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First-order and higher-order inflation expectations: Evidence about households and firms

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Non-technical summary

Research Question

Individuals' expectations about the future value of goods and services play an important role in all economic settings. Yet, given the dynamic nature of interactions in markets, market participants do not only need to estimate fundamental values, but also others' expectations regarding these values. These expectations can vary substantially across individuals or entities, because of different information sets or because of alternative views about the development of the economy. Therefore, higher-order expectations, i.e., the anticipation of the expectations of others, are an important strategic component that is increasingly considered in macroeconomic models. This paper examines the relation between first-order and higher-order inflation expectations of households and firms.

Contribution

We study first-order and higher-order inflation expectations of German households and firms elicited from surveys. In particular, we shed light on the relation between different orders of beliefs and derivate implications for noisy-information models with infinite regress. Moreover, since the elicited data is identical for households and firms, we also provide evidence about whether the relation regarding first-order and higher-order beliefs is identical between the two samples.

Results

First, we find that the average higher-order inflation forecast is almost identical to the average first-order inflation forecast for both households and firms. In general, our data show a positive correlation between first-order and higher-order expectations for households and firms. However, the correlation is not perfect and more pronounced for firms than for households. Second, we show that the cross-sectional standard deviation of higher-order beliefs (disagreement) is substantial in our sample and that the disagreement in first-order beliefs is on average smaller than the disagreement in higher-order beliefs about inflation. Third, the average uncertainty (i.e., the standard deviation of individuals' probabilities across inflation bins) around households' and firms' own inflation forecast is significantly lower than their uncertainty around higher-order beliefs about inflation. This suggests that uncertainty accumulates as individuals extrapolate from their beliefs to what others might believe.

Nichttechnische Zusammenfassung

Fragestellung

Individuelle Erwartungen des künftigen Wertes von Waren und Dienstleistungen spielen in allen Wirtschaftsbereichen eine wichtige Rolle. Dabei müssen Marktteilnehmer jedoch nicht nur die fundamentalen Werte schätzen, sondern auch die Erwartungen anderer in Bezug auf diese Werte. Diese Erwartungen können sich zwischen Personen oder Unternehmen unterscheiden, z. B. weil sie über unterschiedliche Informationen verfügen oder weil sie andere Ansichten über die Entwicklung der Wirtschaft haben. Daher sind Erwartungen höherer Ordnung, d.h. Erwartungen über die Erwartungen anderer, eine wichtige strategische Komponente, die in makroökonomischen Modellen zunehmend Beachtung findet. Dieser Beitrag beleuchtet die Beziehung zwischen Inflationserwartungen erster und höherer Ordnung von Haushalten und Unternehmen.

Beitrag

Wir untersuchen Inflationserwartungen erster und höherer Ordnung von deutschen Haushalten und Unternehmen, ermittelt durch Umfragen. Dabei beleuchten wir die Beziehung zwischen den Erwartungen unterschiedlicher Ordnungen und leiten Implikationen für Noisy-Information-Modelle ab. Da die erhobenen Daten für Haushalte und Unternehmen identisch sind, liefern sie zudem Hinweise auf die Beziehung verschiedener Ordnungen zwischen den beiden Stichproben.

Ergebnisse

Wir finden, dass die Inflationsprognose höherer Ordnung sowohl für Haushalte als auch für Unternehmen mit der Inflationsprognose erster Ordnung positiv korreliert und im Schnitt identisch ist. Die Korrelation ist jedoch nicht perfekt und bei Unternehmen stärker ausgeprägt als bei Haushalten. Zweitens zeigen wir, dass die Standardabweichung der Erwartungen höherer Ordnung in unserer Stichprobe erheblich ist und über der Standardabweichung der ersten Ordnung liegt. Drittens ist die durchschnittliche Unsicherheit (d.h., die Standardabweichung der individuellen Wahrscheinlichkeiten über die Inflationsbereiche hinweg) bei den Inflationsprognosen der Haushalte und Unternehmen deutlich geringer als die Unsicherheit bei den Inflationsprognosen höherer Ordnung. Dies deutet darauf hin, dass die Unsicherheit sich akkumuliert, wenn von der ersten zur höheren Ordnung gewechselt wird.

First-Order and Higher-Order Inflation Expectations: Evidence about Households and Firms

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Abstract: We study first-order and higher-order inflation expectations of German households and firms elicited from surveys. The data allows to shed light on the relation between different orders of beliefs, and to derive implications for noisy-information models with infinite regress. Moreover, since the elicited data is identical for households and firms, it also allows studying whether the relation between first-order and higher-order beliefs differs between the two samples. While we find that this relation is mostly identical between households and firms in our data, we identify differences to previously elicited data in the literature. We discuss potential sources for these differences and their theoretical implications.

Keywords: Inflation expectations, higher-order beliefs, noisy-information models, surveys

JEL: D84, E31, G17

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1. Introduction

Individuals' expectations about the future value of goods and services play an important role in all economic settings. Given the dynamic nature of interactions in markets, market participants often do not only need to estimate fundamental values, but also others' expectations regarding these values. These expectations can vary substantially across individuals or entities, because of different information sets or because of alternative views about the development of the economy.

Higher order expectations, i.e., the anticipation of the expectations of others, are therefore an important strategic component central to many economic scenarios, and have become increasingly emphasized and studied in macroeconomic and finance contexts. For instance, Bacchetta and Van Wincoop (2008) demonstrate that the difference between higher-order and own expectations is important for the link between current asset prices and investors' expectations about future assets' payoffs. Woodford (2002) shows how noisy private information can lead firms to change their prices very gradually because of the slow-moving higher-order beliefs about the actions of other firms. Nimark (2008) highlights the role of higher-order inflation expectations in pricing decisions for generating inflation inertia, i.e. the fact that aggregate inflation responds only gradually to shocks while price changes for goods are quite large. More recent work has emphasized that incorporating higher-order uncertainty in a New Keynesian model reduces the effectiveness of forward guidance which helps to resolve the "forward guidance" puzzle (e.g., Angeletos and Lian 2018; Farhi and Werning 2019; Gabaix 2020). Considering how important higher-order expectations of macroeconomic variables are for monetary policy, surprisingly few studies have explored them empirically until recently – presumably due to data constraints.

This study aims to further our understanding of higher-order macroeconomic expectations by utilizing two large representative surveys of German firms' and households' inflation expectations. The surveys ask households and managers not only about their own expectations (i.e., their first-order expectations), but also what they believe others expect (i.e., their higher-order expectations). Jointly, these surveys allow us to uncover the relationships between first-order and higher-order macroeconomic beliefs that can be used to discipline and test models of higher-order expectations. Using the inflation expectations of firm managers from New Zealand, Coibion et al. (2021, henceforth CGKR) were the first to study these relationships and formulate their results as a set of stylized facts. We test the robustness of these stylized facts in a much larger sample from a much larger economy, and for a different time frame. Additionally,

we provide the first empirical analysis of the relation between first- and higher-order inflation expectations for households. This allows us to document commonalities in how firms and households form higher-order expectations, which helps to guide the empirical search for the determinants and formation of inflation expectations and how these expectations affect financial decisions.

Our analysis is based on data from the Deutsche Bundesbank, in particular the Bundesbank Online Panel Households and the Bundesbank Online Panel Firms. Both panels put strong emphasis on inflation expectations and regularly elicit the one-year-ahead inflation rate via point and via probabilistic predictions. In addition to the regular first-order expectations questions, we added the corresponding questions regarding higher-order expectations for households (March 2021 wave) and for firms (Q4/October to December 2021 wave). These additional modules allow us to establish a number of stylized facts about the relation of first- and higher-order inflation expectations in our unique data sets, which can be summarized as follows.

First, we show that the average higher-order forecast of inflation is almost identical to the average first-order forecast of inflation for both households and firms. In general, our data show a positive correlation between first-order and higher-order beliefs for households and firms. However, the correlation is not perfect and is more pronounced for firms than for households. In other words, firm managers display a higher tendency to believe that competitors have expectations about the future rate of inflation similar to their own. Second, we show that the cross-sectional standard deviation of higher-order beliefs (disagreement) is substantial in our sample and that the disagreement in first-order beliefs is on average smaller than the disagreement in higher-order beliefs about inflation. This finding holds for both firms and households. It suggests that although managers and households disagree about how inflation will develop over the next year, there is even more disagreement about what they think others are predicting inflation to be. Third, the average uncertainty (i.e., the standard deviation of individuals' probabilities across inflation bins) around households' and firms' own inflation forecast is significantly lower than their uncertainty around higher-order beliefs about inflation. This suggests that uncertainty accumulates as individuals extrapolate from their beliefs to what others might believe. Finally, when comparing uncertainty and disagreement, our results consistently show that the average degree of uncertainty is lower than average disagreement, i.e. the cross-sectional dispersion. This finding holds for first- and higher-order expectations as well as for firms and households. As we will show below, we replicate the majority of CGKR's results, yet our results deviate in important ways from the stylized facts put forward by CGKR.

Our article contributes to several strands of literature. First, we contribute to the literature using survey data to understand households' and firms' macroeconomic expectations. For inflation, an extensive literature considers expectations of households (e.g., Bruine de Bruin et al., 2010; D'Acunto et al., 2021a, Conrad et al., 2022), firms (e.g., Coibion et al., 2020a), experts (Genre et al., 2013), investors (e.g., Coibion et al., 2020b), and even central bankers (e.g., Mankiw et al., 2003). However, evidence on higher-order expectations is still scarce and has thus far only been investigated for firm managers. We contribute a set of novel insights into both firms' and households' higher-order inflation expectations based on large representative samples. The established relationships between first- and higher-order expectations are remarkably consistent between our firm and household samples, but differ in important aspects from what CGKR report. Most notably, while CGKR find that both uncertainty and disagreement are higher in first-order than in second-order expectations, we find the opposite. Understanding how heterogeneity in own beliefs compares to the heterogeneity in what others believe is important as it can facilitate sharper tests of macroeconomic models for which subjective beliefs are a driver of economic activity. Mankiw and Reis (2002) as well as Hellwig and Venkateswaran (2009) show that with heterogeneous information, firms face uncertainty not only about aggregate fundamentals but also about the pricing decision of other firms because they can no longer be sure that other firms have the same information. Our results highlight that another source of uncertainty is how agents extrapolate from their own information to what other people believe, even if they share the same information. In essence, our findings imply that uncertainty accrues as individuals move from their own expectations to what other people believe.

Second, our findings contribute to the literature on how agents form expectations in strategic environments. Following early work by Morris and Shin (1998), a growing body of theoretical work emphasizes the potential importance of incomplete information and higher-order thinking for optimal policy and understanding economic dynamics (e.g., Angeletos and La'O, 2009). Establishing the intricate relationship between first- and second-order expectations provides empirical benchmarks for building and testing theories of expectation formation. Applied to inflation expectations, CGKR consider multiple extensions to the static noisy information model of Morris and Shin (2002) to reconcile their stylized facts regarding higher-order expectations with theoretical predictions. In a similar spirit, we consider an extension to the static noisy-information model which introduces heterogeneity in prior beliefs as proposed by Patton and Timmermann (2010). On the one hand, this extension allows us to establish conditions which help to explain the discrepancy between our stylized facts and

CGKR's. On the other hand, this exercise demonstrates that the introduction of long-run priors is a promising candidate to further our understanding of how individuals form higher-order expectations. Consistent with this notion, we present evidence that supports the existence of heterogeneous long-run priors as suggested by Patton and Timmermann (2010).

Our paper proceeds as follows. Section 2 describes the implementation of the surveys and how the first- and higher-order inflation expectations were measured in the respective modules. Section 3 presents our main findings. Section 4 considers the implications for noisy-information models in the light of our empirical rejection of the stylized facts established by CGKR. Section 5 concludes the paper.

2. Data and Survey Design

2.1. The Bundesbank Online Panels for Households and Firms

We use microdata from the Bundesbank Online Panel Households (BOP-HH) and the Bundesbank Online Panel Firms (BOP-F). The BOP-HH (BOP-F) is a regular monthly (quarterly) survey of German households (firms) and was established in April 2019 (June 2020), with the first waves being considered as pilot waves before entering a steady state. The waves in each survey contain both a panel component and a refresher sample, and they consist of a set of core questions as well as additional modules for the investigation of specific topics. The topics concern various expectations and assessments in the economic, political and social domain. In addition, households (firms) answer various questions regarding socio-demographics or household characteristics (firm characteristics). The panels reflect a sample representative for the German population and the German firm sector, respectively.

2.2 Measurement of Inflation Expectations

Both panels put strong emphasis on inflation expectations and regularly elicit the one-year-ahead inflation rate via both point and probabilistic predictions. In addition to conventional first-order expectations, we added the corresponding questions regarding higher-order expectations for households (March 2021 wave) and for firms (Q4/October to December 2021 wave). For the respective periods, the data set contains about 2,300 observations for first-order and higher-order inflation expectations of households and about 2,700 observations for first-order and higher-order inflation expectations of firms.¹ In both cases, the data set contains

¹ Table A1 in Appendix A provides an overview about the distribution of several household characteristics (gender, age, income, education) and firm characteristics (number of employees, turnover, firm sector, firm region).

expectations in the form of point expectations (decimal number with one decimal place) and probabilistic expectations where subjects distribute probability mass over different inflation intervals (Manski, 2004).² Finally, the data also contain a series of waves where three-year, five-year and ten-year ahead point inflation expectations are elicited from households and firms. In Section 3, we concentrate on expectations regarding the one-year ahead inflation rate. In Section 4, we additionally include data on long-term inflation expectations in order explain potential differences between our results and previously found patterns in the literature. We also use the combined data on inflation expectations on different time horizons to inform noisy-information models with infinite regress.

3. Results

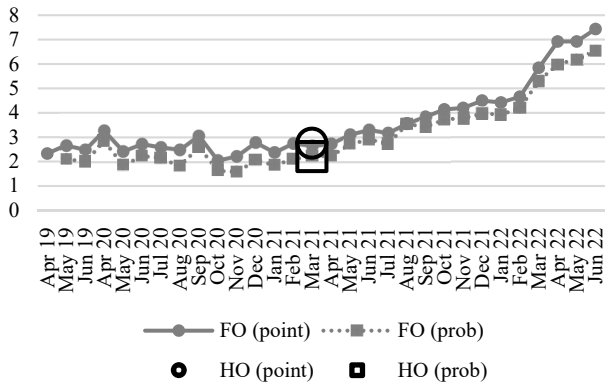
We structure the results as follows. In section 3.1, we analyze the relation between the most important moments of first-order and higher-order inflation expectations. This analysis follows the stylized facts identified by CGKR. We test whether these facts hold in our household and firm samples. In section 3.2, we analyze the prediction error, i.e., the inaccuracy of higher-order expectations, and its relation with individual-level uncertainty in expectations. Section 3.3. studies the role of household and firm characteristics.

