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Technical Paper

The Environmental Multi-Sector DSGE model EMuSe: A technical documentation

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Non-technical summary

Climate change and climate change mitigation policy will have far-reaching implications for macroeconomic developments. Different regions and economic sectors are likely to be affected to very different degrees. In order to adequately gauge the economic implications of climate risks, models with sufficient regional and sectoral differentiation are needed.

For this purpose, we have developed the **E**nvironmental **Mu**lti-**Se**ctor dynamic general equilibrium model *EMuSe*, which is particularly suitable to analysing climate policy-driven adjustment processes, also in an international context. *EMuSe* features several production sectors that are interconnected via input-output linkages. Emissions occur as a by-product of production. The (economic) damage caused by environmental pollution can also be accounted for. The dynamic nature of the model allows us to explicitly picture the transition from an initial to a new steady state.

This paper presents the main features of the benchmark closed-economy flexible-price model, an open-economy extension and a variant of the model with price-setting frictions. The benchmark model is formally derived in detail. We also present selected applications to illustrate key transmission channels of climate change and climate policies in *EMuSe* (such as the effect of an extreme weather event or carbon pricing).

Together with the technical documentation of the model, we provide the program codes for all these model variants. We hope that this is useful for interested researchers and that it fosters a fruitful exchange with potential users, also to further advance the *EMuSe* model.

Nichttechnische Zusammenfassung

Klimawandel und Klimapolitik werden weitreichende Auswirkungen auf die gesamtwirtschaftliche Entwicklung haben. Verschiedene Regionen und Wirtschaftssektoren dürften dabei zum Teil sehr unterschiedlich stark betroffen sein. Um die wirtschaftlichen Auswirkungen von Klimarisiken adäquat abschätzen zu können, werden daher Modelle mit hinreichend regionaler und sektoraler Differenzierung benötigt.

Zu diesem Zweck wurde das *EMuSe*-Modell (**E**nvironmental **Mu**lti-**Se**ctor-Modell) entwickelt, mit dem insbesondere klimapolitische Anpassungsprozesse auch im internationalen Zusammenhang analysiert werden können. *EMuSe* ist ein dynamisches, allgemeines Gleichgewichtsmodell, in dem mehrere Wirtschaftssektoren über Vorleistungsverflechtungen miteinander verbunden sind. Das Modell bildet zudem im Zuge des Produktionsprozesses entstehende Emissionen ab. Aus Umweltverschmutzung resultierende (wirtschaftliche) Schäden können ebenfalls erfasst werden. Der dynamische Charakter des Modells erlaubt es, Übergangspfade hin zu neuen langfristigen Gleichgewichten abzubilden.

In dem vorliegenden Papier wird die Modellversion einer geschlossenen Volkswirtschaft mit flexiblen Preisen ausführlich hergeleitet. Außerdem wird eine Version des Modells mit Preissetzungsfriktionen und eine Variante mit einer offenen Volkswirtschaft präsentiert. Für alle drei Ausführungen werden illustrative Analysen vorgestellt, anhand derer die zentralen Wirkungsmechanismen von Klimawandel und Klimapolitiken im *EMuSe*-Modellrahmen veranschaulicht werden sollen (wie beispielsweise die wirtschaftliche Auswirkung eines Extremwetterereignisses oder einer CO2-Bepreisung).

Mit der technischen Dokumentation geht die Veröffentlichung des gesamten Gleichungssystems sowie des Programm-Codes einher. Wir hoffen, dass dies für interessierte Forschende hilfreich ist und freuen uns auf einen fruchtbaren Austausch – auch, um das *EMuSe*-Modell weiterzuentwickeln.

The **E**nvironmental **Mu**lti-**Se**ctor DSGE model EMuSe: A technical documentation*

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Abstract

Climate change and climate policy will have far-reaching economic implications, thereby also posing new challenges for macroeconomic analysis. This is partly because climate risks have an important global dimension. Moreover, climate change and climate polices are likely to affect different economic sectors to varying degrees. Hence, in order to adequately gauge the macroeconomic implications of climate risks, models with sufficient regional and sectoral differentiation are needed. Against this background, we developed the environmental multi-sector dynamic stochastic general equilibrium model EMuSe. This paper presents the main features of the benchmark closed-economy flexible-price model, an open-economy extension of the model, a variant of the model with price-setting frictions and selected applications to illustrate key transmission channels. In order to give those who are interested the opportunity to gain more experience with EMuSe, the model codes are published together with this documentation.

Keywords: climate risks, DSGE, production linkages, sectoral heterogeneity
JEL Classification E3, E6, F4, H3, Q5

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are a standard tool for quantitative policy analysis in macroeconomics (see Christiano et al., 2018). While monetary and fiscal policy issues are typically at the centre of the research agenda, DSGE models have recently also been adopted to analyse the economic impact of climate change and environmental policies. Environmental dynamic stochastic general equilibrium (E-DSGE) models, as they are known, range from slightly modified standard frameworks primarily designed for studying the effects of environmental policies to integrated assessment type models, which explicitly incorporate the global economy and the climate in a unified framework to assess the macroeconomic consequences of anthropogenic emissions.[1] While the dynamic nature of this class of models makes them well suited to analysing key adjustment processes, E-DSGE models typically feature a high level of sectoral aggregation and often abstract from international linkages (see, e.g., Heutel, 2012; Golosov et al., 2014).

Climate change and climate policy, however, will have far-reaching effects on macroeconomic developments, affecting different sectors of the economy to varying degrees. Moreover, climate change and climate action have an important global dimension. First, regions might be affected very differently by climate risks. Second, without a direct impact, climate change and climate policies are likely to provoke spillover effects via cross-border trade and financial linkages. Hence, models with sufficient regional and sectoral differentiation are needed in order to adequately gauge the macroeconomic implications of climate risks.

To meet these challenges, we developed the **E**nvironmental **Mu**lti-**Se**ctor DSGE model EMuSe. With EMuSe, which is implemented using the software Dynare (under MATLAB),[2] the impact of climate-related adjustment processes can be analysed by taking into account both the sectoral and international dimension. Specifically, EMuSe versions with up to 4 regions and up to 54 sectors have been used.[3]

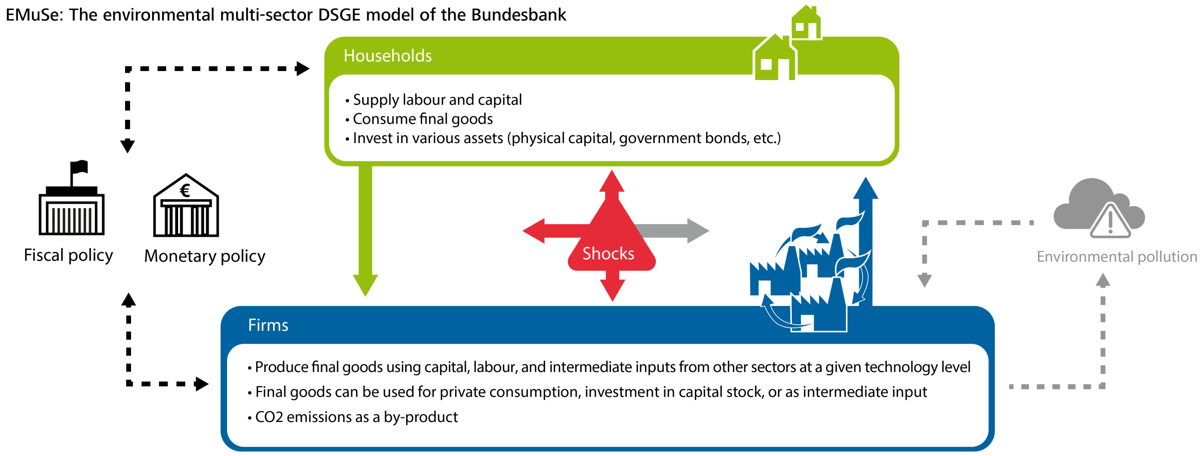
Like prototypical DSGE models, EMuSe features a model block comprising the optimal behaviour of households, a model block describing the optimal behaviour of firms, and a government block characterising the behaviour of monetary and fiscal policy. Each of these blocks is defined by equations derived from the underlying microeconomic structure of the model, i.e., explicit assumptions regarding the specific behaviour of agents as well as the technological, budget and institutional constraints in the economy. The incorporation of expectations about future outcomes provides the source of (forward-looking) dynamic interactions between these interrelated blocks. The model economy can be affected by various types of exogenous processes, referred to as shocks (see Figure 1).

[1] See European Central Bank (2021) for an overview.

[2] See Adjemian et al. (2022).

[3] Technical constraints, however, do not allow us to simultaneously max out both the region and the sector dimension.

Figure 1: Model structure.



Notes: The figure shows a stylised representation of a closed-economy version of EMuSe. Grey shading indicates the environmental module.

While these ingredients are standard in the literature, two features of the EMuSe model stand out: i) a multi-sectoral production structure and ii) the presence of an environmental module. Specifically, in contrast to prototypical production technologies used in DSGE models, we assume that in addition to labour and capital, a bundle of intermediate inputs is needed. This bundle combines output from all sectors using a constant elasticity of substitution (CES) production technology, which implies that the extent to which various inputs are substitutable is limited. Considering sectoral linkages across all sectors allows us to capture a detailed production network. As regards the specification of the environmental module, various approaches can be found in the E-DSGE literature. One common feature, however, is that the production process is related to pollution.[4] In EMuSe, we assume that emissions are a by-product of production. Specifically, emissions in a given sector are proportional to its output. This relationship is captured by the so-called emissions intensity, which is measured as the ratio of emissions to output. In addition, the model may take into account negative pollution externalities in the form of a damage function.[5]

The model's rich sectoral structure leaves many sector-specific production parameters to be specified. For this purpose, we developed a MATLAB-based calibration toolkit that pins down most of the sector-specific production parameters using data from, inter alia, the World Input-Output Database (see Timmer et al., 2015). It allows us to specify the EMuSe model quite flexibly for a custom choice of regions and sectors.

In this paper, we present the main features of the model and selected applications to illus-

[4]The most common modeling strategies in the E-DSGE literature are the inclusion of pollution as an additional production input (see, e.g., Fischer and Springborn, 2011) or its presence as a by-product of the production process (see, e.g., Heutel, 2012; Golosov et al., 2014).

[5]Moreover, the model can be extended to capture pollution abatement efforts.

trate key transmission channels. To give those who are interested an opportunity to gain more experience with EMuSe, we make the program codes for the applications presented here as well as the EMuSe Calibration Toolkit publicly available (for details, see Appendix A.3 and A.4).

The rest of the technical paper is organised as follows. In Section 2, we describe a flexible-price, closed-economy baseline version of the EMuSe model. Section 3 provides information on the parameterisation strategy for the baseline model and its extensions. Section 4 contains an illustrative application of the baseline model. Section 5 details the theoretical setup of an extended model version with nominal price rigidities and Section 6 presents an application of this model version. A multi-region specification of the baseline model is laid out in Section 7. Section 8 shows an application of this framework. Finally, the Appendix provides comprehensive information on the derivation and calibration of the baseline model along with information on the technical implementation.

2 Baseline model

2.1 Overview

The baseline model economy is a closed-economy, flexible-price framework comprising a representative household, a set of $\mathcal{S} = \{1, 2, .., S\}$ production sectors (each containing a perfectly competitive representative firm), perfectly competitive consumption-goods, investment-goods and intermediate-goods retailers, as well as a fiscal authority. The representative household consumes, saves and supplies labour and capital via agencies to the sectoral-goods producer. Labour and capital are not perfectly mobile across sectors. Household income is used for consumption and investment in physical capital. Sectoral output is transformed into bundles of consumption, investment and intermediate goods. This is accomplished by perfectly competitive retailers. Besides renting capital and labour, the representative firm purchases intermediate input bundles. It sets its price equal to marginal costs. There is heterogeneity with respect to factor intensities. Production is assumed to cause emissions, which may differ in their intensity across sectors. The fiscal authority runs a balanced budget by paying out lump-sum transfers which are financed by emissions taxes.

2.2 Representative household

The representative household maximises the stream of expected utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

by choosing a sequence of consumption, labour supply and investment, where $0 < \beta < 1$ is the subjective discount factor. Specifically, the household's instantaneous utility function is given by

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\psi}}{1+\psi},$$

where σ denotes the inverse of the elasticity of intertemporal substitution for consumption, κ_N determines the disutility of labour and ψ represents the inverse of the Frisch labour supply elasticity. The representative household's optimisation problem is subject to the budget constraint

$$P_t^C C_t + \tilde{P}_t^I I_t + B_t = W_t N_t + R_t^k K_{t-1} + R_{t-1}^B B_{t-1} + P_t^C TR_t,$$

where P_t^C is the consumer price index (CPI), $\tilde{P}_t^I$ is the nominal price of a basket of investment goods I_t, W_t is the nominal wage rate, R_t^k is the nominal rental rate of capital K_t and R_{t-1}^B is the gross nominal return on one-period bonds B_{t-1}. TR_t are (real) lump-sum transfers received from the government. In CPI-deflated real terms we get

$$C_t + P_t^I I_t + \frac{B_t}{P_t^C} = w_t N_t + r_t^k K_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t^C} + TR_t,$$

where $P_t^I = \tilde{P}_t^I / P_t^C$, $w_t = W_t / P_t^C$ and $r_t^k = R_t^k / P_t^C$.

The capital accumulation process is represented by the following law of motion

$$K_t = (1-\delta) K_{t-1} + I_t,$$

with δ denoting the rate of depreciation. Moreover, we impose a no-Ponzi-game condition to prevent the household from excessive borrowing.

Hence, the intratemporal first-order condition of the household's optimisation problem is characterised by $\lambda_t = C_t^{-\sigma}$ and $\kappa_N N_t^{\psi} = \lambda_t w_t$. The optimal intertemporal savings decision is given by

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{r_{t+1}^k + (1-\delta) P_{t+1}^I}{P_t^I} \right],$$

and

$$\lambda_t = \beta \mathbb{E}_t \left(\lambda_{t+1} \frac{R_t^B}{\pi_{t+1}^{CPI}} \right),$$

with $\pi_t^{CPI} = P_t^C / P_{t-1}^C$.

