowners.[32] Furthermore, potential buyers may increase their saving efforts in order to finance the down payment when taking on a loan to purchase real estate. Finally, financing private consumption via mortgage equity withdrawal appears to be much less common in Germany than in Anglo-Saxon countries, for example.[33] Against this background, Geiger et al. (2016) point out that the credit-driven house price and consumption booms experienced in countries such as the UK or the US following their credit market liberalisation did not occur in Germany, where liberalisation has been very modest.

2.4 Business investment and credit rationing

Business investment is one of the core investment variables in BbkM-DE. It is the aggregate of private investment in machinery and equipment, private investment in non-residential construction and other private investment. As opposed to the former version of the model, where these categories were modelled separately, the recent version focuses on business investment as a composite. Modelling the composite appears reasonable from the theoretical perspective, considering the representative firm that wants to extend its production capacities and makes decisions on the sum of expenditure for all three investment categories. A decision to increase expenditure on machinery and equipment may only be expedient when accompanied by a corresponding increase in investment in buildings. From a more practical perspective, business investment as an aggregate is one of the key variables in the focus of the projection process. Besides residential investment (Section 2.5) and government investment (Section 2.9), it constitutes the third (and largest) component of gross fixed capital formation for which projections are provided in the regular forecasting exercise.[34]

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30 See Piazzesi and Schneider (2016) for more details on such factors.

31 See Kaas et al. (2020, 2021).

32 According to the EU Statistics on Income and Living Conditions (EU-SILC), only 50% of German households lived in their own property in 2020. This share is significantly below the shares in other euro area countries, e.g. France (64%), Italy (72%; 2019), Spain (75%) or the Netherlands (69%). See the EU-SILC database on the Eurostat webpage.

33 See Calza et al. (2013).

34 Note that the approach applied to obtain the aggregate capital services series used in the aggregate production function (see Section 2.2) requires more granular asset-specific user costs of capital. These – as well as the respective capital stock series – are therefore specified at a more disaggregate level than the investment categories mentioned. The components of business investment are modelled in a top-down approach: Growth

The behavioural equation for business investment has two main characteristics. It represents the optimising behaviour of the representative firm. Therefore, its long-run relationship is derived from the first-order condition for capital input in the production function as presented in Section 2.2. Second, credit rationing can influence investment in the short run through an accelerator mechanism.

Similar to Berben et al. (2018), the **long-run equation for real business investment** (GBAIR) in logs can be inferred from the first-order condition related to capital services under the assumption of a constant capital-output ratio in the long term:

$$gbair_t = yr_t + \sigma_{NZ} \cdot py_t + (\sigma_{KE} - \sigma_{NZ}) \cdot pz_t - \sigma_{KE} \cdot uc_t + ec_t^{GBAIR},$$

where yr_t and py_t denote (the logs of) real total output and its deflator, respectively.[35] Furthermore, uc_t and pz_t are the associated factor prices of capital and the capital-energy bundle as introduced in Section 2.2.[36]

Turning to the short-run part of the investment equation, the modelling approach chosen for BbkM-DE is a result of an examination of the role of credit dynamics in the context of potential investment drivers in Germany. The subsequent paragraphs shed more light on this aspect and explain the path taken for the specification presented here.

Credit supply restrictions, also known as credit rationing, may be an important factor determining the business investment dynamics. A stylised fact is also that small and medium enterprises (SMEs) have more limited access to credit compared to large corporations. At the height of the financial crisis in 2008 and during the subsequent European debt crisis, potential remedies for the restrictive credit supply were widely discussed. This discussion also invoked the need for modelling the role of credit supply within traditional macroeconometric models. Thus, allowing for credit supply effects was also an aim of the major revision of the Bundesbank's macroeconometric model.

Different approaches for modelling credit supply restrictions are possible. For example, Bayoumi and Melander (2008) model macro-financial linkages via credit supply for the US economy. Their approach also proves feasible within the scope of a macroeconometric model. In practice, they use data for capital asset ratios (CAR) of the banking system to explain the variation in survey data on credit supply restrictions, collected from credit officers. The growth of credit granted to enterprises is explained by bank survey data. GDP expenditure aggregates are also influenced by credit growth, while the current CAR stance is a function of GDP growth,

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of private investment in machinery and equipment is directly linked to that of business investment. Other private investment is defined as the product of its exogenous nominal counterpart and the respective deflator, while private non-residential construction investment is derived as the remaining part of the aggregate. These components feed into the individual capital accumulation equations for the different asset types that are used to construct the capital services.

35 Total output equals gross value added plus the value of energy consumption as suggested by Knetsch and Molzahn (2012).

36 User costs of capital (UC) represent an aggregate measure of asset-specific capital prices as percentage of replacement costs. The price variable associated with the capital-energy bundle, PZ, is computed as the corresponding CES composite of the energy and capital cost measures.

accounting for a feedback loop from the business cycle to the financial system. In principle, this approach could easily be implemented in an eclectic environment of a macroeconometric model. Bulligan et al. (2017) follow a similar approach in the BIQM.[37] Instead of credit growth, they explain lending rates through regulatory bank capital dynamics and default probabilities of enterprises subject to the business cycle. Changing macroeconomic conditions impact banks' balance sheets, thus closing the feedback loop.

However, the implementation of these approaches for the German economy turned out to be problematic. First, German data on regulatory bank capital is not available for broad use due to strict data protection laws. Second, the link between bank credit granted to German enterprises and GDP components weakened substantially after the turn of the millennium. Kuzin and Schobert (2015) explain the divergence between banks' credit creation and NA-based value added by the fact that non-financial corporations in Germany have obviously substituted a considerable amount of bank loans with retained earnings. In economic boom phases, companies were able to fall back on their own funds to finance their investments due to their good earnings situation, whereas they were increasingly forced to borrow in periods of crisis because weak domestic and foreign demand depressed their profits. This implies anti-cyclical lending dynamics. If important credit aggregates lack significant correlation with broad measures of economic activity, the above-mentioned modelling cycle can hardly be implemented based on empirical estimates.[38] Therefore, BbkM-DE follows an alternative approach that employs data from the Bank Lending Survey (BLS).[39]

The BLS data provide a detailed insight into the factors of credit rationing in the euro area.[40] The main advantage over other surveys is the diversity of the questions, which allows a distinction to be made between credit rationing factors that are business cycle related and those that are related to banks' balance sheets. In the framework of macroeconomic models, the bank-side factor can be used as an exogenous variable for the simulation of external financial shocks hitting banks' balance sheets. The business cycle related factor, on the other hand, remains endogenous and provides the feedback loop within the model. Let UBLSK and UBLSB denote business cycle related and bank-side credit restriction measures, respectively. Then UBLS is the overall level of **bank credit tightness for enterprises**, which can be modelled as a function of its components:

$$UBLS_t = \beta_1 \cdot UBLSK_t + \beta_2 \cdot UBLSB_t + zz_t^{UBLS} \, ,$$

and finally augments the **short-run part of the business investment equation:**[41]

$$\Delta gbair_t = \beta_0 - \gamma \cdot ec_{t-1}^{GBAIR} - \beta_1 \cdot UBLS_{t-1} + zz_t^{GBAIR}.$$

37 See Miani et al. (2012) for more details on this approach.

38 Using tools from a wavelet analysis, Scharnagl and Mandler (2019) show that there is only rather limited coherence between loans to non-financial corporations and real GDP in Germany (also in comparison to Spain, Italy and France) at business cycle frequencies.

39 See also: Information about the Bank Lending Survey on the Bundesbank's website.

40 For more information about the BLS and its evolution since the beginning of the Global Financial Crisis, see Deutsche Bundesbank (2016b).

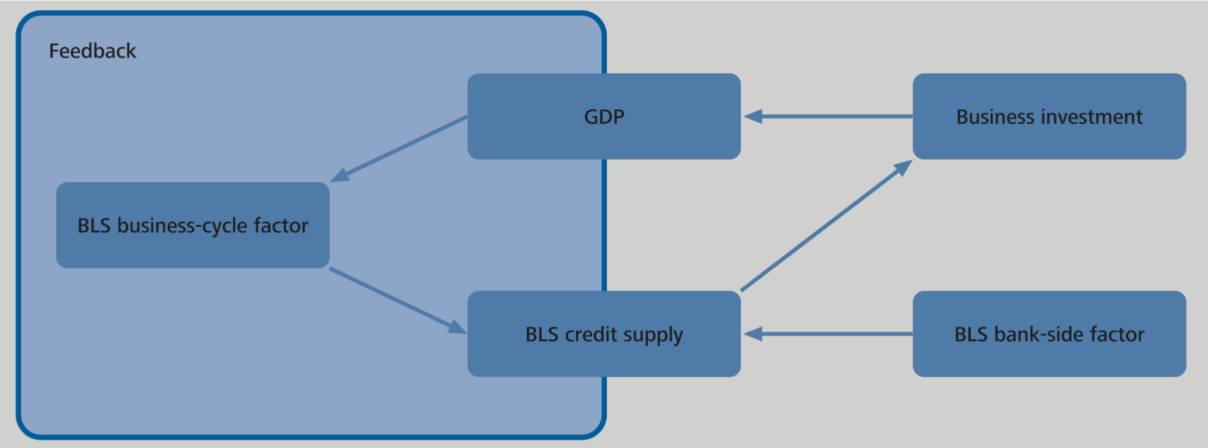
41 BLS data are only available from 2003. In order to allow a longer estimation period, UBLS was extended backwards using information on firm insolvencies.

Increasing bank-side credit rationing (i.e. higher UBLSB), for instance in case of a financial crisis, leads to higher overall credit tightness and hinders investment activities of enterprises. Decreasing investment reduces aggregate economic activity, worsening the business outlook of enterprises. This limits their access to bank credit and thus generates a feedback effect. To close the feedback loop within the model, the **business cycle-related BLS measure** is linked to aggregate economic activity as follows:

$$UBLSK_t = \beta_0 - \sum_{j=0}^{1} \beta_{j+1} \cdot \Delta bipr_{t-j} + zz_t^{UBLSK} ,$$

where BIPR in small letters denotes real GDP in logs. Figure 2 illustrates the accelerator mechanism in the model:

Figure 2: Macro-financial feedback mechanism in BbkM-DE



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However, business investment turned out to be the only series that shows statistically significant correlation with BLS data. Private consumption, house prices and residential investment all fail to exhibit any empirical links with the corresponding BLS series. For this reason, the impact of an isolated financial shock in Germany – under the assumption of no international contagion effects – is rather small. As an illustration: an increase of the bank-side BLS factor (UBLSB), which matches the increase during the financial crisis of 2008, leads to a GDP loss of approximatively only 0.4% compared to the baseline scenario.

2.5 Housing market

Modelling the housing market in Germany is challenging since German unification has had a long-lasting impact. Apartment prices in the cities increased strongly between 1991 and 1995. A remarkable investment boom accompanied this period due to the fact that there had been a strong need to catch up in Eastern Germany, which was additionally fuelled by generous state subsidies for housing.[42] In the subsequent years, a correction to the over-expansion of the construction sector set in, lasting until the middle of the 2000s. The long and persistent cycles make empirical modelling for Germany more cumbersome.

The empirical approach concentrates on the subsequent period from 2003 onwards, which thus also captures the protracted boom period experienced by the real estate markets since 2010.[43] The housing block in BbkM-DE is inspired by a vector error correction model (VECM) of the real estate market, which was introduced in Deutsche Bundesbank (2017).[44] It builds on the theoretical stock-flow model of the housing market as developed by DiPasquale and Wheaton (1994), where both house prices and housing supply can adjust to determine the equilibrium between demand and supply on the residential property market.[45]

The core of BbkM-DE's housing block is a **long-term** relationship between **real house prices** (PWR), real disposable income per household (YVR as real disposable income and NHH as the number of households) and the real interest rate on mortgage loans (RLIWR), estimated in a first step:[46]

$$pwr_t = c_0 + c_1 \cdot (\widetilde{yvr}_t - nhh_t) - c_2 \cdot \widetilde{RLIWR}_t + ec_t^{PWR}.$$

The resulting income and interest rate elasticities (2.8 and -1.7, respectively) are approximately in accordance with the aforementioned VECM.[47]

Growth of real disposable income per household and deviations from the long-run equilibrium lead to price adjustments in the **short run** as estimated in the second step:

$$\Delta pwr_t = \beta_0 - \gamma \cdot ec_{t-1}^{PWR} + \beta_1 \cdot (\Delta\widetilde{yvr}_t - \Delta nhh_t) + zz_t^{PWR},$$

which completes the demand side of the housing block. Note that no particular housing supply variable enters the equations.[48] By decomposing house prices into land and construction

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[42] See Deutsche Bundesbank (2000b).

[43] A detailed description of residential property developments in Germany since 2010 can be found in Deutsche Bundesbank (2020d).

[44] While the VECM is used in the analysis or projection of house prices, the assessment of over or undervaluation on the German housing market is mainly based on a panel estimation of regional house price developments, also taking into account further explanatory factors, such as demographics. For a description of this approach, see Kajuth et al. (2016) or the recent update in Deutsche Bundesbank (2020e).

[45] The modelling approach for both house prices and building permits reflects the economic intuition that the mutual effects of house prices and individual housing demand considerations are independent of the overall number of households in the economy. The volume aggregates (overall number of building permits and real disposable income of households) are therefore included in "per household terms."

[46] Variables are deflated by the price index of private consumption. In line with the definition used in the consumption function, the real interest rate is modelled as the nominal rate from which the smoothed growth rate of the consumption deflator is deducted. Disposable income in the house price equation does not include changes in net equity of households in pension fund reserves.

[47] See also Kajuth (2021), who considers a much longer sample, starting in 1993, and distinguishes between current and expected future income. A simple average of both income elasticities (0.6 and 4.9), however, would yield a similar value, whereas the estimated interest rate elasticity is significantly smaller (-0.8) in comparison with the empirical finding mentioned in the main text. The latter might be explained by the protracted decline in interest rates in the euro area since 2010, which makes up a large part of the sample since 2003, compared to the sample since 1993.

[48] Kajuth (2021) tests productivity in the construction sector in the long run and the change in available building land in the short-run relationship (both as housing supply measures), but does not find a significant impact for

prices, Kajuth (2021) shows that two offsetting effects of a supply expansion are in operation. On the one hand, increasing land supply negatively affects land prices and therefore helps to dampen house prices. On the other hand, construction prices react positively to residential investment and thus put pressure on house prices. The combined price effect of a housing supply expansion through both channels might therefore be weak.

On the supply side of the real estate market, the basic idea is that the level of the housing stock is related to the price level following the setup of the stock-flow model. Therefore, it is price changes rather than the price level that drive residential investment, i.e. the change in stocks, as shown empirically by Kajuth (2021), Lerbs (2014), and Mayer and Somerville (2000).[49] In BbkM-DE, building permits (BAUG) are used as a proxy for changes in the housing stock.[50] Price changes as well as the change in the real interest rate explain the **number of building permits per household**:

$$baug_t - nhh_t = \beta_1 \cdot (baug_{t-1} - nhh_{t-1}) + \beta_2 \cdot \Delta pwr_t - \beta_3 \cdot \Delta R\widetilde{LIW}R_{t-1} + zz_t^{BAUG} .$$

According to the estimation results, price increases indicate higher investment incentives as they lead to a rise in building permits. A reduction of the interest rate on mortgage loans also stimulates the expansion of the housing stock.

Typically, the granting of a building permit and the completion of the investment project do not occur in the same quarter. Available data reveal that completions lag building permits by up to one year. However, expenditure associated with the construction project is already made during the building process. Hence, the transmission period between the application for a building permit and the completion is captured by a moving average term in the model equation of private residential investment. Besides the construction of new buildings, housing investment also consists of investment in existing buildings. The latter is assumed to co-move with real GDP. Therefore, the overall change in **real private residential investment** (IWR) is modelled as a weighted average of the growth rates of building permits and real GDP:

$$\Delta iwr_t = \beta_1 \cdot \left(\sum_{j=0}^{3} \Delta baug_{t-j} / 4 \right) + (1 - \beta_1) \cdot \Delta bipr_t + zz_t^{IWR} .$$

2.6 Labour market

Negotiated wages were selected as a key variable to model wage developments in BbkM-DE in order to take into account the German labour market's institutional characteristics. Although

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either, while residential investment (as another proxy for housing supply) turned out to have a significant but minor effect in the short-run adjustment equation.

[49] Kajuth (2021) compares this approach with a Tobin's Q specification, according to which housing investment would depend on the ratio between house prices and construction costs, and finds more empirical support for the specification in growth rates in German data.

[50] Building permits are used in the regression instead of housing completions as in the original VECM. This is because building permits are available on a monthly basis. As building permits naturally lead housing investment, they help to update the short-term forecast of residential investment during the BMPE process. The series on building permits only refers to the construction of new buildings. The role of construction work in existing buildings as well as the leading property of building permits are taken into account when specifying the model equation for housing investment.

collective bargaining coverage in Germany has steadily decreased from around 85% at the beginning of the 1990s to roughly 60% in recent years according to the OECD (2019), this number is still far above the OECD average. It therefore supports the idea of assigning an important role to negotiated wages in the model.

Furthermore, the Bundesbank maintains a comprehensive and detailed database on negotiated wages in Germany. It covers about half of all employees (i.e. almost 20 million wage earners, salaried employees and civil servants).[51] Source of information are approximately 500 collective wage agreements and regulations on civil servant pay in around 40 sectors. Sector-specific negotiated wages are aggregated across the individual economic sectors to construct monthly index levels for the economy as a whole and for the production sector (including construction). As collectively bargained wage contracts typically have a duration of more than one year, recent wage agreements contain certain information on the future and are useful indicators for forecasting wage developments in Germany.

In BbkM-DE, a behavioural equation is set up for the negotiated monthly wage and salary index level of the overall economy (LTG), converted to an hourly basis per quarter.[52] The equation is derived in the spirit of Blanchard and Katz (1999). It postulates a cointegrating relationship between real wages and labour productivity. Taken in isolation, this would imply a constant labour share in the long run. The specification extends this approach, however, and allows for an episodic violation of this assumption by adding capital intensity (CAPINT) as additional explanatory variable.[53] It therefore captures the fall in the labour share observed over almost two decades before the financial crisis by a trend to substitute capital for labour.[54]

Turning to the short-run dynamics, the wage equation includes measures for labour productivity (LPROD)[55], inflation expectations (INFEXP)[56] and labour market slack (ANSP) similarly to Blanchard and Katz (1999). Labour market tightness is computed as the ratio of the vacancy rate to the unemployment rate.[57] The evolution of negotiated hourly wages is thus described as:[58]

$$\Delta\widetilde{ltg}_t = \beta_0 - \gamma \cdot \left(\widetilde{ltg}_{t-1} - \left(pcp_{t-1} + \widetilde{lprod}_{t-1} - c_1 \cdot \widehat{CAPINT}_{t-1}\right)\right) + \gamma \cdot \Delta\widetilde{lprod}_t + \beta_1 \cdot ANSP_{t-1} + \beta_2 \cdot \widetilde{INFEXP}_{t-1} + zz_t^{LTG}.$$

51 See information on the negotiated pay rate statistics on the Bundesbank's website.

52 To do so, the monthly index level series is divided by the negotiated working time per employee. Note that the series also includes one-off payments.

53 See the suggestion by Pinheiro and Yang (2017).

54 An alternative approach would be to use a measure of the decreasing collective bargaining coverage rate, but this would yield a more off-model explanation of the observed phenomenon.

55 Labour productivity is calculated as the ratio of total output to total working hours, where the latter includes the working hours of the self-employed, who are supposed to have – for reasons of simplification – the same hours per capita as employees.

56 Inflation expectations are proxied by a smoothed growth rate of the private consumption deflator.

57 In order to endogenise the numerator of the indicator for labour market tightness, the model contains an auxiliary equation in which the vacancy rate is tied to labour demand.

58 The equation also laid the ground for a recent analysis in Deutsche Bundesbank (2018a), in which potential dampening effects on wage growth of labour market-oriented immigration, mainly as a consequence of the proceeding EU enlargement in the past decade, were studied.

In line with Blanchard and Katz (1999), the short-run coefficient on labour productivity is restricted to be the EC coefficient with the opposite sign. Loosening this restriction does not result in a significant difference. The estimated value of the coefficient (0.09 as an absolute value) implies a rather slow speed of adjustment of negotiated wages to their expected long-term level.

Since negotiated wages represent the core wage measure in the model, **effective wages** (LGAS) are determined by an EC mechanism, imposing a one-to-one long-run relationship with negotiated hourly wages.[59] The behavioural equation also includes the unemployment gap as a slack measure in order to allow for a cyclical impact in the short-run. The model equation reads as follows:

$$\Delta lgas_t = \beta_0 - \gamma \cdot \left(lgas_{t-1} - \widetilde{ltg}_{t-1}\right) + \sum_{j=0}^{1} \beta_{\mathrm{j}+1} \cdot \widetilde{ltg}_{t-j} - \beta_3 \cdot \widetilde{U}_{t-1} + zz_t^{LGAS} .$$

The approach to modelling wages as described in the paragraphs above can be motivated as a right-to-manage approach. It assumes that employers' and employees' representatives reach consensus after bargaining over wages and that the representative firm then chooses labour input at the negotiated wage rate according to its labour demand function. The latter is derived from the aggregate production function from Section 2.2 and constitutes the core determinant of employment dynamics in the model.

Modelling **labour demand** focusses on employees, while the self-employed are considered as exogenous in BbkM-DE. Since the aggregate production function captures labour input provided both by employees and the self-employed, labour demand, as given in Section 2.2, refers to the broader definition of employment. Therefore, specifying labour demand with respect to the employees would need, in principle, to account for this aspect. This is done by the implicit assumption that labour input in the production function can be expressed as a CES composite combining both working hours by employees and by the self-employed. Additionally, the CES bundle is multiplied by a scaling factor that can be interpreted as efficiency parameter. Based on this approach, the FOC for employees' labour input would add two elements to the labour demand function from Section 2.2: a term reflecting the impact of employees' wages in relation to overall wages and a term measuring the degree of matching efficiency on the labour market. For reasons of simplification and owing to the fact that the self-employed are not modelled explicitly in BbkM-DE, the former element is neglected while the latter is preserved in the following specification. A smoothed measure of the exit rate from unemployment (AGR) augments the long-run relationship of labour demand, describing **working hours by employees** (AVBI) by means of an EC specification:

$$\begin{aligned}\Delta avbi_t = \beta_0 - \gamma \cdot \Big(avbi_{t-1} - \big(yr_t - \sigma_{NZ} \cdot (last_{t-1} - py_{t-1}) + (\sigma_{NZ} - 1) \cdot g_{N,t-1} + c_1 \cdot AGR_{t-1}\big)\Big) \\ + \beta_1 \cdot \Delta yr_t - \beta_2 \cdot \left(\Delta last_t - \Delta py_t - \Delta g_{N,t}\right) + \beta_3 \cdot \Delta avbi_{t-1} + zz_t^{AVBI} ,\end{aligned}$$

[59] Note that LGAS represents gross wage and salary income per hour worked excluding employers' contribution to social security.

where LAST denotes a scaled measure of gross wage and salary income per hour worked including employers' social security contributions, and the short-run part contains the (technology-adjusted) real wage growth rate.

Hours worked per employee (ARST) are modelled in a behavioural equation, imposing that the negotiated working time (including adjustments for part-time work, TA) determines its long-run path. Short-run dynamics are not only driven by the EC mechanism but also by the wage drift and the real GDP growth rate, representing a non-wage related cyclical variable:

$$\Delta arst_t - \Delta ta_t = -\gamma \cdot (arst_{t-1} - ta_{t-1}) + \beta_1 \cdot \Delta bipr_t - \beta_2 \cdot \left(\Delta lgas_t - \Delta \widetilde{ltg}_t\right) + \beta_3 \cdot (\Delta arst_{t-2} - \Delta ta_{t-2}) + zz_t^{ARST} .$$

The number of employees can thus be computed by dividing the total working hours of employees by the hours worked per employee.

Finally, the endogenous labour supply is explicitly accounted for in BbkM-DE in a behavioural equation for the participation rate. In the model context, the participation rate is measured as the share of employees in the total population.[60] It determines the total labour force, which is a key driver of potential output in the model via the production function presented in Section 2.2. Hence, the behavioural equation of the participation rate also acts as transmission channel of its explanatory factors to potential output.

In the long run, the **participation rate of the employees** (EQU) is anchored to the share of the working-age population (WOBA) in the total population (WOBE) in order to control for demographic developments. Additionally, a household's choice to supply labour in the long run depends on the real net per capita wage, i.e. net wages (LN) deflated by the consumption deflator and the number of employees (B1). Further, the participation rate in the total population is bound to increase with a growing share of part-time employees (TZQ), which also affects the short-run dynamics. The two-step EC specification is written as:

$$equ_t = c_0 + (woba_t - wobe_t) + c_1 \cdot tzq_t + c_2 \cdot (ln_t - b1_t - pcp_t) + ec_t^{EQU}$$

and

$$\Delta equ_t = -\gamma \cdot ec_{t-1}^{EQU} + \beta_1 \cdot (\Delta woba_t - \Delta wobe_t) + (1 - \beta_1) \cdot \Delta equ_{t-1} + \beta_2 \cdot \Delta tzq_t + zz_t^{EQU} .$$

2.7 Domestic price setting

Previous versions of the macroeconometric model employed a central price equation as core element of the price block. The latter was constructed to ensure that the development of the central price variable (the deflator of domestic demand) also set the medium-run behaviour of other prices within the model. The current approach presented here differs from the previous

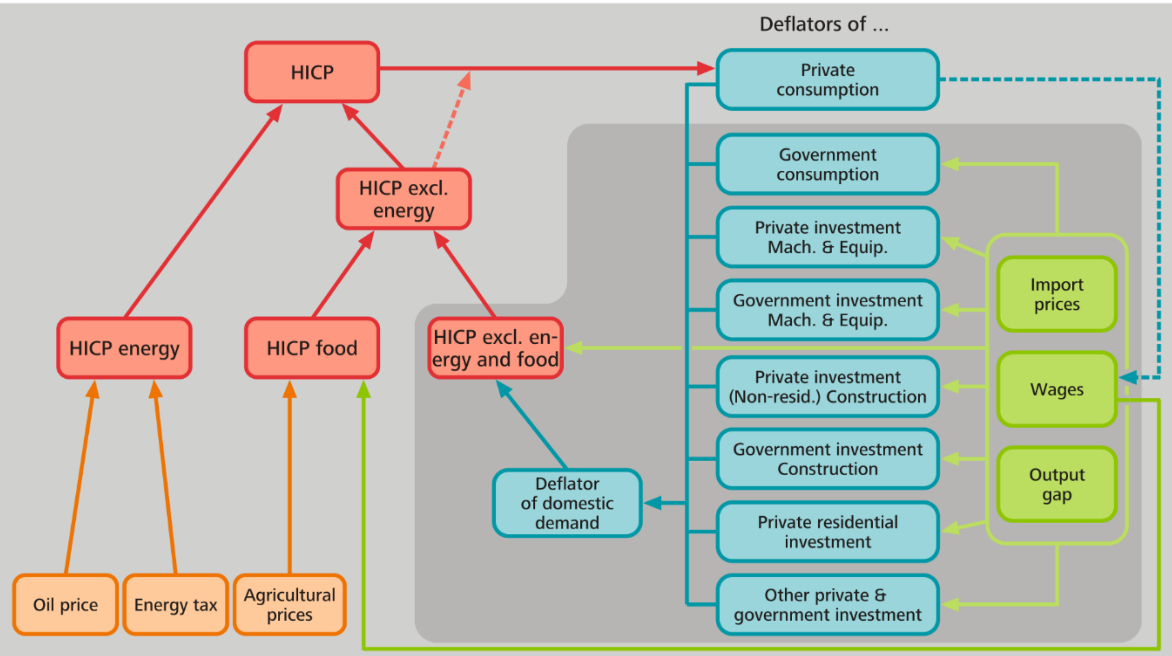
60 Since self-employment takes an exogenous role in BbkM-DE, it is not included in the households' endogenous choice to supply labour.

version in two ways.[61] First, the idea of a central price variable is abandoned. Second, the price equations are no longer estimated as individual equations but in a system. In addition, the aim was to set up a unified framework that brings together both types of price measures: deflators of the GDP expenditure components and those HICP components for which simulation output needs to be frequently produced.

The decentralised modelling approach for the price block in BbkM-DE serves several purposes. First, it helps align the HICP model equations with specifications from satellite models used by the price experts. Second, it warrants a tight connection between the reactions of the private consumption deflator and the HICP in model simulations. Third, it maintains the idea of generating common trend behaviour of the main price variables in BbkM-DE. Fourth, the approach allows for differentiated treatment of deflator- or HICP-component-specific aspects, e.g. regarding the impact of indirect taxes, as will be shown in Section 2.9.

The individual equations of the price block largely contain the same explanatory variables in order to ensure a sufficient degree of homogeneity and thus comparability. The output gap, wages and import prices remain the main determinants of price developments. This ensures that price variables are largely synchronised over the longer term. Individual differences in the dynamics of the respective price variables are taken into account in the estimation of the behavioural equations by means of equation-specific coefficients and potentially by adding additional explanatory variables to the extent that they are relevant for individual price measures. Figure 3 shows the conceptual structure of the price block.

Figure 3: Structure of the domestic price block in BbkM-DE



Deutsche Bundesbank

61 A brief description of this approach can also be found in Deutsche Bundesbank (2019e).

This illustration indicates that behavioural equations are specified for all individual deflators of domestic demand except for the deflator of private consumption. Instead, the HICP core rate (i.e. HICP excluding energy and food) is modelled in accordance with the equation setup for the other deflators. A direct link between the HICP and deflator variables is established by adding the deflator of domestic demand as an additional explanatory variable in the short-run equation for the HICP core variable. A feedback from the HICP to the deflators is, in turn, generated by defining that the private consumption deflator grows in line with the overall HICP. Partial deviations from this one-to-one link are allowed for when energy and non-energy-related consumer prices evolve differently.[62]

The behavioural equations for the **deflators and the core inflation measure** contain the output gap as a measure of the overall degree of economic slack (*GAP*) as well as wages (in terms of nominal unit labour costs, *NULC*) and import prices (P^{im}) in order to capture the direct influence of the cost side on price setting. The equations can thus be interpreted as extended Phillips curves. Ultimately, the basic idea behind this approach is that price variables follow a general price trend that is shaped by conditions on labour and goods markets.

In contrast to the usual modelling of a Phillips curve, the equations are augmented by long-term relationships of each respective price variable with wages and import prices. Depending on the findings, individual equations are supplemented with further regressors and endogenous lags. The behavioural equations therefore display EC models for each relevant price variable $\tilde{P}_t^{(i)}$ (in logs):

$$\Delta\tilde{p}_t^{(i)} = \beta_0^{(i)} - \gamma^{(i)} ec_{t-1}^{(i)} + \beta^{(i)\prime} \boldsymbol{x}_{t-j}^{(i)} + zz_t^{(i)},$$

whereby $ec_t^{(i)}$ represents the EC term.

The full set of explanatory variables is

$$\boldsymbol{x}^{(i)} \subset \left\{GAP, \Delta\widetilde{nulc}, \Delta p^{im}, \Delta\widetilde{pinv}, \Delta R^{LIU}, \mu^{Bau}\right\},$$

whereby the selection and lag structure used differs slightly across the equations and lowercase letters indicate logs. *PINV* is the deflator of domestic demand.[63] R^{LIU} denotes the interest rate on long-term corporate loans.[64] μ^{Bau} corresponds to the inverse of the wage share in the construction sector in order to capture the sector's exceptional profit margins, particularly those observed in the recent past.

