DEUTSCHE BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 01/2023

Make-up strategies with incomplete markets and bounded rationality

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Internet http://www.bundesbank.de

ISBN 978–3–95729–932–1
ISSN 2749–2958

Non-technical summary

Research question

The stabilisation properties of make-up strategies, such as average inflation targeting (AIT), are usually quite strong in representative-agent New Keynesian (RANK) models, especially when interest rate policy is subject to an effective lower bound (ELB). By assuming rational expectations and complete financial markets, this class of models might, however, exaggerate the effectiveness of such strategies. In this paper, we analyse to what extent the stabilisation benefits of make-up strategies depend on these two modelling assumptions.

Contribution

For our quantitative analysis, we extend a heterogeneous-agent New Keynesian (HANK) model by allowing for an occasionally-binding ELB and non-rational expectations. Based on stochastic model simulations, we contrast the macroeconomic stabilisation properties of an inflation targeting (IT) strategy, which sets the interest rate by targeting a value for the current inflation rate, with those of an AIT strategy, which targets the average inflation rate and therefore displays a make-up element. To simulate our model, we develop a numerical algorithm that can simultaneously handle household heterogeneity, non-rational expectation formation and an occasionally-binding ELB.

Results

We obtain four main results. First, our simulations predict that AIT outperforms IT in terms of inflation and output stabilisation when there is an ELB. Second, the relative performance of AIT hardly differs between our HANK model and a corresponding RANK model, suggesting that the complete-markets assumption is not particularly important for the effectiveness of make-up strategies. Third, although we find that AIT tends to mitigate the negative consequences of an ELB better than IT even under bounded rationality, the relative gains become quite small for low cognitive ability levels – including values consistent with micro-evidence. Fourth, whereas market incompleteness and bounded rationality complement each other in attenuating the effects of forward guidance, our model does not indicate such an interaction for the stabilisation properties of make-up strategies.

Nichttechnische Zusammenfassung

Fragestellung

Die Stabilisierungswirkungen vergangenheitsabhängiger geldpolitischer Strategien, wie z.B. einer durchschnittlichen Inflationssteuerung (Average Inflation Targeting: AIT), fallen in neukeynesianischen Modellen mit einem repräsentativen Haushalt zumeist ausgesprochen stark aus. Dies gilt insbesondere dann, wenn die Zinspolitik einer Zinsuntergrenze (ZUG) unterliegt. Aufgrund der Annahme rationaler Erwartungen und vollständiger Kapitalmärkte könnte diese Modellklasse die tatsächliche Effektivität solcher Strategien überzeichnen. In der vorliegenden Arbeit untersuchen wir die Bedeutung der beiden Modellannahmen für die Stabilisierungswirkungen vergangenheitsabhängiger Strategien.

Beitrag

Unsere quantitative Analyse beruht auf einem neukeynesianischen Modell mit heterogenen Haushalten, das wir durch eine endogen bindende ZUG und nicht-rationale Erwartungsbildung erweitern. Auf Basis stochastischer Modellsimulationen vergleichen wir die Stabilisierungswirkungen einer Inflationssteuerung (Inflation Targeting: IT) mit denen von AIT. Während die Zinssetzung bei IT ein Inflationsziel für die laufende Periode anstrebt, wird bei AIT ein Zielwert für den Durchschnitt der Inflationrate angestrebt, wobei dieser über die laufende und vergangene Perioden berechnet wird, so dass eine Vergangheitsabhängigkeit entsteht. Für die Simulation des Modells entwickeln wir einen numerischen Algorithmus, der es erlaubt, simultan heterogene Haushalte, nicht-rationale Erwartungsbildung und eine endogen bindende ZUG zu berücksichtigen.

Ergebnisse

Die Modellanalyse liefert vier Resultate. Erstens zeigen unsere Simulationen, dass AIT über bessere Stabilisierungseigenschaften verfügt als IT, wenn die Zinspolitik einer ZUG unterliegt. Zweitens hängen diese Stabilisierungsvorteile von AIT gegenüber IT nicht von der Annahme (un)vollständiger Kapitalmärkte ab. Drittens kann AIT die negativen Konsequenzen einer ZUG zwar auch bei nicht-rationaler Erwartungsbildung besser abfedern als IT, jedoch fallen die Stabilisierungsvorteile recht klein aus, wenn die kognitiven Fähigkeiten der Wirtschaftssubjekte – in Einklang mit Mikro-Evidenz – gering sind. Viertens beobachten wir, dass sich unvollständige Kapitalmärkte und beschränkte Rationalität nicht in einer Weise ergänzen, die die Stabilsierungsvorteile vergangenheitsabhängiger Strategien zusätzlich abschwächt. Gleichwohl finden wir, dass in unserem Modell eine solche Interaktion hinsichtlich der Wirkung einer Zins-Forward-Guidance besteht.

Deutsche Bundesbank Discussion Paper No 01/2023

Make-Up Strategies with Incomplete Markets and Bounded Rationality*

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December 23, 2022

Abstract

We study the impact of market incompleteness and bounded rationality on the effectiveness of make-up strategies. To do so, we simulate a heterogeneous-agent New Keynesian (HANK) model with reflective expectations and an occasionally-binding effective lower bound (ELB) on the policy rate. Our simulations show that make-up strategies can mitigate the negative consequences of the ELB for inflation and real economic activity. This result holds both for our HANK model as well as a corresponding representative-agent (RANK) model with complete markets, suggesting that market (in)completeness is not important for the effectiveness of make-up strategies. However, the stabilisation benefits of make-up strategies are small when agents' cognitive ability is consistent with micro-evidence. This result is independent of market (in)completeness, emphasising the importance of rational expectations for make-up strategies. Furthermore, while market incompleteness and bounded rationality complement each other in attenuating the effects of forward guidance in our model, we do not observe such a complementarity with respect to the benefits of make-up strategies.

Keywords: Make-Up Strategies, Incomplete Markets, Bounded Rationality, HANK, Effective Lower Bound, Average Inflation Targeting

JEL Classification: C63, D31, E21, E31, E52, E58, E70

*We thank Klaus Adam, Bence Bardóczy, Keshav Dogra, Refet Gürkaynak, Moritz Lenel, Ralph Lütticke, Ludwig Straub, Harald Uhlig, Maximilian Weiß, Mirko Wiederholt, Andreas Worms, and audiences at the European Central Bank, the Deutsche Bundesbank, the June 2022 Meeting of the Working Group on Econometric Modelling (WGEM), the 2022 Econometric Society European Meeting and the Fall 2022 Midwest Macroeconomics Meeting for helpful comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily represent those of the Deutsche Bundesbank, the European Central Bank or the Eurosystem.

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1 Introduction

In August 2020, the Federal Reserve announced its revised monetary policy strategy. Among the most striking changes was a shift to a flexible form of average inflation targeting (AIT). In theory, a make-up strategy like AIT can be helpful in a low-rate environment, which is characterised by an elevated risk of nominal rates hitting an effective lower bound (ELB).[1] The key idea behind AIT is that if the inflation target is not met today, monetary policy promises to "make up" for this in the future. If inflation is currently below target, monetary policy commits to steer inflation above target in the future to ensure that the inflation target is met on average. If credible and well understood, this commitment to raise future inflation already lowers expected real rates today and thereby stimulates current economic activity – even when nominal rates are currently stuck at the ELB.

Typically, the benefits of make-up strategies are quite strong in representative-agent New Keynesian (RANK) models, which are frequently used for monetary policy analysis.[2] However, there are at least two major concerns about the robustness of this finding. First, it is unclear whether the performance of make-up strategies is robust to deviations from rational expectations. Intuitively, if agents do not fully understand that an inflation undershoot today implies an inflation overshoot tomorrow, their behaviour is not going to respond as much as under rational expectations.[3] Second, the effectiveness of make-up strategies crucially depends on the willingness and ability of households to substitute consumption over time. By assuming complete markets, RANK models usually exhibit a counterfactually strong sensitivity of household consumption to interest rate changes (see e.g. Vissing-Jørgensen, 2002; Kaplan et al., 2014) and likely exaggerate the power of make-up strategies as a result.

Motivated by these concerns, we study how bounded rationality and market incompleteness influence the effectiveness of make-up strategies. To do so, we develop a quantitative heterogeneous-agent New Keynesian (HANK) model with incomplete markets, heterogeneous households, nominal price and wage rigidities, an occasionally-binding ELB and bounded rationality. The introduction of market incompleteness and borrowing constraints into the New Keynesian model gives rise to household behaviour that is consistent with empirical evidence (see e.g. Kaplan and Violante, 2018). In particular, household consumption becomes less sensitive to interest rate changes and more responsive to changes in temporary income. As a result, under incomplete markets, monetary policy is largely transmitted through indirect general equilibrium effects that affect household income and less via direct effects of interest rate changes (see Kaplan et al., 2018). Market incompleteness may furthermore interact with bounded rationality in a way that can attenuate the macroeco-

[1]In principle, unconventional monetary policies, such as asset purchases, forward guidance or negative interest rates, can substitute for further interest rate cuts at an ELB. An alternative to the use of additional instruments is to raise the inflation target, which can reduce the probability of hitting the ELB (see e.g. Coibion et al., 2012; Andrade et al., 2018; Blanco, 2021). We do not discuss these policy options in this paper.

[2]See e.g. Work Stream on the Price Stability Objective (2021).

[3]See the literature review at the end of this section for details.

nomic effects of announcements about future interest rates (see Farhi and Werning, 2019). This interaction could thus have immediate consequences for the performance of make-up strategies as well.

We model bounded rationality by assuming that agents form reflective expectations (see García-Schmidt and Woodford, 2019).[4] The concept of reflective expectations can be viewed as a "smooth" version of level-k thinking, which is supported by experimental evidence (see Farhi and Werning, 2019).[5] In both cases, agents form beliefs about future aggregate outcomes based on iterative reasoning. More specifically, agents with a certain cognitive ability level believe that the economy behaves as if it were populated by agents that are slightly less intelligent than they are themselves. The higher the cognitive ability level is, the more agents are able to understand the general equilibrium effects of aggregate shocks to the economy. In the limit, as the cognitive ability level approaches infinity, rational expectations emerge. Under level-k thinking, cognitive ability levels are discrete valued and the belief formation operates in discrete steps (or rounds). By contrast, under reflective expectations, cognitive ability is continuous valued and belief formation follows a differential equation.[6]

To study the impact of bounded rationality and market incompleteness on the performance of make-up strategies, we simulate our model for two monetary policy strategies. The first one is an inflation targeting (IT) strategy, which is modelled via a standard (inertial) Taylor rule commonly used for monetary policy analysis. The second one is an AIT strategy, which we model via a slightly modified Taylor rule that responds to deviations of the average inflation rate from the inflation target. Both interest rate rules are subject to an ELB constraint, which binds endogenously in our model economy that is subject to random aggregate demand and cost-push shocks. To solve and simulate our model, we build on recent advances in the literature that use a sequence-space approximation for heterogeneous-agent models (see Boppart et al., 2018; Auclert et al., 2021) and show how to apply this approach to a model with iterative belief formation and an occasionally-binding ELB.

We obtain four main results. First, our model predicts that AIT outperforms IT in terms of inflation and output stabilisation when there is an ELB. By managing agents' expectations, AIT reduces ELB incidences and negative biases in inflation and real economic activity observed under IT. These findings hold for our model regardless of whether markets are complete and agents are rational. Second, although we calibrate our HANK model to match

[4]The model features three types of forward-looking agents: households, intermediate-good firms and labour unions.

[5]A RANK model economy with agents that form reflective expectations can be reinterpreted as an economy with a continuum of "level-k agents" that are heterogeneous with respect to their cognitive ability (see García-Schmidt and Woodford, 2019). This property will help us identify empirically plausible values for agents' cognitive abilities.

[6]A key advantage of using reflective expectations rather than level-k thinking as in Farhi and Werning (2019) is that the model is not as susceptible to oscillatory feedback between different rounds of reasoning. Such feedback can cause a model to exhibit explosive behaviour after only a few rounds (see e.g. García-Schmidt and Woodford, 2019; Angeletos and Sastry, 2021; Bianchi-Vimercati et al., 2021). Avoiding such oscillatory behaviour, which can be viewed as an artifact of level-k thinking, makes the computation of models more stable, particularly for larger ones.

an empirically plausible average marginal propensity to consume (MPC) out of temporary income, the relative performance of AIT versus IT hardly differs between our HANK model and the corresponding RANK model.[7] This result suggests that the complete-markets assumption may not be particularly important for the effectiveness of make-up policies. Third, while we find that AIT tends to mitigate the consequences of the ELB better than IT even under reflective expectations, the relative gains become quite small for low cognitive ability levels. Importantly, this includes cognitive ability levels consistent with empirical and experimental evidence. This finding reflects that make-up strategies cannot play out their advantages very well if agents do not fully understand how they operate. Fourth, our model does not indicate that market incompleteness and bounded rationality complement each other in attenuating the ability of make-up strategies to steer the economy. However, our model predicts such a complementarity in the context of interest rate forward guidance, consistent with Farhi and Werning (2019). While these two results may appear to be at odds with each other at first sight, we show how to reconcile them in this paper.

Literature Our paper contributes to three strands of literature. First, it contributes to the growing HANK literature that integrates household heterogeneity and incomplete markets (see İmrohoroğlu, 1992; Huggett, 1993; Aiyagari, 1994) into a New Keynesian environment (see Galí, 2015).[8] Within the HANK literature, our paper is particularly related to Farhi and Werning (2019), Auclert et al. (2020) and Grimaud (2021), who allow for deviations from rational expectations, as well as to Feiveson et al. (2020), Dobrew et al. (2021) and Djeutem et al. (2022), who look at the interaction between household heterogeneity and make-up policies under rational expectations.[9] While the last two studies consider an occasionally-binding ELB, the former ones do not. Other papers that study HANK models with an occasionally-binding ELB constraint are Schaab (2020), Fernández-Villaverde et al. (2021), Lee (2021) and McKay and Wieland (2021). However, they restrict attention to standard interest rate rules without make-up elements.

Second, our paper contributes to the literature on make-up monetary policy strategies (see e.g. Svensson, 1999; Arias et al., 2020).[10] Specifically, it contributes to recent studies

[7] As in Hagedorn et al. (2019), we consider a HANK model with nominal wage stickiness and a calibration that attenuates the power of anticipated future monetary accommodation, which AIT relies on. In contrast to these authors, we however allow for positive (nominal) household debt as in Ferrante and Paustian (2019), such that there are redistributive effects that can potentially increase the effectiveness of future interest rate changes by reallocating resources between indebted high- and saving low-MPC households.

[8] See Kaplan and Violante (2018) for a recent survey. Prominent examples of this literature include Werning (2015), McKay et al. (2016), McKay and Reis (2016), Kaplan et al. (2018), Bayer et al. (2019), Hagedorn et al. (2019), Acharya and Dogra (2020), Bhandari et al. (2021), Gornemann et al. (2021) and Luetticke (2021). A complementary literature exists on New Keynesian models with heterogeneous firms and financial frictions (see e.g. Jeenas, 2020; Ottonello and Winberry, 2020; Jungherr et al., 2021).

[9] See also Pfäuti and Seyrich (2022) who introduce cognitive discounting à la Gabaix (2020) into the tractable HANK (THANK) model proposed by Bilbiie (2021). Related, but not within a HANK context, are also Meh et al. (2010) and Bergman et al. (2022). Whereas the former analyse price-level targeting for an overlapping-generations model, the latter study average inflation targeting for a RANK model with heterogeneous workers and a frictional labour market.

[10] Price-level targeting as a particular type of make-up strategy is known to have a number of desirable properties in the standard RANK model. For instance, Vestin (2006) shows that, under certain conditions, the

that highlight the limitations of make-up strategies in New Keynesian models with bounded rationality (see e.g. Budianto et al., 2020; Honkapohja and Mitra, 2020; Mele et al., 2020; Erceg et al., 2021; Bodenstein et al., 2022; Dupraz et al., 2022).[11] In contrast to these studies, our analysis is not restricted to a RANK environment but allows for household heterogeneity and market incompleteness. Farhi and Werning (2019) show that the interaction between bounded rationality and market incompleteness can strongly attenuate the power of forward guidance, which aims to affect current economic activity by managing agents' expectations. Since this type of expectations management is also at the heart of make-up strategies, an analysis of such strategies that allows for bounded rationality but does not take into account market incompleteness could be misleading. To model bounded rationality, we build on recent work that introduces behavioural or informational frictions into the textbook RANK model to eliminate some of its puzzling features (see Angeletos and Lian, 2018; Woodford, 2018; Farhi and Werning, 2019; García-Schmidt and Woodford, 2019; Gabaix, 2020).[12] As in these papers, we consider forward-looking expectation formation that is limited with respect to how much information agents use. Specifically, we use the approach introduced by García-Schmidt and Woodford (2019) and study its implications for the effectiveness of make-up policies in a HANK context.[13]

Third, from a methodological perspective, our paper contributes to recent studies that solve heterogeneous-agent models by using a local model approximation in the sequence space (see Boppart et al., 2018; Auclert et al., 2021) rather than the recursive state space (see Reiter, 2009; Bayer and Luetticke, 2020). In particular, we show how to leverage the sequence-space approximation to solve a HANK model with non-rational expectations and an occasionally-binding ELB, using anticipated monetary policy shocks (see e.g. Bodenstein et al., 2013; Holden, 2016).[14]

Layout The remainder of the paper is organised as follows. Section 2 introduces the model. Section 3 describes the model calibration. Section 4 covers our numerical solution approach. Section 5 presents and discusses the results. Section 6 concludes.

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optimal Ramsey policy can be implemented as the optimal discretionary policy if the central bank's objective includes a price-level rather than an inflation target. Furthermore, price-level targeting can approximate the optimal Ramsey policy prescription for ELB episodes (see e.g. Eggertsson and Woodford, 2003).

[11] See Eusepi and Preston (2018) for a recent survey of monetary policy in RANK models with adaptive learning.

[12] Well-known examples of such features are the forward guidance puzzle (see Del Negro et al., 2015), which states that anticipated future interest rate cuts have implausibly large contemporary effects on economic activity, and the reversal puzzle (see Carlstrom et al., 2015; Gerke et al., 2020), which states that an interest rate peg leads to a counterintuitive contraction in inflation.

[13] Bersson et al. (2019) consider level-k thinking as well as reflective expectations for a RANK model with an ELB constraint. In the paper, the authors look at optimal monetary policy and selected experiments, focussing on a deterministic environment. By contrast, we provide a quantitative analysis of a HANK model based on stochastic simulations. Iovino and Sergeyev (2021) study central bank balance sheet policies when agents have reflective expectations.

[14] Auclert et al. (2020) also use a sequence-space approximation for a HANK model with non-rational expectations. In contrast to our paper, the model considers sticky household expectations rather than an iterative belief formation and does not consider stochastic simulations with an occasionally-binding ELB constraint.

2 Model

In this section, we extend a standard New Keynesian model with nominal price and wage rigidities (see Galí, 2015) by including idiosyncratic household income risk, incomplete financial markets and non-rational expectations. We also allow for an occasionally-binding effective lower bound (ELB) on the short-term nominal interest rate, which gives rise to ELB episodes of endogenous length and severity. Although we are ultimately interested in analysing the model with random aggregate shocks and non-rational expectations, we provide the model formulation only for deterministic aggregate shocks and do not impose restrictions on how expectations are formed. Building on the model formulation laid out in this section, we study stochastic model simulations under non-rational expectation in Section 5. Since these simulations will be carried out based on a linearised model version (see Section 4), where certainty equivalence holds, we refrain from spelling out how agents form expectations in the presence of aggregate uncertainty for the nonlinear model in a general way.[15]

2.1 Economy

This section describes the individual components of the model economy.

Households The economy is populated by a unit-mass continuum of infinitely-lived households, indexed by $i \in [0,1]$, that derive utility from consumption and leisure. Households face idiosyncratic income risk due to time-varying labour productivity $z_{i,t}$, which affects the size of individual labour earnings, $w_t z_{i,t} N_t$. Idiosyncratic productivity follows a first-order discrete Markov process. We normalise its support to obtain an average productivity level of one for the economy, i.e. $\sum_i \Pr(z_i) z_i = 1$ with $\Pr(z_i)$ denoting the time-invariant share of households with z_i. In each period, households receive firm profits due to ownership of intermediate-good firms in the economy. Shares in these firms are not tradable and thus fully illiquid. Aggregate firm dividends d_t are distributed across households in proportion to their individual productivity relative to average labour productivity.[16] Households can only imperfectly insure themselves against income risk by saving (or borrowing) via a nominal non-state contingent bond, $\tilde{b}_{i,t+s}$, subject to the ad-hoc borrowing constraint $\tilde{b}_{i,t+s} \geq \underline{b}$.[17]

Following Farhi and Werning (2019), we assume that households can observe aggregate model variables in the current period t when deciding on their consumption-savings plans. However, for future periods $t+s$, $s>0$, households rely on – potentially non-rational – beliefs for these variables, which is captured by the use of index e. Let Ω_t denote the set

[15] See Appendix B for details on how the model formulation in this section relates to the numerical model solution. As in Evans et al. (2022), one could also view the setting presented in this section as one where agents face aggregate uncertainty but rely on point expectations of future aggregate variables.

[16] This proportional dividend rule reduces the impact that distributional effects have on aggregate fluctuations (see e.g. Hagedorn et al., 2019).

[17] We express individual holdings of nominal bonds in real terms, $\tilde{b}_{i,t} = \tilde{B}_{i,t}/P_t$, where $\tilde{B}_{i,t}$ denotes non-normalised nominal bond holdings and P_t the aggregate price level.

of relevant *endogenous* aggregate variables in period t and Ω^e_{t+s} denote the respective set of period-t beliefs about these variables in period $t+s$, $s>0$. For notational convenience, define the convention that $\Omega^e_{t+s}=\Omega_t$ for $s=0$. The tilde symbol used for the sequences $\{\tilde{c}_{i,t+s}\}_{s=0}^{\infty}$ and $\{\tilde{b}_{i,t+s}\}_{s=0}^{\infty}$ indicates that these are plans, made conditional on being in period t and individual states $(b_{i,t-1},z_{i,t})$, which do not have to materialise in equilibrium, even though consumption and bond holdings are controlled by the household.

Due to sticky nominal wages, household labour supply is entirely demand-determined in the model. As we explain in greater detail below, all household types work the same amount of hours, L_t, which – due to inefficiencies caused by wage dispersion – will not generally coincide with the amount of labour demanded by firms at aggregate real wage w_t, denoted as N_t. Taking as given hours worked and labour demanded, households form beliefs about the respective future values, L^e_{t+s} and N^e_{t+s}, as well as about future values of the aggregate real wage, w^e_{t+s}, aggregate firm dividends, d^e_{t+s}, the (gross) inflation rate, Π^e_{t+s}, and the (gross) nominal interest rate, R^e_{t+s-1}. While households form beliefs about endogenous aggregate variables, they have perfect foresight with respect to the time path of *exogenous* aggregate variables, which are summarised by the vector S_{t+s} that also contains the household discount factor β_{t+s}, $s\geq 0$.

At date t, the decision problem of an individual household i with bond holdings $b_{i,t-1}$ involves choosing a plan for consumption and bond holdings, $\{\tilde{c}_{i,t+s},\tilde{b}_{i,t+s}\}_{s=0}^{\infty}$, to maximise expected lifetime utility,

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\left(\prod_{k=0}^{s}\beta_{t+k-1}\right)\left\{u\left(\tilde{c}_{i,t+s}\right)-v\left(L^e_{t+s}\right)\right\}\right],$$

with felicity functions $u(c)=(c^{1-\sigma}-1)/(1-\sigma)$ and $v(l)=\chi l^{1+\eta}/(1+\eta)$, subject to the actual budget constraint for the current period t, $\tilde{c}_{i,t}+\tilde{b}_{i,t}=b_{i,t-1}R_{t-1}\Pi_t^{-1}+w_t z_{i,t}N_t+d_t z_{i,t}$, expected future period budget constraints, $\tilde{c}_{i,t+s}+\tilde{b}_{i,t+s}=\tilde{b}_{i,t-1+s}R^e_{t-1+s}(\Pi^e_{t+s})^{-1}+w^e_{t+s}z_{i,t+s}N^e_{t+s}+d^e_{t+s}z_{i,t+s}$, for $s>0$, as well as borrowing limits $\tilde{b}_{i,t+s}\geq\underline{b}$, for $s\geq 0$.[18]

It will be useful to define demand functions for individual bond holdings and consumption, given by $x_t(b,z)=x\left(b_{i,t-1},z_{i,t};\{S_{t+s}\}_{s=0}^{\infty},\Omega_{t-1},\Omega_t,\{\Omega^e_{t+1+s}\}_{s=0}^{\infty}\right)$, $x\in\{b,c\}$. These functions solve the household problem at date t and constitute the policy functions for $\tilde{b}_{i,t}$ and $\tilde{c}_{i,t}$, respectively. Furthermore, let $\Psi_{i,t-1}$ denote the beginning-of-period joint distribution of bonds $b_{i,t-1}$ and labour productivity $z_{i,t}$ for period t. In this paper, there is no public debt and bonds are therefore in zero net supply.[19] The bond market clearing condition hence is given by $\int_i b_{i,t}d\Psi_{i,t-1}=0$, with $b_{i,t}=b_t\left(b_{i,t-1},z_{i,t}\right)$.