2.3 Consumption-goods and investment-goods retailer

The representative household demands bundles of consumption and investment goods C_t and I_t, which are traded at prices P_t^C and P_t^I, respectively. Following Bouakez et al. (2023), the production technology of a perfectly competitive, representative retailer that bundles sector-level consumption goods, $C_{s,t}$, of the S sectors is given by

$$C_t = \left(\sum_{s=1}^{S} \psi_{C,s}^{1-\sigma_C} C_{s,t}^{\sigma_C} \right)^{\frac{1}{\sigma_C}}.$$

The parameters $\psi_{C,s}$ and σ_C determine the weight of sectoral output from sector s in the consumption-goods bundle and the elasticity of substitution between sector-level consumption goods, respectively. The representative consumption-goods retailer's optimisation problem can be written as

$$\max_{C_{s,t}} P_t^C C_t - \sum_{s=1}^{S} \tilde{P}_{s,t} C_{s,t},$$

where $\tilde{P}_{s,t}$ is the producer price of sectoral good $s \in \mathcal{S}$. Taking into account the CES-bundling technology leads to the following first-order condition:

$$C_{s,t} = \psi_{C,s} P_{s,t}^{-\frac{1}{(1-\sigma_C)}} C_t,$$

where $P_{s,t} = \tilde{P}_{s,t} / P_t^C$. By plugging this expression into the CES consumption-goods aggregator gives

$$P_t^C = \left[\sum_{s=1}^{S} \psi_{C,s} \tilde{P}_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}} \right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

which implies

$$1 = \left[\sum_{s=1}^{S} \psi_{C,s} P_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}} \right]^{-\frac{(1-\sigma_C)}{\sigma_C}}.$$

Hence, CPI inflation can be expressed as

$$\pi_t^{CPI} = \left[\sum_{s=1}^{S} \psi_{C,s} (\pi_{s,t}^{PPI} P_{s,t-1})^{-\frac{(\sigma_C)}{1-\sigma_C}} \right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

with $\pi_{s,t}^{PPI} = \tilde{P}_{s,t} / \tilde{P}_{s,t-1}$.

For investment goods, a similar bundling technology is assumed:

$$I_t = \left(\sum_{s=1}^{S} \psi_{I,s}^{1-\sigma_I} I_{s,t}^{\sigma_I} \right)^{\frac{1}{\sigma_I}},$$

where $\psi_{I,s}$ reflects the weight of the sectoral output good from sector s in the investment-goods bundle and σ_I determines the elasticity of substitution among the output from the different sectors, respectively. The optimisation problem of the investment-goods bundler (in CPI-deflated real terms) is

$$\max_{I_{s,t}} P_t^I I_t - \sum_{s=1}^{S} P_{s,t} I_{s,t},$$

where P_t^I is the CPI-deflated investment price. The first-order condition implies, that

$$I_{s,t} = \psi_{I,s} \left(\frac{P_{s,t}}{P_t^I} \right)^{-\frac{1}{(1-\sigma_I)}} I_t,$$

while the price index is given by

$$P_t^I = \left[\sum_{s=1}^{S} \psi_{I,s} (P_{s,t})^{-\frac{\sigma_I}{(1-\sigma_I)}} \right]^{-\frac{(1-\sigma_I)}{\sigma_I}}.$$

2.4 Labour and capital

Labour is not perfectly mobile across sectors. We assume that a perfectly competitive, representative labour agency hires the total amount of labour, N_t, from the household at the CPI-deflated real wage w_t and sells it to intermediate-goods producers operating in S different sectors, such that

$$N_t = \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}},$$

where $\omega_{N,s}$ is the weight attached to labour provided to sector $s \in \mathcal{S}$ and ν_N determines the elasticity of substitution of labour across sectors, capturing the degree of labour mobility. The labour agency's optimisation problem can be written as

$$\max_{N_{s,t}} \sum_{s=1}^{S} w_{s,t} N_{s,t} - w_t N_t,$$

which leads to the following first-order condition characterising the sector-specific supply of labour types

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t} \right)^{-\frac{1}{(1-\nu_N)}} N_t \quad \forall s \in \mathcal{S}.$$

After plugging this expression into the CES aggregator of labour goods, we obtain the aggregate wage index

$$w_t = \left[\sum_{s=1}^{S} \omega_{N,s} w_{s,t}^{-\frac{\nu_N}{(1-\nu_N)}} \right]^{-\frac{(1-\nu_N)}{\nu_N}}.$$

A similar proceeding for the capital agency yields

$$K_{s,t} = \omega_{K,s} \left(\frac{r^K_{s,t+1}}{r^K_{t+1}} \right)^{-\frac{1}{(1-\nu_K)}} K_t \quad \forall s \in \mathcal{S},$$

and

$$r^K_t = \left[\sum_{s=1}^{S} \omega_{K,s} (r^K_{s,t})^{-\frac{\nu_K}{(1-\nu_K)}} \right]^{-\frac{(1-\nu_K)}{\nu_K}}.$$

2.5 Production

In each sector $s \in \mathcal{S}$, a perfectly competitive firm produces sectoral output $y_{s,t}$ by combining labour $N_{s,t}$, capital $K_{s,t-1}$ and a bundle of intermediate inputs $H_{s,t}$, according to the constant returns to scale production technology

$$y_{s,t} = [1 - D(M_t)] \, \varepsilon_{s,t} \left(K^{1-\alpha_{N,s}}_{s,t-1} N^{\alpha_{N,s}}_{s,t} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}},$$

where $\varepsilon_{s,t}$ is total factor productivity, while $1 > \alpha_{N,s} > 0$ and $1 > \alpha_{H,s} > 0$ determine the elasticity of output with respect to capital, labour and intermediate inputs. The damage function $D(M_t)$ captures output losses stemming from the atmospheric concentration of carbon dioxide M_t.[6] In the closed-economy version of EMuSe, sectoral output is sold at price $P_{s,t}$ to consumption-goods, investment-goods and intermediate-goods retailers. Emissions $Z_{s,t}$ are a by-product of production and assumed to be proportional to output. Hence, $Z_{s,t} = \kappa_s y_{s,t}$, where $\kappa_s \in [0, \infty)$ represents the emissions intensity.[7] The pollution stock M_t evolves according to

$$M_t = (1 - \rho^M) M_{t-1} + \sum_{s=1}^{S} Z_{s,t} + Z^*_t,$$

with $\rho^M \in (0, 1)$ denoting a linear decay rate (see, e.g., Heutel, 2012; Golosov et al., 2014) and Z^*_t capturing current period emissions from the rest of the world. For illustrative purposes, we follow Heutel (2012) and assume that emissions-induced damage is given by

$$D(M_t) = \gamma_0 + \gamma_1 M_t + \gamma_2 M_t^2,$$

where γ_0, γ_1 and γ_2 are damage parameters governing the intensity of the negative environmental externality on production.[8]

[6] In the literature, various damage function specifications are used. Depending on the assumed functional relationship and the parametrisation, there can be major differences in terms of probable economic losses. Damage functions in macroeconomic climate models are therefore a contentious topic. See, inter alia, Weitzman (2012).

[7] The emissions intensity can also be specified as a stochastic process or as explicitly depending on (dirty) energy input as in Hinterlang et al. (2022). The former specification, for example, would allow to feed pathways for the emissions intensity into the model as a reduced-form representation of "green" technological progress.

[8] In analyses that do not take into account environmental damage, the damage parameters are set to zero.

The firm's optimisation problem can be described by a two-step process. First, the firm minimises its costs $w_{s,t}N_{s,t}+r^k_{s,t}K_{s,t-1}+P^H_{s,t}H_{s,t}$ subject to the constant returns to scale production technology. Taking factor prices as given, we get the following first-order conditions for labour, capital and intermediate inputs:

$$
\begin{aligned}
w_{s,t} &= \alpha_{H,s}\alpha_{N,s}mc_{s,t}\frac{y_{s,t}}{N_{s,t}}, \\
r^k_{s,t} &= \alpha_{H,s}(1-\alpha_{N,s})mc_{s,t}\frac{y_{s,t}}{K_{s,t-1}}, \\
P^H_{s,t} &= (1-\alpha_{H,s})mc_{s,t}\frac{y_{s,t}}{Hs,t},
\end{aligned}
$$

where $mc_{s,t}$ is the shadow cost of producing an additional unit of output in sector s.

Second, the representative firm chooses $y_{s,t}$ to maximise its profits

$$\max_{y_{st}} \Pi_{s,t} = P_{s,t}y_{s,t} - mc_{s,t}y_{s,t} - P^{em}_t \kappa_s y_{s,t},$$

where P^{em}_t denotes a potential, government-mandated real price for pollution.

The first-order condition of the problem is

$$P_{s,t} = \tilde{mc}_{s,t},$$

where $\tilde{mc}_{s,t} = mc_{s,t} + \kappa_s P^{em}_t$ implies that the real marginal costs also include emissions costs.

2.6 Intermediate-goods retailer

Similar to the production of consumer and capital goods, a perfectly competitive intermediate-goods retailer purchases output, $H_{s,j,t}$, from representative firms operating in various sectors $j \in \mathcal{S}$ at a (CPI-deflated) price $P_{j,t}$ and transforms it into a bundle of sector-specific intermediate-goods bundles $H_{s,t}$. Those bundles are sold, in turn, to a representative firm in sector s at a price $P^H_{s,t}$. We assume the following production technology:

$$H_{s,t} = \left(\sum_{j=1}^{S} \psi_{H,s,j}^{1-\sigma_{H,s}} H_{s,j,t}^{\sigma_{H,s}}\right)^{\frac{1}{\sigma_{H,s}}} \quad \forall j \in \mathcal{S} \text{ and } \forall s \in \mathcal{S},$$

where $\psi_{H,s,j}$ denotes the weight of each input in the intermediate-goods bundle and $\sigma_{H,s}$ governs their elasticity of substitution. The optimisation problem can thus be written as

$$\max_{H_{s,j,t}} P^H_{s,t}H_{s,t} - \sum_{j=1}^{S} P_{j,t}H_{s,j,t} \quad \forall j \in \mathcal{S} \text{ and } \forall s \in \mathcal{S},$$

which implies

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P^H_{s,t}} \right)^{-\frac{1}{\left(1-\sigma_{H,s}\right)}} H_{s,t} \quad \forall j \in \mathcal{S} \text{ and } \forall s \in \mathcal{S}$$

and

$$P^H_{s,t} = \left[\sum_{j=1}^{S} \psi_{H,s,j} \left(P_{j,t}\right)^{-\frac{\sigma_{H,s}}{\left(1-\sigma_{H,s}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s}\right)}{\sigma_{H,s}}} \qquad \forall j \in \mathcal{S} \text{ and } \forall s \in \mathcal{S}.$$

2.7 Policy

For the sake of simplicity, we assume that the fiscal authority runs a balanced budget in each period:

$$TR_t = P^{em}_t \sum_{s=1}^{S} Z_{s,t} + \frac{B_t}{P^C_t} - \frac{R^B_{t-1} B_{t-1}}{P^C_t}.$$

2.8 Market clearing and aggregation

In each sector s product market clearing implies

$$P_{s,t} y_{s,t} = P_{s,t} C_{s,t} + P_{s,t} I_{s,t} + \sum_{\tilde{s}=1}^{S} P_{s,t} H_{\tilde{s},s,t}.$$

At the aggregate level, it holds that

$$Y^{va}_t = C_t + P^I_t I_t.$$

In addition, the market-clearing condition $B_t = B_{t-1} = 0$ must hold.

3 Calibrating the model

The model parameters can be partitioned into three subsets. The first comprises (general) parameters related to the aggregate economy. The second subset of parameters captures heterogeneity on the production side by allowing, inter alia, for sector-specific factor intensities, input-output linkages and contributions to final demand. The final group of parameters refers to the environmental module of the model. While, in particular, the general parameters are taken from the literature, sector-specific production and environmental parameters are largely determined with the help of the EMuSe Calibration Toolkit. This toolkit extracts and aggregates data from the most recent release of the World Input-Output Database (WIOD) (see Timmer et al., 2015) and the Environmental Accounts published by the European Commission (see Corsatea et al., 2019) to pin down most of EMuSe's sector-specific parameters for a custom choice of regions and sectors. A detailed description of the EMuSe Calibration Toolkit is presented in

Appendix A.3. An overview of the calibrated parameters of the baseline model can be found in Appendix A.2.

4 Application 1: Macroeconomic impact of different climate policy designs

Climate policy measures can have significant macroeconomic effects, the specifics of which will hinge, inter alia, on their precise design. In the following example, the baseline model presented in Section 2 is used to illustrate the macroeconomic effects of different climate policy scenarios. To this end, climate transition scenarios provided by the Network of Central Banks and Supervisors for Greening the Financial System (NGFS) are employed. These NGFS scenarios can be grouped into three broad categories: Orderly transition scenarios, disorderly transition scenarios and "hot house world" scenarios.[9] Orderly transition scenarios assume that climate policies are introduced relatively early and become gradually more stringent over time, whereas disorderly scenarios feature higher transition risks due to policies being delayed or diverging across countries and sectors. Hot house world scenarios assume that climate policies are implemented only in a few jurisdictions, and that global efforts are insufficient to halt global warming. The various scenarios deliver, inter alia, region- and country-specific trajectories for the carbon price as well as paths for emissions intensities, i.e., the emissions-to-output-ratio.

In the subsequent analysis, NGFS pathways for emissions prices and emissions intensities for an orderly and a disorderly transition scenario are fed into the baseline version of the EMuSe model,[10] which is parametrised to depict the EU and the United Kingdom (see Appendix A.2 for details).[11] In the orderly scenario, climate policy's level of intervention increases gradually intending to limit global warming to below $2°$C compared with pre-industrial levels. In case of a disorderly transition, it is, by contrast, assumed that far-reaching climate action is not implemented until 2030 – but then kicks in more severely to limit warming to below $2°$C (see Figure 2).