² Appendix B contains an overview of the exact wording of all the questions regarding first-order and higher-order inflation expectations we use in our analyses.

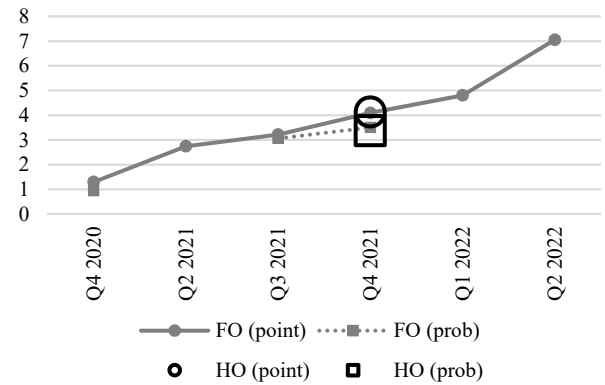
Figure 1. Overview about descriptive results

Panel A. Mean inflation expectations (in %)

A1. Households

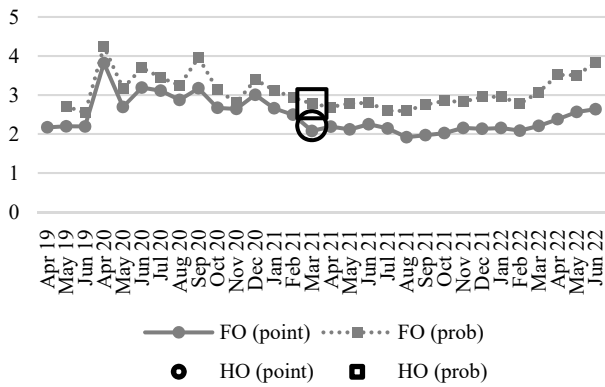


A2. Firms

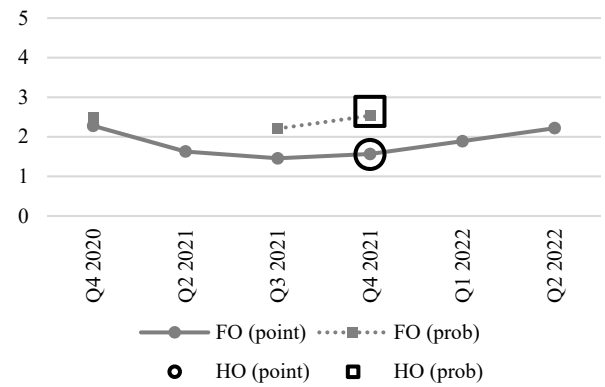


Panel B. Disagreement (in %)

B1. Households

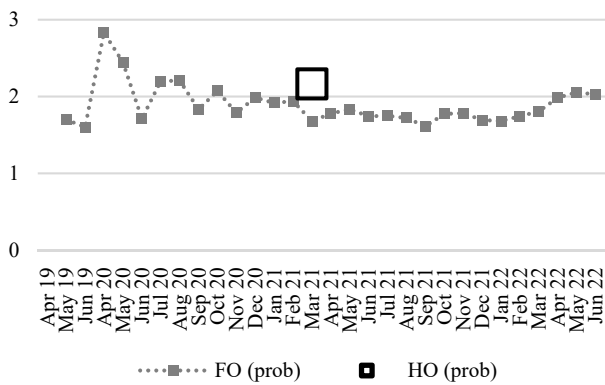


B2. Firms

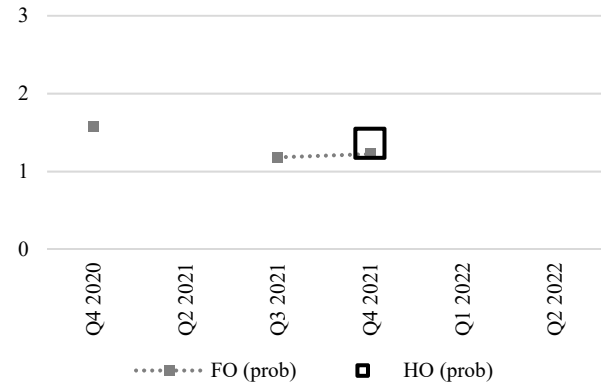


Panel C. Uncertainty (in %)

C1. Households



C2. Firms



Notes: FO = first-order, HO = higher-order. The data from Panel A and Panel B contain both the point and the probabilistic expectation. Data on point expectations is truncated below -12 and above +12. The implied mean derived from the probabilistic estimation is calculated using a value of -16 and 16, respectively, for the outer bins. Panel B represents the between-subject disagreement measured as the standard deviation of inflation expectations. Uncertainty in Panel C is measured as the standard deviation of the probabilistic inflation expectations.

3.1. First-Order and Higher-Order Inflation Expectations

Figure 1 contains descriptive results of all waves. It shows the available panel history of mean first-order inflation expectations, disagreement (i.e., the cross-sectional standard deviation), and uncertainty (i.e., the standard deviation of the probabilistic inflation expectations) of both households and firms. In addition, the figure shows the corresponding values regarding higher-order expectations for the respective waves, i.e., for households in March 2021 and for firms in Q4 2021. Table 1 provides the main findings.

Table 1. First-order and higher-order inflation expectations

	Obs. (1)	Mean (2)	Disagreement (3)	Uncertainty (4)	Correlation (5)
Households:					
Point Expectations					
First-order	2,306	2.65	2.08	–	1.00
Higher- order	2,306	2.75	2.21		0.59
Difference (<i>p</i> -value for equality of moment)		–0.10 (0.02)	–0.13 (<0.01)	–	
Households:					
Probabilistic Expectations					
First-order	2,310	2.24	2.78	1.68	1.00
Higher- order	2,310	2.20	2.78	2.16	0.67
Difference (<i>p</i> -value for equality of moment)		0.04 (0.39)	0.00 (0.99)	–0.48 (<0.01)	
Firms:					
Point Expectations					
First-order	2,681	4.09	1.57	–	1.00
Higher- order	2,681	4.13	1.55		0.71
Difference (<i>p</i> -value for equality of moment)		–0.03 (0.16)	0.02 (0.60)	–	
Firms:					
Probabilistic Expectations					
First-order	2,718	3.51	2.54	1.22	1.00
Higher- order	2,718	3.37	2.64	1.36	0.68
Difference (<i>p</i> -value for equality of moment)		0.14 (0.01)	–0.10 (0.04)	–0.14 (<0.01)	

Notes: The table reports basic moments of first-order and higher-order inflation expectations. Household data has been elicited in March 2021 and firm data between October and December 2021. In order to avoid extreme values, the data on point expectations is truncated below -12 and above +12. The implied mean from the probabilistic estimation in column (2) is calculated using a value of -16 and 16, respectively, for the outer bins. Disagreement in column (3) reports the cross-sectional standard deviation of mean inflation forecasts. Uncertainty in column (4) refers to the standard deviation of the reported probability distribution for future inflation using a value of -16 and 16, respectively, for the outer bins. Column (5) reports the correlation with first-order expectations. Tests for the equality of moments in column (2) and (4) are based on Wilcoxon signed-rank tests. Tests for the equality of disagreement in column (3) are based on Levene's test of homogeneity of variances.

3.1.1 Means and Correlations

For households, we find the first-order explicit point expectation to be slightly lower than the means derived from higher-order expectation (2.65 vs. 2.75, $p = 0.02$). The implied first-order

and higher-order mean expectation derived from the probabilistic measure do not differ (2.24 vs. 2.20, $p = 0.39$). For firms, the explicit first-order and higher-order point expectations do not differ (4.09 vs. 4.13, $p = 0.16$), but the first-order implied mean expectation is slightly higher than the higher-order implied point expectation (3.51 vs. 3.37, $p = 0.01$). Taken together, first-order expectations are neither consistently higher nor lower than higher-order expectations; apparent differences are generally small in absolute terms and their economic relevance may be questioned. The data is consistent with CGKR, who likewise find that for firms average first-order inflation expectations do not differ systematically from average higher-order inflation expectations. Our data generalizes this finding to households and indicates that it is also insensitive to the elicitation method (point versus probabilistic mean expectation).³

After comparing first-order and higher-order expectations on the aggregate level, we analyze the coherence between first-order and higher-order inflation expectations on the individual level. Figure 2, Panel A, shows the distribution of individual-level differences between mean first-order and higher-order point expectations.⁴ For both households and firms, the difference between first-order and higher-order point expectations is symmetrically distributed around zero, with zero being the modal difference on the individual level. However, the coherence between first-order and higher-order expectations is lower for households than for firms; the distribution of differences is less dispersed for firms and first-order and higher-order expectations are identical more often (28.3% for households and 39.1% for firms; $p < 0.001$, Mann-Whitney-U-Test). Panel B shows the analogous analysis for probabilistic expectations by comparing the distribution of within-respondent differences in probabilities assigned to first- and higher-order beliefs. The data shows that most households and most firms do not assign identical weights to the same bins for first-order and higher-order expectations. Instead, they assign relatively more weight to the middle bins for first-order beliefs and relative more weight to the outer bins for higher-order beliefs. Specifically, the shapes of the red lines indicate that the probability distribution referring to first-order beliefs is more concentrated than the probability distribution referring to higher-order beliefs. Also, as in the case of point expectations, the coherence between first-order and higher-order expectations tends to be higher for firms than for households: First, the red line is flatter for firms than for households; second,

³ In Appendix C we provide some insights on the relationship between the point predictions and implied means of the respective probabilistic measures.

⁴ Figure D1 in Appendix D shows the distributions of first- and higher-order expectations separately.

while households assign identical weights to the respective bins in 17.0% of all cases, firms do so in 33.9% ($p < 0.001$, Mann-Whitney-U-Test).⁵

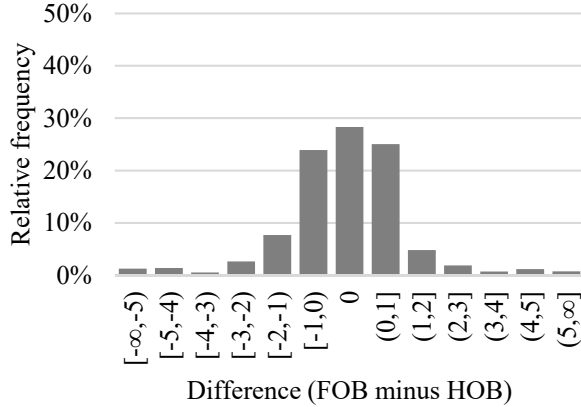
Result 1. CGKR Fact 1 is confirmed. In the aggregate, first-order and higher-order inflation expectations closely correspond for both households and for firms. However, on the individual level, there is a fair amount of incoherence between first-order and higher-order expectations. For both point and probabilistic beliefs, first-order and higher-order expectations diverge more often for households than for firms.

⁵ Note that subjects that exhibit the identical probability distributions for first-order and higher-order expectations fall into the white area in each bin in Panel B of Figure 2. Figure D2 in Appendix D contains analyses about the distribution of differences between first-order and higher-order inflation expectations by subsamples.

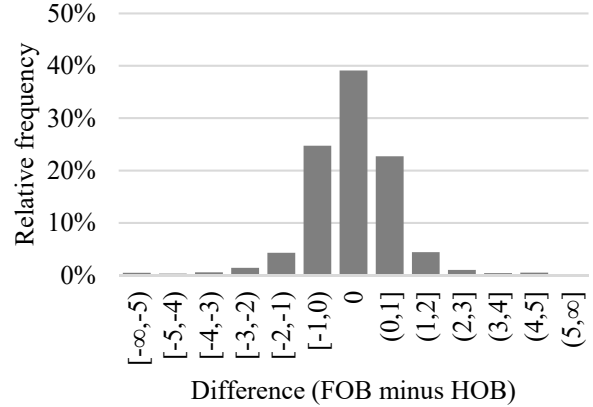
Figure 2. Distribution of differences between first-order and higher-order inflation expectations

Panel A. Point expectations: Difference between mean first-order and higher-order expectations

A1. Households

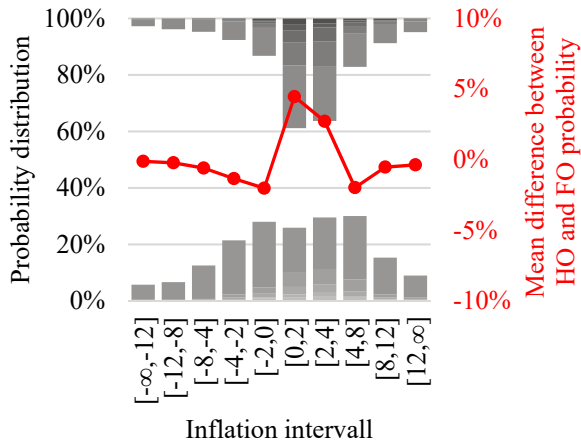


A2. Firms

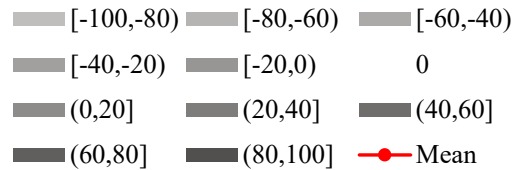
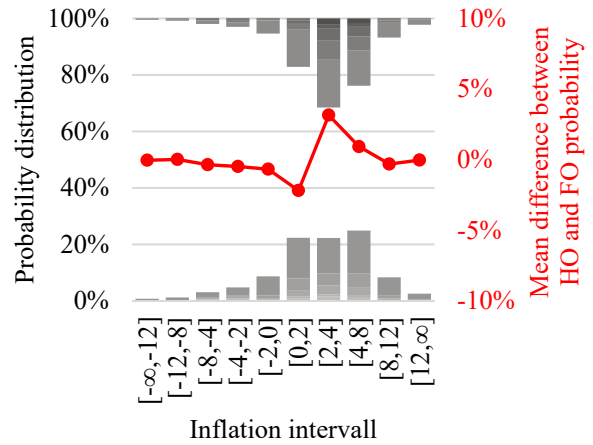


Panel B. Probabilistic expectations: Distribution of within-respondent differences in probabilities assigned to first- and higher-order beliefs

B1. Households



B2. Firms



Notes: Panel A refers to households' and firms' point expectations. The figure shows the distribution of the differences between first-order and higher-order point estimation (i.e., first-order point expectation minus higher-order point expectation). Panel B refers to households' and firms' probabilistic expectations. The white area reflects the category zero in the legend. The red circles are the average difference (first-order minus higher-order expectation) in probability assigned to a specific bin. The sum of the red circles is 100% and the flatter the red line, the higher is the correspondence between first-order and higher-order inflations expectations.