Note that without idiosyncratic income risk and borrowing constraints, i.e. if $z_{i,t+s}=1$,

[18] The expectations operator $\mathbb{E}_t[\cdot]$ computes household expectations with respect to idiosyncratic labour productivity.

[19] In RANK models, lump-sum taxes ensure Ricardian equivalence holds, allowing a separation of monetary and fiscal policy in the presence of positive public debt. Such a separation is no longer possible in HANK models. Even if monetary policy transmission does not depend on fiscal policy in the corresponding RANK model, fiscal policy and the details of how it is specified would play a non-trivial role in our HANK model (see e.g. Kaplan et al., 2018). Given that this would make it more complicated to understand the role of market (in)completeness for make-up strategies, we disregard fiscal policy in this paper altogether.

for $s \geq 0$, as well as $\underline{b} = -\infty$, individual consumption $\tilde{c}_{i,t}$ can be expressed in closed form as

$$\tilde{c}_{i,t} = \frac{b_{i,t-1}\frac{R_{t-1}}{\Pi_t} + w_t N_t + d_t + \sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)\left(w^e_{t+s}N^e_{t+s} + d^e_{t+s}\right)}{1 + \sum_{s=1}^{\infty}\prod_{u=1}^{s}\beta_{t+u-1}^{1/\sigma}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{1/\sigma-1}},$$

which illustrates that individual consumption in period t is a fraction of the sum of initial wealth and the present-value of lifetime income as perceived by the household at date t.[20] Given $\tilde{c}_{i,t}$, bond holdings $\tilde{b}_{i,t}$ are then determined residually via the household period budget constraint $\tilde{b}_{i,t} = b_{i,t-1}R_{t-1}\Pi_t^{-1} + w_t z_{i,t} N_t + d_t z_{i,t} - \tilde{c}_{i,t}$.

Goods market A representative firm combines intermediate production inputs $Y_{j,t}$, supplied by a unit-mass continuum of monopolistic firms, indexed by $j \in [0,1]$, to produce the final good $Y_t = \left(\int_j Y_{j,t}{}^{(\theta_p-1)/\theta_p} dj\right)^{\theta_p/(\theta_p-1)}$. Profit maximisation by the firms yields $Y_{j,t} = (P_{j,t}/P_t)^{-\theta_p} Y_t$ as the demand function for intermediate good j. The price of the final good is $P_t = \left(\int_j P_{j,t}{}^{1-\theta_p} dj\right)^{1/(1-\theta_p)}$, where $P_{j,t}$ denotes the price of input $Y_{j,t}$ set by intermediate-good firm j. The intermediate good $Y_{j,t}$ is produced by firm j with production function $Y_{j,t} = N_{j,t}$. Real marginal costs are equal across firms and given by w_t.

Intermediate-good firms set prices subject to a Calvo-type friction. Only a randomly selected fraction $(1-\xi_p)$ of firms is therefore able to reset its price $P_{j,t}$ in each period, taking into account future price stickiness. For the remaining fraction of firms that cannot optimally reset their prices (ξ_p), the intermediate-good price is updated according to $P_{j,t} = P_{j,t-1}\left[\Pi^{\iota_p}(\Pi^e_{t-1})^{1-\iota_p}\right]$, where Π denotes the long-run inflation rate and $\iota_p \in [0,1]$ how prices are indexed. When setting its price at date t, firm j can observe the endogenous variables for the current period but relies on beliefs about their future values. Specifically, the relevant beliefs involve the aggregate price level, P^e_{t+s}, aggregate production, Y^e_{t+s}, and the aggregate real wage, w^e_{t+s}. Firms do, however, possess perfect foresight with respect to fluctuations in exogenous aggregate variables. For simplicity, we assume that firms discount future profits with constant discount factor β, which is the long-run value of household discount factor β_t. We assume that intermediate firms have to pay a fixed cost $\Phi \geq 0$ in each period.

An intermediate-good firm j that can reset its price in period t chooses $P_{j,t}$ to maximise

$$\sum_{s=0}^{\infty}(\beta\xi_p)^s[(P_{j,t}/P^e_{t+s} - \lambda_{t+s}w^e_{t+s})Y_{j,t+s} - \Phi],$$

subject to the demand function $Y_{j,t} = (P_{j,t}/P_t)^{-\theta_p} Y_t$ and the indexing rule $P_{j,t+s} = P_{j,t} \times \prod_{u=1}^{s}\left[\Pi^{\iota_p}(\Pi^e_{t+u-1})^{1-\iota_p}\right]$, which updates prices until they can again be re-set in the future. We introduce the random variable λ_t to allow for cost-push shocks.[21]

We focus on a symmetric equilibrium in which all firms that can reset their prices do so in the same way. Let P^*_t denote the common reset price and define $\Pi^*_t = P^*_t/P_{t-1}$ as well

[20] The derivation of the optimal decision rules shown in this section can be found in Appendix A.

[21] As in Chen et al. (2012), we assume that the mark-up shifter λ_t shows up in the intermediate-good firm problem, affecting price setting, but not in actual profits. This formulation is similar to how Hagedorn et al. (2019) treat Rotemberg-type price adjustment costs in the goods market.

as $\Pi_t = P_t/P_{t-1}$. It is straightforward to show that the first-order condition for P_t^* can be written as

$$\Pi_t^* = \frac{\theta_p}{\theta_p - 1} \frac{\lambda_t w_t Y_t + \sum_{s=1}^{\infty} (\beta \xi_p)^s \left(\prod_{u=1}^{s} \frac{\Pi^{\iota_p} (\Pi_{t+u-1}^e)^{1-\iota_p}}{\Pi_{t+u}^e} \right)^{-\theta_p} \lambda_{t+s} w_{t+s}^e Y_{t+s}^e}{Y_t + \sum_{s=1}^{\infty} (\beta \xi_p)^s \left(\prod_{u=1}^{s} \frac{\Pi^{\iota_p} (\Pi_{t+u-1}^e)^{1-\iota_p}}{\Pi_{t+u}^e} \right)^{1-\theta_p} Y_{t+s}^e} \Pi_t,$$

with inflation Π_t satisfying ${\Pi_t}^{1-\theta_p} = (1-\xi_p)(\Pi_t^*)^{1-\theta_p} + \xi_p \left(\Pi^{\iota_p} (\Pi_{t-1})^{1-\iota_p} \right)^{1-\theta_p}$.

Due to price dispersion, $\mu_{p,t}$, aggregate output in the economy satisfies $\mu_{p,t} Y_t = N_t$ and is hence distorted for $\mu_{p,t} > 1$. The law of motion for price dispersion is given by $\mu_{p,t} = \Pi_t^{\theta_p} \left[(1-\xi_p)(\Pi_t^*)^{-\theta_p} + \xi_p \left(\Pi^{\iota_p} (\Pi_{t-1}^e)^{1-\iota_p} \right)^{-\theta_p} \mu_{p,t-1} \right]$. Goods market clearing requires $Y_t = \int_i c_{i,t} d\Psi_{i,t-1} + \Phi$, where $c_{i,t} = c_t(b_{i,t-1}, z_{i,t})$. Finally, aggregate firm profits are $d_t = Y_t - w_t N_t - \Phi$.

Labour market There is a unit-one continuum of labour unions, indexed by $k \in [0,1]$, that aggregate labour supplied by households to specialised labour services $N_{k,t}$ which are then offered to a representative labour packer at nominal wage $W_{k,t}$. These labour services are in turn combined by the labour packer to generate the composite labour input N_t according to a CES function, $N_t = \left(\int_k {N_{k,t}}^{(\theta_w - 1)/\theta_w} dk \right)^{\theta_w/(\theta_w - 1)}$, and sold to intermediate-good firms at the nominal wage W_t.[22] Profit maximisation by the labour packer yields the demand function for labour services of union k, $N_{k,t} = (W_{k,t}/W_t)^{-\theta_w} N_t$. The aggregate wage satisfies $W_t = \left(\int_k {W_{k,t}}^{1-\theta_w} dk \right)^{1/(1-\theta_w)}$.

Each union k generates labour services by aggregating efficiency units of labour supplied by households, $N_{k,t} = \int_i z_{i,t} n_{i,k,t} d\Psi_{i,t-1}$. Household i is employed by union k according to the uniform rule $n_{i,k,t} = N_{k,t}$, implying that all households supply the *same* amount of labour to union k and hence in total. Total hours worked by household i are $l_{i,t} = \int_k n_{i,k,t} dk$ and – as will be shown below – equal across households, i.e. $l_{i,t} = L_t$.

Unions are subject to nominal rigidities à la Calvo when setting wage $W_{k,t}$ for labour services $N_{k,t}$. Similar to the goods market, only a fraction $1 - \xi_w$ of randomly selected unions can thus optimally reset the wage $W_{k,t}$ in each period, whereas the remaining fraction cannot re-optimise and updates the wage based on the indexing rule $W_{k,t+s} = W_{k,t} \times \prod_{u=1}^{s} \left[\Pi^{\iota_w} (\Pi_{t+u-1}^e)^{1-\iota_w} \right]$, with $\iota_w \in [0,1]$. Like the households and the intermediate-good firms, labour unions form beliefs about endogenous aggregate variables, $\left\{ \Omega_{t+1+s}^e \right\}_{s=0}^{\infty}$, but have perfect foresight regarding exogenous aggregate variables, $\{S_{t+s}\}_{s=0}^{\infty}$.

Taking as given household consumption-saving decisions and anticipating future wage stickiness, a union k that can reset its wage at date t chooses $W_{k,t}$ to maximise

$$\sum_{s=0}^{\infty} (\beta \xi_w)^s \left\{ MU_{t+s}^e (W_{k,t+s}/P_{t+s}^e) N_{k,t+s} - MV_{t+s}^e N_{k,t+s} \right\},$$

with $MU_{t+s} = \int z_{i,t+s} {c_{i,t+s}}^{-\sigma} d\Psi_{i,t+s-1}$ and $MV_{t+s} = \int \chi {l_{i,t+s}}^{\eta} d\Psi_{i,t+s-1}$, subject to labour

[22] The structure of the labour market follows Auclert et al. (2020), who extend the sticky-wage framework developed in Erceg et al. (2000) and Schmitt-Grohé and Uribe (2006) to a HANK model.

service demand $N_{k,t+s} = \left(W_{k,t+s}/W^e_{t+s}\right)^{-\theta_w} N^e_{t+s}$ and the indexing rule for wages.

In a symmetric equilibrium, all unions that can reset their wages choose to do so in the same way. Define W^*_t as the common nominal reset wage for these unions. The unions' first-order condition for the nominal wage W^*_t can be written in real terms as

$$w^*_t = \frac{\theta_w}{\theta_w - 1} \frac{N_t w_t^{\theta_w} MV_t + \sum_{s=1}^{\infty} (\beta \xi_w)^s \left(\prod_{u=1}^{s} \frac{\Pi^{\iota_w} (\Pi^e_{t+u-1})^{1-\iota_w}}{\Pi^e_{t+u}}\right)^{-\theta_w} N^e_{t+s} \left(w^e_{t+s}\right)^{\theta_w} MV^e_{t+s}}{N_t w_t^{\theta_w} MU_t + \sum_{s=1}^{\infty} (\beta \xi_w)^s \left(\prod_{u=1}^{s} \frac{\Pi^{\iota_w} (\Pi^e_{t+u-1})^{1-\iota_w}}{\Pi^e_{t+u}}\right)^{1-\theta_w} N^e_{t+s} (w^e_{t+s})^{\theta_w} MU^e_{t+s}},$$

where $w^*_t = W^*_t / P_t$.

Using the uniform labour demand rule $n_{i,k,t} = N_{k,t}$ and the demand function for labour services offered by union k, $N_{k,t} = (W_{k,t}/W_t)^{-\theta_w} N_t$, total hours for household i, $l_{i,t} = \int_k n_{i,k,t} dk$, can be written as $l_{i,t} = \mu_{w,t} N_t$, with wage dispersion $\mu_{w,t}$ evolving according to the law of motion $\mu_{w,t} = (1-\xi_w)\left(w^*_t/w_t\right)^{-\theta_w} + \xi_w (w_{t-1}/w_t)^{-\theta_w} \left(\Pi^{\iota_w} (\Pi^e_{t-1})^{1-\iota_w}/\Pi_t\right)^{-\theta_w} \mu_{w,t-1}$. It is straightforward to see that – as noted earlier – all households work the same amount of hours $l_{i,t} = L_t$, which is different from composite labour input N_t for $\mu_{w,t} > 1$. It therefore holds that $L_t = \mu_{w,t} N_t$. Finally, the aggregate real wage satisfies $w_t{}^{1-\theta_w} = (1-\xi_w)\left(w^*_t\right)^{1-\theta_w} + \xi_w \left(\Pi^{\iota_w} (\Pi^e_{t-1})^{1-\iota_w} w_{t-1}/\Pi_t\right)^{1-\theta_w}$.

Monetary policy The central bank sets the nominal interest rate R_t based on an instrument rule and subject to an ELB, $R_t = \max\left\{\tilde{R}_t, R_{ELB}\right\}$. For the shadow rate $\tilde{R}_t$, we consider a slightly modified inertial Taylor rule,

$$\frac{\tilde{R}_t}{R} = \left(\frac{\tilde{R}_{t-1}}{R}\right)^{\rho_R} \left(\left(\frac{\Pi^{(T)}_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right)^{1-\rho_R},$$

where R denotes the (quarterly) long-run nominal interest rate, Π the (quarterly) inflation target, Y the long-run output level, and $\Pi^{(T)}_t = \left(\prod_{k=1}^{T} \Pi_{t-k+1}\right)^{1/T}$ the average (quarterly) inflation rate over the past T periods.

This rule captures two types of monetary policy strategies. For $T = 1$, the rule reduces to a standard Taylor rule that targets the current inflation rate. We refer to this case as an inflation targeting (IT) strategy. This is the case usually considered in the New Keynesian literature to model monetary policy. For $T > 1$, the policy rule targets the average inflation rate over the last T periods. We refer to this case as an average inflation targeting (AIT) strategy. Under this specification, missing the inflation target in a given period has implications for how interest rates will be set in future periods. Specifically, monetary policy is committed to make up for past inflation misses until the average inflation rate meets the target again. As is well known (see Svensson, 1999; Arias et al., 2020), this make-up element can help to stabilise inflation more effectively by guiding agents' expectations, particularly when monetary policy is constrained by an ELB.

As a measure of real economic activity, output enters the interest rate rule as well. This assumption is standard and implies that the central bank does not follow a strict (A)IT strategy. Instead, it follows a flexible version of it that reflects potential policy trade-offs, e.g. due to a dual mandate. This is consistent with the monetary policy of the Fed, both before and after its recent strategy revision. The interest rate rule also includes an interest rate smoothing component, which captures that interest rates tend to be adjusted only gradually in practice (see e.g. Nakata and Schmidt, 2019).

2.2 Equilibrium

In this section, we define the model equilibrium. Having derived the relevant equilibrium conditions for arbitrary beliefs in the previous section, we start by defining a temporary equilibrium for the model.[23] Building on this, we then define the equilibrium under rational and reflective expectations.

Temporary equilibrium We define a temporary equilibrium as follows. Let $\Omega = (\mu_p, \mu_w, \Pi, \{\Pi_-\}, \Pi^*, \Psi(\cdot), d, L, MU, MV, N, R, \tilde{R}, w, w^*, Y)$ denote the set of endogenous aggregate variables and $S = (\beta, \lambda)$ the set of exogenous aggregate variables.[24] Given initial values Ω_{t-1}, S_{t-1} and beliefs $\left\{\Omega^e_{t+1+s}\right\}_{s=0}^{\infty}$, a temporary equilibrium consists of sequences $\{S_{t+s}, \Omega_{t+s}, b_{t+s}(\cdot), c_{t+s}(\cdot)\}_{s=0}^{\infty}$, such that households, firms and unions solve their respective optimisation problems, markets clear and the wealth distribution is consistent with household actions at every date $t+s$, $s \geq 0$.

Rational-expectations equilibrium The rational-expectations equilibrium provides us with an important benchmark for our model analysis under non-rational expectations. We can define a rational-expectations equilibrium simply as a temporary equilibrium that satisfies perfect foresight (see Farhi and Werning, 2019). Specifically, a rational-expectations equilibrium is given by $\{S_{t+s}, \Omega_{t+s}, b_{t+s}(b,z), c_{t+s}(b,z), \Omega^e_{t+s}\}_{s=0}^{\infty}$, such that $\{S_{t+s}, \Omega_{t+s}, b_{t+s}(b,z), c_{t+s}(b,z)\}_{s=0}^{\infty}$ is a temporary equilibrium for beliefs $\left\{\Omega^e_{t+s}\right\}_{s=0}^{\infty}$ and $\left\{\Omega^e_{t+s}\right\}_{s=0}^{\infty} = \{\Omega_{t+s}\}_{s=0}^{\infty}$ holds for $s \geq 0$ (perfect foresight).

Reflective-expectations equilibrium To model bounded rationality, we consider a reflective-expectations equilibrium (see García-Schmidt and Woodford, 2019). The concept of reflective expectations is closely related to level-k thinking, which is analysed in a HANK context by Farhi and Werning (2019). In both cases, beliefs are updated based on a recursive scheme after an aggregate shock is realised. While both concepts are closely related, level-k thinking is more well known and easier to understand than the more general concept of

[23] The temporary equilibrium concept goes back to Hicks (1939), Lindahl (1939) and Grandmont (1977). More recently, Preston (2005) and Woodford (2013), as well as Farhi and Werning (2019) and García-Schmidt and Woodford (2019), have applied this concept for the analysis of New Keynesian models with bounded rationality.

[24] By including the lagged inflation rates $\{\Pi_{-s,t}\}_{s\in\{1,2,\dots,T-2\}}$, where $\Pi_{-s,t} = \Pi_{-(s-1),t-1}$ for $s \in \{1,2,\dots,T-2\}$ and $\Pi_{-0,t-1} = \Pi_{t-1}$, in the vector of endogenous variables, Ω_{t-1} and S_t contain all state variables needed to determine Ω_t, since we can write $\Pi^{(T)}_t = \left(\Pi_t \times \prod_{s=0}^{T-2} \Pi_{-s,t-1}\right)^{1/T}$.

reflective expectations. We thus first describe level-k thinking in the context of the model and then turn to reflective expectations.

Under level-k thinking, the beliefs of agents with the discrete level of reasoning $k > 1$ about outcomes in period $t+s$, $s > 0$, denoted as $\Omega^{e,k}_{t+s}$, are given by the equilibrium outcomes for that period in an economy where all agents possess the level of reasoning $k-1$, denoted as Ω^{k-1}_{t+s}.[25] Starting with some initial beliefs held by agents in a "level-1 economy" in period t about outcomes in period $t+s$, $\Omega^{e,1}_{t+s}$, beliefs held by "level-k" agents in period t about outcomes in period $t+s$ are thus determined by the discrete recursion $\Omega^{e,k}_{t+s} = \Omega^{k-1}_{t+s}$. Intuitively, a naive level-1 agent responds to changes in the exogenous aggregate variables (e.g. an increase in patience) but fails to take into account any general equilibrium effects of the shock. For $k > 1$, level-k agents take general equilibrium effects into account but only imperfectly. The more rounds of reasoning k an agent goes through, the closer his beliefs are to reality, with beliefs approaching rational expectations for $k \to \infty$.

Reflective expectation formation operates similarly but does not proceed in discrete steps. Instead, beliefs are updated continuously. Let n denote the – now continuous – level of cognitive ability under reflective expectations. Beliefs about outcomes in period $t+s$ held by agents with cognitive ability level n in period t, denoted as $\Omega^{e,n}_{t+s}$, are now formed based on the differential equation $d\Omega^{e,n}_{t+s}/dn = \Omega^{n}_{t+s} - \Omega^{e,n}_{t+s}$, where Ω^{n}_{t+s} denotes equilibrium outcomes in period $t+s$ for an economy populated by n-type agents.

To illustrate the connection between reflective expectations and level-k thinking, it is helpful to consider a discrete approximation of the continuous updating process,

$$\Omega^{e,n+dn}_{t+s} \cong dn \times \Omega^{n}_{t+s} + (1-dn) \times \Omega^{e,n}_{t+s},$$

where $dn > 0$ is a very small real number. Beliefs of agents with cognitive ability $n+dn$ about outcomes in period $t+s$ can hence be viewed as a convex combination of equilibrium outcomes in period $t+s$ for an economy populated by n-type agents, Ω^{n}_{t+s}, and beliefs for period $t+s$ held by agents in such an economy, $\Omega^{e,n}_{t+s}$. Compared to level-k thinking, which is effectively nested for $dn = 1$ in the expression above, updating of reflective beliefs occurs in a smooth manner since $dn \approx 0$ implies an updating weight of approximately one for $\Omega^{e,n}_{t+s}$. This smoothness avoids jumps in expectations that can take place during belief updating under level-k thinking and result in oscillatory behaviour of the economy over time (see García-Schmidt and Woodford, 2019). As noted by Angeletos and Sastry (2021), such behaviour can be considered an artifact of discrete belief updating under level-k thinking without deeper economic meaning.[26]

Formally, we can define the reflective-expectations equilibrium as follows. Let $\{S, \Omega, b(b,z), c(b,z), \Omega^{e}\}_{t=0}^{\infty}$ be a rational-expectations equilibrium where all variables stay at the steady state forever. Define initial beliefs $\{\Omega^{e,0}_{t+s}\}_{s=0}^{\infty} = \{\Omega\}_{s=0}^{\infty}$, such that, in period

[25] Equilibrium outcomes in period $t+s$ in turn depend on beliefs for all periods going forward, i.e. $\{\Omega^{e,k-1}_{t+s+j}\}_{j=0}^{\infty}$.

[26] In addition to eliminating this model feature, the smooth belief formation under reflective expectations also improves the stability of the numerical model solution.

t, the economy is expected to remain at the steady state in all future periods.[27] Given a sequence $\{S_{t+s}\}_{s=0}^{\infty}$, a reflective-expectations equilibrium consists of a belief-updating process, $d\Omega_{t+s}^{e,n}/dn = \Omega_{t+s}^{n} - \Omega_{t+s}^{e,n}$, and sequences indexed by level of reflective reasoning $n \geq 0$, $\left\{S_{t+s}, \Omega_{t+s}^{n}, b_{t+s}^{n}(\cdot), c_{t+s}^{n}(\cdot)\right\}_{s=0}^{\infty}$, such that each $\left\{S_{t+s}, \Omega_{t+s}^{n}, b_{t+s}^{n}(\cdot), c_{t+s}^{n}(\cdot)\right\}_{s=0}^{\infty}$ constitutes a temporary equilibrium for beliefs $\left\{\Omega_{t+s}^{e,n}\right\}_{s=0}^{\infty}$, with $n = n^{*} > 0$ being the actual level of reflective reasoning in the economy. For notational convenience, we use the convention $\Omega_{t+s} = \Omega_{t+s}^{n^{*}}$, $s \in \{0, 1, 2, \ldots, T\}$, for the reflective-expectations case. Rational expectations are nested as a special case for the limit $n^{*} \to \infty$.

García-Schmidt and Woodford (2019) show that period-t beliefs about outcomes in period $t+s$ held by agents in a reflective-expectations equilibrium with the continuous level of reasoning n, $\Omega_{t+s}^{e,n}$, are equal to the average beliefs held in a level-k economy where agents with the discrete level of reasoning $k \geq 1$ are distributed across the population according to a Poisson distribution with mean n,

$$\Omega_{t+s}^{e,n} = \sum_{k=1}^{\infty} \frac{n^{k-1} \exp(-n)}{(k-1)!} \Omega_{t+s}^{e,k}.$$

The beliefs held by level-k thinkers for period $t+s$, $\Omega_{t+s}^{e,k}$, are in this case determined by the discrete recursion $\Omega_{t+s}^{e,k} = \Omega_{t+s}^{k-1}$.

