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Cross-predictability of returns: An equilibrium perspective

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ABSTRACT

Directed cash flow linkages give rise to cross-predictability of returns between linked assets solely through equilibrium pricing effects in a frictionless, informationally efficient, and complete market. The risk-based mechanism builds on fundamental principles of economically linked assets. In a consumption-based equilibrium asset pricing model which allows for closed-form solutions, expected excess returns are tied to differences in the strengths of cash flow linkages. Our equilibrium mechanism generates cross-predictability under two conditions. First, cash flow linkages must have a direction. Second, there must be cross-sectional heterogeneity in the strengths of these linkages.

Keywords: Cross-predictability, equilibrium asset pricing, economically linked assets, directed cash flow linkages, consumption-based asset pricing

JEL: G10, G11, G12

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1 Introduction

Cross-predictability of returns, i.e., the observation that returns of one asset (equity return of a firm, industry, country, etc.) in period t have predictive power for the returns of another, economically linked asset in period $t+1$, is a pervasive empirical pattern. It has been documented in various empirical settings and sample periods.[1] The ample evidence and the fact that cross-predictability still exists today suggest that it is driven by principles of equilibrium. Building on this idea, we show that cross-predictability can arise solely through risk-based equilibrium pricing effects, while maintaining the assumptions of market efficiency and market completeness.

The mechanism builds on directed cash flow linkages between assets. A large strand of literature has documented that the existence of linkages between firms, industries, or countries can turn microeconomic shocks into aggregate fluctuations.[2] Specifically, links have a direction: it matters whether a link runs from asset i to asset j or the other way around. We show that this directionality, and the associated amplification of cash flow shocks, has implications for equilibrium risk premia that give rise to cross-predictability of returns.

As a natural starting point for setting up a cross-predictability long-short strategy, the universe of assets needs to be divided into two (not necessarily disjunct) subsets. *Signal assets* are those whose returns serve as input for portfolio formation, the sorting criterion. *Investment assets* are those in which the investor goes long or short to implement the strategy. We study cross-predictability through average returns of long-short strategies constructed from investment assets, which are pairwise connected to signal assets. Our equilibrium explanation of cross-predictability builds on the fundamental assumption that cash flow linkages between signal and investment assets are directed. In a nutshell, we find that directed cash flow linkages give rise to interdependencies between *expected* (or average realized) excess returns of signal and investment assets in equilibrium. Differences in the strengths of cash flow linkages determine both the expected return differentials of signal assets up to time t and the expected return differentials of investment assets after time t.

Suppose that cash flow links go from signal to investment assets. By this, we mean that negative shocks to the cash flows of signal assets increase the probability of subsequent

[1] Examples include Cohen and Frazzini (2008), Hou (2007), Menzly and Ozbas (2010), Albuquerque, Ramadorai, and Watugala (2015), Lee, Sun, Wang, and Zhang (2019), and Huang, Lee, Song, and Xiang (2022). The most recent evidence is provided by Yan and Yu (2023) and by Avramov, Ge, Li, and Linton (2026), based on U.S. data until 2018 and 2024, respectively. Kelly, Malamud, and Pedersen (2023) generalize the notion of cross-predictability and exploit it to reverse-engineer so-called principal portfolios without specifying the fundamental link itself.

[2] See, for instance, the seminal contributions by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2021).

negative shocks to the cash flows of investment assets. We show that, even though cash flow shocks propagate with a time lag and may not even be observed in the time series, expected returns reflect this risk at any time since markets are efficient. Specifically, the equilibrium expected excess return differential between signal assets is positive exactly when the expected excess return differential of the respective economically linked investment assets is positive as well. The differentials represent two sides of the same coin, but they are positive for different reasons.

In the case described above, signal assets have the capacity to spread cash flow shocks throughout the economy. This makes their own cash flows more systematic overall, which increases the risk premia of the signal assets in equilibrium. As a result, the *difference* in signal assets' expected excess returns reflects *differences* in the strengths of cash flow linkages, i.e., signal assets' capacity to spread shocks.

Investment assets receive cash flow shocks in the sense that their own cash flows become riskier after a cash flow shock of a signal asset. Investment assets therefore earn an extra risk premium in equilibrium since their valuation ratios drop upon shocks of the respective signal assets. Additionally, risk premia of investment assets are subject to a hedging effect: when a shock propagates through the economy, an investment asset that is not directly exposed becomes relatively more attractive and its valuation ratio increases. Being shielded from the propagation of aggregate shocks makes an investment asset earn a correspondingly lower risk premium in equilibrium. Crucially, the corresponding compression in risk premia is larger for the investment asset linked to the weaker signal asset. In the cross-section, the receiving and hedging channels thus reinforce rather than offset each other: both raise the expected excess return of the investment asset linked to the stronger signal asset relative to the one linked to the weaker signal asset. Consequently, the *difference* in investment assets' expected excess returns reflects *differences* in the strengths of cash flow linkages through these two mutually reinforcing channels.

To sum up, both expected return differentials depend on the same parameters capturing the strengths of cash flow linkages and are thus tied together in equilibrium. As a result, the expected (or average realized) return differentials of signal assets up to time t are positive exactly when the expected (or average realized) return differentials of investment assets after time t are positive.

Put differently, we show that our equilibrium mechanism generates cross-predictability under two key conditions. First, cash flow linkages must have a direction. Importantly, while it makes a difference for individual risk premia whether a link goes from a signal asset to an investment asset or the other way around, cross-predictability arises in both cases, and also when there are two links in opposing directions. Second, there must be cross-sectional differences in the strengths of cash flow linkages. If all cash flow linkages

in the economy are of equal magnitude (or if there are no cash flow linkages at all), cross-predictability will not arise through the mechanism explained above.[3]

Studying signal and investment assets jointly in one economy is the first step towards understanding cross-predictability. To this end, we tailor the tractable consumption-based equilibrium asset pricing model of Branger, Konermann, Meinerding, and Schlag (2021) to our needs, which builds on the idea of a granular economy put forward by Gabaix (2011). The model specifically features the propagation of cash flow shocks through arbitrary directed networks. Notably, its closed-form solution, in conjunction with a series expansion, allows us to decompose equilibrium risk premia of *individual* assets into spreading, receiving, and hedging channels. We show that these three channels become interrelated in a setting with *pairwise connected* signal and investment assets, and we explain this interrelation precisely in closed form. The spreading channel of the signal assets, on the one hand, and the mutually reinforcing receiving and hedging channel of the investment assets, on the other hand, represent two sides of the same coin. This interrelation leads to commonality in expected (or average realized) returns of connected signal and investment assets, and this commonality is at the heart of our equilibrium pricing mechanism.