To interpret the results, it is helpful to keep in mind that in the EMuSe model a carbon price acts like an additional cost component of firms, which they take into account when deciding on optimal quantities (and prices).[12] Specifically, emissions costs are a product of the carbon price, the emissions intensity and output – with the two former affecting the firms' marginal

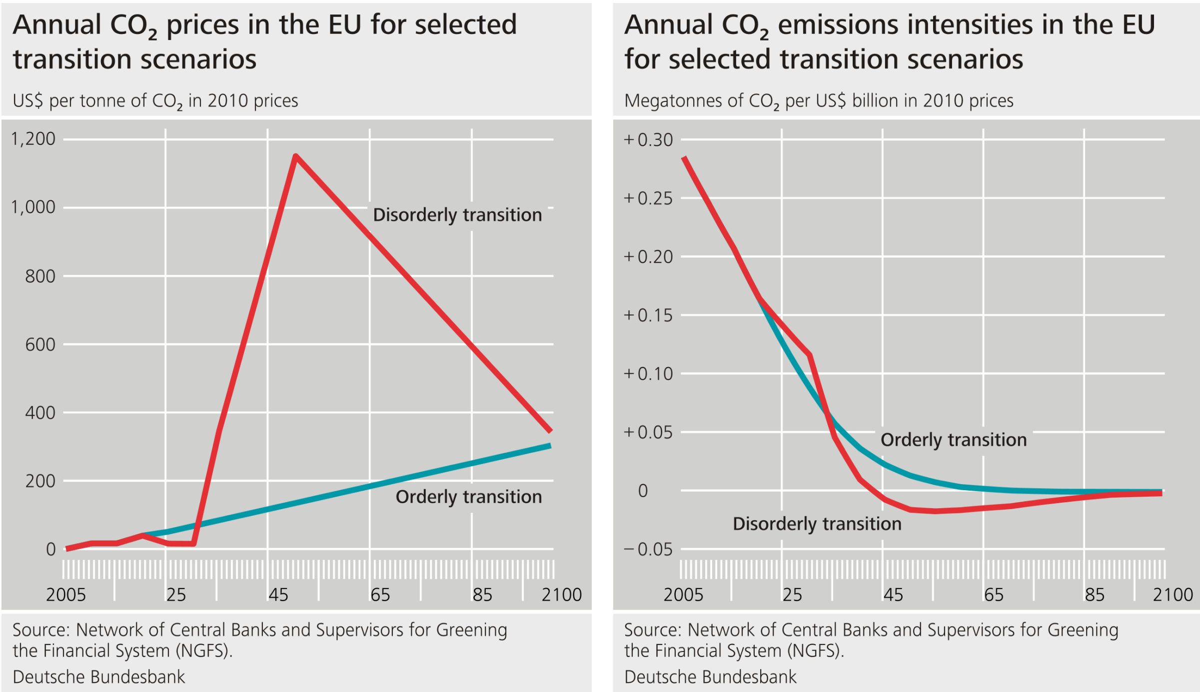
[9] NGFS transition pathways are modelled using well-established Integrated Assessment Models, which combine economic, energy, land-use and climate modules to provide coherent scenarios. Specifically, transition scenarios generated by the Potsdam Institute for Climate Impact Research's REMIND-MAgPIE model are used here. For more details see `https://www.ngfs.net/en`.

[10] In the applications presented here, we abstract from pollution damage.

[11] To this end, the carbon price as well as the emissions intensity are specified as exogenous stochastic processes. The latter might be interpreted as the development of "green technological progress", since it characterises the scenario-specific path of the emissions-to-output ratio.

[12] A model version with nominal price rigidities is presented in Section 5.

Figure 2: NGFS climate policy transition scenarios



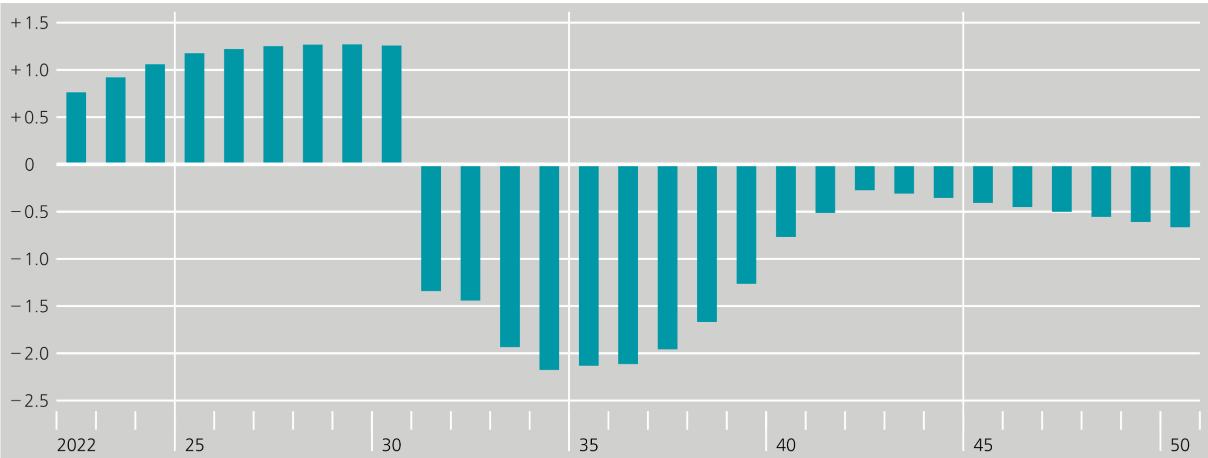
production costs. The simulations show that climate policy design can substantially impact on aggregate output (see Figure 3). In the orderly scenario, emissions costs are markedly higher in the beginning than in the disorderly transition. This difference is mainly driven by the earlier carbon price increase in the orderly transition scenario, which depresses aggregate output. From the year 2030, however, the situation reverses. The substantial increase in the carbon price, which is needed to curb global warming to less than $2°$C in the disorderly transition scenario, implies that emissions costs from that point onwards are significantly higher than they would have been under an orderly transition. Hence, aggregate output is considerably lower now than in the orderly transition scenario.

Due to its multi-sector structure, the EMuSe model also allows us to analyse the impact of the transition scenarios at the sectoral level. The simulation exercise highlights, that the burden resulting from climate policy measures can vary substantially across the sectors of the economy. Specifically, when looking at the policy-induced transition pathway in 2050, the energy sector, which has a comparatively high emissions intensity, is particularly affected by the sharp increase of the carbon price in the disorderly scenario (see Figure 4). The extent to which this development also affects other sectors depends to a large degree on the production linkages.

Figure 3: Impact of climate policies on aggregate output

Difference in real gross value added between a disorderly and an orderly transition form 2022 until 2050 in the EU*

Percentage points

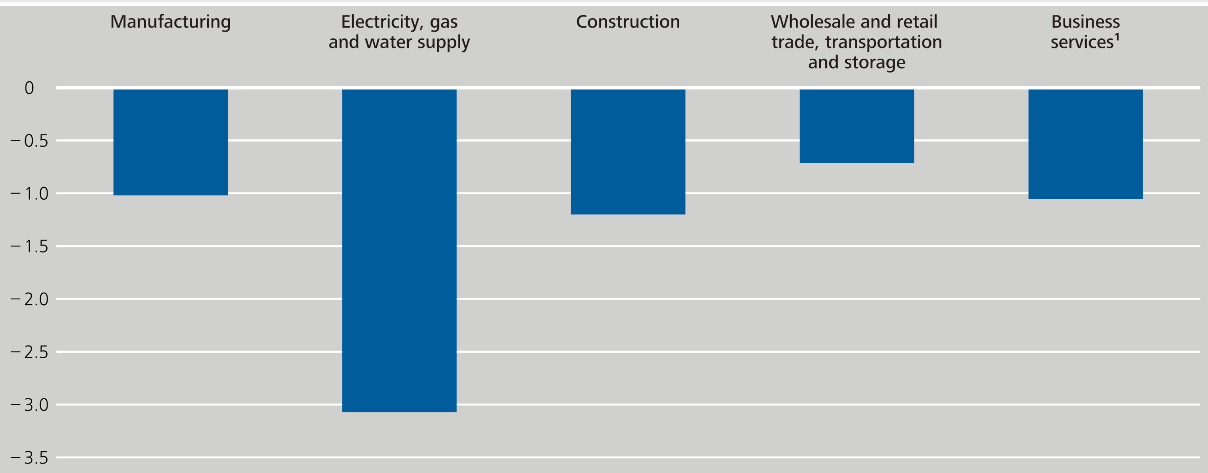


Source: Bundesbank calculations based on the DSGE model EMuSe and transition scenarios provided by the NGFS. * The chart shows the difference in real gross value added expressed in relation to the baseline between a disorderly and an orderly transition. The model is calibrated for the European Union together with the United Kingdom.
Deutsche Bundesbank

Figure 4: Impact of climate policies on sectoral output

Difference in real gross output between a disorderly and an orderly transition in 2050*

Percentage points



Source: Bundesbank calculations based on the DSGE model EMuSe and transition scenarios provided by the NGFS. * The chart shows the difference in sectoral real gross output expressed in relation to the baseline between a disorderly and an orderly transition. The model is calibrated for the European Union together with the United Kingdom. **1** NACE sections M to N: professional, scientific and technical activities as weil as administrative and support service activities.
Deutsche Bundesbank

5 Model version with price-setting friction

5.1 Overview

This section describes an extension of the baseline model, in which we include nominal price rigidities. To this end, the perfectly competitive producers in each sector $s \in \mathcal{S} = \{1, 2, .., S\}$ are replaced by a continuum of monopolistically competitive firms, indexed by $z \in [0, 1]$, which face convex price adjustment costs. In each sector, the output of the monopolistically competitive producers is bundled by a perfectly competitive firm. This model extension also features a monetary authority, which conducts monetary policy by setting the nominal interest rate according to a Taylor-type rule.[13]

5.2 Representative household

Monopolistically competitive firms are owned by the representative household. Consequently, (nominal) dividends $\Pi_t = \int_0^1 \Pi_t(z) dz$ received from these firms are an additional income source for the household. The budget constraint, in CPI-deflated real terms, is modified to reflect that additional income

$$C_t + P_t^I I_t + \frac{B_t}{P_t^C} = w_t N_t + r_t^k K_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t^C} + TR_t + \frac{\Pi_t}{P_t^C}.$$

5.3 Consumption-goods and investment-goods retailer

As in the baseline model, perfectly competitive, representative retailers bundle sector-level consumption and investment goods. These bundles are sold to the representative household.

5.4 Labour and capital

The derivation of labour and capital supply is carried out in the same way as in the baseline model.

5.5 Production

Perfectly competitive producers In a given sector, $y_{s,t}$ is produced by a representative firm that bundles the output of the monopolistically competitive firms $y_{s,t}(z)$ in a perfectly competitive environment. The production technology of that bundler is a constant returns to scale production function

$$y_{s,t} = \left[\int_0^1 y_{s,t}(z)^{\frac{\theta_s^P - 1}{\theta_s^P}} \, dz \right]^{\frac{\theta_s^P}{(\theta_s^P - 1)}},$$

[13] Following Woodford (2003), we assume that prices are measured in terms of a unit of account called "money", but the economy is cashless otherwise.

where $\theta_s^P > 1$ represents the elasticity of substitution between intermediate goods $y_{s,t}(z)$ produced by a continuum of monopolistically competitive firms. Profit maximisation leads to the following demand function:

$$y_{s,t}(z) = \left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t}}\right]^{-\theta_s^P} y_{s,t},$$

with $\tilde{P}_{s,t}(z)$ denoting the nominal price of $y_{s,t}(z)$. The price of $y_{s,t}$ is given by

$$\tilde{P}_{s,t} = \left[\int_0^1 \tilde{P}_{s,t}(z)^{1-\theta_s^P} dz\right]^{1/(1-\theta_s^P)}.$$

Monopolistically competitive firms Each good $y_{s,t}(z)$ is produced by a single monopolistically competitive firm according to the constant returns to scale production technology

$$y_{s,t}(z) = [1 - D(M_t)]\,\varepsilon_{s,t}\left[K_{s,t-1}(z)^{1-\alpha_{N,s}} N_{s,t}(z)^{\alpha_{N,s}}\right]^{\alpha_{H,s}} [H_{s,t}(z)]^{1-\alpha_{H,s}},$$

which closely follows the specification in the baseline model.

Although each firm z exerts some market power, it acts as a price taker in the factor markets. Moreover, the adjustment of the firm's nominal price $\tilde{P}_{s,t}(z)$ is assumed to be costly, where the cost function is convex in the size of the price adjustment. Following Rotemberg (1982), these costs are defined as

$$\frac{\kappa_s^P}{2}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t-1}(z)} - 1\right]^2 \tilde{P}_{s,t} y_{s,t},$$

where $\kappa_s^P \geq 0$ governs the size of price adjustment costs. The firm's optimisation problem can again be described by a two-step process. First, the firm minimises its costs $w_{s,t}N_{s,t}(z) + r_{s,t}^k K_{s,t-1}(z) + P_{s,t}^H H_{s,t}(z)$ subject to the constant returns to scale production technology. Taking factor prices as given, we obtain the first-order conditions for labour, capital and intermediate inputs as in the baseline model.