[62] This reflects deviations in the movements of the HICP and the deflator of private consumption that are the result of different weights of the energy component in both price measures.

[63] In the BbkM-DE equation, the deflator of domestic demand is tax-adjusted. The use of tax-adjusted price variables in the model is explained at the end of this section. The construction of the adjustment factor for PINV follows the approach for the deflator of final demand as described in footnote 74 in the subsequent section.

[64] The interest rate serves as a proxy for the user costs of capital, which have a positive effect on price developments. The user costs of capital are determined, inter alia, by the (model-exogenous) interest rate for corporate loans and the (model-endogenous) capital goods prices. Capital goods prices, in turn, are explained in the model using the deflators of fixed capital investment, because they are empirically closely related. In order to avoid a circular link when modelling the effects of user costs of capital on the investment deflators, the interest rate enters the respective equations.

The long-term relationship for price variable *i* reads as:

$$ec_t^{(i)} = \tilde{p}_t^{(i)} - \left(c_0^{(i)} + c_1^{(i)} \cdot \widetilde{nulc}_t + c_2^{(i)} \cdot p_t^{im}\right).$$

This specification is applied to the deflators of private investment in machinery and equipment (PIAU), in (non-residential) construction (PIBU), in residential construction (PIW) and in other capital goods (PISU), to the deflator of government consumption (PCS), to the deflators of government investment in machinery and equipment (PIAS), in construction (PIBS) and in other capital goods (PIST) as well as to the HICP excluding energy and food (HVPIXF). The price equations are estimated by applying a two-stage procedure based on Engle and Granger (1987), in which – in addition to the conventional method – in a second step, any dependencies between the system's equations are taken into account using a system approach. First, the long-term relationships are estimated separately for each price variable using OLS.[65] Conditioning on the OLS estimates of the long-term relationships, the overall system is estimated by deploying the seemingly unrelated regressions (SUR) method in order to account for correlations between the residuals of the individual price equations.[66] In most equations, the estimated EC coefficient is rather small, implying a slow adjustment of the price variables to their respective long-term path. In addition, estimated coefficients for the output gap are also low, resulting in a rather small contribution of slack to inflation developments. This is in line with findings presented in Deutsche Bundesbank (2016a) for analyses of the HICP headline and the HICP core rate, testing a number of slack measures.

While the HICP core rate is modelled in line with the approach described above, the specifications of the other two HICP components included in BbkM-DE are set up individually. **HICP food** (HVPIF) is specified as an EC equation, in which hourly wages and EU farm-gate agricultural prices of the European Commission's DG Agri (DGFI) determine the long-term equilibrium:

$$\widetilde{hvpif}_t = c_0 + c_1 \cdot last_t + c_2 \cdot dgfi_t + ec_t^{HVPIF},$$

where the latter also explains the short-run dynamics:

$$\Delta\widetilde{hvpif}_t = \beta_0 - \gamma \cdot ec_{t-1}^{HVPIF} + \beta_1 \cdot \Delta dgfi_t + zz_t^{HVPIF}.$$

The modelling of the **HICP energy component** (HVPIE) allows for non-linear effects of the oil price. For this purpose, the equation is specified in levels and not in log differences. The equation is based on modelling approaches applied by the Bundesbank's price experts, who

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65 The deflator of total imports is used in the equation for HICP core inflation as an indicator of the price effects coming from abroad, whereas the price index of energy imports is used in the equations for the deflators. Moreover, the series for deflators of private and government investment in machinery and equipment are adjusted before the estimation with a correction factor. Both have shown an unusual U-shaped pattern in the past, which can be attributed to a significant (but decreasing) price drop for equipment investment in the category "data processing equipment, electronic and optical products" since 1991. When applying this correction factor, it is assumed that the effect of the above-mentioned negative trend on the overall deflator of equipment investment is temporary and has been slowly phasing out since 2010.

66 Details for individual equations can be found in Annex II.

work with disaggregated monthly HICP series. Under the assumption that all the non-oil components are either much less volatile or depend less on the oil price (such as gas prices), the HICP energy (in level differences) is explained by its own lags and the contemporaneous change in crude oil prices:

$$\Delta\widetilde{HVPIE}_t = \beta_1 \cdot \Delta(POIL_t \cdot ER_t + ENST_t) + \sum_{j=1}^{4} \beta_{j+1} \cdot \Delta\widetilde{HVPIE}_{t-j} + zz_t^{HVPIE} ,$$

where the oil price in US dollar (POIL) is converted into domestic currency via the bilateral euro/US dollar exchange rate (ER), and energy taxes (ENST) charged as a quantity tax in cents per litre are added.

Note that this last aspect is the only case in the model where **indirect taxes** enter explicitly as a regressor in the behavioural equation of a price variable. In the other cases, the role of taxes is accounted for mechanically by constructing a specific correction factor $TIPS_t^{(i)}$ for each price measure. The basic idea is that by multiplying the corresponding price variable with this correction factor, all the relevant tax effects are removed from the original series $P_t^{(i)}$. Individual behavioural equations as described above are specified for the tax-adjusted price measures $\tilde{P}_t^{(i)} \left(= P_t^{(i)} \cdot TIPS_t^{(i)}\right)$. As the original price variable $\left(P_t^{(i)}\right)$ is by definition obtained by dividing the tax-adjusted series by the aforementioned correction factor, indirect tax effects are mechanically included.

The individual **tax correction factors** are defined as follows:

$$TIPS_t^{(i)} = 1 - \tau_{A,t}^{(i)} - \tau_{B,t}^{(i)} ,$$

where $\tau_{A,t}^{(i)}$ captures the respective effects of the value added tax (VAT) and $\tau_{B,t}^{(i)}$ corresponds to a quantity tax component, which either encompasses the influence of energy taxes on the investment deflators or effects of drinks and tobacco-related consumption taxes on HICP food.[67]

2.8 Foreign trade

The foreign trade block of BbkM-DE comprises behavioural equations for real exports of goods and services, for real energy imports and for real non-energy imports of goods of services. A

[67] The VAT component takes into account the individual share of goods that are subject to VAT and also distinguishes between the regular and reduced tax rate in case of the HICP core rate and of the HICP food component. Furthermore, the fact that a change in the VAT rate is not necessarily fully passed on to the consumer is also taken into account. In addition, the degree to which indirect tax changes are passed on to consumers can depend on the economic situation and whether the specific change is temporary or permanent, and can be sector-specific. Since a permanent tax change is to be simulated in the BMEs, the last permanent change in the German VAT rate from 1 January 1 2007 is chosen in the modelling approach as the benchmark for the degree of transmission. Therefore, an additional parameter is introduced that determines the VAT pass-through to HVPIXF. It is calibrated at a value of 5/8, implying roughly the impact to overall HICP as assessed by Deutsche Bundesbank (2008a). According to Deutsche Bundesbank (2020f), the pass-through of the recent temporary reduction in VAT on the core rate and overall HICP is estimated at around 50 to 60%.

separation between energy and non-energy allows the considerably lower price elasticity of energy imports compared to that of non-energy imports to be taken into account. Furthermore, the block includes behavioural equations for the respective deflators as well as some equations for some minor national accounts components that are required to define the net lending position of foreign countries and hence for the closing of the model. BbkM-DE does not consider a regional decomposition of foreign trade. This is done via a top-down approach in a satellite model, which specifies behavioural equations for extra-euro area imports and exports as well as the respective deflators. Definitions set out in the satellite model derive the intra-euro area counterparts.

In BbkM-DE, **real exports of goods and services** are determined by real foreign demand and a measure of price competitiveness of German exports, the real effective exchange rate (RAW). In the long run, real exports are assumed to move in line with real foreign demand, which is defined as Germany's trading partners' weighted import demand. This indicator (WDR) is provided to Eurosystem NCBs by the ECB in the common macroeconomic projection exercises. Its path over the projection horizon combines import projections for extra-euro area countries that are agreed upon by Eurosystem experts in the Working Group on Forecasting as well as euro area countries' import projections of the respective national central banks. Thus, they are one tool to ensure the consistency of euro area countries' trade projections.[68] The model additionally takes into account the fact that the expansion of global value chains (GVCs) is likely to have affected German exports beyond what is captured by the unit elasticity with respect to the traditional foreign demand measure, as described above. The integration of international production processes that has led to rising trade in intermediates is proxied by the ratio of global real imports to global real GDP (GLOBIS) and included as an additional explanatory variable in the model's export equation.[69] Over the projection period, it can be extended by the global import and GDP projections agreed upon by the Eurosystem experts in the Working Group on Forecasting. The equation for German real exports (EXR) is set up in an EC framework as follows:

$$\Delta exr_t = \beta_0 - \gamma \cdot \left(exr_{t-1} - \left(wdr_{t-1} - c_1 \cdot raw_{t-1} + c_2 \cdot \widetilde{globis}_{t-1}\right)\right) + \beta_1 \cdot \Delta wdr_t + zz_t^{EXR} ,$$

where the demand elasticity of real exports is restricted to unity in the long run in order to ensure that exports grow in line with foreign markets over the long term.[70] WDR is model-exogenous, while for simulations, RAW is defined as the (model-exogenous) nominal effective exchange rate multiplied by the ratio of domestic to (model-exogenous) foreign competitors'

68 For information on the construction of the WDR variable, see Hubrich and Karlsson (2010).

69 The ECB Working Group on Global Value Chains (2019) associates the development of the global trade-to income-relationship – in particular the strong rise observed until the global financial crisis – with the growing importance of GVCs. It finds empirical evidence for an impact of GVC participation on the import elasticity to aggregate demand. Global imports are measured in gross terms, while GDP is computed in value-added terms. GVCs tend to increase global import volumes relative to GDP since intermediate goods traded within GVCs are counted several times in world trade, but they should be counted only once from a value-added perspective.

70 If estimated, the implied long-run demand elasticity would be slightly above one.

prices, i.e. the deflator of Germany's final demand divided by an indicator of weighted final demand deflators of Germany's trading partners.[71]

On average over the past three decades, **non-energy imports** have accounted for about 93% of total German imports. Similar to exports, a measure of import demand is constructed and used as the key driver of non-energy import dynamics. Information from input-output tables is used to estimate the import shares of final demand components, which are found to be very heterogeneous. For example, exports and investment in machinery and equipment exhibit import shares of more than 40% in the latest figures, which is twice as large as the import content of private consumption at around 20%. The lowest import shares are found for construction investment at around 15% and for government consumption at below 10%. The import content of inventories is assumed to lie at 50%. The aggregate indicator of import demand (WER) is calculated as the sum of real quantities multiplied by their respective import contents. Real non-energy imports are assumed to develop in line with real import demand in the long run. Therefore, the EC equation that describes the evolution of non-energy imports imposes a unit coefficient as done in the export equation.[72] The long-run relationship of the equation also includes a relative price measure in order to capture the effects of the price differential between imports and domestic production: the deflator of non-energy imports (PNEIM) in relation to the (tax-adjusted) deflator of final demand (PEV). The behavioural equation of real non-energy imports (NEIMR) reads as follows:

$$\Delta neimr_t = \beta_0 - \gamma \cdot \left(neimr_{t-1} - \left(wer_{t-1} - c_1 \cdot (pneim_{t-1} - \widetilde{pev}_{t-1})\right)\right) + \beta_1 \cdot \Delta wer_t + zz_t^{NEIMR} .$$

Note that the import shares give an account of how globalisation and the associated process of international division of labour have proceeded. In the projections, they play a non-negligible role for the projection baseline as the assumptions about their future path notably affect the outlook for imports and therefore the GDP forecast. They are extrapolated over the most recent period based on aggregate information on the ratio of imports to final demand. Beyond the available national accounts data, the individual upward trends of import shares are mechanically extended, assuming, however, that the upward trend will become flatter in the future.

[71] The series on nominal and real effective exchange rates reflect the (nominal or real) external value of the German economy compared to the 37 most important trading partners, which is the weighted average of the individual bilateral components (see Deutsche Bundesbank (2020c)). They thus represent a measure of the German economy's price competitiveness compared to the group of countries under consideration. The corresponding weights are based on Germany's foreign trade with the respective partner country in relation to the total German trade volume with the associated group of countries. Indirect trade flows are also included, which takes into account the fact that countries are not only competitors on their own markets, but also compete with each other in the rest of the word. The computational method corresponds to the procedure of the trade consistency exercise of the ECB for calculating the foreign competitors' prices and the nominal effective exchange rates on the export side during the BMPE process (see Hubrich and Karlsson (2010)).

[72] Note that the indicator is, in principle, related to total imports. It does not differentiate between non-energy and energy import demand as the information used to calculate the import shares encompasses both categories. However, using the indicator as determinant of only non-energy imports seems appropriate given the high share of non-energy imports in total imports.

Energy imports in BbkM-DE refer to oil and gas imports. With energy as the third input factor in the aggregate production function, energy demand seems to be a natural candidate as the key determinant of energy imports. The long-run relationship for **energy demand** (ENERGR) is derived from the optimisation problem of the representative firm operating along the production function given in Section 2.2. The corresponding behavioural equation displays the typical EC pattern:

$$\Delta energr_t = \beta_0 - \gamma \cdot \big(energr_{t-1} - (yr_{t-1} + \sigma_{NZ} \cdot py_{t-1} + (\sigma_{KE} - \sigma_{NZ}) \cdot pz_{t-1} - \sigma_{KE} \cdot pe_{t-1})\big) + \beta_1 \cdot \Delta yr_t - \beta_2 \cdot \Delta pimog_t + zz_t^{ENERGR} ,$$

where the deflator of energy imports (PIMOG) acts as the short-run determinant instead of the energy deflator (PE) due to the lack of statistical significance of the latter in this specification.

Energy demand enters the behavioural equation of **real energy imports** (IMOGR) as a key determinant in the long-run relationship, assuming a one-to-one movement between both variables.[73] Relative prices impact energy imports in the short-run:

$$\Delta imogr_t = \beta_0 - \gamma \cdot (imogr_{t-1} - energr_{t-1}) - \beta_1 \cdot (\Delta pimog_t - \Delta \widetilde{pev}_t) + zz_t^{IMOGR} .$$

The model equation of the **export deflator** (PEX) assumes that firms set prices as a mark-up over marginal costs. The latter are proxied by a weighted average of domestic and foreign price indices, i.e. the (tax-adjusted) deflator of final demand and foreign competitors' prices (PEVF) converted into domestic currency with the nominal effective exchange rate (NAW).[74] The specification additionally takes into account that the exchange rate pass-through on export prices has potentially changed since the mid-90s as a result of the higher import share of German exports. This is captured by an additional indicator (the nominal effective exchange rate multiplied by the trend of the import share of exports, EXIMT) in the long-run relationship for the export deflator, which is encompassed in the following EC specification:

$$\Delta pex_t = \beta_0 - \gamma \cdot \big(pex_{t-1} - (c_1 \cdot \widetilde{pev}_{t-1} + (1 - c_1) \cdot (pevf_{t-1} - naw_{t-1}) - c_2 \cdot EXIMT_{t-1} \cdot naw_{t-1})\big) + \beta_1 \cdot \Delta \widetilde{pev}_t + \beta_2 \cdot (\Delta pevf_t - \Delta naw_t) + \beta_3 \cdot \Delta pex_{t-1} + zz_t^{PEX} .$$

73 The assumption that energy imports react proportionally to a change in energy demand is controversial because it neglects the role of the energy mix. However, given that the model does not treat the energy mix endogenously, the assumption should not be crucial.

74 Note that using a tax-adjusted measure of the deflator of final demand (instead of the original series, PEV) ensures that relevant tax effects do not affect the setting of trade prices. In particular, the correction factor ensures that tax effects other than those originating from energy taxes do not have a first-round impact on the domestic price measure included in the equation of the export and the non-energy import deflator. For that purpose, the correction factor is constructed as follows:

$$TIPS_t = 1 - \sum_{i_1} \omega_t^{[i_1]} \cdot \tau_{A,t}^{[i_1]} - \sum_{i_2} \omega_t^{[i_2]} \cdot \tau_{B,t}^{[i_2]} ,$$

where each individual ω denotes the corresponding nominal shares of the respective category associated with $i_1 \in \{HVPIXF, HVPIF, HVPIE, CS, IW, IST\}$ and $i_2 \in \{HVPIF\}$ in final demand. See also Figure 4 at the end of Section 2.9.

Consistent with the quantity measures, two behavioural equations for import prices are modelled. The **deflator of non-energy imports** mirrors the standard approach of the export deflator but accounts for an explicit role of non-energy commodity prices (PCOM) and adds an indicator that reflects the impact of structural shifts within import demand. Empirical data on its composition show that the associated share of private consumption has steadily decreased (though at a decelerating pace) in Germany since the beginning of the 1990s, while prices of consumer goods have increased in relation to the deflator of final demand. Given the reduction in the consumption share of import demand, the price setting of the representative firm abroad should thus have been less strongly geared towards domestic competitors in the consumer goods sector than is reflected in the domestic price measure, i.e. the deflator of final demand. Therefore, the long-term part of the price equation contains a measure capturing the trend of the share of consumption in import demand (CWER). The final one-step specification reads as:

$$\Delta pneim_t = \beta_0 - \gamma \cdot \big(pneim_{t-1} - (c_1 \cdot (pevf_{t-1} - naw_{t-1}) + c_2 \cdot (pcom_{t-1} + er_{t-1}) + (1 - c_1 - c_2) \cdot \widetilde{pev}_{t-1} + c_3 \cdot CWER_{t-1})\big) + \beta_1 \cdot (\Delta pevf_t - \Delta naw_t) + \beta_2 \cdot \Delta \widetilde{pev}_t + zz_t^{PNEIM} .$$

The growth rate of the **energy import deflator** is modelled as a moving average of the percentage change of the crude oil price denominated in euro over the latest four periods:

$$\Delta pimog_t = \sum_{j=0}^{3} \beta_{j+1} \cdot \big(\Delta poil_{t-j} + \Delta er_{t-j}\big) + zz_t^{PIMOG} .$$

2.9 Fiscal block

Alongside households, the business sector and the rest of the world, the government block represents one of the four sector accounts in BbkM-DE. The government block itself consists of two parts – the expenditure and revenue sides – whose difference yields the net lending or borrowing of the public sector. The general structure of these two parts is explained below.[75]

In the projection exercises, the experts of the Bundesbank’s Public Finances Division develop the forecasts for the relevant fiscal variables in an iterative process that ensures consistency between the fiscal and the overall macroeconomic outlook. At each stage of the process, fiscal projections are integrated into the model via residual adjustment. Nevertheless, a comprehensively specified fiscal block is beneficial as it enables model simulations in order to quantify the impact of fiscal measures on the economy and assess their contribution to the baseline forecast.[76] Furthermore, it also allows simulation exercises outside of the regular projection process.[77]

[75] Please see Annex I for an overview of the government account in BbkM-DE.
[76] For example, see Deutsche Bundesbank (2019f) on the effects of the 2021 extension of the carbon-pricing scheme in Germany.
[77] See Bursian et al. (2020), for instance.

The **expenditure side** of the fiscal block is divided into six categories: government consumption (CS), monetary transfer payments (TRNS), subsidies (SUBV), interest payments by the public sector (ZINS), government investment (IST) and a residual item (SRSS). While the latter four entities make up a rather small proportion of total expenditure, government consumption and monetary transfer payments comprise more than three-quarters and thus represent the most important expenditure components. They are modelled in nominal terms.

The behavioural equation for **government consumption** in BbkM-DE makes use of the fact that compensation of employees is one of its main components in the national accounts.[78] By assuming that public wages are closely linked to the compensation of employees in the private sector, it appears plausible to use the latter as an explanatory variable for government consumption. In fact, there is a tight empirical relationship between the ratio of government consumption to GDP and the labour share. Both series exhibit nonstationary behaviour over the observation period and a cointegrating relationship can be confirmed. This provides the framework for the behavioural equation of government consumption (in relation to nominal potential GDP, $\widetilde{BIP^{POT}}$). Besides the labour share that serves as a determinant in the long-run, the fiscal balance ratio (FS, in relation to potential nominal GDP) affects the government consumption ratio in the short-run. This element ensures a fiscal reaction to previous government budget developments: A higher deficit will result in a reduction of government consumption.

The complete specification reads as follows:

$$\Delta\left(\frac{CS_t}{\widetilde{BIP_t^{Pot}}}\right) = \beta_0 - \gamma \cdot \left(\frac{CS_{t-1}}{\widetilde{BIP_{t-1}^{Pot}}} - c_1 \cdot \widetilde{LSHARE}_{t-1}\right) + \beta_1 \cdot \Delta\left(\frac{CS_{t-4}}{\widetilde{BIP_{t-4}^{Pot}}}\right) + \beta_2 \cdot \Delta\left(\frac{FS_{t-4}}{\widetilde{BIP_{t-4}^{Pot}}}\right) + zz_t^{CS},$$

where $\widetilde{LSHARE}_t$ denotes the measure for the aforementioned labour share and is computed as gross salaries and wages in the overall economy in relation to potential GDP multiplied by the number of employees in the public sector in relation to the overall economy.[79] The lagged government consumption ratio and the fiscal balance ratio enter the equation with a lag of four quarters to account for the fact that fiscal budget decisions for the current year are typically made in the previous one and might focus on a multi-year horizon.

Government monetary transfers are the second largest component on the expenditure side. In BbkM-DE, we directly link the monetary transfers paid by the government to the counterpart of the households' account, i.e. monetary transfers obtained by households (TRN):

$$\Delta trns_t = \beta_1 \cdot \Delta trns_{t-1} + \beta_2 \cdot \Delta trn_t + zz_t^{TRNS}.$$

78 Other main components of government consumption are social transfers in kind and intermediate inputs.

79 As can be seen from Section 2.6, the labour market block in the model does not differentiate between the private and the public sector. Therefore, the conventional labour share in the model refers to wages in the overall economy. Multiplying by the ratio of employees in the public sector and the overall economy should compensate for that to some extent in the empirical specification as it allows for effects on the labour share due to diverging employment dynamics.

Note that both TRN and TRNS do not perfectly match, mainly because TRN is measured net of social contributions on social benefits, motor vehicle tax from households and public charges, and TRN additionally includes social benefits from abroad and from private social insurance schemes. The above specification does not impose a one-to-one relationship: β_2 is estimated significantly below unity but approximately reflects the average ratio of TRN and TRNS. The conceptual differences between both series are not further accounted for in BbkM-DE.

A large part of the **monetary transfer payments received by households** stems from the statutory pension insurance scheme. Pension adjustments are typically oriented to wage developments. In addition, monetary transfers to households encompass child-related payments. These are usually adjusted in the course of the regular assessment of the minimum subsistence level, which is also based on the general wage development. Given these two main components of monetary transfers received by households it is gross wages (LG) per employee (B1, residents' concept) multiplied by the non-working age population (WOBS) that serves as their key determinant in BbkM-DE. Furthermore, they are driven by the unemployment rate in order to capture the role of unemployment benefits in monetary transfers paid to households. This leads to the following long-run relationship estimated in the first step:

$$trn_t = c_0 + c_1 \cdot (lg_t - b1_t + wobs_t) + c_2 \cdot ARLQN_t + ec_t^{TRN} ,$$

where ARLQN denotes the smoothed unemployment rate as the long-term counterpart of the actual unemployment rate (ALRQ).

The resulting residuals are then used in the second step to estimate the short-run dynamics:

$$\Delta trn_t = \beta_0 - \gamma \cdot ec_{t-1}^{TRN} - \beta_1 \cdot \Delta bipr_{t-1} + \beta_2 \cdot \Delta ARLQ_t + zz_t^{TRN} ,$$

where the real GDP growth rate is included to account for additional cyclical effects that are reflected by neither the EC adjustment nor the unemployment rate.

Interest payments by the government are defined as:

$$ZINS_t = \frac{BVS_t}{4} \cdot RZIN_t ,$$

where BVS denotes gross indebtedness of the government and RZIN the **associated interest rate**, which is assumed to evolve as a smoothed average of the long-term government bond yield:

$$RZIN_t = \beta_1 \cdot RZIN_{t-1} + (1 - \beta_1) \cdot \sum_{j=1}^{4} \frac{RL_{t-j}}{4} + zz_t^{RZIN} .$$

Subsidies paid by the German government are modelled to move in line with economic activity as measured by GDP. Overutilisation (underutilisation) of capacities, measured by the output gap (GAP), will lead to lower (higher) growth of subsidies in relation to potential GDP:[80]

$$\frac{SUBV_t}{\widetilde{BIP}_t^{Pot}} = \beta_0 - \beta_1 \cdot GAP_t + zz_t^{SUBV} .$$

Finally, government investment in machinery and equipment (IAS), non-residential construction (IBS) and other fixed assets (ISS) are all treated as exogenous in the model. However, their counterparts in real terms represent endogenous variables due to the modelling of the associated deflators. The remaining residual item (SRSS) on the expenditure side comprises several positions: capital transfers, current transfers other than interest, other production duties paid by the government, net acquisition of non-financial non-produced assets and changes in inventories. Given the broad range of these positions but their minor share in government expenditures, SRSS is also exogenous in BbkM-DE.

The **revenue side** of the government block includes three major positions: social insurance contributions (SOZS), direct taxes (TDIR) and indirect taxes (TBSP). Each of these is further subdivided. As in the case of government expenditure, the revenue components are modelled in nominal terms. Two remaining entities on the revenue side remain exogenous: property income of government (GST) and a residual item (VKSS) collecting capital transfers, current transfers other than interest and subsidies received by the government.

Among the major income positions, **social security contributions** provide the largest contribution to the government account. BbkM-DE distinguishes between (a) social security contributions of employees (SZAF) that drive a wedge between the compensation of employees and gross wages and salaries and (b) social security contributions of employees (SOZN). Deducting SOZN as well as income taxes from gross wages and salaries leaves net wages and salaries. Apart from the social security contributions employers and employees pay to the national social security system, both series also include the amounts paid to private social insurance schemes. The **social contributions that the government receives** (SOZS) do not only consist of those paid by employees and employers but, in addition, contain social security contributions paid by others, such as self-employed persons or benefit recipients. Therefore, SOZS does not perfectly match the sum of SOZN and SZAF.[81] Historical data show an increasing gap in the nineties, which began to reduce after the millennium and has almost vanished since 2010. However, in general, both series move closely in line with one another. The modelling approach therefore guarantees a complete transmission of changes in employers' and employees' social security contributions to the government account in simulations:

$$\Delta sozs_t = \Delta soz_t , \qquad \text{where } SOZ_t = SZAF_t + SOZN_t .$$

[80] As the ratio of subsidies to GDP reveals a strong negative trend for the period between the mid-1990s and 2006, the model equation is only estimated from 2007 onwards.

[81] The different components of SOZS and SOZ, respectively, amount to around 15%.

Social security contributions of employers and employees in BbkM-DE are explained by average gross wages and salaries per employee and an average social security contribution rate (SOZB).[82] SOZB is the sum over contribution rates to the statutory pension insurance scheme, the statutory health insurance scheme, the public long-term care insurance scheme and the Federal Employment Agency. The rates are provided by the Bundesbank's fiscal experts.[83] They are used both for employees' and employers' contributions, because the statutory social security system in Germany aims to distribute the burden equally between both groups, even though the actual contribution rates may differ slightly. The model equations are estimated in the two-stage EC setup.

The **long-run equation** for the **employers' social security contributions** is as follows:

$$szaf_t - b1_t = c_0 + c_1 \cdot \left(lg_t - b1_t + \widetilde{sozb}_t\right) + ec_t^{SZAF} .$$

Apart from the EC term, average wages also explain the corresponding **short-term** dynamics:

$$\Delta szaf_t - \Delta b1_t = -\gamma \cdot ec_{t-1}^{SZAF} + \beta_1 \cdot (\Delta lg_t - \Delta b1_t) + zz_t^{SZAF} .$$

For the modelling of **employees' social security contributions**, a further institutional feature is taken into account. In 2002, a government-subsidised private pension scheme was introduced in Germany ("Riester pension plan") in order to encourage households to make additional savings for their retirement.[84] The privately funded pension was meant to compensate for the decline in the pension level provided under the statutory pension insurance scheme. In the following decade, the number of Riester contracts grew strongly, with a positive effect on employees' social security contributions. To account for this development, the corresponding model equation is augmented by means of variables RIEA, denoting the number of additional funded pensions contracts, and RIES, denoting the contribution rate to these contracts (as percentage of gross wage income).[85] The **long-run relationship** therefore reads as follows:

$$sozn_t - b1_t = c_0 + c_1 \cdot \left(lg_t - b1_t + \widetilde{sozb}_t\right) + c_2 \cdot \left(\frac{LG_t}{B1_t} \cdot \frac{RIEA_t}{B1_t} \cdot RIES_t\right) + ec_t^{SOZN} ,$$

where the second term captures the average Riester contributions paid per employee. Since the Riester-related components take zero values in the beginning part of the estimation sample, they are included in levels rather than in logs.

In the **short-run,** employees' social security contributions adjust to average wage developments, changes in the contribution rate as well as the first stage residuals:

$$\Delta sozn_t - \Delta b1_t = \beta_0 - \gamma \cdot ec_{t-1}^{SOZN} + \beta_1 \cdot (\Delta lg_t - \Delta b1_t) + \beta_2 \cdot \Delta\widetilde{sozb}_t + zz_t^{SOZN} .$$

82 Details such as contribution thresholds are not taken into account.

83 The contribution rate to the statutory health insurance scheme is computed as member-weighted average over all health insurance institutions. In the estimations, SOZB is adjusted with a dummy variable capturing the temporary shift of equal financing by employers and by employees regarding the supplementary contribution to the statutory health insurance scheme between 2005 (Q3) and 2018. A comprehensive analysis of the statutory pension system in Germany based on an OLG model setup can be found in Schön (2020)

84 See Deutsche Bundesbank (2008b) for more information about the "Riester pension".

85 The Federal Ministry of Labour and Social Affairs provides a regular update on the stock of Riester contracts. The minimum contribution rate required to receive public subsidies increased from 1% of gross income in 2002/3 to 4% from 2008 onwards.

Direct tax revenues comprise wage tax (LOST) and other direct tax revenues (TDSO). The **wage tax** is the most important direct tax with a share of more than 50%. In general, there are two methods for an empirical modelling of wage tax revenues. The first method employs an average tax rate, calculated as a ratio of wage tax revenues and the corresponding tax base (gross wages and salaries). In this case, the average tax rate is considered as an exogenous policy variable, which is used to calculate wage tax revenues given the endogenous tax base. Simulation experiments could be performed by comparing scenarios with different average tax rates. In the alternative method, the wage tax revenues are modelled as a function of per capita wages and the number of employees. However, both methods entail similar drawbacks: On the one hand, estimating changes in the average tax rate and appropriately taking into account changes in the tax progression is not a trivial task. On the other hand, calculating elasticities of the tax revenue with respect to per capita wages and employee number requires elaborate empirical analysis.

BbkM-DE specifies the wage tax revenue as a function of per capita wages and number of employees. The elasticities used (ε^{LOST01} for per capita wages and ε^{LOST02} for the number of employees) correspond to the elasticities applied by the "Working Group on tax revenue estimates" at the Federal Ministry of Finance. The wage tax revenue is modelled as follows:

$$\Delta lost_t = (1-\beta_1)\cdot(\beta_0 + \varepsilon^{LOST01}\cdot(\Delta lg_t - \Delta b1_t) + \varepsilon^{LOST02}\cdot \Delta b1_t)\cdot + \beta_1 \cdot \Delta lost_{t-1} + zz_t^{LOST} \ ,$$

where only the regression constant and the smoothing coefficient are estimated.