A key contribution of our paper lies in designing the relevant modeling assumptions in such a way that this key mechanism can be formalized tractably and transparently in closed form. After that, we analyze the role of specific modeling assumptions. First, all theorems in our paper also hold when cash flow shocks are positive. This alleviates the concern that our key mechanism implies the magnitude of cross-predictability to covary with aggregate macroeconomic fluctuations. There is no particular form of procyclicality or countercyclicality of cross-predictability hard-wired into our model. Second, we show that the term explaining cross-predictability in our model is positive for any length of the signal and investment period. At any horizon, cross-sectional differences in the strength of cash flow linkages are the determining quantity.

By showing how cross-predictability arises through risk-based equilibrium pricing effects, while maintaining the assumptions of market efficiency and market completeness, our paper closes an important gap in the literature. Some authors view cross-predictability as evidence for mispricing due to informational or other frictions. Based on the seminal work of Lo and MacKinlay (1990), the literature focuses on slow information diffusion, information discreteness, market segmentation, limits to arbitrage, bounded rationality, or financial frictions, addressing cross-predictability at rather short horizons. We do not want to argue against the existence of market inefficiencies, in particular because our model is frictionless, not a model of mispricing. Our point is that frictions are not *necessary* to

[3]In the language of our model, these conditions read: (1) $\beta_i > 0$ and $\beta_{i'} > 0$ for at least two assets i, i' and (2) $\beta_i - \beta_{i'} > 0$ for these two assets.

generate cross-predictability. While the empirical literature explicitly acknowledges the existence of fundamental linkages between firms, industries, or countries, it overlooks their equilibrium implications. Our risk-based explanation builds on Carvalho, Nirei, Saito, and Tahbaz-Salehi (2021), who provide rich empirical evidence that such fundamental linkages lead to delayed propagation of cash flow shocks. We go one step further and prove that cross-predictability can arise from shock propagation risk through equilibrium effects only, which manifest in the spreading, receiving and hedging channels discussed above. Even though cash flow shocks propagate with a time lag and may not even be observed in the time series, expected returns reflect this risk at all times since markets are efficient. The representative agent in our model has rational expectations and full information and is not subject to any frictions; nonetheless, cross-predictability arises. Our mechanism, which is consistent with the empirical evidence of "commonality in momentum" presented in Burt and Hrdlicka (2021), also generates cross-predictability at longer horizons.

Broadly speaking, our paper links three previously unconnected strands of the literature. The first strand of the literature has documented and analyzed cross-predictability for various instances of connectedness at the level of fundamentals, such as customer-supplier relations between firms studied in Cohen and Frazzini (2008).[4] The second strand of literature shows how intersectoral input-output linkages may turn microeconomic shocks into aggregate fluctuations and macroeconomic tail risks. It is motivated, for instance, by the theoretical work of Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). Taking this literature as given, our model starts from the assumption that directed cash flow linkages exist. The third strand of literature is equilibrium asset pricing. We build on long-run risk models as pioneered by Bansal and Yaron (2004) and models featuring rare disasters, e.g., Wachter (2013). The solution of our equilibrium model builds on the exponentially affine framework of Eraker and Shaliastovich (2008). The modeling of cash flow shock propagation follows Branger, Konermann, Meinerding, and Schlag (2021). Risk premia in production networks are studied by Herskovic (2018), Gofman, Segal, and Wu (2020), Grigoris, Hu, and Segal (2022), and Ramirez (2024). Conditional lead and lag structures between aggregate growth and industry-level cash flow growth have been con-

[4]Further candidate linkages at the firm level include competition between firms, geographic proximity, or technological similarity, which are analyzed by Cohen and Lou (2012), Schlag and Zeng (2019), Parsons, Sabbatucci, and Titman (2020), Cao, Chordia, and Lin (2016), Huang, Lee, Song, and Xiang (2022), Scherbina and Schlusche (2020), Lee, Sun, Wang, and Zhang (2019), and Dou and Wu (2023) among others. Hou (2007), Hong, Torous, and Valkanov (2007), and Menzly and Ozbas (2010) study cross-predictability at the industry level, e.g., along input-output linkages. The international asset pricing literature, e.g., Rizova (2013), Albuquerque, Ramadorai, and Watugala (2015), and Huang (2015), has focused on linkages between countries. Kelly, Malamud, and Pedersen (2023) exploit cross-predictability to reverse-engineer so-called principal portfolios.

sidered by Croce, Marchuk, and Schlag (2023). However, none of these papers focus on cross-predictability.

2 Decomposing risk premia of individual assets with directed cash flow links

We want to show that cross-predictability can arise solely through risk-based general equilibrium pricing effects, while maintaining the assumptions of market efficiency and market completeness. For this purpose, we study expected excess returns in a parsimonious model with the following four features. First, the model distinguishes between signal and investment assets. *Signal assets* are assets whose returns serve as input for portfolio formation. These returns are used as a sorting criterion or appear on the right-hand side of regressions. Based on this input, an investor then combines the *investment assets* in a long-short portfolio to implement a long-short strategy. Second, the model allows for cash flow linkages between assets which are, by construction, directed.[5] This is motivated by Carvalho, Nirei, Saito, and Tahbaz-Salehi (2021) who provide rich empirical evidence that fundamental linkages lead to delayed propagation of cash flow shocks. Third, we embed these ingredients in a tractable general equilibrium asset pricing model. To this end, we tailor the model of Branger, Konermann, Meinerding, and Schlag (2021) to our needs. Fourth, by construction, the model excludes all cross-sectional heterogeneity among assets except for the strengths of the directed cash flow linkages. This ensures that our results are driven solely by these linkages, rather than by other differences in asset characteristics.