Second, convex price-adjustment costs make the firm's optimisation problem dynamic. Therefore, each firm chooses $y_{s,t}(z)$ and $\tilde{P}_{s,t}(z)$ in order to maximise its real total market value

$$\max_{y_{s,t}(z),\tilde{P}_{s,t}(z)} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{\Pi_t(z)}{P_t^C}\right]$$

subject to the demand function for intermediate goods. As in the baseline model, λ_t measures period t marginal utility of the representative household provided by an additional unit of dividend income and $\tilde{mc}_{s,t} = mc_{s,t} + P_t^{em}\kappa_s$. Firms' profits are defined as

$$\frac{\Pi_t(z)}{P_t^C} = \frac{\tilde{P}_{s,t}(z)}{P_t^C} y_{s,t}(z) - \tilde{mc}_{s,t} y_{s,t}(z) - \frac{\kappa_s^P}{2}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t-1}(z)} - 1\right]^2 \frac{\tilde{P}_{s,t}}{P_t^C} y_{s,t}.$$

Plugging in the demand for output of firm z, the dynamic optimisation problem can be expressed as

$$\max_{\tilde{P}_{s,t}(z)} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\lambda_t\Bigg\{\frac{\tilde{P}_{s,t}(z)}{P_t^C}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t}}\right]^{-\theta_s^P} y_{s,t} \\ - \tilde{mc}_{s,t}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t}}\right]^{-\theta_s^P} y_{s,t} - \frac{\kappa_s^P}{2}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t-1}(z)}-1\right]^2\frac{\tilde{P}_{s,t}}{P_t^C}y_{s,t}\Bigg\}.$$

Setting the associated first order condition equal to zero and taking expectations yields

$$(1-\theta_s^P)\frac{1}{P_t^C}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t}}\right]^{-\theta_s^P} y_{s,t} + \theta_s^P\frac{\tilde{mc}_{s,t}}{\tilde{P}_{s,t}}\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t}}\right]^{-\theta_s^P-1} y_{s,t} \\ - \kappa_s^P\left[\frac{\tilde{P}_{s,t}(z)}{\tilde{P}_{s,t-1}(z)}-1\right]\frac{1}{\tilde{P}_{s,t-1}(z)}\frac{\tilde{P}_{s,t}}{P_t^C}y_{s,t} \\ + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\kappa_s^P\left[\frac{\tilde{P}_{s,t+1}(z)}{\tilde{P}_{s,t}(z)}-1\right]\frac{\tilde{P}_{s,t+1}(z)}{\tilde{P}_{s,t}(z)^2}\frac{\tilde{P}_{s,t+1}}{P_{t+1}^C}y_{s,t+1}\right\} = 0.$$

In a symmetric equilibrium, this simplifies to

$$(1-\theta_s^P)\frac{y_{s,t}}{P_t^C} + \theta_s^P\frac{\tilde{mc}_{s,t}y_{s,t}}{\tilde{P}_{s,t}} \\ - \kappa_s^P\left(\pi_{s,t}^{PPI}-1\right)\pi_{s,t}^{PPI}\frac{y_{s,t}}{P_t^C} \\ + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa_s^P\left(\pi_{s,t+1}^{PPI}-1\right)\left(\pi_{s,t+1}^{PPI}\right)^2\frac{y_{s,t+1}}{P_{t+1}^C}\right] = 0,$$

where $\pi_{s,t}^{PPI} = \tilde{P}_{s,t}/\tilde{P}_{s,t-1}$. Simplifying the above equation further using the relations $P_{s,t} = \tilde{P}_{s,t}/P_t^C$ and $\pi_{t+1}^{CPI} = P_{t+1}^C/P_t^C$, the sector-specific pricing decision is given by

$$(1-\theta_s^P) + \theta_s^P\frac{\tilde{mc}_{s,t}}{P_{s,t}} - \kappa_s^P\left(\pi_{s,t}^{PPI}-1\right)\pi_{s,t}^{PPI} \\ + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa_s^P\left(\pi_{s,t+1}^{PPI}-1\right)\frac{\left(\pi_{s,t+1}^{PPI}\right)^2}{\pi_{t+1}^{CPI}}\frac{y_{s,t+1}}{y_{s,t}}\right] = 0.$$

5.6 Policy

As before, the fiscal authority runs a balanced budget in each period. In contrast to the baseline version, however, the presence of a monetary authority is now assumed. Specifically, monetary policy is described by a modified Taylor rule of the form

$$\ln\left(\frac{R_t^B}{R^B}\right) = \varphi_R\ln\left(\frac{R_{t-1}^B}{R^B}\right) + (1-\varphi_R)\left[\varphi_\pi\ln\left(\frac{\pi_t^{CPI}}{\pi^{CPI}}\right)\right].$$

Hence, the monetary authority gradually adjusts the short-term nominal interest rate in response to deviations of current gross inflation π_t^{CPI} from its steady state, where φ_R and φ_π are the response coefficients.

5.7 Intermediate-goods retailer

Intermediate-goods bundles are produced and traded in the same way as in the baseline model.

5.8 Market clearing and aggregation

In each sector s product market clearing implies

$$P_{s,t}y_{s,t} = P_{s,t}C_{s,t} + P_{s,t}I_{s,t} + \sum_{\tilde{s}=1}^{S} P_{s,t}H_{\tilde{s},s,t} + \frac{\kappa_s^P}{2}\left(\pi_{s,t}^{PPI} - 1\right)^2 P_{s,t}y_{s,t}.$$

At the aggregate level, it holds that

$$Y_t^{va} = C_t + P_t^I I_t.$$

In addition, the market-clearing conditions $B_t = B_{t-1} = 0$ must hold.

6 Application 2: Macroeconomic effects of extreme weather events

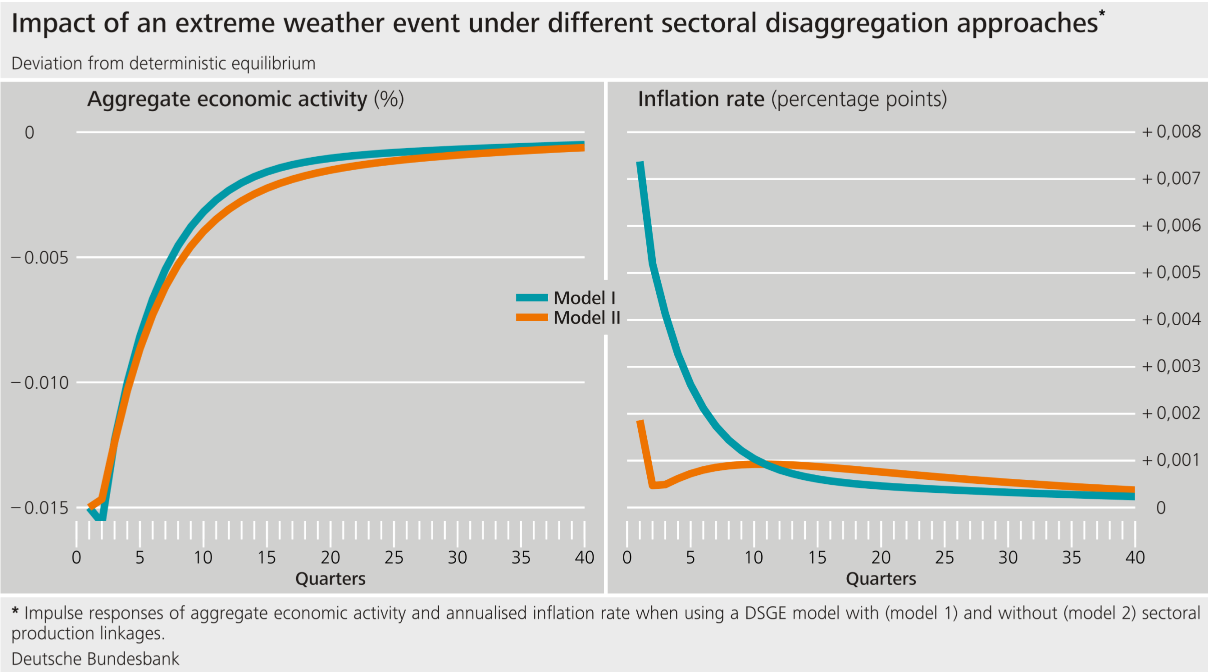
While the EMuSe model is primarily designed to assess the macroeconomic effects of transition risks, it can also be used to analyse the impact of physical risks – for example, in the form of extreme weather events. Specifically, this application illustrates the impact of a weather-induced supply shock in EMuSe and compares its impact on aggregate value added and inflation with the (quarterly) impulse response functions of a one-sector framework. The model versions, which are again parametrised to depict the EU along with the United Kingdom, feature nominal price rigidities as well as a standard Taylor-type monetary policy rule, but factor out trade linkages.[14] The extreme weather event is modelled as a temporary negative supply shock, the strength of which is specified such that when the shock occurs, value added falls by 0.015% in the first quarter (i.e., 0.06% on an annual basis) in both model versions.[15]

The impulse responses show that the degree of sectoral disaggregation can have a substantial impact on the simulated impact of physical risks. Specifically, the magnitude of the

[14] The parametrisation of the model is consistent with that in Section 4. For ease of exposition, a uniform EU monetary policy is assumed here. The response coefficients are set to $\varphi_R = 0.8$ and $\varphi_\pi = 1.5$. For illustrative purposes, we set $\kappa_s^P = 75$, while the calibration of θ_s^P draws on Christopoulou and Vermeulen (2012).

[15] This calibration is based on the estimates of Dafermos et al. (2021). In the multi-sector variant the negative supply shock is assumed to hit only the agricultural sector.

Figure 5: Impact of an extreme weather event on aggregate output



* Impulse responses of aggregate economic activity and annualised inflation rate when using a DSGE model with (model 1) and without (model 2) sectoral production linkages.
Deutsche Bundesbank

consumer price response following a weather-induced supply-side shock is markedly weaker in the model version without sectoral linkages than in the multi-sectoral variant.

7 Model version with international relations

7.1 Overview

The EMuSe model can also be used to analyse the macroeconomic implications of climate risks in an international context. The multi-region extension of the baseline model comprises $\mathcal{S} = \{1, 2, .., S\}$ production sectors and $\mathcal{C} = \{a, b, ..\}$ regions. As in the baseline model, the representative household receives income from financial assets and the provision of labour and capital. Labour and physical capital are immobile internationally and only imperfectly mobile across sectors. International capital mobility is modelled by trade in international interest-bearing assets. To further account for international linkages, it is assumed that model agents in region $co \in \mathcal{C}$ can purchase domestic output as well as goods from abroad. World population is normalised to unity such that ω^{co} indicates the (relative) population size of region co.

7.2 Representative household

In contrast to the baseline model, it is now assumed that the representative household can also purchase internationally traded nominal assets $NFA_{co,t}$. Hence, the budget constraint becomes

$$C_{co,t} + P^I_{co,t} I_{co,t} + \frac{B_{co,t}}{P^C_{co,t}} + \frac{NFA_{co,t}}{P^C_{co,t}} = w_{co,t} N_{co,t} + r^k_{co,t} K_{co,t-1} + \frac{R^B_{co,t-1} B_{co,t-1}}{P^C_{co,t}} + \frac{R^*_{co,t-1} NFA_{co,t-1}}{P^C_{co,t}} + TR_{co,t},$$

where $R^*_{co,t}$ is the gross nominal interest rate on regional holdings on net foreign assets (denominated in domestic currency), which is assumed to include a risk premium. Compared to the baseline model, the introduction of internationally traded assets leads to an additional first-order condition of the form:

$$\lambda_{co,t} = \beta \mathbb{E}_t \left(\lambda_{co,t+1} \frac{R^*_{co,t}}{\pi^{CPI}_{co,t+1}} \right).$$ [16]

7.3 Consumption-goods and investment-goods retailer

The representative household demands bundles of consumption and investment goods $C_{co,t}$ and $I_{co,t}$, which are traded at prices $P^C_{co,t}$ and $P^I_{co,t}$, respectively. The production technology of a perfectly competitive, representative retailer that bundles sector-level consumption goods, $C_{s,co,t}$, is given by

$$C_{co,t} = \left(\sum_{s=1}^{S} \psi_{C,s,co}^{1-\sigma_C} C_{s,co,t}^{\sigma_C} \right)^{\frac{1}{\sigma_C}} \quad \forall co \in \mathcal{C}.$$

As before, the parameters $\psi_{C,s,co}$ and σ_C determine the consumption utility value and the elasticity of substitution between sector-level consumption-goods. The representative consumption-goods retailer's optimisation problem can be written as

$$\max_{C_{s,co,t}} P^C_{co,t} \left(\sum_{s=1}^{S} \psi_{C,s,co}^{1-\sigma_C} C_{s,co,t}^{\sigma_C} \right)^{\frac{1}{\sigma_C}} - \sum_{s=1}^{S} \tilde{P}^C_{s,co,t} C_{s,co,t} \quad \forall co \in \mathcal{C},$$

where $\tilde{P}^C_{s,co,t}$ is the price of sectoral consumption-good $s \in \mathcal{S}$. Taking into account the bundling technology, this leads to the following first-order condition:

$$C_{s,co,t} = \psi_{C,s,co} \left(P^C_{s,co,t} \right)^{-\frac{1}{(1-\sigma_C)}} C_{co,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

[16] Depending on the denomination of the internationally traded assets (domestic vs foreign currency), the first-order condition might additionally take into account exchange rate movements.

where $P^C_{s,co,t} = \tilde{P}^C_{s,co,t}/P^C_{co,t}$. By plugging this expression into the CES-aggregator for consumption goods, it can be shown that

$$P^C_{co,t} = \left[\sum_{s=1}^{S} \psi_{C,s,co}(\tilde{P}^C_{s,co,t})^{-\frac{\sigma_C}{(1-\sigma_C)}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}} \qquad \forall co \in \mathcal{C}.$$

Hence, CPI inflation can be expressed as

$$\pi^{CPI}_{co,t} = \left[\sum_{s=1}^{S} \psi_{C,s,co}(\pi^{CPI}_{s,co,t}P^C_{s,co,t-1})^{-\frac{(\sigma_C)}{1-\sigma_C}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}} \qquad \forall co \in \mathcal{C},$$

with $\pi^{CPI}_{co,t} = P^C_{co,t}/P^C_{co,t-1}$ and $\pi^{CPI}_{s,co,t} = \tilde{P}^C_{s,co,t}/\tilde{P}^C_{s,co,t-1}$.

When international relations are included in the model, international consumption-goods retailers that combine nationally and internationally produced goods of a specific sector also exist. Specifically, a representative international retailer combines goods produced in sector s from all countries $co2 \in \mathcal{C}$, taking into account the sector-specific preference bias of region co towards goods produced in region $co2$, $hb_{C,s,co,co2}$, and the elasticity of substitution $\sigma_{C,s,co}$:

$$C_{s,co,t} = \left[\sum_{co2\in\mathcal{C}} hb^{(1-\sigma_{C,s,co})}_{C,s,co,co2} C^{\sigma_{C,s,co}}_{s,co,co2,t}\right]^{\frac{1}{\sigma_{C,s,co}}} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

with $\sum_{co2\in\mathcal{C}} hb_{C,s,co,co2} = 1$.[17]

Taking CPI-deflated sectoral prices $P_{s,co,co2,t}$ as given, the optimisation problem of the retailer can be expressed as

$$\max_{C_{s,co,co2,t}} P^C_{s,co,t}\left[\sum_{co2\in\mathcal{C}} hb^{(1-\sigma_{C,s,co})}_{C,s,co,co2} C^{\sigma_{C,s,co}}_{s,co,co2,t}\right]^{\frac{1}{\sigma_{C,s,co}}} - \sum_{co2\in\mathcal{C}} P_{s,co,co2,t}C_{s,co,co2,t},$$

which leads to the following first-order condition:

$$C_{s,co,co2,t} = hb_{C,s,co,co2}\left(\frac{P^C_{s,co,t}}{P_{s,co,co2,t}}\right)^{\frac{1}{\left(1-\sigma_{C,s,co}\right)}} C_{s,co,t}.$$

By plugging this expression into the CES aggregator, the following price index can be derived:

$$P^C_{s,co,t} = \left[\sum_{co2\in\mathcal{C}} hb_{C,s,co,co2} P^{-\frac{\sigma_{C,s,co}}{\left(1-\sigma_{C,s,co}\right)}}_{s,co,co2,t}\right]^{-\frac{\left(1-\sigma_{C,s,co}\right)}{\sigma_{C,s,co}}}.$$

[17]The indexes co and $co2$ represent the country in which the consumption good (produced in sector s) is consumed and produced, respectively. Hence, a good is produced domestically if $co2 = co$ while $co2 \neq co$ implies that a good is produced abroad.