Total profit of the private sector is the major explanatory variable for **other direct taxes** (TDSO), which comprise, inter alia, assessed taxes on earnings, local business tax, corporation tax and investment income tax. However, free estimation of the long-run relationship between these two variables results in a tax elasticity significantly greater than one, which would imply explosive behaviour of the government income side. For this reason, the tax elasticity of total profits of the private sector (GU) in the long-run part of EC equation is restricted to one.[86] The estimated equation reads as follows:

$$\Delta tdso_t = \beta_0 - \gamma\cdot(tdso_{t-1} - gu_{t-1}) + \beta_1\cdot GAP_t + \beta_2\cdot\Delta tdso_{t-1} + \beta_3\cdot\Delta tdso_{t-8} + \beta_4 \cdot \Delta tdso_{t-11} + \beta_5\cdot GSTRA_{t-4} + zz_t^{TDSO} \ .$$

Including the output gap as an explanatory variable also allows for cyclical adjustments in the revenues. The exogenous variable GSTRA accounts for effects of legal changes in the tax system and is provided by the Bundesbank's fiscal experts. Over the forecasting period, information about announced tax reform changes is thus also captured (if defined in sufficient detail and with a high likelihood of being implemented). The estimation sample of this equation starts from 2004, since the corporate tax reform in Germany, implemented shortly after the turn of the millennium, changed the dynamics of the time series drastically.

[86] Note that the empirical literature is not clear about the long-run elasticities of profit-related taxes in Germany (see e.g. Koester and Priesmeier (2012)) and estimates might also depend on the observation period, which is rather short for the BbkM-DE equation.

Turning to **indirect tax revenues**, the model differentiates between value-added taxes (VAT, model variable: UST), which contributes almost two-thirds, and other indirect taxes (TBSO). The latter is further subdivided: Its most important component, energy tax revenue (TBENST), is treated explicitly in BbkM-DE. In Germany, tax legislation separates electricity from energy tax. The handling of electricity tax in the model is, however, much simpler and is also explained below. The remaining indirect tax income of the government (TBREST) stems from a variety of further indirect taxes.

The modelling of **VAT** takes into account the individual shares to which private consumption (CP), government consumption (CS), residential investment (IW) and government investment (IST) are subject to VAT.[87] The respective shares, $\phi^i \in (0,1)$, are estimated and provided by the Bundesbank's fiscal experts. They are used as weights in the summation over the relevant aggregates in order to obtain the VAT revenues:

$$UST_t = CP_t \cdot \left(\phi^{CP} \cdot \frac{MWST_t}{1+MWST_t} + \phi^{CPR} \cdot \frac{MWSTR_t}{1+MWSTR_t}\right) + CS_t \cdot \phi^{CS} \cdot \frac{MWST_t}{1+MWST_t} + IW_t \cdot \phi^{IW} \cdot \frac{MWST_t}{1+MWST_t} + IST_t \cdot \phi^{IST} \cdot \frac{MWST_t}{1+MWST_t} + \beta_0 - \beta_1 \cdot TREND + zz_t^{UST} .$$

The equation also accounts for the fact that certain consumption goods and services are taxed at a reduced VAT rate (MWSTR), while others are subject to taxation at the regular rate (MWST). However, the above approximation overestimates VAT revenues to some extent. The gap between actual and estimated revenues exhibits a negative, relatively stable linear trend that is captured by an extra term in the equation.

Energy tax revenues arise predominantly from diesel and petrol fuels, while natural gas and heating oil contribute much less. Time series for the average energy tax rates (in euro per litre of crude oil) paid by households (ENST) and firms (ENSTU) are constructed based on disaggregate energy consumption data and energy source-specific regular tax rates from the Federal Statistical Office. Energy tax revenues (TBENST) can then be calculated by multiplying the estimated energy tax rates by the respective consumption path of petroleum-based fuels and natural gas (ENMOEGHH for households and ENMOEGU for firms, in litres of oil respectively). These sector-specific energy consumption measures are directly derived from overall energy consumption in BbkM-DE by using the time-variant shares of the consumers (i.e. households and firms) and energy carriers (i.e. fuel and gas) as calculated with the AGEB data.[88] For the behavioural equation, a constant is estimated in order to correct for the remaining gap between actual energy tax revenue and its approximation:

$$TBENST_t = \beta_0 + ENST_t \cdot ENMOEGHH_t + ENSTU_t \cdot ENMOEGU_t + zz_t^{TBENST} .$$

87 In the case of commercial investment, companies can deduct input taxes. They therefore do not enter the modelling approach to approximate VAT revenues.

88 Normally, the respective shares would be fixed at constant values in simulations. Shifts in the energy mix could nonetheless be simulated via exogenous changes of these values.

After mineral oil and gas, electricity is the third largest contributor to final energy consumption in Germany. It is taxed separately from the aforementioned energy sources and the associated government income amounts to less than one-seventh of energy taxes. Therefore, **electricity tax** revenue is modelled in a simpler way by imposing a proportional relationship to energy taxes.[89] This is done by means of replacing the electricity tax within the definition of **other indirect taxes** by an estimated parameter (β_1) as follows:

$$TBSO_t = TBENST_t \cdot (1 + \beta_1) + TBREST_t + zz_t^{TBSO} ,$$

thus assuming perfect correlation between energy and electricity tax income.

The **residual part of other indirect tax revenue** comprises several different taxes. Since a substantial (although not fully comprehensive) share is attributed to consumption-related (quantity) taxes (e.g. taxes levied on tobacco and alcohol), the remaining indirect tax revenues are modelled as a function of real private consumption. The EC equation imposes a one-to-one relationship in the long run in order to avoid destabilising dynamics:

$$\Delta tbrest_t = \beta_0 - \gamma \cdot (tbrest_{t-1} - cpr_{t-1}) + \beta_1 \cdot \Delta tbrest_{t-3} + zz_t^{TBREST} .$$

Given that most excise taxes subsumed in this group are levied on goods whose prices are part of the HICP food component, an average excise tax rate (VBST) is constructed in order to capture associated price effects in the model framework.[90]

Overall, BbkM-DE contains five indirect tax rates (MWST, MWSTR, ENST, ENSTU and VBST), which induce more than four-fifths of overall indirect tax income of the government. To account for their impact on prices in the model, these tax rates are incorporated into the mechanical correction factors of the relevant price variables (introduced in Sections 2.7 and 2.8). The following graph illustrates the transmission channels of indirect taxes to prices in BbkM-DE.[91]

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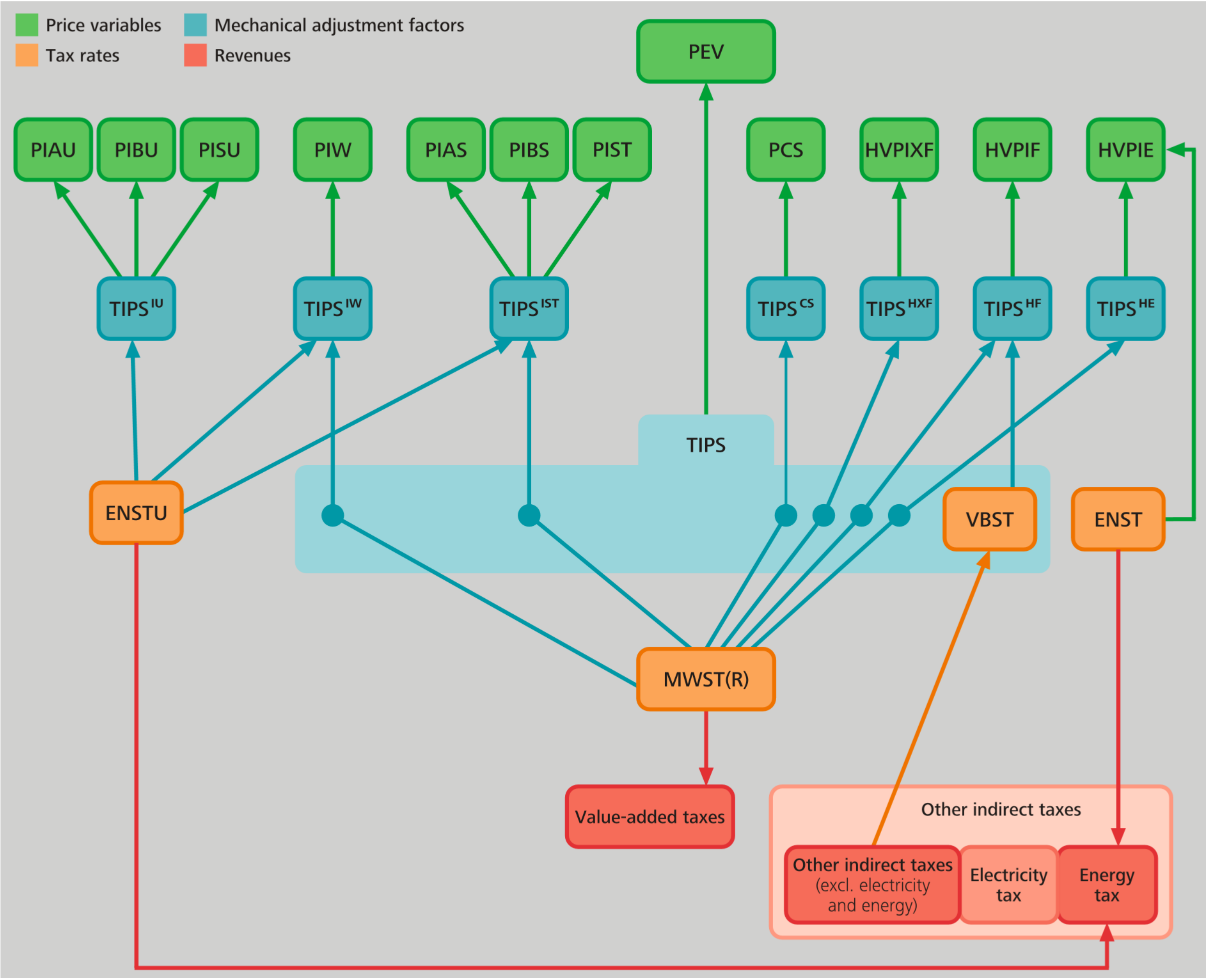
89 The proportionality can be verified in historical data, but should be reconsidered in the context of future energy policy measures.

90 It is calculated as the excise taxes on foodstuff contained in TBREST in relation to real private consumption expenditure that is spent on food. For reasons of simplification, the numerator is computed as constant share of TBREST and the denominator as CPR multiplied by the food weight in the overall HICP.

91 The price variables shown are: deflators of private and government investment in machinery and equipment (PIAU and PIAS), deflators of private investment in non-residential and in residential construction (PIBU and PIW), deflator of government investment in construction (PIBS), deflators of other private and other government investment (PISU and PIST), deflator of final demand (PEV), deflator of government consumption (PCS), and the HICP core measure (HVPIXF) as well as the HICP food (HVPIF) and the HICP energy component (HVPIE). TIPS indicates the corresponding mechanical adjustment factors.

Figure 4: Transmission channels of indirect tax rates in BbkM-DE



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2.10 Financial block

Although the financial block is not unimportant per se, it plays only a minor role in BbkM-DE. There are several reasons for this. First, as the macroeconomic projections are conditional on the Eurosystem's common technical assumptions for interest and exchange rates, key financial variables are exogenised in the projection exercise. Second, the impact of financial market variables on private consumption expenditure via wealth effects is estimated to be considerably smaller in Germany compared to results obtained, for example, in studies for Anglo-Saxon countries. In BbkM-DE's behavioural equation for private consumption, the marginal propensity to consume out of financial wealth is very low as compared to the income determinants. Third, the share of consumer credits in total loans to households is low[92], and they are unlikely to have a notable impact on private consumption. Fourth, the role of housing

92 Only about 12% (on average over the past two decades) of the stock of loans to households are for consumption purposes.

wealth for aggregate consumption behaviour is ambiguous, as explained in Section 2.3). On the other hand, the financing of housing investment via mortgage loans, which constitute the vast majority of loans to households and generally possess a long-term maturity, play a role in the modelling of the housing market, indirectly via lending rates (see Section 2.5). Overall, the modelling of financial market variables in BbkM-DE focusses on the most relevant variables for the transmission of financial market developments to aggregate demand, namely net financial wealth and long-term interest rates.

In line with the national accounts definition, **net lending of households** (FH) corresponds to their disposable income (YV, including adjustment for the change in net equity of households in pension funds reserves) less private consumption expenditures (CP), net investment of households (IP) and net capital transfer payments of households (SVPH):

$$FH_t = YV_t - CP_t - SVPH_t - IP_t \ .$$

Net lending of households in terms of the national accounts – corresponds to its conceptual counterpart in the financial accounts: the difference between the acquisition of financial assets and the external financing of households.[93] Financial flows in the financial accounts are valued at transaction prices. The stock of financial assets is evaluated at market prices (or at least estimated near-market prices). Consequently, a change in stocks from one point in time to another reflects not only the net lending over this period but also valuation changes to the stock of assets held at the beginning of the period considered.

A variable is introduced into the model framework that aims to capture the aforementioned **valuation adjustments** in order to account for the stock-flow discrepancies:

$$NGV_t = NGV_{t-1} + FH_t + BEW_t \ ,$$

where the stock of the households' net financial wealth (NGV) at the end of period *t* is equal to the sum of the stock at the end of the previous period, net lending of households within period *t* and the **valuation effects on net financial wealth of households at market prices** (BEW). This equation can be rearranged to obtain an expression for BEW_t/NGV_{t-1}, which could be interpreted as a rate of return on net financial wealth corrected for the net lending-induced change in stocks. It can be shown empirically that this rate of return has evolved very much in line with the growth rate of the German equity price index CDAX over the past few decades, which is therefore used as one determinant in the behavioural equation that integrates BEW into the model.[94]

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93 For details on the financial accounts, see Deutsche Bundesbank (2019c).

94 Given that the potential impact of interest rates on valuation changes is ambiguous, the interest rate is not included in the model equation. On the one hand, a lower interest rate increases the present value of the future payment obligations of insurance companies, for which they augment their actuarial reserves. The insurance companies report these actuarial reserves as technical reserves, therefore increasing the claims of households against the insurance companies. On the other hand, higher technical reserves of the insurance companies are likely to result in lower profit participation of households (assigned to other claims in the households' financial account). Hence, valuation effects due to interest rate changes may only be shifted within the financial assets of households. Theoretically, a change in the interest rate could also have a negative impact on valuation effects

To stabilise the long-run behaviour of the households' valuation effects on their net financial wealth in the model, the behavioural equation is set up in real terms by using the deflator of private consumption (PCP) as follows:

$$\frac{BEW_t}{NGV_{t-1}} = \beta_0 + \beta_1 \cdot \Delta cdax_t + (1-\beta_1) \cdot \Delta pbip_t + zz_t^{BEW} .$$

CDAX is the main driver of the valuation effects on net financial wealth. Its long-term behaviour can be loosely derived from the typical relationship between stock prices, dividends and the interest rate as postulated in the Gordon growth model. Under the simplifying assumptions that the risk premium and payout ratio are constant, the (log of) firms' profits (GU) and the real long-term interest rate (RLR) determine (the log of) CDAX in the long run as follows:[95]

$$cdax_t = c_1 \cdot gu_t - c_2 \cdot \widetilde{RLR}_t + ec_t^{CDAX} .$$

Apart from the EC term, the changes in the real short-term interest rate (RSR) and in the output gap (GAP) determine the short-run dynamics of CDAX. Additionally, the equation is augmented with changes in the growth rate of listed companies (WRAG).[96] The variable aims to capture the period of extraordinary stock price movements during the "new economy boom" in the late 1990s and its aftermath. It was hallmarked by accelerating growth in the number of listed companies, followed by a deceleration in the subsequent years.[97] The resulting specification of the stock price equation reads as:

$$\Delta cdax_t = -\gamma \cdot ec_{t-1}^{CDAX} - \beta_1 \cdot \Delta \widetilde{RSR}_{t-1} + \beta_2 \cdot \Delta GAP_{t-1} + \beta_3 \cdot \Delta WRAG_t + zz_t^{CDAX} .$$

Besides the short-term nominal interest rate (RS, represented by the 3-month Euribor), and the long-term nominal interest rate (RL, represented by the yield on government bonds with a residual maturity of nine to ten years), BbkM-DE contains the **bank interest rate on long-term loans to non-financial corporations** (RLIU) and the **interest rate on long-term household mortgage loans** (RLIW). Both determine asset-specific user costs of capital, and the interest rate on mortgage loans is also a key determinant in the housing market block (see Section 2.5).

via debt securities held by the households. Given their low share in the asset portfolio of households in Germany (6% on average since 1991), this can, however, also be neglected.

95 Presuming a constant expected dividend growth rate, *g*, in the Gordon growth model, the stock price equals $D \cdot (1+g)/(r-g)$, where the expected rate of return, *r*, can be written as the sum of a riskless interest rate and the risk premium and the dividend, *D*, can be specified as proportion of the firm's earnings (see, e.g., Deutsche Bundesbank (2003b)). Note that Shiller (2007) suggests using the real interest rate rather than the nominal rate when assuming a constant dividend growth rate in the Gordon growth model framework. The equation is estimated via dynamic OLS and includes no leads and up to one lagged first differences based on the statistical information criterion. For simplification, the contemporaneous and the lagged log difference of CDAX that are part of the exact specification are not shown in the equation above. A regression constant turned out to be insignificant.

96 The original series on the growth rate of listed firms reveals a seasonal pattern, particularly in the boom period, indicating some kind of end-of-year rally. Before including it in the regression, an exponential filter is deployed to smooth the series appropriately.

97 See Deutsche Bundesbank (2003a).

Based on interest rate pass-through models estimated using monthly data,[98] the experts from the Bundesbank's Monetary Policy and Analysis Division provide forecasts for these two bank interest rates as part of the projection exercises. These are directly fed into the model as exogenous variables. The monetary policy experts compute reactions of both rates to the shocks on RS and RL, also for the simulation of the basic model elasticities (see Section 4).

To optionally provide endogenous reactions of RLIU and RLIW to changes in the Euribor and the yield of long-term government bond within BbkM-DE, behavioural equations for both bank interest rates were integrated into the model. Both equations use the model's short-term policy rate and the long-term government bond yield as reference rates. The equations are specified such that the impulse responses closely mirror the results provided by the monetary policy experts for the BME simulations. The behavioural equations for the bank rates on loans to non-financial corporations and on loans to households for house purchase in EC form are:

$$\Delta RLIU_t = \beta_0 - \gamma \cdot \left(RLIU_{t-1} - (c_1 \cdot RL_{t-1} + c_2 \cdot RS_{t-1})\right) + \beta_1 \cdot \Delta RL_t + \beta_2 \cdot \Delta RS_t + zz_t^{RLIU}$$

and

$$\Delta RLIW_t = \beta_0 - \gamma \cdot \left(RLIW_{t-1} - (c_1 \cdot RL_{t-1} + c_2 \cdot RS_{t-1})\right) + \beta_1 \cdot \Delta RL_t + \beta_2 \cdot \Delta RL_{t-1} + \beta_3 \cdot \Delta RS_t + zz_t^{RLIW} .$$

Finally, the pass-through of the policy rate to the **long-term government bond yield** is also modelled as an EC mechanism, restricting the long-term yield to move in line with the sum of the policy rate and a term premium (TPREM0) in the long run:[99]

$$\Delta RL_t = -\gamma \cdot \left(RL_{t-1} - (RS_{t-1} + TPREM0)\right) + \beta_1 \cdot \Delta RS_t + \beta_2 \cdot \Delta RS_{t-1} + \beta_3 \cdot \Delta RL_{t-1} + zz_t^{RL} .$$

[98] The bank rates on loans are each explained in EC type equations by two reference interest rates, i.e. a short-term and a long-term interest rate. The estimation is based on monthly data from 2003 onwards, and the reference interest rates may differ depending on the individual bank rate. These two aspects makes it difficult to integrate the equations directly into BbkM-DE. For an application of this approach with focus on the euro area, see Deutsche Bundesbank (2019b).

[99] Currently, the term premium is constant and calculated as the average difference between the long-term government bond yield and the policy rate over the estimation period. In general, it could also be endogenised appropriately.

3 Growth decomposition of key variables

The aim of this chapter is to familiarise the reader with the concept of growth decomposition and provide additional insights into BbkM-DE and its application. To avoid overburdening this documentation with a decomposition for each behavioural equation, the chapter concentrates on four key model variables (exports, negotiated wages, HICP inflation, private consumption) that play an outstanding role for macroeconomic developments. The extent to which off-model information drives their individual forecast in the projection exercise varies a great deal. This also has an influence on how equations are specified and which explanatory variables are selected. Hence, BbkM-DE's quantitative findings, e.g. its growth contributions, should be assessed against this background.

Growth decompositions of BbkM-DE's endogenous variables can be obtained not only for the historical observation sample but are – in the context of the projections – also extended to the forecast period. They provide helpful information when assessing the development of certain variables at the current juncture and are often consulted in the projection exercise where they help derive the narrative underlying the baseline projection and reveal the driving forces.

In technical terms, an explanatory variable's contribution to the growth of the selected variable is the difference between the actual growth of the selected variable and its hypothetical growth if the explanatory variable had been constant over the entire observation period. This computation is performed for all explanatory variables as well as the residuals. Adding up all the individual contributions then yields (approximately) the growth rate of the considered variable.

As the BbkM-DE equations typically specify the dependent variable in first log difference, it seems natural to look at the quarter-on-quarter growth rates, which is usually the case in the projection exercise. However, the resulting graphs can reveal a relatively volatile picture. Instead, the following growth decompositions focus on year-on-year rates, which provide a smoother profile, permitting a clearer illustration of the underlying drivers. The growth decompositions presented below cover the same observation period for all four variables.[100]

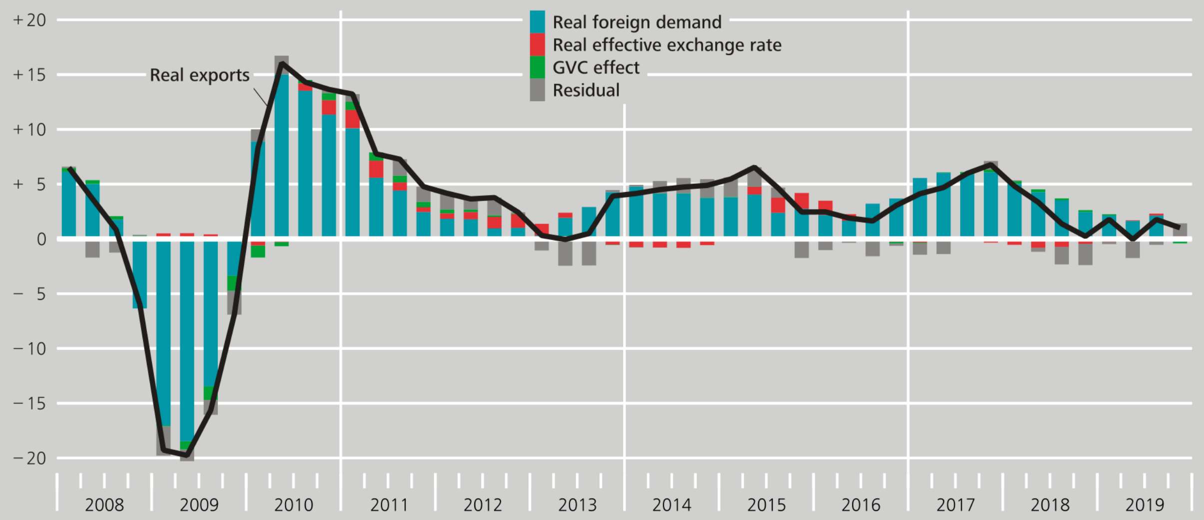
Real exports represent an important variable for German GDP growth, given the large share of export-oriented firms, as also reflected in exports' substantial share in final demand. Hence, they play a central role in the projection exercise. Although BbkM-DE's export equation can be considered a relatively simple specification, it seems reasonable and appropriate as the growth decomposition shows a rather high degree of explanatory power. In the past decades, German export dynamics have been very much in line with foreign demand, which is part of the set of external assumptions that are derived from the import projections of Eurosystem's experts for

[100] Note that this period does not correspond to the respective estimation sample, which is longer in all cases, but would be too granular to serve as an illustrative example here. Therefore, the period shown here is a compromise, and is at least long enough to include the 2008 financial crisis. The most recent quarters are left out as these were likewise not included in the estimation. Only the final part of this chapter refers to the quarters beyond 2019 Q4 as it specifically addresses pandemic-related issues for private consumption.

Germany's trading partners. Therefore, the forecast of German export growth is mainly determined by exogenous information.

Figure 5: Contributions to real export growth

Year-on-year rate in %



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While even the huge drop in German exports during the financial crisis can be largely traced back to the slump in import demand by Germany's trading partners, price competiveness contributed to export growth only in certain periods and to a much lower extent. The rising internationalisation of production processes contributed fairly continuously to real export growth over the earlier part of the estimation period from the second half of the 1990s up until the Global Financial Crisis, most of which is not shown in Figure 5. Some impact from the GVC measure is visible, however, in the aftermath of the crisis: negative initially as global trade plummeted and then positive in the recovery phase afterwards. There were also a few periods in which German export growth was overestimated or underestimated based on the BbkM-DE equation. In particular in 2013 and in 2018/2019, when export growth on a year-on-year basis turned (almost) negative, additional factors not captured in the aggregate BbkM-DE specification came into play, mirroring underlying structural and compositional effects. Germany exports a relatively high share of investment goods. Hence, when global demand for investment goods was relatively weak in 2012/2013, German exports were dampened beyond what could be captured by its trading partners' aggregate import development. In particular, exports to China decelerated due to China's first realignment efforts towards a more consumption-oriented economy.[101] In the second half of 2018, German car producers faced supply delays resulting from their difficulties with the implementation of the new emissions test, the Worldwide Harmonised Light Vehicle Test Procedure (WLTP).[102] The weak second quarter

[101] See Deutsche Bundesbank (2013). Note that the fact the German manufactures have shifted more of their car production to China can additionally explain the weaker German export performance but this should be reflected in the foreign demand indicator used in the export equation.

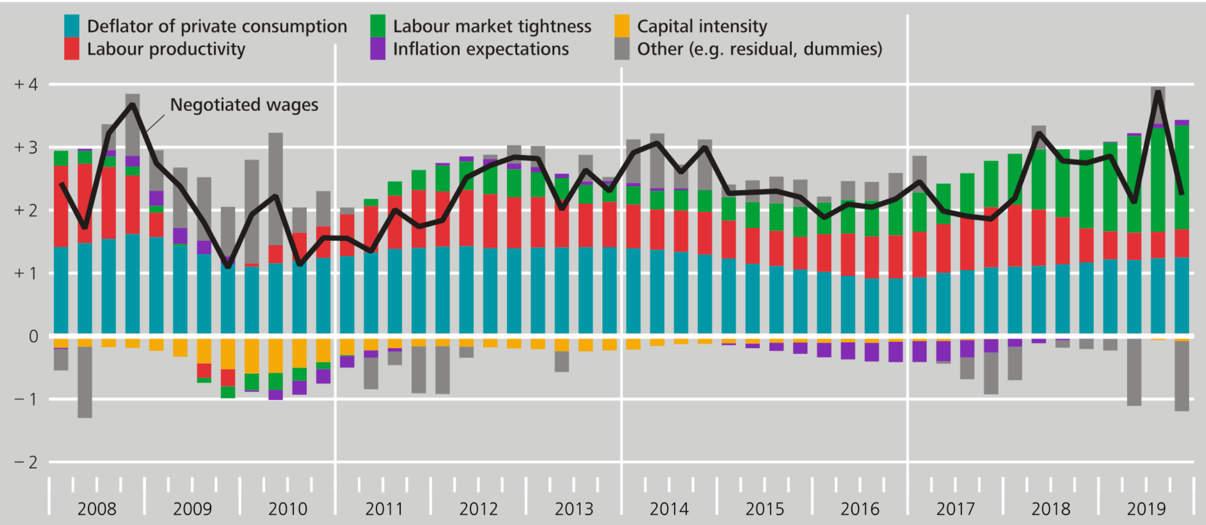
[102] See Deutsche Bundesbank (2018c).

of 2019 may also reflect a one-off factor as German product purchases in the UK were brought forward in advance of the original Brexit date at the end of March 2019.[103]

Negotiated wages represent the central wage measure in the projection exercise. Their forecast is largely based on the information stemming from the Bundesbank's negotiated pay rate statistics. Projections of negotiated wages factor in all past pay agreements included in this database. These agreements are extrapolated beyond their contractual term by taking into account the overall economic situation and industry-specific features but also available information on labour unions' wage claims and counter offers by the employers' associations. The BbkM-DE equation offers an aggregate perspective and helps interlink the wage growth forecast with the outlook for real activity and inflation in the projection exercise. This perspective is illustrated in the growth decomposition in Figure 6.[104]

Figure 6: Contributions to the growth of negotiated wages

Year-on-year rate in %



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Turning to the more general picture, several findings can be mentioned. Actual inflation contributed the most, and in a relatively stable manner, to negotiated wage growth over the observation period. Labour productivity represented the second most important driver before it was outpaced by the contribution of labour market tightness, whose impact on wage growth increased over the recent past due to growing pressures from the resilient performance of the German labour market.

103 See Deutsche Bundesbank (2019d).

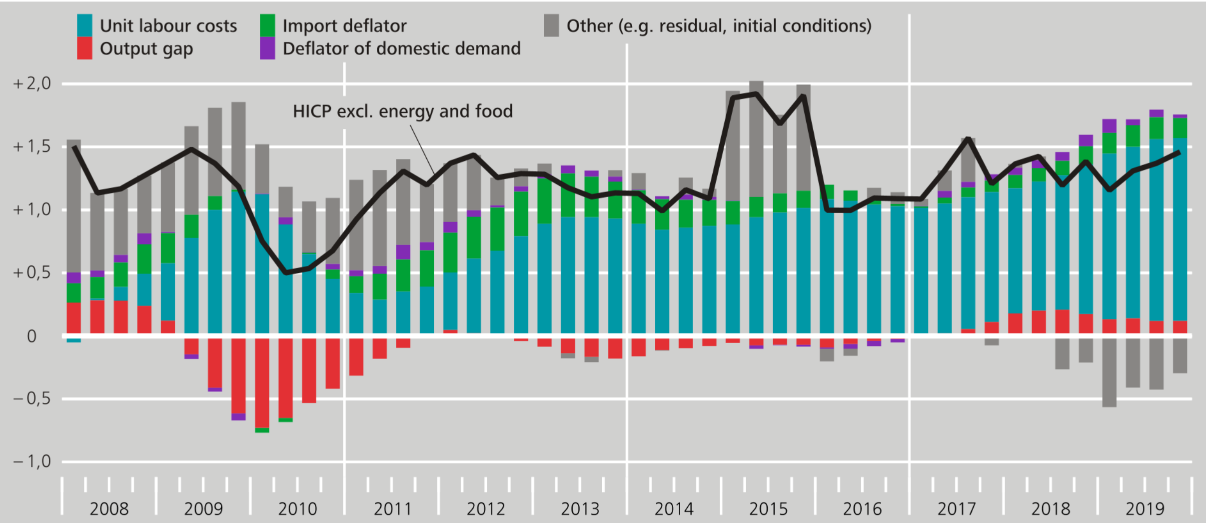
104 Note that not all individual contributions are shown explicitly, but some are combined to provide a clearer picture. The time series on negotiated wages exhibits several outlier periods for which impulse dummy variables are included in the BbkM-DE equation. These reflect extra payments in specific branches in a particular month and can have a strong impact on the profile of the series. For instance, the third quarter of 2019 was affected by new collective special payments in the metal-working and electrical engineering industry in July 2019. These led to an industry-specific annual growth rate in that month of more than 20% if taken in isolation.