Figure 1 illustrates our model of directed cash flow linkages. There are $2n$ assets in a Lucas endowment economy: n signal assets $(S_1, \ldots, S_n)$ and n investment assets $(I_1, \ldots, I_n)$. We translate the notion of *directed cash flow linkages* from signal to investment assets into the modeling assumption that an exogenous negative shock to the cash flow of a signal asset increases the *risk* of subsequent shocks to the cash flow of a linked investment asset, but does not necessarily have any immediate consequences. Specifically, the cash flows are affected by occasional jumps, i.e., the log cash flows of S_j and I_j $(j = 1, \ldots, n)$ have dynamics

$$dy_{Sj,t} = \mu\, dt + L\, dN_{Sj,t}, \tag{1}$$

$$dy_{Ij,t} = \mu\, dt + L\, dN_{Ij,t}. \tag{2}$$

[5]Undirected cash flow links are nested as a special case in which there are directed linkages of the same strength going in opposite directions.

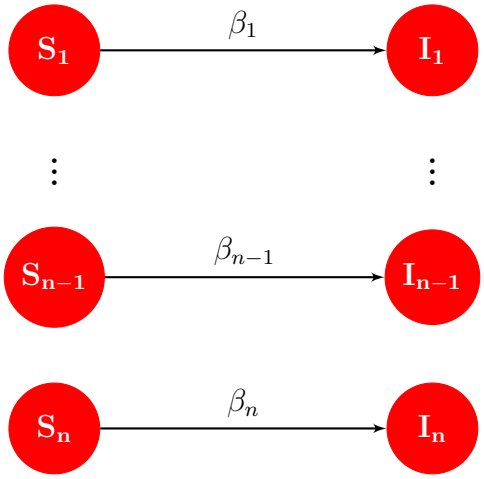


Figure 1
Directed cash flow linkages in Equations (3) and (4)

with a constant drift rate μ and a constant jump size $L < 0$.[6] We assume the following dynamics for the stochastic jump intensities $\ell_{Sj,t}$ and $\ell_{Ij,t}$:

$$d\ell_{Sj,t} = \kappa \left(\bar{\ell}_S - \ell_{Sj,t}\right) dt, \tag{3}$$

$$d\ell_{Ij,t} = \kappa \left(\bar{\ell}_I - \ell_{Ij,t}\right) dt + \beta_j \, dN_{Sj,t}, \tag{4}$$

where $\beta_j > 0$ represents a discrete change in the jump intensity $\ell_{Ij,t}$ of *investment* asset j induced by a jump in *signal* asset j.[7]

Equations (1) to (4) formalize the dynamic shock propagation mechanism. With $\beta_j > 0$, a downward jump in the cash flow of signal asset S_j immediately increases the jump intensity of the cash flow of investment asset I_j by the amount β_j. An initial cash flow shock in S_j thus increases the *probability* of future cash flow shocks to I_j, but it is unknown when (and if at all) these shocks will *materialize*. The shock propagation parameters β_j completely determine the cash flow links in the economy. The dynamics in Equations (3) and (4) reflect the idea behind Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) in the sense that cash flow shocks $N_{Sj,t}$ become systematic and cannot be "diversified away." All jumps impact aggregate consumption with a jump size $K < 0$, i.e.,

[6] Following Branger, Konermann, Meinerding, and Schlag (2021), we abstract from diffusive shocks in the model because this allows us to isolate the key risk premia channels arising from directed cash flow linkages in closed form.

[7] These dynamics represent a special case of so-called "mutually exciting jump processes" studied, e.g., in Aït-Sahalia, Cacho-Diaz, and Laeven (2015) and Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015).

log aggregate consumption $y_t = \ln Y_t$ follows[8]

$$dy_t \quad = \quad \mu\, dt + \sum_{j=1}^{n} K\, dN_{Sj,t} + \sum_{j=1}^{n} K\, dN_{Ij,t}.$$

The assumption that all cash flow shocks feed into aggregate consumption shocks is a prerequisite for the mechanism that we suggest. It guarantees that directed cash flow linkages translate into fluctuations in aggregate consumption. Conceptually, this idea can be grounded in the granular economy framework of Gabaix (2011).

The economy is populated by a representative agent with an infinite planning horizon. The agent has full information, she observes the level of the jump intensities at any time and knows all parameters. We assume that the agent has recursive preferences so that the risk generated by state variables (in this case the intensities $\ell_{Sj,t}$ and $\ell_{Ij,t}$) will be priced in equilibrium. δ is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. We assume $\gamma > 1$ and $\psi > 1$, so that $\theta < 0$. In particular, with $\gamma > \frac{1}{\psi}$, the representative agent has a preference for early resolution of uncertainty. The technical details of the equilibrium solution are provided in Appendix A.

To investigate the profitability of long-short strategies based on cross-predictability, we need to understand how this model involving cash flow risks and shock propagation generates cross-sectional differences in average returns. In equilibrium, the local expected excess return of asset i can be written as

$$\begin{aligned} \mathrm{EER}_i \quad &= \quad \frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t & (5) \\ &= \quad \sum_{j=1}^{n} \ell_{Sj,t}\,\mathrm{MPJR}_{Sj}\,\mathrm{JEXP}_{i,Sj} + \sum_{j=1}^{n} \ell_{Ij,t}\,\mathrm{MPJR}_{Ij}\,\mathrm{JEXP}_{i,Ij}, & (6) \end{aligned}$$

i.e., it is the sum of asset i's risk premia for each jump process N_{Sj} or N_{Ij}. Each jump risk premium is given by the product of jump intensity $\ell_{Sj,t}$ (or $\ell_{Ij,t}$), market price of risk MPJR_{Sj} (or MPJR_{Ij}), and jump exposure $\mathrm{JEXP}_{i,Sj}$ (or $\mathrm{JEXP}_{i,Ij}$). The pricing kernel summarizes the equilibrium compensation for all systematic risks in the economy. Specifically, its dynamics define the market prices of jump risk for MPJR_{Sj} and MPJR_{Ij}:

$$\frac{dM_t}{M_t} \quad = \quad \ldots\, dt - \sum_{j=1}^{n} \mathrm{MPJR}_{Sj}\,\left(dN_{Sj,t} - \ell_{Sj,t}\, dt\right) - \sum_{j=1}^{n} \mathrm{MPJR}_{Ij}\,\left(dN_{Ij,t} - \ell_{Ij,t}\, dt\right). \quad (7)$$

[8]The assumption that $K < 0$ is made for ease of exposition. As we show in Sections 3 and 4, the main results in Theorems 2, 5, and 6 also hold for $L > K > 0$.