3 Calibration

We calibrate the model for the United States at a quarterly frequency. The parameter values are summarised in Table 1. Parameters related to the monetary policy rule are specified as follows. We assume an annual inflation target $(\Pi)^4$ of 2%, an effective lower bound $(R_{ELB})^4$ of 0.125% (see Nakata and Schmidt, 2019) and the rather standard policy rule parameters $(\phi_{\Pi}, \phi_{Y}, \rho_{R}) = (1.5, 1, 0.75)$.[28] Consistent with estimates reported in Bernanke (2020), we target an annual long-run real rate of 1% and implement it via the (long-run) household discount factor. For the RANK model, the required β-value is 0.998, whereas it is 0.993 for the HANK model.[29]

We choose a standard value of 2 for the coefficient of relative risk aversion σ and a Frisch elasticity $1/\eta$ of 0.5 (see Chetty et al., 2011). For the RANK model, we normalise the labour disutility parameter χ to 1 and set it equal to 1.023 for the corresponding HANK model, such that both model versions display the same steady-state output level. The household borrowing limit is set to -1.667, which equals 5 times the monthly household labour income (see Ferrante and Paustian, 2019). We set the elasticity of substitution between in-

[27] It is of course possible to consider initial beliefs based on more general rational-expectations equilibria (see e.g. Farhi and Werning, 2019). We assume this particular equilibrium because our numerical model solution makes this assumption as well.

[28] As suggested by Brayton et al. (1997) and Taylor (1999), we use an output response coefficient of 1, which is somewhat higher compared to parameter values often considered in the literature (see e.g. Taylor, 1993). See also Arias et al. (2020).

[29] The discount factor is lower for the HANK model calibration because the households' precautionary saving motive pushes down the real rate for a given discount factor (see e.g. Huggett, 1993).

Table 1: Model parameters for HANK model

Parameter	Description	Value
β	Discount factor	0.993*
χ	Weight labour disutility	1.023*
Φ	Fixed cost of production	0.167
ϕ_Π	Inflation coeff. policy rule	1.5
ϕ_Y	Output coeff. policy rule	1
η	Inverse Frisch elasticity	2
ι_p	Price indexation	1
ι_w	Wage indexation	1
λ	Price mark-up shifter	1
$(\Pi)^4$	Long-run inflation target (annual)	1.02
Π_w	Long-run nominal wage inflation	1.005
θ_p	Price elast. of subst. int. goods	6
θ_w	Wage elast. of subst. labour services	6
ρ_β	Persist. discount factor	0.85
ρ_λ	Persist. mark-up shifter	0
ρ_z	Persist. idiosync. productivity	0.966
ρ_R	Interest rate smoothing	0.75
σ	Coeff. of relative risk aversion	2
σ_β	Std. dev. discount factor shock	0.006
σ_λ	Std. dev. mark-up shock	0.118
σ_z	Std. dev. idiosync. prod. shock	0.052
ξ_p	Calvo price-setting	0.85
ξ_w	Calvo wage-setting	0.85
$\underline{b}$	Household borrowing limit	-1.667
$(R_{ELB})^4$	ELB (annual)	1.001

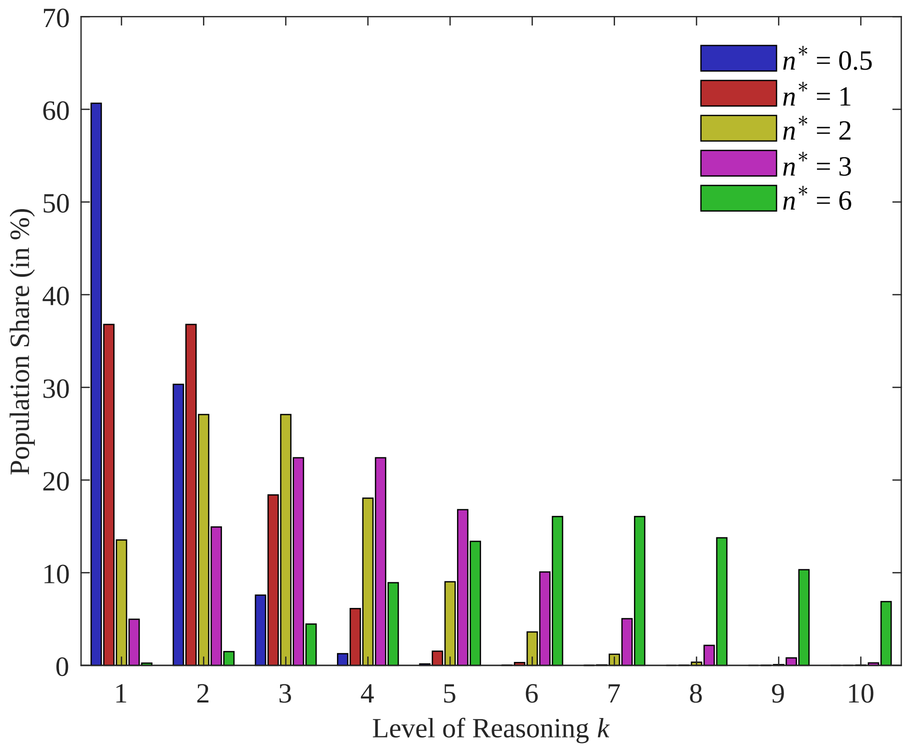
* For the RANK model, we use $(\beta, \chi) = (0.998, 1)$ instead.

termediate goods θ_p and the Calvo price-setting parameter ξ_p to 6 and 0.85, respectively (see McKay et al., 2016), and normalise the degree of price indexation to $\iota_p = 1$. For the labour market, we use the same parameter values as for the goods market, i.e. $\theta_w = \theta_p$, $\xi_w = \xi_p$ and $\iota_w = \iota_p$. As in Hagedorn et al. (2019), we calibrate the fixed production cost to obtain zero firm profits in the steady state, which is accomplished with $\Phi = 0.167$.

Idiosyncratic productivity $z_{i,t}$ follows a log-normal AR(1) process with persistence parameter ρ_z and shock standard deviation σ_z. We set $\rho_z = 0.966$ (see McKay et al., 2016) and – similar to Fernández-Villaverde et al. (2021) – calibrate the shock standard deviation to match an average annual marginal propensity to consume (MPC) of 0.55 (see Auclert et al., 2020), implying $\sigma_z = 0.052$.[30] We also consider log-normal AR(1) processes for the household discount factor and the mark-up shifter, given by $\beta_t = \beta^{1-\rho_\beta} {\beta_{t-1}}^{\rho_\beta} \exp(\varepsilon_{\beta,t})$ and $\lambda_t = \lambda^{1-\rho_\lambda} {\lambda_{t-1}}^{\rho_\lambda} \exp(\varepsilon_{\lambda,t})$, respectively. The i.i.d. shock $\varepsilon_{x,t}$ is drawn from a normal distribution with zero mean and standard deviation σ_x, $x \in \{\beta, \lambda\}$. The persistence parameters

[30]Our model calibration yields a Gini coefficient of 0.80 in the steady state, which is slightly lower than the value of 0.87 reported by Kuhn and Ríos-Rull (2016) based on 2013 US data on liquid wealth.

Figure 1: Distribution of level-k agents for different cognitive ability levels n^*



ρ_β and ρ_λ are set to 0.85 and 0, respectively (see Nakata and Schmidt, 2019). We calibrate the shock standard deviations to match statistics for the rational-expectations RANK model with an IT interest rate rule, i.e. for $T = 1$. To obtain an ELB frequency of 20%, we set σ_β to 0.0062. For σ_λ, we choose a value of 0.118 to match the standard deviation of annualised US inflation for the time period 1997:Q3-2017:Q2, which is 0.52% (see Nakata and Schmidt, 2019).[31] When analysing AIT, we consider specifications with averaging windows of $T = 16$ and $T = 32$ quarters, i.e. 4 and 8 years, for the average inflation target.[32] We refer to these specifications as AIT4 and AIT8, respectively.

For the reflective-expectations case, we consider the cognitive ability levels $n^* \in \{0.5, 1, 2, 3, 6\}$. Based on the interpretation proposed by García-Schmidt and Woodford (2019) (see Section 2.2), different n^*-values imply different (Poisson) distributions of level-k agents in the population. Figure 1 displays the population shares associated with the values that we consider for the simulation exercise. Experimental evidence typically suggests that most probability mass is covered by $k \in \{1, 2, 3\}$ (see e.g. Nagel, 1995; Costa-Gomes and Crawford, 2006; Arad and Rubinstein, 2012), which is consistent with the distributions predicted by $n^* \in \{0.5, 1, 2\}$.[33] As argued by Farhi and Werning (2019), experimental evidence relies

[31]For simplicity, both shock standard deviations are calibrated to match the respective targets under the assumption that the other aggregate shock is not operative. They are hence not jointly calibrated.

[32]The remaining model parameters are kept unchanged in this case.

[33]There are, however, exceptions. For example, Kneeland (2015)'s experimental evidence shows substantial

on games that are substantially simpler than the environment agents are facing in macroeconomic models. As a result, the k-values relevant for our quantitative model are likely lower than the ones found in the experimental literature. Whereas the n^*-values 0.5, 1 and – perhaps – 2 can be considered empirically plausible, the values 3 and 6 suggest a level of cognitive ability that is hard to reconcile with (micro-level) evidence. Nevertheless, we consider these higher values as well to investigate how large n^* has to be in order for the economy to behave as in the rational-expectations case.

4 Numerical solution

In theory, the extended path method is a natural candidate for stochastic simulations of a model with reflective expectations.[34] However, the method is not well suited for our particular application, which not only involves iterative belief formation but also an ELB and household heterogeneity.[35] In a stochastic simulation, the model economy is hit with new unanticipated aggregate shocks in each simulation period going forward. When using the extended path method for such a simulation, one has to numerically compute a new nonlinear transition path back to the steady state for each simulation period. In practice, these computations become already quite slow for our rather simple RANK model when allowing for iterative belief formation and an occasionally-binding ELB. In a model that also includes incomplete markets, such that the entire wealth distribution becomes a relevant aggregate state variable, these speed issues become prohibitive.

To overcome this difficulty, we use a numerical solution approach that is based on a local approximation of the model in the sequence space (see Boppart et al., 2018; Auclert et al., 2021) and takes into account the occasionally-binding ELB constraint by using anticipated monetary policy shocks (see Bodenstein et al., 2013; Holden, 2016).[36] The algorithm consists of two steps. The first step involves the computation of impulse response functions for the endogenous aggregate variables to shocks to the exogenous aggregate model variables. These shocks hit the economy in the deterministic steady state and are assumed to be unanticipated, transitory and small. Under these assumptions, Boppart et al. (2018)

probability mass also at $k = 4$, which is in line with the distribution implied by $n^* = 2$. Similarly, based on firm survey data, Coibion et al. (2021) find levels of reasoning $k \in \{3,4,5\}$ to be well represented.

[34]See Heer and Maußner (2009) for a recent treatment of the method. As discussed in García-Schmidt and Woodford (2015), there is a direct link between the belief formation process under reflective expectation (or level-k thinking) and the extended path method, which was pioneered by Fair and Taylor (1983). However, García-Schmidt and Woodford (2015, 2019), as well as Farhi and Werning (2019) and Bianchi-Vimercati et al. (2021), only consider one-time MIT shocks and do not employ the extended path algorithm to perform model simulations with randomly drawn aggregate shocks. Assuming that agents rely on point expectations of future aggregate variables, which effectively implies certainty equivalence, it is straightforward to set up the extended path algorithm based on the equilibrium definitions from Section 2 (see Appendix B). See Qiu (2018) for an alternative approach that uses a recursive model formulation for a New Keynesian model with level-k thinking and reduced-form household heterogeneity.

[35]We cannot use standard perturbation techniques (see e.g. Schmitt-Grohé and Uribe, 2004; Reiter, 2009) for the HANK model with reflective expectations due to the iterative nature of belief formation.

[36]McKay and Wieland (2021) also use anticipated monetary policy shocks to enforce the ELB for simulations of a continuous-time HANK model with durable consumption.

Table 2: Results for RANK model with rational expectations (RE-RANK)

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT (w/o ELB)	0	0	2.00	0.78	0.00	0.88
AIT4 (w/o ELB)	0	0	2.00	0.70	0.00	0.95
AIT8 (w/o ELB)	0	0	2.00	0.69	0.00	0.97
IT	20.18	6.78	1.95	0.87	-0.11	1.17
AIT4	17.11	6.65	1.98	0.74	-0.07	1.14
AIT8	16.71	6.42	1.99	0.72	-0.06	1.14

show that the computed impulse response functions can be viewed as a linear approximation of the model in the sequence space. In the second step, we use this remarkable result to perform Monte Carlo simulations for our model based on the linearised model. Given that the model approximation is a local one, it does not take into account the ELB constraint on the nominal interest rate. To enforce the ELB for the model simulations, we use news shocks to the policy rate (see Holden, 2016). In principle, applying this method requires the assumption of perfect foresight to be satisfied. However, we show how to apply it to our model with reflective expectations and demonstrate the numerical accuracy of the approach. Details about our numerical solution and its accuracy can be found in Appendix B.

5 Results

This section presents and discusses the results of our quantitative model analysis.

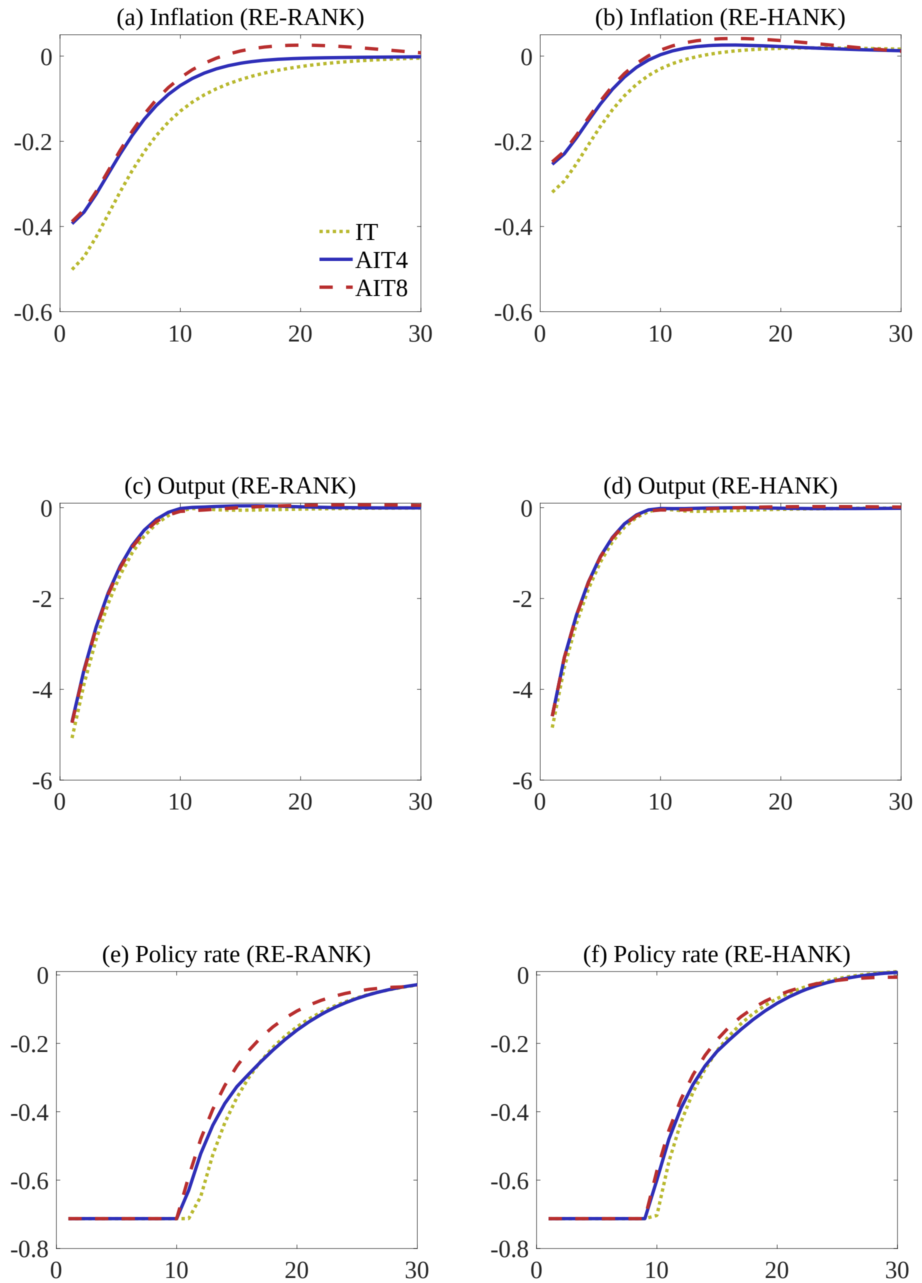
5.1 Stochastic simulations

In this section, we explore the impact of market incompleteness and bounded rationality on the effectiveness of AIT vis-à-vis IT based on stochastic simulations.[37] To isolate the contribution of the two features, we proceed in four steps. To establish a baseline, we start with a comparison of IT and AIT for the standard RANK model with complete markets and rational expectations (RE-RANK). Then we relax the complete-markets assumption and compare the two strategies for the rational-expectations HANK (RE-HANK) model. After that, we provide a comparison of IT and AIT for the RANK model with bounded rationality (BR-RANK), highlighting the role played by the rational-expectations assumption. Finally, we compare the two monetary policy strategies for the full (BR-HANK) model, abandoning both complete markets and rational expectations.

RANK with rational expectations A comparison of IT and AIT for the RE-RANK model without an ELB constraint already illustrates a key advantage of make-up monetary

[37] We conduct 25 simulations with 40,000 periods each (excluding 1,000 additional burn-in periods). For each of the 25 simulations, we then calculate moments for variables of interest and report the average of these moments across simulations.

Figure 2: ELB episode under rational expectations



Notes: IRFs are for a large demand shock ($\varepsilon_{\beta,t} = 5\sigma_\beta$) and in percentage deviations from steady state.

Table 3: Results for HANK model with rational expectations (RE-HANK)

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT (w/o ELB)	0	0	2.00	0.59	0.00	0.89
AIT4 (w/o ELB)	0	0	2.00	0.57	0.00	0.91
AIT8 (w/o ELB)	0	0	2.00	0.57	0.00	0.92
IT	16.57	5.45	1.93	0.65	-0.07	1.11
AIT4	14.40	5.79	1.97	0.59	-0.05	1.07
AIT8	14.07	5.65	1.98	0.59	-0.05	1.07

policy strategies emphasised in the literature: improved inflation stabilisation (see Table 2). Specifically, under AIT4, the standard deviation of inflation is about 10% (8 basis points) lower relative to IT. Extending the averaging window by an additional 4 years (16 model periods) further improves inflation stabilisation, but the additional improvement relative to AIT4 is only 1 basis point. The lower inflation volatility under AIT is achieved by successfully managing agents' expectations. If the average inflation target is missed in a given period, AIT prescribes that monetary policy will make up for this in the future. For instance, if average 4-year inflation is below target today, monetary policy is committed to be more expansionary in the subsequent periods. This commitment in turn drives up current expectations for inflation and real economic activity, providing additional stimulus today by encouraging households to increase consumption, firms to raise prices and unions to increase wages. Due to this "automatic stabiliser", all of this is accomplished without the need to move interest rates as much as in the IT case, which contributes to the improvement in price stabilisation. However, history dependence has the opposite impact on output volatility, which goes up as the length of the averaging window is increased. This highlights a potential downside of make-up strategies.[38]

The commitment to make up for past misses is particularly valuable when the economy is at the ELB and monetary policy cannot offer additional stimulus by cutting current interest rates (see e.g. Eggertsson and Woodford, 2003). This can be illustrated by looking at impulse response functions for a large contractionary demand shock. This shock immediately pushes the policy rate to the ELB (see left panels in Figure 2), causing strong contractions in inflation and output. However, inflation drops substantially less during the ELB episode for AIT (solid blue and dashed red lines) relative to IT (dotted yellow line). Furthermore, under AIT, the policy rate does not stay at the ELB as long as under IT. These observations highlight the automatic stabiliser property of make-up strategies mentioned above. When average inflation is below target during an ELB episode, the AIT rule promises to keep interest rates "lower for longer", providing additional stimulus that the IT rule does not. Note that, although AIT generally promises to make up for past inflation misses, it only leads to

[38] In contrast to the dampening impact on the volatility of inflation, we find that – in the absence of an ELB constraint – the qualitative effect of history dependence on output volatility depends on the degree of interest rate smoothing, ρ_R.

Table 4: Results for RANK model with reflective expectations (BR-RANK w/o ELB)

	Std. inflation (in %)			Std. output (in %)		
Cognitive ability n^*	IT	AIT4	AIT8	IT	AIT4	AIT8
0.5	0.64	0.64	0.64	1.26	1.27	1.28
1	0.68	0.68	0.68	0.96	1.00	1.02
2	0.74	0.72	0.74	0.89	0.95	0.98
3	0.77	0.72	0.74	0.88	0.95	0.98
6	0.78	0.70	0.69	0.88	0.95	0.97
∞	0.78	0.70	0.69	0.88	0.95	0.97

an actual inflation overshoot above target in Figure 2 for an averaging window of 8 years. In the stochastic model simulations, the lack of monetary stimulus at the ELB is reflected in a negative bias in inflation and output as well as heightened macroeconomic volatility. Consistent with Figure 2, these negative effects are mitigated by an AIT rule. It is worth noting that with an ELB constraint, inflation and output volatility are both lower under AIT, such that AIT leads to unambiguous improvements over IT in terms of macroeconomic stabilisation.

HANK with rational expectations Compared to the RE-RANK model, inflation and output do not behave much differently in the RE-HANK model – with or without the ELB (see Table 3). The only sizable difference involves a generally lower incidence and duration of ELB episodes in the incomplete markets case, as well as a generally lower degree of macroeconomic volatility. Importantly, the relative performance of AIT vs. IT does not change compared to the RE-RANK model, both qualitatively and quantitatively (see also right panels in Figure 2). These results likely reflect counteracting forces with respect to how market incompleteness affects the way AIT can manage economic activity by steering agents' expectations. On the one hand, the sensitivity of household consumption to promised future interest rate cuts is diminished by the presence of borrowing-constrained households (see McKay et al., 2016; Hagedorn et al., 2019). On the other hand, the marginal propensity to consume out of temporary income is substantially higher in the HANK model. Compared to the complete-markets case, this raises the importance of indirect monetary policy effects that stimulate economic activity via changes in disposable household income. For instance, redistributional effects that transfer wealth from low-MPC savers to high-MPC borrowers could in principle raise the potential of future interest rate cuts to provide economic stimulus (see e.g. Ferrante and Paustian, 2019). For our model, these counteracting forces appear to roughly cancel each other out, such that market incompleteness does not matter much for the effectiveness of make-up strategies.[39]

[39] As shown in Section 5.3, this finding is robust with respect to various assumptions made about the model, such as the denomination of debt and the cyclicality of household income risk.