Hence, it follows that

$$\pi^{CPI}_{s,co,t} P^{C}_{s,co,t-1} = \left[\sum_{co2 \in \mathcal{C}} hb_{C,s,co,co2} (\pi^{PPI}_{s,co,co2,t} P_{s,co,co2,t-1})^{-\frac{\sigma_{C,s,co}}{\left(1-\sigma_{C,s,co}\right)}} \right]^{-\frac{\left(1-\sigma_{C,s,co}\right)}{\sigma_{C,s,co}}},$$

with $\pi^{CPI}_{s,co,t} = P^{C}_{s,co,t} / P^{C}_{s,co,t-1}$.

The production process of the investment-good bundle $I_{co,t}$, that the household can purchase is similar to that of the consumption-good bundle. A perfectly competitive, representative investment-goods retailer transforms goods purchased from producers operating in the various sectors into a generic investment-good basket and sells it to households. The production technology is given by

$$I_{co,t} = \left(\sum_{s=1}^{S} \psi^{1-\sigma_I}_{I,s,co} I^{\sigma_I}_{s,co,t} \right)^{\frac{1}{\sigma_I}} \quad \forall co \in \mathcal{C}.$$

Thereby, the parameter $\psi_{I,s,co}$ reflects the weight of sectoral investment-good $I_{s,co,t}$ in the investment-goods basket and σ_I determines the elasticity of substitution among investment-goods from different sectors. The investment-goods retailer's optimisation problem can be written as

$$\max_{I_{s,co,t}} P^{I}_{co,t} \left(\sum_{s=1}^{S} \psi^{1-\sigma_I}_{I,s,co} I^{\sigma_I}_{s,co,t} \right)^{\frac{1}{\sigma_I}} - \sum_{s=1}^{S} P_{s,co,t} I_{s,co,t} \quad \forall co \in \mathcal{C},$$

which leads to the following first-order condition:

$$I_{s,co,t} = \psi_{I,s,co} \left(\frac{P_{s,co,t}}{P^{I}_{co,t}} \right)^{-\frac{1}{(1-\sigma_I)}} I_t \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

with $P_{s,co,t}$ and $P^{I}_{co,t}$ both denoting CPI-deflated relative prices.

By plugging this expression into the constant elasticity of substitution aggregator, the following price index for investment-goods can be derived:

$$P^{I}_{co,t} = \left[\sum_{s=1}^{S} \psi_{I,s,co} (P^{I}_{s,co,t})^{-\frac{\sigma_I}{(1-\sigma_I)}} \right]^{-\frac{(1-\sigma_I)}{\sigma_I}} \quad \forall co \in \mathcal{C}.$$

To account for international linkages we proceed analogously to consumer-goods production. The production technology of domestic investment-goods retailers is given by

$$I_{s,co,t} = \left[\sum_{co2 \in \mathcal{C}} hb^{(1-\sigma_{I,s,co})}_{I,s,co,co2} I^{\sigma_{I,s,co}}_{s,co,co2,t} \right]^{\frac{1}{\sigma_{I,s,co}}} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

with $\sum_{co2 \in \mathcal{C}} hb_{I,s,co,co2} = 1$.

Hence, the CES aggregator for each sector $s \in \mathcal{S}$ and for each country $co \in \mathcal{C}$ aggregates investment-goods from all countries $co2 \in \mathcal{C}$, weighting them by the preference parameter $hb_{I,s,co,co2}$ and taking into account the elasticity of substitution between goods produced at home ($co2 = co$) and abroad ($co2 \neq co$), which is determined by $\sigma_{I,s,co}$.[18] Taking sectoral prices $P_{s,co,co2,t}$ as given, the optimisation problem can be expressed as

$$\max_{I_{s,co,co2,t}} P^I_{s,co,t} \left[\sum_{co2 \in \mathcal{C}} hb_{I,s,co,co2}^{(1-\sigma_{I,s,co})} I_{s,co,co2,t}^{\sigma_{I,s,co}} \right]^{\frac{1}{\sigma_{I,s,co}}} - \sum_{co2 \in \mathcal{C}} P_{s,co,co2,t} I_{s,co,co2,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

The first order condition of this optimisation problem is

$$I_{s,co,co2,t} = hb_{I,s,co,co2} \left(\frac{P^C_{s,co,t}}{P_{s,co,co2,t}} \right)^{\frac{1}{\left(1-\sigma_{I,s,co}\right)}} I_{s,co,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co, co2 \in \mathcal{C}$$

with the corresponding price index

$$P^I_{s,co,t} = \left[\sum_{co2 \in \mathcal{C}} hb_{I,s,co,co2} P_{s,co,co2,t}^{-\frac{\sigma_{I,s,co}}{\left(1-\sigma_{I,s,co}\right)}} \right]^{-\frac{\left(1-\sigma_{I,s,co}\right)}{\sigma_{I,s,co}}} \qquad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

7.4 Labour and capital

As outlined above, labour and the physical capital stock are not mobile across regions. Hence, the derivation of labour and capital supply is carried out in exactly the same way as in the baseline model.[19]

7.5 Production

The production block in each region is essentially similar to that of the baseline model.

7.6 Intermediate-goods retailer

Similar to consumer goods and investment goods, intermediate inputs can be sourced both domestically and internationally. Specifically, denoting by $H_{s,j,co,t}$ the quantity of goods used in sector s of region co purchased from producers in sector j , the intermediate-input retailer's production technology is

$$H_{s,co,t} = \left(\sum_{j=1}^{S} \psi_{H,s,j,co}^{1-\sigma_{H,s,co}} H_{s,j,co,t}^{\sigma_{H,s,co}} \right)^{\frac{1}{\sigma_{H,s,co}}}, \; j \in \mathcal{S} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

[18] It holds that $\sum_{co2 \in \mathcal{C}} hb_{I,s,co,co2} = 1$.

[19] Domestic households, however, can take the detour via international assets to participate in foreign capital investments.

Hence, the CES aggregator for each sector $s \in \mathcal{S}$ and for each country $co \in \mathcal{C}$ aggregates the intermediate goods from all sectors $j \in \mathcal{S}$, after weighting them by the parameter $\psi_{H,s,j,co}$ and taking into account the elasticity of substitution between those intermediate goods. The latter is determined by $\sigma_{H,s,co}$ and may differ across countries and sectors. The intermediate-input retailer's optimisation problem can be written as

$$\max_{H_{s,j,co,t}} P^{H}_{s,co,t} \left(\sum_{j=1}^{S} \psi_{H,s,j,co}^{1-\sigma_{H,s,co}} H_{s,j,co,t}^{\sigma_{H,s,co}} \right)^{\frac{1}{\sigma_{H,s,co}}} - \sum_{j=1}^{S} P^{HH}_{s,j,co,t} H_{s,j,co,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

which leads to the following first-order condition characterising the demand for gross output from sector j used in the production of intermediate goods for sector s

$$H_{s,j,co,t} = \psi_{H,s,j,co} \left(\frac{P^{HH}_{s,j,co,t}}{P^{H}_{s,co,t}} \right)^{-\frac{1}{\left(1-\sigma_{H,s,co}\right)}} H_{s,co,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C},$$

with $P^{HH}_{s,j,co,t}$ and $P^{H}_{s,co,t}$ denoting CPI-deflated relative prices.

After plugging this expression into the constant elasticity of substitution aggregator of intermediate goods, we obtain an expression for the price of intermediate goods

$$P^{H}_{s,co,t} = \left[\sum_{j=1}^{S} \psi_{H,s,j,co} (P^{HH}_{s,j,co,t})^{-\frac{\sigma_{H,s,co}}{\left(1-\sigma_{H,s,co}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s,co}\right)}{\sigma_{H,s,co}}} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

To account for international linkages, intermediate-goods bundles purchased by domestic firms are a CES composite of the form

$$H_{s,j,co,t} = \left[\sum_{co2 \in \mathcal{C}} hb_{H,s,j,co,co2}^{(1-\sigma_{H,s,co})} H_{s,j,co,co2,t}^{\sigma_{H,s,co}} \right]^{\frac{1}{\sigma_{H,s,co}}},$$

with $\sum_{co2 \in \mathcal{C}} hb_{H,s,co,co2} = 1 \forall s, j \in \mathcal{S}$ and $\forall co \in \mathcal{C}$.

The variable $H_{s,j,co,co2,t}$ is the amount of intermediate goods used in sector $s \in \mathcal{S}$ of country $co \in \mathcal{C}$, produced in sector $j \in \mathcal{S}$ of country $co2 \in \mathcal{C}$. The parameter $hb_{H,s,j,co,co2}$ is a weight in the basket of intermediate goods destined for sector s in country co and $\sigma_{H,s,co}$ determines the elasticity of substitution between the different intermediate goods (different with respect to both sector-specific and country-specific origin).

Taking sectoral producer prices $P_{j,co,co2,t}$ as given, the optimisation problem can be expressed as

$$\max_{H_{s,j,co,co2,t}} P^{HH}_{s,j,co,t} H_{s,j,co,t} - \sum_{co2 \in \mathcal{C}} P_{j,co,co2,t} H_{s,j,co,co2,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

For each sectoral intermediate demand, we have

$$H_{s,j,co,co2,t} = hb_{H,s,j,co,co2} \left(\frac{P^{HH}_{s,j,co,t}}{P_{j,co,co2,t}} \right)^{\frac{1}{\left(1-\sigma_{HH,s}\right)}} H_{s,j,co,t} \quad \forall s \in \mathcal{S} \text{ and } \forall co, co2 \in \mathcal{C},$$

with price index

$$P^{HH}_{s,j,co,t} = \left[\sum_{co2 \in \mathcal{C}} hb_{H,s,co,co2} (P_{j,co,co2,t})^{-\frac{\sigma_{H,s,co}}{\left(1-\sigma_{H,s,co}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s,co}\right)}{\sigma_{H,s,co}}} \qquad \forall s \in \mathcal{S} \text{ and } \forall co \in \mathcal{C}.$$

7.7 Policy

As in the baseline setup, the fiscal authority sets transfers to run a balanced budget each period:

$$TR_{co,t} = P^{em}_{co,t} \sum_{s=1}^{S} Z_{s,co,t} + \frac{B_{co,t}}{P^C_{co,t}} - \frac{R^B_{co,t-1} B_{co,t-1}}{P^C_{co,t}}.$$

7.8 International linkages, market clearing and aggregation

For the ease of exposition, a two-region setup is described here, i.e. $\mathcal{C} = \{a, b\}$. It is assumed that the law of one price holds in each sector. In other words, after accounting for the exchange rate, the price of a sectoral good produced and used in country a is equal to the real exchange rate times the price of the same good that is produced in a but used in b. Formally, this is expressed as

$$P_{s,b,a,t} = rer_{ba,t} P_{s,a,a,t} \quad \forall s \in \mathcal{S},$$
$$P_{s,a,b,t} = \frac{1}{rer_{ba,t}} P_{s,b,b,t} \quad \forall s \in \mathcal{S}.$$

For the producer price inflation, the following relations have to hold:

$$\pi^{PPI}_{b,a,t} = \frac{rer_{ba,t}}{rer_{ba,t-1}} \pi^{PPI}_{a,a,t},$$
$$\pi^{PPI}_{a,b,t} = \frac{rer_{ba,t-1}}{rer_{ba,t}} \pi^{PPI}_{b,b,t}.$$

The sectoral trade balance for region co is

$$TB_{s,co,t} = \frac{P_{s,co,co,t}}{\omega^{co}} \sum_{co2 \neq co} \omega^{co2} \left(C_{s,co2,co,t} + I_{s,co2,co,t} + \sum_{j=1}^{S} H_{s,j,co2,co,t} \right) - \sum_{co2 \neq co} P_{s,co,co2,t} \left(C_{s,co,co2,t} + I_{s,co,co2,t} + \sum_{j=1}^{S} H_{s,j,co,co2,t} \right),$$

where ω^{co} indicates a region's relative size.[20] The aggregate trade balance of region co is, then, given by

$$TB_{co,t} = \sum_{s=1}^{S} TB_{s,co,t}.$$

If denominated in domestic currency, net foreign assets evolve according to

$$nfa_{co,t} = \frac{R^*_{co,t-1}}{\pi^{CPI}_{co,t}} nfa_{co,t-1} + TB_{co,t},$$

with $nfa_{co,t} = NFA_{co,t}/P^C_{co,t}$.

In line with Schmitt-Grohé and Uribe (2003), it is assumed that $R^*_{co,t}$ depends on the net foreign asset position. In particular, we follow Stähler and Thomas (2012) in assuming that the nominal interest rate is given by

$$R^*_{co,t-1} = R_{w,t-1} \exp \left[-\Psi_2 \left(\frac{nfa_{co,t-1} - nfa_{co}}{Y^{va}_{co,t-1}} \right) \right],$$

where nfa_{co} denotes the steady-state level of foreign debt, R_w is the world interest rate and $\Psi_2 > 0$.