Jump exposures are the response of the price of asset i to a shock in the cash flow of the respective signal and investment asset.

$$dR_{i,t} \;=\; \ldots dt + \sum_{j=1}^{n} \mathrm{JEXP}_{i,Sj}\, dN_{Sj,t} + \sum_{j=1}^{n} \mathrm{JEXP}_{i,Ij}\, dN_{Ij,t}. \tag{8}$$

Both the market prices of jump risk and the jump exposures are non-linear functions of the model parameters, including the shock propagation parameters β_j. They depend on coefficients which are solutions of non-linear equations given in Appendix A. Motivated by Walden (2019) and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2021), we derive theorems through a Taylor series expansion and a first-order approximation. The superscript ** denotes that polynomial terms of order 2 or higher in the coefficients β are omitted.

The key quantities we analyze in this paper are the expected excess returns of signal asset S_i and investment asset I_i which can be rewritten and decomposed as follows

$$\mathrm{EER}^{**}_{Si,t} \;=\; \underbrace{\mathcal{CF}_t}_{\text{cash flow channel}} + \underbrace{\mathcal{S}_t\,\beta_i}_{\text{spreading channel}} + \underbrace{\mathcal{H}_{Si,t} \sum_{j=1}^{n} \beta_j}_{\text{hedging channel}}, \tag{9}$$

$$\mathrm{EER}^{**}_{Ii,t} \;=\; \underbrace{\mathcal{CF}_t}_{\text{cash flow channel}} + \underbrace{\mathcal{R}_{Ii,t}\,\beta_i}_{\text{receiving channel}} + \underbrace{\mathcal{H}_{Ii,t} \sum_{j=1,\, j\neq i}^{n} \beta_j}_{\text{hedging channel}}, \tag{10}$$

where $\mathcal{CF}_t = -\ell_t\,\mathcal{A}\,L$, $\mathcal{S}_t = -\ell_t\,L\,\mathcal{B}$, $\mathcal{R}_{Ii,t} = -\ell_t\,\mathcal{A}\,\mathcal{D}_{Ii}$, $\mathcal{H}_{Si,t} = -\ell_t\,\mathcal{A}\,\mathcal{C}_{Si}$, and $\mathcal{H}_{Ii,t} = -\ell_t\,\mathcal{A}\,\mathcal{C}_{Ii}$. The coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}_{Si}$, $\mathcal{C}_{Ii}$ and $\mathcal{D}_{Ii}$ are given in Appendix A. We have the following theorem regarding the signs of the coefficients.

Theorem 1. *Under the assumptions*

(A1) $0 < \kappa < 1, \quad -\ln(2) < K, \quad L < K < 0, \quad \gamma > 2\psi - 1$

(A2) $\ell_{S1,t} = \ldots = \ell_{Sn,t} = \ell_{I1,t} = \ldots = \ell_{In,t} = \ell_t$

we have $\mathcal{CF}_t > 0, \quad \mathcal{S}_t > 0, \quad \mathcal{R}_{Ii,t} > 0, \quad \mathcal{H}_{Si,t} < 0,$ *and* $\quad \mathcal{H}_{Ii,t} < 0 \quad$ *for all* i.

Proof: See Appendix B.1.

The mild parameter Assumptions (A1) are made for convenience. They are sufficient, but not necessary for the statements in Theorem 1.[9] Assumption (A2) makes sure that

[9]The assumption that $0 < \kappa < 1$ allows us to shorten the proofs. $-\ln(2) < K$ is a mild bound on the jump size of consumption which is in line with the estimates in Barro (2006). $L < K < 0$ states that the jump size in individual cash flows is more negative than the jump size in aggregate consumption, in line

the cross-sectional differences can arise through cash flow linkages only and not through differences in current levels of jump intensities.

Equation (9) shows that the expected excess return of signal asset S_i comprises three terms. (i) $\mathcal{CF}_t$ denotes the positive risk premium for jumps in the cash flow of asset S_i itself, irrespective of any shock propagation and dynamics of state variables. (ii) $\mathcal{S}_t\,\beta_i$ captures the positive risk premium arising through the *spreading channel*. With $\beta_i > 0$, jumps in signal asset S_i can propagate to investment asset I_i. These shocks thus have an impact on the rest of the economy and increase the risk of subsequent shocks elsewhere. They are more systematic and carry a higher (i.e., more negative) market price of risk in equilibrium. (iii) The last term, $\mathcal{H}_{Si,t}\sum_{j=1}^{n}\beta_j$, is negative and represents the *hedging channel*. In general, if a shock has the potential to spread throughout the economy, *unconnected* assets become relatively more attractive and their valuation ratios increase. Specifically, when a shock propagates from a signal asset to an investment asset, this reasoning applies not only to all other signal assets, but also to the signal asset from which the shock originated. Stated differently, asset S_i itself can be viewed as a hedging device against shock propagation risk. As indicated by $\mathcal{H}_{Si,t} < 0$, this hedging property lowers S_i's total risk premium in equilibrium.