The inflation outlook is a key element of the Eurosystem's projection exercise. For the Eurosystem's Narrow Inflation Projection Exercise (NIPE), NCBs provide forecasts over a horizon of 11 months for overall HICP and fourteen components (non-energy industrial goods, services, energy and (optionally) seven subcomponents, unprocessed food and processed food, tobacco, rents, administered prices).[105] The Bundesbank's price experts operate with an even more disaggregate approach of more than 20 HICP components at a monthly frequency, whereas the BbkM-DE equations rely on quarterly data and differentiate only between three HICP components. While the NIPE forecast is explicitly used for the short-term outlook, in the bi-annual BMPEs, the inflation projection beyond the first year represents a combined perspective of both NIPE models and BbkM-DE.[106] To do so, inflation forecasts for the second and third year from the NIPE tools are cross-checked with the information from BbkM-DE that takes into account interdependencies between the nominal and real side in a broad macro outlook.

The growth decomposition based on the BbkM-DE equation for **HICP excluding energy and food** that is shown in Figure 7 reveals that unit labour costs have been the dominant determining factor over the past two decades. The contribution of the output gap becomes visible but remains modest. Even in recent years with a relatively high positive output gap, its impact remained fairly muted according to the BbkM-DE equation. Import prices contributed particularly strongly to core inflation during their recovery after the financial crisis.

Figure 7: Contributions to growth of HICP excl. energy and food

Year-on-year rate in %



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105 See European Central Bank (2016) for more organisational details about the projection exercise in the Eurosystem.

106 In the bi-annual Macroeconomic Projection Exercises (MPEs), the medium-term price outlook is derived by ECB experts.

The relatively large contribution stemming from other factors (grey bars) are of a technical nature that requires further explanation. The grey bars not only contain the residuals' contribution, i.e. the part that cannot be explained by all the variables included in the equation, but also comprise the effects of the initial conditions. The latter are inherent in the way of computing the growth contributions and are reflected in a (positive) deviation between the observed growth rate of the dependent variable and the sum of all contributions.[107] Usually, the initial conditions do not become apparent, because they phase out quickly. However, due to the very low speed of the adjustment coefficient in BbkM-DE's behavioural equation for core inflation, the initial conditions account for more than half of the grey-marked contributions in the first years shown. From 2012 onwards, they counterbalance the pure residuals' contribution, but their effect is decreasing further and only at a few hundredth percentage points by the end of the observation period. Therefore, to some degree, the grey bars conceal parts of the model equation's overestimation of the core rate in the last decade. Note that the stepwise pattern in 2015 is largely due to a one-off statistical effect in the underlying time series.[108]

Real private consumption expenditures are key for the overall macroeconomic outlook given their large share in German GDP. In the projection exercise, their forecast is neither heavily dependent on exogenous assumptions (such as real exports) nor strongly based on satellite approaches (such as negotiated wages and core HICP). Instead, private consumption represents a highly endogenous variable in BbkM-DE.

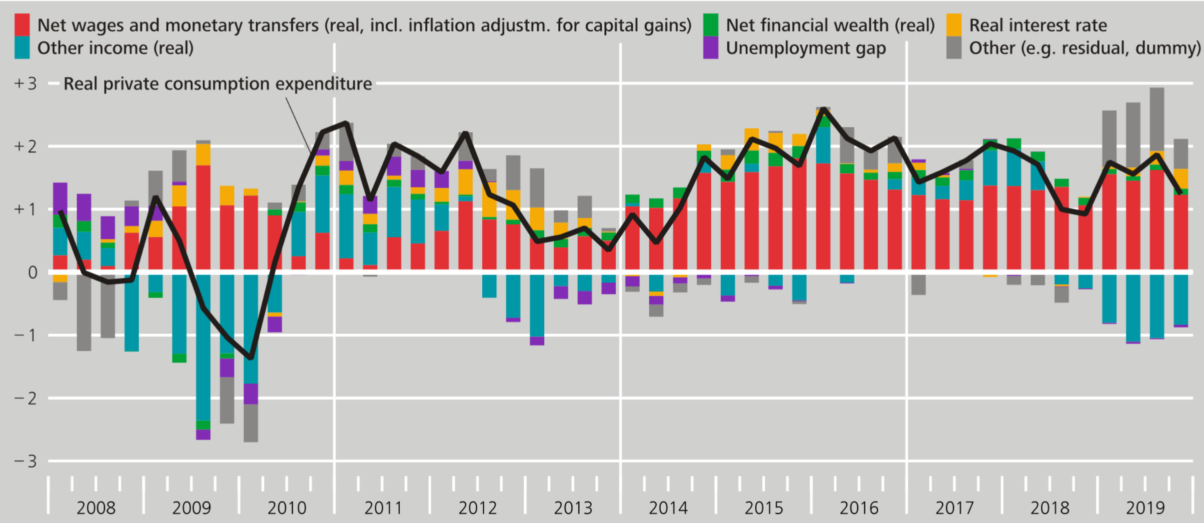
Decomposing the growth rate of real private consumption based on the BbkM-DE equation (see Figure 8) discloses what has already been indicated by the respective MPCs described in Section 2.3. Labour income was the main driver over the selected observation period with some exception during the 2008 financial crisis and its aftermath. While other income pushed down private consumption during the crisis, it also underpinned its recovery afterwards. By contrast, financial wealth and the real interest rate contributed much less to the overall private consumption dynamics. In the latter case, the continuous positive contribution shows some impact of the negative trend in the interest rate development, however.

[107] Note that the initial conditions do not necessarily reflect parts that cannot be explained by the other variables as it is the case for the residuals. Instead, they represent shares that cannot be associated specifically with the determinants in the equation, neither the variables nor the residuals, as a consequence of this numerical procedure.

[108] The 2019 revision of the sub-index for package holidays by the German Federal Statistical Office back to 2015 in combination with the chain-linking method used resulted in distorted year-on-year rates for headline and core HICP rates in 2015; see Deutsche Bundesbank (2019a).

Figure 8: Contributions to real private consumption growth

Year-on-year rate in %



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Model-based growth contributions such as those presented above have proven helpful when assessing drivers of economic developments in the past and at the current juncture as well as for developing a projection baseline. The findings focus on the period before the outbreak of the recent COVID-19 pandemic, which had an extraordinary impact on economic activity in the recent quarters. Private consumption expenditure plummeted in particular, mainly driven by the exceptional nature of the pandemic-related containment measures. Decomposing growth in private consumption in 2020 therefore yields huge residual contributions, as these developments cannot be explained by standard macro modelling approaches such as the BbkM-DE consumption equation. The following box describes some of the steps undertaken to deal with COVID-19 in the projections – with a particular focus on private consumption.

Box: COVID-19-related challenges in recent projection exercises

The COVID-19 pandemic and its large, persistent, and pervasive economic consequences pose challenges to all macroeconometric models that rely on empirical estimates of historic relationships. There is no simple solution to these challenges, as we have seen only two pandemics in the post-World War II era: the “Asian flu” in 1957-58 and the “Hong Kong flu” in 1968-69 (no lockdowns were enforced, though). Nevertheless, models can prove helpful for structuring the economic analysis. In the June 2020 projection exercise, assessing the size of the slump in economic activity in the second quarter was a pressing task.[109] Valuable information for the development of the projection baseline was derived from a comparison of the pre-pandemic December 2019 projection with a number of BbkM-DE-simulation outcomes that captured the overall impact of the individual shocks hitting the Germany economy simultaneously. Besides the substantial slump in private consumption expenditure because of

[109] See Deutsche Bundesbank (2020a) for details on the June 2020 outlook for Germany.

the lockdown, these shocks included the collapse in foreign demand, investment restraint as a result of higher uncertainty and, lastly, fiscal measures taken in order to ease the impact of the pandemic on households and firms. The size of the individual shocks had to be calibrated using off-model assessments and information. While, for instance, the decline in export markets was the outcome of projections for the international environment by Eurosystem experts, the potential drop in consumption was calibrated using short-term indicators and assessments of activity at a sectoral level.[110]

A year later, in the June 2021 projection exercise, national accounts data for five quarters that were affected by COVID-19 were available.[111] While the impact of the pandemic on real disposable income had remained very contained because of strong government support for employees and firms, consumption had declined sharply and had only recovered somewhat in quarters in which containment measures had been eased to a certain extent. Households had therefore accumulated a large volume of savings since the beginning of 2020. BbkM-DE's behavioural equation was not able to capture the explanatory factors and most of the observed slump in private consumption was attributed to the contribution of negative residuals. In order to develop an informed projection for private consumption it was, however, valuable to assess the motives behind the large accumulation of savings: Was this merely enforced due to the lack of consumption opportunities, i.e. constrained supply, or to what extent did households also refrain from spending for precautionary reasons, thus reducing their demand voluntarily?

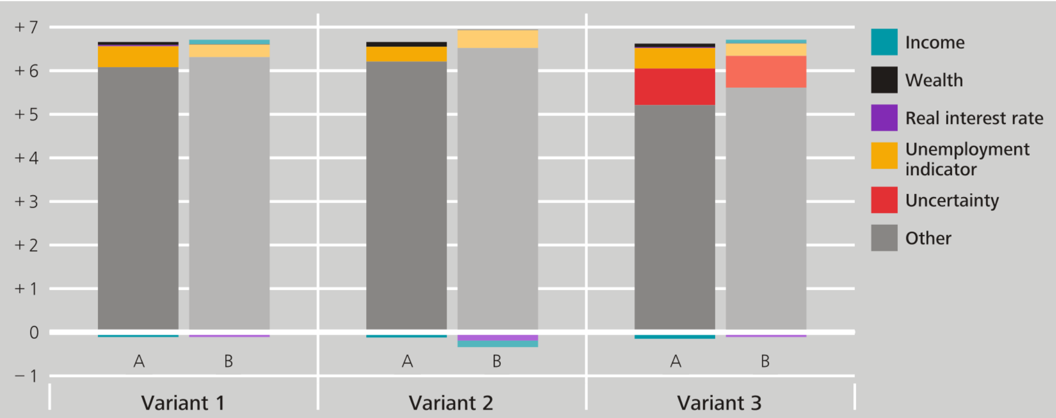
BbkM-DE's consumption equation was used as a starting point to quantify the explanatory content of households' precautionary saving motive. The analysis was based on three variants of the consumption equation. Variant 1 corresponded to the equation as currently used in the model.[112] It includes the unemployment gap as a proxy for the expectations of future labour market and related income developments. Variant 2 replaced the unemployment gap with unemployment expectations (over the next 12 months) taken from the European Commission's consumer survey.[113] Variant 3 extended the model's consumption equation (i.e. Variant 1) with an explanatory variable that aims to capture the uncertainty about future labour market and associated income developments. The underlying idea is that risk-averse employees limit their consumption if the situation on the labour market and thus future income is less predictable. The analysis therefore considered not only an "expectation effect" (worsened prospects) but also an "uncertainty effect" (less predictability) as a source for possible precautionary savings during the pandemic. The uncertainty effect was meant to capture households' caution with respect to labour market outcomes and the associated possibility of reduced income prospects.[114] This type of uncertainty can be measured empirically by the variation in forecast

[110] For more information about the approaches taken for the June 2020 BMPE, please see Box 15 in Work stream on Eurosystem Modelling (2021).
[111] More details can be found in Deutsche Bundesbank (2021a).
[112] In order to decompose the change in the saving ratio in this approach, the consumption equation is augmented by two equations to define the saving ratio so that the change of the latter can be attributed to the consumption drivers.
[113] See also Dossche and Zlatanos (2020).
[114] See Lugilde et al. (2018).

errors for labour market developments. In BbkM-DE, the latter is mainly determined by the dynamics of employees' total hours worked. Therefore, the absolute values of residuals from this behavioural equation were used as the measure of variation of the aforementioned forecast error. As a robustness check, a behavioural equation for the saving ratio of households was also estimated directly. In addition to the components of disposable income, it included the real interest rate, the stock price index and the unemployment rate as explanatory variables. Similarly to the consumption equation approach, a second variant (with the series on expected unemployment as a substitute for the actual) and a third variant (with the residuals from the model's behavioural equation for total hours worked as a supplement) of the saving ratio equation were estimated.[115]

Figure 9: Change in the saving ratio in 2020*

Contributions in PP



* Results are based on analyses conducted during the June 2021 BMPE. "A" corresponds to consumption equation approach, "B" refers to saving ratio equation.

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The results obtained for each behavioural equation described above indicated a limited impact of the precautionary motive on the change in the saving ratio in 2020 (see Figure 9). Variant 3 thereby provided an upper limit of around 15 to 20%.[116] The unexplained part dominated the changes of the saving ratio by more than 90% or (in case of Variant 3) 80 to 85%, respectively.[117] This was interpreted as the extent of forced savings in the context of the June 2021 projection exercise. However, the approaches described above were very closely related in that they each relied on the estimation of a single equation in error correction form with a

[115] Like for the consumption equation, the estimated coefficients in the saving ratio equation were also based on observation samples ending in 2019 Q4.

[116] Other sources of income uncertainty, e.g. those resulting from volatility in stock markets or other well-known uncertainty measures, turned out to be less significant in the analysis and thus back the interpretation that the findings associated with Variant 3 reflect an upper bound of the impact of the precautionary saving motive. In particular, these alternative explanatory factors that can also be associated with uncertainty are financial market uncertainty (based on the volatility index of the German stock market, VDAX), economic policy uncertainty (Baker et al (2016)) or macroeconomic uncertainty (Meinen and Röhe (2017)).

[117] These results largely confirm the findings of the Banque de France and the ECB. See blog of the Banque de France and Dossche and Zlatanos (2020).

similar set of variables. Different approaches and a more disaggregated perspective provided complementary insights into the question of saving motives.

For the June 2021 projection, in particular, the rich information from the Bundesbank Online Panel survey conducted among households (BOP-HH) in March 2021 provided not only differentiated information on saving motives according to income and age groups. Specifically, it gave an outlook on the extent to which the surveyed households plan to spend the accumulated savings when the pandemic is overcome. This information was an important basis for developing the projection baseline for aggregate private consumption beyond 2021.[118]

[118] For more details, see Deutsche Bundesbank (2021b).

4 Basic model elasticities of the macroeconometric model

The basic model elasticities (BMEs) offer an insight into the transmission mechanisms of a number of isolated permanent shocks through BbkM-DE. Each of these sets of elasticities is the outcome of a full-model simulation, assuming that all exogenous variables other than the one subject to the shock remain unaffected. The BMEs are computed as deviations between the variables' paths in a hypothetical shock scenario and a baseline scenario over a horizon of four years. In this exercise, the model simulation usually starts at the current juncture. Feedback effects from foreign countries are not taken into account, e.g. the BMEs for the interest rate shocks do not feature the impact on Germany's foreign trade variables stemming from changes in import demand and export prices of other euro area countries. Furthermore, monetary and fiscal rules are switched off, whereas model-endogenous reactions of automatic fiscal stabilisers, such as taxes and social contributions as well as monetary unemployment expenses are included. This framework corresponds to the one applied to derive national BMEs. These serve as an input for a tool that combines elasticities of euro area/EU countries and is regularly applied in the macroeconomic projection process of the Eurosystem.[119] While BMEs, as they are shown here, can also act as an analytical device in order to check the model's simulation properties, these setups are unlikely to be observed in reality. However, the orthogonality of shocks allows sets of BMEs to be combined in order to derive model-based effects of scenarios that would normally feature the simultaneous occurrence of a number of shocks. This tool is frequently deployed in the projection process in order to assess the impact of the combined revision of assumptions (for interest rates, exchange rates, oil prices, external markets etc.). Bundesbank staff also use it for quick assessments of very recent developments, e.g. the announcement of a new fiscal policy package, when there is no time to conduct a comprehensive simulation study (see Deutsche Bundesbank (2020b)).

When interpreting the results, a number of caveats need to be taken into account. The potential origin of shocks is not reflected in these isolated simulations. In order to take into account the external environment more adequately, BMEs need to be used in combination. If, for example, changes in oil prices are demand-driven, one would expect simultaneous adjustments of real demand from abroad. The economic impacts of shocks depicted here are based on estimated elasticities of the model's behavioural equations. Therefore, they represent average relationships between variables observed over the estimation period and would be suited to indicating effects of limited movements of external variables, for instance. Very large shocks, by contrast, are more likely to result in non-linear economic impacts, which are taken into account neither in the linearised application of BMEs nor in BbkM-DE itself. The model's simulation properties are broadly linear. The impact of oil price shocks as well as shocks to the bilateral euro/US dollar exchange rate that are transmitted via the HICP energy component are, however, level-dependent. For more information, see Section 4.3. These BMEs represent

[119] See also European Central Bank (2016).

the current state of estimations and assessment. They are frequently re-assessed and subject to change with re-estimations of behavioural equations and/or revisions of the model in the light of new analyses and empirical findings. As the BMEs are applied over the projection period, they are meant to appropriately reflect characteristics of the current business cycle. Therefore, BMEs need not be pure model outcomes, but may also reflect expert judgment by NCB staff to a certain extent.[120]

4.1 Foreign demand

Table 1 shows the impact of a permanent 1% rise in the real import demand of Germany's trading partners. Germany's real exports increase almost to the same extent, reflecting the elasticity of one with respect to foreign demand. The impact on real exports levels off slightly after the third year as rising domestic prices in conjunction with the – by assumption – unaffected foreign prices dampen the competitiveness of German exports. Stronger final demand provides incentives for firms to expand their investment expenditure and labour demand. Households use the additional income to increase their spending on consumption as well as on housing investment, whilst also being able to save a higher share of their income. Given the relatively high import content of exports and of machinery and equipment investment, imports also rise considerably as a reaction to the external demand shock. Overall, the impact on real GDP peaks in the second and the third year of the simulation horizon.

On the nominal side, wages also increase due to higher labour demand that pushes down the unemployment rate. In the short term, however, the strong growth in productivity stemming from the higher output outweighs the increase in compensation. Unit labour costs therefore decline in the first year before starting to exceed their baseline levels over the remainder of the simulation horizon. Prices eventually begin to increase above baseline levels as firms react to the higher wage costs as well as the higher capacity utilisation. Export prices rise due to the higher domestic price level, as does the import deflator due to pricing-to-market effects. The increase in aggregate demand as well as higher employment and wages boost government tax and social security income, resulting in a stronger fiscal balance and a gradual decrease in the public debt ratio over the simulation horizon.

The foreign demand shock underlying this simulation includes demand from trading partners both inside and outside the euro area. If the initial shock was restricted to demand from outside the euro area, this would still affect around 64% of German exports, a share that is notably higher than for other large euro area countries such as France, Italy or Spain. The relatively high share of exports in German final demand, which amounts to around one-third, is an additional reason why the direct impact of a global demand shock would be stronger for the German economy than for most of its euro area trading partners.

[120] The shocks for which BMEs are presented in Section 4 are only a subset of those applied for the full BME set that is regularly provided in the Eurosystem. The results shown here aim to give a broad overview of BbkM-DE's simulation properties. In a similar manner, Aldama and Ouvrard (2020) show a multitude of BMEs for the French economy, for instance.

Table 1: Impact of a 1% increase in real foreign demand

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	0.42	0.43	0.40	0.38
Private consumption	0.13	0.23	0.23	0.24
Public consumption	0.00	0.00	0.00	0.00
Total Investment	0.20	0.32	0.38	0.43
Business investment	0.15	0.31	0.37	0.42
Public investment	0.00	0.00	0.00	0.00
Private housing investment	0.36	0.46	0.53	0.60
Exports	1.07	1.00	0.96	0.91
Imports	0.58	0.66	0.66	0.67
Prices				
HICP	0.00	0.05	0.12	0.21
HICP excl. energy and food	0.01	0.05	0.13	0.23
GDP deflator	0.00	0.05	0.19	0.35
Export deflator	0.02	0.05	0.16	0.28
Import deflator	0.01	0.03	0.10	0.18
Labour market				
Unemployment rate *(deviation in pp)*	-0.09	-0.11	-0.13	-0.11
Total employment	0.10	0.13	0.17	0.16
Unit labour costs	-0.15	0.09	0.37	0.67
Compensation per employee	0.17	0.39	0.61	0.89
Productivity	0.32	0.30	0.24	0.22
Real compensation per employee	0.16	0.34	0.48	0.67
Households				
Real disposable income	0.15	0.29	0.30	0.34
Saving ratio *(deviation in pp)*	0.01	0.05	0.07	0.09
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	0.18	0.25	0.27	0.27
Gross debt *(in % of GDP, deviation in pp)*	-0.09	-0.31	-0.56	-0.81

4.2 Foreign competitors' prices

The impact of a sustained 1% rise in foreign competitors' prices is depicted in Table 2. Import prices increase, resulting in a certain upsurge of the domestic price level. Given the higher level of domestic prices and higher prices in foreign markets (as well as unaffected exchange rates), German exporters also raise their prices. They do so, however, to a smaller extent compared to the rise in import prices, because the export deflator's estimated elasticity with respect to foreign prices lies considerably below that of the import deflator.

As a consequence of higher competitiveness, German exports exceed their baseline levels. In view of higher final demand, firms augment their investment expenditure and labour demand. Households benefit from higher disposable income, which is the result of more employment as well as an increase in wages. They have more financial resources available and increase both consumption expenditure and residential investment. Imports only fall below their baseline levels to a small extent. The negative impact of higher import prices relative to the domestic price level is largely outweighed by higher real import demand that stems in particular from the increase in exports and business investment, both of which exhibit a relatively high import content. The fiscal balance benefits from higher aggregate demand as well as labour income, leading to a fall in public debt.

Table 2: Impact of a 1% increase of foreign competitors' prices

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	0.10	0.25	0.28	0.28
Private consumption	0.02	0.12	0.14	0.14
Public consumption	0.00	0.00	0.00	0.00
Total Investment	0.04	0.15	0.23	0.28
Business investment	0.02	0.14	0.22	0.27
Public investment	0.00	0.00	0.00	0.00
Private housing investment	0.08	0.24	0.33	0.38
Exports	0.16	0.32	0.33	0.31
Imports	0.00	0.03	0.04	0.05
Prices				
HICP	0.00	0.03	0.07	0.14
HICP excl. energy and food	0.01	0.03	0.09	0.16
GDP deflator	0.02	0.04	0.08	0.18
Export deflator	0.30	0.34	0.36	0.42
Import deflator	0.35	0.39	0.41	0.46
Labour market				
Unemployment rate *(deviation in pp)*	-0.03	-0.06	-0.07	-0.07
Total employment	0.03	0.07	0.09	0.10
Unit labour costs	-0.04	-0.01	0.13	0.34
Compensation per employee	0.03	0.17	0.33	0.52
Productivity	0.07	0.18	0.19	0.18
Real compensation per employee	0.03	0.14	0.25	0.38
Households				
Real disposable income	0.02	0.13	0.18	0.20
Saving ratio *(deviation in pp)*	0.00	0.01	0.03	0.05
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	0.04	0.13	0.17	0.18
Gross debt *(in % of GDP, deviation in pp)*	-0.01	-0.11	-0.26	-0.43

4.3 Oil prices

In contrast to the value-added tax that is levied on the value of a good or service, the petroleum tax is a quantity tax charged per litre of mineral oil (or megawatt hour of gas). Thus, when crude oil prices rise, the share of taxes in the final sales prices declines. This means that the elasticity of consumer prices with respect to crude oil prices increases with the level of oil prices. This will also affect the reaction of the real side of the economy to changes in oil prices. In projection applications of the BMEs, four sets of elasticities for crude oil baseline prices (€30, €55, €85 and €115 per barrel of Brent crude oil) are made available owing to the level dependency of the oil price elasticities. The elasticities shown in Table 3 depict those of a sustained 10% increase in crude oil prices starting from the baseline level that is closest to the one currently prevailing, i.e. for a baseline price of €85 per barrel of Brent crude oil.[121] The elasticities shown for the HICP energy component are those directly obtained by the Bundesbank's inflation experts when applying the detailed bottom-up price projection

[121] For the selection of baseline levels mentioned, the impact of the 10% crude oil price increase on the HICP energy component in year 4 of the simulation horizon varies between 1.9% when starting from a baseline level of 30 euros per barrel and around 3.5% when starting from a baseline level of €115. For the given size of the shock, the divergence of effects on overall HICP and real GDP is considerably smaller (varying between 0.25% and 0.46% for HICP and between -0.11% and -0.17% for real GDP in year 4, respectively).

approach that aggregates from sub-categories of the energy component using monthly data. These effects are directly imposed on the model variable when running the simulation. The specification of the model's behavioural equation for the HICP energy component, however, closely follows the modelling approaches taken for the disaggregated monthly price projections. The reactions to oil price shocks from both approaches are frequently cross-checked and are very close.

As indicated above, the increase in oil prices directly affects the energy component of consumer prices. Higher energy costs will eventually also entail some increase of other consumer price components, resulting in a contained rise of the HICP core component. The cost push for energy input will affect demand for this production factor by directly raising the deflator of energy imports in BbkM-DE. Through this channel, investment deflators also increase to some extent as firms face higher energy costs.

Table 3: Impact of a 10% increase in Brent crude oil prices (initial level: €85 per barrel)

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.04	-0.10	-0.13	-0.16
Private consumption	-0.11	-0.16	-0.18	-0.22
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.01	-0.04	-0.04	-0.06
Business investment	0.00	-0.01	0.00	-0.01
Public investment	0.00	0.00	0.00	0.00
Private housing investment	-0.05	-0.11	-0.13	-0.19
Exports	-0.01	-0.06	-0.09	-0.11
Imports	-0.07	-0.06	-0.06	-0.07
Prices				
HICP	0.19	0.33	0.38	0.41
HICP excl. energy and food	0.01	0.04	0.07	0.09
HICP energy	1.67	2.80	3.04	3.14
GDP deflator	-0.02	0.09	0.15	0.19
Export deflator	0.07	0.19	0.23	0.26
Import deflator	0.38	0.50	0.52	0.54
Labour market				
Unemployment rate *(deviation in pp)*	-0.01	-0.01	0.03	0.04
Total employment	0.01	0.00	-0.04	-0.06
Unit labour costs	0.04	0.16	0.22	0.22
Compensation per employee	0.00	0.06	0.14	0.13
Productivity	-0.05	-0.10	-0.08	-0.10
Real compensation per employee	-0.14	-0.18	-0.15	-0.19
Households				
Real disposable income	-0.15	-0.26	-0.27	-0.29
Saving ratio *(deviation in pp)*	-0.04	-0.08	-0.07	-0.06
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.02	-0.05	-0.08	-0.11
Gross debt *(in % of GDP, deviation in pp)*	0.01	0.04	0.11	0.20

The increase in consumer prices dampens households' real disposable income and hence their real consumption expenditure as well as housing investment. Households save a lower share of their income compared to baseline. Business investment also falls below its baseline level due to the lower final demand. Exports decrease as higher domestic prices in conjunction with the – by definition – unaffected foreign prices reduce their competitiveness. The dampening effect of higher energy import prices on overall real imports remains contained, given the relatively low price elasticity of energy imports as well as their small share in overall imports that amounts to around 6%.[122] Due to the reduced aggregate demand, the fiscal balance weakens and public debt rises somewhat over the simulation horizon.

4.4 Euro/US dollar exchange rate

The exchange rate BMEs distinguish between a sustained 10% appreciation of the euro against the US dollar, while all other bilateral rates remain unchanged, and a sustained 10% appreciation against all other currencies (apart from the US dollar). This allows the US dollar's particular role for foreign trade as the invoicing currency for most commodity trade in world markets to be taken into account. In a linearised application, the impact of both simulations would have to be added in order to approximate the effect of a pure 10% nominal effective appreciation of the euro.

BbkM-DE contains both the bilateral euro/US dollar exchange rate as well as the nominal effective exchange rate in national definition as exogenous variables. It takes into account the other 18 member countries of the euro area as well as 19 important trading partners of the German economy outside the euro area. Therefore, an appreciation of the euro against the US dollar is implemented by setting two shocks simultaneously: 1.) on the bilateral rate and 2.) on the nominal effective exchange rate, scaled by the weight of the US dollar in the nominal rate in national definition vis-à-vis 37 trading partners, which is 0.105.

An appreciation of the euro against the US dollar directly reduces consumer prices via the HICP energy component. Energy imports become cheaper. The deflator of non-energy imports also falls due to lower euro prices for non-energy commodity imports and non-commodity imports. The latter effect is taken into account directly via the impact of the (weighted) nominal effective exchange rate on non-energy imports.

Real private consumption expenditure increases, as households benefit from lower consumer prices. Given the appreciation of the euro, however, German exporters lose competitiveness in their sales markets and exports decrease below baseline levels, as does investment due to the overall drop in final demand. Imports rise above baseline levels because the impact of the decline in import prices offsets that of the lower real import demand. The labour market reacts in line with the overall dip in GDP as both employment and compensation per employee fall below the baseline. Government tax and social security income decline, resulting in a deterioration of the fiscal balance. The public debt ratio exceeds its baseline level.

[122] Average for 2020 based on real aggregates.

Table 4: Impact of a 10% appreciation of the euro against the US dollar

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.09	-0.28	-0.35	-0.35
Private consumption	0.09	0.07	0.03	0.02
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.03	-0.17	-0.28	-0.35
Business investment	-0.02	-0.17	-0.32	-0.41
Public investment	0.00	0.00	0.00	0.01
Private housing investment	-0.06	-0.22	-0.32	-0.36
Exports	-0.16	-0.28	-0.27	-0.22
Imports	0.13	0.33	0.41	0.42
Prices				
HICP	-0.16	-0.26	-0.36	-0.49
HICP excl. energy and food	-0.02	-0.08	-0.18	-0.31
GDP deflator	0.09	0.08	-0.03	-0.20
Export deflator	-0.40	-0.56	-0.65	-0.79
Import deflator	-0.96	-1.32	-1.41	-1.49
Labour market				
Unemployment rate *(deviation in pp)*	0.00	0.05	0.08	0.08
Total employment	0.00	-0.04	-0.08	-0.09
Unit labour costs	0.05	0.03	-0.19	-0.52
Compensation per employee	-0.03	-0.21	-0.47	-0.78
Productivity	-0.09	-0.24	-0.27	-0.26
Real compensation per employee	0.08	0.00	-0.16	-0.36
Households				
Real disposable income	0.12	0.13	0.08	0.03
Saving ratio *(deviation in pp)*	0.03	0.05	0.04	0.01
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.01	-0.08	-0.15	-0.18
Gross debt *(in % of GDP, deviation in pp)*	0.00	0.05	0.17	0.34

4.5 Nominal effective exchange rate (excl. US dollar)

A sustained 10% appreciation of the euro against all other trading partners' currencies, apart from the US dollar, constitutes a shock to the nominal effective exchange rate scaled by the weight of extra-euro area trading partners excluding the US, which is at around 0.46. The shock on the effective exchange rate is therefore more than four times larger than the one implemented for the pure US dollar shock described in Section 4.4. At the same time, with the bilateral euro/US dollar exchange rate remaining unaffected, there are no direct effects via dollar-invoiced commodities on consumer and energy import prices in the simulation results. Non-energy import prices, however, decrease considerably due to the euro appreciation, which is gradually transmitted through the model's price block to other demand deflators and consumer prices. The drop in capacity utilisation – caused by lower aggregate demand – additionally dampens price developments.

