Discussion Paper

Deutsche Bundesbank
No 19/2024

Listening to the noise: On price efficiency with dynamic trading

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Internet http://www.bundesbank.de

ISBN 978–3–95729–994–9
ISSN 2941–7503

Non-technical summary

Research Question

Since the start of the COVID-19 pandemic, a global boom in retail investing has occurred in financial markets. In light of this boom, professional investors such as hedge funds and banks have been analyzing the communication behavior of retail investors in stock message boards, extracting sentiment indicators that influence their own trading behavior (so-called *social sentiment investing*). Resorting to a theoretical model, this paper explores the impact of social sentiment investing on financial market bubbles. Our research question is the following: Does social sentiment investing counteract or amplify financial market bubbles that stem from retail investors' trading behavior?

Contribution

To the best of our knowledge, our research paper is the first to address the impact of social sentiment investing in financial markets within a theoretical model. Notably, there already exists a relevant strand of literature that deals with the trading behavior of professional investors collecting information on retail trading. However, all relevant publications have so far disregarded the phenomenon of social sentiment investing. Our paper thus provides new theoretical insights that are relevant to the aforementioned strand of literature.

Results

The central result of our theoretical analysis says that social sentiment investing potentially destabilizes financial markets and makes financial bubbles even grow further. This outcome contrasts sharply with the conventional wisdom derived from the relevant literature that professional investors who collect information on retail trading unequivocally stabilize financial markets. Our theoretical model reveals that social sentiment investing makes professional investors engage in so-called front running, which potentially contributes to distorting market prices.

Nichttechnische Zusammenfassung

Fragestellung

Auf Finanzmärkten ist seit der Corona-Pandemie ein Kleinanleger-Boom zu beobachten. Professionelle Anleger wie Hedgefonds und Banken analysieren seitdem das Kommunikationsverhalten von Kleinanlegern auf Aktienforen und extrahieren hierbei Sentiment-Indikatoren, die sie bei ihrem eigenen Handelsverhalten auf Finanzmärkten berücksichtigen (sogenanntes *Social Sentiment Investing*). Dieses Papier untersucht in einem theoretischen Modell, welchen Einfluss Social Sentiment Investing auf Blasenbildungen in Finanzmärkten ausübt. Unsere Forschungsfrage lautet: Konterkariert Social Sentiment Investing Finanzmarktblasen, die auf dem Handelsverhalten von Kleinanlegern beruhen, oder verstärkt es diese womöglich?

Beitrag

In unserem Forschungspapier wird sich erstmals innerhalb eines theoretischen Modells mit den Auswirkungen von Social Sentiment Investing auf Finanzmärkten beschäftigt. Es existieren bereits einige Beiträge in der relevanten Literatur, welche die Effekte des Handelsverhaltens von professionellen Investoren, die Informationen über Kleinanleger sammeln, erforschen. Jedoch lassen sämtliche einschlägige Publikationen Social Sentiment Investing bisher außer Acht. Unser Papier liefert somit neue Erkenntnisse, die für den erwähnten Literaturstrang von Relevanz sind.

Ergebnisse

Das zentrale Ergebnis unserer theoretischen Analyse ist, dass sich Social Sentiment Investing potenziell destabilisierend auf Finanzmärkte auswirkt und Blasenbildungen zusätzlich befeuern kann. Dieses Resultat kontrastiert in starkem Maße mit dem gängigen Resultat in der theoretischen Finanzmarktforschung, dass professionelle Händler, die Informationen über Kleinanleger sammeln, stets stabilisierend auf Finanzmärkte wirken. Unser theoretisches Modell offenbart, dass professionelle Investoren mithilfe von Social Sentiment Investing sogenanntes *Front-running* betreiben und hierdurch Marktpreise verzerren können.

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Listening to the Noise: On Price Efficiency with Dynamic Trading*

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Abstract

This paper shows that, in the canonical dynamic rational expectations equilibrium model, public information about *future* noise trading is potentially detrimental to contemporaneous price efficiency. Our result supports concerns that social sentiment investing, sparked by growing availability of big data and advances in the way of processing it, exacerbates, rather than ameliorates, the negative impact of noise trading on price efficiency.

JEL classification: G12, G14
Keywords: social sentiment investing, price efficiency, noise trading, information aggregation

*Declaration of interest: none.

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1 Introduction

This paper investigates the impact of a public signal about future noise trading in a dynamic noisy rational expectations equilibrium (REE) model. We demonstrate that an *increase* in the precision of public information about future noise trader demand potentially *reduces* the efficiency of the current asset price as a signal about the asset's fundamental value in the unique linear equilibrium of the model. In particular, the relation between signal precision and price efficiency cannot be monotonically increasing, but can be monotonically decreasing. That is, informed trading based on signals about noise trading has the potential to aggravate, rather than ameliorate, the negative impact of noise trading on price efficiency. This contrasts sharply with the results of two related strands of the REE literature, viz., that information about noise increases price efficiency in a static REE (see Ganguli and Yang, 2009; Manzano and Vives, 2011; Marmora and Rytchkov, 2018; Zeng et al., 2018) and that public information about fundamentals increases price efficiency in a dynamic REE (see Gao, 2008). Our result lends support to Goldstein et al.'s (2021, p. 3222) conjecture that "[A]lthough big data provides more information for sophisticated players such as institutional investors and firms, the impact of big data may not always be positive."

Our analysis of public information about noise trading is motivated by two current developments going on in the finance industry: rapid growth in the amount of data related to retail investing propagated via social media and intensifying use of these data by professional investors based on new methods in big data processing. Information about retail traders' investments comes from money blogs, the online financial press, search queries, and, in particular, stock message boards. The January 2021 *GameStop* short squeeze is a popular case in point. The buy orders that caused the *GameStop* stock to rise from $ 20 to $ 480 were coordinated via stock message boards such as *WallStreetBets*.[1] Subsequently, the shares of *AMC Entertainment* and *BlackBerry* became meme stocks, with soaring prices after going viral online.[2] Financial analysts and professional investors make use of advances in textual analysis, machine learning, and computing power in order to exploit the big data propagated in the social media for the development of new investment strategies. Social sentiment investing (or "Trading on Twitter", see Sul et al., 2017) takes on different forms. Analysts produce

[1] See https://www.businessinsider.com/melvin-capital-lost-53-percent-january-after-gamestop-shares-skyrocketed-2021-1.

[2] See https://www.thebalance.com/what-is-a-meme-stock-5118074.

and sell social sentiment indicators, such as S-scores, which yield quantitative information about social media users' mood or sentiment and can be used as an input in the development of trading strategies. Other market participants do not disclose the output of their big data analyses of retail investing, but use it to devise their own investment strategies. In March 2021, *VanEck*, an investment management company, launched the first social sentiment ETF, called "BUZZ", based on textual analysis of X (then Twitter) and several other social media platforms.[3] The growing importance of sentiment investing has been acknowledged by SEC chairman Gary Gensler in his testimony before the U.S. House Committee on Financial Services on May 6, 2021 relating to the recent volatility in the prices of *GameStop* and other meme stocks:

> "it's no longer just retail investors or even humans who are following these online conversations, but institutional investors and their algorithms. Developments in machine learning, data analytics, and natural language processing have allowed sophisticated investors to monitor various forms of public communication to see relationships between words and prices. This practice, called sentiment analysis, has picked up steam in the last couple of years, and it has grown to include online communities."[4]

The framework we use in order to analyze the impact of information about future social sentiment on price efficiency is the canonical dynamic REE model with noise traders. The noise trader approach to finance has been popularized by Black (1986) and Shleifer and Summers (1990).[5] Following Barber et al. (2009), Foucault et al. (2011), Peress and Schmidt (2019, 2021), Barber et al. (2022), and Eaton et al. (2022), among others, we interpret noise traders as retail investors acting on social sentiment. Total noise trader demand is exogenous. That is, we do not model the interactions between different noise traders caused by the dissemination of social sentiment (see Semenova and Winkler, 2021), but treat noise trader demand as the aggregate outcome of these social interactions.

We introduce information about current and future noise trading as costless public signals.

[3]See `https://www.cnbc.com/2021/03/04/buzz-etf-tracking-social-media-sentiment-launches-thursday-amid-reddit-manias-in-stocks.html`.

[4]See `https://www.sec.gov/news/testimony/gensler-testimony-20210505`.

[5]Shocks to net asset supply can alternatively be interpreted as shocks to rational investors' asset endowments. The two alternative interpretations lead to similar results. Given our focus on information about noise, we adopt the noise trader interpretation. Vives (2008, Section 4.4) explains how fluctuations in net supply can be endogenized by incorporating risk-averse hedgers.

The fact that advances in machine learning algorithms, textual analysis, and computing power allow quick and cheap evaluation of sentiment data (cf. Zhou, 2018, Subsection 4.3) justifies the assumption that observing the signals involves little cost for the rational traders. The fact that the data stem from the same publicly available internet sources supports the assumption that different traders observe the same signal. The assumption that social sentiment reveals information about *future* noise trading is motivated by evidence that social sentiment indicators have predictive power for future prices and stock returns. This has been shown, e.g., for postings on Yahoo! message boards (see Wysocki, 1998; Antweiler and Frank, 2004; Das and Chen, 2007), Facebook's Gross National Happiness Index (Karabulut, 2013; Siganos et al., 2014), internet search behaviour of private households (Da et al., 2015), and Tweets (Sul et al., 2017; Duz and Tas, 2020). Zhou (2018, p. 250) notes that "[I]n comparison with market- and survey-based measures, ... measures based on textual analysis perform better by far" (Baker and Wurgler, 2006, is the classic study of market-based sentiment indicators).

Price efficiency is a central outcome measure for financial markets with asymmetric information. High price efficiency means that market participants that do not have access to other signals than prices still have available valuable information for informed portfolio decisions. Following Hayek (1945), it is often held that price efficiency also contributes to an efficient allocation of resources (i.e., to real efficiency) by providing accurate signals to investors. While recent research has shown that the nexus between price efficiency and real efficiency is not as close as originally suspected (see Bond et al., 2012; Goldstein and Yang, 2017, Section 4), the argument still carries weight. Finally, the extraction of information from market prices can be helpful for regulators and other policymakers. We focus on the most commonly used metric for price efficiency, viz., the inverse of the variance of fundamentals conditional on prices (see Goldstein and Yang, 2017, p. 106; see also Vives, 2008, pp. 121-122; Ganguli und Yang, 2009, p. 99; Manzano and Vives, 2011, Subsection 4; Marmora und Rytchkov, 2018, Section 4.1; Zeng et al., 2018, Section 3; Farboodi und Veldkamp, 2020, Section 2; among others). We show that the use of alternative measures of price efficiency proposed in the literature (e.g., by Grossman and Stiglitz, 1980; Mendel and Shleifer, 2012; Li, 2022) leads to similar conclusions. Bai et al. (2016) and Farboodi et al. (2022) provide interesting recent attempts to quantify price efficiency empirically. They find that growth of finance was accompanied by increases in price efficiency in the U.S. since the 1960s. Interestingly, however, the descriptive statistics in Farboodi et al. (2022, Table 2, p. 3115) show a strong

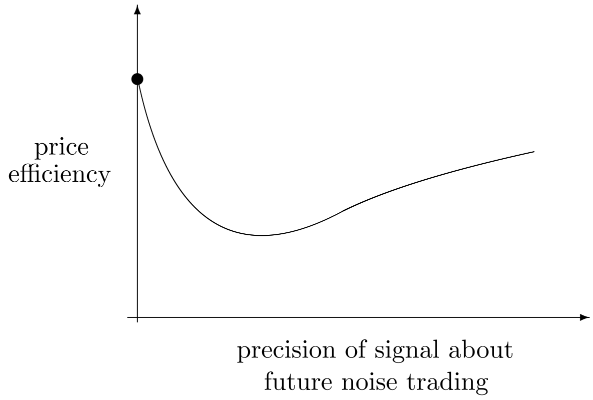

Figure 1: The impact of an increase in the precision of a signal about future noise trading is not unambiguously positive

decline in price efficiency in the 2010s (the end of their sample), which, consistently with our model, coincides with the rise of sentiment investing.