Besides the above mentioned cash flow channel $\mathcal{CF}_t$, the expected excess return of investment asset I_i given in Equation (10) comprises two additional terms. (i) $\mathcal{R}_{Ii,t}\,\beta_i$ describes the positive risk premium due to the *receiving channel*. If $\beta_i > 0$, then jumps in the cash flow of asset S_i make asset I_i riskier. The possibility of such jumps gives rise to an extra risk premium for asset I_i in equilibrium. (ii) $\mathcal{H}_{Ii,t}\sum_{j=1,\,j\neq i}^{n}\beta_j$ captures the previously discussed hedging channel. Investment asset I_i can be viewed as a hedging device against any shock propagation risk in the economy, other than the one originating from signal asset S_i. Upon a cash flow shock in signal asset S_j $(j \neq i)$, investment asset I_i becomes relatively more attractive and its valuation ratio increases. Again, since $\mathcal{H}_{Ii,t} < 0$, the hedging property lowers I_i's total risk premium in equilibrium.

3 Risk premia of pairwise connected signal and investment assets

From the dissection of the risk premia of *individual* assets into different channels, we can now deduce the main theorem of our paper concerning the risk premia differentials of *pairwise connected* assets. The equilibrium expected excess return differential between

with the concept of levered consumption dating back to Abel (1999). As we show in Sections 3 and 4, the main results in Theorems 2, 5, and 6 also hold for $L > K > 0$. $\gamma > 2\psi - 1$ is a mild restriction on the preference parameters which is satisfied for standard parameter choices, such as $\psi = 1.5$ and $\gamma > 2$ or $\gamma = 10$ and $\psi < 5.5$.

two signal assets is positive exactly when the expected excess return differential of the respective economically linked investment assets is positive as well.

As outlined in Section 2, we assume the existence of a directed cash flow link from signal asset S_i to investment asset I_i for all i. To derive statements about cross-predictability, we analyze the difference in expected excess returns between two investment assets I_i and $I_{i'}$ as well as between the respective signal assets S_i and $S_{i'}$, as shown in Figure 2.

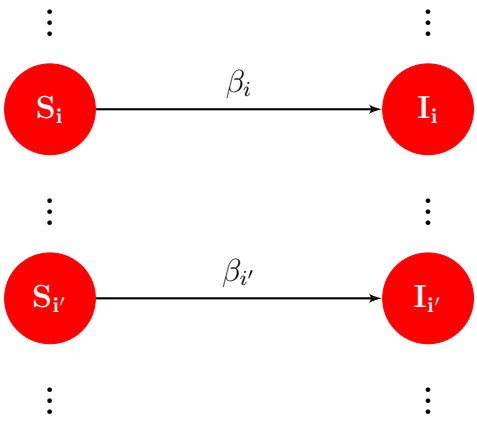


Figure 2
Directed cash flow linkages in Theorem 2

Without loss of generality, we rely on the notational convention that signal assets are always ordered in such a way that $\text{EER}^{**}_{S1,t} > \ldots > \text{EER}^{**}_{Sn,t}$. We then have:

Theorem 2. *Assuming (A1) and (A2) as well as*

(A3) $\mathcal{C}_{Ii} = \mathcal{C}_{Ii'}$ *and* $\mathcal{C}_{Si} = \mathcal{C}_{Si'}$ *for all* i, i'

the difference between the expected excess returns of two signal assets S_i *and* $S_{i'}$ *or between two investment assets* I_i *and* $I_{i'}$ *is given by*

$$EER^{**}_{Si,t} - EER^{**}_{Si',t} = \underbrace{\mathcal{S}_t \, (\beta_i - \beta_{i'})}_{\textit{spreading channel}} . \tag{11}$$

$$EER^{**}_{Ii,t} - EER^{**}_{Ii',t} = \underbrace{(\mathcal{R}_{I,t} - \mathcal{H}_{I,t}) \, (\beta_i - \beta_{i'})}_{\textit{receiving + hedging channel}} \tag{12}$$

Since $\mathcal{S}_t > 0$*, the notational convention implies that* $\beta_i - \beta_{i'} > 0$*. From* $\mathcal{R}_{I,t} - \mathcal{H}_{I,t} > 0$*, we thus conclude that*

$$EER^{**}_{Ii,t} - EER^{**}_{Ii',t} > 0 \quad \textit{for } i < i'. \tag{13}$$

Proof: See Appendix B.2.

Assumption (A3) states that the steady-state price-cash flow ratios of all investment assets are identical. Similarly, the steady-state price-cash flow ratios of all signal assets are identical.[10] This assumption is made for convenience. It is sufficient, but not necessary for the statements in Theorem 2 and allows us to tease out the effect as cleanly as possible. The assumption also implies that $\mathcal{D}_{Ii} = \mathcal{D}_{Ii'}$ for all i and i'. We can then define $\mathcal{R}_{I,t} := \mathcal{R}_{Ii,t} = \mathcal{R}_{Ii',t}$, $\mathcal{H}_{S,t} := \mathcal{H}_{Si,t} = \mathcal{H}_{Si',t}$ and $\mathcal{H}_{I,t} := \mathcal{H}_{Ii,t} = \mathcal{H}_{Ii',t}$. Importantly, while we assume symmetry among investment assets and among signal assets, we do not impose any relation between the price-cash flow ratios of signal and investment assets.

Theorem 2 states that the difference between the shock propagation parameters $\beta_i - \beta_{i'}$ is the key determinant of both expected excess return differentials. But the difference $\beta_i - \beta_{i'}$ plays different roles in Equations (11) and (12). From Theorem 1 in the previous section, we know that the expected returns of the signal assets comprise three terms: cash flow channel, spreading channel, and hedging channel. The cash flow channel is the same for all assets in the economy because their cash flows share the same jump size. Moreover, the hedging channels for two signal assets exactly offset because of Assumption (A3). Since they do not receive shocks from other assets, *each* signal asset serves as a hedging device for *all* shocks that propagate through the economy. Therefore, the expected excess return differential reflects the difference in the spreading channels of the two signal assets, which is captured by the difference $\beta_i - \beta_{i'}$. Since, by notational convention, $\mathrm{EER}^{**}_{Si,\,t} > \mathrm{EER}^{**}_{Si',\,t}$ for $i < i'$, we can deduce that $\beta_i > \beta_{i'}$, i.e., the spreading channel is more pronounced for signal asset S_i than for $S_{i'}$.