Figure 3: Results for RANK model with reflective expectations (BR-RANK)

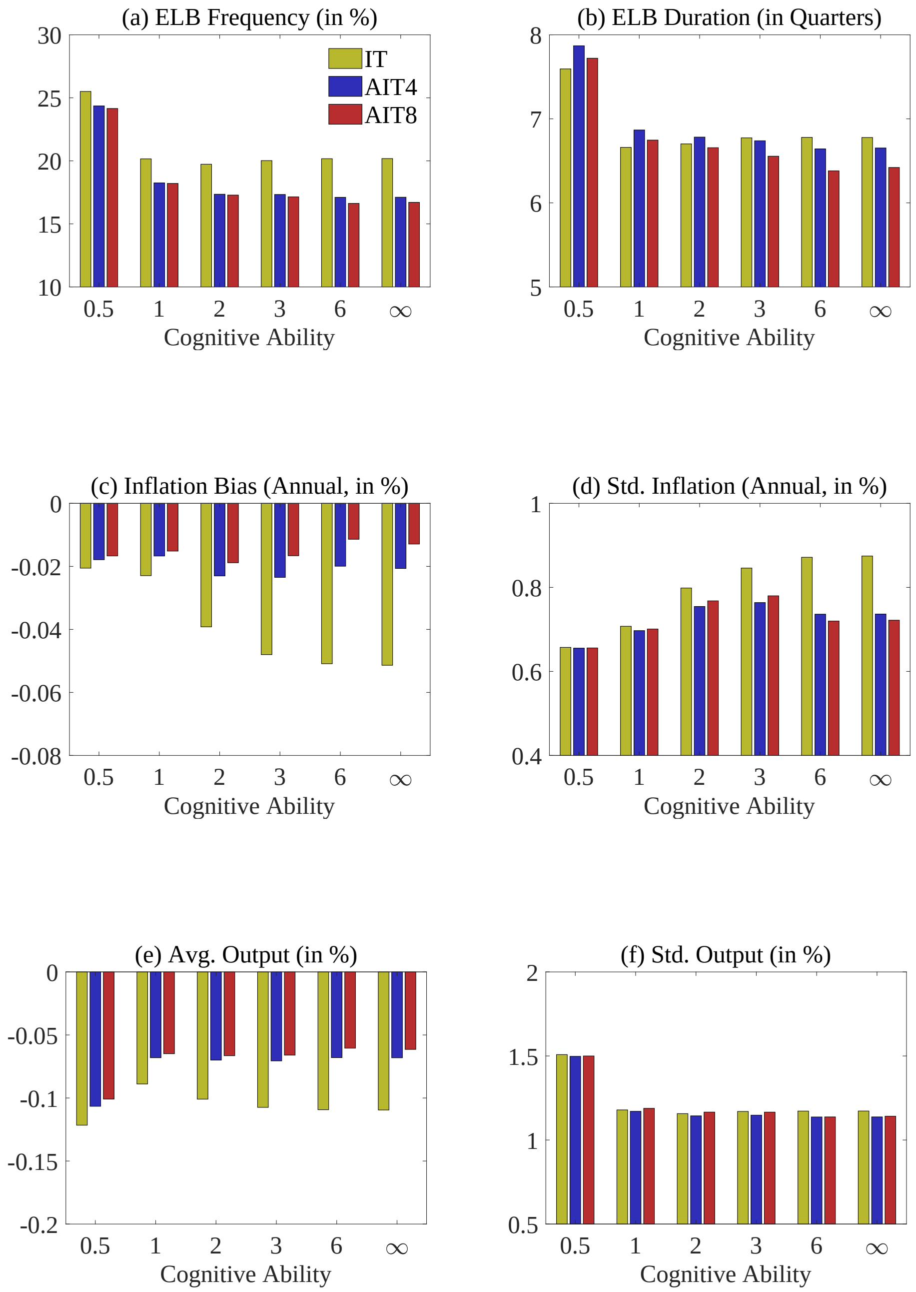
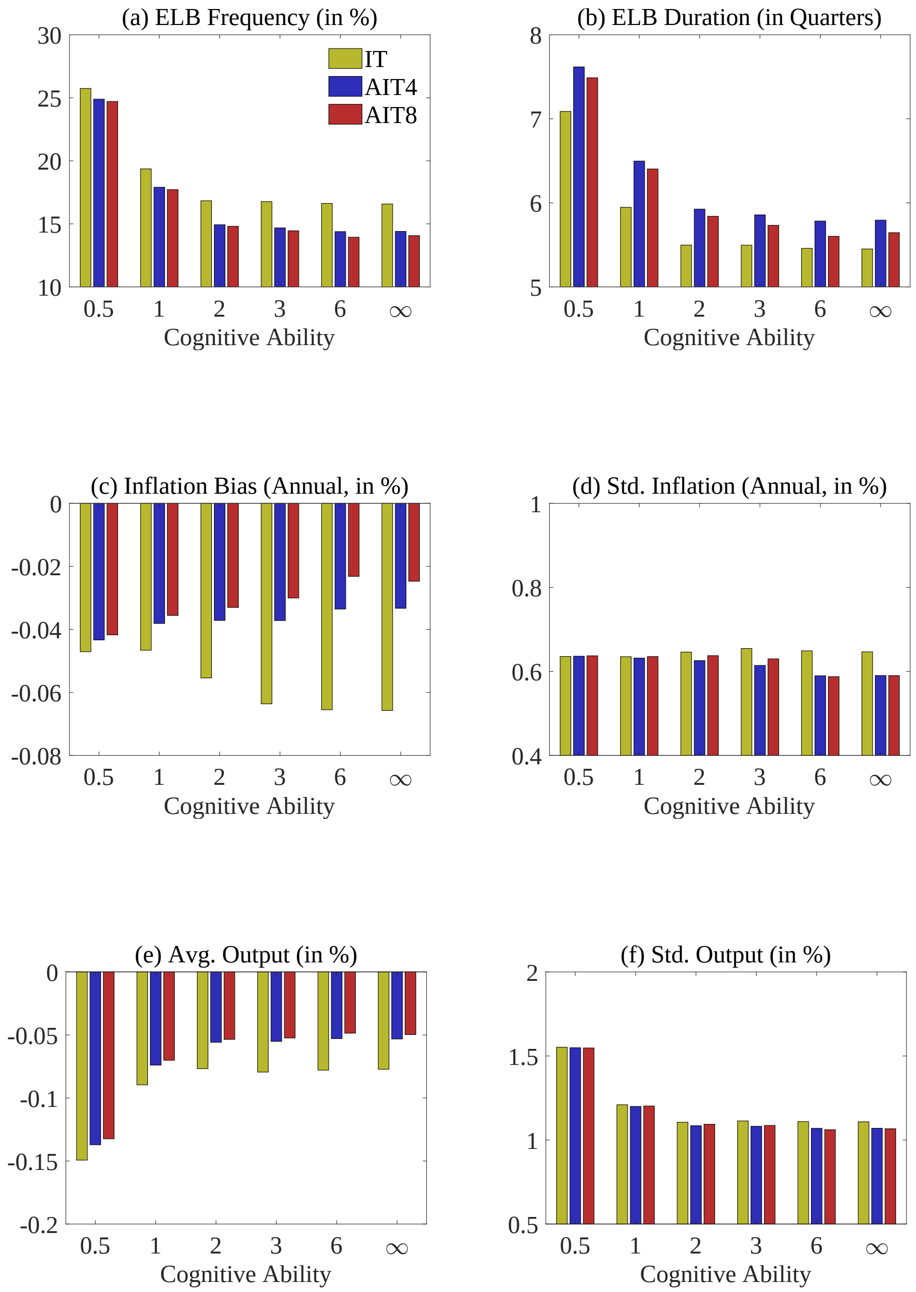


Figure 4: Results for HANK model with reflective expectations (BR-HANK)



RANK with reflective expectations In contrast to market incompleteness, bounded rationality has a clearly visible impact on model predictions and the effectiveness of AIT. First consider the BR-RANK model without an ELB constraint (see Table 4).[40] Under IT, inflation tends to be more volatile than under AIT. Furthermore, cognitive ability n^* increases inflation volatility under IT, whereas n^* has a hump-shaped impact on inflation volatility under AIT. With respect to output, we observe that it is more stable when cognitive ability is high and becomes more volatile as the degree of history dependence goes up – in line with the findings for the RE-RANK model. The model properties are virtually the same for the BR-RANK model with $n^* = 6$ and the RE-RANK model (depicted in Table 4 as $n^* = \infty$), and are already close to the rational-expectations case for $n^* = 3$.[41]

When monetary policy faces an ELB constraint, inflation is more stable under an AIT rule (see Figure 3). Inflation volatility furthermore increases with cognitive ability under IT, whereas n^* again exhibits a hump-shaped impact on the standard deviation of inflation under AIT. By contrast, output is almost equally volatile under IT and AIT. In addition, output volatility is higher for low degrees of cognitive ability ($n^* = 0.5$) but hardly different between intermediate and high cognitive ability levels. Note that inflation and output tend to be only slightly less volatile under AIT8 than AIT4, illustrating decreasing returns to history dependence under AIT.

The better macroeconomic stabilisation properties of AIT are reflected in a lower incidence of ELB episodes. Importantly, by acting as an "automatic stabiliser", AIT reduces the first-order consequences of the ELB observed under IT, as measured by downward biases in average inflation and output, for all considered degrees of cognitive ability. However, the improvement vis-à-vis IT is quite small for degrees of cognitive ability that are in line with empirical and experimental evidence, i.e. $n^* \in \{0.5, 1, 2\}$.

HANK with reflective expectations Similar to the rational-expectations case, the behaviour of the HANK model economy is also close to that of the corresponding RANK model economy under reflective expectations (see Figure 4). This observation holds for both monetary policy strategies, and regardless of the agents' cognitive ability and the presence of an ELB constraint (see Table 5). This underscores our previous findings: Whereas market (in)completeness does not matter for the relative performance of AIT vs. IT in our model, the expectations process clearly does.

In addition, we do not find an interaction between market incompleteness and bounded rationality with respect to the attenuation of make-up strategies. Figure 5 illustrates this finding by plotting the inflation and output biases under AIT relative to IT. The relationship between the relative biases and cognitive ability n^*, given by the solid blue (AIT4)

[40] As under rational expectations, impulse responses for a demand-shock induced ELB episode paint a picture that is in line with the simulation-based results (see Figures 14 to 16 in Appendix C).

[41] This observation is reassuring from a computational perspective since we use a numerical equation solver for the rational-expectation model versions but the iterative procedure sketched in Appendix B for the cases with bounded rationality. As predicted by theory, both approaches yield the same results if n^* is sufficiently large.

Table 5: Results for HANK model with reflective expectations (BR-HANK w/o ELB)

	Std. inflation (in %)			Std. output (in %)		
Cognitive ability n^*	IT	AIT4	AIT8	IT	AIT4	AIT8
0.5	0.62	0.62	0.62	1.29	1.30	1.30
1	0.61	0.61	0.62	1.00	1.02	1.03
2	0.60	0.60	0.61	0.90	0.92	0.93
3	0.60	0.58	0.60	0.89	0.92	0.93
6	0.59	0.57	0.57	0.89	0.91	0.92
∞	0.59	0.57	0.57	0.89	0.91	0.92

and dashed red (AIT8) lines, is almost the same in the RANK model (left panels) and the HANK model (right panels). The only notable difference is a small upward shift of the lines in the HANK model, which reflects that make-up strategies are slightly less effective in lowering the biases under incomplete markets. A positive (negative) complementarity between market incompleteness and bounded rationality would require the lines to be steeper (flatter) for the HANK model, showing that bounded rationality dampens the effectiveness of make-up strategies more (less) in this case relative to the RANK model with complete markets. However, Figure 5 documents that the marginal impact of a change in n^* on the benefits of AIT is almost the same in the RANK and HANK models.

5.2 (Non-)Interaction between incomplete markets and bounded rationality

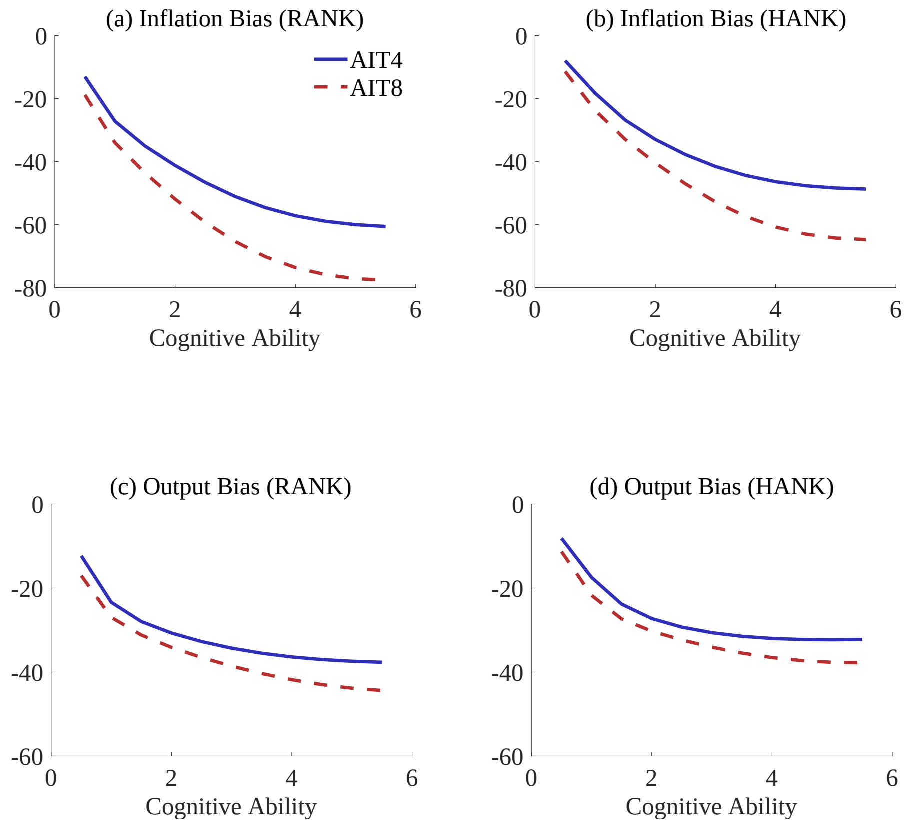
Farhi and Werning (2019) show that incomplete markets and bounded rationality can strongly attenuate the power of forward guidance when combined, but only have small attenuating effects when considered in isolation. In other words, there is a positive complementarity. Since make-up strategies operate via a built-in commitment to future interest rates, it might be surprising to see that our simulations do not indicate such a relationship between the incomplete markets and bounded rationality. In this section, we show how to reconcile our findings with those in Farhi and Werning (2019).

Forward guidance We start by performing a forward guidance experiment for different cognitive ability levels and conditional on market (in)completeness. Running this experiment allows us to check the existence of a complementarity between market incompleteness and bounded rationality with respect to forward guidance à la Farhi and Werning (2019). Given that we can indeed verify this property for our model, we can rule out differences between our model and the one used in Farhi and Werning (2019) as causes of a missing complementarity in the context of make-up strategies.[42] We show the results of the experiment in Figures 6 and 7.

Figure 6 plots the period-t responses of output and inflation to a date-t announcement

[42] Apart from small differences in terms of calibration, our model mainly differs from the quantitative sticky-price model employed in Farhi and Werning (2019) by featuring nominal wage stickiness, allowing for positive and nominal household debt as well as excluding outside liquidity.

Figure 5: Negative inflation and output biases for stochastic simulations

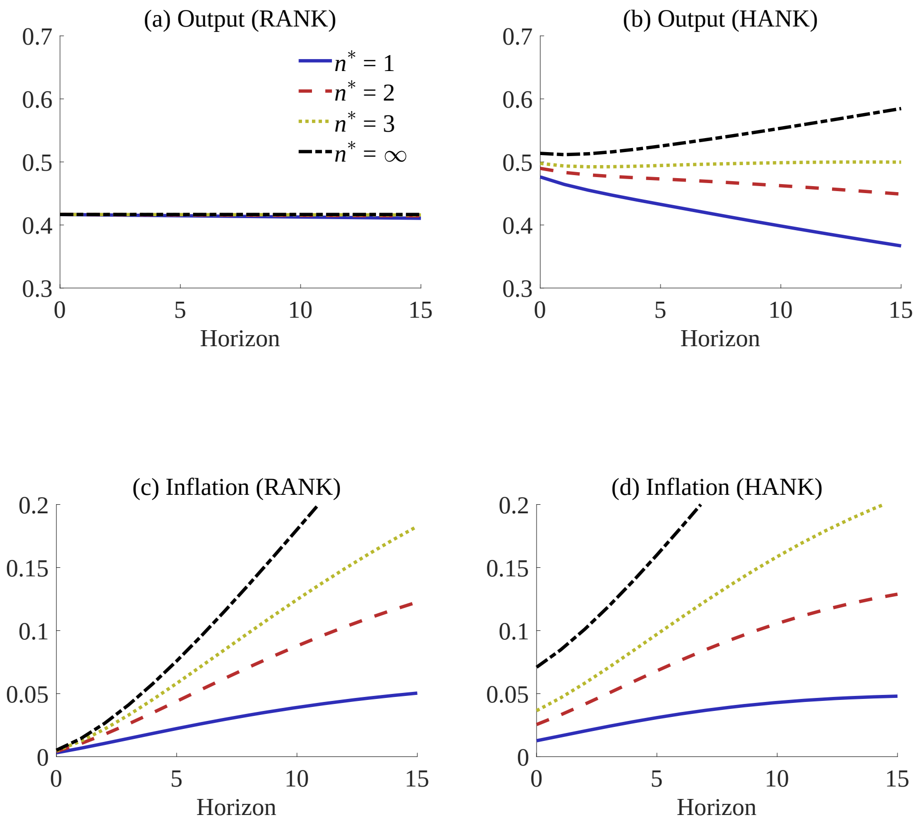


Notes: The biases are expressed as percentage deviations from the respective values under inflation targeting.

of a one-percent *real* rate cut in period $t+\tau$ (see McKay et al., 2016). First, consider the RANK model (left panels). Under rational expectations ($n^* = \infty$), the responsiveness of output does not depend on the horizon of the real rate cut realisation τ, whereas the inflation response exponentially increases with it. These properties constitute what is usually referred to as the "forward guidance puzzle" (see Del Negro et al., 2015; McKay et al., 2016), as they are difficult to square with economic intuition. By contrast, under reflective expectations, the output response declines with τ and inflation responds less strongly to real rate cuts further away in the future compared to $n^* = \infty$. What is more, the less sophisticated agents are, the more the output and inflation responses are attenuated. While the output response in the RANK model hardly changes with time horizon τ at all, the marginal effect of τ on the inflation response is declining for sufficiently low cognitive ability levels n^*. As a result, inflation remains bounded for $\tau \to \infty$ in this case. In that sense, bounded rationality can successfully solve the forward guidance puzzle, although its quantitative impact is not particularly large, which is in line with Farhi and Werning (2019).

In the HANK model (right panels), the output response declines with τ when agents'

Figure 6: Forward guidance experiment



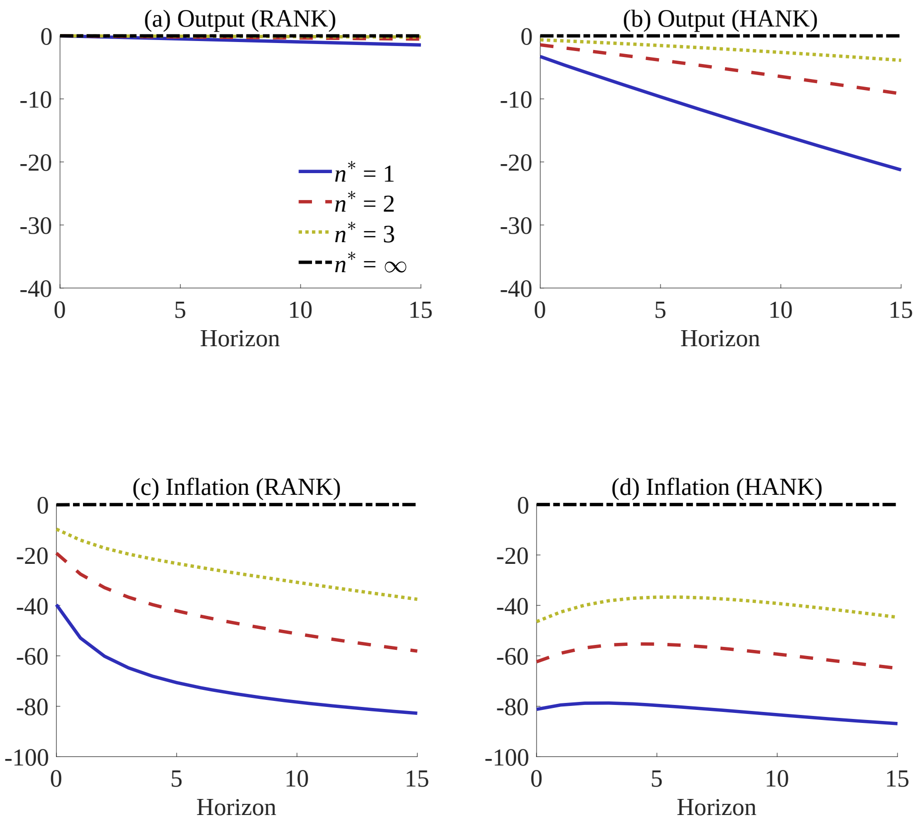
Notes: The figure plots the percentage deviations of the respective variables from steady state upon announcement of a one-percent real rate cut in τ periods.

cognitive ability is sufficiently low, but increases with the horizon under rational expectations. As in the RANK model, the inflation response remains bounded for $\tau \to \infty$ when n^* is low, but not for $n^* = \infty$. Market incompleteness thus amplifies the forward guidance puzzle for our model calibration under rational expectations.[43] Note that forward guidance generally has stronger output and inflation effects in the HANK model relative to the RANK model if the time horizon τ is short, reflecting strong indirect effects of monetary policy that are are not (as) important under complete markets (see Kaplan et al., 2018).

To visualise the complementarity between market incompleteness and bounded rationality in the context of forward guidance, it is helpful to normalise the responses depicted in Figure 6 by the respective values under rational expectations. Given that the output and inflation responses in the RANK and HANK models differ in size, these normalised re-

[43] As first pointed out by Werning (2015), this property depends on various model features, the cyclicality of household income risk and aggregate liquidity in particular. In our case, a redistribution of resources from savers (low MPCs) to borrowers (high MPCs) via inflation is crucial for making forward guidance more powerful under incomplete markets. When this redistribution of wealth is absent because bonds are indexed to inflation, the output effect declines with the horizon τ in all cases, including $n^* = \infty$.

Figure 7: Forward guidance experiment (normalised)



Notes: The figure plots the responses from Figure 6 but in percent of the respective responses under rational expectations $n^* = \infty$.

sponses illustrate the impact of bounded rationality on the power of forward guidance in a more transparent way.[44] Figure 7 plots these normalised responses for different cognitive ability levels and time horizons τ. By comparing these relative responses between RANK (left panels) and HANK (right panels), we observe that the attenuating effect of bounded rationality on the interest rate sensitivity of output and inflation is stronger under incomplete markets and increasing with time horizon τ.[45] In line with Farhi and Werning (2019), these differences between the RANK and HANK models reveal a positive complementarity between market incompleteness and bounded rationality with respect to the attenuation of forward guidance.

Discussion Forward guidance, as analysed in the previous section, is modelled as an unanticipated but perfectly observable news shock. As such, it constitutes a direct and uncondi-

[44] Farhi and Werning (2019) construct and calibrate their model such that the output and inflation responses do not differ between the RANK and HANK models under rational expectations. This property makes it more straightforward to see how market incompleteness and bounded rationality interact.

[45] Farhi and Werning (2019) refer to the former effect as *mitigation effect* and the latter one as *horizon effect*.

tional commitment to a future interest change, which agents are going to adjust their individual behaviour to. This particular behavioural response is the direct (or partial equilibrium) effect of the interest rate shock. In addition, the agents' behaviour reflects beliefs about the consequences of the interest rate shock for the future evolution of the aggregate economy. The corresponding response constitutes the indirect (or general equilibrium) effect of the interest rate change. The more sophisticated agents are, the better they understand these indirect effects and adjust their behaviour accordingly. Even the most naive agent (n close to 0) who does not adjust his expectations about the aggregate economy at all after the shock is announced, will, however, respond to the interest rate change itself. Bounded rationality therefore attenuates the indirect general equilibrium effects of the interest rate shock but not its direct effects, as agents can observe the interest rate change by assumption.

As shown by Farhi and Werning (2019) and verified for our model (see Figures 6 and 7), relaxing the rational-expectations assumption only mildly weakens the output effects of forward guidance under complete markets, even for quite large deviations from rational expectations. This finding reflects that in the RANK model, it is largely the direct effect of the interest rate change rather than its indirect effects that matter for the consumption response (see Kaplan et al., 2018; Hagedorn et al., 2019).[46] Given that the interest rate change itself is observed by households regardless of their cognitive ability n, this direct effect – and hence the dominant transmission channel of forward guidance under complete markets – is not affected when the rational-expectations assumption is relaxed for the RANK model. By contrast, relaxing the complete-markets assumption by introducing occasionally-binding borrowing constraints weakens the direct effect of monetary policy. However, at the same time, it also raises the importance of indirect effects compared to the RANK case. A priori, it is therefore not obvious whether forward guidance is attenuated or even amplified in this case.[47] When combined, bounded rationality and market incompleteness jointly weaken direct and indirect effects, which yields a positive complementarity with respect to the attenuation of forward guidance.