Price competitiveness of exports declines due to the stronger euro and exports fall notably below their baseline level. Firms reduce their investment expenditure as well as their labour demand. With the unemployment rate increasing and the lower aggregate price level, wages drop as well. Households smooth some of the reduction in disposable income by lowering their saving ratio, but also spend notably less on consumption and housing investment. Despite the strong fall of the import deflator, imports decline below baseline levels in the second half of the simulation period as the dip in real import demand more than outweighs the competitiveness gains. Overall, the impact on real final demand as well as the fiscal balance and public debt is much larger than that of a pure US dollar shock. In both simulations, the maximum effect on demand components is reached in the third year of the simulation period.

Table 5: Impact of a 10% increase in the nominal effective exchange rate excl. USD

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.46	-1.14	-1.28	-1.26
Private consumption	-0.10	-0.56	-0.71	-0.75
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.18	-0.70	-1.06	-1.29
Business investment	-0.10	-0.64	-1.03	-1.27
Public investment	0.00	0.00	0.00	0.01
Private housing investment	-0.38	-1.07	-1.50	-1.80
Exports	-0.74	-1.43	-1.47	-1.36
Imports	0.00	-0.15	-0.23	-0.27
Prices				
HICP	-0.02	-0.13	-0.34	-0.64
HICP excl. energy and food	-0.03	-0.16	-0.40	-0.73
GDP deflator	-0.13	-0.27	-0.54	-1.00
Export deflator	-1.41	-1.78	-2.00	-2.36
Import deflator	-1.57	-1.79	-1.94	-2.18
Labour market				
Unemployment rate *(deviation in pp)*	0.14	0.33	0.39	0.39
Total employment	-0.15	-0.35	-0.45	-0.50
Unit labour costs	0.16	0.01	-0.67	-1.59
Compensation per employee	-0.15	-0.78	-1.49	-2.34
Productivity	-0.31	-0.78	-0.83	-0.76
Real compensation per employee	-0.12	-0.65	-1.14	-1.68
Households				
Real disposable income	-0.11	-0.63	-0.88	-1.02
Saving ratio *(deviation in pp)*	0.00	-0.06	-0.15	-0.24
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.20	-0.61	-0.82	-0.89
Gross debt *(in % of GDP, deviation in pp)*	0.07	0.51	1.21	2.01

4.6 Short-term interest rate

The BMEs provide separate sets of elasticities for a permanent 100 bp rise in the short-term interest rate (3-month Euribor) and for a permanent 100 bp rise in the nominal long-term interest rate (yield on 10-year government bonds). Especially since the short-term rate shock is unlikely to occur in isolation, both sets need to be applied in combination when aiming to assess the model's response to an interest rate shock. When considering a monetary policy shock, one would also need to take account of potential exchange rate effects, which are – by definition – not part of the responses presented here, either.

The impact of the 100 bp increase in the Euribor on bank lending rates for corporate loans and mortgages to households are included, however. These are directly taken from interest rate pass-through models based on monthly data that are used in the projection process. The model's behavioural equations for both lending rates would yield similar results.

The rise in bank lending rates dampens business investment via higher user costs of capital. In addition, higher lending rates reduce house prices and the number of building permits below their baseline levels, resulting in lower housing investment. The overall effect of the isolated short-term interest rate shock remains contained, though, and prices hardly react over the simulation horizon.

Table 6: Impact of a 100 bp increase in the short-term interest rate

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.02	-0.07	-0.10	-0.11
Private consumption	-0.01	-0.05	-0.06	-0.06
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.07	-0.27	-0.42	-0.53
Business investment	-0.09	-0.32	-0.54	-0.71
Public investment	0.00	0.00	0.00	0.00
Private housing investment	-0.05	-0.27	-0.34	-0.37
Exports	0.00	0.00	0.00	0.00
Imports	-0.01	-0.05	-0.08	-0.10
Prices				
HICP	0.00	0.00	-0.01	-0.02
HICP excl. energy and food	0.00	0.00	-0.01	-0.02
GDP deflator	0.00	0.01	0.01	-0.02
Export deflator	0.00	0.01	0.01	-0.01
Import deflator	0.00	0.01	0.00	-0.01
Labour market				
Unemployment rate *(deviation in pp)*	0.00	0.01	0.02	0.03
Total employment	0.00	-0.01	-0.02	-0.03
Unit labour costs	0.01	0.03	0.01	-0.04
Compensation per employee	0.00	-0.03	-0.07	-0.12
Productivity	-0.02	-0.06	-0.07	-0.08
Real compensation per employee	0.00	-0.03	-0.06	-0.10
Households				
Real disposable income	0.00	-0.03	-0.05	-0.07
Saving ratio *(deviation in pp)*	0.01	0.02	0.00	-0.01
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	0.00	-0.03	-0.05	-0.06
Gross debt *(in % of GDP, deviation in pp)*	0.00	0.02	0.06	0.11

4.7 Long-term interest rate

A sustained 100 bp increase of government bond yields affects aggregate demand via the same channels as the Euribor shock. However, as implied by the Bundesbank's interest rate pass-through models, the impact of the long-term interest rate shock on bank lending rates is considerably larger than the reaction to the Euribor shock described in Section 4.6. Therefore, business and residential investment also respond much more strongly. In addition, households' incentives to save rise with higher government bond yields, thus dampening private consumption expenditure. Overall, the GDP effect in year 4 of the simulation period is about four times larger than the impact of the short-term interest rate shock. The unemployment rate rises with lower labour demand and consumer prices fall below baseline levels from year 2 onwards. The fiscal balance weakens and the debt ratio exceeds its baseline level, not only as a result of lower tax and social security income of the government, but also due to higher interest payments on public debt.

Table 7: Impact of a 100 bp increase in the long-term interest rate

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.15	-0.38	-0.38	-0.33
Private consumption	-0.24	-0.49	-0.49	-0.43
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.24	-0.86	-1.01	-1.06
Business investment	-0.14	-0.54	-0.81	-0.95
Public investment	0.00	0.00	0.00	0.00
Private housing investment	-0.53	-1.73	-1.78	-1.66
Exports	0.00	-0.01	0.00	0.04
Imports	-0.09	-0.26	-0.28	-0.27
Prices				
HICP	0.00	-0.01	-0.06	-0.12
HICP excl. energy and food	0.00	-0.01	-0.06	-0.13
GDP deflator	0.00	0.01	-0.07	-0.20
Export deflator	0.01	0.03	-0.03	-0.14
Import deflator	0.01	0.02	-0.02	-0.09
Labour market				
Unemployment rate *(deviation in pp)*	0.02	0.07	0.10	0.10
Total employment	-0.02	-0.08	-0.12	-0.13
Unit labour costs	0.08	0.09	-0.13	-0.40
Compensation per employee	-0.05	-0.21	-0.38	-0.59
Productivity	-0.13	-0.30	-0.26	-0.19
Real compensation per employee	-0.04	-0.20	-0.32	-0.46
Households				
Real disposable income	-0.03	-0.12	-0.08	-0.02
Saving ratio *(deviation in pp)*	0.18	0.33	0.36	0.37
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.09	-0.36	-0.52	-0.58
Gross debt *(in % of GDP, deviation in pp)*	0.03	0.29	0.74	1.27

4.8 Government consumption

The shock is set to real government consumption in order to increase the ex ante ratio of nominal government consumption to GDP by 1 percentage point over the full simulation period of four years. The ex post ratio, as the outcome of the simulations, however, also reflects the model-endogenous reactions of aggregate demand and prices and can thus deviate from the ex ante value.[123] By definition, the higher public consumption results in an expansion of real GDP.[124] Companies increase their investment due to the higher final demand. Labour demand rises and the unemployment rate falls below baseline levels. Wages increase, eventually resulting in a notable upswing of unit labour costs, which is reflected in a higher price level. Households benefit from additional disposable income due to the labour market and wage

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123 The shock is meant to affect intermediate government consumption in contrast to e.g. assuming a shock to employment of the government sector. The simulation therefore takes into account that the additional expenditure has a higher import content than overall government consumption.

124 Please note that in accordance with the commonly agreed simulation setup, all fiscal expenditure shocks considered in the following are debt financed and therefore considerably increase the public debt level, as shown in the tables. A simultaneous tax increase in order to finance the additional public expenditures would substantially reduce the economic impact.

effects and are able to increase their consumption and housing investment expenditure as well as their saving ratio. Exports, on the other hand, are dampened by a loss in price competitiveness. The ex post fiscal balance does not deteriorate by the full amount of the shock for two reasons. Higher aggregate demand and labour income simultaneously raise government tax and social security revenue. In addition, nominal GDP in the denominator of the ex post fiscal ratios used to depict the fiscal effects lies considerably above baseline nominal GDP as it includes all endogenous reactions to the original government consumption shock.

Table 8: Impact of an increase in government consumption by 1% of GDP

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	1.03	1.15	1.11	1.05
Private consumption	0.29	0.54	0.56	0.60
Public consumption	4.52	4.81	4.87	4.84
Total Investment	0.47	0.82	0.99	1.12
Business investment	0.35	0.79	0.97	1.11
Public investment	0.00	0.00	0.00	-0.01
Private housing investment	0.87	1.18	1.38	1.56
Exports	0.00	0.02	-0.05	-0.18
Imports	0.86	1.08	1.10	1.12
Prices				
HICP	0.01	0.10	0.29	0.53
HICP excl. energy and food	0.01	0.11	0.31	0.57
GDP deflator	-0.04	0.03	0.38	0.82
Export deflator	-0.04	-0.01	0.26	0.59
Import deflator	-0.04	-0.01	0.18	0.40
Labour market				
Unemployment rate *(deviation in pp)*	-0.19	-0.26	-0.33	-0.28
Total employment	0.20	0.30	0.41	0.41
Unit labour costs	-0.43	0.10	0.83	1.69
Compensation per employee	0.39	0.95	1.54	2.34
Productivity	0.83	0.85	0.70	0.64
Real compensation per employee	0.38	0.85	1.24	1.79
Households				
Real disposable income	0.33	0.67	0.75	0.87
Saving ratio *(deviation in pp)*	0.04	0.12	0.16	0.24
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.53	-0.32	-0.23	-0.22
Gross debt *(in % of GDP, deviation in pp)*	0.39	0.78	1.02	1.22

4.9 Government investment

As BbkM-DE distinguishes between government construction, equipment and other investment, the shock is set to all categories, in real terms, according to their share in overall government investment in order to permanently raise the ex ante ratio of nominal public investment to GDP by 1 percentage point. As for government consumption, the investment shock by definition directly increases GDP. Given that more than half of government investment expenditure goes into construction, which has a lower import share compared to equipment and other investment, imports rise by less than they do in response to the intermediate consumption shock. Overall, the impact on GDP lies slightly below the one described in Section 4.8. Potential output increases by more due to the direct effect of government investment on capital services. Therefore, capacity utilisation expands less than compared to the public consumption shock. Thus, both prices and wages also rise somewhat less. The deterioration of the fiscal balance and rise of the debt ratio are slightly smaller compared to the government consumption shock.

Table 9: Impact of an increase in government investment by 1% of GDP

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	0.98	1.06	0.99	0.91
Private consumption	0.26	0.46	0.46	0.46
Public consumption	0.00	0.00	0.00	0.00
Total Investment	4.99	5.26	5.34	5.41
Business investment	0.34	0.74	0.88	0.98
Public investment	39.07	39.72	39.74	40.12
Private housing investment	0.84	1.07	1.21	1.32
Exports	0.01	0.01	-0.06	-0.17
Imports	0.84	1.02	1.02	1.02
Prices				
HICP	0.00	0.08	0.24	0.44
HICP excl. energy and food	0.00	0.08	0.26	0.46
GDP deflator	-0.03	0.05	0.38	0.75
Export deflator	-0.08	-0.04	0.20	0.49
Import deflator	-0.07	-0.03	0.14	0.33
Labour market				
Unemployment rate *(deviation in pp)*	-0.16	-0.25	-0.32	-0.28
Total employment	0.17	0.29	0.40	0.39
Unit labour costs	-0.44	0.07	0.72	1.46
Compensation per employee	0.37	0.84	1.32	1.99
Productivity	0.81	0.77	0.59	0.52
Real compensation per employee	0.36	0.75	1.08	1.54
Households				
Real disposable income	0.30	0.57	0.60	0.68
Saving ratio *(deviation in pp)*	0.04	0.10	0.12	0.19
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.49	-0.31	-0.27	-0.29
Gross debt *(in % of GDP, deviation in pp)*	0.37	0.74	1.01	1.25

4.10 Direct taxes and social security contributions

The shock is set to raise the sum of direct taxes and social security contributions by employees and employers ex ante by 1% of baseline GDP over the entire simulation period of four years. It is distributed across categories according to their shares in the overall amount. For income taxes of employees as well as other direct taxes, the shock is implemented via the respective behavioural equations' residuals, while the higher social security contributions of employees and employers are both imposed via adequate shocks to the contribution rate.

Higher income taxes as well as social security contributions of employees directly diminish the net labour income component of disposable income. The increase in other direct taxes, which include corporate and capital income taxes, on the other hand, dampens firms' operating surplus, which feeds into the component "other income" of disposable household income with some delay. The impact on the latter over the simulation horizon considered here is hence more limited. Overall, households react to the lower income at their disposal by reducing their expenditure for consumption and housing investment and by lowering their saving ratio.

Table 10: Impact of an increase in the sum of direct taxes and SSC by 1% of GDP

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.43	-0.59	-0.59	-0.60
Private consumption	-0.90	-1.10	-1.09	-1.14
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.25	-0.58	-0.73	-0.82
Business investment	-0.13	-0.37	-0.46	-0.52
Public investment	0.00	0.00	0.00	0.00
Private housing investment	-0.56	-1.18	-1.49	-1.68
Exports	-0.01	-0.04	-0.04	-0.01
Imports	-0.27	-0.41	-0.41	-0.42
Prices				
HICP	0.03	0.05	0.01	-0.03
HICP excl. energy and food	0.02	0.04	0.00	-0.05
GDP deflator	0.06	0.14	0.05	-0.05
Export deflator	0.06	0.14	0.07	-0.01
Import deflator	0.05	0.09	0.04	0.00
Labour market				
Unemployment rate *(deviation in pp)*	0.25	0.12	0.09	0.06
Total employment	-0.29	-0.26	-0.33	-0.39
Unit labour costs	0.55	0.40	0.22	0.06
Compensation per employee	0.41	0.07	-0.05	-0.15
Productivity	-0.15	-0.34	-0.27	-0.21
Real compensation per employee	0.38	0.01	-0.06	-0.12
Households				
Real disposable income	-1.25	-1.59	-1.67	-1.74
Saving ratio *(deviation in pp)*	-0.30	-0.44	-0.52	-0.54
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	0.74	0.58	0.56	0.56
Gross debt *(in % of GDP, deviation in pp)*	-0.50	-1.09	-1.62	-2.11

Higher social security contributions of employers increase employers' labour costs and therefore reduce their profits. Hence, aggregate demand is affected via the same lagged channel of other income previously described. Primarily, however, higher employers' contributions raise unit labour costs, which leads to an increase of the domestic price level. The resulting reduction of real disposable income further dampens consumption and residential investment expenditure of households. Exports fall below baseline levels due to the loss of price competitiveness. Corporates cut their investment expenditure as result of the lower aggregate demand. In combination with the higher labour costs this also leads firms to reduce labour demand. The increase in the unemployment rate gradually eases over the simulation horizon as lower net wages – due to higher employees' income taxes and social security contributions – induce households to adjust their labour market participation downwards. The ex post fiscal balance ratio improves less than the initial shock on direct tax and social security income, because the dampening impact on government revenues due to the endogenous reaction of lower aggregate demand and labour income is not fully offset by the shock-induced lower nominal GDP in the denominator of the ex post fiscal ratios.

4.11 Indirect taxes

Indirect tax income is set to increase ex ante by 1% of baseline GDP until the end of the simulation period. Higher tax payments are imposed on value-added taxes and other taxes on goods and production charges according to their shares in overall indirect taxes. Value-added taxes are directly adjusted via regular and reduced tax rates, and the rise of energy taxes is set via an add-on to the quantity tax. The shock on the remaining indirect taxes is implemented through the residuals of the respective behavioural equation.

In order to obtain the required size of the ex ante additional tax income, both the regular and the reduced value-added tax rates need to increase by around 1.8 percentage points. Higher value-added tax rates directly lead to an upsurge in all HICP components and therefore the consumption deflator, as well as the deflators of government consumption and residential investment. The sizes of the individual impacts reflect the extent to which each GDP or HICP component is subject to taxation at the regular and reduced value-added tax rates. The required ex ante increase in energy taxes is obtained by raising the quantity tax by around 2.6 cents per litre. Higher energy taxes result in an additional increase of the HICP energy component and are also captured in investment deflators. Higher excise duties on food products additionally affect the HICP food component. Overall, higher consumer prices directly diminish households' real disposable income. In addition, higher other indirect taxes and energy taxes paid by firms dampen corporate profits, which is also reflected in the other income component of real disposable income of households. These react to the overall drop in real financial means available by cutting their expenditure for consumption and residential investment and by compensating for some of the real income losses by reducing their saving ratio. Firms adjust their investment spending downwards as a reaction to the lower aggregate demand. Exports fall below baseline levels, as the increase in energy taxes boosts corporate production costs, thus leading to a decline in price competitiveness. Employers reduce their labour demand and the unemployment rate rises. The reaction of wages to the indirect tax shock remains limited over the first three years of the simulation horizon because higher consumer prices largely offset the downward impact of lower aggregate demand. Compensation per employee only decreases more notably below baseline levels in the last year of the simulation period.

The improvement of the fiscal balance declines faster over the simulation horizon compared to the effects of the direct tax shock described in Section 4.10, because the larger decline in aggregate demand endogenously resulting from the indirect tax shock has a stronger negative impact on government revenues. In addition, as opposed to the direct tax shock, nominal GDP increases due to the strong price effects of higher indirect taxes. This also reduces the ex post fiscal ratios used to derive the impact on government finances shown in the table below.

Table 11: Impact of an increase in indirect taxes by 1% of GDP

deviation in % from baseline level	Year 1	Year 2	Year 3	Year 4
Real economic activity				
Real GDP	-0.48	-0.57	-0.69	-0.76
Private consumption	-0.92	-1.01	-1.23	-1.34
Public consumption	0.00	0.00	0.00	0.00
Total Investment	-0.26	-0.37	-0.53	-0.75
Business investment	-0.15	-0.31	-0.37	-0.48
Public investment	0.00	0.00	0.00	0.00
Private housing investment	-0.55	-0.60	-0.94	-1.41
Exports	-0.12	-0.26	-0.27	-0.27
Imports	-0.33	-0.47	-0.55	-0.61
Prices				
HICP	1.18	1.23	1.25	1.24
HICP excl. energy and food	0.63	0.63	0.62	0.59
HICP energy	2.77	3.32	3.54	3.65
HICP food	2.36	2.30	2.35	2.34
GDP deflator	0.94	0.96	0.94	0.90
Export deflator	0.14	0.16	0.14	0.10
Import deflator	0.11	0.11	0.09	0.07
Labour market				
Unemployment rate *(deviation in pp)*	0.14	0.30	0.36	0.30
Total employment	-0.17	-0.39	-0.49	-0.46
Unit labour costs	0.27	0.35	0.35	0.12
Compensation per employee	-0.05	0.16	0.15	-0.18
Productivity	-0.32	-0.19	-0.20	-0.29
Real compensation per employee	-1.17	-0.99	-1.02	-1.33
Households				
Real disposable income	-1.28	-1.48	-1.61	-1.76
Saving ratio *(deviation in pp)*	-0.32	-0.43	-0.35	-0.38
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	0.70	0.58	0.47	0.38
Gross debt *(in % of GDP, deviation in pp)*	-0.43	-1.02	-1.50	-1.85

4.12 Monetary transfer payments of the government to households

Social benefits to households are set to permanently rise by ex ante 1% of baseline GDP. The shock is implemented via the residuals in the model's respective behavioural equation. The increase in monetary social benefits directly boosts the financial means available to households in an income category that is characterised by a notably higher spending elasticity compared to that of other income. Households use the additional disposable income by expanding their expenditure on consumption and residential investment. In addition, they save a higher share of their income. Firms raise their investment and labour demand as a reaction to the higher final demand. The unemployment rate declines. At the same time, wages grow markedly over the projection horizon, which is eventually reflected in the increase of unit labour costs. In conjunction with the higher capacity utilisation, this results in a notable upsurge of the domestic price level over the second half of the simulation horizon. In year 4, the dampening impact on competitiveness stemming from the higher price level begins to reduce the positive effect on aggregate demand of the expansionary fiscal shock. The increase of monetary transfer payments generates less model-endogenous additional tax and social security income compared to the expansionary fiscal shocks on government consumption and investment.

Therefore, the deterioration of the fiscal balance is larger and also declines to a smaller extent over the projection horizon when setting the shock to monetary transfer payments. The reactions of fiscal ratios as shown in the table below are also stronger than those depicted in Section 4.10 and 4.11 because nominal GDP (in the denominator) rises by less compared to the fiscal expansions via government consumption or investment.

Table 12: Impact of an increase in monetary transfer payments by 1% of GDP

deviation in % from baseline level	**Year 1**	**Year 2**	**Year 3**	**Year 4**
Real economic activity				
Real GDP	0.59	0.74	0.74	0.71
Private consumption	1.23	1.40	1.45	1.49
Public consumption	0.00	0.00	0.00	0.00
Total Investment	0.37	0.78	0.99	1.11
Business investment	0.20	0.48	0.62	0.73
Public investment	0.00	0.00	0.00	0.00
Private housing investment	0.81	1.61	2.05	2.28
Exports	0.00	0.01	-0.04	-0.12
Imports	0.38	0.53	0.57	0.59
Prices				
HICP	0.00	0.05	0.17	0.33
HICP excl. energy and food	0.00	0.06	0.19	0.35
GDP deflator	-0.01	0.04	0.26	0.54
Export deflator	-0.03	-0.01	0.15	0.36
Import deflator	-0.03	0.00	0.11	0.25
Labour market				
Unemployment rate *(deviation in pp)*	-0.10	-0.16	-0.22	-0.19
Total employment	0.11	0.19	0.27	0.27
Unit labour costs	-0.26	0.01	0.47	1.02
Compensation per employee	0.22	0.56	0.95	1.47
Productivity	0.49	0.55	0.48	0.44
Real compensation per employee	0.22	0.51	0.77	1.13
Households				
Real disposable income	1.88	2.11	2.20	2.30
Saving ratio *(deviation in pp)*	0.54	0.62	0.65	0.71
Fiscal development				
Balance *(in % of GDP, deviation in pp)*	-0.80	-0.69	-0.61	-0.59
Gross debt *(in % of GDP, deviation in pp)*	0.52	1.25	1.86	2.40

5 Concluding remarks and way forward

This paper offers a comprehensive overview of the Bundesbank's macroeconometric model. The model is one of the main vehicles and coordinating tools used in the Macroeconomic Analysis and Projections Division of the Directorate General Economics to produce the biannual macroeconomic forecast of the German economy and to conduct simulation exercises around the projection baseline. As a semi-structural model, it has a rather traditional character: over the past few decades, such models were typically designed at policy institutions for broad macroeconomic analyses. Still, the model has proven that it can provide valuable input for the daily work of the division. The Bundesbank's macroeconomic projection is not a pure model-based forecast. It is the outcome of an extensive process incorporating various satellite approaches deployed by the Bundesbank's experts for various parts of the economy, who thus all contribute substantially to the projection. The information provided stems from a great number of sources, such as short-term forecasts, disaggregate inflation forecasts and fiscal projections, but also includes a large body of valuable expert knowledge that cannot be captured in pure model-based instruments. The macroeconometric model's key role is to provide a comprehensive framework bringing together this large amount of information and to help develop a consistent narrative for the projection. Considering the high number of variables modelled, the interdependencies taken into account and the model applications, it can be concluded that BbkM-DE serves many purposes. This means, however, that when deciding on steps in the model's maintenance process and in the further development of BbkM-DE, one is constantly faced with trade-offs among which a sensible compromise needs to be found.

Although the macroeconometric model mainly represents standard modelling approaches for its various blocks, some model properties can be highlighted as they reflect less conventional characteristics. These refer to the specification of the production function that not only has a CES functional form but also includes energy as a third input factor. The model's domestic price block combines the GDP deflator and consumer price components in a disaggregated and coherent setup. The fiscal block also features a notable degree of disaggregation that goes beyond the scope required for the BMEs, for instance. In combination with the price block, it also permits a differentiated modelling of indirect tax effects such as energy and value-added taxes.

Of course, some model properties also limit its field of application and may understate transmission channels that have become increasingly important over the years, such as the role of expectations in the context of forward guidance policy by the ECB's Governing Council. In addition, the increased use of survey-based data (see Beckmann and Schmidt (2020), for instance) helps to better measure expectations of economic agents.[125] As a result, the requirements for macroeconomic models to realistically map such aspects of economic

[125] See also the Bundesbank's website regarding the survey on firms' expectations in Germany.

behaviour are likely to increase significantly in the future. In this context, Banque de France[126] and ECB staff[127] have recently developed semi-structural models in the spirit of the FRB/US model,[128] where greater emphasis is placed on the modelling of expectations. As these models lay the ground for modern semi-structural model development, Bundesbank staff are also exploring this modelling approach and are planning to go in this direction.

This is just one example showing that maintaining a large macroeconometric model such as BbkM-DE should be considered an ongoing process. The documentation can only present a snapshot at the cut-off date of its publication. Model maintenance and further development will proceed, with alterations likely in the near future. This is even more evident since the outbreak of the COVID-19 pandemic last year, causing huge economic upheavals. The implications for empirical and theoretical modelling, such as huge outliers in the time series or potential structural shifts in the behaviour of economic agents, are still open questions.[129] This paper has only touched briefly on this aspect. Nevertheless, modellers and forecasters need to tackle those challenges in a timely manner in order to provide a reliable assessment for policy makers. It also shows the need for macroeconometric models that offer an appropriate degree of flexibility. This has been exemplified by applications of BbkM-DE in recent projection exercises.

[126] See Lemoine et al. (2019).

[127] See Angelini et al. (2019).

[128] See Brayton and Tinsley (1996) and the Federal Reserve's website for a recent model documentation.

[129] However, this is not to disregard the number of valuable papers published since last year in this context, which explain how to address this issue from an economic and econometric perspective. As regards semi-structural modelling, Angelini et al (2020) conducted pioneering work by combining the ECB's main model for the euro area (ECB-BASE) with an epidemiological module.

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Annex I: Sector accounts in BbkM-DE

Households

Private consumption	CP	Gross wage income	LG
Investment of households	IP	Other (net) income	GNEH
Net capital transfer payments of households	SVPH	Net monetary transfers received	TRN
		./. Wage taxes	./. LOST
		./. Employees' social security contributions	./. SOZN
Financial balance	FH	./. Transfers of households abroad	./. VERR
		Change in net equity of pension fund reserves	BV
Disposable income	YV	Disposable income	YV

Government

Government consumption	CS	Direct taxes	TDIR
Government investment	IST	Indirect taxes	TBSP
Monetary transfer payments	TRNS	Social security contributions	SOZS
Subsidies	SUBV	Property income of government	GST
Interest payments	ZINS	Sales and other revenue of government	VKSS
Residual item in government financial account	SRSS		
Financial balance	FS		

Foreign countries

Exports	EX	Transfer payments of households to foreign countries	VERR
./. Imports	./. IM	Transfers of private firms to foreign countries	ARSF
Net current transfers from abroad	SEVE	Current account balance	LBS
Residual item in the current account (incl. capital transfers)	DLBS		

Firms[130]

Fixed investment of firms	GBAI	Depreciation allowances	D
Private residential investment excl. investment of households	IW – IP	Gross profit income of private sector	GU
Inventory investment	V	./. Other direct taxes	./. TDSO
		./. Transfers of private firms to foreign countries	./. ARSF
		./. Other (net) income of households	./. GNEH
Financial balance	FU	./. Change in net equity of pension fund reserves	./. BV
		Net capital transfer payments of households	SVPH

Income and expenditure

Wage income	L	Private consumption	CP
Profit income	GW	Government consumption	CS
Indirect taxes	TBSP	Fixed investment	BAI
./. Subsidies	./. SUBV	Inventory investment	V
Depreciation allowances	D	Exports	EX
./. Net current transfers from abroad	./. SEVE	./. Imports	./ IM
Gross domestic product	BIP	Gross domestic product	BIP

Financial balances[131]

Households	FH	Current account balance	LBS
Government	FS	./. Residual item in the current account (incl. capital transfers)	./. DLBS
Firms	FU		

130 Please note that in order to exactly balance the accounts, the following “residual positions” would need to be included on the liabilities side of the firms’ account: 1. Employees’ and employers’ social security contributions ./. social security contributions received by government [SOZ-SOZS]; 2. Monetary transfer payments of the government ./. net monetary transfers received by households [TRNS-TRN]; 3. Residual item in government financial account ./. sales and other revenue of government [SRSS-VKSS].

131 Please note that [LBS-DLBS] on the credit side equals the financial balance of foreign countries (-FA, if defined from Germany’s perspective).

Annex II: BbkM-DE's equations[132]

Behavioural (and auxiliary) equations in the main text[133]

Potential total output (YPOT):[134]

$$ln\left(\frac{YPOT_t}{Y_0}\right) = \underset{(9.60)}{0.02} + \left(\frac{\sigma_{NZ}}{\sigma_{NZ}-1}\right) \cdot ln\left(\alpha_0 \cdot \left(\left(e^{(gn_1 \cdot \tilde{t} + gn_2 \cdot \tilde{t}^2)}\right) \cdot \frac{AVPOT_t}{N_0}\right)^{\frac{\sigma_{NZ}-1}{\sigma_{NZ}}} + (1-\alpha_0) \cdot \left(\frac{Z_t}{Z_0}\right)^{\frac{\sigma_{NZ}-1}{\sigma_{NZ}}}\right) + AFYPOT_t$$

with

$$\frac{Z_t}{Z_0} = \left(\gamma_0 \cdot \left(\frac{SCSER_t}{K_0}\right)^{\frac{\sigma_{KE}-1}{\sigma_{KE}}} + (1-\gamma_0) \cdot \left(\frac{ENERGR_t}{E_0}\right)^{\frac{\sigma_{KE}-1}{\sigma_{KE}}}\right)^{\frac{\sigma_{KE}}{\sigma_{KE}-1}}$$

where $\sigma_{NZ} = \underset{(6.47)}{0.15}$, $\sigma_{KE} = \underset{(27.22)}{0.34}$, $gn_1 = \underset{(38.92)}{0.003}$ and $gn_2 = -\underset{(15.89)}{0.00003}$

Estimation sample: 1995Q1-2019Q4

Real private consumption (CPR):

$$\Delta ln\left(\frac{CPR_t}{WOBE_t}\right) = \underset{(4.73)}{0.35} - \underset{(5.24)}{0.47} \cdot \left(ln\left(\frac{CPR_{t-1}}{WOBE_{t-1}}\right) - \left(\underset{(11.31)}{0.47} \cdot ln\left(\frac{LN_{t-1} + TRN_{t-1}}{PCP_{t-1}/100 \cdot WOBE_{t-1}}\right) + \underset{(6.29)}{0.17} \cdot ln\left(\frac{GNEH_{t-1} + BV_{t-1} - VERR_{t-1}}{PCP_{t-1}/100 \cdot WOBE_{t-1}}\right) - \underset{(3.77)}{0.08} \cdot \theta_{95} \cdot \sum_{i=4}^{7} \frac{PCPD_{t-i}}{400} + \underset{(2.23)}{0.03} \cdot ln\left(\frac{NGVD_{t-1}}{PCP_{t-1}/100 \cdot WOBE_{t-1}}\right)\right)\right) - \underset{(2.16)}{0.20} \cdot \left(\frac{RL_{t-1} - PCPD_{t-1}}{100}\right) - \underset{(4.38)}{0.25} \cdot \left(\frac{ARLQ_{t-1} - ARLQN_{t-1}}{100}\right) + \underset{(8.43)}{0.50} \cdot \Delta ln\left(\frac{LN_t + TRN_t}{PCP_t/100 \cdot WOBE_t}\right) + \underset{(7.82)}{0.21} \cdot \Delta ln\left(\frac{GNEH_t + BV_t - VERR_t}{PCP_t/100 \cdot WOBE_t}\right) - \underset{(5.27)}{0.01} \cdot DUM082_t + zz_t^{CPR}$$

Estimation sample: 1995Q1-2019Q4

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[132] Estimated coefficients and associated (absolute) t-values in the equation for potential output and in the short-run equations of the domestic prices block result from SUR estimations. Absolute t-values for individually estimated equations (except for first-stage EC specifications) are based on heteroscedasticity and autocorrelation robust standard errors. The t-values are shown in parentheses below the estimated coefficients.