In each sector s and region co, product market clearing implies

$$P_{s,co,t} y_{s,co,t} = P^C_{s,co,t} C_{s,co,t} + P^I_{s,co,t} I_{s,co,t} + TB_{s,co,t} + \sum_{j=1}^{S} P^{HH}_{j,s,co,t} H_{j,s,co,t}.$$

At the aggregate level, it holds that

$$Y^{va}_{co,t} = C_{co,t} + P^I_{co,t} I_{co,t} + TB_{co,t}.$$

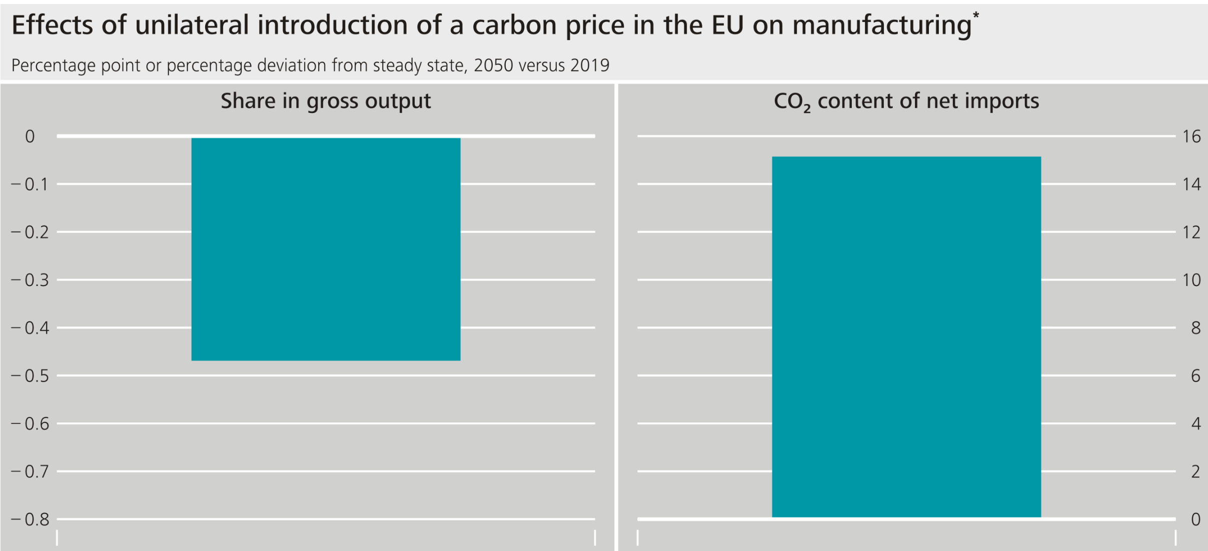
Expressed in (regional) per-capita terms, total regional emissions are given by $Z_{co,t} = \sum_{s=1}^{S} \omega^{co} Z_{s,co,t}$, world emissions by $Z_t = \sum_{co \in \mathcal{C}} Z_{co,t}$.

[20]The aggregate size of all regions is normalised to unity, i.e. $\sum_{co \in \mathcal{C}} \omega^{co} = 1$.

8 Application 3: International climate policy and its macroeconomic impact

So far, only simulations with closed-economy versions of EMuSe have been considered. However, it is not just domestic climate action that is expected to exert a considerable influence on macroeconomic developments; climate policy abroad is also likely to be highly influential. For example, unilateral policy measures could lead to goods with high emissions and energy intensity profiles increasingly being sourced from overseas – a phenomenon frequently termed "carbon leakage". This can have persistent effects on the sectoral structure of an economy, in turn affecting macroeconomic developments and the efficiency of climate policy measures. This aspect is illustrated in the subsequent application in which an open-economy version of EMuSe with two regions is employed – one calibrated to the EU and the other capturing the rest of the world. The exercise analyses the repercussions of a unilateral introduction of a carbon price in the EU, consistent with the orderly NGFS transition scenario (see Figure 2), while the rest of the world takes no climate policy action. For illustrative purposes, we focus on the manufacturing sector in the EU.

Figure 6: Unilateral implementation of climate policies



Source: Bundesbank calculations based on a two-region version of the DSGE model EMuSe and transition scenarios provided by NGFS. * The model is calibrated for the European Union (together with the United Kingdom) and for the rest of the world. The simulation period extends from 2005 to 2100. In the rest of the world, no carbon price is introduced. The path of the carbon price in the EU assumed for the simulation is based on an orderly NGFS transition scenario, in which climate policy aims to limit the rise in the global average temperature to below 2°C above pre-industrial levels.
Deutsche Bundesbank

The simulation results show that the share of gross output accounted for by the EU manufacturing sector would drop noticeably in case of a unilateral introduction of a carbon price in the EU (see Figure 6). Moreover, the right-hand panel of Figure 6 shows that this is partly because the sector's emissions intensive products are now being increasingly sourced from the rest of the world. Hence, emissions would be shifted abroad.[21]

9 Concluding remarks and way forward

Both climate change and climate policy will have far-reaching economic implications, posing new challenges to macroeconomic analysis. First, different sectors of the economy are affected differently by physical and transition risks. Second, both types of risk have an important global dimension. Hence, in order to adequately gauge the macroeconomic implications of climate change and climate policies, models with sufficient regional and sectoral differentiation are needed. The **E**nvironmental **Mu**lti-**Se**ctor DSGE model EMuSe was developed to meet these requirements. It allows the impact of climate-related adjustment processes to be analysed, taking into account both the sectoral and international dimension.

With this in mind, it should be emphasised that EMuSe is a work in progress that will be continuously developed. For example, the baseline model has recently been extended to highlight the specifics of energy, including a breakdown of energy into green and brown components (see Hinterlang et al., 2022).[22] In this context, the environmental module has also been developed further by assuming that sector-specific emissions-to-output ratios vary endogenously as a function of fossil energy input.

The aim of this paper is to present the EMuSe model and to provide suggestions for potential areas of application. By publishing the program codes including the EMuSe Calibration Toolkit, we also hope to foster a fruitful exchange with potential users in order to further advance the EMuSe-project.

[21] See also Ernst et al. (2023) on the economic implications of border adjustment mechanisms and climate clubs using a three-region version of the EMuSe model.

[22] This model version explicitly distinguishes between energy and non-energy output, both with respect to the consumption as well as intermediate-goods bundles.

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A Appendix

A.1 Derivation of the baseline model

A.1.1 The model

The appendix contains further details on the theoretical model. Subsection A.1.1 provides the derivation of the model equations, while subsection A.1.2 summarises the nonlinear equilibrium conditions.

- Households:
  The representative household chooses $\{C_t, B_t, I_t, K_t, N_t\}_{t=0}^{\infty}$ to maximise utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\psi}}{1+\psi} \right)$$

  subject to the budget constraint

$$C_t + P_t^I I_t + \frac{B_t}{P_t^C} = w_t N_t + r_t^k K_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t^C} + TR_t$$

  and the law of motion for capital

$$K_t = (1-\delta) K_{t-1} + I_t.$$

  Hence, the Lagrangian can be written as

$$\begin{aligned} \Lambda = \mathbb{E}_0 \sum_{t=0}^{\infty} \Bigg\{ & \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\psi}}{1+\psi} \right) \\ & - \beta^t \lambda_t \Bigg[C_t + P_t^I \left[K_t - (1-\delta) K_{t-1} \right] + \frac{B_t}{P_t^C} \\ & - w_t N_t - r_t^k K_{t-1} - \frac{R_{t-1}^B B_{t-1}}{P_t^C} - TR_t \Bigg] \Bigg\}. \end{aligned}$$

  The first-order conditions corresponding to this problem are

$$\Lambda_C = C_t^{-\sigma} - \lambda_t = 0,$$

$$\Lambda_N = \kappa_N N_t^{\psi} - \lambda_t w_t = 0,$$

$$\Lambda_K = \lambda_t - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{r_{t+1}^k + (1-\delta) P_{t+1}^I}{P_t^I} \right] = 0,$$

$$\Lambda_B = \lambda_t - \beta \mathbb{E}_t \left(\lambda_{t+1} \frac{R_t^B}{\pi_{t+1}^{CPI}} \right) = 0$$

and

$$\begin{aligned} \Lambda_\lambda = C_t + P_t^I \left[K_t - (1-\delta) K_{t-1} \right] + \frac{B_t}{P_t^C} \\ - w_t N_t - r_t^k K_{t-1} - \frac{R_{t-1}^B B_{t-1}}{P_t^C} - TR_t = 0. \end{aligned}$$

Finally, we impose the standard transversality conditions to guarantee that bonds and capital do not grow too quickly:

$$\lim_{t \to \infty} \beta^t \lambda_t \frac{B_t}{P_t^C} = 0,$$
$$\lim_{t \to \infty} \beta^t \lambda_t K_t = 0.$$

- Consumption-goods and investment-goods retailers:

The representative consumption-goods retailer optimisation problem is given by

$$\max_{C_{s,t}} P_t^C C_t - \sum_{s=1}^{S} \tilde{P}_{s,t} C_{s,t},$$

subject to the technology constraint

$$C_t = \left(\sum_{s=1}^{S} \psi_{C,s}^{1-\sigma_C} C_{s,t}^{\sigma_C} \right)^{\frac{1}{\sigma_C}}.$$

Therefore, the retailer's optimisation problem can be written as

$$\max_{C_{s,t}} P_t^C \left(\sum_{s=1}^{S} \psi_{C,s}^{1-\sigma_C} C_{s,t}^{\sigma_C} \right)^{\frac{1}{\sigma_C}} - \sum_{s=1}^{S} \tilde{P}_{s,t} C_{s,t},$$

which leads to the following first-order condition characterising the demand for consumption goods:

$$C_{s,t} = \psi_{C,s} P_{s,t}^{-\frac{1}{(1-\sigma_C)}} C_t,$$

with $P_{s,t} = \tilde{P}_{s,t} / P_t^C$.

By plugging this expression into the constant elasticity of substitution aggregator of consumption goods it can be shown that

$$P_t^C = \left[\sum_{s=1}^{S} \psi_{C,s} \tilde{P}_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

with $P_{s,t} = \tilde{P}_{s,t}/P_t^C$.

Hence, CPI inflation, $\pi_t^{CPI} = P_t^C/P_{t-1}^C$, can be expressed as

$$\pi_t^{CPI} = \left[\sum_{s=1}^{S} \psi_{C,s} (\pi_{s,t}^{PPI} P_{s,t-1})^{-\frac{(\sigma_C)}{1-\sigma_C}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

with $\pi_{s,t}^{PPI} = \tilde{P}_{s,t}/\tilde{P}_{s,t-1}$.

Analogously, the optimisation problem of the representative investment-goods retailer is given by

$$\max_{I_{s,t}} P_t^I I_t - \sum_{s=1}^{S} P_{s,t} I_{s,t},$$

subject to the technology constraint

$$I_t = \left(\sum_{s=1}^{S} \psi_{I,s}^{1-\sigma_I} I_{s,t}^{\sigma_I}\right)^{\frac{1}{\sigma_I}}.$$

Therefore, the retailer's optimisation problem can be written as

$$\max_{I_{s,t}} P_t^I \left(\sum_{s=1}^{S} \psi_{I,s}^{1-\sigma_I} I_{s,t}^{\sigma_I}\right)^{\frac{1}{\sigma_I}} - \sum_{s=1}^{S} P_{s,t} I_{s,t}.$$

The first-order condition corresponding to this problem is

$$I_{s,t} = \psi_{I,s} \left[\frac{P_{s,t}}{P_t^I}\right]^{-\left(\frac{1}{1-\sigma_I}\right)} I_t,$$

while the price index is given by

$$P_t^I = \left[\sum_{s=1}^{S} \psi_{I,s} (P_{s,t})^{-\frac{\sigma_I}{(1-\sigma_I)}}\right]^{-\frac{(1-\sigma_I)}{\sigma_I}}.$$

- Labour and capital supply

  The optimisation problem of the representative labour agency can be written as

$$\max_{N_{s,t}} \sum_{s=1}^{S} w_{s,t} N_{s,t} - w_t N_t,$$

  subject to the technology constraint

$$N_t = \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}}$$

  or more compactly

$$\max_{N_{s,t}} \sum_{s=1}^{S} w_{s,t} N_{s,t} - w_t \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}},$$

  which leads to the following first-order condition:

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t} \right)^{-\frac{1}{(1-\nu_N)}} N_t \quad \forall s \in \mathcal{S}.$$

  By plugging this expression into the CES aggregator of labour goods, we get the aggregate wage index:

$$w_t = \left[\sum_{s=1}^{S} \omega_{N,s} w_{s,t}^{-\frac{\nu_N}{(1-\nu_N)}} \right]^{-\frac{(1-\nu_N)}{\nu_N}}.$$

  Analogously, the optimisation problem related to the supply of capital can be written as

$$\max_{K_{s,t-1}} \sum_{s=1}^{S} r_{s,t}^K K_{s,t-1} - r_t^K K_{t-1},$$

  subject to the technology constraint

$$K_t = \left(\sum_{s=1}^{S} \omega_{K,s}^{1-\nu_K} K_{s,t}^{\nu_K} \right)^{\frac{1}{\nu_K}}.$$

  Hence, the capital agency's optimisation problem can be expressed as

$$\max_{K_{s,t-1}} \sum_{s=1}^{S} r_{s,t}^K K_{s,t-1} - r_t^K \left(\sum_{s=1}^{S} \omega_{K,s}^{1-\nu_K} K_{s,t-1}^{\nu_K} \right)^{\frac{1}{\nu_K}},$$

which leads to the following first-order condition:

$$K_{s,t} = \omega_{K,s} \left(\frac{r^K_{s,t+1}}{r^K_{t+1}} \right)^{-\frac{1}{(1-\nu_K)}} K_t \quad \forall s \in \mathcal{S}.$$

By plugging this expression into the CES aggregator of capital goods, we get:

$$r^K_t = \left[\sum_{s=1}^{S} \omega_{K,s} (r^K_{s,t})^{-\frac{\nu_K}{(1-\nu_K)}} \right]^{-\frac{(1-\nu_K)}{\nu_K}}.$$

- Production

  First, the representative firm in each sector s minimises its costs

$$w_{s,t} N_{s,t} + r^k_{s,t} K_{s,t-1} + P^H_{s,t} H_{s,t}$$

  subject to the constant returns to scale production technology

$$y_{s,t} = [1 - D(M_t)] \varepsilon_{s,t} \left(K^{1-\alpha_{N,s}}_{s,t-1} N^{\alpha_{N,s}}_{s,t} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}},$$

  where

$$D(M_t) = \gamma_0 + \gamma_1 M_t + \gamma_2 M^2_t.$$

  Therefore, the optimisation problem can be written as

$$\min_{N_{s,t}, K_{s,t-1}, H_{s,t}} w_{s,t} N_{s,t} + r^k_{s,t} K_{s,t-1} + P^H_{s,t} H_{s,t} + mc_{s,t} \left(y_{s,t} - [1 - D(M_t)] \varepsilon_{s,t} \left(K^{1-\alpha_{N,s}}_{s,t-1} N^{\alpha_{N,s}}_{s,t} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}} \right).$$

  The first-order conditions corresponding to this problem are:

$$\begin{aligned} w_{s,t} &= \alpha_{H,s} \alpha_{N,s} mc_{s,t} \frac{y_{s,t}}{N_{s,t}}, \\ r^k_{s,t} &= \alpha_{H,s} (1 - \alpha_{N,s}) mc_{s,t} \frac{y_{s,t}}{K_{s,t-1}}, \\ P^H_{s,t} &= (1 - \alpha_{H,s}) mc_{s,t} \frac{y_{s,t}}{Hs,t}. \end{aligned}$$

  Second, the representative firm chooses $y_{s,t}$ to maximise its profits

$$\max_{y_{st}} \Pi_{s,t} = P_{s,t} y_{s,t} - mc_{s,t} y_{s,t} - P^{em}_t \kappa_s y_{s,t}.$$

The first-order condition of the problem is

$$P_{s,t} = \tilde{mc}_{s,t},$$

with $\tilde{mc}_{s,t} = mc_{s,t} + \kappa_s P_t^{em}$.