In contrast to forward guidance, make-up strategies are rule-based and do not offer information about future interest rates that is directly observable to all types of agents. Instead, they only provide an indirect and conditional commitment to future interest rate changes. Specifically, whether monetary policy commits to adjust future interest rates depends on whether current average inflation meets the target or not. Future interest rate changes therefore have to be triggered first by a macroeconomic shock. While such a shock can be observed by the agents, whether and to what extent it will affect future interest rate movements has to be figured out by agents together with all other general equilibrium effects.

[46]The direct effect of the interest rate change can further be decomposed into an income and a substitution effect. In the RANK model, it is primarily the latter that drives the size of the household consumption response and the aggregate output response as a result. This can be seen by looking at the real rate elasticity of output in the announcement period of a real rate shock. It is given as $(C/Y)\sigma^{-1}$ for the RE-RANK model, highlighting the tight link between the intertemporal elasticity of substitution σ^{-1} and the output response. The coefficient C/Y reflects that output does not equal consumption in the steady state if there is a fixed production cost $\Phi > 0$.

[47]As discussed above, our quantitative model implies that forward guidance is more effective in the HANK model for $n^* = \infty$.

In contrast to the forward guidance experiment, the transmission of monetary policy under an interest rate rule hence operates entirely via indirect general equilibrium effects.[48] Given that these effects are not directly observable to the agents, bounded rationality has a symmetric impact on how make-up strategies affect the economy under complete and incomplete markets. A complementarity as observed for forward guidance is therefore absent (or too small to notice), as it relies on market incompleteness and bounded rationality attenuating different types of effects.[49]

5.3 Sensitivity

In this section, we assess the sensitivity of our simulation results with respect to different model assumptions. First, we allow for real instead of nominal household debt. Second, we consider a model calibration where goods prices and nominal wages are partially indexed to lagged inflation. Third, we allow household income risk to change endogenously with macroeconomic conditions. Fourth, we consider model versions where only one type of forward-looking agent is boundedly rational. Fifth, we investigate the role of interest rate smoothing for our results.

Debt denomination When household debt is nominal, a deflationary shock redistributes resources from high-MPC borrowers to low-MPC savers. This transfer of wealth increases the severity of ELB episodes and raises the benefits of policies that attenuate deflationary or disinflationary spirals. When debt is real, this (surprise) redistribution is absent and one would expect the benefits of make-up strategies to be lower compared to a scenario with nominal household debt (see Table 3). While we confirm this intuition (see Table 6 in Appendix C), the quantitative differences between RE-HANK model versions with nominal and real debt are of negligible size. The relative performance of IT and AIT thus is not affected by the denomination of household debt.

Price and wage indexation For our baseline calibration, we assumed price- and wage-setting to be entirely forward-looking, i.e. $\iota_p = \iota_w = 1$. We now assume that prices and wages of non-resetting firms and unions are partially indexed to lagged inflation ($\iota_p = \iota_w = 0.9$).[50] This assumption introduces inertia into price- and wage-setting behaviour and thereby reduces the relative importance of expected price and wage inflation for the economy. Although one might expect AIT to lose some of its stabilising power due to an

[48]Only the aggregate shocks themselves have a direct effect on the agents' behaviour in this case.

[49]In line with results presented by Bianchi-Vimercati et al. (2021) for unconventional fiscal policies in a RANK model, discretionary and perfectly observable interest rate cuts are more effective at stimulating an economy when agents are boundedly rational agents than rule-based tax cuts. Closely related, Angeletos and Sastry (2021) emphasise the benefits of communicating monetary policy by specifying targets for variables of interest (like inflation or output) rather than instruments (like interest rates) when the public faces cognitive constraints.

[50]For ι_p-values below 0.9, we found that for some model versions, there is no equilibrium when the economy hits the ELB during some simulations, as identified by Holden (2016)'s method (see Section B.2). To compare all relevant model versions, we therefore took the lowest ι_p-value to ensure that this issue did not occur for any of the considered model versions.

impaired ability to manage agents' expectations, the relative performance of IT and AIT does not change (see Tables 7 and 8 in Appendix C).

Cyclical income risk The literature has emphasised the importance of cyclical household income risk for monetary policy transmission in general and forward guidance in particular (see e.g. Werning, 2015; Acharya and Dogra, 2020). Specifically, whereas countercyclical income risk strengthens the power of forward guidance in a HANK context compared to the complete markets case, procyclical income risk attenuates it. The intuition is straightforward. Countercyclical (procyclical) income risk raises (lowers) MPCs in response to an expansionary (contractionary) monetary policy, which amplifies (dampens) the economy's response by strengthening (weakening) indirect effects. To allow income risk to systematically change with aggregate economic conditions in our model, we let individual household productivity depend on aggregate output by adopting a reduced form relationship as in Auclert and Rognlie (2020).[51] In particular, we assume that individual productivity is given as $z_i\Gamma(z_i, Y_t)$, with

$$\Gamma(z_i, Y_t) = \frac{z_i^{\zeta \log(Y_t/Y)}}{\sum_j \Pr(z_j)\, z_j^{1+\zeta \log(Y_t/Y)}}.$$

This functional form implies that the cross-sectional standard deviation of log-household productivity is $[1+\zeta \log(Y_t/Y)] \times \mathrm{SD}(\log z_i)$. Parameter ζ directly governs the cyclicality of idiosyncratic income risk. In the previous sections we assumed $\zeta = 0$, implying acyclical income risk. For $\zeta < 0$, the model features countercyclical income risk, whereas income risk becomes procyclical for $\zeta > 0$.

Tables 9 and 10 in Appendix C show simulation statistics for RE-HANK model versions with $\zeta = -1$ and $\zeta = 1$, respectively. As expected, countercyclical income risk raises macroeconomic volatility in general and results in stronger inflation and output biases. The latter reflects stronger contractions at the aggregate level during ELB episodes compared to $\zeta = 0$ (see Table 3). The opposite observations can be made when income risk is procyclical. Importantly, these differences relative to the case with acyclical income risk apply uniformly across policy rules. The relative performance of make-up strategies is therefore not affected by the cyclicality of income risk.

Heterogeneous cognitive ability So far, we assumed that households, intermediate-good firms and labour unions all possess the same degree of cognitive ability. Because of this assumption, one cannot tell which of the three types of agents matters most for the impact of bounded rationality on the effectiveness of make-up strategies.[52] To clarify this issue, we solve and simulate our model under the assumption that only one of the three aforementioned agents is non-rational. Specifically, we assume that this particular type of agent is

[51] Bayer et al. (2020) model the cyclicality of income risk in a reduced-form way as well. Their model estimation implies a positive impact of aggregate income on individual income risk.

[52] Moreover, a recent empirical literature on subjective expectations shows that different types of agents possess different degrees of rationality (see e.g. Weber et al., 2022).

characterised by $n = 2$, while the other ones are perfectly rational. As argued in Section 3, a value of 2 is the highest cognitive ability level n that can be considered in line with micro-evidence.[53] Figures 17 and 18 in Appendix C show that the model statistics slightly vary depending on which type of agent is assumed to be non-rational. However, the size of these differences is quantitatively small. Bounded rationality therefore has a quite uniform impact on monetary policy transmission under all three policy strategies. The results of this paper thus do not depend on which type of agent is boundedly rational.

Interest rate smoothing While an interest rate rule with interest rate smoothing is standard in the literature, it could potentially distort the comparison between IT and AIT because under interest rate smoothing even IT features a history-dependent element. We therefore simulate the model also without interest rate smoothing ($\rho_R = 0$) to assess the robustness of our findings along this dimension. To enhance comparability, we recalibrate the demand shock volatility relative to Section 3 to again match an ELB frequency of 20% for the RE-RANK case. The resulting value is $\sigma_\beta = 0.0047$. As can be seen in Figures 19 and 20 in Appendix C, the main findings of this paper (see Section 5.1) do not change once interest rate smoothing is abandoned.

6 Conclusion

In this paper, we have studied to what extent the effectiveness of make-up strategies depends on the assumptions of rational expectations and complete markets. Specifically, we have performed simulations for a HANK model that was extended to allow for non-rational expectation formation and an occasionally-binding ELB constraint. Our simulations suggest that the power of make-up strategies does not particularly depend on whether markets are complete. By contrast, if agents are quite limited in their ability to process the macroeconomic consequences of aggregate shocks, the effectiveness of make-up strategies suffers, which can make them almost as useful as standard inflation targeting for addressing the adverse effects of an ELB. Since we observe that, for a given level of cognitive ability, the relative performance of AIT and IT hardly differs between our HANK model and a corresponding RANK model, one can argue that (bounded) rationality matters more for the effectiveness of make-up policies than market (in)completeness. This observation also implies that market incompleteness and bounded rationality do not complement each other in attenuating the benefits of make-up strategies.

[53]This value is also the lowest one where we did not encounter non-existence of equilibrium during some ELB episodes for any of the considered model versions.

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A Appendix: Derivation of optimal decision rules

This section derives the optimal decision rules shown in Section 2.1.

Households Define income $y_t = w_t N_t + d_t$, which is taken as given by all households. The household period budget constraint is

$$b_{i,t-1}\frac{R_{t-1}}{\Pi_t} + y_t = \tilde{b}_{i,t} + \tilde{c}_{i,t},$$

for current period t and

$$\tilde{b}_{i,t+s-1}\frac{R^e_{t+s-1}}{\Pi^e_{t+s}} + y^e_{t+s} = \tilde{b}_{i,t+s} + \tilde{c}_{i,t+s},$$

for future periods $t+s$, $s>0$.

Repeated forward substitution of (planned) end-of-period bonds via

$$\tilde{b}_{i,t+s-1} = \left(\tilde{b}_{i,t+s} + \tilde{c}_{i,t+s} - y^e_{t+s}\right)\left(\frac{R^e_{t+s-1}}{\Pi^e_{t+s}}\right)^{-1},$$

yields the intertemporal budget constraint,

$$b_{i,t-1}\frac{R_{t-1}}{\Pi_t} = \sum_{s=0}^{\infty}\left(\prod\nolimits_{u=0}^{s}\left(\frac{R^e_{t+u}}{\Pi^e_{t+u+1}}\right)^{-1}\right)\left(c_{i,t+s} - y_{t+s}\right),$$

or

$$\tilde{c}_{i,t} + \sum_{s=1}^{\infty}\left(\prod\nolimits_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)\tilde{c}_{i,t+s} = b_{i,t-1}\frac{R_{t-1}}{\Pi_t} + y_t + \sum_{s=1}^{\infty}\left(\prod\nolimits_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)y^e_{t+s}.$$

Household i maximises expected lifetime utility,

$$\sum\nolimits_{s=0}^{\infty}\left(\prod\nolimits_{k=0}^{s}\beta_{t+k-1}\right)\left\{u\left(\tilde{c}_{i,t+s}\right) - v\left(L^e_{t+s}\right)\right\},$$

by choosing consumption sequence $\{\tilde{c}_{i,t+s}\}_{s=0}^{\infty}$ subject to the intertemporal budget constraint. With felicity function $u(c) = (c^{1-\sigma}-1)/(1-\sigma)$, the first-order condition for (planned) consumption in period $t+s$ is

$$\tilde{c}_{i,t}^{-\sigma} = \tilde{c}_{i,t+s}^{-\sigma}\prod\nolimits_{u=1}^{s}\beta_{t+u-1}\frac{R^e_{t+u-1}}{\Pi^e_{t+u}},$$

or

$$\tilde{c}_{i,t+s} = \tilde{c}_{i,t}\left(\prod\nolimits_{u=1}^{s}\beta_{t+u-1}\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{1/\sigma}.$$

By using this condition to replace $\tilde{c}_{i,t+s}$ in the intertemporal household budget constraint, we obtain

$$\tilde{c}_{i,t}+\tilde{c}_{i,t}\sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)\left(\prod_{u=1}^{s}\beta_{t+u-1}\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{1/\sigma}$$
$$=b_{i,t-1}\frac{R_{t-1}}{\Pi_t}+y_t+\sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)y^e_{t+s}.$$

Solving for $\tilde{c}_{i,t}$ then yields

$$\tilde{c}_{i,t}=\frac{b_{i,t-1}\frac{R_{t-1}}{\Pi_t}+y_t+\sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)y^e_{t+s}}{1+\sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)\left(\prod_{u=1}^{s}\beta_{t+u-1}\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{1/\sigma}},$$

or simplified

$$\tilde{c}_{i,t}=\frac{b_{i,t-1}\frac{R_{t-1}}{\Pi_t}+y_t+\sum_{s=1}^{\infty}\left(\prod_{u=1}^{s}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{-1}\right)y^e_{t+s}}{1+\sum_{s=1}^{\infty}\prod_{u=1}^{s}\beta^{1/\sigma}_{t+u-1}\left(\frac{R^e_{t+u-1}}{\Pi^e_{t+u}}\right)^{1/\sigma-1}}.$$

Intermediate-good firms In period t, intermediate-good firm j sets price $P_{j,t}$ to maximise the expected present value of its profits,

$$\sum_{s=0}^{\infty}(\beta\xi_p)^s\left[\begin{array}{l}(P^*_t)^{1-\theta_p}\left(\frac{\prod_{u=1}^{s}\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{P^e_{t+s}}\right)^{1-\theta_p}Y^e_{t+s}\\ -\lambda_{t+s}w^e_{t+s}(P^*_t)^{-\theta_p}\left(\frac{\prod_{u=1}^{s}\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{P^e_{t+s}}\right)^{-\theta_p}Y^e_{t+s}-\Phi\end{array}\right],$$

where we used that $Y_{j,t+s}=(P_{j,t+s}/P_{t+s})^{-\theta_p}Y_{t+s}$ and $P_{j,t+s}=P_{j,t}\times\prod_{u=1}^{s}\left[\Pi^{\iota_p}(\Pi^e_{t+u-1})^{1-\iota_p}\right]$.

Let P^*_t denote the optimal reset price, which is the same across all firms j that can adjust prices in period t. The first-order condition for P^*_t can be written as

$$P^*_t=\frac{\theta_p}{\theta_p-1}\frac{\sum_{s=0}^{\infty}(\beta\xi_p)^s\left(\frac{\prod_{u=1}^{s}\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{P^e_{t+s}}\right)^{-\theta_p}\lambda_{t+s}w^e_{t+s}Y^e_{t+s}}{\sum_{s=0}^{\infty}(\beta\xi_p)^s\left(\frac{\prod_{u=1}^{s}\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{P^e_{t+s}}\right)^{1-\theta_p}Y^e_{t+s}}.$$

Define optimal reset inflation as $\Pi^*_{t+s}=P^*_{t+s}/P_{t+s-1}$. Using $P^e_{t+s}=P_t\times\prod_{u=1}^{s}\Pi^e_{t+u}$ and $\Pi_{t+s}=P_{t+s}/P_{t+s-1}$, we can write the expression above as

$$\Pi^*_t=\frac{\theta_p}{\theta_p-1}\frac{\sum_{s=0}^{\infty}(\beta\xi_p)^s\left(\prod_{u=1}^{s}\frac{\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{\Pi^e_{t+u}}\right)^{-\theta_p}\lambda_{t+s}w^e_{t+s}Y^e_{t+s}}{\sum_{s=0}^{\infty}(\beta\xi_p)^s\left(\prod_{u=1}^{s}\frac{\Pi^{\iota_p}\left(\Pi^e_{t+u-1}\right)^{1-\iota_p}}{\Pi^e_{t+u}}\right)^{1-\theta_p}Y^e_{t+s}}\Pi_t.$$

Labour unions Labour union k sets nominal wage $W_{k,t}$ to maximise the expected average lifetime utility of its members, $\sum_{s=0}^{\infty}(\beta\xi_w)^s\left\{MU^e_{t+s}(W_{k,t+s}/P^e_{t+s})N_{k,t+s}-MV^e_{t+s}N_{k,t+s}\right\}$, subject to the labour demand condition $N_{k,t+s}=(W_{k,t+s}/W_{t+s})^{-\theta_w}N_{t+s}$ and the wage indexing rule $W_{k,t+s}=W_{k,t}\times\prod_{u=1}^{s}\left[\Pi^{\iota_w}(\Pi^e_{t+u-1})^{1-\iota_w}\right]$.

Using the two constraints to eliminate $N_{k,t+s}$ and $W_{k,t+s}$ in the union's objective yields

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s\left[\begin{array}{l}\left(P^e_{t+s}\right)^{-1}(W_{k,t})^{1-\theta_w}\left(\prod_{u=1}^{s}\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}\right)^{1-\theta_w}N^e_{t+s}\left(W^e_{t+s}\right)^{\theta_w}MU^e_{t+s}\\-(W_{k,t})^{-\theta_w}\left(\prod_{u=1}^{s}\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}\right)^{-\theta_w}N^e_{t+s}\left(W^e_{t+s}\right)^{\theta_w}MV^e_{t+s}\end{array}\right].$$

Let W_t^* denote the nominal wage chosen by all unions that can reset their wages in period t. The first-order condition for W_t^* can be written as

$$W_t^*=\frac{\theta_w}{\theta_w-1}\frac{\sum\limits_{s=0}^{\infty}(\beta\xi_w)^s\left(\prod_{u=1}^{s}\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}\right)^{-\theta_w}N^e_{t+s}\left(W^e_{t+s}\right)^{\theta_w}MV^e_{t+s}}{\sum\limits_{s=0}^{\infty}(\beta\xi_w)^s(P^e_{t+s})^{-1}\left(\prod_{u=1}^{s}\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}\right)^{1-\theta_w}N^e_{t+s}(W^e_{t+s})^{\theta_w}MU^e_{t+s}},$$

which in turn can be rearranged to

$$w_t^*=\frac{\theta_w}{\theta_w-1}\frac{\sum\limits_{s=0}^{\infty}(\beta\xi_w)^s\left(\prod_{u=1}^{s}\frac{\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}}{\Pi^e_{t+u}}\right)^{-\theta_w}N^e_{t+s}\left(w^e_{t+s}\right)^{\theta_w}MV^e_{t+s}}{\sum\limits_{s=0}^{\infty}(\beta\xi_w)^s\left(\prod_{u=1}^{s}\frac{\Pi^{\iota_w}\left(\Pi^e_{t+u-1}\right)^{1-\iota_w}}{\Pi^e_{t+u}}\right)^{1-\theta_w}N^e_{t+s}(w^e_{t+s})^{\theta_w}MU^e_{t+s}},$$

by using $P^e_{t+s}=P_t\times\prod_{u=1}^{s}\Pi^e_{t+u}$, $w_{t+s}=W_{t+s}/P_{t+s}$ and $w_t^*=W_t^*/P_t$.

B Appendix: Numerical solution

In this section, we describe our numerical solution approach.

B.1 Extended path method

Despite its shortcomings (see Section 4), the extended path method provides a natural conceptual starting point and a useful benchmark for our local method.[54] We first describe the method for the (standard) case with rational expectations. Then, we present a more general version that can handle reflective expectations as well.

Rational expectations Suppose that the economy enters period $t = 0$ at its deterministic steady state, i.e. $\Omega_{-1} = \Omega$ and $S_{-1} = S$. In period $t = 0$, the economy is then hit by unanticipated shocks that move the exogenous aggregate variables S_0 and therefore the endogenous aggregate ones Ω_0 away from their respective steady state values. If the economy does not experience shocks in later periods $t > 0$, the assumption of rational expectations implies perfect foresight. Date-0 beliefs about the transition path back to the steady state must therefore coincide with the actual evolution of the economy, i.e. $\left\{\Omega^e_{t+s}\right\}_{s=0}^{\infty} = \{\Omega_t\}_{t=0}^{\infty}$. Assuming the economy will have returned to the steady state at date $t = T+1$, the computation of sequence $\{\Omega_t\}_{t=0}^{T}$ boils down to solving a nonlinear equation system, which can e.g. be accomplished by using a numerical solver in combination with a good initial guess.[55] While the computation of this sequence can be time-consuming even under rational expectations, especially when there is household heterogeneity, it is straightforward to implement conceptually. Note that this step requires tracking the evolution of the infinite-dimensional wealth distribution $\{\Psi_t(b,z)\}_{t=0}^{T}$ in a tractable way. As is common in the literature, we do so using a discrete approximation of this distribution based on a finite-dimensional histogram (see Reiter, 2009; Young, 2010; Bayer and Luetticke, 2020).

Now, suppose that we are not interested in the economy's response to unanticipated one-time (MIT) shocks in period $t = 0$, but in running a stochastic model simulation. In this case, the sequence $\{\Omega_t\}_{t=0}^{T}$ is only needed to determine the beliefs $\left\{\Omega^e_{t+s}\right\}_{s=0}^{T}$ held by agents at date t, which are necessary to pin down the actual values for the current period $t = 0$, Ω_0. To compute the actual values for $t = 1$, i.e. Ω_1, we proceed as just described for $t = 0$, except that the initial values now are given by Ω_0 and S_0, instead of $\Omega_{-1} = \Omega$ and $S_{-1} = S$. New unanticipated shocks then again hit the exogenous states, S_1. Under certainty equivalence, i.e. ignoring the possibility of future shocks hitting the economy, perfect foresight applies again and the shocks give rise to a new deterministic sequence $\{S_t\}_{t=1}^{T}$. Since (rational) beliefs adjust to this new information, one needs to compute a new sequence of beliefs $\left\{\Omega^e_{t+s}\right\}_{s=0}^{T}$ for $t = 1$. This can again be accomplished by using a nonlinear equation solver to calculate the deterministic sequence $\{\Omega_{t+s}\}_{s=0}^{T}$ for $t = 1$, which coincides with beliefs

[54] See Christiano et al. (2016) and Erceg et al. (2021) for recent applications of the extended path method.

[55] For the rational-expectations case, one can e.g. use perturbation methods to obtain a good initial guess, both with as well as without household heterogeneity (see e.g. Dobrew et al., 2021).

$\left\{\Omega^e_{t+s}\right\}_{s=0}^{T}$ in a rational-expectations equilibrium under certainty equivalence. From this sequence, we again only keep the values Ω_1, which are the actually realised values in period $t = 1$ that – together with S_1 – serve as initial values for the next period, $t = 2$, where we will proceed as for $t = 1$. We repeat these steps for a sufficiently high number of periods $\tilde{T}$ to obtain times series for the model variables of length $\tilde{T}$. Based on these time series, we can calculate model statistics for selected variables and assess the quantitative model properties for different monetary policy rules. Note that, since we use a nonlinear equation solver for the computations, it is straightforward to impose the ELB constraint for the monetary policy rule. However, such a nonlinearity slows down the numerical computation.

Reflective expectations Under reflective rather than rational expectations, we can proceed as sketched, except that beliefs $\left\{\Omega^e_{t+s}\right\}_{s=0}^{T}$ do not generally coincide with the actual evolution of the respective variables under perfect foresight (and certainty equivalence) anymore. As a result, we cannot directly solve for the entire sequence as before and – similar to Farhi and Werning (2019) and García-Schmidt and Woodford (2019) – use an iterative procedure to determine Ω_t for each time period t instead, taking as given initial values Ω_{t-1} and S_{t-1}. Specifically, we apply the following algorithm:

1. Let n^* denote the actual level of cognitive ability that we want the economy to exhibit. Choose a small number $\gamma > 0$. The necessary number of iterations with respect to cognitive ability then is n^*/γ. Set the initial time index to $t = 0$. Set the initial values to $\Omega_{t-1} = \Omega$ and $S_{t-1} = S$.

2. Draw period-t shocks ε_t. Given S_{t-1}, ε_t and the law of motion for S_{t+s}, calculate the time path $\{S_{t+s}\}_{s=0}^{T}$. Set the initial level of reflective reasoning to $n = \gamma$ and initialise beliefs $\left\{\Omega^{e,n}_{t+s}\right\}_{s=0}^{T}$.

3. Given Ω_{t-1}, $\{S_{t+s}\}_{s=0}^{T}$ and $\left\{\Omega^{e,n}_{t+s}\right\}_{s=0}^{T}$, compute the individual policy functions for the households, firms and unions for $s \in \{0, 1, 2, \dots, T\}$. While closed-form solutions are available for the reset price and the reset wage, this is only true for individual household consumption and bond holdings in the RANK model. For the HANK model, one has to compute the individual household policies numerically via backward induction, using a global solution method like the endogenous grid method (see Carroll, 2006).[56]

4. Go forward in time to compute the sequence $\left\{\Omega^n_{t+s}\right\}_{s=0}^{T}$:

    a. Set $s = 0$ and $\Omega^n_{t-1} = \Omega_{t-1}$.

    b. Compute Ω^n_{t+s}, given Ω^n_{t+s-1} and the policies computed in Step 3.

    c. Set $s = s + 1$.

    d. If $s \leq T$ go to Step 4b. If $s > T$, go to Step 5.