[133] In chronological order as mentioned in the main text.

[134] To estimate the equation, actual total output is used. Note that labour-specific technology trend is supposed to evolve linearly in the simulation/forecasting period according to $gn_{0TB} + gn_{1TB} \cdot \tilde{t}$, where the parameters are set such that the growth rate in the last observation period is held constant over the simulation period. To differentiate between estimation (i.e. quadratic trend) and simulation period (i.e. linear trend) a shift dummy (DUMTB0) is used. The same holds for the labour demand equation (see AVBI).

Real commercial gross fixed capital formation (GBAIR):

$$\Delta ln(GBAIR_t) = \underset{(1.96)}{-0.26} - \underset{(2.03)}{0.14} \cdot \Big(ln(GBAIR_{t-1}) - \big(ln(YR_{t-1}) + \sigma_{NZ} \cdot ln(PY_{t-1}) + (\sigma_{KE} - \sigma_{NZ}) \cdot ln(PZ_{t-1}) - \sigma_{KE} \cdot ln(UC_{t-1})\big)\Big) - \underset{(3.11)}{0.0004} \cdot UBLS_{t-1} + zz_t^{GBAIR}$$

Estimation sample: 1992Q4-2019Q4

BLS credit standards for firms (UBLS):

$$UBLS_t = \underset{(6.11)}{0.58} \cdot UBLSK_t + \underset{(3.64)}{0.59} \cdot UBLSB_t + zz_t^{UBLS}$$

Estimation sample: 2002Q4-2019Q4

BLS credit standards for firms, business cycle factor (UBLSK):

$$UBLSK_t = \underset{(3.94)}{9.07} - \underset{(5.07)}{754.43} \cdot \Delta ln(BIPR_t) - \underset{(4.70)}{438.01} \cdot \Delta ln(BIPR_{t-1}) + zz_t^{UBLSK}$$

Estimation sample: 2002Q4-2019Q4

Real residential property price (PWR):

$$ln(PWR_t) = \underset{(28.12)}{17.35} + \underset{(20.48)}{2.80} \cdot ln\left(\frac{YV_t - BV_t}{\frac{PCP_t}{100} \cdot NHH_t}\right) - \underset{(5.68)}{1.74} \cdot \left(\frac{RLIW_t - PCPD_t}{100}\right) + ec_t^{PWR}$$

Estimation sample: 2003Q1-2019Q4

$$\Delta ln(PWR_t) = \underset{(2.81)}{0.004} - \underset{(1.84)}{0.11} \cdot ec_{t-1}^{PWR} + \underset{(2.11)}{0.44} \cdot \Delta ln\left(\frac{YV_t - BV_t}{\frac{PCP_t}{100} \cdot NHH_t}\right) + zz_t^{PWR}$$

Estimation sample: 2003Q2-2019Q4

Number of building permits (BAUG):

$$ln\left(\frac{BAUG_t}{NHH_t}\right) = \underset{(36.67)}{0.96} \cdot ln\left(\frac{BAUG_{t-1}}{NHH_{t-1}}\right) + \underset{(1.94)}{2.23} \cdot \Delta ln(PWR_t) - \underset{(2.28)}{10.80} \cdot \Delta\left(\frac{RLIW_{t-1} - PCP_{t-1}}{100}\right) - \underset{(6.13)}{0.18} \cdot DUM051_t + \underset{(16.12)}{0.15} \cdot DUM111_t - \underset{(7.10)}{0.15} \cdot DUM121_t + zz_t^{BAUG}$$

Estimation sample: 2004Q2-2019Q4

Real private investment in residential construction (IWR):

$$\Delta ln(IWR_t) = \underset{(3.49)}{0.19} \cdot \sum_{i=0}^{3} \Delta ln(BAUG_{t-i})/4 + \left(1 - \underset{(3.49)}{0.19}\right) \cdot \Delta ln(BIPR_t) + \underset{(34.13)}{0.05} \cdot DUM102_t + \underset{(32.81)}{0.06} \cdot DUM111_t + zz_t^{IWR}$$

Estimation sample: 2004Q1-2019Q4

Negotiated monthly wage and salary level (LTG):

$$\Delta ln\left(\frac{LTG_t \cdot (1-AFTZ_t)}{TA_t}\right) = -\underset{(4.17)}{0.46} - \underset{(4.20)}{0.09} \cdot \left(ln\left(\frac{LTG_{t-1}\cdot(1-AFTZ_{t-1})}{TA_{t-1}}\right) - \left(ln\left(\frac{PCP_{t-1}}{100}\right) + ln\left(\frac{YR_{t-1}}{AVBI_{t-1} + ARST_{t-1}\cdot\frac{SELB_{t-1}}{1000}}\right) - \underset{(4.64)}{0.002}\cdot\frac{SCSER_{t-1}}{AVBI_{t-1}}\right)\right) + \underset{(4.20)}{0.09} \cdot \Delta ln\left(\frac{YR_t}{AVBI_t + ARST_t\cdot\frac{SELB_t}{1000}}\right) + \underset{(4.57)}{0.01}\cdot ANSP_{t-1} + \underset{(2.60)}{0.26}\cdot\frac{PCPD_{t-1}}{100} - \underset{(12.92)}{0.01} \cdot (DUM063_t + DUM071_t + DUM073_t) + \underset{(8.87)}{0.01}\cdot DUM193 - \underset{(15.44)}{0.02}\cdot DUM194 + zz_t^{LTG}$$

Estimation sample: 1995Q1-2019Q4

Gross wage and salary income per hour worked (excl. employers' contribution to social security, LGAS):

$$\Delta ln(LGAS_t) = \underset{(2.30)}{0.37} - \underset{(2.30)}{0.06}\cdot\left(ln(LGAS_{t-1}) - ln\left(\frac{LTG_{t-1}\cdot(1-AFTZ_{t-1})}{TA_{t-1}}\right)\right) + \underset{(4.82)}{0.49} \cdot \Delta ln\left(\frac{LTG_t\cdot(1-AFTZ_t)}{TA_t}\right) + \underset{(5.12)}{0.67}\cdot\Delta ln\left(\frac{LTG_{t-1}\cdot(1-AFTZ_{t-1})}{TA_{t-1}}\right) - \underset{(3.80)}{0.17} \cdot \left(\frac{ARLQ_{t-1} - ARLQN_{t-1}}{100}\right) + zz_t^{LGAS}$$

Estimation sample: 1992Q3-2019Q4

Total hours worked by employees (domestic concept, AVBI):

$$\Delta ln(AVBI_t) = -\underset{(1.93)}{0.37} - \underset{(1.87)}{0.09} \cdot \left(ln(AVBI_{t-1}) - \left(ln(YR_{t-1}) - \sigma_{NZ}\cdot ln\left(\frac{LAST_{t-1}}{PY_{t-1}}\right) + (\sigma_{NZ}-1)\cdot(gn_1\cdot(\tilde{t}-1) + gn_2\cdot(\tilde{t}-1)^2) + \underset{(3.67)}{0.04} \cdot AGR_{t-1}\right)\right) + \underset{(6.68)}{0.36}\cdot\Delta ln(YR_t) - \underset{(6.65)}{0.40}\cdot\Delta ln\left(\frac{LAST_t}{PY_t\cdot e^{(gn_1\cdot\tilde{t}+gn_2\cdot\tilde{t}^2)}}\right) + \underset{(2.75)}{0.24}\cdot\Delta ln(AVBI_{t-1}) + zz_t^{AVBI}$$

Estimation sample: 1992Q2-2019Q4

Hours worked per employee (ARST):

$$\Delta ln\left(\frac{ARST_t}{TA_t}\right) = -\underset{(3.75)}{0.05}\cdot ln\left(\frac{ARST_{t-1}}{TA_{t-1}}\right) + \underset{(7.80)}{0.21}\cdot\Delta ln(BIPR_t) - \underset{(5.44)}{0.35}\cdot\Delta ln\left(\frac{LGAS_t\cdot TA_t}{LTG_t\cdot(1-AFTZ_t)}\right) + \underset{(2.29)}{0.14} \cdot\Delta ln\left(\frac{ARST_{t-2}}{TA_{t-2}}\right) - \underset{(11.28)}{0.01}\cdot DUM051_t + zz_t^{ARST}$$

Estimation sample: 1999Q1-2019Q4

Labour force participation rate (EQU):

$$ln(EQU_t) = -\underset{(12.19)}{0.57} + ln\left(\frac{WOBA_t}{WOBE_t}\right) + \underset{(30.02)}{0.20} \cdot ln(TZQ_t) + \underset{(11.43)}{0.29} \cdot ln\left(\frac{LN_t}{PCP_t/100 \cdot B1_t}\right) + ec_t^{EQU}$$

Estimation sample: 1993Q1-2019Q4

$$\Delta ln(EQU_t) = -\underset{(1.54)}{0.03} \cdot ec_{t-1}^{EQU} + \underset{(4.84)}{0.51} \cdot \Delta ln\left(\frac{WOBA_t}{WOBE_t}\right) + \left(1 - \underset{(4.84)}{0.51}\right) \cdot \Delta ln(EQU_{t-1}) + \underset{(2.80)}{0.06} \cdot \Delta ln(TZQ_t) + \underset{(9.52)}{0.01} \cdot DUM993_t + zz_t^{EQU}$$

Estimation sample: 1993Q2-2019Q4

Deflator of private investment in machinery and equipment (PIAU):

$$ln(PIAU_t \cdot TIPSIU_t \cdot AFODV_t) = \underset{(202.52)}{4.63} + \underset{(30.20)}{0.52} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(23.86)}{0.07} \cdot ln(PIMOG_t) + ec_t^{PIAU}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIAU_t \cdot TIPSIU_t) = -\underset{(3.42)}{0.002} - \underset{(2.22)}{0.06} \cdot ec_{t-1}^{PIAU} + \underset{(0.94)}{0.02} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(4.52)}{0.16} \cdot \Delta ln(PIM_t) + \underset{(4.04)}{0.31} \cdot \sum_{i=2}^{5} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + zz_t^{PIAU}$$

Estimation sample: 1992Q4-2019Q4

Deflator of private investment in (non-residential) construction (PIBU):

$$ln(PIBU_t \cdot TIPSIU_t) = \underset{(105.81)}{5.16} + \underset{(35.49)}{1.31} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(10.95)}{0.06} \cdot ln(PIMOG_t) + ec_t^{PIBU}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIBU_t \cdot TIPSIU_t) = -\underset{(3.55)}{0.01} - \underset{(1.75)}{0.03} \cdot ec_{t-1}^{PIBU} + \underset{(3.13)}{0.08} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(6.60)}{0.21} \cdot \Delta ln(PIM_t) + \underset{(0.94)}{0.06} \cdot \sum_{i=2}^{5} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + \underset{(5.95)}{0.02} \cdot ln(MUBAU_{t-1}) + \underset{(1.76)}{0.24} \cdot \Delta\left(\frac{RLIU_{t-2}}{100}\right) + \underset{(2.31)}{0.13} \cdot \Delta ln(PIBU_{t-1} \cdot TIPSIU_{t-1}) + \underset{(6.00)}{0.02} \cdot DUM071_t + zz_t^{PIBU}$$

Estimation sample: 1992Q4-2019Q4

Deflator of private investment in residential construction (PIW):

$$ln(PIW_t \cdot TIPSIW_t) = \underset{(121.05)}{5.06} + \underset{(38.60)}{1.22} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(7.40)}{0.04} \cdot ln(PIMOG_t) + ec_t^{PIW}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIW_t \cdot TIPSIW_t) = -\underset{(4.90)}{0.01} - \underset{(1.83)}{0.03} \cdot ec_{t-1}^{PIW} + \underset{(2.86)}{0.07} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(5.28)}{0.15} \cdot \Delta ln(PIM_t) + \underset{(1.94)}{0.12} \cdot \sum_{i=1}^{4} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + \underset{(8.03)}{0.03} \cdot ln(MUBAU_{t-1}) + \underset{(2.00)}{0.24} \cdot \Delta\left(\frac{RLIU_{t-2}}{100}\right) + zz_t^{PIW}$$

Estimation sample: 1992Q4-2019Q4

Deflator of other private investment (PISU):

$$ln(PISU_t \cdot TIPSIU_t) = \underset{(158.41)}{4.76} + \underset{(31.76)}{0.72} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(18.33)}{0.07} \cdot ln(PIMOG_t) + ec_t^{PISU}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PISU_t \cdot TIPSIU_t) = \underset{(13.46)}{0.003} - \underset{(3.47)}{0.05} \cdot ec_{t-1}^{PISU} + \underset{(2.85)}{0.04} \cdot \left(\frac{GAP_t}{100}\right) + \underset{(7.26)}{0.14} \cdot \Delta ln(PIM_t) + zz_t^{PISU}$$

Estimation sample: 1992Q4-2019Q4

Deflator of government consumption (PCS):

$$ln(PCS_t \cdot TIPSCS_t) = \underset{(236.88)}{4.93} + \underset{(48.64)}{0.77} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(9.75)}{0.02} \cdot ln(PIMOG_t) + ec_t^{PCS}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PCS_t \cdot TIPSCS_t) = \underset{(4.00)}{0.002} - \underset{(4.07)}{0.19} \cdot ec_{t-1}^{PCS} + \underset{(2.20)}{0.07} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(1.69)}{0.18} \cdot \sum_{i=1}^{4} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 - \underset{(5.99)}{0.03} \cdot DUM931_t + zz_t^{PCS}$$

Estimation sample: 1992Q4-2019Q4

Deflator of government investment in machinery and equipment (PIAS):

$$ln(PIAS_t \cdot TIPSIST_t \cdot AFODV_t) = \underset{(160.09)}{4.57} + \underset{(34.62)}{0.75} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(23.07)}{0.08} \cdot ln(PIMOG_t) + ec_t^{PIAS}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIAS_t \cdot TIPSIST_t) = -\underset{(2.17)}{0.001} - \underset{(1.69)}{0.04} \cdot ec_{t-1}^{PIAS} + \underset{(0.37)}{0.009} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(4.70)}{0.17} \cdot \Delta ln(PIM_t) + \underset{(4.27)}{0.32} \cdot \sum_{i=2}^{5} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + zz_t^{PIAS}$$

Estimation sample: 1992Q4-2019Q4

Deflator of government investment in construction (PIBS):

$$ln(PIBS_t \cdot TIPSIST_t) = \underset{(102.37)}{5.05} + \underset{(31.98)}{1.20} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(7.25)}{0.04} \cdot ln(PIMOG_t) + ec_t^{PIBS}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIBS_t \cdot TIPSIST_t) = -\underset{(4.57)}{0.01} - \underset{(2.30)}{0.03} \cdot ec_{t-1}^{PIBS} + \underset{(3.25)}{0.07} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(5.99)}{0.15} \cdot \Delta ln(PIM_t) + \underset{(1.47)}{0.08} \cdot \sum_{i=1}^{4} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + \underset{(6.51)}{0.02} \cdot ln(MUBAU_{t-1}) + \underset{(1.21)}{0.13} \cdot \Delta\left(\frac{RLIU_{t-2}}{100}\right) + \underset{(4.64)}{0.27} \cdot \Delta ln(PIBS_{t-1} \cdot TIPSIST_{t-1}) + zz_t^{PIBS}$$

Estimation sample: 1992Q4-2019Q4

Deflator of other government investment (PIST):

$$ln(PIST_t \cdot TIPSIST_t) = \underset{(135.52)}{4.84} + \underset{(15.05)}{0.41} \cdot ln\left(\frac{L_t}{BIPR_t}\right) - \underset{(5.42)}{0.02} \cdot ln(PIMOG_t) + ec_t^{PIST}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(PIST_t \cdot TIPSIST_t) = -\underset{(2.65)}{0.12} \cdot ec_{t-1}^{PIST} + \underset{(1.62)}{0.07} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(2.40)}{0.13} \cdot \Delta ln(PIM_t) + \underset{(2.22)}{0.23} \cdot \sum_{i=3}^{5} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/3 + zz_t^{PIST}$$

Estimation sample: 1992Q4-2019Q4

Harmonised consumer prices, all items excl. energy and food (HVPIXF):

$$ln(HVPIXF_t \cdot TIPSHXF_t) = \underset{(7.53)}{3.18} + \underset{(18.62)}{0.79} \cdot ln\left(\frac{L_t}{BIPR_t}\right) + \underset{(4.66)}{0.41} \cdot ln(PIM_t) + ec_t^{HVPIXF}$$

Estimation sample: 1991Q1-2019Q4

$$\Delta ln(HVPIXF_t \cdot TIPSHXF_t) = \underset{(9.34)}{0.002} - \underset{(3.67)}{0.02} \cdot ec_{t-1}^{HVPIXF} + \underset{(2.96)}{0.03} \cdot \left(\frac{GAP_{t-1}}{100}\right) + \underset{(2.51)}{0.09} \cdot \sum_{i=1}^{4} \Delta ln\left(\frac{L_{t-i}}{BIPR_{t-i}}\right)/4 + \underset{(1.54)}{0.07} \cdot \Delta ln(PINV_{t-1} \cdot TIPSINLV_{t-1}) + \underset{(4.85)}{0.01} \cdot DUM041_t + \underset{(6.13)}{0.01} \cdot DUM151_t + zz_t^{HVPIXF}$$

Estimation sample: 1992Q4-2019Q4

Harmonised consumer prices, food component (HVPIF):

$$ln(HVPIF_t \cdot TIPSHF_t) = \underset{(6.38)}{0.62} + \underset{(17.04)}{0.63} \cdot ln(LAST_t) + \underset{(9.90)}{0.19} \cdot ln(DGFI_t) + ec_t^{HVPIF}$$

Estimation sample: 1998Q1-2019Q4

$$\Delta ln(HVPIF_t \cdot TIPSHF_t) = \underset{(4.36)}{0.004} - \underset{(4.40)}{0.15} \cdot ec_{t-1}^{HVPIF} + \underset{(1.65)}{0.04} \cdot \Delta ln(DGFI_t) + zz_t^{HVPIF}$$

Estimation sample: 1998Q2-2019Q4

Harmonised consumer prices, energy component (HVPIE):

$$\Delta(HVPIE_t \cdot TIPSHE_t) = \underset{(17.79)}{40.01} \cdot \Delta\left(WE\_USDPOIL_t \cdot \frac{ER_t}{159} + ENST_t\right) + \underset{(4.18)}{0.19} \cdot \Delta(HVPIE_{t-1} \cdot TIPSHE_{t-1}) + \underset{(1.07)}{0.05} \cdot \Delta(HVPIE_{t-2} \cdot TIPSHE_{t-2}) + \underset{(2.44)}{0.11} \cdot \Delta(HVPIE_{t-3} \cdot TIPSHE_{t-3}) + \underset{(2.66)}{0.13} \cdot \Delta(HVPIE_{t-4} \cdot TIPSHE_{t-4}) + \underset{(22.78)}{1.82} \cdot DUM031_t + \underset{(3.91)}{2.14} \cdot DUM084_t - \underset{(7.09)}{2.10} \cdot DUM092_t + zz_t^{HVPIE}$$

Estimation sample: 1995Q2-2019Q4

Real exports of goods and services (EXR):

$$\Delta ln(EXR_t) = \underset{(4.84)}{3.04} - \underset{(5.33)}{0.39} \cdot \left(ln(EXR_{t-1}) - \left(ln(DE\_WDR_{t-1}) - \underset{(4.32)}{0.40} \cdot ln\left(WE\_RAW_{t-1_{t-1}}\right) + \underset{(2.52)}{0.14} \cdot ln\left(\frac{WE\_IMPR_{t-1}}{WE\_GDPR_{t-1}}\right)\right)\right) + \underset{(10.50)}{1.12} \cdot \Delta ln(DE\_WDR_t) + zz_t^{EXR}$$

Estimation sample: 1996Q2-2019Q4

Real non-oil and non-gas imports (NEIMR):

$$\Delta ln(NEIMR_t) = \underset{(2.71)}{0.01} - \underset{(3.54)}{0.35} \cdot \left(ln(NEIMR_{t-1}) - \left(ln(WER_{t-1}) - \underset{(12.30)}{0.90} \cdot ln\left(\frac{PNEIM_{t-1}}{PEV_{t-1} \cdot TIPS_{t-1}}\right)\right)\right) + \underset{(10.91)}{0.80} \cdot \Delta ln(WER_t) + zz_t^{NEIMR}$$

Estimation sample: 1998Q1-2017Q4

Real energy input (ENERGR):

$$\Delta ln(ENERGR_t) = -\underset{(1.83)}{0.14} - \underset{(1.80)}{0.06} \cdot \left(ln(ENERGR_{t-1}) - \left(ln(YR_{t-1}) + \sigma_{NZ} \cdot ln(PY_{t-1}) + (\sigma_{KE} - \sigma_{NZ}) \cdot ln(PZ_{t-1}) - \sigma_{KE} \cdot ln(PE_{t-1})\right)\right) + \underset{(3.35)}{0.77} \cdot \Delta ln(YR_t) - \underset{(2.17)}{0.04} \cdot \Delta ln(PIMOG_t) + zz_t^{ENERGR}$$

Estimation sample: 1997Q1-2019Q4

Real oil and gas imports (IMOGR):

$$\Delta ln(IMOGR_t) = -\underset{(3.59)}{0.16} - \underset{(3.78)}{0.64} \cdot \left(ln(IMOGR_{t-1}) - ln(ENERGR_{t-1})\right) - \underset{(4.91)}{0.28} \cdot \Delta ln\left(\frac{PIMOG_t}{PEV_t \cdot TIPS_t}\right) + zz_t^{IMOGR}$$

Estimation sample: 2002Q1-2019Q4

Deflator of exports of goods and services (PEX):

$$\Delta ln(PEX_t) = \underset{(1.71)}{0.09} - \underset{(2.32)}{0.07} \cdot \left(log(PEX_{t-1}) - \left(\underset{(6.04)}{0.82} \cdot ln(PEV_{t-1} \cdot TIPS_{t-1}) + \left(1 - \underset{(6.04)}{0.82}\right) \cdot ln\left(\frac{WE\_PEVF_{t-1}}{WE\_NAW_{t-1}}\right) - \underset{(5.36)}{0.21} \cdot EXIMTREND_{t-1} \cdot ln(WE\_NAW_{t-1})\right)\right) + \underset{(6.05)}{0.67} \cdot \Delta ln(PEV_t \cdot TIPS_t) + \underset{(5.63)}{0.15} \cdot \Delta ln\left(\frac{WE\_PEVF_t}{WE\_NAW_t}\right) + \underset{(5.10)}{0.27} \cdot \Delta ln(PEX_{t-1}) + zz_t^{PEX}$$

Estimation sample: 1996Q2-2019Q4

Deflator of non-oil and non-gas imports (PNEIM):

$$\Delta ln(PNEIM_t) = \underset{(3.43)}{0.50} - \underset{(3.90)}{0.43} \cdot \left(ln(PNEIM_{t-1}) - \left(\underset{(7.33)}{0.32} \cdot ln\left(\frac{WE\_PEVF_{t-1}}{WE\_NAW_{t-1}}\right) + \underset{(6.60)}{0.05} \cdot ln(WE\_PCOM_{t-1} \cdot ER_{t-1}) + \left(1 - \underset{(7.33)}{0.32} - \underset{(6.60)}{0.05}\right) \cdot ln(PEV_{t-1} \cdot TIPS_{t-1}) + \underset{(33.26)}{0.82} \cdot CWER_{t-1}\right)\right) + \underset{(5.39)}{0.25} \cdot \Delta ln\left(\frac{WE\_PEVF_t}{WE\_NAW_t}\right) + \underset{(3.02)}{0.69} \cdot \Delta ln(PEV_t \cdot TIPS_t) + zz_t^{PNEIM}$$

Estimation sample: 2000Q1-2019Q4

Deflator of oil and gas imports (PIMOG):

$$\Delta ln(PIMOG_t) = \underset{(30.30)}{0.56} \cdot \Delta ln(WE\_USDPOIL_t \cdot ER_t) + \underset{(10.39)}{0.14} \cdot \Delta ln(WE\_USDPOIL_{t-1} \cdot ER_{t-1}) + \underset{(6.33)}{0.15} \cdot \Delta ln(WE\_USDPOIL_{t-2} \cdot ER_{t-2}) + \underset{(2.38)}{0.04} \cdot \Delta ln(WE\_USDPOIL_{t-3} \cdot ER_{t-3}) + zz_t^{PIMOG}$$

Estimation sample: 1996Q1-2019Q4

Nominal government consumption (CS):

$$\Delta\left(\frac{CS_t}{PBIP_t/100 \cdot BIPQ_t}\right) = \underset{(2.49)}{0.02} - \underset{(3.42)}{0.22} \cdot \left(\left(\frac{CS_{t-1}}{PBIP_{t-1}/100 \cdot BIPQ_{t-1}}\right) - \underset{(6.92)}{1.18} \cdot \left(\frac{LG_{t-1} \cdot (1 - BIPRIV_{t-1}/BI_{t-1})}{PBIP_{t-1}/100 \cdot BIPQ_{t-1}}\right)\right) + \underset{(2.35)}{0.20} \cdot \Delta\left(\frac{CS_{t-4}}{PBIP_{t-4}/100 \cdot BIPQ_{t-4}}\right) + \underset{(1.96)}{0.01} \cdot \Delta\left(\frac{FS_{t-4}}{PBIP_{t-4}/100 \cdot BIPQ_{t-4}}\right) + zz_t^{CS}$$

Estimation sample: 1993Q3-2019Q4

Monetary transfers paid by the government (TRNS):

$$\Delta ln(TRNS_t) = \underset{(3.20)}{0.17} \cdot \Delta ln(TRNS_{t-1}) + \underset{(14.63)}{0.84} \cdot \Delta ln(TRN_t) + zz_t^{TRNS}$$

Estimation sample: 1992Q1-2019Q4

Monetary transfer payments received by households (TRN):

$$ln(TRN_t) = -\underset{(10.76)}{1.19} + \underset{(54.48)}{1.05} \cdot ln\left(\frac{LG_t \cdot WOBS_t}{B1_t}\right) + \underset{(11.55)}{2.48} \cdot \left(\frac{ARLQN_t}{100}\right) + ec_t^{TRN}$$

Estimation sample: 1992Q2-2019Q4

$$\Delta ln(TRN_t) = \underset{(7.34)}{0.01} - \underset{(2.66)}{0.11} \cdot ec_{t-1}^{TRN} - \underset{(1.90)}{0.24} \cdot \Delta ln(BIPR_{t-1}) + \underset{(3.29)}{1.94} \cdot \Delta\left(\frac{ARLQ_t}{100}\right) + zz_t^{TRN}$$

Estimation sample: 1992Q3-2019Q4

Interest rate on government debt (RZIN):

$$RZIN_t = \underset{(48.62)}{0.93} \cdot RZIN_{t-1} + \left(1 - \underset{(48.62)}{0.93}\right) \cdot \sum_{i=1}^{4} \frac{RL_{t-i}}{4} + zz_t^{RZIN}$$

Estimation sample: 1992Q1-2019Q4

Government subsidies to private firms (SUBV):

$$\frac{SUBV_t}{PBIP_t/100 \cdot BIPQ_t} = \underset{(35.26)}{0.01} - \underset{(2.15)}{0.04} \cdot \left(\frac{GAP_t}{100}\right) + zz_t^{SUBV}$$

Estimation sample: 2007Q1-2019Q4

Social security contributions received by government (auxiliary equation, SOZS):

$$\Delta ln(SOZS_t) = \Delta ln(SOZ_t) + zz_t^{SOZS}$$

Employers' contributions to social security (SZAF):

$$ln\left(\frac{SZAF_t}{B1_t}\right) = -\underset{(51.48)}{4.28} + \underset{(57.34)}{0.84} \cdot ln\left(\frac{LG_t \cdot (SOZB_t - DUM053184_t)}{B1_t}\right) + ec_t^{SZAF}$$

Estimation sample: 1995Q1-2019Q4

$$\Delta ln\left(\frac{SZAF_t}{B1_t}\right) = -\underset{(3.41)}{0.27} \cdot ec_{t-1}^{SZAF} + \underset{(3.87)}{0.61} \cdot \Delta ln\left(\frac{LG_t}{B1_t}\right) + zz_t^{SZAF}$$

Estimation sample: 1995Q2-2019Q4

Employees' social security contributions (SOZN):

$$ln\left(\frac{SOZN_t}{B1_t}\right) = -\underset{(46.61)}{4.67} + \underset{(46.41)}{0.84} \cdot ln\left(\frac{LG_t \cdot (SOZB_t + DUM053184_t)}{B1_t}\right) + \underset{(28.81)}{0.01} \cdot \left(\frac{LG_t}{B1_t} \cdot \frac{RIEA_t}{B1_t} \cdot RIES_t\right) + ec_t^{SOZN}$$

Estimation sample: 1995Q1-2019Q4

$$\Delta ln\left(\frac{SOZN_t}{B1_t}\right) = \underset{(1.52)}{0.003} - \underset{(6.36)}{0.63} \cdot ec_{t-1}^{SOZN} + \underset{(2.37)}{0.62} \cdot \Delta ln\left(\frac{LG_t}{B1_t}\right) + \underset{(9.44)}{0.79} \cdot \Delta ln(SOZB_t + DUM053184_t) + zz_t^{SOZN}$$

Estimation sample: 1995Q2-2019Q4

Tax on wage and salary income (LOST):

$$\Delta ln(LOST_t) = \left(1 - \underset{(2.89)}{0.19}\right) \cdot \left(-\underset{(3.12)}{0.004} + ELOST01 \cdot \Delta ln\left(\frac{LG_t}{B1_t}\right) + ELOST02 \cdot \Delta ln(B1_t)\right) + \underset{(2.89)}{0.19} \cdot \Delta ln(LOST_{t-1}) + zz_t^{LOST}$$

where $ELOST01 = 1.8$ and $ELOST02 = 1$

Estimation sample: 1995Q1-2019Q4

Other direct taxes (TDSO):

$$\Delta ln(TDSO_t) = -\underset{(3.47)}{0.16} - \underset{(4.30)}{0.13} \cdot \left(ln(TDSO_{t-1}) - ln(GU_{t-1})\right) + \underset{(1.80)}{0.55} \cdot \left(\frac{GAP_t}{100}\right) - \underset{(4.41)}{0.52} \cdot \Delta ln(TDSO_{t-1}) - \underset{(3.53)}{0.27} \cdot c_4 \Delta ln(TDSO_{t-8}) - \underset{(2.04)}{0.25} \cdot \Delta ln(TDSO_{t-11}) + \underset{(3.92)}{0.03} \cdot GSTRA_{t-4} + zz_t^{TDSO}$$