According to Theorem 1, the expected excess returns of investment assets comprise cash flow channel, receiving channel and hedging channel. Taking differences, the cash flow channels cancel out again, but the receiving and the hedging channels do not. Asset I_i's receiving channel depends on β_i and $I_{i'}$'s receiving channel on $\beta_{i'}$, so that their difference is a multiple of $\beta_i - \beta_{i'}$. With respect to the hedging channel, asset I_i (or $I_{i'}$, respectively) serves as a hedging device for all shocks except the ones originating from S_i (or $S_{i'}$). As a result, the expected excess return differential of two investment assets I_i and $I_{i'}$ contains also the difference in the hedging channels of these two assets, which depend on the difference $\beta_i - \beta_{i'}$. From the analysis of the signal assets, we know that $\beta_i > \beta_{i'}$. Since $\mathcal{R}_{I,t} > 0$ and $\mathcal{H}_{I,t} < 0$, we find that $\mathrm{EER}^{**}_{Ii,\,t} > \mathrm{EER}^{**}_{Ii',\,t}$ for $i < i'$.

The spreading channel of the signal assets on the one hand and the receiving and hedging channel of the investment assets on the other hand thus represent two sides of the

[10] As Appendix A.2 shows, price-cash flow ratios are affine functions of jump intensities, $v_{Si,t} = A_{Si} + (C_{Si,S1}, \ldots, C_{Si,Sn}, C_{Si,I1}, \ldots, C_{Si,In})\, \ell_t$. Assuming that the long-run means of the jump intensities are identical, (A3) is satisfied if the coefficients A_{Si}, $C_{Si,S1}$, …, $C_{Si,Sn}$, $C_{Si,I1}$, …, $C_{Si,In}$ are equal across signal assets. The same applies to investment assets.

same coin: equilibrium asset pricing. The expected excess return of signal asset S_i exceeds the one of $S_{i'}$ exactly when the expected excess return of investment asset I_i exceeds the one of $I_{i'}$, but for different reasons. S_i's higher expected excess return reflects its larger capacity to spread cash flow shocks throughout the economy, which makes its own cash flows more systematic. I_i's higher expected excess return comes from two other sources. First, I_i receives cash flow shocks from asset S_i, giving it a larger exposure to aggregate cash flow shocks, resulting in a larger risk premium through the receiving channel. Second, the stronger receiving channel of asset I_i is reinforced by a stronger hedging channel. Asset I_i is "immune" to receiving cash flow shocks from any signal asset other than S_i. The resulting hedging channel of asset I_i is captured by the term $\mathcal{H}_{I,t}\,\beta_{i'} < 0$, the respective hedging channel of asset $I_{i'}$ is captured by $\mathcal{H}_{I,t}\,\beta_i < 0$, all other hedging terms cancel out when taking the difference between I_i and $I_{i'}$. From $\beta_i > \beta_{i'}$, we see that the hedging channel is *less negative* for asset i than for asset i'. That is, both the receiving channel and the hedging channel contribute positively to the expected excess return differential of the investment assets.

For the sake of completeness, the following Corollary 1 highlights the *necessity* of the two key conditions mentioned in the introduction.

Corollary 1. *Under the assumptions of Theorem 2, expected excess return differentials of signal and investment assets are non-zero if and only if there is cross-sectional heterogeneity in β_i. They are zero if all β_i are equal and in particular if all β_i are zero, i.e., there are no directed cash flow linkages.*

As the final step of our argumentation, we explain why Theorem 2 implies cross-predictability. In a nutshell, cross-predictability refers to the observation that the expected (or average realized) returns of signal assets up to time t have predictive power for the expected (or average realized) returns of investment assets after time t. We discuss this specific feature within our model framework in greater detail in Section 5. The predictive power gives rise to profitable long-short investment strategies. The profitability of such a cross-predictability strategy rests on the average return differentials between signal assets and between investment assets over two different time periods: signal asset returns are evaluated over a sorting period before and up until time t, investment assets are analyzed over an investment period after time t. We have the following theorem:

Theorem 3. *Consider the following long-short investment strategy. At time t, the signal assets $S_1, \ldots, S_n$ are ordered according to their realized returns over a sorting period $[t-\Delta,\, t]$, $R_{S1,[t-\Delta,\,t]}, \ldots, R_{Sn,[t-\Delta,\,t]}$. The same ordering is then applied to the investment assets $I_1, \ldots I_n$. Over the investment period $[t,\, t+\Delta]$, the investor goes long into the top N investment assets and short into the bottom N investment assets for some cut-off threshold N. Assume (A1), (A2), and (A3) as well as*

(A4) Δ is small enough for the following approximations to hold:

$$R_{Sj,[t-\Delta,\,t]} = EER^{**}_{Sj,t} + \int_{t-\Delta}^{t} \varepsilon_{Sj,u}\, du, \tag{14}$$

$$R_{Ij,[t,\,t+\Delta]} = EER^{**}_{Ij,t} + \int_{t}^{t+\Delta} \varepsilon_{Ij,u}\, du, \tag{15}$$

where $\varepsilon_{Sj,t}$ and $\varepsilon_{Ij,t}$ denote zero mean noise processes for $j = 1, \ldots, n$, which are identically distributed across assets.

Then, this long-short portfolio strategy has a positive average return.