Correlation analysis corroborates these findings. We find a positive correlation between first-order and higher-order expectations across both samples and both elicitation methods. However, correlations are clearly lower than 1. For households, we find correlation coefficients of 0.59 and 0.67 for the point and the implied mean estimates, respectively. For firms, the

corresponding values are 0.71 and 0.68. The correlation between first-order and higher-order point predictions is lower for households than for firms (0.59 vs. 0.71; $p < 0.001$), while the correlation between the probabilistic expectations does not differ between households and firms (0.67 vs. 0.68; $p = 0.39$).⁶

Our data is consistent with CGKR, who likewise find that first-order and higher-order inflation expectations are positively but not perfectly correlated, and the sizes of correlation coefficients are very similar to those found by CGKR. Our data generalizes their finding to households.

Result 2. CGKR Fact 2 is confirmed. The correlation between first-order and higher-order expectations is positive though not perfect, for both households and firms and for both point and probabilistic expectations. For point expectations, the correlation between first-order and higher-order expectations is higher for firms than for households.

3.1.2. Disagreement

For households, the cross-sectional disagreement is lower for first- than for higher-order expectations in the case of the point predictions (2.08 vs. 2.21; $p < 0.01$, Levene's test of homogeneity of variances), and identical for first- and higher-order expectations based on implied means from the probabilistic measure (2.78 vs. 2.78; $p = 0.99$, Levene's test of homogeneity of variances). For firms, the disagreement in point predictions is identical (1.57 vs. 1.55; $p = 0.60$, Levene's test of homogeneity of variances), but disagreement in the implied means is significantly lower for first- than for higher-order expectations (2.54 vs. 2.64; $p = 0.04$, Levene's test of homogeneity of variances). Our data thus indicates that disagreement regarding first-order and higher-order expectations is rather comparable. If anything, disagreement regarding first-order expectations is lower than for higher-order expectations. Importantly, in none of the comparisons, we find that disagreement regarding higher-order beliefs would be smaller than disagreement regarding first-order beliefs. This finding stands in contrast to CGKR. In their data, disagreement in first-order inflation expectations is greater than in higher-order inflation expectations. We cannot replicate this pattern in either sample or elicitation method.

⁶ To compare correlations, we use Cohen et al.'s (2014) procedure.

Result 3. CGKR Fact 3 is not confirmed. The cross-sectional disagreement of first-order beliefs is lower than the disagreement of higher-order beliefs, both for households and firms.

3.1.3. Uncertainty

We calculate individual-level uncertainty for the probabilistic expectations measure. Our data shows that the average uncertainty is lower for first-order than for higher-order expectations, for both households and firms (households: 1.68 vs. 2.16, $p < 0.01$; firms: 1.22 vs. 1.36, $p < 0.01$; Wilcoxon signed-rank tests).⁷ Another possible way to measure the degree of uncertainty inherent in expectations is to look at the correlation between point predictions and implied means from the probabilistic measure.⁸ For first-order expectations, we find that the correlation is 0.61 for households and 0.47 for firms. For higher-order expectations, the correlation is 0.47 for households and 0.39 for firms. Both for households and firms the correlation is stronger for first-order expectations (households: 0.61 vs. 0.47, $p < 0.001$; firms: 0.47 vs. 0.39, $p < 0.001$). That is, the coherence between different elicitation procedures is higher for first-order expectations than for higher-order expectations. This is consistent with the finding that uncertainty is larger in higher-order expectations.

Our data stands in contrast to CGKR's, who find that average uncertainty in first-order inflation expectations is greater than in higher-order inflation expectations. In fact, our results point in the opposite direction in a consistent manner for both households and firms. Note, however, that in CGKR's follow-up wave (see their Table II), they find the same pattern as in our data, i.e., that uncertainty of higher-order inflation expectation is higher than the uncertainty of first-order inflation expectations.

Result 4. CGKR Fact 4 is not confirmed. Both for households and firms, the average uncertainty on the individual level is lower for first- than for higher-order expectations.

⁷ Note that the relation between uncertainty of first-order and higher-order beliefs determines the shape of the red line in Panel B of Figure 2. Specifically, in our data, the average of the individual-level distributions of first-order beliefs is less dispersed than the corresponding distribution of higher-order beliefs. Figure D2 in Appendix D depicts the same data as Figure 2, Panel B, but for different household and firm subgroups.

⁸ The assumption here is that the less concentrated the stated probability distribution becomes on the individual level, the higher is the probability that the implied mean from this distribution deviates from the point estimation, e.g., because the distribution becomes asymmetric.

3.1.4. Comparison between Disagreement and Uncertainty

We find that the average degree of uncertainty is consistently lower than the cross-sectional dispersion or variance. This is true for both disagreement in point predictions and disagreement in implied means, and holds for first- and higher-order expectations and for both households and firms (households: first-order: point prediction 2.08 / implied mean 2.78 vs. uncertainty 1.68; higher-order: point prediction 2.21 / implied mean 2.78 vs. uncertainty 2.16; firms: first-order: point prediction 1.57 / implied mean 2.54 vs. uncertainty 1.22; higher-order: point prediction 1.55 / implied mean 2.64 vs. uncertainty 1.36). All differences are statistically significant at the 0.1%-level (Wilcoxon signed-rank tests). This finding is consistent with the results reported by CGKR.

Result 5. CGKR Fact 5 is confirmed. The average level of uncertainty is smaller than the cross-sectional dispersion in inflation expectations for both households and firms, for both point and probabilistic expectations and for both first- and higher-order expectations.

Two key findings emerge from our analyses in Section 3.1. First, we only replicate facts 1, 2, and 5 of CGKR, and find opposite patterns for fact 3 and 4. Second, our pattern emerges highly consistently for our large samples of firms and households. We discuss the implications in Section 4. In Section 3.2. and 3.3. we take a closer look at the determinants of the prediction errors in higher-order expectations, and the role of heterogeneity in firm and household characteristics.

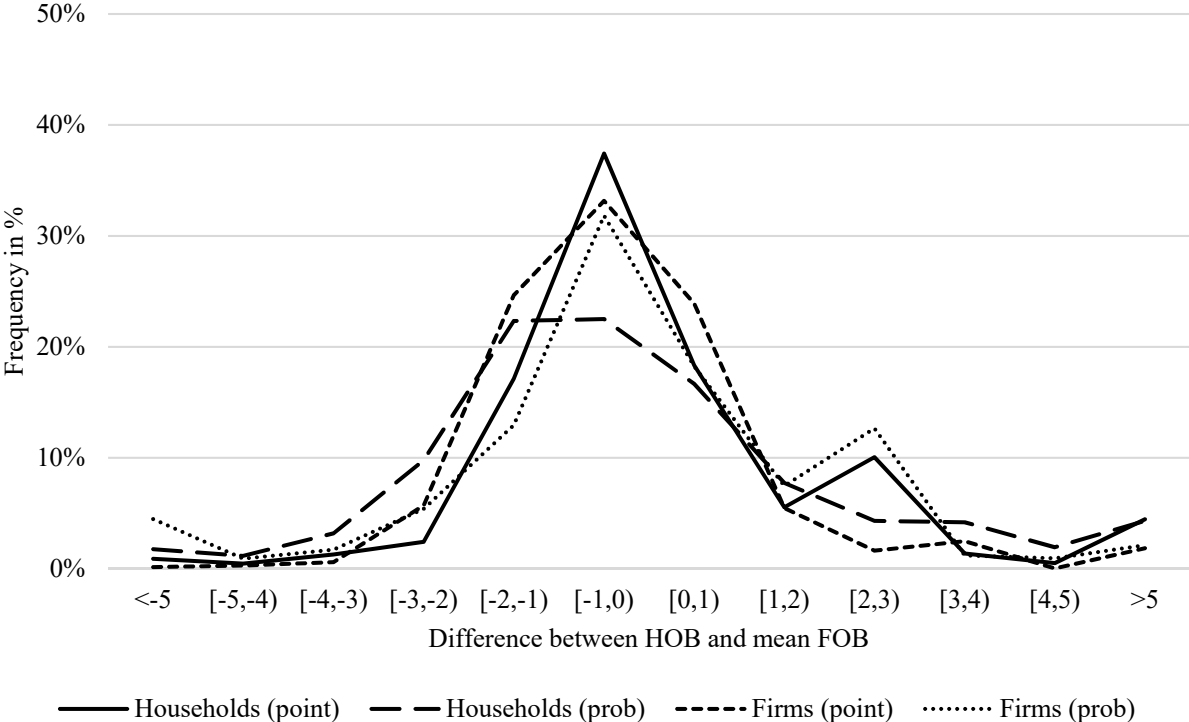
3.2 Prediction Error of Higher-Order Inflation Expectations

Next, we are interested in how accurate households and firms are in predicting the mean expectations of others. We analyze the individual prediction error, measured as the difference between individual higher-order beliefs and the actual mean first-order expectation. The smaller the difference, the more accurate the prediction. Depending on the elicitation method, we either use the average of the point predictions or the average of the implied means to determine the actual mean first-order expectation. Figure 3 shows the distribution of the prediction error on the population level. To compare the magnitude of prediction error, we use the absolute value, thereby neglecting the direction of error (i.e., whether higher-order beliefs overestimate or underestimate actual average first-order expectations). We find that predictions are better for firms than for households. Households exhibit an absolute value of prediction error of 1.44 (1.85) in the point (probabilistic) measure. Firms exhibit an absolute value of prediction error

of 1.08 (1.71) in the point (probabilistic) measure. Firms are less prone to error for both elicitation methods (point: 1.44 vs. 1.08, $p < 0.001$; probabilistic: 1.84 vs. 1.71, $p < 0.001$, Mann-Whitney-U-Tests). Moreover, comparing the two elicitation methods shows that both for households and firms, higher-order point expectations are less prone to error than higher-order probabilistic expectations (households: 1.44 vs. 1.85, $p < 0.001$; probabilistic: 1.08 vs. 1.71, $p < 0.001$, Wilcoxon signed-rank tests). However, one needs to keep in mind Result 1, which shows that on average neither households nor firms overestimate or underestimate the actual mean first-order inflation expectations.

Result 6. Higher-order expectations are more accurate for firms than for households, and more accurate for point predictions than for implied means derived from the probabilistic measure.

Figure 3. Prediction error of higher-order expectations



Notes: Prediction error is measured as the difference between the higher-order belief of an individual respondent and the actual mean first-order expectation (i.e., higher-order belief minus mean first-order belief).

We are also interested in the relation between individual level uncertainty and prediction error (Table 2). Accordingly, we conduct correlation analysis between uncertainty and the absolute value of the prediction error. The positive correlations show that the more uncertain households and firms are, the greater is the prediction error of their higher-order expectations.

Moreover, the relation between uncertainty in (both first-order and higher-order) inflation expectations and the extent of prediction error is stronger in households than in firms.

Result 7. Uncertainty and prediction error are positively correlated. The relation between uncertainty and the prediction error is stronger in households than in firms.

Table 2. Correlation between uncertainty and prediction error

	Households		Firms	
	Uncertainty of first-order expectation	Uncertainty of higher-order expectation	Uncertainty of first-order expectation	Uncertainty of higher-order expectation
Prediction error of point inflation expectation	0.22***	0.23***	0.07***	0.05**
Prediction error of probabilistic inflation expectation	0.17***	0.10***	0.05**	0.03

Notes: In this analysis, prediction error is measured as the absolute difference between the higher-order belief of an individual and the actual mean first-order expectation. The smaller that difference, the better the prediction. We report Pearson’s correlation coefficients. *, **, *** indicates significance at the 5%, 1%, and 0.1% level.

3.3 Sample Splits and Household and Firm Characteristics

We next make use of the available household and firm characteristics to (i) probe whether results 1-5 are robust to sample splits, and to (ii) study whether characteristics affect expectations and uncertainty about expectations. We find that the reported results regarding the relation between first-order and higher-order expectations consistently occur across various household and firm subsamples (see Table A1 for households and Table A2 for firms in Appendix A). While the results thus replicate in each subsample, expectations may still be affected by heterogeneity in household and firm characteristics. Table 3 contains regression analyses of mean inflation expectations and uncertainty on household characteristics. The results indicate that means of both first-order and higher-order inflation expectations tend to be higher for female respondents. This is consistent with Bruine de Bruin et al. (2010) and D’Acunto et al. (2021b), who find that being female is associated with higher first-order inflation expectations.⁹ Moreover, consistent with Das et al. (2020), we find that first-order and higher-order inflation expectations decrease with income and are also lower if respondents have a university degree. There is no association with age. The results further show that uncertainty is positively related to being female, but negatively related to age and income. University

⁹ D’Acunto et al. (2021a) find that this can be explained by females having higher exposure to grocery prices.

education does not seem to affect uncertainty. There are no qualitative differences between the results for first-order and higher-order expectations.

Table 4 contains regression analyses of mean inflation expectations and uncertainty on firm characteristics. Mean point expectations (both first-order and higher-order) are decreasing in the number of employees and are lower in West Germany than in East Germany. These differences, however, do not show up for the probabilistic measure. Firm characteristics also do not seem to be associated with differences in uncertainty.

Result 8. The relation between first-order and higher-order inflation expectations is robust to sample splits. Household characteristics affect first- and higher-order expectations in a consistent way. We identify no effects of firm characteristics on expectations.