Our main result, illustrated in Figure 1, is that in the unique REE of our model, an *increase* in the precision of public information about future noise trader demand potentially *reduces* the efficiency of the current asset price about the asset's fundamentals. This holds true both in a version of the dynamic model with long-lived agents (LLA) and in a variant with overlapping generations (OLG) of investors and short-term trading. Contemporaneous price efficiency depends on three factors. First, current noise trader demand reduces price efficiency. Second, by trading against noise traders, rational traders weaken the impact of noise trading. These two effects are also present in a static setting, and higher precision of a signal about contemporaneous noise makes the net effect smaller, thereby raising price efficiency. The additional, third effect present in the dynamic model is due to the fact that, other than in the static model, the return on investment is determined by the resale price rather than fundamentals prior to the final trading date. A signal about future noise helps rational investors anticipate the effect of future noise on the resale price. The ensuing effect on current asset demand comes at the expense of reduced contemporaneous price efficiency. The net effect of an increase in the precision of a signal about future noise via these three channels is possibly negative. A simple example is the special case of the LLA model with no signal about contemporaneous noise trading. The strengths of the two effects also present in the static setting are independent of the precision of the signal about future noise in this case, so that the changes in signal precision affect price efficiency only through the additional, third channel, operating through the resale price. Zero precision implies maximum price efficiency, because this is the only finite value for which traders ignore the signal, so that the additional negative effect vanishes. Generally, price efficiency is higher with a very imprecise than with a very precise signal, meaning that the relation between the two cannot be monotonically increasing, whereas it can be monotonically decreasing. In the OLG version of the model, price efficiency is generally lower than in the LLA model (with identical model parameters), but an increase in the precision of the signal about future noise trading is less likely to reduce price efficiency.

Related literature includes models that investigate the impact of non-fundamental information on price efficiency in REE models, such as Ganguli and Yang (2009), Manzano and Vives (2011), Marmora and Rytchkov (2018), and Zeng et al. (2018). Information about noise is conducive to price efficiency in these models in stable static equilibria. More precise non-fundamental information can lead to a fall in price efficiency in unstable equilibria (see Manzano and Vives, 2011, Section 4). However, price efficiency is always higher in the presence than in the absence of non-fundamental information. We show that these results do not carry over to information about future sentiment in the dynamic setting. Gao (2008) shows that noisy public information about fundamentals is conducive to price efficiency in the dynamic REE model. Our analysis demonstrates that the same does not hold true for non-fundamental information. Farboodi and Veldkamp (2020) analyze a model with growth in information processing capacity that can be used to produce combinations of fundamental and private non-fundamental information (similar as in Marmora and Rytchkov, 2018). They show that not only fundamental information but also the non-fundamental information traders acquire is conducive to price efficiency. Again, this contrasts with our finding that the impact of non-fundamental information on price efficiency can be negative. Finally, our model is related to papers that analyze the interaction between rational traders and noise traders using different setups than the REE setup of Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981). De Long et al. (1990) show that rational investors may drive prices away from fundamentals if noise traders follow a positive feedback investment strategy. Similarly, Madrigal (1996) and Yang and Zhu (2017) show that in a Kyle (1985) setup with endogenous market making and strategic behavior, the presence of a non-fundamental speculator can harm price efficiency. In a similar framework, Sadzik and Woolnough (2021) demonstrate that a trader with information about the persistent component of noise trader demand can destabilize prices by amplifying its impact on prices.

Brunnermeier et al. (2022) show that rational investors may harm price efficiency by speculating on information about noise originating from government intervention in the financial market.

The remainder of this paper is organized as follows. As a benchmark for the subsequent dynamic analysis, Section 2 analyzes the static version of the REE model with a public signal about noise. Sections 3 and 4 analyze the LLA and OLG versions of the dynamic model, respectively. Section 5 concludes. Proofs of propositions are collected in the Appendix.

2 Static model

This section shows that in a static setup, public information about noise trading increases the efficiency of the asset price (cf., e.g., Manzano and Vives, 2011).

Model

Consider a static one-good economy populated by a unit mass of rational investors indexed by the interval $[0, 1]$ and by a set of noise traders. Rational agents value consumption π according to the CARA utility function $-\exp(-\gamma^{-1}\pi)$, where γ (> 0) is their risk tolerance (i.e., the inverse of the degree of absolute risk aversion). There are one risky and one safe asset. The net supply of the risky asset is fixed and normalized to zero for simplicity. One unit of the risky asset pays off θ units of consumption.[6] The safe asset is in perfectly elastic supply. Its rate of return is zero. Initial endowments are also normalized to zero. Agents trade the assets in the financial market. Noise trader demand for the risky asset s is exogenous. Rational agents maximize expected utility conditional on available information. Agent i obtains a private signal $x_i = \theta + \varepsilon_i$ about θ. In addition, there is a public signal about noise trader demand

$$Y = s + \eta \tag{1}$$

observed by each rational trader. The assumption that the signal is costless and the same for each rational trader is motivated in the Introduction by the observation that social sentiment investors search the same internet sources for data, which can be processed at relatively low

[6] The simplifying assumption that the net supply of the risky asset is zero on average is taken from Vives' (2008, Section 4.2) canonical REE model. It implies that, here as well as in the dynamic model in the subsequent section, the (unconditionally) expected asset price coincides with $\mathrm{E}\theta$. With positive mean net supply, there would be a positive risk premium that compensates rational traders for the risk they bear in equilibrium (see Vives, 2008, p. 120).

cost. The random variables $\xi \in \{\theta, s, \varepsilon_i, \eta \,|\, i \in [0,1]\}$ are jointly normally and independently distributed. The means of these variables are normalized to zero. The precision of ξ is denoted τ_ξ (i.e., the variance of ξ is τ_ξ^{-1}). Our focus is on the impact of τ_η on price efficiency.[7]

Rational expectations equilibrium

Suppose the price of the risky asset is a linear function of θ, s, and Y:

$$P = a\theta + bs - cY \tag{2}$$

for real numbers a, b, and c. Investor i extracts information about the asset payoff θ from the two signals she receives as well as from the asset price. The vector of her signals is denoted $I_i = (P, x_i, Y)$. Her investment in the risky asset and her final wealth are denoted D_i and $\pi_i = (\theta - P)D_i$, respectively.

Definition (rational expectations equilibrium): A price function (2) and asset demands D_i, $i \in [0,1]$, are a linear *rational expectations equilibrium (REE)* if D_i maximizes expected utility $\mathrm{E}[-\exp(\gamma^{-1}\pi_i) \,|\, I_i]$ for all $i \in [0,1]$ and the market for the risky asset clears, i.e., $\int_0^1 D_i di + s = 0$.

Expected utility maximization yields the asset demands

$$D_i = \gamma \, \frac{\mathrm{E}(\theta \,|I_i) - P}{\mathrm{var}(\theta \,|\, I_i)}. \tag{3}$$

The market clearing price P contains useful information about fundamentals θ since it aggregates individuals' private signals x_i. The public signal about noise Y also conveys valuable information about θ, even though the two random variables are uncorrelated. This is because it helps to disentangle the impacts of the private signals about fundamentals on the one hand and noise trader demand on the other hand on P. Define

$$P^* \equiv \frac{1}{a}\left[P - \left(\frac{\tau_\eta}{\tau_s + \tau_\eta} b - c\right) Y\right]. \tag{4}$$

Using (1) and (2),

$$P^* = \theta + \frac{1}{\rho} \frac{\tau_s s - \tau_\eta \eta}{\tau_s + \tau_\eta},$$

[7] Incorporating costly information acquisition in an "information market" raises the issue of complementarity and multiplicity of equilibria (see Ganguli and Yang, 2009; Manzano and Vives, 2011; Zeng et al., 2018; Russ, 2022). The absence of complementarities and, hence, multiple equilibria ensures that the comparative statics with respect to τ_η are well defined in our model.

where $\rho \equiv a/b$. P^* is a signal about θ with precision $\rho^2(\tau_s+\tau_\eta)$. It aggregates the information contained in P and Y, so $\mathrm{E}(\theta \,|\, I_i) = \mathrm{E}(\theta \,|\, P^*, x_i)$ and $\mathrm{var}(\theta \,|\, I_i) = \mathrm{var}(\theta \,|\, P^*, x_i)$ (formally, this follows from the projection theorem). The conditional moments in i's asset demand function (3) are thus

$$
\begin{aligned}
\mathrm{E}(\theta \,|\, I_i) &= \frac{\tau_\varepsilon x_i + \rho^2(\tau_s+\tau_\eta)P^*}{\tau_\theta + \tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)} \\
\mathrm{var}(\theta \,|\, I_i) &= \frac{1}{\tau_\theta + \tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)}.
\end{aligned}
$$

Substituting the demands into the asset market clearing condition, applying the strong law of large numbers (i.e., $\int_0^1 \varepsilon_i di = 0$), using the definition of P^*, and solving for P yields

$$
P = \frac{a\tau_\varepsilon\theta + a\gamma^{-1}s + [\rho^2(\tau_s+\tau_\eta)c - a\rho\tau_\eta]\,Y}{a(\tau_\theta+\tau_\varepsilon) + (a-1)\rho^2(\tau_s+\tau_\eta)}.
$$

By matching coefficients with (2), we obtain:

Proposition 1: *There exists a unique linear REE, with*

$$
\begin{aligned}
\rho &= \gamma\tau_\varepsilon \\
a &= \frac{\tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)}{\tau_\theta + \tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)} \\
b &= \frac{a}{\gamma\tau_\varepsilon} \\
c &= \frac{a\rho\tau_\eta}{\tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)}.
\end{aligned}
$$

Proposition 1 provides a closed-form solution for the parameters that characterize a linear REE. The linear REE is unique.

The special case with $\tau_\eta = 0$ is the textbook REE model with no useful information about noise (cf. Vives, 2008, Section 4.2). From Proposition 1 and (4), $c = 0$ and $P^* = P/a$, so $\mathrm{E}(\theta| \, I_i)$ is a weighted sum of the private signal x_i and the asset price P in this case. For $\tau_\eta > 0$, the coefficient of Y in (4) is negative:

$$
-\left(\frac{\tau_\eta}{\tau_s+\tau_\eta}\frac{b}{a} - \frac{c}{a}\right) = -\frac{1}{\rho}\frac{\tau_\eta}{\tau_s+\tau_\eta}\left[1 - \frac{\rho^2(\tau_s+\tau_\eta)}{\tau_\varepsilon + \rho^2(\tau_s+\tau_\eta)}\right] < 0.
$$

Thus, for a given private signal and a given asset price, an increase in the value of the signal

Y decreases P^* and $\mathrm{E}(\theta|\, I_i)$, as it makes it more likely that high noise trader demand rather than sound fundamentals support the asset price. From (3), rational traders' asset demand goes down, and, from (2), the strength of the ensuing drop in the equilibrium market price P is given by c. As one would expect, c is large when the precision of the signal τ_η is high. c/a measures how strongly the asset price reacts to the public signal about noise relative to the average of the private signals about fundamentals (i.e., to $\int_0^1 x_i di = \theta$).

Rational traders are compensated for the risk they carry in an REE. So noise traders underperform the market on average, as their buy orders drive up the price and their short sales drive it down. The presence of the signal about noise Y reduces their expected losses, as it allows rational traders to partly offset the impact of noise trading on the asset price. Formally, this follows from

$$\mathrm{E}[(\theta - P)s] = -\mathrm{E}(Ps) = -(b-c)\frac{1}{\tau_s} < 0$$

(from (2) and Proposition 1) and $b > c$ for $\tau_\eta > 0$.

Price efficiency

The measure of price efficiency we focus on is the inverse of the variance of fundamentals conditional on the price, i.e., $\mathrm{var}^{-1}(\theta \,|\, P)$. Appendix B shows that the use of two alternative measures leads to analogous results, viz., the squared correlation between θ and P (cf. Grossman and Stiglitz, 1980, p. 399; Li, 2022) and the ratio of "good variance" to "bad variance" proposed by Li (2022) (based on Mendel and Shleifer, 2012, p. 314).

Normality of θ and P implies that $\mathrm{var}^{-1}(\theta \,|\, P)$ is non-random. Since $P^{**} \equiv P/a$ is informationally equivalent to P, price efficiency is given by

$$\mathrm{var}^{-1}(\theta|\, P^{**}) = \tau_\theta + \mathrm{var}^{-1}(P^{**}|\, \theta).$$

It is inversely related to

$$\mathrm{var}(P^{**}|\, \theta) = \left(\frac{1}{\rho} - \frac{c}{a}\right)^2 \frac{1}{\tau_s} + \left(\frac{c}{a}\right)^2 \frac{1}{\tau_\eta} \tag{5}$$

(where use is made of (1) and (2)). $\mathrm{var}^{-1}(\theta|\, P^{**})$ is a weighted sum of noise trader demand volatility $1/\tau_s$ and the volatility inherent in the signal about noise trading $1/\tau_\eta$ (from Proposition 1, $c/a < 1/\rho$). We call the respective contributions to the total variance the

"CON (COntemporaneous Noise trading)" and "COMESCON (COMmon Errors in the Signal about COntemporaneous Noise trading)" effects in what follows.