- Intermediate-goods retailers:

  The optimisation problem of the representative intermediate-goods retailer can be expressed as

$$\max_{H_{s,j,t}} P_{s,t}^H H_{s,t} - \sum_{j=1}^{S} P_{j,t} H_{s,j,t},$$

subject to the technology constraint

$$H_{s,t} = \left(\sum_{j=1}^{S} \psi_{H,s,j}^{1-\sigma_{H,s}} H_{s,j,t}^{\sigma_{H,s}} \right)^{\frac{1}{\sigma_{H,s}}}.$$

Hence, the retailer solves

$$\max_{H_{s,j,t}} P_{s,t}^H \left(\sum_{j=1}^{S} \psi_{H,s,j}^{1-\sigma_{H,s}} H_{s,j,t}^{\sigma_{H,s}} \right)^{\frac{1}{\sigma_{H,s}}} - \sum_{j=1}^{S} P_{j,t} H_{s,j,t},$$

which leads to the first-order condition

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P_{s,t}^H} \right)^{-\frac{1}{\left(1-\sigma_{H,s}\right)}} H_{s,t}$$

while the price index is given by

$$P_{s,t}^H = \left[\sum_{j=1}^{S} \psi_{H,s,j} \left(P_{j,t}\right)^{-\frac{\sigma_{H,s}}{\left(1-\sigma_{H,s}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s}\right)}{\sigma_{H,s}}}.$$

- Fiscal authority

$$TR_t = P_t^{em} \sum_{s=1}^{S} Z_{s,t} + \frac{B_t}{P_t^C} - \frac{R_{t-1}^B B_{t-1}}{P_t^C}.$$

- Market clearing

  In each sector s product market clearing implies

$$P_{s,t}y_{s,t} = P^C_{s,t}C_{s,t} + P^I_{s,t}I_{s,t} + \sum_{\tilde{s}=1}^{S} P_{s,h,t}H_{\tilde{s},s,t}.$$

  At the aggregate level it holds that

$$Y^{va}_t = C_t + P^I_t I_t.$$

  In addition, the market-clearing condition $B_t = B_{t-1} = 0$ must hold.

A.1.2 Representing the equilibrium

The model is characterised by the following nonlinear difference equations. We assume that the market clearing condition for the bond market, $B_t = B_{t-1} = 0$, holds for all $t = 0, 1, 2, ...$

$$\lambda_t = C_t^{-\sigma}, \tag{1}$$

$$\kappa_N N_t^{\psi} = \lambda_t w_t, \tag{2}$$

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{r^k_{t+1} + (1-\delta)P^I_{t+1}}{P^I_t} \right], \tag{3}$$

$$\lambda_t = \beta \mathbb{E}_t \left(\lambda_{t+1} \frac{R^B_t}{\pi^{CPI}_{t+1}} \right), \tag{4}$$

$$C_t + P^I_t I_t = w_t N_t + r^k_t K_{t-1} + TR_t, \tag{5}$$

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{6}$$

$$C_{s,t} = \psi_{C,s} P_{s,t}^{-\frac{1}{(1-\sigma_C)}} C_t, \tag{7}$$

$$1 = \left[\sum_{s=1}^{S} \psi_{C,s} P_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}} \right]^{-\frac{(1-\sigma_C)}{\sigma_C}}, \tag{8}$$

$$I_{s,t} = \psi_{I,s} \left(\frac{P_{s,t}}{P^I_t} \right)^{-\left(\frac{1}{1-\sigma_I}\right)} I_t, \tag{9}$$

$$P_t^I = \left[\sum_{s=1}^{S} \psi_{I,s} (P_{s,t})^{-\frac{\sigma_I}{(1-\sigma_I)}} \right]^{-\frac{(1-\sigma_I)}{\sigma_I}}, \tag{10}$$

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t} \right)^{-\left(\frac{1}{1-\nu_N}\right)} N_t, \tag{11}$$

$$w_t = \left[\sum_{s=1}^{S} \omega_{N,s} w_{s,t}^{-\frac{\nu_N}{(1-\nu_N)}} \right]^{-\frac{(1-\nu_N)}{\nu_N}}, \tag{12}$$

$$K_{s,t} = \omega_{K,s} \left(\frac{r_{s,t+1}^K}{r_{t+1}^K} \right)^{-\left(\frac{1}{1-\nu_K}\right)} K_t, \tag{13}$$

$$r_t^K = \left[\sum_{s=1}^{S} \omega_{K,s} (r_{s,t}^K)^{-\frac{\nu_K}{(1-\nu_K)}} \right]^{-\frac{(1-\nu_K)}{\nu_K}}, \tag{14}$$

$$y_{s,t} = [1 - D(M_t)] \varepsilon_{s,t} \left(K_{s,t-1}^{1-\alpha_{N,s}} N_{s,t}^{\alpha_{N,s}} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}}, \tag{15}$$

$$D(M_t) = \gamma_0 + \gamma_1 M_t + \gamma_2 M_t^2, \tag{16}$$

$$w_{s,t} = \alpha_{H,s} \alpha_{N,s} mc_{s,t} \frac{y_{s,t}}{N_{s,t}}, \tag{17}$$

$$r_{s,t}^k = \alpha_{H,s} (1 - \alpha_{N,s}) mc_{s,t} \frac{y_{s,t}}{K_{s,t-1}}, \tag{18}$$

$$P_{s,t}^H = (1 - \alpha_{H,s}) mc_{s,t} \frac{y_{s,t}}{Hs,t}, \tag{19}$$

$$P_{s,t} = \tilde{m}c_{s,t}, \tag{20}$$

$$\tilde{m}c_{s,t} = mc_{s,t} + \kappa_s P_t^{em}, \tag{21}$$

$$Z_{s,t} = \kappa_s y_{s,t}, \tag{22}$$

$$M_t = (1 - \rho^M) M_{t-1} + \sum_{s=1}^{S} Z_{s,t} + Z_t^*, \tag{23}$$

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P^H_{s,t}} \right)^{\left(-\frac{1}{1-\sigma_{H,s}}\right)} H_{s,t}, \tag{24}$$

$$P^H_{s,t} = \left[\sum_{j=1}^{S} \psi_{H,s,j} \left(P_{j,t}\right)^{-\frac{\sigma_{H,s}}{\left(1-\sigma_{H,s}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s}\right)}{\sigma_{H,s}}}, \tag{25}$$

$$TR_t = P^{em}_t \sum_{s=1}^{S} Z_{s,t}, \tag{26}$$

$$P_{s,t} y_{s,t} = P_{s,t} C_{s,t} + P_{s,t} I_{s,t} + \sum_{\tilde{s}=1}^{S} P_{s,h,t} H_{\tilde{s},s,t} \tag{27}$$

and

$$Y^{va}_t = C_t + P^I_t I_t. \tag{28}$$

A.2 Calibration details

The model parameters can be partitioned into three subsets. The first comprises the specification of general parameters related to the aggregate economy, mainly taken from the literature. The second set of parameters captures heterogeneity on the production side by allowing for sector-specific factor intensities, input-output linkages, price rigidities and contributions to final demand. The final group of parameters, namely the carbon intensities, refers to the environmental module of the model. We calibrate the baseline model to the EU27 countries plus the UK.

General parameters The baseline model is calibrated to the annual frequency. We set the discount factor to $\beta = 0.992^4$, which implies an annual interest rate of 3.3%. The intertemporal elasticity of substitution is fixed at a standard value of $\sigma_c = 2$. Along the lines of Coenen et al. (2013), the Frisch elasticity of labour supply is calibrated to 0.5 (i.e. $\psi = 2$). The relative weight of the disutility of labour is set to $\kappa_N = 32.881$ in order to match a targeted aggregate labour supply of $\bar{N} = 0.33$. We assume an annual depreciation rate of 10%, which is a standard choice in the literature (see, for example, Cooley and Prescott, 1995).

Substitution elasticities for goods produced in the different sectors are set as follows. For the consumption basket, we follow Baqaee and Farhi (2019) and choose $\sigma_C = 1 - 1/0.9091$. As regards the investment goods basket it is assumed that $\sigma_I = 1 - 1/0.7511$. For intermediate inputs, we follow Bouakez et al. (2023) and Atalay (2017) by choosing $\sigma_{H,s} = 1 - 1/0.100$. For the substitution elasticities of labour and capital, we follow Bouakez et al. (2023) and set $\nu_N = \nu_K = 2.000$. Table 1 summarises our baseline calibration of general parameters.

Table 1: Baseline calibration of general parameters

Variable/Parameter	Symbol	Value
Discount factor	β	0.992^4
Elasticity of intertemporal substitution	σ	2.000
Inverse of Frisch elasticity of lab. supply	ψ	2.000
Labour disutility scaling	κ^N	32.881
Capital depreciation rate	δ^k	0.100
Substitution elasticities:		
Elasticity of substitution, consumption	σ_C	1−1/0.9091
Elasticity of substitution, investment	σ_I	1−1/0.7511
Elasticity of substitution, intermediates	$\sigma_{H,s}$	1−1/0.1000
Elasticity of substitution, labour	ν_N	2.000
Elasticity of substitution, capital	ν_K	2.000

Notes: The table shows calibrated values for general parameters as described in the main text.

Sector-specific production parameters On the production side of the economy, we distinguish between $S = 10$ sectors, relying on the standard NACE Rev. 2 classification (see Table 2).

Table 2: Sector specification

Sector	
1	Agriculture, forestry and fishing (NACE section A)
2	Mining and quarrying (NACE section B)
3	Manufacturing (NACE section C)
4	Electricity, gas, steam and air conditioning supply (NACE section D)
5	Water supply; sewerage, waste management and remediation activities (NACE section E)
6	Construction (NACE section F)
7	Wholesale and retail trade, transportation and storage, accommodation and food service activities (NACE sections G–I)
8	Information and communication (NACE section J)
9	Professional, scientific, technical, administration and support service activities (NACE sections M–N)
10	Other services (NACE sections R–S)

Notes: The table summarises the sector specification of the baseline model, which relies on the NACE Rev. 2 classification (see Eurostat, 2008).

We allow for several heterogeneities across sectors. Labour and capital are not perfectly mobile across sectors, represented by $\omega_{N,s}$ and $\omega_{K,s}$, respectively. Furthermore, the production technology of intermediate-goods producers differs across sectors as we allow for heterogeneous factor intensities for labour, capital and intermediate inputs. Moreover, all sectors contribute differently to final demand. For each sector s, these parameters are derived using 2005 data from the most recent release of the World Input-Output Database (WIOD) (see Timmer et al., 2015).[23] It includes data on socioeconomic accounts as well as input-output tables

[23]Our calibration toolkit allows us to extract and aggregate WIOD data for a custom choice of years, country and

for 56 sectors and 43 countries. We build an aggregate over the 27 European Union countries plus the UK. While the socioeconomic accounts help us to pin down $\omega_{N,s}$, $\omega_{K,s}$, $\alpha_{N,s}$ and $\alpha_{H,s}$, we can use the provided input-output tables to match inter-sectoral trade shares, $\psi_{H,s,j}$, as well as the sectoral shares in the consumption and investment good bundles, $\psi_{C,s}$ and $\psi_{I,s}$, respectively (see Table 3 and Table 4). In order to determine sector-specific labour and capital supply, we first sum up the number of persons engaged and the nominal capital stock over all sectors, and then compute the respective shares. Dividing the amount of intermediate inputs by gross output per industry yields the factor intensities for intermediate inputs, $1 - \alpha_{H,s}$. In combination with the share of gross output that flows into labour compensation, we can fix the values for $\alpha_{N,s}$.

Table 3: Baseline calibration of sector-specific parameters

Sector	$\alpha_{N,s}$	$\alpha_{H,s}$	$\omega_{N,s}$	$\omega_{K,s}$	$\psi_{C,s}$	$\psi_{I,s}$
1	0.672	0.468	0.084	0.064	0.037	0.005
2	0.260	0.611	0.005	0.018	0.001	0.000
3	0.620	0.304	0.222	0.233	0.352	0.299
4	0.297	0.375	0.008	0.072	0.034	0.003
5	0.464	0.428	0.009	0.054	0.018	0.000
6	0.713	0.370	0.101	0.086	0.011	0.475
7	0.652	0.511	0.336	0.260	0.394	0.085
8	0.552	0.513	0.036	0.061	0.065	0.071
9	0.659	0.549	0.142	0.106	0.029	0.054
10	0.679	0.591	0.057	0.046	0.060	0.006

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The values were computed using 2005 data from the latest vintage of the WIOD.

Parameters $\psi_{H,s,j}$ describe the share of intermediate inputs consumed by sector s that are produced by sector j. To obtain these, we first compute the total sum of intermediate inputs for each sector and then the respective shares of the producing sectors, using the input-output tables. Relying on WIOD's national accounts data, the distribution of final consumption expenditure by households and gross fixed capital formation across sectors can be derived, giving us the CES bundle shares $\psi_{C,s}$ and $\psi_{I,s}$. To facilitate calculations, we normalise relative prices to one in the initial steady state.