Table 3. Household Characteristics and Inflation Expectations

	Mean point expectations		Mean probabilistic expectations		Uncertainty	
	FO	HO	FO	HO	FO	HO
Female	0.161 (0.098)	0.411*** (0.106)	0.353** (0.129)	0.396** (0.129)	0.204* (0.086)	0.352*** (0.096)
Age	0.001 (0.003)	0.004 (0.003)	0.003 (0.004)	0.009* (0.004)	-0.009*** (0.003)	-0.017*** (0.003)
Household income	-0.091*** (0.019)	-0.064*** (0.019)	-0.089*** (0.024)	-0.076** (0.024)	-0.080*** (0.015)	-0.069*** (0.017)
University degree	-0.346*** (0.089)	-0.233* (0.095)	-0.302** (0.115)	-0.363** (0.115)	-0.140 (0.078)	0.020 (0.087)
Constant	3.333*** (0.246)	2.977*** (0.246)	2.737*** (0.332)	2.266*** (0.323)	2.728*** (0.205)	3.490*** (0.229)
N	2,224	2,224	2,233	2,233	2,233	2,233

Notes: OLS regressions. *, **, *** indicates significance at the 5%, 1%, and 0.1% level. FO = first-order, HO = higher-order. Female is an indicator for female gender. Age is an integer (minimum is 16, maximum is 80). Household income has 13 categories (with category 1 referring to income between 0€ and 500€ and category 13 referring to income above 10,000€). College education is an indicator for a university degree. Robust standard errors are clustered on the individual level and reported in parentheses.

Table 4. Firm Characteristics and Inflation Expectations

	Mean point expectations		Mean probabilistic expectations		Uncertainty	
	FO	HO	FO	HO	FO	HO
Employees	-0.192*** (0.040)	-0.189*** (0.043)	-0.118 (0.066)	-0.132 (0.076)	-0.001 (0.027)	0.016 (0.029)
Turnover	0.032 (0.038)	0.062 (0.041)	0.035 (0.061)	0.032 (0.070)	0.015 (0.027)	-0.001 (0.029)
Services	-0.076 (0.064)	-0.023 (0.064)	0.164 (0.106)	0.110 (0.108)	0.010 (0.045)	0.019 (0.048)
West Germany	-0.313** (0.101)	-0.266* (0.104)	-0.027 (0.179)	-0.216 (0.180)	0.029 (0.070)	0.061 (0.072)
Constant	4.744*** (0.122)	4.634*** (0.130)	3.590*** (0.226)	3.681*** (0.228)	1.154*** (0.087)	1.262*** (0.090)
N	2,513	2,513	2,548	2,548	2,548	2,548

Notes: OLS regressions. *, **, *** indicates significance at the 5%, 1%, and 0.1% level. FO = first-order, HO = higher-order. Employees is the number of employees and consists of five categories with category 1 referring to 1-10 employees and category 5 referring to more than 1,000 employees. Turnover consists of five categories with category 1 referring to an annual turnover below 1 million Euro and category 5 referring to an annual turnover above 229 million Euro. Services is an indicator for the firm belonging to the services sector (rather than the industrial or the construction sector). West Germany is an indicator for the firm being based in West Germany (rather than East Germany). Robust standard errors are clustered on the individual level and reported in parentheses.

Tables 5 and 6 show regression analyses of the absolute value of prediction error on household and firm characteristics, respectively. For households, the prediction error is positively associated with being female, but negatively with age, household income and having

a university degree. For firms, the prediction error decreases with the number of employees, and is smaller for firms located in West Germany.

Table 5. Household Characteristics and Prediction Error

	Point	Probabilistic
Female	0.352*** (0.081)	0.427*** (0.096)
Age	-0.008*** (0.002)	-0.002 (0.003)
Household income	-0.030* (0.014)	-0.047** (0.018)
University degree	-0.153* (0.072)	-0.386*** (0.084)
Constant	2.020*** (0.182)	2.309*** (0.244)
N	2,224	2,233

Notes: OLS regressions. *, **, *** indicates significance at the 5%, 1%, and 0.1% level. Female is an indicator for female gender. Age is an integer (minimum is 16, maximum is 80). Household income has 13 categories (with category 1 referring to income between 0€ and 500€ and category 13 referring to income above 10,000€). College education is an indicator for a university degree. Robust standard errors are clustered on the individual level and reported in parentheses.

Table 6. Firm Characteristics and Prediction Error

	Point	Probabilistic
Employees	-0.0756* (0.0303)	-0.140* (0.0583)
Turnover	0.0176 (0.0291)	0.0189 (0.0515)
Services	-0.0517 (0.0458)	-0.169* (0.0817)
West Germany	-0.244** (0.0744)	-0.420** (0.134)
Constant	1.444*** (0.0937)	2.420*** (0.171)
N	2,513	2,548

Notes: OLS regressions. *, **, *** indicates significance at the 5%, 1%, and 0.1% level. Employees is the number of employees and consists of five categories with category 1 referring to 1-10 employees and category 5 referring to more than 1,000 employees. Turnover consists of five categories with category 1 referring to an annual turnover below 1 million Euro and category 5 referring to an annual turnover above 229 million Euro. Services is an indicator for the firm belonging to the services sector (rather than the industrial or the construction sector). West Germany is an indicator for the firm being based in West Germany (rather than East Germany). Robust standard errors are clustered on the individual level and reported in parentheses.

4. Implications for Calibrating Noisy Information Models

The previous section reveals several relationships between first- and higher-order inflation expectations that are remarkably consistent between firms and households. At the same time, we find that some of the documented relationships are inconsistent with the stylized facts of CGKR. Both disagreement and uncertainty in higher-order expectations are greater than in first-order expectations, which is the opposite of what CGKR find.

In this section, we investigate this discrepancy through an extension of the static noisy information model of Morris and Shin (2002). To reconcile their stylized facts about higher-order inflation expectations with theoretical predictions in settings where firms perform infinite

regress in their expectations, CGKR consider multiple extensions of the baseline noisy information model. A promising extension is a model with heterogeneity in long-run priors (Patton and Timmermann, 2010), which we consider in this section. The idea is that forecasters shrink their optimal inflation forecasts toward their prior beliefs about long-run inflation which can reconcile most of CGKR's facts. We focus only on its predictions regarding the relative magnitude of disagreement and uncertainty in first- and higher-order expectations. We present the key intuitions here and refer for detailed derivations to Appendix E.

Suppose firm $i \in [0,1]$ chooses to set its optimal price, p_i , as a linear combination of its expectation of a fundamental, m , and its expectation of the aggregate price level in the economy, \bar{p} :

$$(1) \quad p_i = (1 - \alpha)E_i[m] + \alpha E_i^*[\bar{p}],$$

where $\alpha \in (0,1)$ describes the degree of complementarity in pricing. Additionally, assume that $\bar{p} \stackrel{\text{def}}{=} \int_0^1 p_j dj$, such that individual i can iterate the optimal price equation forward by substituting the average optimal price equation for the aggregate price level:

$$(2) \quad p_i = (1 - \alpha)E_i[m] + \alpha E_i^* \left[\int_0^1 p_j dj \right],$$

Following Patton and Timmermann (2010), we allow individual's "long-run" prior, μ_i , to skew expectations of the aggregate price level in the economy:

$$(3) \quad \begin{aligned} E_i^*[\bar{p}] &= \omega \mu_i + (1 - \omega)E_i[\bar{p}] \\ &= \omega \mu_i + (1 - \omega)\{(1 - \alpha)E_i[\bar{E}[m]] + \alpha E_i[\bar{E}[\bar{p}]]\}, \end{aligned}$$

where $E_i^*[\bar{p}]$ denotes the skewed first-order expectation of \bar{p} and $E_i[\bar{p}] = (1 - \alpha)E_i[\bar{E}[m]] + \alpha E_i[\bar{E}[\bar{p}]]$ denotes the rational (non-skewed) expectation of \bar{p} . Additionally, $\mu_i \sim N(\bar{\mu}, \kappa_\mu^{-1})$, with $\bar{\mu}$ and κ_μ^{-1} measuring the average level and the dispersion of the "long-run" priors, and

$\omega = \frac{\text{var}(E_i[\int p_j dj])}{\gamma^2 + \text{var}(E_i[\int p_j dj])}$ with $\gamma^2 \geq 0$ being a parameter measuring the degree to which an

individual prefers her own "long-run" prior. Further, define the average expectation in the economy for variable m as $\bar{E}[m]$ and let $E_i[\bar{E}[m]]$ be the expectation of individual i about the average expectation in the economy. Similarly, let $E_i(\bar{p})$ denote the first-order expectation about the average price level, and $E_i[\bar{E}[\bar{p}]]$ the second-order expectation about the price level.

We can iterate these expectations to the k th higher-order recursively: $\bar{E}^k[X] = \int_0^1 E_j(\bar{E}^{k-1}[X])dj$.

To characterize how individuals form (higher-order) expectations about the fundamental, we follow CGKR and assume that individuals do not possess full information.¹⁰ Agents face uncertainty concerning the fundamental m , but receive one noisy public signal and one private signal. To simplify equations, we follow Morris and Shin (2002) and assume that m is drawn from an (improper) uniform prior over the real line. Each of the two signals individually reflects the true value of m combined with some noise. The public signal $y = m + \varepsilon$, where $\varepsilon \sim N(0, \kappa_y^{-1})$ is common across firms and the private signal takes the form: $x_i = m + v_i$, with $v_i \sim N(0, \kappa_x^{-1})$, and where κ_x and κ_y denote the precision of each type of signal. In order to obtain an individual expectation of m , firms weight their signals according to the relative noise in each:

$$(4) \quad E_i[m] = \frac{\kappa_y}{\kappa_x + \kappa_y} y + \frac{\kappa_x}{\kappa_x + \kappa_y} x_i = (1 - \delta)y + \delta x_i,$$

where $\delta = \frac{\kappa_x}{\kappa_x + \kappa_y}$. The intuition is straightforward: As the private signal becomes more precise relative to the public signal, the firm places relatively more weight on it in when forming beliefs about the fundamental. It can be shown that firm i 's higher-order expectation about the average expectation of other firms is (with $\bar{E}^k[X]$ denoting the k th higher-order belief):

$$(5) \quad E_i[\bar{E}^k[m]] = (1 - \delta^{k-1})y + \delta^{k-1}E_i[\bar{E}^{k-1}[m]] = (1 - \delta^k)y + \delta^k x_i$$

Equation (5) shows that higher order expectations will depend increasingly more on the public signal as that signal is common across firms.

By repeatedly substituting into the optimal pricing equation (1), one can show that the aggregate price level also becomes an average of increasingly higher-order expectations of the fundamental:

$$(6) \quad p_i = \alpha \omega \mu_i + \alpha \omega \sum_{k=0}^{\infty} \alpha^{k+1} (1 - \omega)^{k+1} E_i[\bar{E}^k[\bar{\mu}]] + (1 - \alpha) \sum_{k=0}^{\infty} \alpha^k (1 - \omega)^k [(1 - \delta^{k+1})y + \delta^{k+1} x_i]$$

An individual firm sets its price as a function of its “long-run” prior μ_i , a sum of progressively higher-order expectations of the average prior $\bar{\mu}$, and a sum of progressively higher-order

¹⁰ In the model, we focus on higher-order expectations of the fundamental while our data comprises higher-order expectations of inflation. This is to ensure direct comparison with the results of CGKR, who study the same model of higher-order expectations of the fundamental despite using data on higher-order inflation expectations.

expectations of the fundamental m . Following CGKR, we assume that individuals do not know the true average prior $\bar{\mu}$, but rather observe a private signal of the mean: $\zeta_i \sim N(\bar{\mu}, \kappa_\zeta^{-1})$. The uncertainty about the aggregate prior, κ_ζ^{-1} , is necessary to bring priors and the dispersion they provide into higher-order expectations. Additionally, with heterogeneity in priors, there are two sources of variation between first- and higher-order expectations. On the one hand, information in private signals differs across individuals. On the other hand, there is a potential discrepancy in individual's own prior beliefs (regarding the long-run prior) and the prior belief they assign to others. Because both sources of uncertainty are assumed to be uncorrelated and the weights on signals are different for first- and higher-order expectations, the cross-sectional correlation between first- and second-order beliefs is imperfect, which is consistent with our stylized fact 1.

Finally, we can proceed to find expressions for cross-sectional disagreement and forecast uncertainty in first- and second-order expectations (see Appendix E). It can be shown that disagreement in second-order expectations ($Var[E_i[\bar{E}[\bar{p}]]]$) is greater than disagreement in first-order expectations ($Var[E_i[\bar{p}]]$) if:

$$(7) \quad \begin{aligned} Var[E_i[\bar{E}[\bar{p}]]] &> Var[E_i[\bar{p}]] \\ \Leftrightarrow (2(1-\omega)(\omega\theta\omega')(\omega'-1) + \omega^2(\omega'^2-2))\kappa_\mu^{-1} \\ &+ \left[\omega \left((\omega + (1-\omega)\theta) + ((1-\omega)\theta) \right) \right] (1 \\ &- \omega')^2 \kappa_\zeta^{-1} + (1-\omega)^2 (\phi_x \delta)^2 (\delta^2 - 1) \kappa_x^{-1} > 0 \end{aligned}$$

Similarly, one can show that uncertainty in second-order expectations ($\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$) is greater than uncertainty in first-order expectations ($\Omega_{\{E_i[\bar{p}]|y\}}$) if:

$$(8) \quad \begin{aligned} \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} &> \Omega_{\{E_i[\bar{p}]|y\}} \\ \Leftrightarrow \left[\omega \left((\omega + (1-\omega)\theta) + ((1-\omega)\theta) \right) \right] (1 - \omega')^2 \kappa_\zeta^{-1} \\ &+ (1-\omega)^2 (\phi_x \delta)^2 (\delta^2 - 1) \kappa_x^{-1} > 0 \end{aligned}$$

In contrast to the basic noisy-information model with infinite regress which CGKR introduce, the extension with heterogenous long-run priors does not make a clear prediction on the relative magnitude of disagreement and uncertainty in first versus second-order expectations. Instead, the prediction depends on the relative magnitudes of κ_ζ^{-1} and κ_x^{-1} . In particular, if κ_x^{-1} and κ_μ^{-1} are large relative to κ_ζ^{-1} the model produces $Var[E_i[\bar{E}[\bar{p}]]] < Var[E_i[\bar{p}]]$, as well as

$\Omega_{\{E_i[\bar{E}[\bar{p}]|y]\}} < \Omega_{\{E_i[\bar{p}]|y\}}$, which is the finding of CGKR. However, if κ_x^{-1} and κ_μ^{-1} are small relative to κ_ζ^{-1} , the model produces $Var[E_i[\bar{E}[\bar{p}]]] > Var[E_i[\bar{p}]]$ as well as $\Omega_{\{E_i[\bar{E}[\bar{p}]|y]\}} > \Omega_{\{E_i[\bar{p}]|y\}}$, which is consistent with our findings. To interpret the directions of both predictions, it is instructive to review what the parameters capture. The parameter κ_x^{-1} measures dispersion in private signals. For instance, when κ_x^{-1} is high, there is a lot of dispersion in private signals. Moving from first-order to second-order beliefs, individuals place less weight on their own priors (and on the private signal), which tends to reduce disagreement and uncertainty in second-order relative to first-order beliefs. κ_μ^{-1} captures dispersion in individuals' long-run priors. A high κ_μ^{-1} indicates a lot heterogeneity in long-run priors about average prices.