An increase in the precision of the signal about noise trading τ_η raises price efficiency by reducing the CON effect (as c/a rises). The additional impact via the COMESCON effect is ambiguous, as rational traders react more strongly to the less volatile signal (i.e., $1/\tau_\eta$ falls, but its coefficient in (5) rises). To determine the net impact on price efficiency, substitute the coefficients in Proposition 1 to get

$$\mathrm{var}^{-1}(\theta|\, P^{**}) = \tau_\theta + \frac{\rho^2\tau_s\left[1+\rho\gamma(\tau_s+\tau_\eta)\right]^2}{1+\rho\gamma\tau_s\left[2+\rho\gamma(\tau_s+\tau_\eta)\right]}.$$

As the fraction on the right-hand side is an increasing function of $1+\rho\gamma(\tau_s+\tau_\eta)$, we have:

Proposition 2: *An increase in the precision of the signal about noise trader demand raises price efficiency:* $\partial[\mathrm{var}^{-1}(\theta|\, P^{**})]/\partial\tau_\eta > 0$.

This confirms the findings of Ganguli and Yang (2009), Manzano and Vives (2011), and Zeng et al. (2018) that more precise information about noise ameliorates its negative impact on price efficiency.

3 Dynamic model

This section shows that higher precision of a public signal about noise trading is not unambiguously beneficial to price efficiency if there is more than one trading date before the asset pays off: more precise information about *future* noise trader demand can make the *contemporaneous* asset price less efficient.

Model

There are now two trading dates t $(=1,2)$ before the assets pay off (cf. Brown and Jennings, 1989). There are a fixed net supply equal to zero of a risky asset that pays off θ and a perfectly elastic supply of a safe asset with a zero rate of return. There is a unit mass of rational investors indexed by the interval $[0,1]$, whose lifespan encompasses both trading dates and the final date, at which the assets pay off. These long-lived agents (LLA) value final-date consumption according to the same CARA utility function as in Section 2. Noise traders demand exogenous and independent amounts of the risky asset s_1 and s_2 at trading dates 1 and 2, respectively (as, e.g., in Allen et al., 2006; Gao, 2008; Farboodi and Veldkamp, 2020; and Farboodi et al., 2023).[8] Rational agent i receives a private signal $x_i = \theta + \varepsilon_i$ about fundamentals at date 1. In addition, each rational agent i observes costless public signals about current and future noise trader demand

$$Y_t = s_t + \eta_t, \quad t = 1, 2, \tag{6}$$

at date 1 (there is no updating of Y_2 at date 2). Information about *future* noise trading is the crucial novel ingredient of the model. As explained in the Introduction, it is motivated by evidence of predictive power of social sentiment indicators. The random variables $\xi \in \{\theta, s_t, \varepsilon_i, \eta_t \,|\, t = 1, 2,\ i \in [0, 1]\}$ are jointly normally and independently distributed. The means are normalized to zero. The precision of ξ_t is denoted $\tau_{\xi t}$.

Equilibrium

Let P_t denote the asset price at trading date t $(= 1, 2)$. Suppose

$$P_1 = a_1\theta + b_1 s_1 - c_{11}Y_1 + c_{12}Y_2 \tag{7}$$

$$P_2 = a_2\theta + b_2 s_2 - c_{21}Y_1 - c_{22}Y_2 + d_2 P_1, \tag{8}$$

for real numbers a_t, b_t, c_{t1}, c_{t2} $(t = 1, 2)$, and d_2. Let $I_{i1} = (P_1, x_i, Y_1, Y_2)$ and $I_{i2} = (P_1, x_i, Y_1, Y_2, P_2)$ denote the vectors of signals available to i at dates 1 and 2, respectively. D_{i1} and D_{i2} denote her asset demands at the two trading dates, and $\pi_i = (P_2 - P_1)D_{i1} + (\theta - P_2)D_{i2}$ is her final wealth.

Definition (dynamic REE): Price functions (7) and (8) and asset demands D_{it}, $t = 1, 2$, $i \in [0, 1]$, are a linear *dynamic REE* if D_{i2} maximizes date-2 expected utility $\mathrm{E}[-\exp(\gamma^{-1}\pi_i) \,|\, I_{i2}]$ and D_{i1} maximizes date-1 expected utility $\mathrm{E}[-\exp(\gamma^{-1}\pi_i) \,|\, I_{i1}]$ given D_{i2} for all $i \in [0, 1]$ and the market for the risky asset clears at both trading dates, i.e., $\int_0^1 D_{it}di + s_t = 0$, $t = 1, 2$.

Agent i's asset demands result from solving her expected utility maximization problem recursively. They are given by Proposition A3 in Brown and Jennings (1989, p. 544) or

[8] On the consequences of persistence in noise trading, see Cespa and Vives (2012, 2015) and Avdis (2016).

Proposition B.1 in Avdis (2016, p. 579):

$$D_{i2} = \gamma \frac{\mathrm{E}(\theta \mid I_{i2}) - P_2}{\mathrm{var}(\theta \mid I_{i2})} \tag{9}$$

$$D_{i1} = \gamma \frac{\mathrm{E}\left[P_2 - h(\theta - P_2) \mid I_{i1}\right] - P_1}{\mathrm{var}\left[P_2 - h(\theta - P_2) \mid I_{i1}\right]}, \tag{10}$$

where

$$h = \frac{\mathrm{cov}(\theta - P_2, P_2 \mid I_{i1})}{\mathrm{var}(\theta - P_2 \,| I_{i1})}.$$

The date-2 demand function (9) is analogous to the demand function (3) in the static model. As the payoff on date-1 investments is given by the date-2 resale price, P_2 rather than θ appears in the conditional moments in (10). While a decrease in P_2 reduces the payoff on date-1 investments, it increases the payoff on date-2 investments. This negative correlation between the payoffs at dates 1 and 2 makes rational traders less reluctant to take risks at the first trading date: for $h < 0$ and $\mathrm{E}[(\theta - P_2)| \, I_{i1}] > 0$, the additional term $\mathrm{E}[-h(\theta - P_2)| \, I_{i1}]$ in (10) contributes positively to date-1 asset demand.

In maximizing their utility, rational agents use the signals they receive to predict θ at date 2 and to predict θ and P_2 at date 1 (see (9) and (10)). Analogous as in the static model, the signals Y_1 and Y_2 about noise trader demand are informative, as they help to disentangle the impacts of fundamentals and noise on prices. Let

$$P_1^* \equiv \frac{1}{a_1}\left[P_1 - \left(\frac{\tau_{\eta 1}}{\tau_{s1} + \tau_{\eta 1}} b_1 - c_{11}\right) Y_1 - c_{12} Y_2\right]$$

$$P_2^* \equiv \frac{1}{a_2}\left[P_2 + c_{21} Y_1 - \left(\frac{\tau_{\eta 2}}{\tau_{s2} + \tau_{\eta 2}} b_2 - c_{22}\right) Y_2 - d_2 P_1\right].$$

Using $\rho_t \equiv a_t / b_t$ and (6), it follows that

$$P_t^* = \theta + \frac{1}{\rho_t} \frac{\tau_{st} s_t - \tau_{\eta t} \eta_t}{\tau_{st} + \tau_{\eta t}}.$$

That is, P_t^* is a signal about θ with precision $\rho_t^2(\tau_{st} + \tau_{\eta t})$. At date 2, analogous as in the static case, (P_1^*, x_i, P_2^*) conveys the same information as $I_{i2} = (P_1, x_i, Y_1, Y_2, P_2)$. At date 1, (P_1^*, x_i) conveys the same information as $I_{i1} = (P_1, x_i, Y_1, Y_2)$. Formally, this follows from the projection theorem.

Substituting the updated moments into (9) and the resulting demands D_{i2} into the date-

2 market clearing condition yields P_2 as a linear function of θ, s_2, Y_1, Y_2, and P_1. The coefficients of these variables are matched with those in (8). Likewise, inserting the date-1 demands (10) with the updated moments into the date-1 market clearing condition yields P_1 as a linear function of θ, s_1, Y_1, and Y_2, whose coefficients are matched with those in (7). Solving the resulting system of equations yields the coefficients in (7) and (8). The algebra, carried out in Appendix A, yields:

Proposition 3: *There exists a unique linear dynamic REE, with*

$$
\begin{aligned}
\rho_1 &= \gamma\tau_\varepsilon \\
\rho_2 &= \gamma\tau_\varepsilon \\
\Delta &= \tau_\theta + \tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1}) + \rho_2^2(\tau_{s2} + \tau_{\eta 2}) \\
a_2 &= \frac{\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})}{\Delta} \\
b_2 &= \frac{1 + \gamma\rho_2(\tau_{s2} + \tau_{\eta 2})}{\gamma\Delta} \\
\Gamma_1 &= \left[\tau_\theta + \tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})\right]^{-1} \\
\Gamma_2 &= (\tau_{s2} + \tau_{\eta 2})^{-1} \\
h &= \frac{a_2(1 - a_2)\Gamma_1 - b_2^2\Gamma_2}{(1 - a_2)^2\Gamma_1 + b_2^2\Gamma_2} \\
a_1 &= \frac{\left[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})\right]\Gamma_1\Gamma_2 b_2^2}{(1 - a_2)^2\Gamma_1 + b_2^2\Gamma_2} + (1 + h)\frac{\rho_1^2(\tau_{s1} + \tau_{\eta 1})}{\Delta} \\
b_1 &= \frac{a_1}{\rho_1} \\
c_{11} &= a_1 \frac{\rho_1\tau_{\eta 1}\left(1 + \frac{1 - a_2}{\Delta b_2^2\Gamma_2}\right)}{\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})\left(1 + \frac{1 - a_2}{\Delta b_2^2\Gamma_2}\right)} \\
c_{12} &= a_1 \frac{\frac{1 - a_2}{b_2}\tau_{\eta 2}\left(1 - \frac{\rho_2}{b_2\Gamma_2\Delta}\right)}{\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})\left(1 + \frac{1 - a_2}{b_2^2\Gamma_2\Delta}\right)} \\
c_{21} &= \frac{-\rho_1^2(\tau_{s1} + \tau_{\eta 1})\frac{c_{11}}{a_1} + \rho_1\tau_{\eta 1}}{\Delta} \\
c_{22} &= \frac{\rho_1^2(\tau_{s1} + \tau_{\eta 1})\frac{c_{12}}{a_1} + \rho_2\tau_{\eta 2}}{\Delta} \\
d_2 &= \frac{\rho_1^2(\tau_{s1} + \tau_{\eta 1})}{a_1\Delta}.
\end{aligned}
$$

As in the static model, the coefficients in the proposition are uniquely determined, so there is a unique linear dynamic REE. The equations in the proposition provide a closed-form solution: whenever a variable appears that is not a primitive of the model, it is determined by the preceding equations.

From $a_2 < 1$ and $b_2\Gamma_2\Delta > \rho_2$ (see the proof of Proposition 4 in Appendix A), it follows that $c_{11} > 0$, $c_{12} > 0$, and $c_{22} > 0$. From (7) and (8), increases in Y_1 thus reduce P_t $(t = 1, 2)$: as in the static model, rational traders' asset demand responds negatively to the signal about *contemporaneous* noise trading, thereby ameliorating the impact of noise trader shocks on asset prices. By contrast, the signal Y_2 rational traders receive at date 1 about *future* noise trader demand raises the date-1 asset price P_1. The prospect of a high resale price due to strong noise trader demand at date 2 encourages rational traders to buy at date 1. More generally, the fact that $c_{12} \neq 0$ means that, in line with the empirical evidence that sentiment indicators predict future returns, the value of Y_2 observed at date 1 has predictive power for the date-2 asset price.