Estimation sample: 2004Q1-2019Q4

Value added tax (UST):

$$UST_t = CP_t \cdot \left(\frac{\phi_0^{CP} \cdot MWST_t}{1 + MWST_t}\right) + CP_t \cdot \left(\frac{\phi_0^{CPR} \cdot MWSTR_t}{1 + MWSTR_t}\right) + CS_t \cdot \left(\frac{\phi_0^{CS} \cdot MWST_t}{1 + MWST_t}\right) + IW_t \cdot \left(\frac{\phi_0^{IW} \cdot MWST_t}{1 + MWST_t}\right) + (IAS_t + IBS_t + ISS_t) \cdot \left(\frac{\phi_0^{IST} \cdot MWST_t}{1 + MWST_t}\right) - \underset{(21.46)}{3.54} - \underset{(18.00)}{0.05} \cdot T\_UST\_99 + zz_t^{UST}$$

Estimation sample: 1999Q1-2019Q4

Energy tax (auxiliary equation, TBENST):

$$TBENST_t = \underset{(10.11)}{0.48} + ENST_t \cdot ENMOEGHH_t + ENSTU_t \cdot ENMOEGU_t + zz_t^{TBENST}$$

Estimation sample: 1991Q1-2019Q4

Other indirect taxes (auxiliary equation, TBSO):

$$TBSO_t = TBENST_t \cdot \left(1 + \underset{(32.13)}{0.16}\right) + TBREST_t + zz_t^{TBSO}$$

Estimation sample: 2000Q1-2019Q4

Other indirect taxes (excl. energy and electricity taxes, TBREST):

$$\Delta ln(TBREST_t) = -\underset{(3.42)}{0.44} - \underset{(3.49)}{0.14} \cdot \left(ln(TBREST_{t-1}) - ln(CPR_{t-1})\right) + \underset{(2.37)}{0.30} \cdot \Delta ln(TBREST_{t-3}) + zz_t^{TBREST}$$

Estimation sample: 2002Q1-2019Q4

Valuation effects on net financial wealth of households at market prices (BEW):

$$\frac{BEW_t}{NGV_{t-1}} = -\underset{(3.28)}{0.004} + \underset{(11.12)}{0.17} \cdot \Delta ln(CDAX_t) + \left(1 - \underset{(11.12)}{0.17}\right) \cdot \Delta ln(PBIP_t) - \underset{(10.06)}{0.08} \cdot DUM9193_t \cdot Q1 + \underset{(5.35)}{0.09} \cdot DUM9193_t \cdot Q4 + zz_t^{BEW}$$

Estimation sample: 1991Q2-2019Q4

Equity price index CDAX:[135]

$$ln(CDAX_t) = \underset{(126.19)}{0.87} \cdot ln(GU_t) - \underset{(4.38)}{5.78} \cdot \left(\frac{RL_t - PCPD_t}{100}\right) - \underset{(1.31)}{0.88} \cdot \Delta ln(GU_t) + \underset{(2.71)}{23.07} \cdot \Delta\left(\frac{RL_t - PCPD_t}{100}\right) - \underset{(0.18)}{0.12} \cdot \Delta log(GU_{t-1}) + \underset{(1.91)}{16.52} \cdot \Delta\left(\frac{RL_{t-1} - PCPD_{t-1}}{100}\right) + ec_t^{CDAX}$$

Estimation sample: 1995Q1-2019Q4

$$\Delta ln(CDAX_t) = -\underset{(4.22)}{0.16} \cdot ec_{t-1}^{CDAX} - \underset{(1.71)}{7.82} \cdot \Delta\left(\frac{RS_{t-1} - PCPD_{t-1}}{100}\right) + \underset{(2.38)}{3.51} \cdot \Delta\left(\frac{GAP_{t-1}}{100}\right) + \underset{(2.66)}{10.01} \cdot \Delta\left(\frac{WRAG_t}{100}\right) + zz_t^{CDAX}$$

Estimation sample: 1995Q2-2019Q4

Long-term bank interest rate on loans to non-financial corporations (RLIU):

$$\Delta RLIU_t = \underset{(4.45)}{0.28} - \underset{(4.62)}{0.17} \cdot \left(RLIU_{t-1} - \left(\underset{(4.90)}{0.45} \cdot RL_{t-1} + \underset{(5.39)}{0.48} \cdot RS_{t-1}\right)\right) + \underset{(6.69)}{0.29} \cdot \Delta RL_t + \underset{(7.90)}{0.24} \cdot \Delta RS_t + \underset{(9.28)}{0.28} \cdot DUM111_t + \underset{(14.06)}{0.27} \cdot DUM112_t + zz_t^{RLIU}$$

Estimation sample: 2003Q1-2019Q4

Long-term bank interest rate on household mortgage loans (RLIW):

$$\Delta RLIW_t = \underset{(2.33)}{0.40} - \underset{(2.26)}{0.25} \cdot \left(RLIW_{t-1} - \left(\underset{(15.75)}{0.69} \cdot RL_{t-1} + \underset{(3.07)}{0.14} \cdot RS_{t-1}\right)\right) + \underset{(11.43)}{0.33} \cdot \Delta RL_t + \underset{(6.70)}{0.31} \cdot \Delta RL_{t-1} + \underset{(1.13)}{0.03} \cdot \Delta RS_t + \underset{(4.41)}{0.09} \cdot DUM034_t + \underset{(7.22)}{0.11} \cdot DUM083_t + zz_t^{RLIW}$$

Estimation sample: 2003Q1-2019Q4

Yield on 10-year government bonds (RL):

$$\Delta RL_t = -\underset{(2.42)}{0.09} \cdot \left(RL_{t-1} - (RS_{t-1} + TPREM_0)\right) + \underset{(5.15)}{0.45} \cdot \Delta RS_t - \underset{(4.63)}{0.37} \cdot \Delta RS_{t-1} + \underset{(2.36)}{0.22} \cdot \Delta RL_{t-1} + zz_t^{RL}$$

Estimation sample: 2003Q1-2019Q4

[135] The long-run specification shown is the result of a dynamic OLS estimation, where the lag length is determined via an information criterion.

Other behavioural or auxiliary equations not explicitly shown in the main text[136]

Transfers of private firms to foreign countries (ARSF):

$$ARSF_t = \underset{(4.57)}{3.92} + \underset{(2.04)}{0.31} \cdot ARSF_{t-4} + \underset{(3.13)}{33.22} \cdot \left(\frac{EX_t - IM_t}{BIP_t}\right) + zz_t^{ARSF}$$

Estimation sample: 1992Q1-2019Q4

Number of employees in private sector (auxiliary equation, domestic concept, BIPRIV):

$$\Delta ln(BIPRIV_t) = \Delta ln(BI_t) + zz_t^{BIPRIV}$$

Real gross value added (auxiliary equation, BWSR):

$$\Delta ln(BWSR_t) = \Delta ln(BIPR_t) + zz_t^{BWSR}$$

Energy input in litres of crude oil (auxiliary equation, ENERGL):

$$ENERGL_t = \underset{(1.15)}{0.01} + \underset{(13918.34)}{2.50} \cdot ENERGR_t + zz_t^{ENERGL}$$

Estimation sample: 1991Q1-2019Q4

Other income (net) of households (GNEH):

$$ln(GNEH_t) = \underset{(4.58)}{0.49} + \underset{(14.82)}{0.71} \cdot ln(GNEH_{t-1}) + \underset{(5.50)}{0.18} \cdot ln(GU_{t-1} - TDSO_{t-1}) + zz_t^{GNEH}$$

Estimation sample: 1995Q1-2019Q4

Real private investment in machinery and equipment (auxiliary equation, IAUR):

$$\Delta ln(IAUR_t) = \underset{(18.25)}{1.48} \cdot \Delta ln(GBAIR_t) + zz_t^{IAUR}$$

Estimation sample: 2000Q1-2019Q4

Net investment of households (IP):

$$\Delta(IP_t) = \underset{(5.72)}{12.02} - \underset{(5.81)}{0.45} \cdot \left(IP_{t-1} - \left(\underset{(10.09)}{0.51} \cdot IW_{t-1} - \underset{(21.62)}{1.24} \cdot DWR_{t-1}\right)\right) + \underset{(3.01)}{0.20} \cdot \Delta(IP_{t-2}) + \underset{(2.48)}{0.25} \cdot \Delta(IP_{t-3}) + zz_t^{IP}$$

Estimation sample: 1992Q1-2019Q4

Taxes on products less subsidies on products (auxiliary equation, NGTST):

$$NGTST_t = \mu_{GTST05} \cdot TBSP_t - \mu_{GTSU05} \cdot SUBV_t + zz_t^{NGTST}$$

[136] In alphabetical order.

Deflator of private consumption (auxiliary equation, PCP):

$$\Delta ln(PCP_t) = \Delta ln(HVPI_t) - \underset{(3.02)}{0.03} \cdot \left(\Delta ln(HVPIE_t) - \Delta ln(HVPIX_t)\right) + zz_t^{PCP}$$

Estimation sample: 1997Q2-2019Q4

Deflator of energy input (auxiliary equation, PE):

$$\Delta ln\left(PE_t \cdot (1 - TAUPEB_t)\right) = \underset{(1.45)}{0.005} + \underset{(15.13)}{0.62} \cdot \Delta ln(PIMOG_t) + zz_t^{PE}$$

Estimation sample: 2000Q1-2019Q4

Number of commuters (PEND):

$$PEND_t = \underset{(3.13)}{0.006} + \underset{(50.25)}{0.95} \cdot PEND_{t-1} + zz_t^{PEND}$$

Estimation sample: 1992Q2-2019Q4

Deflator of non-residential construction capital stock (PKB):

$$\Delta ln(PKB_t) = \underset{(19.89)}{0.69} \cdot \Delta ln(PKB_{t-1}) + \underset{(7.84)}{0.13} \cdot \Delta ln(PIBU_t) + \underset{(4.65)}{0.10} \cdot \Delta ln(PIBU_{t-1}) + \underset{(3.54)}{0.06} \cdot \Delta ln(PIBU_{t-2}) + zz_t^{PKB}$$

Estimation sample: 1992Q2-2019Q4

Deflator of capital stock in cars (PKF):

$$\Delta ln(PKF_t) = \underset{(17.37)}{0.94} \cdot \Delta ln(PKF_{t-1}) + \underset{(1.59)}{0.02} \cdot \Delta ln(PIAU_t) + \underset{(1.94)}{0.02} \cdot \Delta ln(PIAU_{t-1}) + \underset{(1.67)}{0.0004} \cdot DUM0009 + zz_t^{PKF}$$

Estimation sample: 1992Q2-2019Q4

Deflator of capital stock in machinery (PKM):

$$\Delta ln(PKM_t) = \underset{(2.89)}{0.0003} + \underset{(19.31)}{0.79} \cdot \Delta ln(PKM_{t-1}) + \underset{(3.78)}{0.05} \cdot \Delta ln(PIAU_t) + \underset{(5.54)}{0.07} \cdot \Delta ln(PIAU_{t-1}) + \underset{(2.86)}{0.04} \cdot \Delta ln(PIAU_{t-2}) + zz_t^{PKF}$$

Estimation sample: 1992Q2-2019Q4

Deflator of other capital stock (PKS):

$$\Delta ln(PKS_t) = \underset{(39.40)}{0.88} \cdot \Delta ln(PKS_{t-1}) + \underset{(6.76)}{0.12} \cdot \Delta ln(PISU_t) + zz_t^{PKS}$$

Estimation sample: 1992Q2-2019Q4

Deflator of residential construction capital stock (PKW):

$$\Delta ln(PKW_t) = \underset{(9.99)}{0.55} \cdot \Delta ln(PKW_{t-1}) + \underset{(8.48)}{0.14} \cdot \Delta ln(PIW_t) + \underset{(5.99)}{0.14} \cdot \Delta ln(PIW_{t-1}) + \underset{(4.02)}{0.11} \cdot \Delta ln(PIW_{t-2}) + \underset{(2.60)}{0.06} \cdot \Delta ln(PIW_{t-3}) + zz_t^{PKW}$$

Estimation sample: 1992Q2-2019Q4

Net capital transfer payments of households (SVPH):

$$SVPH_t = -\underset{(4.56)}{0.71} + \underset{(9.01)}{0.65} \cdot SVPH_{t-1} + zz_t^{SVPH}$$

Estimation sample: 1991Q2-2019Q4

Nominal inventory investment (V):

$$V_t = \underset{(12.65)}{0.67} \cdot V_{t-1} + \underset{(4.64)}{0.15} \cdot \Delta(END_{t-1} - V_{t-1}) + zz_t^V$$

Estimation sample: 1991Q3-2019Q4

Transfer payments of households to foreign countries (VERR):

$$VERR_t = \underset{(1.56)}{0.10} + \underset{(6.07)}{0.59} \cdot VERR_{t-1} + \underset{(3.04)}{0.33} \cdot VERR_{t-2} + \underset{(1.49)}{1.57} \cdot \Delta ln(BIP_t) + zz_t^{VERR}$$

Estimation sample: 1992Q1-2019Q4

Vacancy rate (auxiliary equation, VQ):

$$\Delta\left(\frac{VQ_t}{100}\right) = \underset{(7.15)}{0.65} \cdot \Delta\left(\frac{VQ_{t-1}}{100}\right) + \underset{(1.69)}{0.03} \cdot \Delta ln(AVBI_{t-1}) + \underset{(16.57)}{0.002} \cdot DUM981_t + \underset{(23.18)}{0.001} \cdot DUM051_t + zz_t^{VQ}$$

Estimation sample: 1992Q3-2019Q4

Real inventory investment (VR):

$$VR_t = \underset{(24.35)}{0.85} \cdot VR_{t-1} + \underset{(3.04)}{0.14} \cdot \Delta(ENDR_{t-1} - VR_{t-1}) + zz_t^{VR}$$

Estimation sample: 1991Q3-2019Q4

Identities[137]

$$AFSELB_t = \left(\frac{SELB_t}{BI_t + SELB_t}\right) \cdot (1 - WOSVT_t)$$

$$AFTZ_t = TZQ_t \cdot (1 - WOTV_t)$$

$$ANSP_t = \frac{VQ_t}{ARLQ_t}$$

$$ARL_t = EW_t - B1_t - SELB_t$$

$$ARLQ_t = \frac{ARL_t}{EW_t} \cdot 100$$

$$ARLQN_t = 0.9 \cdot ARLQN_{t-1} + 0.1 \cdot ARLQ_t$$

$$AUSL_t = \frac{YR_t}{YPOT_t} \cdot 100$$

$$AVPOT_t = \left(EW_t \cdot \left(1 - \frac{ARLQN_t}{100}\right) + PEND_t\right) \cdot (1 - AFSELB_t) \cdot \frac{TA_t}{1000}$$

$$B1_t = BI_t - PEND_t$$

$$BAI_t = IA_t + IB_t + IS_t$$

$$BI_t = \frac{AVBI_t}{ARST_t} \cdot 1000$$

$$BIG_t = BAI_t + V_t$$

$$BIGR_t = BAIR_t + VR_t$$

$$BIP_t = END_t - IM_t$$

$$BIPQ_t = \frac{BIPR_t}{AUSL_t} \cdot 100$$

$$BSP_t = BIP_t + SEVE_t$$

137 In alphabetical order. For identities that did not hold exactly over the past, a residual is included in the equation in order to ensure the tracking of historical series when running counterfactual simulations.

$$BVS_t = BVS_{t-1} - FS_t + zz_t^{BVS}$$

$$BWS_t = BWS_t - NGTST_t$$

$$CCK_t = CCKB_t + CCKF_t + CCKM_t + CCKS_t + CCKW_t$$

$$CCKB_t = \frac{UCKB_t}{100} \cdot KBR_t$$

$$CCKF_t = \frac{UCKF_t}{100} \cdot KFR_t$$

$$CCKM_t = \frac{UCKM_t}{100} \cdot KMR_t$$

$$CCKS_t = \frac{UCKS_t}{100} \cdot KSR_t$$

$$CCKW_t = \frac{UCKW_t}{100} KWR_t$$

$$CP_t = CPR_t \cdot \frac{PCP_t}{100}$$

$$CSER_t = CSER_{t-1} \cdot e^{CSERGR_t/100}$$

$$CSERGR_t = \big(TVQKB_t \cdot \Delta ln(KBR_t) + TVQKF_t \cdot \Delta ln(KFR_t) + TVQKM_t \cdot \Delta ln(KMR_t) + TVQKS_t \cdot \Delta ln(KSR_t) + TVQKW_t \cdot \Delta ln(KWR_t)\big) \cdot 100$$

$$CSR_t = \frac{CS_t}{PCS_t} \cdot 100$$

$$\Delta(D_t) = \Delta\left(\frac{DASQ_t \cdot PIAU_t \cdot KASR_t + DBQ_t \cdot PIBU_t \cdot KBR_t + DWQ_t \cdot PIW_t \cdot KWR_t}{40000}\right) + zz_t^D$$

$$DAR_t = \frac{DAQ_t \cdot KAR_t}{400}$$

$$DASR_t = \frac{DASQ_t \cdot KASR_t}{400}$$

$$DBR_t = \frac{DBQ_t \cdot KBR_t}{400}$$

$$DSR_t = \frac{DSQ_t \cdot KSR_t}{400}$$

$$DWR_t = \frac{DWQ_t \cdot KWR_t}{400}$$

$$END_t = INLV_t + EX_t$$

$$ENERG_t = ENERGR_t \cdot \frac{PE_t}{100}$$

$$ENMOEGHH_t = GAMOEGHH_t \cdot GAMOEG_t \cdot ENERGL_t$$

$$ENMOEGU_t = (1 - GAMOEGHH_t) \cdot GAMOEG_t \cdot ENERGL_t$$

$$EURLPOIL_t = WE\_USDPOIL_t \cdot \frac{ER_t}{159}$$

$$EW_t = EQU_t \cdot WOBE_t + SELB_t$$

$$EX_t = EXR_t \cdot \frac{PEX_t}{100}$$

$$FA_t = -(EX_t - IM_t) + ARSF_t + VERR_t - SEVE_t$$

$$FH_t = YV_t - CP_t - IP_t - SVPH_t$$

$$FS_t = SEIN_t - (CS_t + IAS_t + IBS_t + ISS_t + TRNS_t + SUBV_t + ZINS_t + SRSS_t)$$

$$FU_t = -FH_t - FS_t - FA_t$$

$$GAP_t = AUSL_t - 100$$

$$GBAI_t = IAU_t + IBU_t + ISU_t$$

$$GU_t = GW_t - GST_t + ZINS_t$$

$$GW_t = BSP_t - (L_t + TBSP_t - SUBV_t + D_t)$$

$$IA_t = IAU_t + IAS_t$$

$$IAIS_t = IA_t + IS_t$$

$$IASR_t = \frac{IAS_t}{PIAS_t} \cdot 100$$

$$IAU_t = IAUR_t \cdot \frac{PIAU_t}{100}$$

$$IB_t = IBU_t + IBS_t + IW_t$$

$$IBSR_t = \frac{IBS_t}{PIBS_t} \cdot 100$$

$$IBU_t = IBUR_t \cdot \frac{PIBU_t}{100}$$

$$IM_t = IMOG_t + NEIM_t$$

$$IMOG_t = IMOGR_t \cdot \frac{PIMOG_t}{100}$$

$$INLV_t = CP_t + CS_t + BAI_t + V_t$$

$$IS_t = ISU_t + ISS_t$$

$$ISSR_t = \frac{ISS_t}{PIST_t} \cdot 100$$

$$IST_t = IAS_t + IBS_t + ISS_t$$

$$ISUR_t = \frac{ISU_t}{PISU_t} \cdot 100$$

$$IW_t = IWR_t \cdot \frac{PIW_t}{100}$$

$$KAR_t = 2 \cdot KAR_{t-1} - KAR_{t-2} + \Delta(IAR_t) - \Delta(DAR_t) + zz_t^{KAR}$$

$$KASR_t = 2 \cdot KASR_{t-1} - KASR_{t-2} + \Delta(IAR_t + ISR_t) - \Delta(DASR_t) + zz_t^{KASR}$$

$$KASUR_t = AUA_t \cdot KASR_t$$

$$KASURD_t = \frac{KASUR_t + KASUR_{t-1}}{2}$$

$$KBR_t = 2 \cdot KBR_{t-1} - KBR_{t-2} + \Delta(IBR_t - IWR_t) - \Delta(DBR_t) + zz_t^{KBR}$$

$$KBUR_t = AUB_t \cdot KBR_t$$

$$KBURD_t = \frac{KBUR_t + KBUR_{t-1}}{2}$$

$$KFR_t = KAR_t - KMR_t + zz_t^{KFR}$$

$$KMR_t = KAR_t \cdot \frac{KMRQ_t}{100}$$

$$KSR_t = 2 \cdot KSR_{t-1} - KSR_{t-2} + \Delta(ISR_t) - \Delta(DSR_t) + zz_t^{KSR}$$

$$KWR_t = 2 \cdot KWR_{t-1} - KWR_{t-2} + \Delta(IWR_t) - \Delta(DWR_t) + zz_t^{KWR}$$

$$KWRD_t = \frac{KWR_t + KWR_{t-1}}{2}$$

$$L_t = LG_t + SZAF_t$$

$$LAST_t = \left(\frac{L_t \cdot 1000}{B1_t \cdot ARST_t}\right) \cdot GALAST0$$

$$LBS_t = -FA_t + DLBS_t$$

$$LG_t = \frac{LGAS_t \cdot AVBI_t \cdot B1_t}{BI_t \cdot GALG0}$$

$$LN_t = LG_t - LOST_t - SOZN_t$$

$$LPOT_t = \frac{BWS_t - UC_t/100 \cdot SCSER_t}{AVPOT_t}$$

$$NEIM_t = NEIMR_t \cdot \frac{PNEIM_t}{100}$$

$$NGV_t = NGV_{t-1} + BEW_t + FH_t$$

$$NGVA_t = NGVA_{t-1} + FA_t$$

$$NGVD_t = \frac{NGV_t + NGV_{t-1}}{2}$$

$$NGVH_t = NGVH_{t-1} + FH_t$$

$$NGVS_t = NGVS_{t-1} + FS_t$$

$$NGVU_t = NGVU_{t-1} + FU_t$$

$$PBIP_t = \frac{BIP_t}{BIPR_t} \cdot 100$$

$$PBWS_t = \frac{BWS_t}{BWSR_t} \cdot 100$$

$$PCPD_t = 0.9 \cdot PCPD_{t-1} + 0.1 \cdot \Delta_4 ln(PCP_{t-1}) \cdot 100$$

$$PEV_t = \frac{END_t}{ENDR_t} \cdot 100$$

$$PEVD_t = 0.9 \cdot PEVD_{t-1} + 0.1 \cdot \Delta_4 ln(PEV_{t-1}) \cdot 100$$

$$PIM_t = \frac{IM_t}{IMR_t} \cdot 100$$

$$PINV_t = \frac{INLV_t}{INVR_t} \cdot 100$$

$$PKBD_t = 100 \cdot \Delta_4 ln(PKB_t) + 0.9 \cdot \left(PKBD_{t-1} - 100 \cdot \Delta_4 ln(PKB_t)\right)$$

$$PKFD_t = 100 \cdot \Delta_4 ln(PKF_t) + 0.9 \cdot \left(PKFD_{t-1} - 100 \cdot \Delta_4 ln(PKF_t)\right)$$

$$PKMD_t = 100 \cdot \Delta_4 ln(PKM_t) + 0.9 \cdot \left(PKMD_{t-1} - 100 \cdot \Delta_4 ln(PKM_t)\right)$$

$$PKSD_t = 100 \cdot \Delta_4 ln(PKS_t) + 0.9 \cdot \left(PKSD_{t-1} - 100 \cdot \Delta_4 ln(PKS_t)\right)$$

$$PKWD_t = 100 \cdot \Delta_4 ln(PKW_t) + 0.9 \cdot \left(PKWD_{t-1} - 100 \cdot \Delta_4 ln(PKW_t)\right)$$

$$PW_t = PWR_t \cdot \frac{PCP_t}{100}$$

$$PY_t = \frac{Y_t}{YR_t} \cdot 100$$

$$PZ_t = \left(\gamma_0 \left(\frac{UC_t}{R_0}\right)^{1-\sigma_{KE}} + (1-\gamma_0)\left(\frac{PE_t}{PE_0}\right)^{1-\sigma_{KE}}\right)^{\frac{1}{1-\sigma_{KE}}}$$

$$SCSER_t = CSER_t \cdot \lambda_K$$

$$SEIN_t = TDIR_t + TBSP_t + SOZS_t + GST_t + VKSS_t$$

$$SOZ_t = SOZN_t + SZAF_t$$

$$TA_t = (1 - AFTZ_t) \cdot (KATA_t - TJU_t) \cdot \frac{WOST_t}{5}$$

$$TAUCSA_t = \frac{\phi_0^{CS} \cdot MWST_t}{1 + \phi_0^{CS} \cdot MWST_t}$$

$$TAUHEA_t = \frac{\phi_0^{HE} \cdot MWST_t + \phi_0^{HER} \cdot MWSTR_t}{1 + \phi_0^{HE} \cdot MWST_t + \phi_0^{HER} \cdot MWSTR_t}$$

$$TAUHFA_t = \frac{\phi_0^{HF} \cdot MWST_t + \phi_0^{HFR} \cdot MWSTR_t}{1 + \phi_0^{HF} \cdot MWST_t + \phi_0^{HFR} \cdot MWSTR_t}$$

$$TAUHFB_t = \frac{VBST_t}{HVPIF_t/100}$$

$$TAUIB_t = \frac{(1 - \alpha_0) \cdot (1 - \gamma_0) \cdot GAMOEG_t \cdot ENSTU_t}{WE\_EURPOIL_t + ENSTU_t}$$

$$TAUHXFA_t = \frac{\lambda_{HXF} \cdot (\phi_0^{HXF} \cdot MWST_t + \phi_0^{HXFR} \cdot MWSTR_t)}{1 + \lambda_{HXF} \cdot (\phi_0^{HXF} \cdot MWST_t + \phi_0^{HXFR} \cdot MWSTR_t)}$$

$$TAUISTA_t = \frac{\phi_0^{IST} \cdot MWST_t}{1 + \phi_0^{IST} \cdot MWST_t}$$

$$TAUIWA_t = \frac{\phi_0^{IW} \cdot MWST_t}{1 + \phi_0^{IW} \cdot MWST_t}$$

$$TAUPEB_t = \frac{GAMOEG_t \cdot ENSTU_t}{WE\_EURPOIL_t + ENSTU_t}$$

$$TBSP_t = UST_t + TBSO_t$$

$$TDIR_t = LOST_t + TDSO_t$$

$$TIPS_t = 1 - \frac{CP_t}{END_t} \cdot \left[\left(1 - \frac{GHVPIE_t}{1000} - \frac{GHVPIF_t}{1000}\right) \cdot TAUHXFA_t + \frac{GHVPIF_t}{1000} \cdot (TAUHFA_t + TAUHFB_t) + \frac{GHVPIE_t}{1000} \cdot TAUHEA_t\right] - \frac{CS_t}{END_t} \cdot TAUCSA_t - \frac{IW_t}{END_t} \cdot TAUIWA_t - \frac{IST_t}{END_t} \cdot TAUISTA_t$$

$$TIPSCS_t = 1 - TAUCSA_t$$

$$TIPSHE_t = 1 - TAUHEA_t$$

$$TIPSHF_t = 1 - TAUHFA_t - TAUHFB_t$$

$$TIPSHXF_t = 1 - TAUHXFA_t$$

$$TIPSINLV_t = 1 + (TIPS_t - 1) \cdot \frac{END_t}{INLV_t}$$

$$TIPSIST_t = 1 - TAUISTA_t - TAUIB_t$$

$$TIPSIU_t = 1 - TAUIB_t$$

$$TIPSIW_t = 1 - TAUIWA_t - TAUIB_t$$

$$TVQKB_t = \frac{CCKB_t/CCK_t + CCKB_{t-1}/CCK_{t-1}}{2}$$

$$TVQKF_t = \frac{CCKF_t/CCK_t + CCKF_{t-1}/CCK_{t-1}}{2}$$

$$TVQKM_t = \frac{CCKM_t/CCK_t + CCKM_{t-1}/CCK_{t-1}}{2}$$

$$TVQKS_t = \frac{CCKS_t/CCK_t + CCKS_{t-1}/CCK_{t-1}}{2}$$

$$TVQKW_t = \frac{CCKW_t/CCK_t + CCKW_{t-1}/CCK_{t-1}}{2}$$

$$UC_t = \left(\frac{1}{CCKB0 + CCKF0 + CCKM0 + CCKS0 + CCKW0}\right) \cdot (UCKB_t \cdot CCKB0 + UCKF_t \cdot CCKF0 + UCKM_t \cdot CCKM0 + UCKS_t \cdot CCKS0 + UCKW_t \cdot CCKW0)$$

$$UCKB_t = PKB_t \cdot \left(\frac{RLIU_t - PEVD_t}{100} + \frac{DBNRQ_t - (PKBD_t - PEVD_t)}{100}\right)$$

$$UCKF_t = PKF_t \cdot \left(\frac{RLIU_t - PEVD_t}{100} + \frac{DFNRQ_t - (PKFD_t - PEVD_t)}{100}\right)$$

$$UCKM_t = PKM_t \cdot \left(\frac{RLIU_t - PEVD_t}{100} + \frac{DMNRQ_t - (PKMD_t - PEVD_t)}{100}\right)$$

$$UCKS_t = PKS_t \cdot \left(\frac{RLIU_t - PEVD_t}{100} + \frac{DSNRQ_t - (PKSD_t - PEVD_t)}{100}\right)$$

$$UCKW_t = PKW_t \cdot \left(\frac{RLIU_t - PEVD_t}{100} + \frac{DWNRQ_t - (PKWD_t - PEVD_t)}{100}\right)$$

$$VMH_t = NGVD_t + KWRD_t \cdot \frac{PW_t}{100}$$

$$WER_t = CPIM_t \cdot CPR_t + CSIM_t \cdot CSR_t + IAIM_t \cdot IAISR_t + IBIM_t \cdot IBR_t + EXIM_t \cdot EXR_t + 0.5 \cdot VR_t$$

$$ln(WE\_RAW_t) = ln\left(\frac{WE\_NAW_t \cdot PEV_t}{WE\_PEVF_t}\right) + zz_t^{WE\_RAW}$$

$$WOBS_t = WOBE_t - WOBA_t$$

$$Y_t = BWS_t + ENERG_t$$

$$YV_t = LN_t + GNEH_t + TRN_t - VERR_t + BV_t$$

$$ZINS_t = \frac{BVS_t \cdot RZIN_t}{400}$$

Chain equations[138]

$$PYBAI_{t-1} \cdot BAIR_t = PYIA_{t-1} \cdot IAR_t + PYIB_{t-1} \cdot IBR_t + PYIS_{t-1} \cdot ISR_t$$

$$PYBIP_{t-1} \cdot BIPR_t = PYEND_{t-1} \cdot ENDR_t - PYIM_{t-1} \cdot IMR_t$$

$$PYEND_{t-1} \cdot ENDR_t = PYINV_{t-1} \cdot INVR_t + PYEX_{t-1} \cdot EXR_t$$

$$\begin{aligned} HVPI_t = \Bigg[& GHVPIE_t \cdot \left(\frac{HVPIE_t}{Q1 \cdot HVPIE_{t-1} + Q2 \cdot HVPIE_{t-2} + Q3 \cdot HVPIE_{t-3} + Q4 \cdot HVPIE_{t-4}}\right) \\ & + (1000 - GHVPIE_t) \\ & \cdot \left(\frac{HVPIX_t}{Q1 \cdot HVPIX_{t-1} + Q2 \cdot HVPIX_{t-2} + Q3 \cdot HVPIX_{t-3} + Q4 \cdot HVPIX_{t-4}}\right)\Bigg] \\ & \cdot \left(\frac{Q1 \cdot HVPI_{t-1} + Q2 \cdot HVPI_{t-2} + Q3 \cdot HVPI_{t-3} + Q4 \cdot HVPI_{t-4}}{1000}\right) + zz_t^{HVPI} \end{aligned}$$

$$\begin{aligned} HVPIX_t = \Bigg[& GHVPIF_t \cdot \left(\frac{HVPIF_t}{Q1 \cdot HVPIF_{t-1} + Q2 \cdot HVPIF_{t-2} + Q3 \cdot HVPIF_{t-3} + Q4 \cdot HVPIF_{t-4}}\right) \\ & + (1000 - GHVPIF_t) \\ & \cdot \left(\frac{HVPIXF_t}{Q1 \cdot HVPIXF_{t-1} + Q2 \cdot HVPIXF_{t-2} + Q3 \cdot HVPIXF_{t-3} + Q4 \cdot HVPIXF_{t-4}}\right)\Bigg] \\ & \cdot \left(\frac{Q1 \cdot HVPIX_{t-1} + Q2 \cdot HVPIX_{t-2} + Q3 \cdot HVPIX_{t-3} + Q4 \cdot HVPIX_{t-4}}{1000}\right) + zz_t^{HVPIX} \end{aligned}$$

$$PYIAIS_{t-1} \cdot IAISR_t = PYIA_{t-1} \cdot IAR_t + PYIS_{t-1} \cdot ISR_t$$

$$PYIA_{t-1} \cdot IAR_t = PYIAU_{t-1} \cdot IAUR_t + PYIAS_{t-1} \cdot IASR_t$$

[138] In alphabetical order.