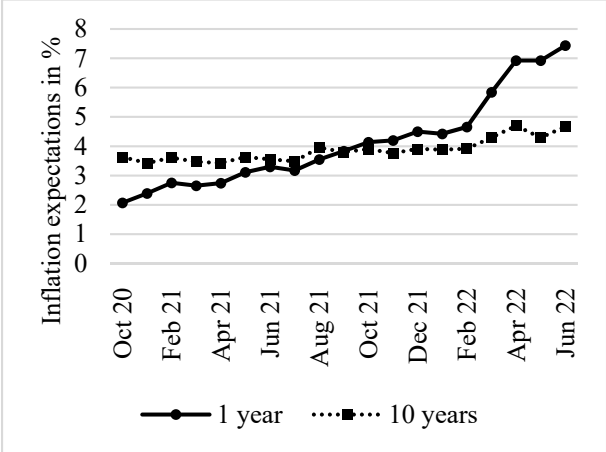
Consider an individual who understands that her first-order belief is skewed by her long-run prior and that others also report skewed first-order beliefs. When thinking about other individuals' beliefs, each individual tries to remove her own "bias" (i.e., her own long-run prior) from the reported value. This tends to reduce dispersion in second-order beliefs relative to first-order beliefs. Finally, κ_ζ^{-1} measures dispersion in beliefs about other individuals' long-run priors (i.e. uncertainty about the aggregate prior). When κ_ζ^{-1} is high, there is a lot of dispersion in beliefs about other individuals' long-run priors. This tends to increase both uncertainty and disagreement in second-order forecasts relative to first-order forecasts, which is opposite to the effect of κ_μ^{-1} . Reproducing our facts would require that the effect of κ_x^{-1} and κ_μ^{-1} be weaker than the effect of κ_ζ^{-1} . Conversely, if the effect of κ_x^{-1} and κ_μ^{-1} is stronger, than the model would reproduce the facts of CGKR.

It is important to note that the relation between uncertainty and disagreement in first- versus higher-order expectations always moves in tandem as they rely on the same conditions: If one finds that disagreement in second-order expectations is higher than disagreement in first-order expectations, one should also find that uncertainty is higher in second-order than in first order expectations. Our finding of higher uncertainty and disagreement in second-order than in first-order expectations is thus consistent with these theoretical predictions but implies different beliefs about long-run priors than the results of CGKR. This reconciles the differences in results between our study and the study by CGKR. Importantly, to generate predictions that match our data, dispersion in beliefs about others' long-run priors needs to be high relative to dispersion in private signals and to dispersion in individuals' own long-run priors. Intuitively, this seems plausible. It suggests that uncertainty aggregates when individuals extrapolate from they believe to what others believe.

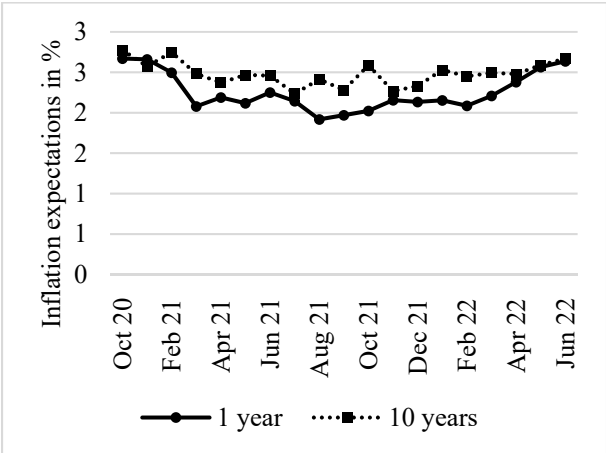
To gain some insight into potential long-run priors, we consider a question of our household panel (BOP-HH) on long-run inflation expectations. For Wave 10 and from Wave 13 onwards, BOP-HH includes questions on the expected inflation rate in 10 years. The question is framed as a point prediction. Given the long time frame, it is unlikely that people have reliable signals to base their expectations on and we argue that these long-run expectations capture long-run priors for inflation expectations. Figure 4 Panel A shows the mean of 10-year inflation expectations, in comparison to the 12 months point prediction of the first-order inflation expectation. Panel B displays disagreement.

Figure 4. Comparison of long-run priors and 12 months point predictions

Panel A. Mean Inflation Expectations (BOP-HH)



Panel B. Disagreement (BOP-HH)



We make two observations. First, 10-year expectations are much more stable than the 12 months expectations. While the 12 month point prediction shows a clear upwards trend over the depicted time frame, long term predictions are mostly flat. Second, results in Panel B display that long-run priors show considerable disagreement and indeed somewhat more disagreement than the point prediction for households. This further supports the existence of heterogenous long-term priors as suggested by CGKR and Patton and Timmermann, (2010). The introduction of heterogenous long-run priors appears to be a promising candidate to rationalize differences between our results and those of CGKR. Future work could fruitfully examine expectations about the long-run priors of others, and how uncertainty and disagreement in these second-order expectations compare to their own long-run priors. This would identify $E_i[\bar{\mu}]$, and in particular κ_{ζ}^{-1} .

5. Conclusion

This article presents novel survey evidence on firms' and households' higher-order inflation expectations. Despite playing an important role in macroeconomic and intertemporal microeconomic models, empirical evidence on them is scarce. Our study provides the first evidence on how households form higher-order beliefs about inflation and how such beliefs compare to their first-order expectations. Additionally, having access to both household and firm first- and higher-order expectations data, we can identify common patterns that allow us to establish a number of stylized facts. These insights can be used to discipline models of higher-order beliefs. For instance, a central implication of both our samples is that uncertainty accrues as individuals extrapolate from their beliefs to what others might know. This is in contrast to the assumption of the baseline noisy information model, which assumes that uncertainty is lower in higher order-expectations as agents put higher weight on a public signal when forming their beliefs. One potential avenue to rationalize this discrepancy is by introducing heterogenous long-run priors which add another source of uncertainty that is not present in first-order expectations. To test such a conjecture empirically, future research would not only need to assess agents' higher-order expectations, but also their uncertainty regarding the long-run priors of others.

Beyond implications for theory, our results also offer more immediate policy guidance. For instance, forward guidance typically aims not only at moving first-order but also higher order-expectations. The common patterns between first- and higher-order expectations among firms and households we identified suggest that such communication-based policy tools are not only

useful for moving expectations of professional forecasters or firm managers but also for the average household.

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Appendix A: Additional Tables

Table A1. Household's moments of first- and higher-order inflation expectations by subsamples

Panel A. Point expectations by households			Obs.	Mean		Disagreement		Uncertainty		Correlation
			(1)	(2)		(3)		(4)		(5)
				FO	HO	FO	HO	FO	HO	
All			2,306	2.65	2.75	2.08	2.21	-	-	0.59
Gender	Male		1,417	2.53	2.55	1.79	1.92	-	-	0.60
	Female		889	2.85	3.07	2.46	2.57	-	-	0.58
Age	18-39		361	2.53	2.60	2.09	2.29	-	-	0.61
	40-59		870	2.68	2.78	2.10	2.34	-	-	0.56
	>59		1,069	2.65	2.77	2.02	2.06	-	-	0.61
Income	<2499		579	3.10	3.17	2.44	2.36	-	-	0.57
	2500-4999		1,172	2.60	2.66	2.01	2.18	-	-	0.57
	>4999		483	2.28	2.50	1.57	1.94	-	-	0.63
Education	No college degree		1,258	2.88	2.92	2.23	2.34	-	-	0.61
	College degree		1,034	2.36	2.53	1.79	2.01	-	-	0.56
Panel B. Probabilistic expectations by households			Obs.	Mean		Disagreement		Uncertainty		Correlation
			(1)	(2)		(3)		(4)		(5)
				FO	HO	FO	HO	FO	HO	
All			2,310	2.24	2.20	2.78	2.78	1.68	2.16	0.67
Gender	Male		1,401	2.07	2.02	2.31	2.38	1.57	2.01	0.64
	Female		909	2.49	2.47	3.36	3.28	1.85	2.41	0.69
Age	18-39		371	1.89	1.73	2.98	2.56	1.89	2.58	0.57
	40-59		885	2.40	2.28	2.92	2.88	1.73	2.32	0.67
	>59		1,048	2.21	2.29	2.56	2.74	1.55	1.88	0.72
Income	<2499		583	2.54	2.55	3.29	3.23	1.92	2.36	0.71
	2500-4999		1,190	2.26	2.19	2.73	2.69	1.67	2.13	0.64
	>4999		470	1.84	1.82	1.86	2.15	1.38	1.96	0.60
Education	No college degree		1,283	2.46	2.45	3.02	3.10	1.79	2.21	0.67
	College degree		1,014	1.96	1.89	2.36	2.21	1.53	2.09	0.66

Notes: FO = first-order, HO = higher-order. The table reports basic moments of first-order and higher-order inflation expectations. In order to avoid extreme values, the data on point expectations in Panel 3A is truncated below -12 and above +12. The implied mean from the probabilistic estimation in Panel 3B, column (2) is calculated using a value of -16 and 16, respectively, for the outer bins. Disagreement in column (3) reports the cross-sectional standard deviation of mean inflation forecasts. Uncertainty in column (4) refers to the standard deviation of the reported probability distribution for future inflation using a value of -16 and 16, respectively, for the outer bins. Column (5) reports the correlation between first-order and higher-order expectations.

Table A2. Firm's moments of first- and higher-order inflation expectations by subsamples

Panel A. Point expectations by firms		Obs.	Mean		Disagreement		Uncertainty		Correlation
		(1)	(2)		(3)		(4)		(5)
			FO	HO	FO	HO	FO	HO	
All		2,681	4.09	4.13	1.57	1.55	-	-	0.71
Employees	1-10	963	4.23	4.23	1.66	1.58	-	-	0.71
	11-50	861	4.20	4.21	1.67	1.67	-	-	0.69
	51-200	498	3.91	3.99	1.35	1.41	-	-	0.72
	201-1000	252	3.72	3.82	1.26	1.31	-	-	0.76
	>1000	79	3.63	3.70	1.15	0.98	-	-	0.75
Turnover	<1 mio	864	4.14	4.14	1.59	1.54	-	-	0.69
	1mio-7 mio	775	4.19	4.22	1.68	1.71	-	-	0.70
	7 mio-34 mio	538	4.08	4.11	1.50	1.39	-	-	0.72
	34 mio-229 mio	323	3.84	3.92	1.34	1.41	-	-	0.74
	>229 mio	134	3.83	4.02	1.32	1.51	-	-	0.82
Sector	Industry	720	4.08	4.10	1.59	1.57	-	-	0.76
	Construction	287	4.17	4.16	1.51	1.60	-	-	0.66
	Services	1,525	4.08	4.13	1.55	1.55	-	-	0.71
Region	West Germany	2,254	4.06	4.09	1.53	1.51	-	-	0.71
	East Germany	331	4.37	4.35	1.74	1.81	-	-	0.78

Panel B. Probabilistic expectations by firms		Obs.	Mean		Disagreement		Uncertainty		Correlation
		(1)	(2)		(3)		(4)		(5)
			FO	HO	FO	HO	FO	HO	
All		2,718	3.51	3.37	2.54	2.64	1.22	1.36	0.68
Employees	1-10	976	3.54	3.44	2.85	2.83	1.20	1.33	0.67
	11-50	883	3.68	3.46	2.52	2.66	1.24	1.39	0.73
	51-200	500	3.40	3.36	2.13	2.29	1.21	1.34	0.74
	201-1000	253	3.06	2.96	2.26	2.46	1.28	1.40	0.61
	>1000	80	3.33	2.86	1.53	2.34	1.29	1.49	0.18
Turnover	<1 mio	877	3.47	3.36	2.78	2.74	1.21	1.34	0.71
	1mio-7 mio	795	3.64	3.46	2.62	2.64	1.22	1.38	0.76
	7 mio-34 mio	544	3.52	3.31	2.32	2.62	1.22	1.35	0.68
	34 mio-229 mio	321	3.25	3.13	2.16	2.38	1.22	1.36	0.54
	>229 mio	135	3.42	3.36	1.78	2.28	1.34	1.44	0.69
Sector	Industry	729	3.29	3.15	2.87	2.84	1.25	1.37	0.72
	Construction	295	3.62	3.57	2.35	2.53	1.14	1.28	0.77
	Services	1,545	3.57	3.41	2.43	2.57	1.22	1.36	0.64
Region	West Germany	2,280	3.49	3.32	2.45	2.57	1.23	1.37	0.65
	East Germany	341	3.51	3.54	3.16	3.15	1.18	1.31	0.83

Notes: FO = first-order, HO = higher-order. The table reports basic moments of first-order and higher-order inflation expectations. In order to avoid extreme values, the data on point expectations in Panel 4A is truncated below -12 and above +12. The implied mean from the probabilistic estimation in Panel 4B, column (2) is calculated using a value of -16 and 16, respectively, for the outer bins. Disagreement in column (3) reports the cross-sectional standard deviation of mean inflation forecasts. Uncertainty in column (4) refers to the standard deviation of the reported probability distribution for future inflation using a value of -16 and 16, respectively, for the outer bins. Column (5) reports the correlation between first-order and higher-order expectations.