Proof: Assumption (A4) has two implications. First, average realized returns of signal assets over the sorting period $[t-\Delta, t]$ equal $\mathrm{EER}^{**}_{Sj,t}$ plus a zero-mean noise integral, i.e., expected excess returns are constant over this interval. Second, average realized returns of investment assets over the investment period $[t, t+\Delta]$ equal $\mathrm{EER}^{**}_{Ij,t}$ plus a zero-mean noise integral, i.e., expected excess returns are constant over this interval. Under the assumptions of Theorem 2, constructing quantile portfolios based on the realized returns of the signal assets prior to t implies that, on average, the investment assets $I_1, \ldots, I_N$ end up in the top portfolio, while the investment assets $I_{n-N+1}, \ldots, I_n$ end up in the bottom portfolio. Inequality (13) then implies that the long-short investment portfolio will have a positive average excess return. □

To understand Assumption (A4), we decompose the realized excess return differentials into expected return and noise components:

$$R_{Sj,[t-\Delta,\,t]} = \int_{t-\Delta}^{t} (\mathrm{EER}_{Sj,u} + \varepsilon_{Sj,u})\, du, \tag{16}$$

$$R_{Ij,[t,\,t+\Delta]} = \int_{t}^{t+\Delta} (\mathrm{EER}_{Ij,u} + \varepsilon_{Ij,u})\, du, \tag{17}$$

where $\varepsilon_{Sj,t}$ and $\varepsilon_{Ij,t}$ denote zero mean noise processes for $j = 1, \ldots, n$. Assumption (A4) in Theorem 3 states that Δ is reasonably small. Our model is written as a continuous-time pure jump model with finite activity Poisson processes. The probability that a jump occurs exactly at time t is zero. Hence, right and left limits coincide with probability one (almost surely). Therefore, we have $\lim_{\Delta\to 0} \mathrm{EER}^{**}_{i,\,[t-\Delta,\,t]} = \lim_{\Delta\to 0} \mathrm{EER}^{**}_{i,\,[t,t+\Delta]} = \mathrm{EER}^{**}_{i,\,t}$ with probability one. Thus, *realized* excess returns over an infinitesimally small time period *on average* approximately equal expected excess returns exactly at time t, justifying the approximations (14) and (15). We emphasize that we make Assumptions (A4) for ease of exposition. In Section 5, we relax it and show that our main results also hold under certain conditions when taking variation in expected excess returns into account.

Assumption (A1) states that all cash flow shocks in our economy are negative. In our opinion, this represents the economically most relevant case. To complete the exposition, however, we point out that the main statements presented above can also be derived in an economy with only positive jumps.

Theorem 4. *The statements from Theorem 2, in particular Inequality* (13)*, also hold when Assumption (A1) is replaced by*

(A1') $0 < \kappa < 1, \quad L > K > 0, \quad \gamma > 2\psi - 1,$

i.e., for positive jump sizes. The same applies to the statements from Theorems 5 and 6 derived further below.

Proof: See Appendix B.3.

The mechanism in Theorem 2 is also related to the empirical concept of "commonality in momentum" documented by Burt and Hrdlicka (2021).

Remark 1. *The equilibrium asset pricing mechanism explained above can serve as a foundation for "commonality in momentum." Momentum can have a common component across economically linked assets in the sense that signal assets have momentum exactly when their associated investment assets have momentum. Assuming that the average realized returns of both signal assets and investment assets before time t are approximately equal to their expected values (similar to Assumption (A4)), Theorems 2 and 3 speak to this idea. Commonality in momentum implies that the ordering of signal-asset past returns coincides with the ordering of investment-asset expected returns, consistent with Theorems 2 and 3.*

Finally, while portfolio sorts can be viewed as a nonparametric way to analyze cross-predictability, Theorem 2 implies that the intuition carries over to parametric regressions. The return differentials load on the same factor $\beta_i - \beta_{i'}$, with loadings independent from i and i'. The slope coefficient in a parametric regression of investment return differentials on signal return differentials will depend on these loadings.

4 Varying the direction of the links

4.1 Links from investment to signal assets

In the previous section, we have formulated the main theorem under the assumption of links going from signal to investment assets. More precisely, we have documented that

the spreading channel of the signal assets on the one hand and the receiving and hedging channel of the investment assets on the other hand represent two sides of the same coin, giving rise to profitable cross-predictability investment strategies in equilibrium.

In this section, we show that these strategies are profitable irrespective of the direction of the links. Whether a link goes from a signal asset to an investment asset or the other way around makes a difference only for *individual* risk premia. But our key result from Theorem 2 continues to hold: The expected excess return differential between two signal assets is positive exactly when the expected excess return differential of the economically linked investment assets is positive. However, the roles of spreading, receiving, and hedging channels are reversed.

To see this, we briefly discuss a slightly different version of the model, in which we replace Equations (3) and (4) with

$$d\ell_{Sj,t} = \kappa \left(\bar{\ell}_{Sj} - \ell_{Sj,t}\right) dt + \beta_j \, dN_{Ij,t}, \tag{18}$$

$$d\ell_{Ij,t} = \kappa \left(\bar{\ell}_{Ij} - \ell_{Ij,t}\right) dt, \tag{19}$$

Shocks can thus propagate from investment assets to signal assets, as visualized in Figure 3.

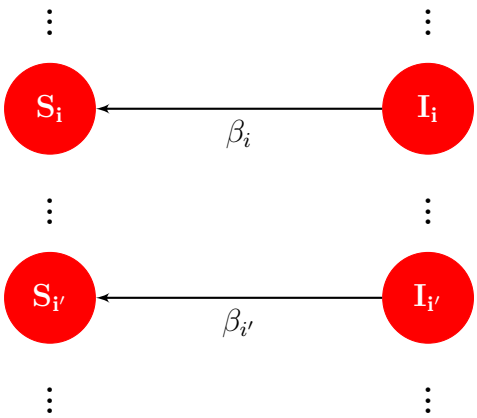


Figure 3
Cash flow linkages in Theorem 5

Similar to Theorem 2, we can then prove:

Theorem 5. *Assuming (A1), (A2), and (A3), the difference between the expected excess returns of two investment assets I_i and $I_{i'}$ (or two signal assets S_i and $S_{i'}$) is given by*

$$EER^{**}_{Ii,\,t} - EER^{**}_{Ii',\,t} = \underbrace{\mathcal{S}_t\ (\beta_i - \beta_{i'})}_{spreading\ channel} \tag{20}$$

$$EER^{**}_{Si,\,t} - EER^{**}_{Si',\,t} = \underbrace{(\mathcal{R}_{S,t} - \mathcal{H}_{S,t})\ (\beta_i - \beta_{i'})}_{receiving\ +\ hedging\ channel}. \tag{21}$$

Since $\mathcal{R}_{S,t} - \mathcal{H}_{S,t} > 0$, the notational convention implies that $\beta_i - \beta_{i'} > 0$. From $\mathcal{S}_t > 0$, we thus conclude that

$$EER^{**}_{Ii,\,t} - EER^{**}_{Ii',\,t} > 0 \quad for\ i < i'. \tag{22}$$

Proof: See Appendix B.4.