[56]We also use the endogenous grid method when directly solving for a sequence under rational expectations.

5. If $n = n^*$, set $\Omega_t = \Omega_t^n$ and go to Step 6. If $n < n^*$, update beliefs, $\left\{\Omega_{t+s}^{e,n+\gamma}\right\}_{s=0}^{T} = \gamma \times \left\{\Omega_{t+s}^{n}\right\}_{s=0}^{T} + (1-\gamma) \times \left\{\Omega_{t+s}^{e,n}\right\}_{s=0}^{T}$, as well as the level of reasoning, $n = n + \gamma$, and go to Step 3.[57]

6. If $t = \tilde{T}$, stop. If $t < \tilde{T}$, update the time index, $t = t + 1$, and go to Step 2.

This procedure yields the simulated time series $\{\Omega_t\}_{t=0}^{\tilde{T}}$ which we can use to calculate statistics for model variables of interest. Step 4b requires the use of a nonlinear equation solver. Given that – for each $n \in \{\gamma, \ldots n^*\}$ – it is only the variables in period t and not an entire sequence that is computed in this step, it usually is not particularly time-consuming on its own. Since it will however be performed n^*/γ times for each $t+s$, $s \in \{0, 1, 2, \ldots, T\}$, the computational burden is much higher relative to the rational-expectations computations sketched earlier, even if n^*/γ is low. While one could in principle use the iterative algorithm above to approximate the rational-expectations case for sufficiently high n^*, it usually is hence not advisable to do so as it would be much slower – as well as less numerically stable – compared to the procedure described above.[58]

One can imagine various ways to initialise beliefs for Step 2. However, we make the simplifying assumption that beliefs are anchored at the steady state in each period, i.e. $\Omega_{t+s}^{n,e} = \Omega$ for $n = \gamma$, $s \in \{0, 1, 2, \ldots, T\}$ and $t \in \left\{0, 1, 2, \ldots, \tilde{T}\right\}$, which is consistent with our definition of a reflective-expectations equilibrium in the previous section. This assumption has two main implications for our analysis. First, when shocks are only drawn for the first period and set to zero thereafter, i.e. $\{\varepsilon_t\}_{t=0}^{\tilde{T}} = \{\varepsilon_t, 0, \ldots, 0\}$ with $\varepsilon_t \neq 0$, where $\varepsilon_t = \left(\varepsilon_{\beta,t}, \varepsilon_{\lambda,t}\right)$, the algorithm outlined above would imply that the sequence $\{\Omega_t\}_{t=0}^{\tilde{T}}$, obtained after applying the entire algorithm above, coincides with the sequence $\{\Omega_{t+s}\}_{s=0}^{T}$ obtained after applying the steps 1 to 5 only once for $t = 0$, assuming $T = \tilde{T}$. By contrast, if initial beliefs $\left\{\Omega_{t+s}^{n,e}\right\}_{s=0}^{T}$, with $n = \gamma$, could change (endogenously) with t, this relationship would not generally hold anymore. Although a time-varying anchoring of beliefs could allow for potentially interesting learning dynamics (see e.g. Bianchi-Vimercati, 2022), we prefer to keep the model analysis simple at this stage. Second, anchoring beliefs at the steady state allows us to apply a local approximation for the reflective-expectations model, facilitating fast model simulation. If initial beliefs were to change as time passes, the local approximation would no longer be appropriate anymore because the anchor for the belief formation would (or at least could) move too far away from the steady state.[59]

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[57] This updating rule reflects that the continuous belief-updating process $d\Omega_{t+s}^{e,n}/dn = \Omega_{t+s}^{n} - \Omega_{t+s}^{e,n}$ can be approximated by $\left(\Omega_{t+s}^{e,n+dn} - \Omega_{t+s}^{e,n}\right)/dn = \Omega_{t+s}^{n} - \Omega_{t+s}^{e,n}$. If $dn = \gamma$, this equation can in turn be written as $\Omega_{t+s}^{e,n+\gamma} = \gamma \times \Omega_{t+s}^{n} + (1-\gamma) \times \Omega_{t+s}^{e,n}$.

[58] However, when there are substantial nonlinearities at the aggregate level, we find that the iterative algorithm can be more stable compared to an approach that directly computes the entire time path for the model variables as sketched above. For instance, we found this to be the case when considering more sophisticated monetary policy rules relative to Section 2, such as asymmetric average inflation targeting or temporary price-level targeting (see e.g. Arias et al., 2020; Erceg et al., 2021).

[59] For $\gamma = 1$, one can also use the algorithm to compute an equilibrium where belief formation is based on level-k thinking (see Farhi and Werning, 2019). However, we find that the algorithm is not numerically stable in this case and does not converge to the rational-expectations solution for $n^* \to \infty$.

B.2 Local approximation method

Our local model approximation builds on the insights of Boppart et al. (2018). Assume that the random variables in the model follow log-normal AR(1) processes, with persistence parameter ρ_x and shock standard deviation σ_x, for $x \in \{\beta, \lambda\}$. Now suppose that we have computed a transition path for the model variables after a small unanticipated one-time (MIT) demand shock $\varepsilon_{\beta,t} = h > 0$ has hit the economy in the steady state at date $t = 0$, giving rise to the (shock-specific) transition path $\{\Omega_{t|\beta}\}_{t=0}^{T}$. Furthermore, suppose that $R_{ELB} = 0$ and – for simplicity – only demand shocks are operative in the model, such that we have $\{\hat{S}_{t+s}\}_{s=0}^{T} = \{\hat{\beta}_{t+s}\}_{s=0}^{T} = \{\varepsilon_{\beta,0}, \rho_\beta \varepsilon_{\beta,0}, \rho_\beta^2 \varepsilon_{\beta,0}, \rho_\beta^3 \varepsilon_{\beta,0}, \ldots, \rho_\beta^T \varepsilon_{\beta,0}\}$. The hat symbol indicates that a variable is expressed in log-deviations from steady state, $\hat{X}_t = \log(X_t/X)$. Following Boppart et al. (2018), we can use

$$\hat{X}_t \cong \sum_{s=0}^{T} \hat{X}_{s|\beta} \varepsilon_{\beta,t-s},$$

as a linear perfect-foresight approximation around the steady state, for $X \in \Omega$, where $\hat{X}_{s|\beta} = (X_{s|\beta} - X)/h$, for $s \in \{0, 1, 2, \ldots, T\}$.[60] Note that if the model is linear and $T = \infty$, the relationship above holds with equality, not just approximately.

To simulate the model, we can now simply draw a sequence of demand shocks and then calculate the model variables' values for each period by using the impulse response coefficients $\{\hat{X}_{t|\beta}\}_{t=0}^{T}$ and the (truncated) shock history for that period via the formula above. Instead of relying on a conventional perturbation of the model based on a recursive formulation, we thus perturb the model in the sequence space, i.e. with respect to the history of past and present aggregate shocks.

There are two key advantages of using this local approximation method in this paper. First, when a model features substantial heterogeneity across agents, the histogram used to approximate the distribution of agents in the economy involves a large number of bins to achieve a reasonable degree of numerical accuracy. As a result, the size of the minimal state space required for a local approximation based on a recursive model formulation is usually going to exceed the number of shocks needed for a model approximation in the (truncated) sequence space in this case. Boppart et al. (2018)'s approach exploits this property and makes it possible to quickly solve and simulate heterogeneous-agent models, requiring only the computation of an impulse response function for each type of aggregate shock, $x \in \{\beta, \lambda\}$.[61] For the full model with all three types of shocks, one can then approximate the

[60] As Boppart et al. (2018) explain, this normalised transition path represents an impulse response function (IRF) that can be used as a numerical derivative of the time path for variable X with respect to a one-time shock. Loosely speaking, the IRF provides a linearisation of the nonlinear model around the deterministic steady state. If the shock is sufficiently small (and the time horizon T sufficiently long), this linearisation yields the same results as the method by Reiter (2009).

[61] Given that the required transition paths need to be calculated only for small unanticipated one-time shocks, the computational cost of this step is not that high, especially under rational expectations.

model variables in period t via

$$\hat{X}_t \cong \sum_{s\in\{0,1,\dots,T\}} \sum_{x\in\{\beta,\lambda\}} \hat{X}_{s|x}\varepsilon_{x,t-s},$$

for $X \in \Omega$.

Second, it is straightforward to apply the sequence-space approximation for models with iterative belief formation (see e.g. Farhi and Werning, 2019; García-Schmidt and Woodford, 2019; Bianchi-Vimercati et al., 2021) because the belief-formation process is directly taken into account when computing the sequences $\left\{\Omega_{t|x}\right\}_{t=0}^{T}$.[62] However, note that, as mentioned earlier, these sequences would be computed under the assumption that beliefs are anchored at the steady state for all periods t.

B.3 Enforcing the ELB constraint

The local approximation discussed in the previous section does not take into account the nonlinearity introduced by the occasionally-binding ELB constraint. Building on Bodenstein et al. (2013) and Holden (2016), we enforce this constraint by using anticipated monetary policy shocks.[63] In principle, the method is only guaranteed to recover the true solution for a linear dynamic model with an occasionally-binding constraint if expectations are rational and perfect foresight applies. Under these assumptions, enforcing the ELB for the expected evolution of the economy then amounts to enforcing it for the actual evolution of the economy as well. This link is, however, broken for our model with non-rational expectations. Nevertheless, as shown below, we find that one can still apply the method and that the resulting approximation is quite good.

Anticipated monetary policy shocks To apply the method, replace the relationship $R_t = \max\left\{\tilde{R}_t, R_{ELB}\right\}$ with $R_t = \tilde{R}_t \prod_{s\in\{0,1,\dots,T\}} \exp\left(\varepsilon_{R,t-s|t}\right)$, where $\varepsilon_{R,t-s|t}$ is an anticipated monetary policy shock that is announced in period $t-s$ but realised in period t.[64] Before we can start the model simulation, we need to compute the $T+1$ transition paths $\left\{\Omega_{t+s|R,k}\right\}_{s=0}^{T}$, $k \in \{0,\dots,T\}$, using the methods described earlier. The sequence $\left\{\Omega_{t+s|R,k}\right\}_{s=0}^{T}$ denotes the transition path for the economy after the anticipated monetary policy shock $\varepsilon_{R,t|t+k} = h_R > 0$ is revealed in period t ("today") to hit the economy in period $t+k$. Based on the computed sequences, we calculate the impulse response coefficients $\left\{\hat{X}_{s|R,k}\right\}_{s=0}^{T}$ with $\hat{X}_{s|R,k} = \left(X_{t+s|R,k} - X\right)/h_R$, for $k \in \{0,\dots,T\}$ and $X \in \Omega$.

[62] By contrast, at least for the HANK model, it does not appear to be feasible to apply standard perturbation techniques to a nonlinear model with iterative belief formation. As in Bianchi-Vimercati et al. (2021), for quite simple models – like the RANK model in this paper – a feasible option would be to first log-linearise the model equations analytically for a given level of reflective reasoning and then apply the iterative belief formation process (see also Bersson et al., 2019; Molavi, 2022).

[63] Laséen and Svensson (2011) have pioneered the use of such anticipated monetary policy shocks to implement given (exogenous) interest rate paths for linearised DSGE models.

[64] Remember that it is the shadow rate $\tilde{R}_t$, not the realised rate R_t, which affects $\tilde{R}_{t+1}$ as a lagged argument via the interest rate rule. As a result, the shadow rate $\tilde{R}_t$ is not directly affected by the monetary policy shocks, as they are introduced to replace the ELB and not as an independent source of uncertainty.

Rational expectations We first illustrate the method for the rational-expectations case, building on the ideas of Bodenstein et al. (2013) and Holden (2016) but relying on a local approximation of the model in the sequence space instead. Suppose we want to find the values for the model variables in a given simulation period t. First, use the local method sketched in Section B.1 to generate the unconstrained (log-linear) perfect-foresight sequence $\{\bar{\Omega}_{t+s}\}_{s=0}^{T}$ by using $\bar{X}_{t+s} = \sum_{m\in\{0,1,...,T\}} \sum_{x\in\{\beta,\lambda\}} \hat{X}_{m|x} \varepsilon_{x,t+s-m}$, for $X \in \Omega$, initially setting $\varepsilon_{R,t|t+k} = 0$ for all k. Second, check whether the sequence $\{\bar{R}_{t+s}\}_{s=0}^{T}$ violates the ELB constraint, i.e. whether $\bar{R}_{t+s} \geq \log(R_{ELB}/R)$ holds for all future periods $s \in \{0,1,\dots,T\}$. If this condition is violated for at least one period s, we look for anticipated monetary policy shocks $\{\varepsilon_{R,t|t+k}\}_{k=0}^{T}$, such that $\hat{R}_{t+s} \geq \log(R_{ELB}/R)$ holds for $\hat{R}_{t+s} = \bar{R}_{t+s} + \sum_{k\in\{0,1,...,T\}} \hat{R}_{s|R,k} \varepsilon_{R,t|t+k}$, with $\varepsilon_{R,t|t+k} \geq 0, k \in \{0,1,\dots,T\}$, as well as $[\hat{R}_{t+s} - \log(R_{ELB}/R)]\,\varepsilon_{R,t|t+s} = 0$.[65] We thus need to find non-negative anticipated monetary policy shocks for period t, such that the resulting perfect-foresight transition path for the policy rate, $\{\hat{R}_{t+s}\}_{s=0}^{T}$, (i) does not violate the ELB constraint and (ii) only features strictly positive policy shocks, $\varepsilon_{R,t|t+s} > 0$, in periods $s \in \{0,1,\dots,T\}$ with a binding ELB constraint. Having found appropriate period-t policy shocks $\{\varepsilon_{R,t|t+s}\}_{s=0}^{T}$, we can compute the entire perfect-foresight path consistent with the ELB constraint, $\{\hat{\Omega}_{t+s}\}_{s=0}^{T}$, by using $\hat{X}_{t+s} = \bar{X}_{t+s} + \sum_{k\in\{0,1,...,T\}} \hat{X}_{s|R,k} \varepsilon_{R,t|t+k}$, for $X \in \Omega$. Having calculated these sequences, we then only use the first entry, $\hat{\Omega}_t$, as it contains the actually realised values for the model variables in simulation period t. Next, we move to the simulation period $t+1$ and proceed as just described for period t.

There are several ways to obtain anticipated policy shocks consistent with the two requirements listed above. Whereas Bodenstein et al. (2013) employ an iterative guess-and-verify scheme that shares similarities with Guerrieri and Iacoviello (2015)'s piecewise-linear method, Holden (2016) proposes an algorithm that solves a mixed-integer linear programming (MILP) problem.[66] Although we find that both methods lead to the same solution for our model, the MILP approach has a number of appealing advantages. Specifically, Holden (2016) proves that a linear dynamic model augmented with anticipated monetary policy shocks yields the same model dynamics as the original model that is linear except for the max-operator. In general, the existence and the uniqueness of a sequence $\{\hat{R}_{t+s}\}_{s=0}^{T}$ consistent with requirements (i)-(ii) are, however, not guaranteed and depend on the local model dynamics, as captured by the impulse response coefficients $\{\hat{R}_{s|R,k}\}_{s=0}^{T}$. Since the approach used by Bodenstein et al. (2013) simply assumes the existence of suitable anticipated monetary policy shocks and then proceeds under this assumption, it is not obvious whether a lack of convergence of their method would be due to numerical issues or the non-existence of a solution (see Holden, 2016). By contrast, the method proposed by Holden (2016) has the appealing feature of being able to detect whether a solution exists.[67] In ad-

[65] Formally, finding a shock sequence that satisfies these conditions is a linear complementarity problem (see Holden, 2016).

[66] See Bodenstein et al. (2009) for details on the relationship between the iterative approaches used by Bodenstein et al. (2013) and Guerrieri and Iacoviello (2015).

[67] For our model, we found that the approach presented in Holden and Paetz (2012), which determines the

dition, it makes it possible to select a particular solution in case of multiplicity, e.g. the solution that minimises the length of an ELB spell.

In the remainder, we closely follow Holden (2016) and refer the interested reader to that paper for a detailed treatment of the method. Define the vector $\mathrm{q} = \left(\bar{R}_t, \bar{R}_{t+1}, \ldots, \bar{R}_{t+T}\right)'$ $-\left(\log\left(R_{ELB}/R\right), \log\left(R_{ELB}/R\right), \ldots, \log\left(R_{ELB}/R\right)\right)'$ and the coefficient matrix $\mathrm{M} = (\mathrm{m}_0, \mathrm{m}_1, \ldots, \mathrm{m}_T)$, which contains the IRFs $\mathrm{m}_s = \left(\hat{R}_{s|R,0}, \hat{R}_{s|R,1}, \ldots, \hat{R}_{s|R,T}\right)'$. For a given scalar $\omega > 0$, we can find suitable policy shocks $\left\{\varepsilon_{R,t|t+k}\right\}_{k=0}^{T}$ by formulating a MILP problem that solves for scalar α, vector $\mathrm{v} = (v_0, v_1, \ldots, v_T)'$, and vector $\mathrm{z} \in \{0,1\}^{T+1}$:

$$\max_{\{\alpha,\mathrm{v},\mathrm{z}\}} \ \alpha \quad \text{s.t.} \quad 0 \leq \alpha, \ \ 0 \leq \mathrm{v} \leq \mathrm{z}\ , \ 0 \leq \alpha \mathrm{q} + \mathrm{M}\mathrm{v} \leq \omega\left(1_{(T+1)\times 1} - \mathrm{z}\right).$$

This type of optimisation problem is well understood in the literature and can be solved efficiently using readily available numerical solvers.[68] How can we recover the anticipated monetary policy shocks from a solution to this problem? If the solution involves $\alpha = 0$, there is no collection of shocks $\left\{\varepsilon_{R,t|t+s}\right\}_{s=0}^{T}$ that satisfies the two requirements (i) and (ii) listed above. If $\alpha > 0$, we can recover the shocks via $\varepsilon_{R,t|t+s} = v_s/\alpha$, for $s \in \{0, 1, \ldots, T\}$. In case different policy shocks are consistent with different model dynamics at the ELB, reflecting equilibrium multiplicity, one can select a particular equilibrium via parameter ω. For instance, $\omega \to 0$ would select anticipated policy shocks that minimise $||\mathrm{q} + \mathrm{M}\mathrm{v}||_\infty$, whereas $\omega \to \infty$ would select the shock sequence minimising $||\mathrm{v}||_\infty$. In addition, by treating ω as an i.i.d. random variable, it is also possible to use sunspot shocks as an equilibrium selection device for the simulation.

Reflective expectations So far, the method described above relied on perfect foresight to enforce the ELB constraint for the linearised model. However, under reflective expectations, perfect foresight does not hold anymore, such that the equivalence between the realised sequences and the expected sequences breaks down. It is therefore not obvious whether and, if so, how one can use anticipated monetary policy shocks to enforce the ELB in this case. In principle, one would now have to enforce the ELB constraint for all levels of reflective reasoning, $n \in \{\gamma, 2\gamma, \ldots, n^*\}$, considered for the belief formation process. Doing so would, however, make the local method as slow as the nonlinear one and therefore undo its key advantage. Instead, we proceed as in the rational-expectations case by assuming that the (unconstrained) actual evolution of the economy populated by agents with cognitive ability n^*, as predicted by the impulse response coefficients and the current history of shocks, coincides with what these agents believe about their economy in general and the interest rate path in particular.

For our purposes, we consider this assumption reasonable for the following reasons. First, if γ is close to zero, the two sequences $\left\{\Omega_{t+s}^{n^*}\right\}_{s=0}^{T}$ and $\left\{\Omega_{t+s}^{n^*,e}\right\}_{s=0}^{T}$ are very close to each other for all $s \in \{0, 1, \ldots, T\}$. As a result, the sequence $\left\{\hat{\Omega}_{t+s}^{n^*}\right\}_{s=0}^{T}$ provides a

required anticipated policy shocks by solving a quadratic programming problem, also leads to the same results as the other two approaches. However, as in Bodenstein et al. (2013), it assumes the existence of a solution.

[68] We use MATLAB's built-in routine `intlinprog`.

reasonable approximation of beliefs $\left\{\hat{\Omega}^{n^*,e}_{t+s}\right\}^T_{s=0}$. Motivated by this insight, we simply use the approach sketched above for the rational-expectations case also for the reflective-expectations case, enforcing the ELB based only on the unconstrained log-linear realised sequence $\left\{\bar{\Omega}_{t+s}\right\}^T_{s=0}$. Second, regardless of whether one considers our conceptual argument compelling or not, we find that by enforcing the ELB only based on $\left\{\bar{\Omega}_{t+s}\right\}^T_{s=0}$, the resulting sequence $\left\{\hat{\Omega}_{t+s}\right\}^T_{s=0}$ consistent with the ELB constraint is quite close to the "ELB-consistent" sequence that a nonlinear perfect-foresight solution would deliver.[69] This observation justifies our approach from a practical perspective and underscores our conceptual argument.[70]

Capturing nonlinearities at the ELB Whereas Bodenstein et al. (2013) and Holden (2016) – as well as Guerrieri and Iacoviello (2015) – use a recursive model formulation as is common in the DSGE literature, we use the sequence representation described above. Doing so offers an additional degree of flexibility, given by the shock size h_R, that can be used to improve the accuracy of the model approximation for periods at the ELB. Usually, the unconstrained model violates the ELB constraint only if the economy experiences a particularly large shock or a sequence of adverse shocks, likely pushing the economy far away from the deterministic steady state. A binding ELB constraint would then amplify these adverse conditions even further, causing a linear approximation in the neighbourhood of the steady state to possibly deliver a poor performance. As Reiter (2018) highlights, the approximation proposed by Boppart et al. (2018) may perform better if the impulse response coefficients are calculated for h-values that are not close to zero. Whether this is indeed the case depends on how nonlinear the original model is and the properties of the stochastic processes, which affect the time spent close to the steady state. Regarding the anticipated monetary policy shocks, one has good reasons to believe that they are strictly positive only when the economy is far away from the steady state. As a result, it is likely that one can improve the accuracy of the approximation by choosing h_R to keep the difference between the nonlinear solution path and the local one low for selected model variables of interest, such as output or inflation, when the economy is at the ELB.

Accuracy of the local solution To assess the numerical performance of our local model approximation, we compute impulse responses for a large demand shock ($\varepsilon_{\beta,t} = 3\sigma_\beta$) with our local method and compare them to impulse responses computed with the nonlinear approach sketched in Section B.1. Figures 8 to 10 show the results for the case without an ELB constraint. Although the size of the shock is quite large, the responses of the interest rate, inflation and output hardly differ across the two solution methods. This observation

[69] We provide details on the numerical accuracy of the local method at the end of this section.