Appendix B: Survey Questions

Table B1. Survey Questions for Households

Item	Wording of Question and Input
1 One-year ahead first-order point inflation expectations qualitative	Do you think inflation or deflation is more likely over the next twelve months? <input type="radio"/> Inflation more likely <input type="radio"/> Deflation more likely
1A One-year ahead first-order point inflation expectations quantitative (if respondent states in the qualitative question “Inflation more likely”)	What do you think the rate of inflation in Germany will roughly be over the next twelve months? [Decimal number with one decimal place]
1B One-year ahead first-order point deflation expectations quantitative (if respondent states in the qualitative question “Deflation more likely”)	What do you think the rate of deflation in Germany will roughly be over the next twelve months? [Decimal number with one decimal place]
2 One-year ahead first-order probabilistic inflation expectations	In your opinion, how likely is it that the rate of inflation will change as follows over the next twelve months? [Subjects have to assign probability weights to the different bins that need to add up to 100%] a) The rate of deflation (opposite of inflation) will be 12% or higher. b) The rate of deflation (opposite of inflation) will be between 8% and less than 12%. c) The rate of deflation (opposite of inflation) will be between 4% and less than 8%. d) The rate of deflation (opposite of inflation) will be between 2% and less than 4%. e) The rate of deflation (opposite of inflation) will be between 0% and less than 2%. f) The rate of inflation will be between 0% and less than 2%. g) The rate of inflation will be between 2% and less than 4%. h) The rate of inflation will be between 4% and less than 8%. i) The rate of inflation will be between 8% and less than 12%. j) The rate of inflation will be 12% or higher.
3 One-year ahead higher-order point inflation expectations qualitative	In your opinion, do the other participants in this survey believe that inflation or deflation is more likely in Germany over the next twelve months? <input type="radio"/> Inflation more likely <input type="radio"/> Deflation more likely

Item	Wording of Question and Input
3A One-year ahead higher-order point inflation expectations quantitative (if respondent states in the qualitative question “Inflation more likely”)	In your opinion, what do the other participants in this survey think the rate of inflation will roughly be over the next twelve months? [Decimal number with one decimal place]
3B One-year ahead higher-order point deflation expectations quantitative (if respondent states in the qualitative question “Deflation more likely”)	In your opinion, what do the other participants in this survey think the rate of deflation will roughly be over the next twelve months? [Decimal number with one decimal place]
4 One-year ahead higher-order probabilistic inflation expectations	In your opinion, how likely do the other participants in this survey think it is that the rate of inflation will change as follows over the next twelve months? [Subjects have to assign probability weights to the different bins that need to add up to 100%] a) The rate of deflation (opposite of inflation) will be 12% or higher. b) The rate of deflation (opposite of inflation) will be between 8% and less than 12%. c) The rate of deflation (opposite of inflation) will be between 4% and less than 8%. d) The rate of deflation (opposite of inflation) will be between 2% and less than 4%. e) The rate of deflation (opposite of inflation) will be between 0% and less than 2%. f) The rate of inflation will be between 0% and less than 2%. g) The rate of inflation will be between 2% and less than 4%. h) The rate of inflation will be between 4% and less than 8%. i) The rate of inflation will be between 8% and less than 12%. j) The rate of inflation will be 12% or higher.
5 Five-years ahead first-order point inflation expectations quantitative	And what value do you think the rate of inflation or deflation will take on average over the next five years? [Decimal number with one decimal place]
6 Ten-years ahead first-order point inflation expectations quantitative	And what value do you think the rate of inflation or deflation will take on average over the next ten years? [Decimal number with one decimal place]

Table B2. Survey Questions for Firms

Item	Wording of Question and Input
1 One-year ahead first-order point inflation expectations quantitative	What do you expect the rate of inflation to be over the next twelve months? [Decimal number with one decimal place]
2 One-year ahead first-order probabilistic inflation expectations	In your opinion, how likely is it that the rate of inflation will change as follows over the next twelve months? [Subjects have to assign probability weights to the different bins that need to add up to 100%] a) The rate of deflation (opposite of inflation) will be 12% or higher. b) The rate of deflation (opposite of inflation) will be between 8% and less than 12%. c) The rate of deflation (opposite of inflation) will be between 4% and less than 8%. d) The rate of deflation (opposite of inflation) will be between 2% and less than 4%. e) The rate of deflation (opposite of inflation) will be between 0% and less than 2%. f) The rate of inflation will be between 0% and less than 2%. g) The rate of inflation will be between 2% and less than 4%. h) The rate of inflation will be between 4% and less than 8%. i) The rate of inflation will be between 8% and less than 12%. j) The rate of inflation will be 12% or higher.
3 One-year ahead higher-order point inflation expectations quantitative	What rate of inflation do you think other enterprises in Germany are expecting on average over the next twelve months? [Decimal number with one decimal place]
4 One-year ahead higher-order probabilistic inflation expectations	In your opinion, how likely do other enterprises in Germany think it is that the rate of inflation will change as follows over the next twelve months? Input: [Subjects have to assign probability weights to the different bins that need to add up to 100%] a) The rate of deflation (opposite of inflation) will be 12% or higher. b) The rate of deflation (opposite of inflation) will be between 8% and less than 12%. c) The rate of deflation (opposite of inflation) will be between 4% and less than 8%. d) The rate of deflation (opposite of inflation) will be between 2% and less than 4%. e) The rate of deflation (opposite of inflation) will be between 0% and less than 2%. f) The rate of inflation will be between 0% and less than 2%. g) The rate of inflation will be between 2% and less than 4%. h) The rate of inflation will be between 4% and less than 8%.

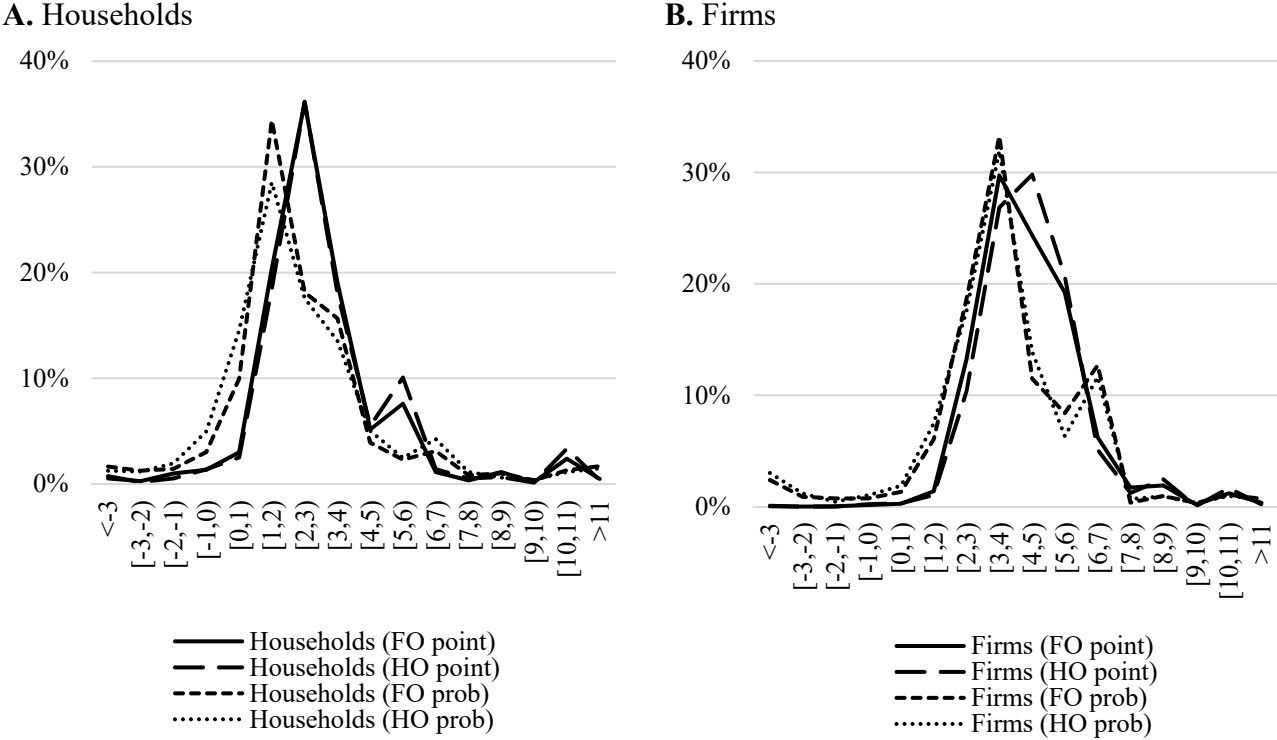
		i) The rate of inflation will be between 8% and less than 12%.
		j) The rate of inflation will be 12% or higher.
5	Three-years ahead first-order point inflation expectations quantitative	What do you expect the rate of inflation to be on average over the next three years? [Decimal number with one decimal place]
6	Five-years ahead first-order point inflation expectations quantitative	What do you expect the rate of inflation to be on average over the next five years? [Decimal number with one decimal place]

Appendix C: Differences between Point and Probabilistic Expectations

Figure 1 shows the values of mean first-order expectations (Panel A) and disagreement (Panel B) over time. First, average expectations elicited via point estimation are consistently higher than the average implied point estimations derived from the probabilistic measure. Second, disagreement is lower among the point estimations than among the implied estimations. Table 1 and Figure C1 zoom in on these aspects and compare the average distribution of mean first-order and higher-order inflation expectations on the individual level for the respective points in time. The differences in mean first-order expectations are statistically significantly different (households: 2.65 vs. 2.24, $p < 0.001$; firms: 4.09 vs. 3.51, $p < 0.001$, Mann-Whitney-U-Test). Likewise, the differences in disagreement of first-order expectations are statistically significantly different (households: 2.08 vs. 2.78, $p < 0.001$; firms: 1.57 vs. 2.54, $p < 0.001$, Levene's test of homogeneity of variances). Conducting the same analyses for higher-order expectations across elicitation methods yields the same results. The results indicate that, both for households and firms, averages of first-order and higher-order expectations are highly congruent within each elicitation method, but differ across elicitation methods.

Our results add to the mixed results from the literature. Consistent with our results, Coibion et al. (2018) find that the point estimation tends to produce higher mean expectations than the implied mean derived from the probabilistic distribution. In contrast to our results, Rich and Tracy (2010) explicitly study the coherence between point and probabilistic inflation expectations and do not find strong and systematic differences.

Figure C1. Overview about distribution of first-order and higher-order inflation expectations



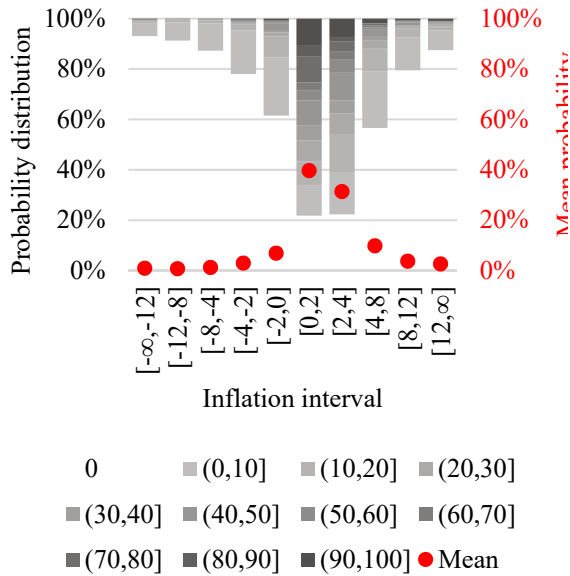
Notes: The figure shows point and probabilistic mean first-order (FO) and higher-order (HO) inflation expectations of households and firms. Household data from Panel A stems from March 2021, firm data from Panel B stems from Q4 2021.

Appendix D: Additional Figures

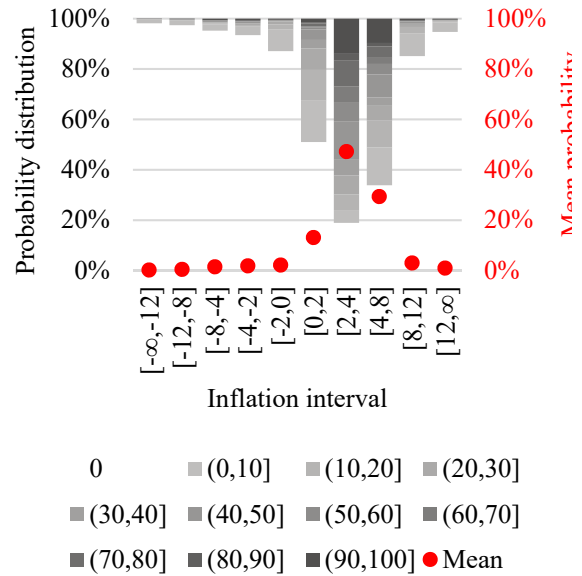
Figure D1. Probability distributions of first-order and higher-order inflation expectations

Panel A. First-order expectations

A1. Households

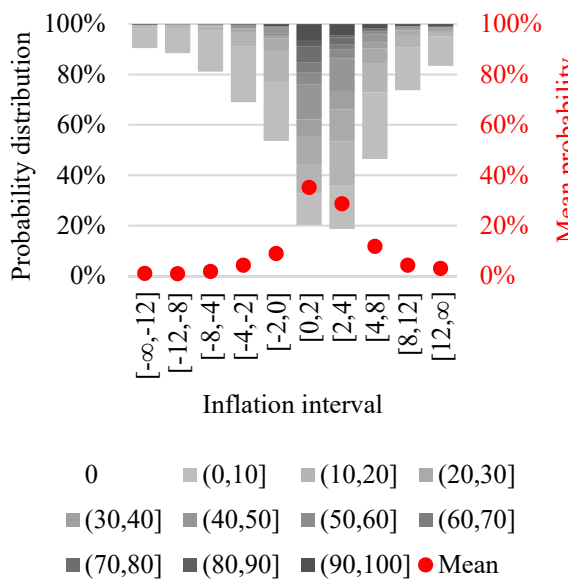


A2. Firms

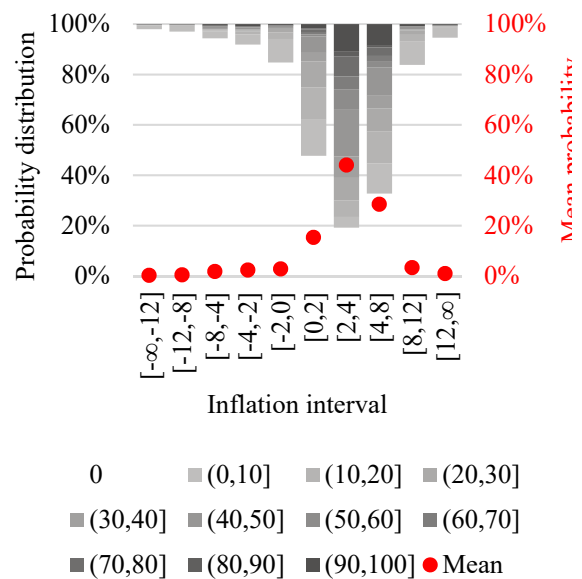


Panel B. Higher-order expectations

B1. Households



B2. Firms

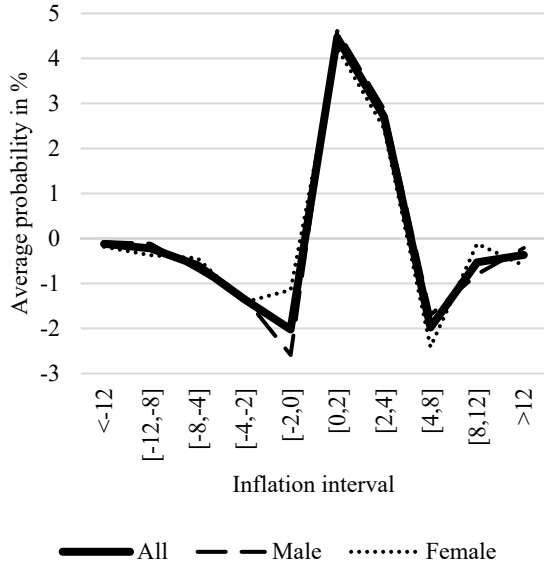


Notes: The red circles are the average probability assigned to a particular bin across all respondents and their sum equals 100%. The shaded areas show the share of respondents reporting a given probability range in a specific inflation interval. The white area refers to the category zero.