Environmental parameters Sector-specific CO2 emissions per unit of output are calibrated using environmental accounts provided by the European Commission that are consistent with the WIOD (see Corsatea et al., 2019). While information on sectoral emissions is available from 2000–2016, we take values from 2014, since the WIOD series on gross output ends in this period and we approximate carbon intensities by dividing emissions by gross output.

sector specifications.

Table 4: Input-Output Matrix

	$\psi_{H,1,j}$	$\psi_{H,2,j}$	$\psi_{H,3,j}$	$\psi_{H,4,j}$	$\psi_{H,5,j}$	$\psi_{H,6,j}$	$\psi_{H,7,j}$	$\psi_{H,8,j}$	$\psi_{H,9,j}$	$\psi_{H,10,j}$
$\psi_{H,s,1}$	0.285	0.004	0.056	0.003	0.002	0.002	0.013	0.001	0.003	0.006
$\psi_{H,s,2}$	0.002	0.129	0.020	0.148	0.003	0.006	0.002	0.001	0.001	0.001
$\psi_{H,s,3}$	0.390	0.309	0.554	0.163	0.267	0.414	0.232	0.200	0.137	0.184
$\psi_{H,s,4}$	0.025	0.081	0.025	0.420	0.044	0.006	0.025	0.015	0.013	0.036
$\psi_{H,s,5}$	0.011	0.012	0.013	0.011	0.222	0.004	0.008	0.004	0.008	0.018
$\psi_{H,s,6}$	0.015	0.051	0.010	0.041	0.053	0.297	0.032	0.020	0.023	0.036
$\psi_{H,s,7}$	0.191	0.223	0.200	0.108	0.149	0.128	0.423	0.146	0.146	0.155
$\psi_{H,s,8}$	0.008	0.026	0.019	0.022	0.042	0.018	0.061	0.345	0.130	0.104
$\psi_{H,s,9}$	0.067	0.155	0.099	0.077	0.206	0.119	0.191	0.231	0.516	0.233
$\psi_{H,s,10}$	0.006	0.010	0.005	0.006	0.013	0.006	0.013	0.037	0.022	0.226

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The values were computed using 2005 data from the latest vintage of the WIOD.

The stock of pollution decays linearly at a rate of $1 - \rho^{EM} = 0.9979$ as in Heutel (2012) and Annicchiarico and Di Dio (2015). In the applications presented here, we abstract from pollution damage. Hence, the parameters of the damage function are set to zero.

Table 5: Baseline calibration of sector-specific emissions intensity

Sector	κ_s
1	0.192
2	0.402
3	0.153
4	2.363
5	0.130
6	0.026
7	0.157
8	0.013
9	0.020
10	0.032

Notes: The table shows calibrated values for the sector-specific emissions intensity. The values were computed using 2005 data from the latest vintage of the WIOD and the WIOD-consistent environmental accounts provided by the European Commission (see Corsatea et al., 2019).

A.3 The EMuSe Calibration Toolkit

A.3.1 Introduction

The EMuSe Calibration Toolkit is a MATLAB-based routine that allows the user to pin down most of EMuSe's sector-specific production parameters for a custom choice of regions and sectors. The routine extracts and aggregates data from the most recent release of the

WIOD (see Timmer et al., 2015) and the Environmental Accounts published by the European Commission (see Corsatea et al., 2019). The WIOD includes data on socioeconomic accounts (SEA) as well as input-output (IO) tables for 43 countries and 56 sectors, relying on the International Standard Industrial Classification revision 4 (ISIC Rev. 4). The first level and the second level of ISIC Rev. 4 (sections and divisions) are identical to sections and divisions of NACE Rev. 2 (see Eurostat, 2008). Since the sectoral breakdown in EMuSe only goes up to the second level, we find this correspondence to hold for the model and label the model sectors according to the *NACE* classification. The WIOD-data can be used to parametrise sector-specific contributions to final demand, capital and labour shares, factor intensities, inter-sectoral trade linkages as well as preference biases regarding the origin of the goods produced. Observations are available for the years 2000–2014. From the Environmental Accounts, the calibration toolkit uses information on sectoral CO2 emissions obtainable for the period 2000–2016. These are used to fix sector-specific emission intensities.

The toolkit comprises the main file *Main_EMuSe_Calibration_Toolkit.m* and two subfolders *Data* and *Functions*. All parts are described in more detail in the following.

A.3.2 Data

The folder *Data* contains several MAT-files including WIOD data and the environmental accounts, which were imported from respective Excel spreadsheets using MATLAB's import function. *WIOD_SEA_dat* covers all data for all years from the SEA and *WIOD_Exchange_rates* contains exchange rates provided by WIOD that are necessary to convert some SEA variables into uniform units (USD). Both are given in table format. World-wide input-output tables and national accounts (NA) for each year are saved as cell arrays in the subfolder *WIOT_mat*. The MAT-file*CO2_emissions* captures sectoral CO2 emissions from the Environmental Accounts in a structure format with a table for each country, covering all years and sectors.

A.3.3 *Main_EMuSe_Calibration_Toolkit.m*

In the following, the MATLAB script $Main\_EMuSe\_Calibration\_Toolkit.m$ is described. The script consists of eight building blocks:

1. In the first block, the two subfolders are added to the current MATLAB path. Moreover, users need to add the path for *WIOT_mat_files* and specify in which years they are interested by adjusting the variables *year_codes*, *years* and the starting year *year* accordingly.[24] Note that selecting only one year shortens computational time substantially since much less data has to be loaded.

[24] This choice only affects IO and NA tables. SEA data and emission intensities are always computed for all available years.

2. The second block loads the data. No changes have to be made here.

3. In the third block, users select up to four regions. In line 58, they have to set *all_countries* dummy to 1 if all WIOD countries are included in the regions. For example, this is the case if one of the regions represents the rest of the world. Otherwise, the dummy is set to zero. In the following lines, users have to define ISO_a, ISO_b, ISO_c, ISO_d using the respective country codes that can also be found in the variable *ISO_SEA*.[25] The variable *ISO_regions* contains the region names as they appear in the final calibration MAT-file. Note that the names in *ISO_regions* cannot be changed, only the respective number of regions has to be adjusted. An example for three regions with Germany, the rest of the euro area and the rest of the world is given in the code.

4. The fourth block concerns the sector choice. The *all_sectors* dummy has to be set to 1 if all 54 sectors are to be modelled. For a description of the NACE codes, see $NACE\_55\_descr$.[26] If the dummy is set to 0, the users have to define the variable $NACE$ in line 127. They can select pre-specified sector aggregates from $NACE\_available$. If users want different sector aggregates than the ones already implemented, they have to add them in the functions $extract\_SEA\_sectors.m$, $extract\_WIOD\_sectors.m$ and $extract\_and\_aggregate\_emissions.m$ accordingly.[27]

5. The fifth block calls the functions $extract\_SEA\_(all)\_sectors.m$ and $aggregate\_countries\_SEA.m$. The former extracts the data from the SEA for the included countries and sectors. The latter aggregates countries into regions and computes $\alpha_{N,s}$, $\alpha_{H,s}$, $\tilde{\omega}_{N,s}$, $\tilde{\omega}_{K,s}$ (see more details below). No changes have to be made here (except when new sector aggregates are added). It writes results into $Calibration.SEA$.

6. The sixth block calls the functions $extract\_WIOD\_(all)\_sectors.m$ and $aggregate\_countries\_WIOD.m$. The former extracts the data from the SEA for the included countries and sectors. The latter aggregates countries into regions and computes $\psi_{C,s}$, $\psi_{I,s}$, $\psi_{H,s,j}$, $hb_{X,s,i,\bar{i}}$ $X \in \{C, I, H\}$ (see more details below). No changes have to be made here (except when new sector aggregates are added). The function writes the results into $Calibration.WIOD$. Moreover, it adds the country codes to the final calibration MAT-file in order to have an overview list of which country is included in which region.

7. The script $Compute\_means$ in block seven computes parameter averages over the years chosen in block one. No changes have to be made here.

[25]Note that all four regional ISO variables have to be defined. If you only want to model three regions, set $ISO\_d = \{\};$. Moreover, note that the IO tables also include the rest of the world (ROW), while the SEA do not.

[26]Note that we exclude the NACE sections T and U (activities of households and extraterritorial organisation and bodies) since these are not available for all countries.

[27]When choosing the sector aggregates in $NACE$, one should keep the same order as they appear in $NACE\_available$.

8. Block eight runs the $Main\_climate.m$ script, which calls the function $extract\_and\_aggregate\_emissions.m$, loading and aggregating CO2 emissions data and computing emission intensities $carb\_int_s$ (see more details below). No changes have to be made here (except when new sector aggregates are added). It writes results into $Calibration.Emissions$.

A.3.4 Functions

The folder $Functions$ contains scripts and subfunctions that are called in the main file. They operate as follows.

$extract\_SEA\_(all)\_sectors.m$

- Inputs: Imported data given in $WIOD\_SEA\_dat.mat$, ISO country code (one country at a time is extracted)
- Output: Structure for each sector, containing the variables value added (VA), gross output (GO), intermediate inputs (II), labour compensation (LAB), number of persons engaged (EMP) and capital stock (K).

$aggregate\_countries\_SEA.m$

- Inputs: Regions and sector specifications ($ISO\_a, ISO\_b, ISO\_c, ISO\_d,$ $ISO\_regions, NACE$), extracted country-specific SEA data ($Data$), exchange rates ($Exchange$)
- Output: Structure for each region and each sector with aggregated values and parameters for all 15 years
- Computations:
  - ▷ Variables with units in national currencies are transformed in USD using $Exchange$ at the beginning
  - ▷ $\alpha_{H,s} = 1 - II_s/GO_s$
  - ▷ $\alpha_{N,s} \cdot \alpha_{H,s} = LAB_s/GO_s$
  - ▷ $\alpha_{N,s} = \alpha_{N,s} \cdot \alpha_{H,s}/\alpha_{H,s}$
  - ▷ $\tilde{\omega}_{N,s} = EMP_s/\sum_s EMP_s$
  - ▷ $\tilde{\omega}_{K,s} = K_s/\sum_s K_s$
  - ▷ $size_s = VA_s/\sum_s VA_s$ (only informational, not used in EMuSe code)

$extract\_WIOD\_(all)\_sectors.m$

- Inputs: Imported data given in $WIOD\_all.mat$ (for a specific year), regions and sector specifications (ISO , $NACE$/ $NACE\_available$)
- Output: Structure for each country, containing the sector-specific values of the national accounts (e.g. consumption and investment) and input-output tables

$aggregate\_countries\_WIOD.m$

- Inputs: Regions and sector specifications ($ISO\_all, ISO\_a, ISO\_b, ISO\_c, ISO\_d, ISO\_regions, NACE$), extracted data ($Data$)
- Output: Structure for each region and year with aggregated values and parameters
- Computations:
  - ▷ $\psi_{X,s} = X_s / \sum_s X_s$, $X \in \{C, I, G\}$
  - ▷ $\psi_{H,s,j} = H_{s,j} / \sum_s H_{s,j}$
  - ▷ Home biases as the share of X, $X \in \{C, I, G, H\}$ that is produced domestically

$Main\_climate.m$

- Script that loads CO2 emissions data, calls the function $extract\_and\_aggregate\_emissions.m$ and computes emissions intensities for the years 2000–2014.
- $extract\_and\_aggregate\_emissions.m$:
  - Inputs: Imported emissions data given in $CO2.mat$, regions and sector specifications ($ISO\_a, ISO\_b, ISO\_c, ISO\_d, ISO\_regions, all\_sectors, NACE, NACE\_available$)
  - Output: Structure with table for each region that reports CO2 emissions for chosen sectors
- Computations:
  - ▷ $carb\_int_s = Z_s/GO_s$ for constant fraction of output or
  - ▷ $carb\_int_s = 1 - \ln(Z_s)/\ln(GO_s)$ for isoelastic form

A.4 The EMuSe Simulation Toolkit

The EMuSe Simulation Toolkit is a Dynare-based routine that allows the user to runs specific simulations. All results were produced using MATLAB 2019b and Dynare 4.5.7. For higher versions of Dynare, please note that the macro language has to be adapted.

In this section, we outline how to reproduce the main results of the application described in Section 4. Specifically, we describe how to perform simulations of the 10-sector, 1-region economy ("_10S_1C"). The routines running other versions/applications of the EMuSe model are analogous. In order to run customised versions of EMuSe with different sectors and/or regions, the mod-files have to be adjusted accordingly.

The file "*Main_orderly_vs_disorderly.m*" in the first folder calls the plotting routines to reproduce the figures shown in Section 4. The subfolders "Orderly" and "Disorderly" contain the model files to run the simulations. More specifically, the directory "EMuSe" in the mentioned subfolders contains the following scripts:

- "*Main_MS_Model.m*": after some housekeeping, loads the calibration data (obtained through the Calibration Toolkit), assigns the sector names (and regions) of the data to those of the model, calls Dynare files to derive the steady state and undertake simulations and calls plotting routines.
- "*SS_MSMod_10S_1C.mod*": derives the steady state of the 10 sector-model; the subfiles "*make_getk_text.m*" and "*make_getk_SS_text.m*" are needed to derive the function "*getk.m*" that calculates the steady states of the capital stock and the intermediate inputs of the model.
- "*MS_1C_Mod.mod*": derives the steady state of the model by making use of "*Vars_params_10S_1C.m*" (which defines variables and parameters), "*SS_MSMod_10S_1C.mod*" (which derives the steady state), "*Equations_10S_1C.mod*" (which contains the model equations) and "*Init_Val_MSMod_10S_1C.mod*" (which assigns the initial steady state-values to the model variables); the Dynare file calculates the steady state and performs model checks as well as a stochastic simulation.
- "*MS_1C_Mod_simul.mod*": uses the same files as "*MS_1C_Mod.mod*" but now performs a deterministic perfect-foresight simulation as described in Section 4.
- "*Save_Results.m*": saves the results of the perfect-foresight simulation in a corresponding folder.
- "*Figure_Simul_MS_PerfForesight.m*": plots selected results of the perfect-foresight simulation.