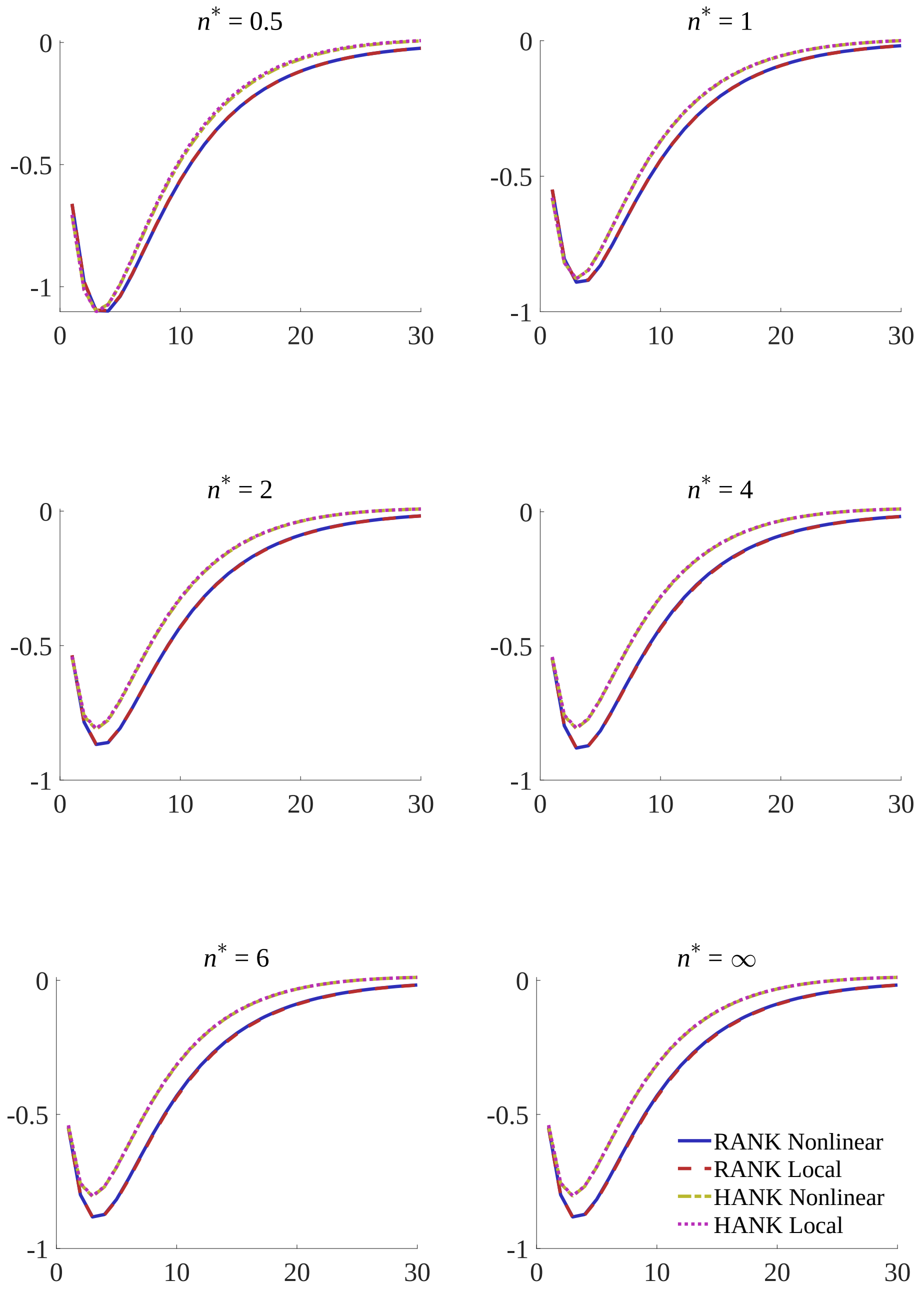
[70] We also assume that the monetary policy shocks, unlike shocks to S_t, are not directly observable by the agents. They are hence only "pseudo-anticipated shocks". This assumption reflects that (i) reflective agents form imperfect expectations about whether the ELB is going to bind or not due to lack of perfect foresight, and (ii) monetary policy shocks do not constitute an exogenous source of aggregate volatility in the model but are determined endogenously to enforce a binding ELB.

holds regardless of assumptions made about cognitive ability and market completeness. With an ELB constraint (see Figures 11 to 13), we notice some small differences for low cognitive ability levels, $n^* \in \{0.5, 1, 2\}$. For low n^*-values, beliefs held by agents and actual outcomes differ to a greater degree. As a result, since Holden (2016)'s approach assumes perfect foresight, enforcing the ELB via anticipated monetary policy shocks works less well in this case than for high n^*-values, reflecting that the model departs more from this assumption. By contrast, when agents are close to being rational, the gap between beliefs and outcomes shrinks and the impulse responses implied by the two solution methods are again very close to each other.[71] Interestingly, for a given n^*, the local solution approach works similarly well in the RANK and HANK models with and without an occasionally-binding ELB. This finding could suggest that our model does not feature sizable nonlinear interactions between the ELB and household heterogeneity.[72]

[71] We use $\gamma = 0.1$ for the computations in this paper (see Section B.1). By lowering γ, the gap between beliefs and outcomes can further be reduced. This in turn lowers the differences between the local and nonlinear impulse responses observed for low n^*-values under an occasionally-binding ELB. However, using lower γ-values did not affect our qualitative findings and had a negligible effect on our quantitative results, while increasing the computation time.

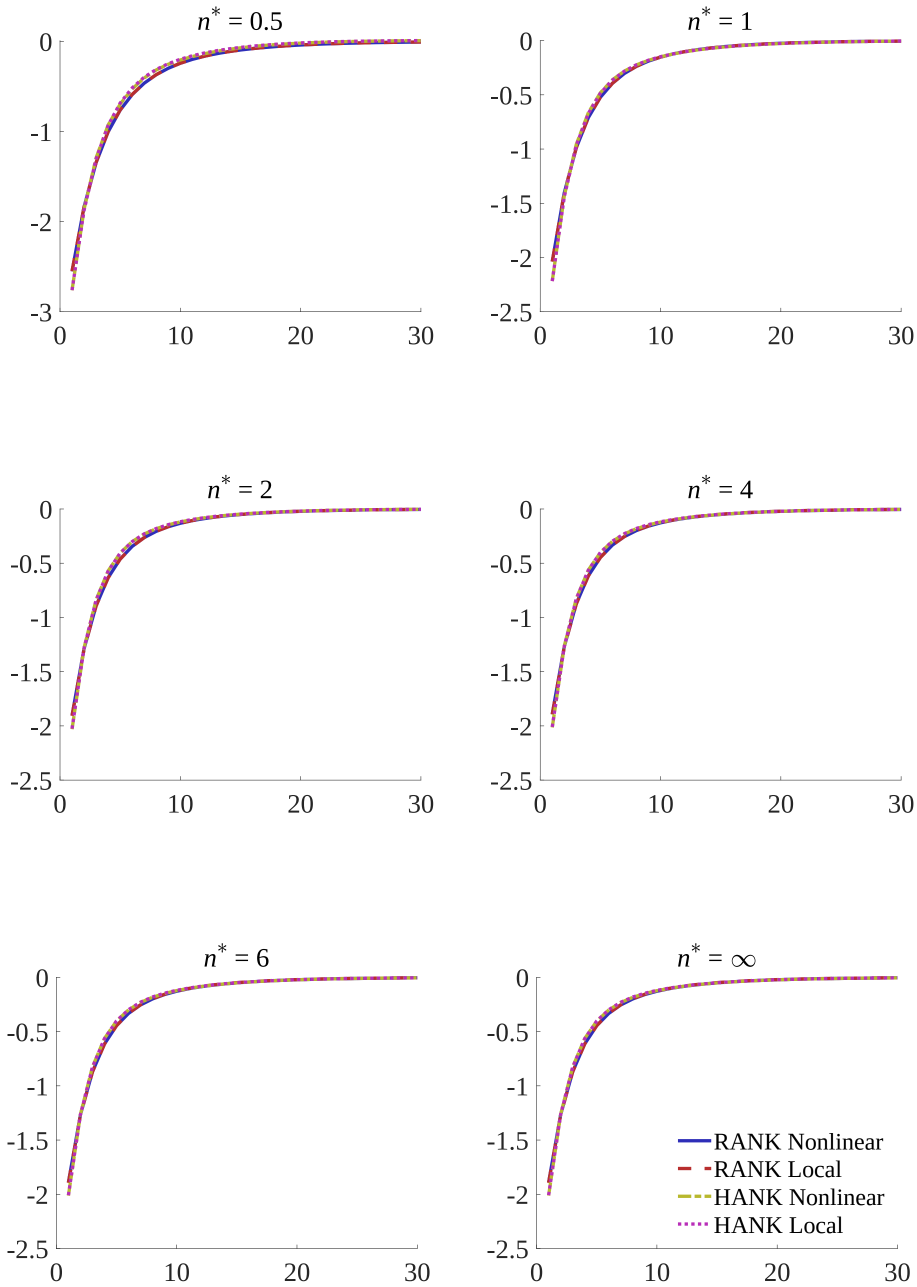
[72] We only consider an MIT shock and therefore do not capture the type of interactions studied by Schaab (2020) and Fernández-Villaverde et al. (2021) for HANK models with aggregate risk.

Figure 8: Response of R_t to a contractionary demand shock (without ELB)



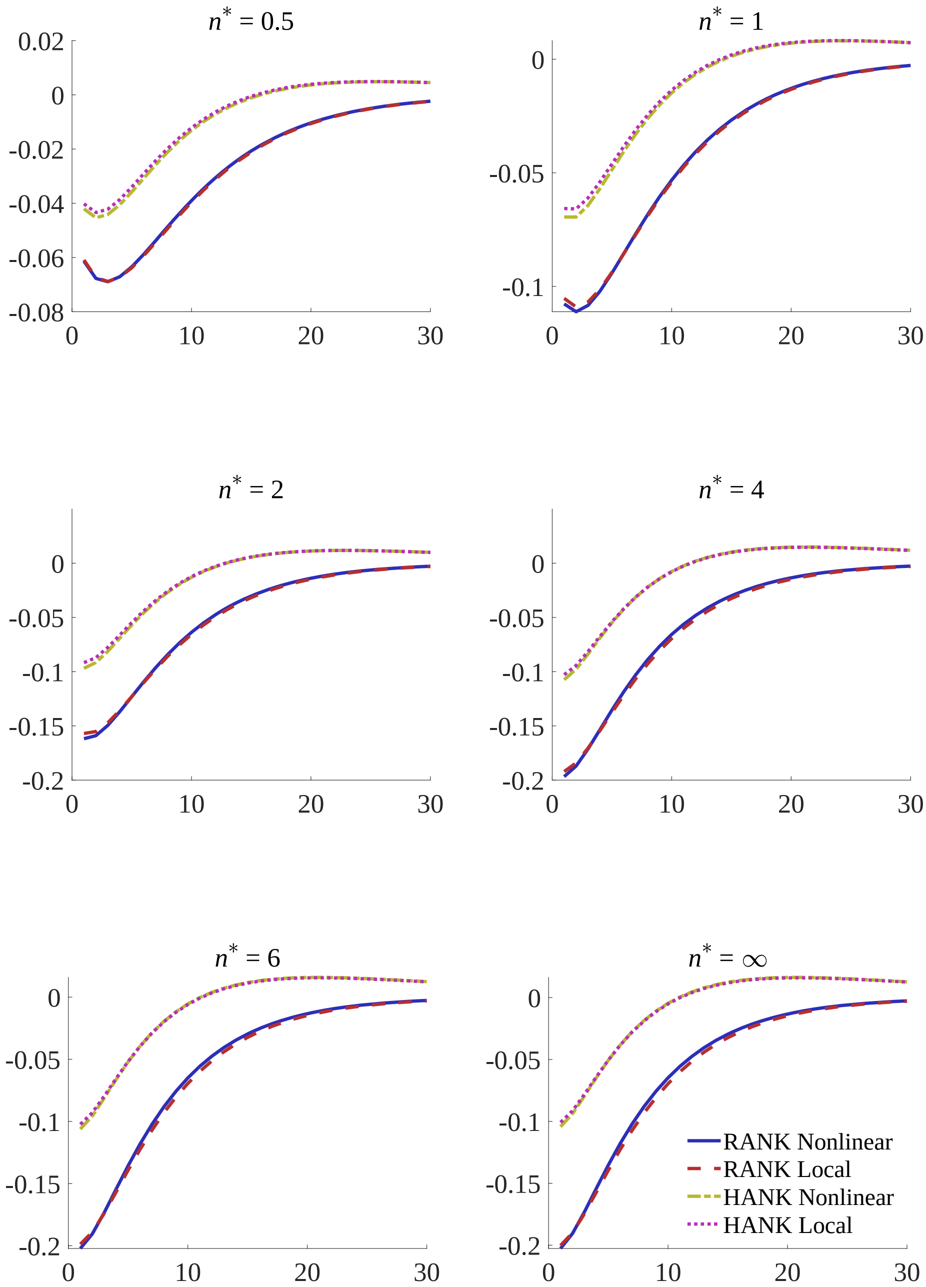
Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

Figure 9: Response of Y_t to a contractionary demand shock (without ELB)



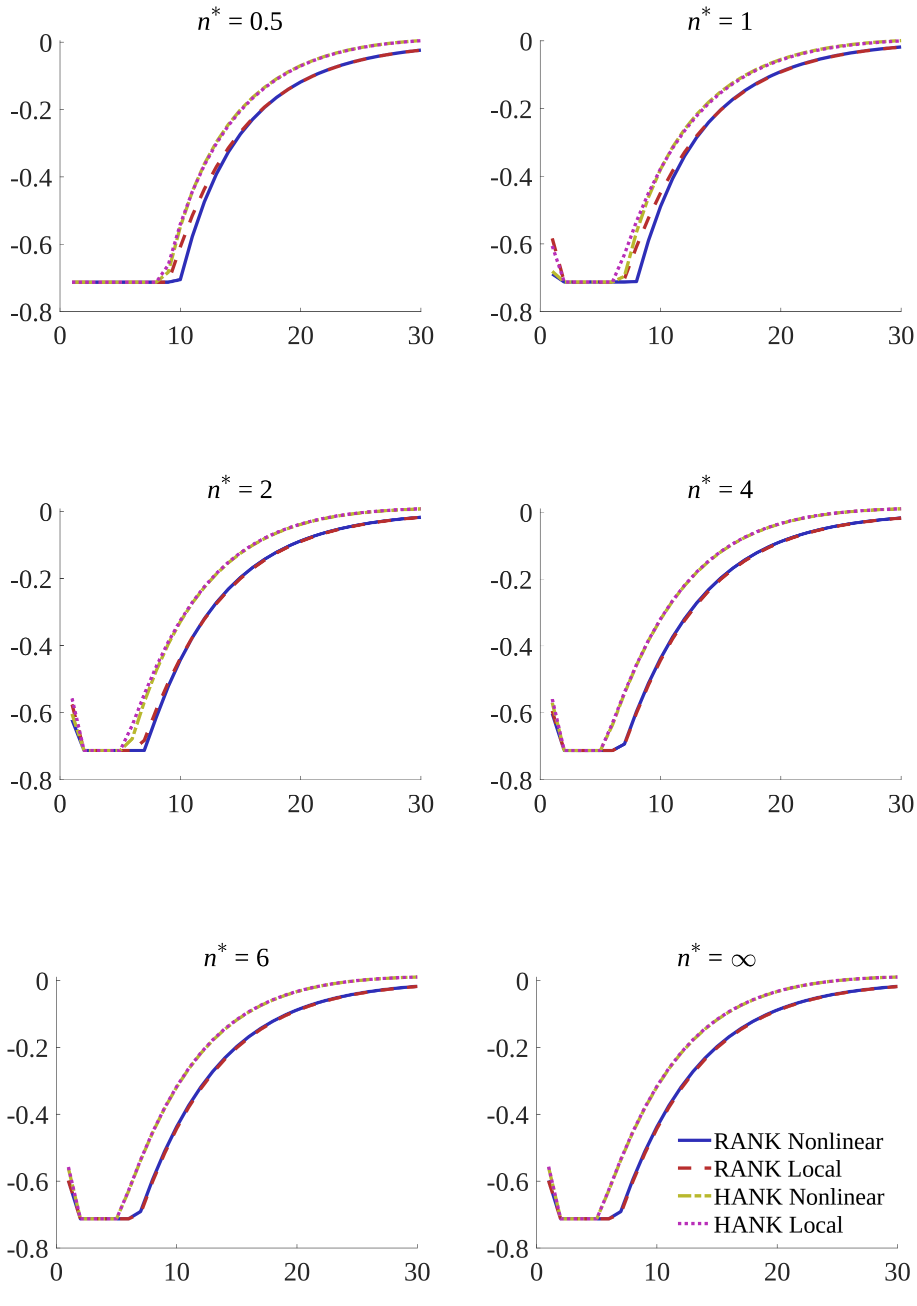
Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

Figure 10: Response of Π_t to a contractionary demand shock (without ELB)



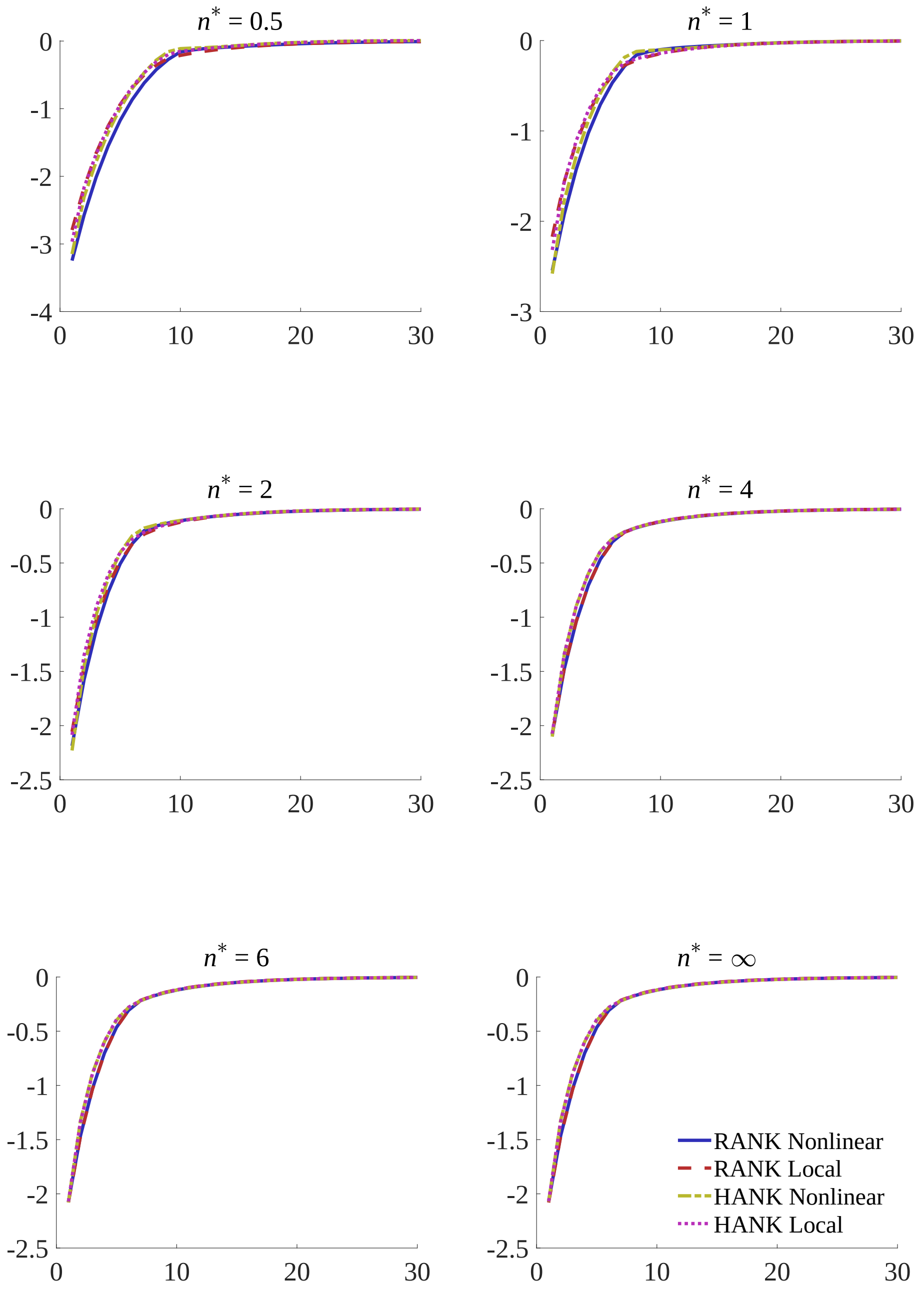
Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

Figure 11: Response of R_t to a contractionary demand shock (with ELB)



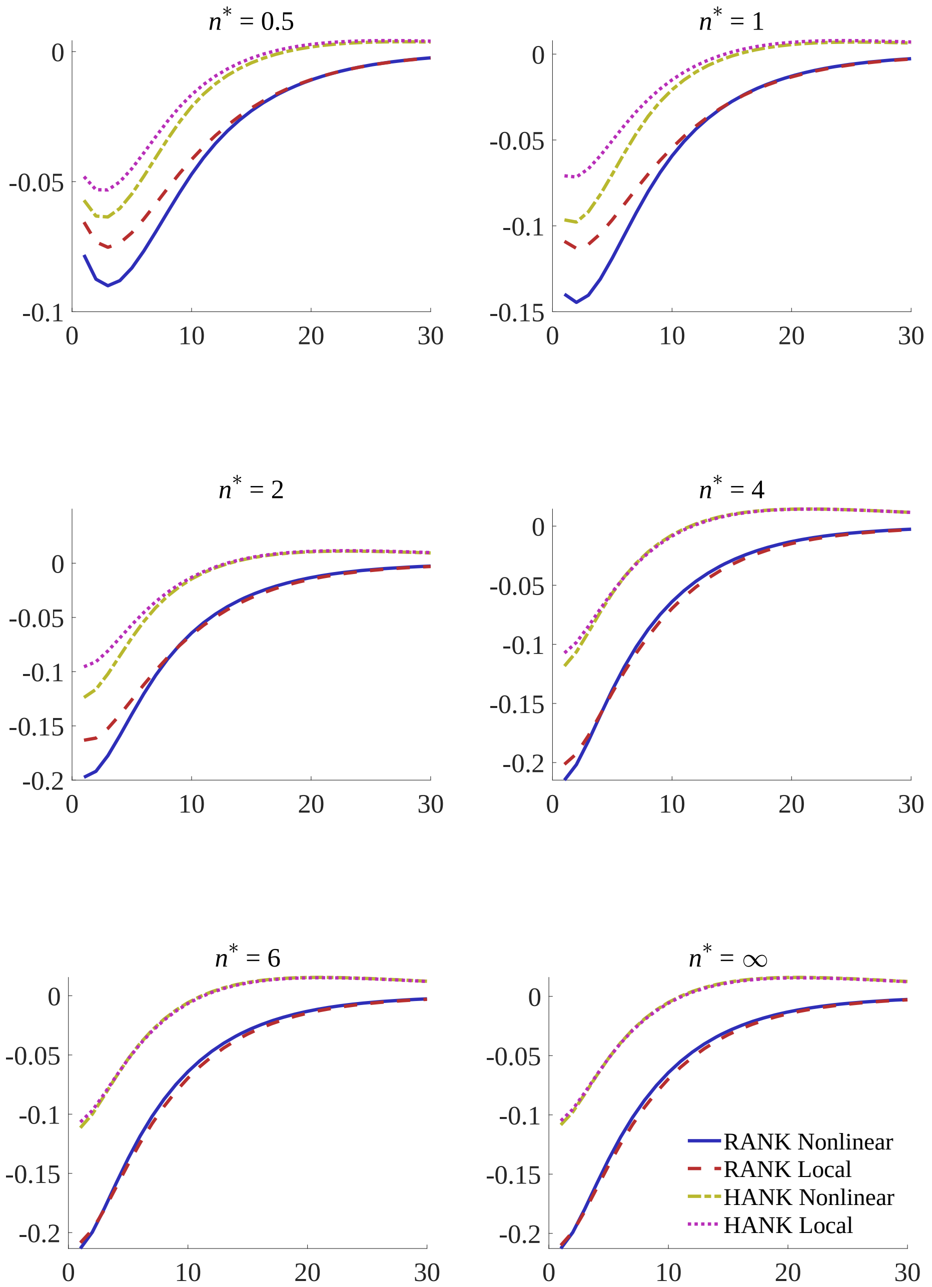
Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

Figure 12: Response of Y_t to a contractionary demand shock (with ELB)



Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

Figure 13: Response of Π_t to a contractionary demand shock (with ELB)



Notes: Responses are computed under the IT rule and expressed in percentage deviations from steady state.