The "Keynesian beauty contest" (KBC) effect identified by Allen et al. (2006) is at work in the dynamic REE. From (9) and the date-2 market clearing condition, the date-2 asset price P_2 depends positively on average expectations of fundamentals $\int_0^1 \mathrm{E}(\theta|\, I_{i2})di$. As a consequence, from (10) and the date-1 market clearing condition, P_1 depends positively on average expectations of average expectations of fundamentals $\int_0^1 \mathrm{E}\,[\int_0^1 \mathrm{E}(\theta|\, I_{i2})di|\, I_{i1}]di$. Allen et al. (2006) show that average expectations violate the law of iterated expectations: the weights on public signals are higher and the weights on private signals are lower in average expectations of average expectations than in average expectations. Applied to the present context, extra weight on public information means that c_{11}/a_1 is "too large".

The fact that $d_2 \neq 0$ means that P_1 also helps to predict P_2, i.e., "technical analysis ... has value" (Brown and Jennings, 1989, p. 527).

Analogous as in the static model, noise traders underperform on average at the final trading date: from (8),

$$\mathrm{E}[(\theta - P_2)s_2] = -\mathrm{E}(P_2 s_2) = -(b_2 - c_{22})\frac{1}{\tau_{s2}} - d_2 c_{12}\frac{1}{\tau_{s2}} < 0.$$

The first term on the right-hand side of the equality captures the effects that are also present in the static model: noise traders lose since their demand drives up the asset price at date 2, and rational traders' contrarian trading based on the signal Y_2 dampens the expected losses.

The second term on the right-hand side of the equality captures an additional, negative effect: as a high value of Y_2 indicates a high resale price P_2, it raises rational traders' date-1 asset demands and the date-1 asset price P_1. As $d_2 > 0$, there is a positive feedback effect on P_2, which lowers noise traders' expected final wealth further.

From (7) and (8), noise traders' unconditional interim expected wealth is

$$\mathrm{E}[(P_2 - P_1)s_1] = \mathrm{E}(P_2 s_1) - \mathrm{E}(P_1 s_1) = -c_{21}\frac{1}{\tau_{s1}} + d_2(b_1 - c_{11})\frac{1}{\tau_{s1}} - (b_1 - c_{11})\frac{1}{\tau_{s1}}.$$

The third term on the far right-hand side is the usual cost effect (i.e., $-\mathrm{E}(P_1 s_1)$): noise traders bid up the price at which they buy assets, and vice versa. The first term captures their expected losses due to the fact that the resale price P_2 depends negatively on their date-1 investment s_1 (from (8) and $c_{21} > 0$). The second term captures an additional effect that works in the opposite direction, raising noise traders' expected interim wealth: an increase in s_1, by raising P_1, also has a positive effect on the resale price P_2 (from (8) and $d_2 > 0$). This positive effect notwithstanding, noise traders' interim expected wealth is generally negative (see Appendix A). The fact that prior to the final trading date, there is an effect that works in the opposite direction suggests that the conditions for noise trader survival are at least less bleak than in the static model. A thorough investigation would require the introduction of more trading dates and the analysis of the whole distribution of noise trader wealth over time, however (see De Long et al., 1991; Kogan et al., 2006).

Price efficiency

We are now in a position to prove our main result: public information about future noise trader demand potentially reduces contemporaneous price efficiency.

Define $P_1^{**} \equiv P_1/a_1$:

$$P_1^{**} \equiv \theta + \frac{1}{\rho_1}s_1 - \frac{c_{11}}{a_1}Y_1 + \frac{c_{12}}{a_1}Y_2.$$

Substituting for Y_t from (6), we obtain price efficiency at date 1:

$$\mathrm{var}^{-1}(\theta \mid P_1^{**}) = \tau_\theta + \left[\left(\frac{1}{\rho_1} - \frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{s1}} + \left(\frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{\eta 1}} + \left(\frac{c_{12}}{a_1}\right)^2 \left(\frac{1}{\tau_{s2}} + \frac{1}{\tau_{\eta 2}}\right)\right]^{-1}, \quad (11)$$

where the term in square brackets is $\mathrm{var}(P_1^{**} \mid \theta)$. As in the static case, the squared correlation between θ and P and the ratio of "good variance" to "bad variance" as measures of price efficiency yield analogous results (see Appendix B). The first two terms in the sum in square brackets in (11) are the CON and COMESCON effects identified in the static model (cf. the right-hand side of (5)). They capture the effects on price efficiency of contemporaneous noise trading and the common error term in the corresponding signal, respectively. The third term in the sum in square brackets has no counterpart in the static case. It captures the impact of the signal about future noise trader demand Y_2 on price efficiency at date 1. We call this additional component of the conditional price variance the "COMSFUN (COMmon Signal about FUture Noise trading)" effect. The weights of the variances $1/\tau_{st}$ and $1/\tau_{\eta t}$ in (11) are determined by ρ_1 and the expressions $(c_{1t}/a_1)^2$ $(t = 1, 2)$, which indicate how strongly the date-1 asset price responds to the signals about noise compared to fundamentals.

To assess the impact of changes in the precision of the signal about future noise trader demand $\tau_{\eta 2}$ on date-1 price efficiency, it is instructive to start with the special case of the model without an informative signal about contemporaneous noise trader demand at date 1, i.e., with $\tau_{\eta 1} = 0$. Loosely speaking, noise traders' social media activity consists of rumors which only allow inferences about their future trading behavior in this case. Price efficiency is maximum in this case if the signal about future noise Y_2 is also completely uninformative:

Proposition 4: *Let $\tau_{\eta 1} = 0$. Then $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ is greater for $\tau_{\eta 2} = 0$ than for any finite $\tau_{\eta 2} > 0$.*

The proof is in Appendix A. P_1 is unrelated to Y_1 if Y_1 is uninformative: $c_{11}/a_1 = (c_{11}/a_1)^2/\tau_{\eta 1} = 0$ (cf. (7)). As a consequence, the CON effect boils down to $1/(\rho_1^2 \tau_{s1})$, which is independent of $\tau_{\eta 2}$, and the COMESCON effect vanishes altogether. So $\tau_{\eta 2}$ affects date-1 price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ only via the COMSFUN effect

$$\left(\frac{c_{12}}{a_1}\right)^2 \left(\frac{1}{\tau_{s2}} + \frac{1}{\tau_{\eta 2}}\right).$$

The COMSFUN effect vanishes for $\tau_{\eta 2} = 0$, as the signal about future noise is also useless (as, from Proposition 3, $c_{12}/a_1 = (c_{12}/a_1)^2/\tau_{\eta 2} = 0$). The assertion of Proposition 4 follows from the fact that the COMSFUN effect is strictly positive for all other finite values of $\tau_{\eta 2}$. Actually, the COMSFUN effect also vanishes in the limit as $\tau_{\eta 2} \to \infty$. The date-2 price becomes perfectly informative in this case: $P_2 = \theta$ (since $a_2 \to 1$, $b_2 \to 1/\rho_2$, $c_{21} \to 0$, $c_{22} \to 1/\rho_2$, and $d_2 \to 0$; see the proof of Proposition 5). As a result, the signal Y_2 about future noise s_2 is of no use in predicting the payoff on date-1 investments (i.e., the resale price P_2) and does not affect date-1 asset demand. So the term $c_{12}Y_2$ drops out of (7) and

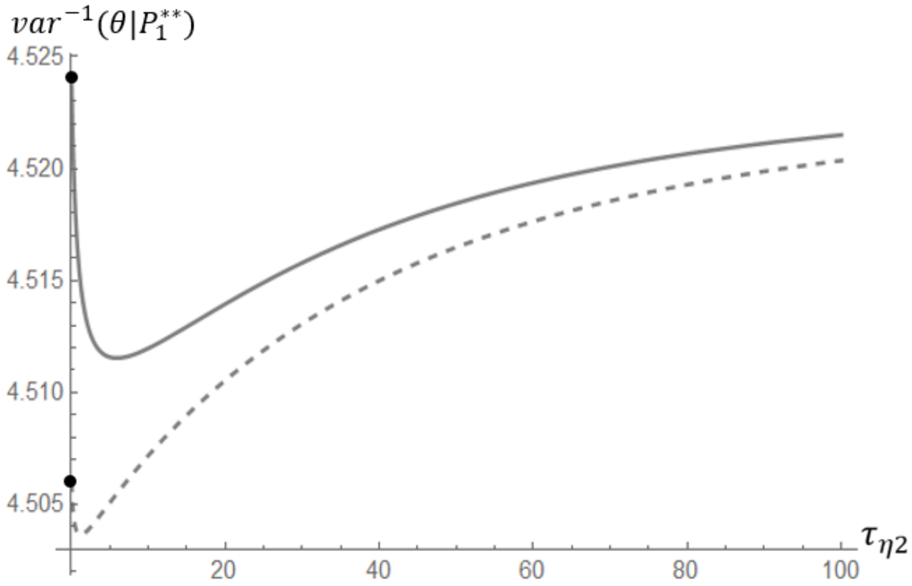

Figure 2: The impact of the precision of public information about future noise trader demand on current price efficiency in the LLA (solid curve) and OLG (dashed curve) models (parameters: $\tau_{\eta 1} = 0$, $\gamma = 2$, $\tau_\theta = 4.5$, $\tau_\varepsilon = 0.1$, $\tau_{s1} = 0.6$, $\tau_{s2} = 2.5$)

the COMSFUN term drops out of (11) (formally, $c_{12}/a_1 \to 0$, as $\tau_{\eta 2}(1-a_2)/b_2$ converges to a constant and $b_2\Gamma_2\Delta \to \rho_2$; see the proof of Proposition 5 in Appendix A).

In fact, the relation between $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ and $\tau_{\eta 2}$ has the U-shape depicted in Figure 2: the marginal impact of increases in $\tau_{\eta 2}$ on price efficiency is negative at first and turns positive at higher levels (see Appendix A for the formal proof). The economic intuition behind this result is the following: As τ_{η_2} rises, the weight traders put on the signal about future noise (relative to fundamentals), expressed by the ratio c_{12}/a_1, is influenced by two counteracting effects. For one thing, as Y_2 predicts future noise trader demand more precisely, agents trade more aggressively on the signal when forming their demand. For another, a more precise Y_2 implies that, at date 2, rational traders offset more of the date-2 noise trader demand. This reduces the noise in the date-2 price, making Y_2 less useful for predicting P_2 at date 1.

When the signal is imprecise, the destabilizing effect dominates and agents trade more aggressively on the signal Y_2 (relative to fundamentals) as τ_{η_2} increases. Consequently, price efficiency deteriorates. However, there exists a point where the stabilizing effect takes over, and agents trade less aggressively on Y_2 at date 1 as the signal gains further in precision. This is conducive to price efficiency.

Turning to the general case (i.e., $\tau_{\eta 1} \geq 0$), the relation between $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ and $\tau_{\eta 2}$ is not necessarily U-shaped. Nevertheless, similar arguments establish that more accurate information about future noise trading cannot be unambiguously conducive to price efficiency: price efficiency cannot be a monotonically increasing function of signal precision, so there generally exists a range of values of signal precision over which it has a negative impact on price efficiency. This follows from the observation that a completely uninformative signal generally leads to higher efficiency than a perfectly informative signal:

Proposition 5: $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ *is no less for* $\tau_{\eta 2} = 0$ *than for* $\tau_{\eta 2} \to \infty$*.*

The proof is in Appendix A. By the same arguments as in the case $\tau_{\eta 1} = 0$, the COMSFUN effect drops out of (11) for $\tau_{\eta 2} \to 0$ (the signal plays no role) and for $\tau_{\eta 2} \to \infty$ (the signal is useless at date 1 because noise trading s_2 does not affect the resale price). So price efficiency at date 1 depends on $\tau_{\eta 2}$ solely via the CON and COMESCON effects

$$\left(\frac{1}{\rho_1} - \frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{s1}} + \left(\frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{\eta 1}}.$$

This sum is smaller and price efficiency is larger for $\tau_{\eta 2} = 0$ than for $\tau_{\eta 2} \to \infty$ (see Appendix A). The economic explanation for this result goes as follows: From Proposition 3, we know that ρ_1 is independent of $\tau_{\eta 2}$. Thus, the result is driven by the ratio c_{11}/a_1, which indicates rational agents' trading intensity against the public signal about contemporaneous noise trading (relative to their trading intensity on fundamentals). Along the proof of Proposition 3.5 in Appendix A, we show that c_{11}/a_1 is greater for $\tau_{\eta 2} = 0$ than as $\tau_{\eta 2} \to \infty$. This implies that agents trade more aggressively against contemporaneous noise (compared to trading on fundamentals) when the signal about future noise is completely imprecise. Date-1 traders use the signal about current noise Y_1 in two ways. On the one hand, Y_1 is contained in P_1^* and used to extract noise inherent in the date-1 market price. Notably, $\tau_{\eta 2}$ does *not* influence how aggressively agents trade on P_1^* (see date-1 traders' demand function in the proof of Proposition 3 in Appendix A). On the other hand, since the public signal Y_1 is also observable at date 2, forecasting P_2 entails forecasting Y_1. Thus, Y_1 directly helps to predict P_2. If Y_2 is perfectly precise, the exact value of θ can be observed at date 2 by disentangling the information conveyed by P_2. Consequently, date-2 rational traders do not use Y_1 to predict fundamentals (i.e., $c_{21} = 0$). As of date 1, this makes Y_1 less useful for predicting P_2 and traders put less weight on the signal when forming date-1 demand than for $\tau_{\eta 2} = 0$. This explains why c_{11}/a_1 is unequivocally larger for $\tau_{\eta 2} = 0$ than as $\tau_{\eta 2} \to \infty$.