$$PYIB_{t-1} \cdot IBR_t = PYIBU_{t-1} \cdot IBUR_t + PYIBS_{t-1} \cdot IBSR_t + PYIW_{t-1} \cdot IWR_t$$

$$PYIBU_{t-1} \cdot IBUR_t = PYGBAI_{t-1} \cdot GBAIR_t - PYIAU_{t-1} \cdot IAUR_t - PYISU_{t-1} \cdot ISUR_t$$

$$PYIM_{t-1} \cdot IMR_t = PYNEIM_{t-1} \cdot NEIMR_t + PYIMOG_{t-1} \cdot IMOGR_t$$

$$PYINV_{t-1} \cdot INVR_t = PYCP_{t-1} \cdot CPR_t + PYCS_{t-1} \cdot CSR_t + PYBIG_{t-1} \cdot BIGR_t$$

$$PYIS_{t-1} \cdot ISR_t = PYISU_{t-1} \cdot ISUR_t + PYISS_{t-1} \cdot ISSR_t$$

$$PYIST_{t-1} \cdot ISTR_t = PYIAS_{t-1} \cdot IASR_t + PYIBS_{t-1} \cdot IBSR_t + PYISS_{t-1} \cdot ISSR_t$$

$$PYY_{t-1} \cdot YR_t = PYBWS_{t-1} \cdot BWSR_t + PYE_{t-1} \cdot ENERGR_t$$

Annual average price indices[139]

$$PYBAI_t = (1 - Q4) \cdot PYBAI_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} BAI_{t-i}}{\sum_{i=0}^{3} BAIR_{t-i}}\right)$$

$$PYBIG_t = (1 - Q4) \cdot PYBIG_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} BIG_{t-i}}{\sum_{i=0}^{3} BIGR_{t-i}}\right)$$

$$PYBIP_t = (1 - Q4) \cdot PYBIP_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} BIP_{t-i}}{\sum_{i=0}^{3} BIPR_{t-i}}\right)$$

$$PYBWS_t = (1 - Q4) \cdot PYBWS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} BWS_{t-i}}{\sum_{i=0}^{3} BWSR_{t-i}}\right)$$

$$PYCP_t = (1 - Q4) \cdot PYCP_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} CP_{t-i}}{\sum_{i=0}^{3} CPR_{t-i}}\right)$$

$$PYCS_t = (1 - Q4) \cdot PYCS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} CS_{t-i}}{\sum_{i=0}^{3} CSR_{t-i}}\right)$$

$$PYE_t = (1 - Q4) \cdot PYE_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} ENERG_{t-i}}{\sum_{i=0}^{3} ENERGR_{t-i}}\right)$$

$$PYEND_t = (1 - Q4) \cdot PYEND_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} END_{t-i}}{\sum_{i=0}^{3} ENDR_{t-i}}\right)$$

$$PYEX_t = (1 - Q4) \cdot PYEX_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} EX_{t-i}}{\sum_{i=0}^{3} EXR_{t-i}}\right)$$

$$PYGBAI_t = (1 - Q4) \cdot PYGBAI_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} GBAI_{t-i}}{\sum_{i=0}^{3} GBAIR_{t-i}}\right)$$

$$PYIA_t = (1 - Q4) \cdot PYIA_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IA_{t-i}}{\sum_{i=0}^{3} IAR_{t-i}}\right)$$

$$PYIAIS_t = (1 - Q4) \cdot PYIAIS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IAIS_{t-i}}{\sum_{i=0}^{3} IAISR_{t-i}}\right)$$

$$PYIAS_t = (1 - Q4) \cdot PYIAS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IAS_{t-i}}{\sum_{i=0}^{3} IASR_{t-i}}\right)$$

[139] In alphabetical order.

$$PYIAU_t = (1 - Q4) \cdot PYIAU_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IAU_{t-i}}{\sum_{i=0}^{3} IAUR_{t-i}}\right)$$

$$PYIB_t = (1 - Q4) \cdot PYIB_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IB_{t-i}}{\sum_{i=0}^{3} IBR_{t-i}}\right)$$

$$PYIBS_t = (1 - Q4) \cdot PYIBS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IBS_{t-i}}{\sum_{i=0}^{3} IBSR_{t-i}}\right)$$

$$PYIBU_t = (1 - Q4) \cdot PYIBU_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IBU_{t-i}}{\sum_{i=0}^{3} IBUR_{t-i}}\right)$$

$$PYIM_t = (1 - Q4) \cdot PYIM_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IM_{t-i}}{\sum_{i=0}^{3} IMR_{t-i}}\right)$$

$$PYIMOG_t = (1 - Q4) \cdot PYIMOG_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IMOG_{t-i}}{\sum_{i=0}^{3} IMOGR_{t-i}}\right)$$

$$PYINV_t = (1 - Q4) \cdot PYINV_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} INLV_{t-i}}{\sum_{i=0}^{3} INVR_{t-i}}\right)$$

$$PYIS_t = (1 - Q4) \cdot PYIS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IS_{t-i}}{\sum_{i=0}^{3} ISR_{t-i}}\right)$$

$$PYISS_t = (1 - Q4) \cdot PYISS_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} ISS_{t-i}}{\sum_{i=0}^{3} ISSR_{t-i}}\right)$$

$$PYIST_t = (1 - Q4) \cdot PYIST_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IST_{t-i}}{\sum_{i=0}^{3} ISTR_{t-i}}\right)$$

$$PYISU_t = (1 - Q4) \cdot PYISU_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} ISU_{t-i}}{\sum_{i=0}^{3} ISUR_{t-i}}\right)$$

$$PYIW_t = (1 - Q4) \cdot PYIW_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} IW_{t-i}}{\sum_{i=0}^{3} IWR_{t-i}}\right)$$

$$PYNEIM_t = (1 - Q4) \cdot PYNEIM_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} NEIM_{t-i}}{\sum_{i=0}^{3} NEIMR_{t-i}}\right)$$

$$PYY_t = (1 - Q4) \cdot PYY_{t-1} + Q4 \cdot \left(\frac{\sum_{i=0}^{3} Y_{t-i}}{\sum_{i=0}^{3} YR_{t-i}}\right)$$

Annex III: List of BbkM-DE's variables

Variable	Description	endog. / exog.
AFODV	Adjustment factor for PIAU and PIAS	X
AFSELB	Adjustment factor for self-employed	N
AFTZ	Adjustment factor for part-time work	N
AFYPOT	Adjustment variable for potential output equation	X
AGR	Exit rate (from unemployment into employment)	X
ANSP	Labour market tightness	N
ARL	Number of unemployed persons	N
ARLQ	Unemployment rate	N
ARLQN	Smoothed unemployment rate	N
ARSF	Transfers of private firms to foreign countries	N
ARST	Hours worked per employee	N
AUA	Firms' share of stock of machinery and equipment and other capital	X
AUB	Firms' share of stock of construction capital	X
AUSL	Utilisation rate of production capacity	N
AVBI	Total hours worked by employees (domestic concept)	N
AVPOT	Potential working hours	N
B1	Number of employees (residents)	N
BAI	Gross fixed investment at current prices	N
BAIR	Real gross fixed investment at (chained) previous year's prices	N
BAUG	Number of building permits	N
BEW	Valuation effects on net financial wealth of households at market prices	N
BI	Number of employees (domestic concept)	N
BIG	Gross investment at current prices	N
BIGR	Real gross investment at (chained) previous year's prices	N
BIP	Gross domestic product at current prices	N
BIPQ	Potential GDP	N
BIPR	Real gross domestic product at (chained) previous year's prices	N
BIPRIV	Number of employees in private sector (domestic concept)	N
BSP	Gross national product at current prices	N
BV	Adjustment for the change in net equity of households in pension funds reserves	X
BVS	Gross indebtedness of government	N

BWS	Gross value added at current prices	N
BWSR	Real gross value added at (chained) previous year's prices	N
CCK	Capital compensation at (chained) previous year's prices	N
CCKB	Capital compensation of non-residential construction at (chained) previous year's prices	N
CCKF	Capital compensation of cars at (chained) previous year's prices	N
CCKM	Capital compensation of machinery at (chained) previous year's prices	N
CCKS	Capital compensation of other capital at (chained) previous year's prices	N
CCKW	Capital compensation of residential construction at (chained) previous year's prices	N
CDAX	Equity price index CDAX	N
CP	Private consumption at current prices	N
CPIM	Import share of private consumption	X
CPR	Real private consumption at (chained) previous year's prices	N
CS	Government consumption at current prices	N
CSER	Capital services	N
CSERGR	Capital services, growth rates	N
CSIM	Import share of government consumption	X
CSR	Real government consumption at (chained) previous year's prices	N
CWER	Proxy for share of private consumption in (rest of) weighted import demand	X
D	Depreciation allowances at current prices	N
DAQ	Depreciation rate for investment in machinery and equipment	X
DAR	Real depreciation allowances investment in machinery and equipment at (chained) previous year's prices	N
DASQ	Depreciation rate for machinery and equipment and other capital	X
DASR	Real depreciation allowances for machinery and equipment and other capital at (chained) previous year's prices	N
DBNRQ	Depreciation rate for net stock of non-residential construction	X
DBQ	Depreciation rate for non-residential construction	X
DBR	Real depreciation allowances for non-residential construction at (chained) previous year's prices	N
DE_WDR	Real world demand indicator for German exports	X

DFNRQ	Depreciation rate for net stock of cars	X
DGFI	DG-AGRI food price index	X
DLBS	Residual item in the current account incl. capital transfers	X
DMNRQ	Depreciation rate for net stock of machinery	X
DSNRQ	Depreciation rate for net stock of other capital	X
DSQ	Depreciation rate for investment in other capital	X
DSR	Real depreciation allowances for other capital at (chained) previous year's prices	N
DUM0009	Impulse Sequence Dummy Variable , 1 from t=2000Q1 to t=2009Q4 , 0 else	X
DUM031	Impulse Dummy Variable , 1 for t=2003Q1 , 0 else	X
DUM034	Impulse Dummy Variable , 1 for t=2003Q4 , 0 else	X
DUM041	Impulse Dummy Variable , 1 for t=2004Q1 , 0 else	X
DUM051	Impulse Dummy Variable , 1 for t=2005Q1 , 0 else	X
DUM053184	Impulse Sequence Dummy Variable , 0 for t <2005Q3, 0.45 from 2005Q3 to 2018Q4, 0 later	X
DUM063	Impulse Dummy Variable , 1 for t=2006Q3 , 0 else	X
DUM071	Impulse Dummy Variable , 1 for t=2007Q1 , 0 else	X
DUM073	Impulse Dummy Variable , 1 for t=2007Q3 , 0 else	X
DUM082	Impulse Dummy Variable , 1 for t=2008Q2 , 0 else	X
DUM083	Impulse Dummy Variable , 1 for t=2008Q3 , 0 else	X
DUM084	Impulse Dummy Variable , 1 for t=2008Q4 , 0 else	X
DUM092	Impulse Dummy Variable , 1 for t=2009Q2 , 0 else	X
DUM102	Impulse Dummy Variable , 1 for t=2010Q2 , 0 else	X
DUM111	Impulse Dummy Variable , 1 for t=2011Q1 , 0 else	X
DUM112	Impulse Dummy Variable , 1 for t=2011Q2 , 0 else	X
DUM121	Impulse Dummy Variable , 1 for t=2012Q1 , 0 else	X
DUM151	Impulse Dummy Variable , 1 for t=2015Q1 , 0 else	X
DUM193	Impulse Dummy Variable , 1 for t=2019Q3 , 0 else	X
DUM194	Impulse Dummy Variable , 1 for t=2019Q4 , 0 else	X
DUM9193	Impulse Sequence Dummy Variable , 1 from t=1991Q1 to t=1993Q4 , 0 else	X
DUM931	Impulse Dummy Variable , 1 for t=1993Q1 , 0 else	X
DUM981	Impulse Dummy Variable , 1 for t=1998Q1 , 0 else	X
DUM993	Impulse Dummy Variable , 1 for t=1999Q3 , 0 else	X
DUMTB0	Shift Dummy Variable, 1 for t before or equal current and 0 for t after current quarter	X
DWNRQ	Depreciation rate for net stock of residential construction	X
DWQ	Depreciation rate for residential construction	X

DWR	Real depreciation allowances for residential construction at (chained) previous year's prices	N
END	Final demand at current prices	N
ENDR	Real final demand at (chained) previous year's prices	N
ENERG	Energy input at current prices	N
ENERGL	Energy input in litres of crude oil	N
ENERGR	Real energy input at (chained) previous year's prices	N
ENMOEGHH	Energy consumption related to mineral oil and gas, households	N
ENMOEGU	Energy consumption related to mineral oil and gas, firms	N
ENST	Energy tax rate, households	X
ENSTU	Energy tax rate, firms	X
EQU	Labour force participation rate	N
ER	Euro/US dollar exchange rate	X
EURLPOIL	Crude oil price (in euro per litre)	N
EW	Total labour force (residents)	N
EX	Exports of goods and services at current prices	N
EXIM	Import share of exports	X
EXIMTREND	Trend proxy for import share of exports	X
EXR	Real exports of goods and services at (chained) previous year's prices	N
FA	Net lending of foreign countries (from the view of foreign countries)	N
FH	Net lending of households	N
FS	Net lending of government	N
FU	Net lending of private firms	N
GAMOEG	Share of mineral oil and gas in total energy consumption	X
GAMOEGHH	Share of households in energy consumption related to mineral oil and gas	X
GAP	Output gap	N
GBAI	Commercial gross fixed capital formation at current prices	N
GBAIR	Real commercial gross fixed capital formation at (chained) previous year's prices	N
GHVPIE	HICP – Energy, Sub-index weight	X
GHVPIF	HICP – Food, Sub-index weight	X
GNEH	Other income (net) of households	N
GST	Property income of government	X
GSTRA	Tax change effects, other direct taxes	X
GU	Gross profit income of private sector	N

GW	Gross profit and proprietary income	N
HVPI	Harmonised index of consumer prices, all items	N
HVPIE	Harmonised index of consumer prices, energy	N
HVPIF	Harmonised index of consumer prices, food	N
HVPIX	Harmonised index of consumer prices, all items excl. energy	N
HVPIXF	Harmonised index of consumer prices, all items excl. energy and food	N
IA	Gross investment in machinery and equipment at current prices	N
IAIM	Import share of investment in machinery and equipment	X
IAIS	Investment in machinery and equipment and other investment at current prices	N
IAISR	Real investment in machinery and equipment and other investment at (chained) previous year's prices	N
IAR	Real gross investment in machinery and equipment at (chained) previous year's prices	N
IAS	Government investment in machinery and equipment at current prices	X
IASR	Real government investment in machinery and equipment at (chained) previous year's prices	N
IAU	Private investment in machinery and equipment at current prices	N
IAUR	Real private investment in machinery and equipment at (chained) previous year's prices	N
IB	Investment in construction at current prices	N
IBIM	Import share of investment in construction	X
IBR	Real investment in construction at (chained) previous year's prices	N
IBS	Government investment in construction at current prices	X
IBSR	Real government investment in construction at (chained) previous year's prices	N
IBU	Private investment in non-residential construction at current prices	N
IBUR	Real private investment in non-residential construction at (chained) previous year's prices	N
IM	Imports of goods and services at current prices	N
IMOG	Imports of oil and gas at current prices	N
IMOGR	Real imports of oil and gas at (chained) previous year's prices	N
IMR	Imports of goods and services at (chained) previous year's prices	N
INLV	Domestic demand at current prices	N

INVR	Real domestic demand at (chained) previous year’s prices	N
IP	Net investment of households at current prices	N
IS	Other investment at current prices	N
ISR	Real other investment at (chained) previous year’s prices	N
ISS	Other government investment at current prices	X
ISSR	Real other government investment at (chained) previous year’s prices	N
IST	Government investment at current prices	N
ISTR	Real government investment in machinery and equipment at (chained) previous year’s prices	N
ISU	Other private investment at current prices	X
ISUR	Real other private investment at (chained) previous year’s prices	N
IW	Private investment in residential construction at current prices	N
IWR	Real private investment in residential construction at (chained) previous year’s prices	N
KAR	Real capital stock in machinery and equipment at (chained) previous year’s prices	N
KASR	Real capital stock in machinery and equipment and other capital at (chained) previous year’s prices	N
KASUR	Real private capital stock in machinery and equipment and other capital at (chained) previous year’s prices	N
KASURD	Smoothed real private capital stock in machinery and equipment and other capital at (chained) previous year’s prices	N
KATA	Potential working days	X
KBR	Real capital stock in non-residential construction at (chained) previous year’s prices	N
KBUR	Real private capital stock in non-residential construction at (chained) previous year’s prices	N
KBURD	Smoothed real capital stock in non-residential construction at (chained) previous year’s prices	N
KFR	Real capital stock in cars at (chained) previous year’s prices	N
KMR	Real capital stock in machinery at (chained) previous year’s prices	N
KMRQ	Machinery capital quote with respect to machinery and equipment	X
KSR	Real capital stock in other capital at (chained) previous year’s prices	N

KWR	Real capital stock in residential property at (chained) previous year’s prices	N
KWRD	Smoothed real capital stock in residential property at (chained) previous year’s prices	N
L	Gross wages and salaries	N
LAST	Gross wage and salary income per hour worked	N
LBS	Current account	N
LG	Gross wages excluding employers’ contribution to social security	N
LGAS	Gross wage and salary income per hour worked (excl. employers’ contribution to social security)	N
LN	Net wage and salary income	N
LOST	Tax on wage and salary income	N
LPOT	Gross wage income per potential working hour	N
LTG	Negotiated monthly wage and salary level	N
MUBAU	Proxy for mark-up in construction sector	X
MWST	Value-added tax rate	X
MWSTR	Reduced value-added tax rate	X
NEIM	Non-oil and non-gas imports at current prices	N
NEIMR	Real non-oil and non-gas imports at (chained) previous year’s prices	N
NGTST	Taxes on products less subsidies on products	N
NGV	Net financial wealth of households at market values	N
NGVA	Net financial wealth of foreign countries	N
NGVD	Smoothed net financial wealth of households at market values	N
NGVH	Net financial wealth of households	N
NGVS	Net financial wealth of the government	N
NGVU	Net financial wealth of firms	N
NHH	Number of households	X
PBIP	Deflator of gross domestic product	N
PBWS	Deflator of gross value added	N
PCP	Deflator of private consumption	N
PCPD	Smoothed inflation rate of private consumption	N
PCS	Deflator of government consumption	N
PE	Deflator of energy input	N
PEND	Commuters	N
PEV	Deflator of final demand	N
PEVD	Smoothed inflation rate of final demand	N
PEX	Deflator of exports of goods and services	N
PIAS	Deflator of government investment in machinery and equipment	N

PIAU	Deflator of private investment in machinery and equipment	N
PIBS	Deflator of government investment in construction	N
PIBU	Deflator of private investment in non-residential construction	N
PIM	Deflator of imports of goods and services	N
PIMOG	Deflator of imports of oil and gas	N
PINV	Deflator of domestic demand	N
PIST	Deflator of other government investment	N
PISU	Deflator of other private investment	N
PIW	Deflator of private investment in residential construction	N
PKB	Deflator of non-residential construction capital stock	N
PKBD	Smoothed capital gains of non-residential construction stock	N
PKF	Deflator of capital stock in cars	N
PKFD	Smoothed capital gains of car stock	N
PKM	Deflator of capital stock in machinery	N
PKMD	Smoothed capital gains of machinery stock	N
PKS	Deflator of other capital stock	N
PKSD	Smoothed capital gains of other capital stock	N
PKW	Deflator of residential construction capital stock	N
PKWD	Smoothed capital gains of residential construction stock	N
PNEIM	Deflator of non-oil and non-gas imports	N
PW	Residential property prices	N
PWR	Real residential property prices	N
PY	Deflator of total output	N
PYBAI	Average annual price of gross fixed investment	N
PYBIG	Average annual price of gross investment	N
PYBIP	Average annual price of gross domestic product	N
PYBWS	Average annual price of gross value added	N
PYCP	Average annual price of private consumption	N
PYCS	Average annual price of government consumption	N
PYE	Average annual price of energy input	N
PYEND	Average annual price of final demand	N
PYEX	Average annual price of exports	N
PYGBAI	Average annual price of commercial gross fixed capital formation	N
PYIA	Average annual price of investment in machinery and equipment	N
PYIAIS	Average annual price of investment in machinery and equipment and other investment	N

PYIAS	Average annual price of government investment in machinery and equipment	N
PYIAU	Average annual price of private investment in machinery and equipment	N
PYIB	Average annual price of investment in construction	N
PYIBS	Average annual price of government investment in construction	N
PYIBU	Average annual price of private investment in non-residential construction	N
PYIM	Average annual price of imports	N
PYIMOG	Average annual price of oil and gas imports	N
PYINV	Average annual price of domestic demand	N
PYIS	Average annual price of other investment	N
PYISS	Average annual price of other government investment	N
PYIST	Average annual price of government investment	N
PYISU	Average annual price of other private investment	N
PYIW	Average annual price of investment in residential construction	N
PYNEIM	Average annual price of non-oil and non-gas imports	N
PYY	Average annual price of total output	N
PZ	Price index for capital-energy bundle	N
Q1	Dummy variable for 1st quarter: 1 for Q1, 0 else	X
Q2	Dummy variable for 2nd quarter: 1 for Q2, 0 else	X
Q3	Dummy variable for 3rd quarter: 1 for Q3, 0 else	X
Q4	Dummy variable for 4th quarter: 1 for Q4, 0 else	X
RIEA	Number of RIESTER agreements concluded	X
RIES	Contribution rate in RIESTER agreements	X
RL	Yield on 10-year government bonds	N
RLIU	Long-term bank interest rate on loans to non-financial corporations	N
RLIW	Long-term bank interest rate on household mortgage loans	N
RS	Interest rate for 3-month funds	X
RZIN	Interest rate on government debt	N
SCSER	Scaled capital services	N
SEIN	Government revenue	N
SELB	Number of self-employed persons	X
SEVE	Net current transfers from abroad	X
SOZ	Employers’ and employees’ contributions to social security	N
SOZB	Rate of social security contributions	X
SOZN	Employees’ social security contributions	N

SOZS	Social security contributions received by government	N
SRSS	Residual item in the financial account of the government	X
SUBV	Government subsidies to private firms	N
SVPH	Net capital transfer payments of households	N
SZAF	Employers' contributions to social security	N
T_TILDE	Time trend in production function	X
T_UST_99	Time trend, beginning in 1999Q1	X
TA	Negotiated working time per employee adjusted for part-time work	N
TAUCSA	VAT component of deflator of government consumption	N
TAUHEA	VAT component wrt HICP energy	N
TAUHFA	VAT component wrt HICP food	N
TAUHFB	Consumption tax component wrt HICP food	N
TAUHXFA	VAT component wrt HICP excl. energy and food	N
TAUIB	Energy tax component of investment deflators	N
TAUISTA	VAT component of deflator of government investment	N
TAUIWA	VAT component of deflator of residential investment	N
TAUPEB	Energy tax component of deflator of energy input	N
TBENST	Energy taxes	N
TBREST	Other indirect taxes (excl. energy and electricity taxes)	N
TBSO	Other indirect taxes	N
TBSP	Production and import duties	N
TDIR	Direct taxes	N
TDSO	Other direct taxes	N
TIPS	Average indirect tax factor for deflator of final demand	N
TIPSCS	Indirect tax factor for deflator of government consumption	N
TIPSHE	Indirect tax factor wrt HICP energy	N
TIPSHF	Indirect tax factor wrt HICP food	N
TIPSHXF	Indirect tax factor wrt HICP excl. energy and food	N
TIPSINLV	Average indirect tax factor for deflator of domestic demand	N
TIPSIST	Indirect tax factor for deflator of government investment	N
TIPSIU	Indirect tax factor for deflator of coporate investment	N
TIPSIW	Indirect tax factor for deflator of residential investment	N
TJU	Vacation days	X
TRN	Monetary transfer payments received by households	N
TRNS	Monetary transfers paid by the government	N
TVQKB	Törnqvist weights of non-residential construction (for calculation of capital services)	N
TVQKF	Törnqvist weights of cars (for calculation of capital services)	N

TVQKM	Törnqvist weights of machinery (for calculation of capital services)	N
TVQKS	Törnqvist weights of other capital (for calculation of capital services)	N
TVQKW	Törnqvist weights of residential construction (for calculation of capital services)	N
TZQ	Share of part-time employees	X
UBLS	BLS credit standards for firms	N
UBLSB	BLS credit standards for firms, bank-side factor	X
UBLSK	BLS credit standards for firms, business cycle factor	N
UC	User costs of capital	N
UCKB	User costs of capital wrt non-residential construction	N
UCKF	User costs of capital wrt cars	N
UCKM	User costs of capital wrt machinery	N
UCKS	User costs of other capital	N
UCKW	User costs of capital wrt residential construction	N
UST	Value-added tax	N
V	Inventory investment at current prices	N
VBST[140]	(Average) consumption tax on food-related consumption	X
VERR	Transfer payments of households to foreign countries	N
VKSS	Sales and other revenue of the government	X
VMH	Wealth of households at market values	N
VQ	Vacancy rate	N
VR	Real inventory investment at (chained) previous year's prices	N
WE_GDPR	Real world GDP	X
WE_IMPR	Real world imports	X
WE_NAW	Nominal effective exchange rate of the euro for Germany	X
WE_PCOM	Commodity price index, excl. energy	X
WE_PEVF	Weighted prices of foreign trade partners	X
WE_RAW	Real effective exchange rate of the euro for Germany	N
WE_USDPOIL	Price for 1 barrel Brent, Great Britain	X
WER	Weighted real import demand	N
WOBA	Population between 15 and 65 years old	X
WOBE	Population	X
WOBS	Population below 15 and above 65 years old	N
WOST	Standard working hours	X
WOSVT	Ratio of weekly hours worked by self-employed persons and by full-time and part-time employees	X

[140] Only endogenous when shocks on food-related consumption tax revenues are simulated.

WOTV	Ratio of weekly hours worked by part-time employees and by full-time employees	X
WRAG	(Smoothed) growth rate of the number of listed companies	X
Y	Total output at current prices	N
YPOT	Potential total output at (chained) previous year's prices	N
YR	Real total output at (chained) previous year's prices	N
YV	Disposable income of households	N
ZINS	Interest paid by government	N

List of parameters

Parameter	Description
α_0	Mean (over estimation period) of (modified) labour income ratio
CCKB0	Average annual value of CCKB in base year for calculation of aggregate user costs of capital
CCKF0	Average annual value of CCKF in base year for calculation of aggregate user costs of capital
CCKM0	Average annual value of CCKM in base year for calculation of aggregate user costs of capital
CCKS0	Average annual value of CCKS in base year for calculation of aggregate user costs of capital
CCKW0	Average annual value of CCKW in base year for calculation of aggregate user costs of capital
E_0	Mean (over estimation period) of energy input
GALAST0	Mean of LAST in base year, required when converting L into LAST
GALG0	Mean of LGAS in base year, required when converting LGAS into LG
γ_0	Mean (over estimation period) of capital income ratio (in relation to energy)
K_0	Mean (over estimation period) of capital input
λ_K	Scaling parameter for capital services
λ_{HXF}	Assumed degree to which value-added tax changes are passed on to HICP core
μ_{GTST05}	Mean (from 2005) of taxes on products in relation to production and import taxes (from the government account)
μ_{GTSU05}	Mean (from 2005) of subsidies on products in relation to production and import taxes (from the government account)
N_0	Mean (over estimation period) of labour input
PE_o	Mean (over estimation period) of the price of energy input

ϕ_0^{CP}	Mean (from 1999) of the share of goods in private consumption that are taxed with the regular VAT rate
ϕ_0^{CPR}	Mean (from 1999) of the share of goods in private consumption that are taxed with the reduced VAT rate
ϕ_0^{CS}	Mean (from 1999) of the share of goods in public consumption that are taxed with the regular VAT rate
ϕ_0^{HE}	Assumed share of goods in HICP energy that are taxed with the regular VAT rate
ϕ_0^{HER}	Assumed share of goods in HICP energy that are taxed with the reduced VAT rate
ϕ_0^{HF}	Assumed share of goods in HICP food that are taxed with the regular VAT rate
ϕ_0^{HFR}	Assumed share of goods in HICP food that are taxed with the reduced VAT rate
ϕ_0^{HXF}	Mean (from 1999) of the share of goods in HICP excl. energy and food that are taxed with the regular VAT rate
ϕ_0^{HXFR}	Mean (from 1999) of the share of goods in HICP excl. energy and food that are taxed with the reduced VAT rate
ϕ_0^{IST}	Mean (from 1999) of the share of goods in government investment that are taxed with the regular VAT rate
ϕ_0^{IW}	Mean (from 1999) of the share of goods in residential investment that are taxed with the regular VAT rate
R_0	Mean (over estimation period) of the user costs of capital
$TPREM0$	Mean of term premium (i.e. mean of difference between RL and RS over estimation period of behavioural equation for RL)
θ_{95}	Mean (over the estimation period) of the ratio of net financial wealth (end of previous year) to disposable income of households
Y_0	Mean (over estimation period) of total output