C Appendix: Additional results

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT	16.29	5.60	1.94	0.64	-0.07	1.10
AIT4	14.31	5.80	1.97	0.59	-0.05	1.06
AIT8	13.99	5.67	1.98	0.59	-0.05	1.06

Table 6: Results for RE-HANK model with real household debt

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT	20.60	6.92	1.94	0.95	-0.12	1.19
AIT4	17.26	6.67	1.98	0.79	-0.07	1.15
AIT8	16.82	6.42	1.99	0.77	-0.06	1.15

Table 7: Results for RE-RANK model with different price / wage indexation ($\iota_p = \iota_w = 0.9$)

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT	16.90	5.57	1.92	0.71	-0.08	1.12
AIT4	14.49	5.80	1.96	0.63	-0.05	1.08
AIT8	14.12	5.64	1.97	0.63	-0.05	1.07

Table 8: Results for RE-HANK model with different price / wage indexation ($\iota_p = \iota_w = 0.9$)

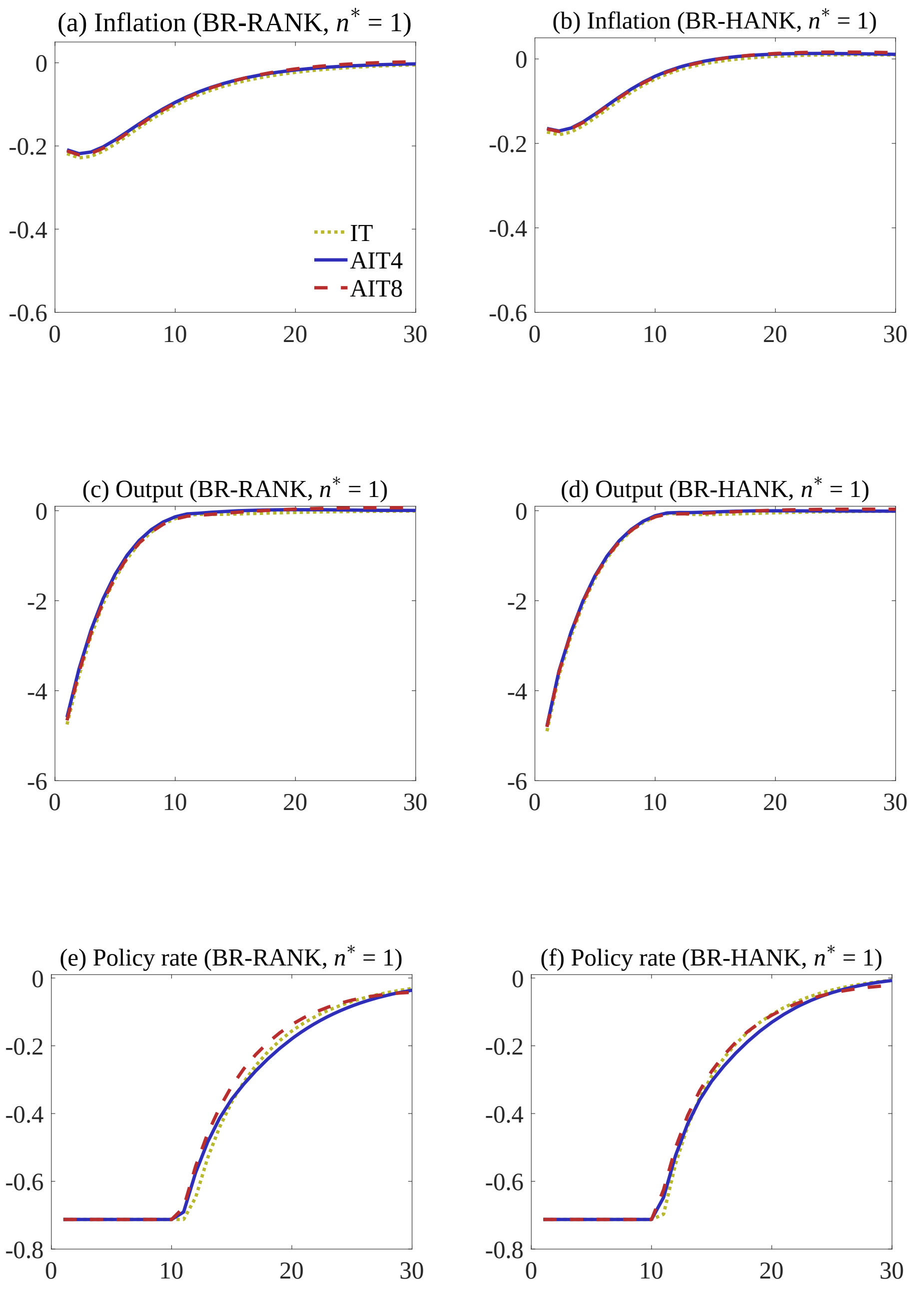
	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT	17.28	5.58	1.92	0.67	-0.09	1.17
AIT4	15.08	5.90	1.96	0.60	-0.06	1.10
AIT8	14.72	5.75	1.97	0.60	-0.05	1.10

Table 9: Results for RE-HANK model with countercyclical income risk ($\zeta = -1$)

	ELB incidence		Inflation (%)		Output (%)	
Policy rule	Freq. (%)	Avg. duration (quarters)	Avg.	Std.	Avg.	Std.
IT	15.90	5.31	1.95	0.63	-0.07	1.06
AIT4	13.85	5.72	1.97	0.58	-0.05	1.03
AIT8	13.52	5.59	1.98	0.58	-0.04	1.02

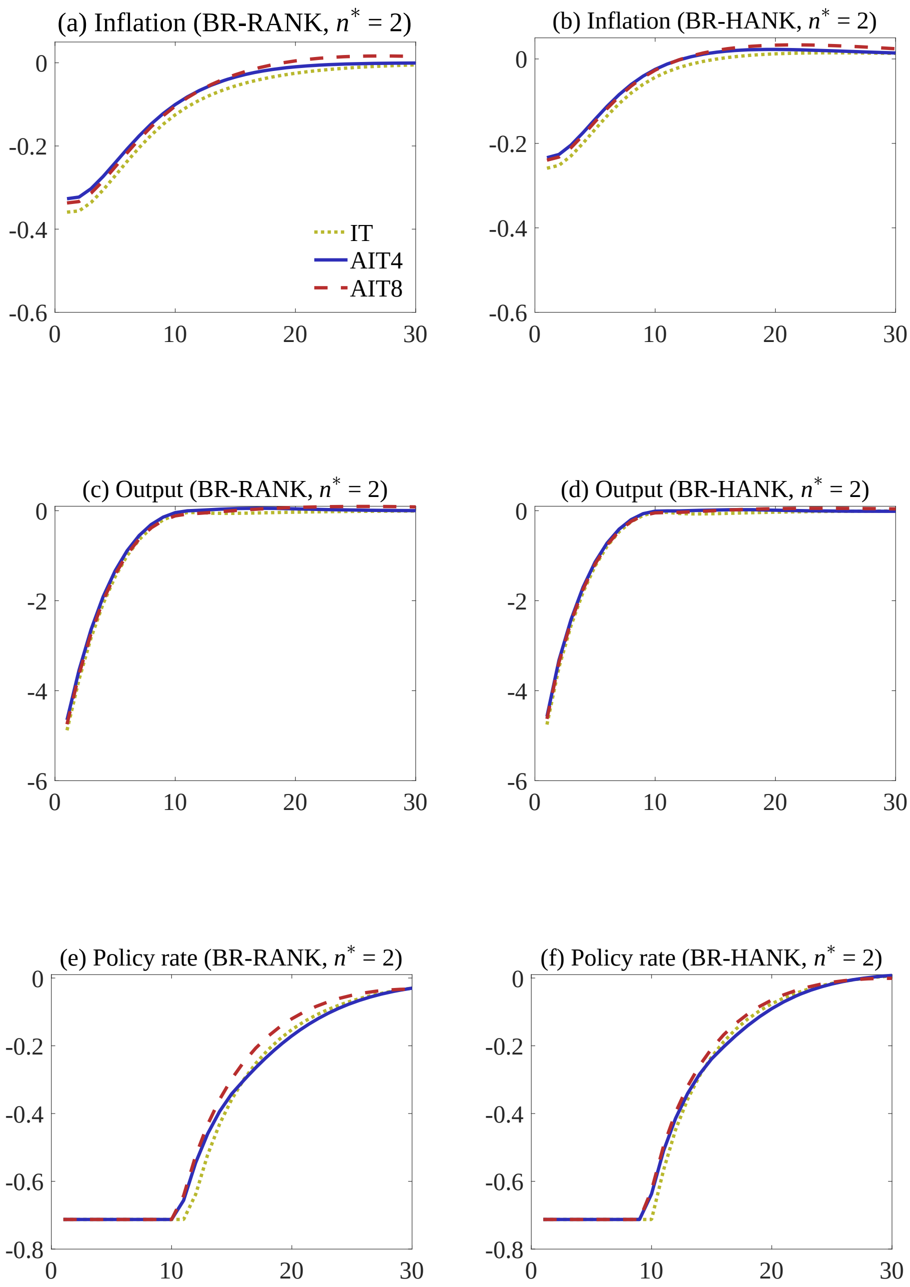
Table 10: Results for RE-HANK model with procyclical income risk ($\zeta = 1$)

Figure 14: ELB episode under reflective expectations ($n^* = 1$)



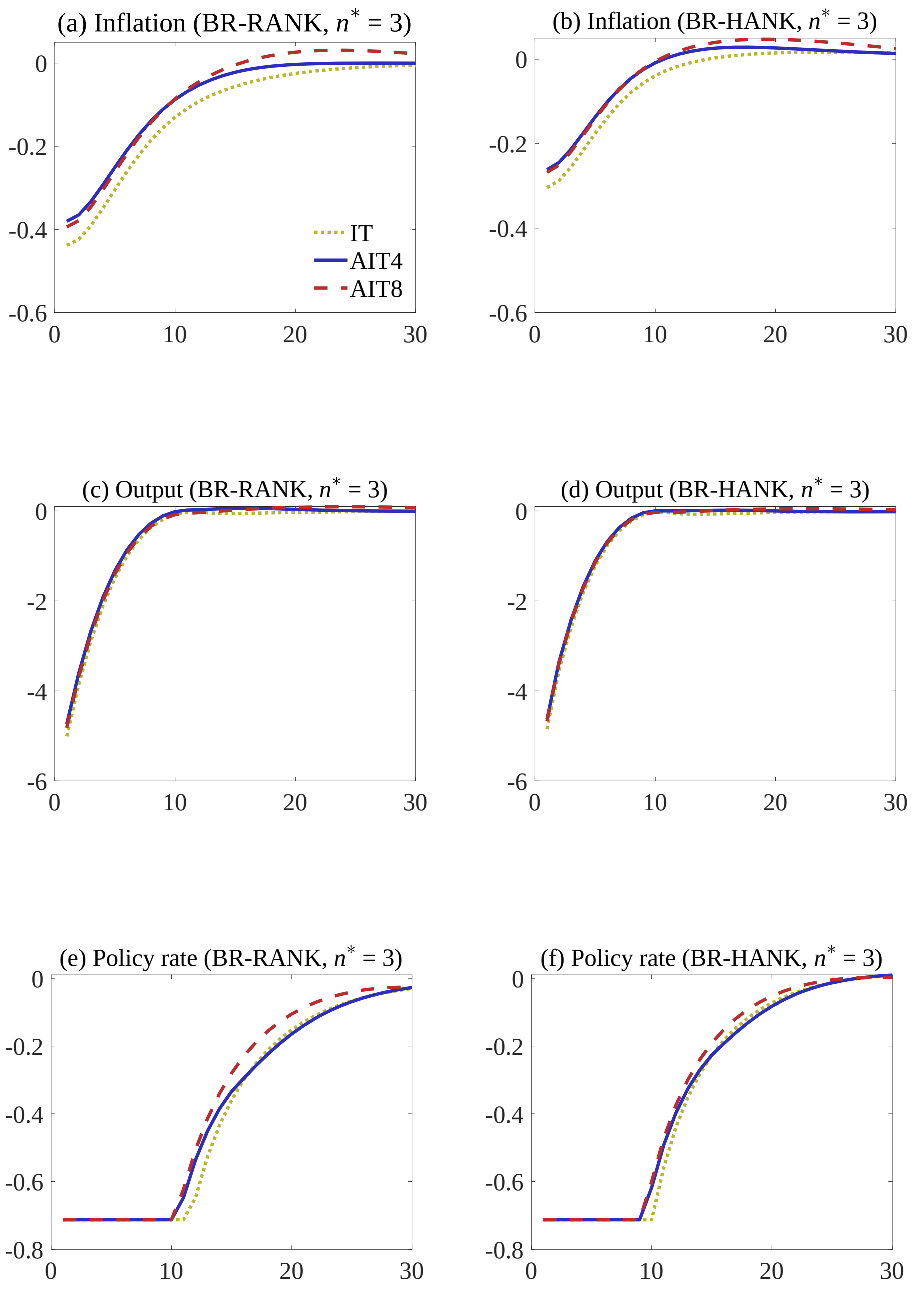
Notes: IRFs are for a large demand shock ($\varepsilon_{\beta,t} = 5\sigma_\beta$) and in percentage deviations from steady state.

Figure 15: ELB episode under reflective expectations ($n^* = 2$)



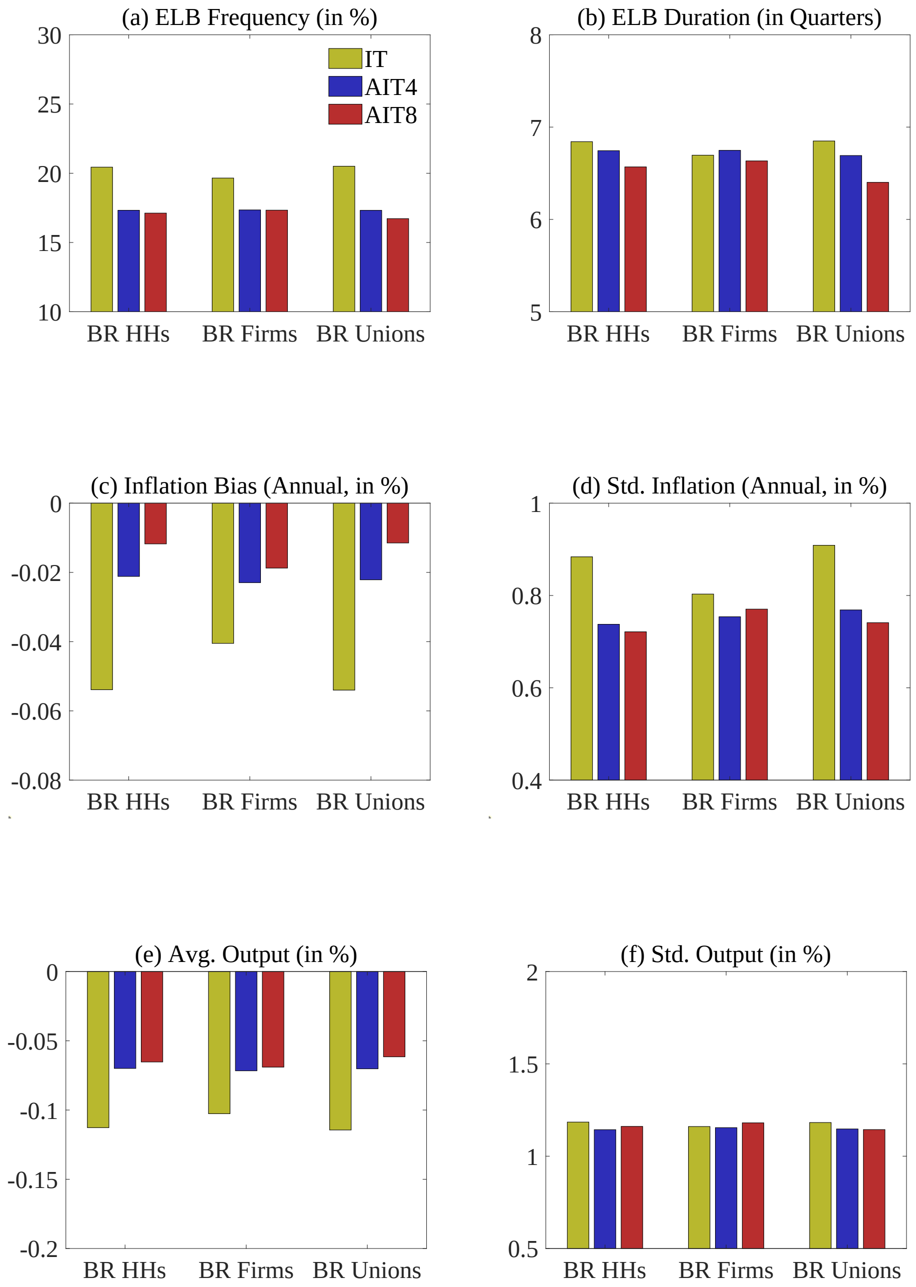
Notes: IRFs are for a large demand shock ($\varepsilon_{\beta,t} = 5\sigma_\beta$) and in percentage deviations from steady state.

Figure 16: ELB episode under reflective expectations ($n^* = 3$)



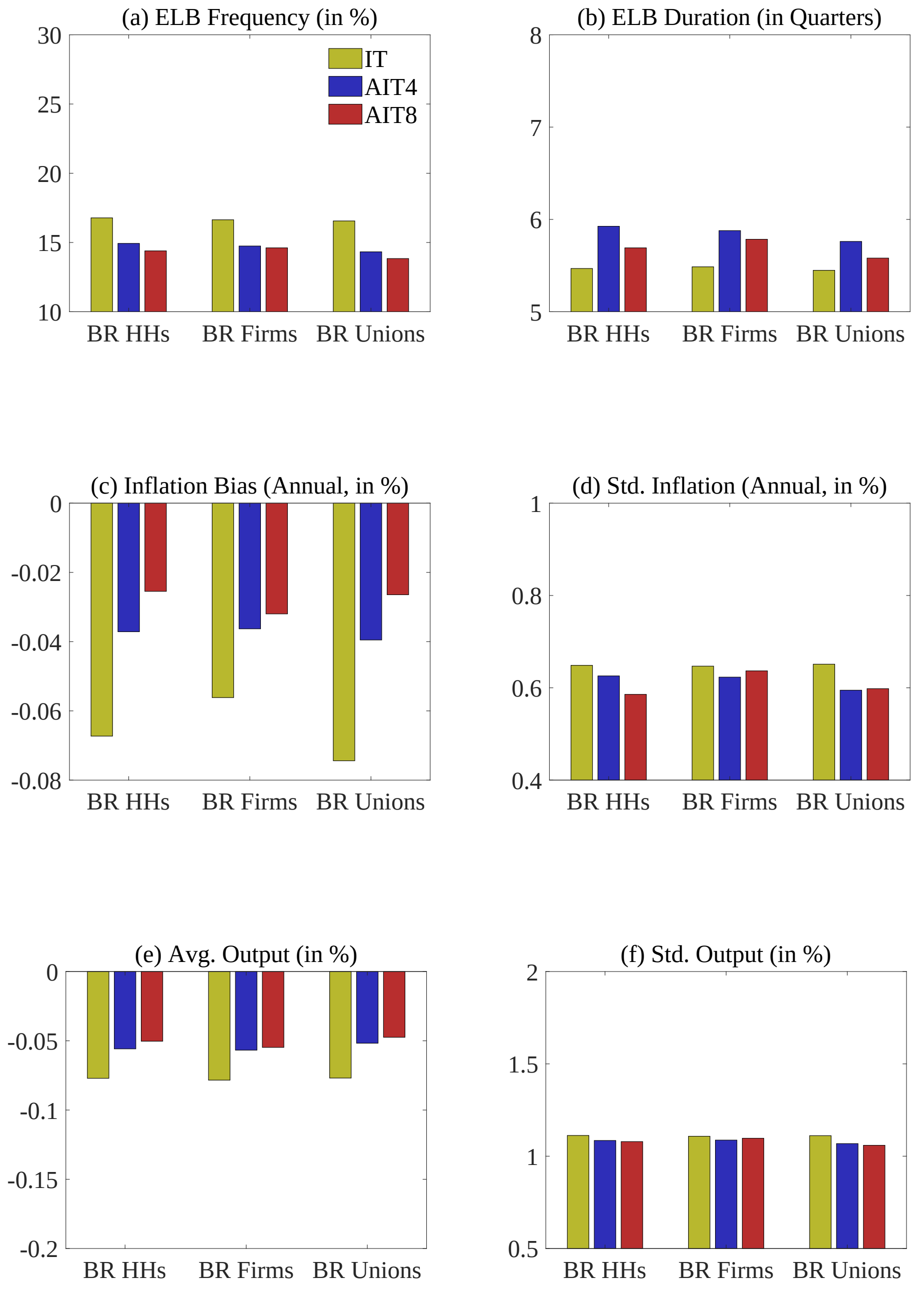
Notes: IRFs are for a large demand shock ($\varepsilon_{\beta,t} = 5\sigma_\beta$) and in percentage deviations from steady state.

Figure 17: Results for RANK model with only one type of non-rational agent



Notes: The non-rational type of agent is assumed to possess cognitive ability $n = 2$.

Figure 18: Results for HANK model with only one type of non-rational agent



Notes: The non-rational type of agent is assumed to possess cognitive ability $n = 2$.

Figure 19: Results for BR-RANK without interest rate smoothing ($\rho_R = 0$)

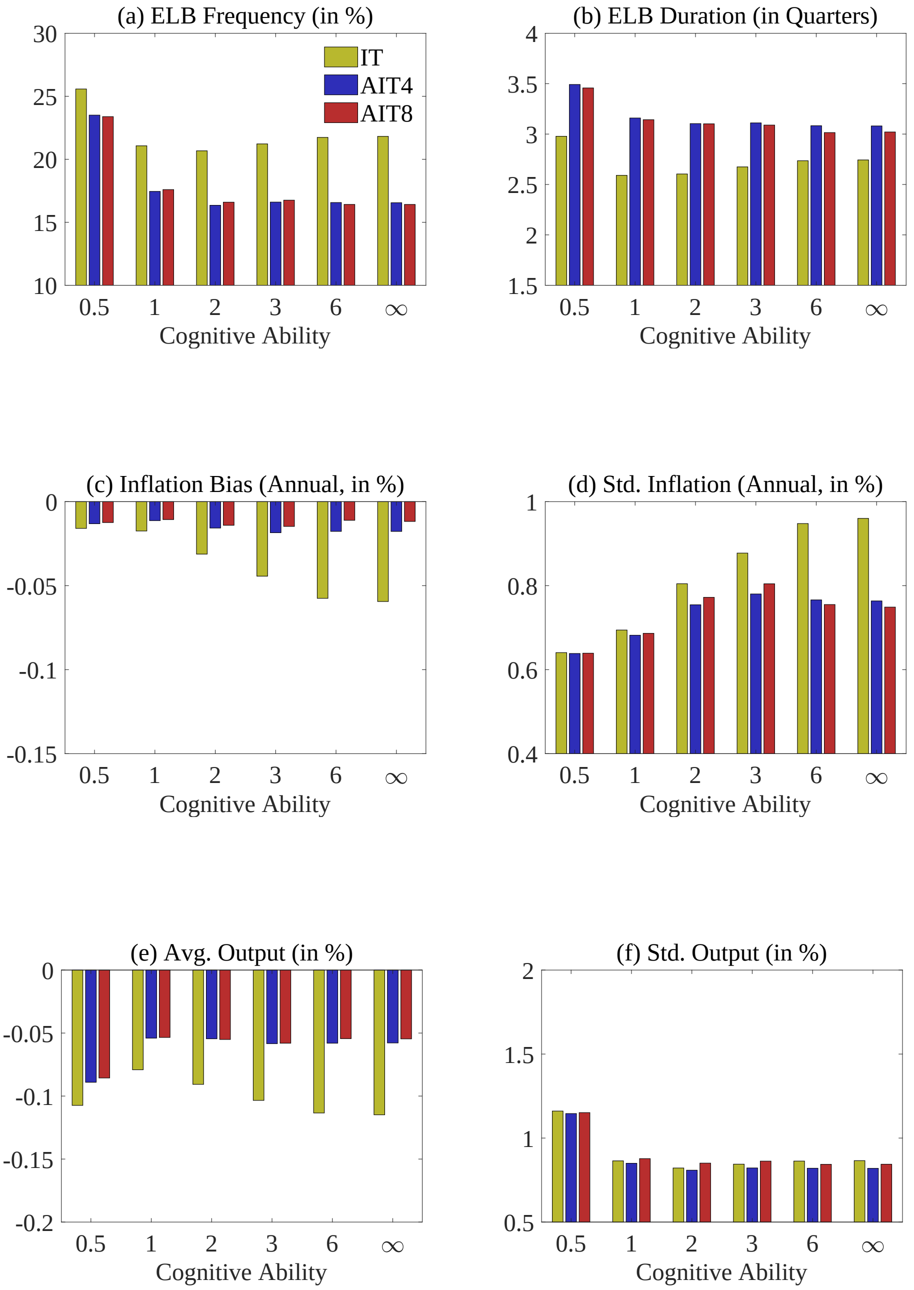


Figure 20: Results for BR-HANK without interest rate smoothing ($\rho_R = 0$)