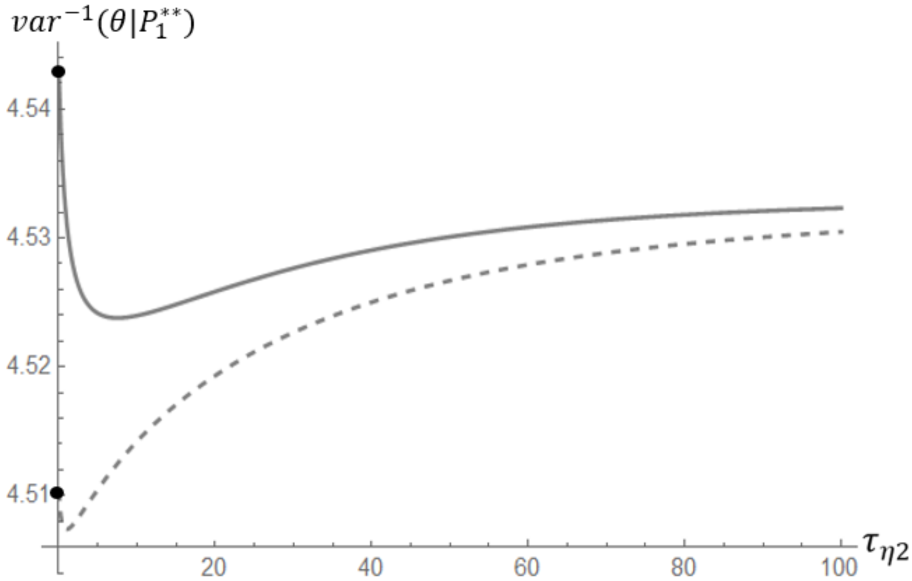

Figure 3: The impact of the precision of public information about future noise trader demand on current price efficiency in the LLA (solid curve) and OLG (dashed curve) models (parameters: $\tau_{\eta 1} = 2.5$, $\gamma = 2$, $\tau_\theta = 4$, $\tau_\varepsilon = 0.8$, $\tau_{s1} = 0.01$, $\tau_{s2} = 3.5$)

The fact that c_{11}/a_1 is greater in the absence of information about future noise trading implies that the CON effect is less pronounced for $\tau_{\eta_2} = 0$ than as $\tau_{\eta_2} \to \infty$, raising price efficiency. The COMESCON effect, by contrast, is clearly more pronounced for $\tau_{\eta_2} = 0$, which harms price efficiency. Thus, as τ_{η_2} switches from infinity to zero, the impact of more aggressive trading against Y_1 on price efficiency is two-edged. Nevertheless, the result in Proposition 5 demonstrates that the stabilizing impact coming from the reduction in the CON effect dominates the destabilizing impact generated by the increase in the COMESCON effect.

Figure 3 illustrates the result contained in Proposition 5. Interestingly, while the relation between precision of the signal about date-2 noise and date-1 price efficiency thus cannot be monotonically increasing, there exist parameterizations such that it is *monotonically decreasing* (see Figure 4).

Price efficiency tends to go down when the precision of the signal about future noise rises from zero to small positive values. This is necessarily true for $\tau_{\eta 1} = 0$, in which case price efficiency is maximum for $\tau_{\eta 2} = 0$ (see Proposition 4). Numerical analysis confirms this property for a wide range of parameters, including those underlying Figures 3 and 4. The following proposition gives the necessary and sufficient condition:

Proposition 6: $\partial[\mathrm{var}^{-1}(\theta \mid P_1^{**})]/\partial\tau_{\eta 2} < 0$ *for* $\tau_{\eta 2} = 0$ *exactly if*

$$-\frac{2\tau_{\eta 1}\tau_\varepsilon^2(1-\gamma\rho_2\tau_{s2})}{\tau_{s1}\left[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})(1+\phi_{10})\right]}+\frac{\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})}{1+\gamma\rho_2\tau_{s2}}>0,$$

where

$$\phi_{10}=\frac{\gamma^2\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]\tau_{s2}}{(1+\gamma\rho_2\tau_{s2})^2}.$$

The proof is in Appendix A. The first term on the left-hand side of the inequality in Proposition 6 represents the (adjusted) impact of a marginal increase in $\tau_{\eta 2}$, starting from zero, on the sum of the CON and COMESCON effects in (11). The second term is the (adjusted) marginal change in the COMSFUN effect. As one would expect, the impact on the COMSFUN effect is positive and, hence, detrimental to price efficiency. The total effect on price efficiency is negative for $\gamma\rho_2\tau_{s2}$ large enough. A simple sufficient condition is $\gamma\rho_2\tau_{s2} > 1$. In this case, increases in $\tau_{\eta 2}$ make date-1 agents trade less aggressively against the public signal Y_1, strengthening the CON effect and weakening the COMESCON effect in (11) (see Appendix A). The net effect is positive, adding to the COMSFUN effect and reducing price efficiency, because a high value of $\gamma\rho_2\tau_{s2}$ implies that c_{11}/a_1 is small (see the proof of Proposition 5 in Appendix A), so that the weight of the CON effect is large compared to the COMESCON effect in (11).

In sum, the impact of more accurate information about future noise trading on price efficiency is not unequivocally positive. Higher precision necessarily has a negative impact on price efficiency over a range of relatively low values, and possibly throughout. This challenges the conventional wisdom, derived from static models, that more non-fundamental information is conducive to the conveyance of information via asset prices and supports concerns about the impact of social sentiment investing on the informational content of asset prices.

We also checked the implications of the model for price efficiency at date 2 and the impact of changes in the precision of the signal about contemporaneous noise trader demand $\tau_{\eta 1}$ on date-1 price efficiency. Higher precision $\tau_{\eta 2}$ potentially reduces not only date-1 price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**})$, but also date-2 price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**}, P_2^{**})$. This reinforces our conclusion that non-fundamental information potentially makes asset prices less informative. Perhaps surprisingly, an increase in $\tau_{\eta 1}$ can also decrease date-1 price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**})$. However, this can only happen when there is also an informative signal about future noise,

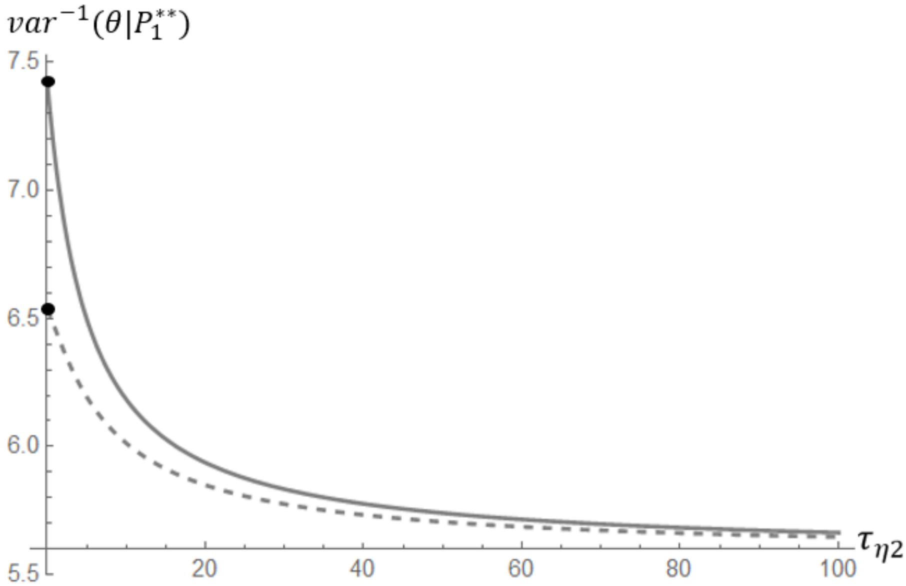

Figure 4: The impact of the precision of public information about future noise trader demand on current price efficiency in the LLA (solid curve) and OLG (dashed curve) models (parameters: $\tau_{\eta 1} = 0.5$, $\gamma = 2$, $\tau_\theta = 4.5$, $\tau_\varepsilon = 0.1$, $\tau_{s1} = 0.6$, $\tau_{s2} = 2.5$)

i.e., when $\tau_{\eta 2} > 0$.[9] So, unlike a signal about contemporaneous noise, a signal about future noise is a necessary prerequisite for a negative impact of more precise non-fundamental information on price efficiency.

4 Short-term trading

Following Allen et al. (2006) and others (e.g., Brown and Jennings, 1989; Gao, 2008; Cespa and Vives, 2015; Farboodi and Veldkamp, 2020), this section replaces the long-lived traders in the model of Section 3 with two overlapping generations (OLG) of short-lived traders. This has become a standard way of modeling short-term trading by agents that enter and exit markets in quick succession and are, therefore, predominantly interested in price changes, not in fundamentals. We show that, as in the model with LLA, the impact of an increase in the precision of the signal about future noise on price efficiency can be negative.

Model

The model is the same as in Section 3 except that there are now two generations of rational

[9] Proofs of the stated results are available on request.

traders, one that enters the market at date 1 and exits at date 2 and another one that enters at date 2 and lives until date 3. Each generation of investors has unit mass, and each investor is characterized by the same coefficient of constant absolute risk aversion γ^{-1}. Each rational agent i obtains a private signal $x_i = \theta + \varepsilon_i$ about θ. As before, the signals are costless. The exogenous random variables are jointly normally and independently distributed with zero mean and the established notation for precisions.

Equilibrium

The final wealth of a first generation investor who invests D_{i1} in the risky asset $\pi_{i1} = (P_2 - P_1)D_{i1}$ is determined by the resale price P_2 and does not depend directly on fundamentals θ. Investors that enter the market at date 2 invest D_{i2} and obtain final wealth $\pi_{i2} = (\theta - P_2)D_{i2}$. A dynamic REE is defined in the same way as in the LLA version of the model except that both generations of rational agents maximize their respective expected utilities.