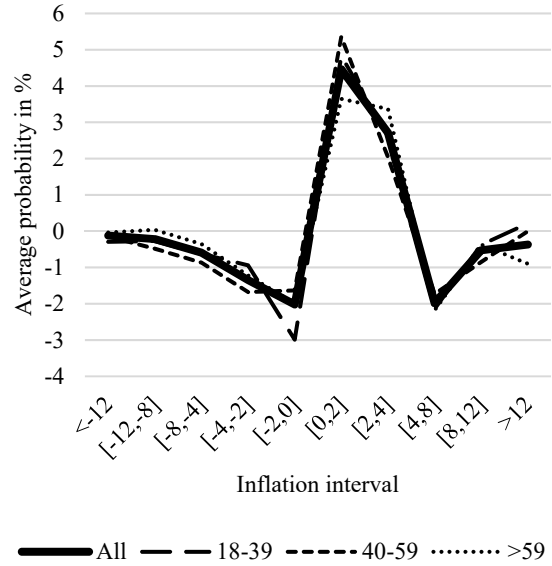
Figure D2. Distribution of differences between first-order and higher-order inflation expectations by subsamples

Panel A. Households

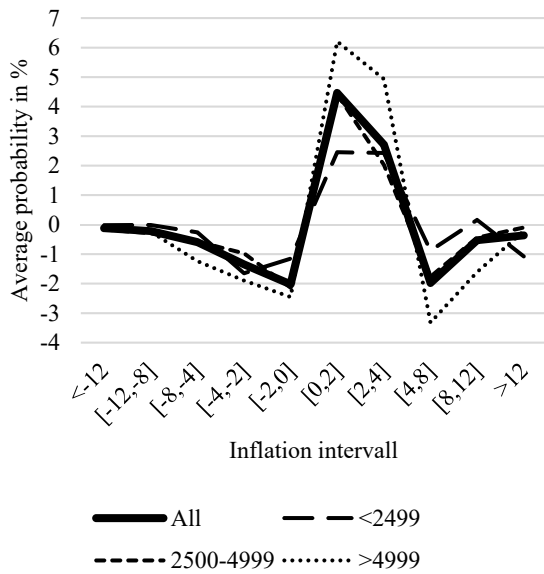
A1. Difference by gender



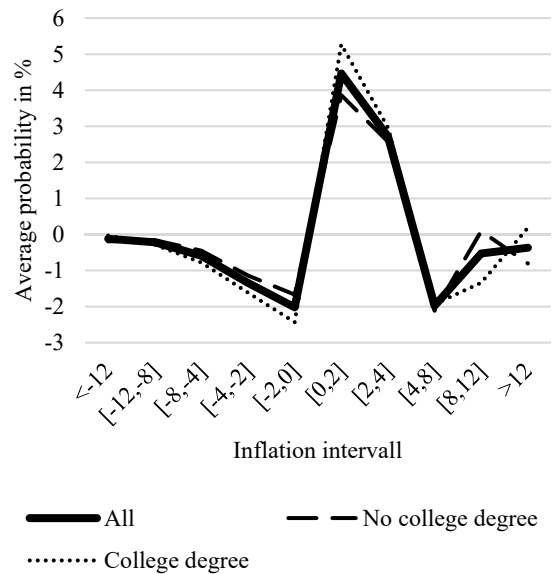
A2. Difference by age



A3. Difference by household income

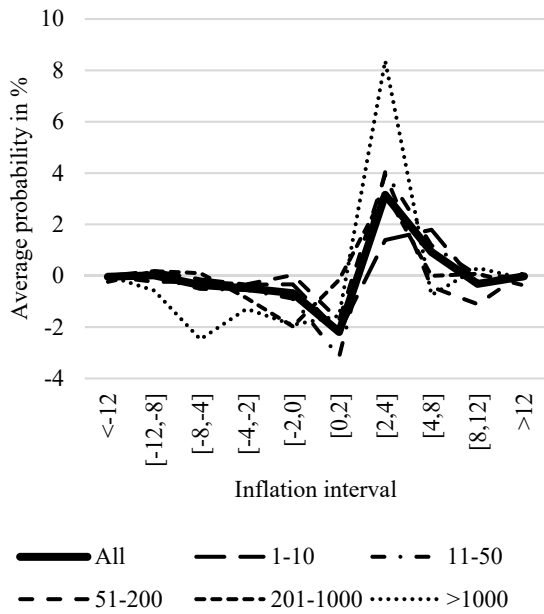


A4. Difference by education

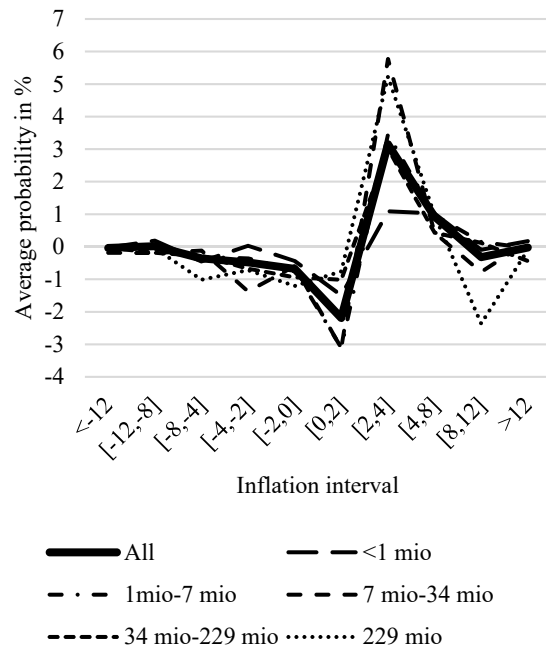


Panel B. Firms

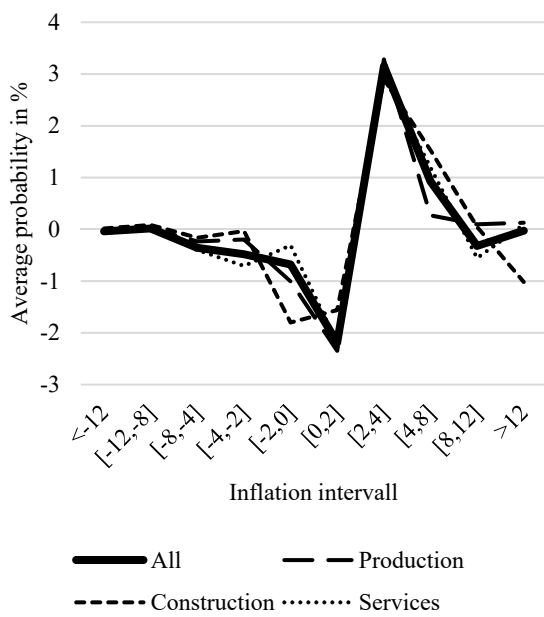
B1. Difference by number of employees



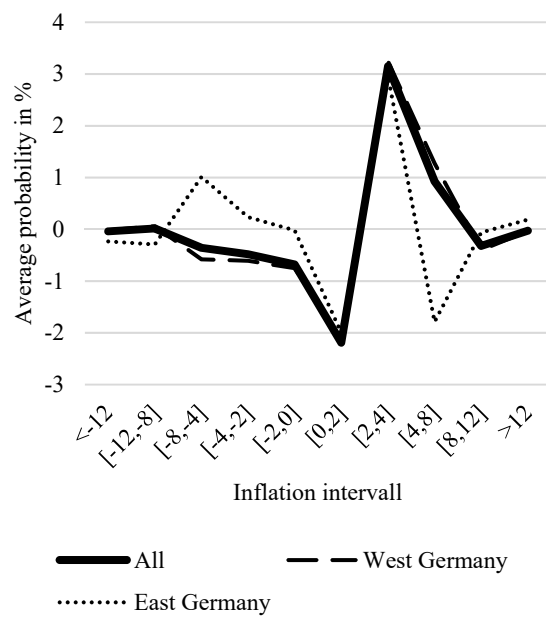
B2. Difference by turnover



B3. Difference by firm sector



B4. Difference by region



Notes: The lines indicate the difference (first-order minus higher-order expectation) in probability assigned to a specific bin.

Appendix E: Derivation of a Noisy Information Model with Heterogenous Long-Run Priors

In the following, we use a modified version of the noisy information model of Morris and Shin (2002) to demonstrate how the established relationship between first- and higher-order expectations in our surveys compare with theoretical predictions. The extension follows earlier work by Patton and Timmermann (2010) and introduces heterogeneity in prior beliefs. The derivations presented in this section directly build on a similar exercise reported in Coibion et al. (2021)'s online appendix.

Suppose firm $i \in [0,1]$ chooses to set its optimal price, p_i , as a linear combination of its expectation of a fundamental, m , and its expectation of the aggregate price level in the economy, \bar{p} :

$$(1) \quad p_i = (1 - \alpha)E_i[m] + \alpha E_i^*[\bar{p}],$$

where $\alpha \in (0,1)$ describes the degree of complementarity in pricing. Additionally, assume that $\bar{p} \stackrel{\text{def}}{=} \int_0^1 p_j dj$, such that individual i can iterate the optimal price equation forward by substituting the average optimal price equation for the aggregate price level:

$$(1') \quad p_i = (1 - \alpha)E_i[m] + \alpha E_i^* \left[\int_0^1 p_j dj \right],$$

Following Patton and Timmermann (2010), we allow manager's "long-run" prior, μ_i , to skew expectations of the aggregate price level in the economy:

$$(2) \quad \begin{aligned} E_i^*[\bar{p}] &= \omega \mu_i + (1 - \omega)E_i[\bar{p}] \\ &= \omega \mu_i + (1 - \omega)\{(1 - \alpha)E_i[\bar{E}[m]] + \alpha E_i[\bar{E}[\bar{p}]]\}, \end{aligned}$$

where $E_i^*[\bar{p}]$ denotes the skewed first-order expectation of \bar{p} , $E_i[\bar{p}] = (1 - \alpha)E_i[\bar{E}[m]] + \alpha E_i[\bar{E}[\bar{p}]]$ denotes the mathematically correct non-skewed expectation of \bar{p} . Additionally,

$\mu_i \sim N(\bar{\mu}, \kappa_\mu^{-1})$, and $\omega = \frac{\text{var}(E_i[\int p_j dj])}{\gamma^2 + \text{var}(E_i[\int p_j dj])}$ with $\gamma^2 \geq 0$ being a parameter measuring the degree

to which a firm manager prefers her own "long-run" prior, and $\bar{\mu}$ and κ_μ^{-1} measuring the average level and the dispersion of the "long-run" priors. Further, define the average expectation in the economy for variable m as $\bar{E}[m]$ and let $E_i[\bar{E}[m]]$ be the expectation of individual i about the average expectation in the economy. Similarly, let $E_i(\bar{p})$ denote the first-order expectation about the average price level, and $E_i[\bar{E}[\bar{p}]]$ the second-order expectation about the price level. We can iterate these expectations to the k th higher-order recursively:

$$\bar{E}^k[X] = \int_0^1 E_j(\bar{E}^{k-1}[X])dj.$$

To characterize how individuals form (higher-order) expectations about the fundamental, we follow Coibion et al. (2021) and assume that individuals do not possess full information. Instead of observing m perfectly, they receive one noisy public signal and one private signal. Each signal individually reflects the true value of m combined with some noise. In particular, the public signal takes the form: $y = m + \varepsilon$, where $\varepsilon \sim N(0, \kappa_y^{-1})$ and is common across firms. Moreover, each firm i also receives its own private signal about m : $x_i = m + v_i$, with $v_i \sim N(0, \kappa_x^{-1})$, and where κ_x and κ_y denote the precision of each type of signal. In order to obtain an individual expectation of m , firms weight their signals according to the relative noise in each:

$$(3) \quad E_i[m] = \frac{\kappa_y}{\kappa_x + \kappa_y} y + \frac{\kappa_x}{\kappa_x + \kappa_y} x_i = (1 - \delta)y + \delta x_i,$$

where $\delta = \frac{\kappa_x}{\kappa_x + \kappa_y}$. The intuition is straightforward, as the private signal becomes more precise relative to the public signal, the firm places relatively more weight on it in when forming beliefs about the fundamental. Aggregating Equation (3) across firms gives the average expectation about the fundamental in the economy:

$$(4) \quad \bar{E}[m] = (1 - \delta)y + \delta m = \int_0^1 E_j(m) dj$$

Firm i 's expectation about the average expectation of other managers in the economy is:

$$(5) \quad E_i[\bar{E}[m]] = (1 - \delta)y + \delta E_i[m] = (1 - \delta^2)y + \delta^2 x_i$$