Let the asset prices be given by (7) and (8). Solving the model backwards yields a unique linear dynamic REE (see Appendix A):

Proposition 7: *There exists a unique linear dynamic REE, with*

$$\begin{aligned}
\rho_1 &= \frac{\gamma^2\tau_\varepsilon(\tau_{s2}+\tau_{\eta2})}{1+\gamma^2\tau_\varepsilon(\tau_{s2}+\tau_{\eta2})}\gamma\tau_\varepsilon \\
\rho_2 &= \gamma\tau_\varepsilon \\
\Delta &= \tau_\theta + \tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta1}) + \rho_2^2(\tau_{s2}+\tau_{\eta2}) \\
a_1 &= \frac{\rho_1^2(\tau_{s1}+\tau_{\eta1})(\Delta+\tau_\varepsilon) + \tau_\varepsilon\left[\tau_\varepsilon + \rho_2^2(\tau_{s2}+\tau_{\eta2})\right]}{\Delta\left[\tau_\theta+\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta1})\right]} \\
b_1 &= \frac{a_1}{\rho_1} \\
c_{11} &= \frac{(\Delta+\tau_\varepsilon)\rho_1\tau_{\eta1}}{\Delta[\tau_\theta+\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta1})]} \\
c_{12} &= \frac{\tau_{\eta2}}{\gamma\Delta(\tau_{s2}+\tau_{\eta2})} \\
a_2 &= \frac{\tau_\varepsilon+\rho_2^2(\tau_{s2}+\tau_{\eta2})}{\Delta} \\
b_2 &= \frac{1+\gamma\rho_2(\tau_{s2}+\tau_{\eta2})}{\gamma\Delta} \\
c_{21} &= \frac{-\rho_1^2(\tau_{s1}+\tau_{\eta1})\frac{c_{11}}{a_1}+\rho_1\tau_{\eta1}}{\Delta} \\
c_{22} &= \frac{\rho_2\tau_{\eta2}+\rho_1^2(\tau_{s1}+\tau_{\eta1})\frac{c_{12}}{a_1}}{\Delta}
\end{aligned}$$

$$d_2 = \frac{\rho_1^2(\tau_{s1} + \tau_{\eta 1})}{a_1 \Delta}.$$

Equilibrium shares several properties with the LLA model of Section 3. Proposition 7 provides a closed-form solution for the unique linear REE. The expressions for ρ_2, a_2, b_2, c_{21}, c_{22}, and d_2 are the same as in the LLA model. This is because the analysis of date-2 trading is identical in both models. Demand responds negatively to the signal about contemporaneous noise trading (as $c_{11} > 0$ and $c_{22} > 0$) but positively to the signal about future noise trading (as $c_{12} > 0$). The latter effect is due to the fact that a high value of the signal indicates price pressure emanating from high noise trader demand at date 2. $c_{21} \neq 0$ and $d_2 \neq 0$ mean that the signal about date-1 noise trader demand and the date-1 price have predictive power for the date-2 price level. The remarks on noise traders' expected final wealth $\mathrm{E}[(\theta - P_2)s_2]$ and expected interim wealth $\mathrm{E}[(P_2 - P_1)s_1]$ following Proposition 3 in Section 3 hold without modification (see Appendix A for the computations). A difference compared to the LLA model, which is important for price efficiency, is that ρ_1 ($= a_1/b_1$) depends (positively) on the precision of the non-fundamental signal $\tau_{\eta 2}$.

Price efficiency

As in Section 3, price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ is given by equation (11), where $P_1^{**} \equiv P_1/a_1$ and the coefficients ρ_1, a_1, c_{11}, and c_{12} are now given by Proposition 7 (Appendix B shows that the alternative metrics yield analogous conclusions). So the same three effects (CON, COMESCON, and COMSFUN) determine price efficiency.

Proposition 8: *For $\tau_{\eta 1} = 0$, $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ is smaller for $\tau_{\eta 2} = 0$ than for $\tau_{\eta 2} \to \infty$. For all $\tau_{\eta 1} \geq 0$, the limit of $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ as $\tau_{\eta 2} \to \infty$ is equal to the limit in the model with LLA.*

The proof is in Appendix A. Suppose $\tau_{\eta 1} = 0$. Evidently, from Proposition 7, $c_{11}/a_1 = (c_{11}/a_1)^2/\tau_{\eta 1} = 0$ in this case. As in the LLA model, the COMESCON effect drops out of (11) and price efficiency is inversely related to

$$\frac{1}{\rho_1^2 \tau_{s1}} + \left(\frac{c_{12}}{a_1}\right)^2 \left(\frac{1}{\tau_{s2}} + \frac{1}{\tau_{\eta 2}}\right).$$

The first term is the CON effect, the second one is the COMSFUN effect. Contrary to the LLA model, the CON effect is not independent of $\tau_{\eta 2}$: increases in $\tau_{\eta 2}$ raise ρ_1, thereby weakening the CON effect and raising price efficiency $\mathrm{var}^{-1}(\theta \mid P_1^{**})$. This is why more precise information about future noise is less likely to reduce contemporaneous price efficiency than in the model with LLA. As in the LLA model, the COMSFUN effect vanishes for $\tau_{\eta 2} = 0$ and for $\tau_{\eta 2} \to \infty$ (see the proof of Proposition 8 in Appendix A). Taken together, it follows that, contrary to the model with LLA, price efficiency is higher for $\tau_{\eta 2} \to \infty$ than for $\tau_{\eta 2} = 0$ (see also Figure 2). For all $\tau_{\eta 1} \geq 0$, ρ_1 converges to the constant equilibrium value of the LLA model, viz., $\gamma\tau_\varepsilon$ (see Propositions 3 and 7). As a consequence, price efficiency and the impact of changes in $\tau_{\eta 2}$ on price efficiency are similar in the two versions of the model (see Figures 2 and 3).

Numerical analysis of the case $\tau_{\eta 1} > 0$ shows that price efficiency is generally lower in the OLG model than in the LLA model. Jointly with the fact that, from Proposition 8, the difference goes to zero as $\tau_{\eta 2}$ grows large, this means that the marginal impact of increases in $\tau_{\eta 2}$ tends to be larger. This more optimistic assessment of the impact of more precise non-fundamental information notwithstanding, as in the LLA case, there exist parameters such that $\mathrm{var}^{-1}(\theta \,|\, P_1^{**})$ is a monotonically *decreasing* function of $\tau_{\eta 2}$ (see Figure 4).

5 Conclusion

The growing availability of big data has a profound impact on many parts of social life and economic activity. This holds true in particular for activity in financial markets. As a rule of thumb, the dissemination of information allows individual market participants to make more accurate financial decisions and leads to more efficient market outcomes. In line with this, the availability of more data on economic fundamentals makes asset prices more informative in theory. But can we be confident that more data on noise trading collected by social sentiment investors also makes asset prices more efficient? Given the obvious problems of coming up with a sound empirical answer to this question, one might be interested in what theory says to start with. If one tackles the problem from a static perspective, one might be confident that the rule of thumb also holds: better information about noise allows sentiment investors to trade more aggressively against noise trader shocks, making asset prices more efficient. Unfortunately, this optimistic assessment of the impact of sentiment investing on price efficiency does not generally carry over to a dynamic setup. The dynamic perspective acknowledges that, as most assets are traded frequently before maturity and between coupon or dividend payments, resale prices are the main determinant of returns on investment. Information about future noise moves the current price of an asset because rational investors anticipate the impact of future noise on future asset prices, i.e., the prices at which they resell assets not held up to maturity. The ensuing additional conditional price volatility reduces the informativeness of the current asset price as an indicator of fundamentals. So taking the dynamic perspective leads to a more pessimistic assessment of the impact of social sentiment trading on price efficiency.

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A Proofs

Proof of Proposition 3: The first two conditional moments of θ at date 2 are

$$\mathrm{E}(\theta \mid I_{i2}) = \frac{\tau_\varepsilon \, x_i + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \, P_1^* + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2}) \, P_2^*}{\tau_\theta + \tau_\varepsilon + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2})}$$

$$\mathrm{var}^{-1}(\theta \mid I_{i2}) = \tau_\theta + \tau_\varepsilon + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2}) \, .$$

Trader i's date-2 demand (adjusted by her risk tolerance) can be written as

$$\frac{D_{i2}}{\gamma} = \tau_\varepsilon \, x_i + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \, P_1^* + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2}) \, P_2^* - \Delta P_2,$$

where $\Delta \equiv \mathrm{var}^{-1}(\theta \mid I_{i2})$. Market clearing at date 2 and the strong law of large numbers (i.e., $\int_0^1 x_i \, di = \theta$) imply

$$\begin{aligned}
0 &= \frac{s_2}{\gamma} + \int_0^1 \frac{D_{i2}}{\gamma} di \\
&= \frac{s_2}{\gamma} + \tau_\varepsilon \, \theta + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \left(\frac{P_1 + c_{11} Y_1 - c_{12} Y_2}{a_1} - \frac{1}{\rho_1} \frac{\tau_{\eta 1}}{\tau_{s1} + \tau_{\eta 1}} Y_1 \right) \\
&\quad + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2}) \left[\theta + \frac{1}{\rho_2} \left(s_2 - \frac{\tau_{\eta 2}}{\tau_{s2} + \tau_{\eta 2}} Y_2 \right) \right] - \Delta P_2.
\end{aligned}$$

Solving for P_2 yields

$$\begin{aligned}
P_2 = \frac{1}{\Delta} \Bigg\{ & \left[\tau_\varepsilon + \rho_2^2 \, (\tau_{s2} + \tau_{\eta 2}) \right] \theta + \left[\gamma^{-1} + \rho_2 \, (\tau_{s2} + \tau_{\eta 2}) \right] s_2 \\
& - \left[\rho_1 \tau_{\eta 1} - \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \, \frac{c_{11}}{a_1} \right] Y_1 - \left[\rho_2 \tau_{\eta 2} + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \, \frac{c_{12}}{a_1} \right] Y_2 \\
& + \rho_1^2 \, (\tau_{s1} + \tau_{\eta 1}) \, \frac{1}{a_1} P_1 \Bigg\}.
\end{aligned}$$

Matching coefficients and some algebra yield the expressions for a_2, b_2, c_{21}, c_{22}, d_2, and ρ_2 in the proposition.

Turning to date 1, after defining $\Gamma_1 \equiv \mathrm{var}(\theta \mid I_{i1})$ and $\Gamma_2 \equiv \mathrm{var}(s_2 \mid I_{i1})$, we have

$$\mathrm{cov}(\theta - P_2, P_2 \mid I_{i1}) = a_2 (1 - a_2) \Gamma_1 - b_2^2 \Gamma_2$$

$$\mathrm{var}(\theta - P_2 \mid I_{i1}) = (1 - a_2)^2 \Gamma_1 + b_2^2 \Gamma_2,$$

so that

$$h = \frac{a_2(1-a_2)\Gamma_1 - b_2^2\Gamma_2}{(1-a_2)^2\Gamma_1 + b_2^2\Gamma_2}.$$

Using

$$\begin{aligned}
\mathrm{E}[P_2 - h(\theta - P_2) \,|\, I_{i1}] = & \; [(1+h)a_2 - h]\, \mathrm{E}(\theta \,|\, I_{i1}) \\
& + (1+h)\, [b_2 \mathrm{E}(s_2 \,|\, I_{i1}) - c_{21} Y_1 - c_{22} Y_2 + d_2 P_1]
\end{aligned}$$

$$\mathrm{E}(\theta \,|\, I_{i1}) = \frac{\tau_\varepsilon\, x_i + \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})\, P_1^*}{\tau_\theta + \tau_\varepsilon + \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})}$$

$$\mathrm{E}(s_2 \,|\, I_{i1}) = \frac{\tau_{\eta 2}}{\tau_{s2} + \tau_{\eta 2}} Y_2$$

$$\mathrm{var}[P_2 - h(\theta - P_2) \,|\, I_{i1}] = \frac{b_2^2 \Gamma_1 \Gamma_2}{(1-a_2)^2\Gamma_1 + b_2^2\Gamma_2},$$

trader i's date-1 demand (adjusted by her risk tolerance) becomes

$$\begin{aligned}
\frac{D_{i1}}{\gamma} = & \; \tau_\varepsilon\, x_i + \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})\, P_1^* + \frac{1-a_2}{b_2^2\Gamma_2}\left(b_2 \frac{\tau_{\eta 2}}{\tau_{s2} + \tau_{\eta 2}} Y_2 - c_{21} Y_1 - c_{22} Y_2 + d_2 P_1\right) \\
& - \frac{(1-a_2)^2\Gamma_1 + b_2^2\Gamma_2}{b_2^2\Gamma_1\Gamma_2} P_1.
\end{aligned}$$

Market clearing in the first period yields

$$\begin{aligned}
0 = & \; \frac{s_1}{\gamma} + \int_0^1 \frac{D_{i1}}{\gamma} di \\
= & \; \tau_\varepsilon\, \theta + \frac{s_1}{\gamma} - \left[\rho_1 \tau_{\eta 1} - \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})\, \frac{c_{11}}{a_1} + \frac{1-a_2}{b_2^2\Gamma_2}\, c_{21}\right] Y_1 \\
& + \left[\frac{1-a_2}{b_2^2\Gamma_2}\left(b_2 \frac{\tau_{\eta 2}}{\tau_{s2} + \tau_{\eta 2}} - c_{22}\right) - \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})\, \frac{c_{12}}{a_1}\right] Y_2 \\
& - \left[\frac{(1-a_2)^2\Gamma_1 + b_2^2\Gamma_2}{b_2^2\Gamma_1\Gamma_2} - \frac{1-a_2}{b_2^2\Gamma_2}\, d_2 - \rho_1^2\,(\tau_{s1} + \tau_{\eta 1})\, \frac{1}{a_1}\right] P_1.
\end{aligned}$$

Solving for P_1 and matching coefficients gives the expressions for a_1, b_1, c_{11}, c_{12}, and ρ_1 in the proposition. q.e.d.

Proof that $\mathrm{E}[(P_2 - P_1)s_1] < 0$ *in the LLA model:*
From Proposition 3,

$$-c_{21}\frac{1}{\tau_{s1}} + d_2(b_1 - c_{11})\frac{1}{\tau_{s1}} = \frac{\rho_1}{\Delta}$$

and

$$(b_1 - c_{11})\frac{1}{\tau_{s1}} = a_1 \frac{\tau_\varepsilon + \rho_1^2 \tau_{s1}\left(1 + \frac{1-a_2}{\Delta b_2^2 \Gamma_2}\right)}{\rho_1 \tau_{s1}\left[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})\left(1 + \frac{1-a_2}{\Delta b_2^2 \Gamma_2}\right)\right]}.$$

Plugging this into the expression for noise traders' interim expected wealth in the main text, letting

$$\begin{aligned} D &\equiv 1 + \gamma^2 \tau_\varepsilon (\tau_{s2} + \tau_{\eta 2}) \\ E &\equiv \tau_\theta + \rho_1^2(\tau_{s1} + \tau_{\eta 1}), \end{aligned}$$

and simplifying terms, we get

$$\mathrm{E}[(P_2 - P_1)s_1] = -\frac{E\gamma^2(\tau_{s2} + \tau_{\eta 2})\rho_1^2 \tau_{s1}[\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})] + D^2[\tau_\varepsilon \Delta + \rho_1^2 \tau_{s1}\rho_2^2(\tau_{s2} + \tau_{\eta 2})]}{\rho_1 \tau_{s1}\Delta\left\{E^2\gamma^2(\tau_{s2} + \tau_{\eta 2}) + D^2[\tau_\theta + \tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})]\right\}} < 0.$$

q.e.d.

Proof of Proposition 4: From the formulas in Proposition 3, for $\tau_{\eta 1} = 0$, we obtain:

$$\begin{aligned} \frac{1 - a_2}{b_2} &= \gamma \frac{\tau_\theta + \rho_1^2 \tau_{s1}}{1 + \gamma \rho_2(\tau_{s2} + \tau_{\eta 2})} \\ b_2 \Gamma_2 \Delta &= \frac{1 + \gamma \rho_2(\tau_{s2} + \tau_{\eta 2})}{\gamma(\tau_{s2} + \tau_{\eta 2})} \end{aligned}$$

We note in passing that $b_2\Gamma_2\Delta > \rho_2$. It follows that

$$\begin{aligned} \frac{c_{11}}{a_1} &= 0 \\ \left(\frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{\eta 1}} &= 0 \end{aligned}$$

for $\tau_{\eta 1} = 0$. That $c_{12}/a_1 = 0$ if $\tau_{\eta 2} = 0$ follows from the fact that

$$\frac{\rho_2}{b_2 \Gamma_2 \Delta} = \frac{\gamma \rho_2(\tau_{s2} + \tau_{\eta 2})}{1 + \gamma \rho_2(\tau_{s2} + \tau_{\eta 2})} < 1.$$

Since c_{12}/a_1 is positive for all other finite values of $\tau_{\eta 2}$, $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ is greater for $\tau_{\eta 2} = 0$ than for any finite $\tau_{\eta 2} > 0$. q.e.d.

Proof that $\mathrm{var}^{-1}(\theta \mid P_1^{**})$ *is U-shaped in* $\tau_{\eta 2}$ *for* $\tau_{\eta 1} = 0$:
Define

$$A_3 \equiv \left(\frac{c_{12}}{a_1}\right)^2 \left(\frac{1}{\tau_{s2}} + \frac{1}{\tau_{\eta 2}}\right).$$

Then, we can show that

$$\frac{\partial A_3}{\partial \tau_{\eta 2}} = \frac{\gamma^2(\tau_\theta + \rho_1^2\tau_{s1})^2\left(-b_3\tau_{\eta 2}^3 - b_2\tau_{\eta 2}^2 + b_1\tau_{\eta 2} + b_0\right)}{\tau_{s2}k_3^3},$$

where

$$\begin{aligned}
b_3 &\equiv 2\gamma^2\rho_2^2(\tau_\epsilon + \rho_1^2\tau_{s1}),\\
b_2 &\equiv 3\gamma^2\rho_2^2\tau_{s2}(\tau_\epsilon + \rho_1^2\tau_{s1}),\\
b_1 &\equiv 2(1 + \gamma\rho_2\tau_{s2})(\tau_\epsilon + \rho_1^2\tau_{s1}) + \rho_1^2\tau_{s1}\gamma(\tau_\theta + \rho_1^2\tau_{s1})\tau_{s2},\\
b_0 &\equiv \tau_{s2}[(1 + \gamma\rho_2\tau_{s2})^2(\tau_\epsilon + \rho_1^2\tau_{s1}) + \rho_1^2\tau_{s1}\gamma(\tau_\theta + \rho_1^2\tau_{s1})\tau_{s2}].
\end{aligned}$$

Note that the term in brackets in the numerator of the derivative is a cubic polynomial in $\tau_{\eta 2}$. To determine the number of positive real roots, we use Descartes' rule of signs. Since the cubic exhibits one sign change, it possesses exactly one positive real root, $\bar{\tau}_{\eta 2}$ say. This, in return, implies that the unique extremum of A_3 lies at $\tau_{\eta 2} = \bar{\tau}_{\eta 2}$. As $\partial A_3/\partial\tau_{\eta 2} > 0$ for $\tau_{\eta 2} = 0$ and $\partial A_3/\partial\tau_{\eta 2} < 0$ for $\tau_{\eta 2}$ large enough, A_3 has a global maximum at $\tau_{\eta 2} = \bar{\tau}_{\eta 2}$. Since date-1 price efficiency is inversely related to A_3, it is decreasing (resp., increasing) in $\tau_{\eta 2}$ for $\tau_{\eta 2} \lesseqgtr \bar{\tau}_{\eta 2}$. q.e.d.

Proof of Proposition 5: Define

$$\begin{aligned}
A_1 &\equiv \left(\frac{1}{\rho_1} - \frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{s1}}\\
A_2 &\equiv \left(\frac{c_{11}}{a_1}\right)^2 \frac{1}{\tau_{\eta 1}}\\
A_3 &\equiv \left(\frac{c_{12}}{a_1}\right)^2 \left(\frac{1}{\tau_{s2}} + \frac{1}{\tau_{\eta 2}}\right),
\end{aligned}$$

so that $\mathrm{var}^{-1}(\theta \mid P_1^{**}) = \tau_\theta + (A_1 + A_2 + A_3)^{-1}$. A_1, A_2, and A_3 are the CON, COMESCON, and COMSFUN effects defined in the text, respectively. From Proposition 3,

$$\begin{aligned}
\frac{1-a_2}{b_2} &\to 0 \\
\frac{1-a_2}{b_2}\tau_{\eta 2} &\to \frac{\tau_\theta + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}{\rho_2} \\
b_2\Gamma_2\Delta &\to \rho_2 \\
\frac{c_{11}}{a_1} &\to \frac{\rho_1\tau_{\eta 1}}{\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})} \\
\frac{c_{12}}{a_1} &\to 0 \\
A_1 &\to \frac{1}{\rho_1^2\tau_{s1}}\left[\frac{\tau_\varepsilon + \rho_1^2\tau_{s1}}{\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}\right]^2 \\
A_2 &\to \frac{1}{\rho_1^2\tau_{\eta 1}}\left[\frac{\rho_1^2\tau_{\eta 1}}{\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}\right]^2 \\
A_3 &\to 0
\end{aligned}$$

as $\tau_{\eta 2} \to \infty$ and

$$\begin{aligned}
\frac{1-a_2}{b_2} &= \gamma\frac{\tau_\theta + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}{1+\gamma\rho_2\tau_{s2}} \\
b_2\Gamma_2\Delta &= \frac{1+\gamma\rho_2\tau_{s2}}{\gamma\tau_{s2}} \\
\frac{c_{11}}{a_1} &= \frac{\rho_1\tau_{\eta 1}}{B\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})} \\
\frac{c_{12}}{a_1} &= 0 \\
A_1 &= \frac{1}{\rho_1^2\tau_{s1}}\left[\frac{B\tau_\varepsilon + \rho_1^2\tau_{s1}}{B\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}\right]^2 \\
A_2 &= \frac{1}{\rho_1^2\tau_{\eta 1}}\left[\frac{\rho_1^2\tau_{\eta 1}}{B\tau_\varepsilon + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}\right]^2 \\
A_3 &= 0
\end{aligned}$$

for $\tau_{\eta 2} = 0$, where

$$B \equiv \left[1 + \gamma^2\tau_{s2}\frac{\tau_\theta + \rho_1^2(\tau_{s1}+\tau_{\eta 1})}{(1+\gamma\rho_2\tau_{s2})^2}\right]^{-1} < 1.$$

We note in passing that c_{11}/a_1 is greater for $\tau_{\eta 2} = 0$ than for $\tau_{\eta 2} \to \infty$. We also note that increases in $\gamma\rho_2\tau_{s2}$ raise B and reduce c_{11}/a_1 for $\tau_{\eta 2} = 0$. It is easily checked that $A_1+A_2+A_3$ is no greater for $\tau_{\eta 2} = 0$ than for $\tau_{\eta 2} \to \infty$. q.e.d.

Proof of Proposition 6:
$\partial[\text{var}^{-1}(\theta \mid P_1^{**})]/\partial\tau_{\eta 2} < 0$ for $\tau_{\eta 2} = 0$ exactly if

$$\sum_{i=1}^{3} \frac{\partial A_i}{\partial \tau_{\eta 2}}\bigg|_{\tau_{\eta 2}\,=\,0} > 0.$$

Define

$$\phi_1 \equiv \frac{1 - a_2}{b_2^2 \Gamma_2 \Delta},$$

so that

$$A_1 = \left\{ \frac{\tau_\varepsilon + \rho_1^2 \tau_{s1}(1 + \phi_1)}{\rho_1[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})(1 + \phi_1)]} \right\}^2 \frac{1}{\tau_{s1}}.$$

Differentiating with respect to $\tau_{\eta 2}$ yields

$$\frac{\partial A_1}{\partial \tau_{\eta 2}} = -\frac{2\tau_{\eta 1}\tau_\varepsilon \left[\tau_\varepsilon + \rho_1^2 \tau_{s1}(1 + \phi_1)\right] \frac{\partial \phi_1}{\partial \tau_{\eta 2}}}{\tau_{s1} \left[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})(1 + \phi_1)\right]^3}.$$

Likewise, differentiating

$$A_2 = \tau_{\eta 1} \left[\frac{\rho_1(1 + \phi_1)}{\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})(1 + \phi_1)} \right]^2$$

gives

$$\frac{\partial A_2}{\partial \tau_{\eta 2}} = \frac{2\rho_1^2 \tau_{\eta 1}\tau_\varepsilon(1 + \phi_1)\frac{\partial \phi_1}{\partial \tau_{\eta 2}}}{\left[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})(1 + \phi_1)\right]^3}.$$

Hence,

$$\sum_{i=1}^{2} \frac{\partial A_i}{\partial \tau_{\eta 2}} = -\frac{2\tau_{\eta 1}\tau_\varepsilon^2 \frac{\partial \phi_1}{\partial \tau_{\eta 2}}}{\tau_{s1}\left[\tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})(1 + \phi_1)\right]^3}.$$

Using the coefficients in Proposition 3, ϕ_1 can be written as

$$\phi_1 = \frac{\gamma^2[\tau_\theta + \rho_1^2(\tau_{s1} + \tau_{\eta 1})](\tau_{s2} + \tau_{\eta 2})}{[1 + \gamma\rho_2(\tau_{s2} + \tau_{\eta 2})]^2}.$$

So

$$\frac{\partial\phi_1}{\partial\tau_{\eta 2}} = \frac{\gamma^2\left[1-\gamma\rho_2(\tau_{s2}+\tau_{\eta 2})\right]\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]}{\left[1+\gamma\rho_2(\tau_{s2}+\tau_{\eta 2})\right]^3}.$$

Letting

$$\phi_{10} \equiv \phi_1|_{\tau_{\eta 2=0}} = \frac{\gamma^2\tau_{s2}\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]}{(1+\gamma\rho_2\tau_{s2})^2},$$

it follows that

$$\sum_{i=1}^{2}\left.\frac{\partial A_i}{\partial\tau_{\eta 2}}\right|_{\tau_{\eta 2}=0} = -\frac{2\gamma^2\tau_{\eta 1}\tau_\varepsilon^2(1-\gamma\rho_2\tau_{s2})\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]}{\tau_{s1}(1+\gamma\rho_2\tau_{s2})^3\left[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})(1+\phi_{10})\right]^3}.$$