By continuing to substitute $E_i[m]$ for m , one can obtain progressively higher-order expectations of m to find:

$$(6) \quad E_i[\bar{E}^k[m]] = (1 - \delta^{k-1})y + \delta^{k-1} E_i[\bar{E}^{k-1}[m]] = (1 - \delta^k)y + \delta^k x_i$$

Similarly, let $E_i[\bar{p}]$ be the first-order (own) expectation about the price level, and $E_i[\bar{E}[\bar{p}]]$ the higher-order expectation about the price level (i.e. beliefs regarding other managers' beliefs). The average (own) expectation about the price level can be written as:

$$(7) \quad \bar{E}[\bar{p}] = \omega \bar{\mu} + (1 - \omega)\{(1 - \alpha)\bar{E}^2[m] + \alpha \bar{E}^2[\bar{p}]\},$$

Following Coibion et al. (2021), we assume that only first-order expectations are skewed directly. As such, a managers' higher-order expectation is:

$$(8) \quad E_i[\bar{E}[\bar{p}]] = \omega E_i[\bar{\mu}] + (1 - \omega)\{(1 - \alpha)E_i[\bar{E}^2[m]] + \alpha E_i[\bar{E}^2[\bar{p}]]\},$$

Continuing this logic, we can identify further higher-order expectations:

$$(9) \quad \bar{E}^2[\bar{p}] = \omega \bar{E}[\bar{\mu}] + (1 - \omega)\{(1 - \alpha)\bar{E}^3[m] + \alpha \bar{E}^3[\bar{p}]\},$$

$$(10) \quad E_i[\bar{E}^2[\bar{p}]] = \omega E_i[\bar{E}[\bar{\mu}]] + (1 - \omega)\{(1 - \alpha)E_i[\bar{E}^3[m]] + \alpha E_i[\bar{E}^3[\bar{p}]]\},$$

By repeated substitutions in Equation (1), the aggregate price level becomes an average of progressively higher-order expectations of the fundamental, weighted by the complementarities present at each step:

$$(11) \quad \begin{aligned} p_i &= (1 - \alpha)E_i[m] + \alpha\omega\mu_i + \alpha(1 - \omega)(1 - \alpha)E_i[\bar{E}[m]] \\ &\quad + \alpha^2(1 - \omega)E_i[\bar{E}[p]] \\ &= (1 - \alpha)E_i[m] + \alpha\omega\mu_i + \alpha(1 - \omega)(1 - \alpha)E_i[\bar{E}[m]] \\ &\quad + \alpha^2(1 - \omega)\omega E_i[\bar{\mu}] + \alpha^2(1 - \omega)^2(1 - \alpha)E_i[\bar{E}^2[m]] \\ &\quad + \alpha^3(1 - \omega)^2\omega E_i[\bar{E}[\bar{\mu}]] \\ &\quad + \alpha^3(1 - \omega)^3(1 - \alpha)E_i[\bar{E}^3[m]] + \dots, \end{aligned}$$

which can be rewritten as:

$$(12) \quad \begin{aligned} p_i &= \alpha\omega\mu_i + \alpha\omega \sum_{k=0}^{\infty} \alpha^{k+1}(1 - \omega)^{k+1} E_i[\bar{E}^k[\bar{\mu}]] + (1 \\ &\quad - \alpha) \sum_{k=0}^{\infty} \alpha^k(1 - \omega)^k [(1 - \delta^{k+1})y + \delta^{k+1}x_i] \end{aligned}$$

Equations (11) and (12) reflect that optimal decisions of firms depend not just on their expectations of the fundamental, but also what they think others think about the fundamental, and so on.

Following Coibion et al. (2021), we next impose some structure on $E_i[\bar{E}^k[\bar{\mu}]]$. Since the optimal price depends on the individual's expectations of the average prior, $\bar{\mu}$, we allow this mean to be unknown, but let each manager observe a private signal of the mean: $\zeta_i \sim N(\bar{\mu}, \kappa_{\zeta}^{-1})$.

We assume that the manager's own "long-run" prior skews her view of the aggregate prior:

$$(13) \quad E_i[\bar{\mu}] = w'\mu_i + (1 - w')\zeta_i$$

where $w' = \frac{\kappa_{\zeta}^{-1}}{(\gamma')^2 + \kappa_{\zeta}^{-1}}$ and $(\gamma')^2 \geq 0$ is again a parameter measuring the degree to which a firm manager prefers her own "long-run" prior when forming beliefs about $\bar{\mu}$. Given that the average of expectations $E_i[\bar{\mu}]$ in Equation (13) is $\bar{E}[\bar{\mu}] = \bar{\mu}$, it follows that $E_i[\bar{E}[\bar{\mu}]] = w'\mu_i + (1 - w')\zeta_i$ and $\bar{E}^2[\bar{\mu}] = \bar{\mu}$. Continuing this logic and by using repeated substitutions, we can show that the expectation for all orders of expectations of the aggregate prior are the same, i.e.

$\bar{E}^k[\bar{\mu}] = \bar{\mu}$ for any k . Replacing $\bar{E}^k[\bar{\mu}] = \bar{\mu}$ in the optimal pricing equation (12), the formula can be rewritten as:

$$(14) \quad p_i = \alpha\omega\mu_i + \alpha\omega \sum_{k=0}^{\infty} \alpha^{k+1}(1-\omega)^{k+1}(w'\mu_i + (1-w')\zeta_i) + (1 - \alpha) \sum_{k=0}^{\infty} \alpha^k(1-\omega)^k[(1-\delta^{k+1})y + \delta^{k+1}x_i]$$

Coibion et al. (2021) proceeds to rewrite Equation (13) in terms of strategies:

$$(15) \quad p_i = \phi_\mu\mu_i + \phi_\zeta\zeta_i + \phi_x x_i + \phi_y y$$

where

$$(16) \quad \phi_\mu = \alpha\omega \left[1 + \frac{\alpha(1-\omega)}{1-\alpha(1-\omega)} \omega' \right]$$

$$(17) \quad \phi_\zeta = \alpha\omega \left[\frac{\alpha(1-\omega)}{1-\alpha(1-\omega)} (1-\omega') \right]$$

$$(18) \quad \phi_x = \frac{\delta(1-\alpha)}{1-\alpha\delta(1-\omega)}$$

$$(19) \quad \phi_y = \frac{(1-\alpha)}{1-\alpha(1-\omega)} - \frac{\delta(1-\alpha)}{1-\alpha\delta(1-\omega)}$$

We can simplify notation by defining

$$(20) \quad \theta \stackrel{\text{def}}{=} \phi_\mu + \phi_\zeta = \frac{\alpha\omega}{1-\alpha(1-\omega)}$$

such that

$$(21) \quad 1 - \theta = \phi_x + \phi_y = \frac{(1-\alpha)}{1-\alpha(1-\omega)}$$

If long-run priors do not matter, we would receive $\theta = 0$, as both $\omega = \omega' = 0$. Given Equation (15) and substituting for θ , the aggregate price level is:

$$(22) \quad \bar{p} = \theta\bar{\mu} + \phi_x m + \phi_y y.$$

Expectations about the aggregate price level are formed in line with Equation (22), with the weight assigned to the “long-run” prior:

$$(23) \quad \begin{aligned} E_i(\bar{p}) &= \omega\mu_i + (1-\omega)[\theta E_i[\bar{\mu}] + (1-\theta)E_i(m)] \\ &= \omega\mu_i + (1-\omega)[\theta E_i[\bar{\mu}] + \phi_x \delta x_i + ((1-\phi_x\delta) - \theta)y] \end{aligned}$$

$$= (\omega + (1 - \omega)\theta w')\mu_i + (1 - \omega)\theta(1 - \omega')\zeta_i + (1 - \omega)[\phi_x\delta x_i + ((1 - \phi_x\delta) - \theta)y]$$

The average expected price is thus:

$$(24) \quad \bar{E}[\bar{p}] = (\omega + (1 - \omega)\theta)\bar{\mu} + (1 - \omega)[\phi_x\delta E_i[m] + ((1 - \phi_x\delta) - \theta)y]$$

Manager i then beliefs other managers to believe:

$$(25) \quad \begin{aligned} E_i[\bar{E}[\bar{p}]] &= (\omega + (1 - \omega)\theta)E_i[\bar{\mu}] \\ &\quad + (1 - \omega)[\phi_x\delta E_i[m] + ((1 - \phi_x\delta) - \theta)y] \\ &= (\omega + (1 - \omega)\theta)\omega'\mu_i + (\omega + (1 - \omega)\theta)(1 - \omega')\zeta_i + (1 - \omega)[\phi_x\delta^2 x_i + ((1 - \phi_x\delta^2) - \theta)y] \end{aligned}$$

Average higher-order expectations are thus:

$$(26) \quad \bar{E}^2[\bar{p}] = (\omega + (1 - \omega)\theta)\bar{\mu} + (1 - \omega)[\phi_x\delta^2 m + ((1 - \phi_x\delta^2) - \theta)y]$$

The difference between average higher-order and first-order expectations is given by:

$$(27) \quad \bar{E}^2[\bar{p}] - \bar{E}[\bar{p}] = (1 - \omega)\phi_x\delta(1 - \delta)(y - m) > 0$$

We follow Coibion et al. (2021) by using Equations (23) – (26) to find expressions for cross-sectional disagreement and for forecast uncertainty:

$$(28) \quad \begin{aligned} Var[E_i[\bar{p}]] &= (\omega + (1 - \omega)\theta\omega')^2\kappa_\mu^{-1} + ((1 - \omega)\theta(1 - \omega'))^2\kappa_\zeta^{-1} \\ &\quad + (1 - \omega)^2(\phi_x\delta)^2\kappa_x^{-1} \end{aligned}$$

$$(29) \quad \begin{aligned} Var[E_i[\bar{E}[\bar{p}]]] &= ((\omega + (1 - \omega)\theta)\omega')^2\kappa_\mu^{-1} \\ &\quad + ((\omega + (1 - \omega)\theta)(1 - \omega'))^2\kappa_\zeta^{-1} \\ &\quad + (1 - \omega)^2(\phi_x\delta^2)^2\kappa_x^{-1} \end{aligned}$$

$$(30) \quad \Omega_{\{E_i[\bar{p}]\}|\mathcal{Y}} = ((1 - \omega)\theta(1 - \omega'))^2\kappa_\zeta^{-1} + (1 - \omega)^2(\phi_x\delta)^2\kappa_x^{-1}$$

$$(31) \quad \begin{aligned} \Omega_{\{E_i[\bar{E}[\bar{p}]]\}|\mathcal{Y}} &= ((\omega + (1 - \omega)\theta)(1 - \omega'))^2\kappa_\zeta^{-1} \\ &\quad + (1 - \omega)^2(\phi_x\delta^2)^2\kappa_x^{-1} \end{aligned}$$

The derivations in Equations (30) and (31) assume that each manager knows his own “long-run” prior with certainty and is not considering that his “long-run” prior differs from the aggregate prior. Importantly, by using Equations (28) and (29), we can investigate the relative magnitude of disagreement in first-order and higher-order expectations. In a similar spirit, we

can use Equations (30) and (31) to investigate the relative magnitude of uncertainty in first-order and higher-order expectations. We start by deriving predictions regarding disagreement in first- and higher-order expectations:

$$\begin{aligned}
(32) \quad \text{Var} [E_i[\bar{E}[\bar{p}]]] - \text{Var}[E_i[\bar{p}]] &= (2(1 - \omega)(\omega\theta\omega')(\omega' - 1) + \omega^2(\omega'^2 - 2))\kappa_\mu^{-1} \\
&+ \left[\omega \left((\omega + (1 - \omega)\theta) + ((1 - \omega)\theta) \right) \right] (1 \\
&- \omega')^2 \kappa_\zeta^{-1} + (1 - \omega)^2 (\phi_x \delta)^2 (\delta^2 - 1) \kappa_x^{-1}
\end{aligned}$$

Next, we derive predictions regarding uncertainty in first- and higher-order expectations:

$$\begin{aligned}
(33) \quad \Omega_{\{E_i[\bar{E}[\bar{p}]|y]\}} - \Omega_{\{E_i[\bar{p}]|y\}} &= \left[\omega \left((\omega + (1 - \omega)\theta) + ((1 - \omega)\theta) \right) \right] (1 \\
&- \omega')^2 \kappa_\zeta^{-1} + (1 - \omega)^2 (\phi_x \delta)^2 (\delta^2 - 1) \kappa_x^{-1}
\end{aligned}$$

Importantly, the expressions for disagreement and uncertainty in Equations (32) and (33) do not make a clear prediction regarding the relative magnitude of uncertainty in first- and higher-order expectations. In contrast, in the baseline model of Coibion et al. (2021), uncertainty (disagreement) in higher-order expectations is always lower than uncertainty (disagreement) in first-order expectations. In the extension with heterogenous “long-run” priors, this relation is ambiguous and depends on relative magnitudes of κ_ζ^{-1} and κ_x^{-1} . In particular, if κ_x^{-1} and κ_μ^{-1} are large relative to κ_ζ^{-1} the model produces $\Omega_{\{E_i[\bar{E}[\bar{p}]|y]\}} < \Omega_{\{E_i[\bar{p}]|y\}}$ as well as $\text{Var} [E_i[\bar{E}[\bar{p}]]] < \text{Var}[E_i[\bar{p}]]$, which is the finding of Coibion et al. (2021). However, if κ_x^{-1} and κ_μ^{-1} are small relative to κ_ζ^{-1} , the model produces $\Omega_{\{E_i[\bar{E}[\bar{p}]|y]\}} > \Omega_{\{E_i[\bar{p}]|y\}}$ as well as $\text{Var} [E_i[\bar{E}[\bar{p}]]] > \text{Var}[E_i[\bar{p}]]$, which is consistent with our findings. It is important to note that the relation between uncertainty and disagreement in first- versus higher-order expectations always moves in tandem as they rely on the same conditions.