Differentiating

$$A_3 = \tau_{\eta 2}\frac{\gamma^2[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})]^2(\tau_{s2}+\tau_{\eta 2})}{\tau_{s2}[1+\gamma\rho_2(\tau_{s2}+\tau_{\eta 2})]^4[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\,(1+\phi_1)]^2}$$

gives

$$\begin{aligned}\frac{\partial A_3}{\partial\tau_{\eta 2}} = {} & \frac{\gamma^2[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})]^2(\tau_{s2}+\tau_{\eta 2})}{\tau_{s2}[1+\gamma\rho_2(\tau_{s2}+\tau_{\eta 2})]^4[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\,(1+\phi_1)]^2} \\ & + \tau_{\eta 2}\frac{\partial}{\partial\tau_{\eta 2}}\left\{\frac{\gamma^2[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})]^2(\tau_{s2}+\tau_{\eta 2})}{\tau_{s2}[1+\gamma\rho_2(\tau_{s2}+\tau_{\eta 2})]^4[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\,(1+\phi_1)]^2}\right\}.\end{aligned}$$

Evaluating this derivative at $\tau_{\eta 2}=0$, we obtain

$$\left.\frac{\partial A_3}{\partial\tau_{\eta 2}}\right|_{\tau_{\eta 2}=0} = \frac{\gamma^2\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]^2}{(1+\gamma\rho_2\tau_{s2})^4\left[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\,(1+\phi_{10})\right]^2}.$$

Therefore,

$$\begin{aligned}\sum_{i=1}^{3}\left.\frac{\partial A_i}{\partial\tau_{\eta 2}}\right|_{\tau_{\eta 2}=0} \;=\; & \frac{\gamma^2\left[\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})\right]}{(1+\gamma\rho_2\tau_{s2})^3\left[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})(1+\phi_{10})\right]^2} \\ & \cdot\left\{-\frac{2\tau_{\eta 1}\tau_\varepsilon^2(1-\gamma\rho_2\tau_{s2})}{\tau_{s1}\left[\tau_\varepsilon+\rho_1^2(\tau_{s1}+\tau_{\eta 1})(1+\phi_{10})\right]}+\frac{\tau_\theta+\rho_1^2(\tau_{s1}+\tau_{\eta 1})}{1+\gamma\rho_2\tau_{s2}}\right\}.\end{aligned}$$

The condition in the proposition follows immediately. q.e.d.

Proof of Proposition 7:

The vectors of signals available to rational agents born at dates 1 and 2 are $I_{i1} = (P_1, x_i, Y_1, Y_2)$ and $I_{i2} = (P_1, x_i, Y_1, Y_2, P_2)$, respectively. Define P_1^* and P_2^* as in Section 3. The analysis of date 2 in Section 3 goes through unchanged. So the expressions for ρ_2, a_2, b_2, c_{21}, c_{22}, and d_2 in Proposition 3 hold true.

The asset demands of the first-generation rational traders are

$$D_{i1} = \gamma \frac{\mathrm{E}(P_2 \,|\, I_{1i}) - P_1}{\mathrm{var}(P_2 \,|\, I_{1i})}.$$

The conditional moments of P_2 as of date 1 are:

$$\begin{aligned}
\mathrm{E}(P_2 \,|\, I_{1i}) &= a_2 \Gamma_1 \left[\tau_\varepsilon \, x_i + \rho_1^2 (\tau_{s1} + \tau_{\eta 1}) P_1^*\right] + b_2 \Gamma_2 \tau_{\eta 2} Y_2 - c_{21} Y_1 - c_{22} Y_2 + d_2 P_1 \\
\mathrm{var}(P_2 \,|\, I_{1i}) &= a_2^2 \Gamma_1 + b_2^2 \Gamma_2,
\end{aligned}$$

where Γ_1 and Γ_2 are defined as in Proposition 3. These expressions can be used to rewrite the date-1 market clearing condition $\int_0^1 D_{i1} di + s_1 = 0$ as

$$\begin{aligned}
0 = {} & a_2 \Gamma_1 \tau_\varepsilon \, \theta \\
& + \frac{a_2^2 \Gamma_1 + b_2^2 \Gamma_2}{\gamma} s_1 - \left\{ a_2 \Gamma_1 \left[\rho_1 \tau_{\eta 1} - \rho_1^2 (\tau_{s1} + \tau_{\eta 1}) \frac{c_{11}}{a_1} \right] + c_{21} \right\} Y_1 \\
& + \left[-a_2 \Gamma_1 \rho_1^2 (\tau_{s1} + \tau_{\eta 1}) \frac{c_{12}}{a_1} + b_2 \Gamma_2 \tau_{\eta 2} - c_{22} \right] Y_2 \\
& - \left[1 - d_2 - \frac{a_2 \Gamma_1 \rho_1^2 (\tau_{s1} + \tau_{\eta 1})}{a_1} \right] P_1.
\end{aligned}$$

Solving for P_1 and matching coefficients with (7) yields the expressions for ρ_1, a_1, b_1, c_{11}, and c_{12} in the proposition. q.e.d.

Proof that $\mathrm{E}[(P_2 - P_1) s_1] < 0$ *in the OLG model:*

From Proposition 7, we get the same expression for $-c_{21}/\tau_{s1} + d_2(b_1 - c_{11})/\tau_{s1}$ as in the LLA model and

$$(b_1 - c_{11}) \frac{1}{\tau_{s1}} = \frac{\rho_1^2 \tau_{s1} (\Delta + \tau_\varepsilon) + \tau_\varepsilon \left[\tau_\varepsilon + \rho_2^2 (\tau_{s2} + \tau_{\eta 2})\right]}{\Delta \rho_1 \tau_{s1} \left[\tau_\theta + \tau_\varepsilon + \rho_1^2 (\tau_{s1} + \tau_{\eta 1})\right]}.$$

Plugging this into the expression for noise traders' interim expected wealth in the main text and simplifying terms, we get

$$\frac{\rho_1}{\Delta} - (b_1 - c_{11}) \frac{1}{\tau_{s1}} = - \frac{(\rho_1^2 \tau_{s1} + \tau_\varepsilon) \left[\tau_\varepsilon + \rho_2^2 (\tau_{s2} + \tau_{\eta 2})\right]}{\Delta \rho_1 \tau_{s1} \left[\tau_\theta + \tau_\varepsilon + \rho_1^2 (\tau_{s1} + \tau_{\eta 1})\right]} < 0.$$

q.e.d.

Proof of Proposition 8:

Define A_i ($i = 1, 2, 3$) as in the proof of Proposition 5. Using the coefficients in Proposition 7, one obtains

$$\begin{aligned}
A_1 &= \frac{1}{\rho_1^2 \tau_{s1}} \left\{ \frac{\rho_1^2 \tau_{s1}(\Delta + \tau_\varepsilon) + \tau_\varepsilon[\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})]}{\rho_1^2(\tau_{s1} + \tau_{\eta 1})(\Delta + \tau_\varepsilon) + \tau_\varepsilon[\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})]} \right\}^2 \\
A_2 &= \tau_{\eta 1} \left\{ \frac{\rho_1(\Delta + \tau_\varepsilon)}{\rho_1^2(\tau_{s1} + \tau_{\eta 1})(\Delta + \tau_\varepsilon) + \tau_\varepsilon[\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})]} \right\}^2 \\
A_3 &= \frac{\tau_{\eta 2}}{\tau_{s2}(\tau_{s2} + \tau_{\eta 2})} \left(\frac{\tau_\theta + \tau_\varepsilon + \rho_1^2(\tau_{s1} + \tau_{\eta 1})}{\gamma \left\{ \rho_1^2(\tau_{s1} + \tau_{\eta 1})(\Delta + \tau_\varepsilon) + \tau_\varepsilon[\tau_\varepsilon + \rho_2^2(\tau_{s2} + \tau_{\eta 2})] \right\}} \right)^2,
\end{aligned}$$

where Δ is defined as in Proposition 3.

As $\tau_{\eta 2} \to \infty$, we have

$$A_1 + A_2 \to \frac{[(\gamma\tau_\varepsilon)^2 \tau_{s1} + \tau_\varepsilon]^2 + \tau_{\eta 1}\tau_{s1}(\gamma\tau_\varepsilon)^4}{(\gamma\tau_\varepsilon)^2 \tau_{s1} \left[(\gamma\tau_\varepsilon)^2(\tau_{s1} + \tau_{\eta 1}) + \tau_\varepsilon\right]^2}$$

and $A_3 \to 0$. These limits are identical to those in the model of Section 3 (cf. the proof of Proposition 5). For $\tau_{\eta 2} = 0$, we have

$$A_1 + A_2 = \frac{(\rho_{10}^2 \tau_{s1} + \tau_\varepsilon C)^2 + \tau_{\eta 1}\tau_{s1}\rho_{10}^4}{\rho_{10}^2 \tau_{s1} \left[\rho_{10}^2(\tau_{s1} + \tau_{\eta 1}) + \tau_\varepsilon C\right]^2},$$

where

$$\begin{aligned}
\rho_{10} &\equiv \gamma\tau_\varepsilon \frac{\gamma^2 \tau_\varepsilon \tau_{s2}}{1 + \gamma^2 \tau_\varepsilon \tau_{s2}} \\
C &\equiv \frac{\tau_\varepsilon + \rho_2^2 \tau_{s2}}{\tau_\varepsilon + \Delta_0} \\
\Delta_0 &\equiv \tau_\theta + \tau_\varepsilon + \rho_{10}^2(\tau_{s1} + \tau_{\eta 1}) + \rho_2^2 \tau_{s2}
\end{aligned}$$

and $\rho_2 = \gamma\tau_\varepsilon$ is given by Proposition 7, and $A_3 = 0$.

For $\tau_{\eta 1} = 0$, the expressions for $A_1 + A_2$ simplify to

$$A_1 + A_2 \to \frac{1}{(\gamma\tau_\varepsilon)^2 \tau_{s1}}$$

for $\tau_{\eta 2} \to \infty$ and

$$A_1 + A_2 = \frac{1}{\rho_{10}^2 \tau_{s1}} > \frac{1}{(\gamma\tau_\varepsilon)^2 \tau_{s1}}$$

for $\tau_{\eta 2} = 0$. q.e.d.

B Alternative measures of price efficiency

Grossman and Stiglitz (1980, p. 399) and Li (2022), based on Mendel and Shleifer (2012, p. 314), propose alternative measures of price efficiency, viz., $\text{corr}^2(\theta, P)$ and $\text{var}(P|\, s, \eta)/\text{var}(P|\, \theta)$, respectively.

In the static model of Section 2, from (2),

$$\begin{aligned} \text{cov}(\theta, P) &= a\frac{1}{\tau_\theta} \\ \text{var}(P) &= a^2\frac{1}{\tau_\theta} + (b-c)^2\frac{1}{\tau_s} + c^2\frac{1}{\tau_\eta} \\ \text{var}(P|\, s, \eta) &= a^2\frac{1}{\tau_\theta} \\ \text{var}(P|\, \theta) &= (b-c)^2\frac{1}{\tau_s} + c^2\frac{1}{\tau_\eta}. \end{aligned}$$

Using Proposition 1, we obtain

$$\text{corr}^2(\theta, P) = \frac{\text{cov}^2(\theta, P)}{\text{var}(\theta)\text{var}(P)} = \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \frac{1}{\text{var}^{-1}(\theta|\, P) - \tau_\theta}} \tag{B.1}$$

and

$$\frac{\text{var}(P|\, s, \eta)}{\text{var}(P|\, \theta)} = \frac{\text{var}^{-1}(\theta|\, P)}{\tau_\theta} - 1. \tag{B.2}$$

For given τ_θ, both alternative measures of price efficiency are increasing functions of price efficiency $\text{var}^{-1}(\theta|\, P)$. So, given τ_θ, the three different notions of price efficiency move in lockstep when $\tau_{\eta 2}$ changes. In the dynamic model of Section 3, using (7) instead of (2) and Proposition 3 (in the LLA version) or Proposition 7 (in the OLG version) instead of Proposition 1, one obtains the same expressions (B.1) and (B.2) for the alternative measures of price efficiency, with P_1^{**} replacing P.