As before, Theorem 5 states that the difference between the shock propagation parameters $\beta_i - \beta_{i'}$ is the key determinant of both expected excess return differentials. But the difference $\beta_i - \beta_{i'}$ plays different roles in Equations (20) and (21), i.e., the logic of the proof is reversed.

In contrast to Theorem 2, the expected excess return differential of two signal assets now reflects the difference in their receiving and hedging channels, which is again captured by the difference $\beta_i - \beta_{i'}$. Since, by notational convention, $\text{EER}^{**}_{Si,\,t} > \text{EER}^{**}_{Si',\,t}$ for $i < i'$, we can deduce that $\beta_i - \beta_{i'} > 0$, i.e., the combined receiving and hedging channel is more pronounced for signal asset S_i than for $S_{i'}$.

The expected excess return differential of two investment assets I_i and $I_{i'}$ reflects the differences in the spreading channels of these two assets, which both depend on the difference $\beta_i - \beta_{i'}$ again. From the analysis of the signal assets, we know that $\beta_i - \beta_{i'} > 0$. Since $\mathcal{S}_t > 0$, we find that $\text{EER}^{**}_{Ii,\,t} > \text{EER}^{**}_{Ii',\,t}$ for $i < i'$.

As a result, the expected excess return of signal asset S_i exceeds the one of $S_{i'}$ exactly when the expected excess return of investment asset I_i exceeds the one of $I_{i'}$. S_i's higher expected excess return reflects its larger exposure to aggregate cash flow shocks, resulting in a larger risk premium through the receiving channel, as well as a *less negative* hedging channel. I_i's higher expected excess return captures its larger capacity to spread cash flow shocks throughout the economy, which makes its own cash flows more systematic.

4.2 Links in both directions

Lastly, we also show that the key result from Theorem 2 also applies in an economy in which shocks can propagate in both directions, from signal asset to investment assets and

vice versa. To see this, we introduce a slightly different notation and replace Equations (3) and (4) with

$$d\ell_{Sj,t} = \kappa \left(\bar{\ell}_{Sj} - \ell_{Sj,t}\right) dt + \beta_{S_j,I_j}\, dN_{Ij,t}, \tag{23}$$

$$d\ell_{Ij,t} = \kappa \left(\bar{\ell}_{Ij} - \ell_{Ij,t}\right) dt + \beta_{I_j,S_j}\, dN_{Sj,t}, \tag{24}$$

and the shock propagation is visualized in Figure 4.

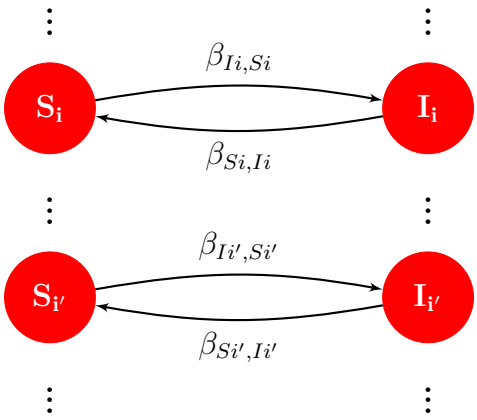


Figure 4
Cash flow linkages in Theorem 6

Similar to Theorem 2, we can then prove:

Theorem 6. *Assuming (A1), (A2), and (A3), the difference between the expected excess returns of two investment assets I_i and $I_{i'}$ (or two signal assets S_i and $S_{i'}$) is given by*

$$EER^{**}_{Ii,t} - EER^{**}_{Ii',t} = \underbrace{\mathcal{S}_t\,(\beta_{Si,Ii} - \beta_{Si',Ii'})}_{\textit{spreading channel}} + \underbrace{(\mathcal{R}_{I,t} - \mathcal{H}_{I,t})\,(\beta_{Ii,Si} - \beta_{Ii',Si'})}_{\textit{receiving + hedging channel}}. \tag{25}$$

$$EER^{**}_{Si,t} - EER^{**}_{Si',t} = \underbrace{\mathcal{S}_t\,(\beta_{Ii,Si} - \beta_{Ii',Si'})}_{\textit{spreading channel}} + \underbrace{(\mathcal{R}_{S,t} - \mathcal{H}_{S,t})\,(\beta_{Si,Ii} - \beta_{Si',Ii'})}_{\textit{receiving + hedging channel}}. \tag{26}$$

Assume that $(\beta_{Ii,Si} - \beta_{Ii',Si'}) \cdot (\beta_{Si,Ii} - \beta_{Si',Ii'}) > 0$, i.e., that $\beta_{Ii,Si} - \beta_{Ii',Si'}$ and $\beta_{Si,Ii} - \beta_{Si',Ii'}$ have the same sign. Since $\mathcal{R}_{S,t} - \mathcal{H}_{S,t} > 0$ and $\mathcal{S}_t > 0$, the notational convention implies that $\beta_{Si,Ii} - \beta_{Si',Ii'} > 0$ and $\beta_{Ii,Si} - \beta_{Ii',Si'} > 0$. From $\mathcal{R}_{I,t} - \mathcal{H}_{I,t} > 0$ and $\mathcal{S}_t > 0$, we thus conclude that

$$EER^{**}_{Ii,t} - EER^{**}_{Ii',t} > 0 \quad \textit{for } i < i'. \tag{27}$$

Proof: See Appendix B.5.

Theorem 6 is the combination of Theorems 2 and 5. If shocks can propagate from signal to investment assets and vice versa, then the expected excess return differentials of signal assets and of investment assets both comprise all three channels: spreading, receiving, and hedging channel. As before, the differentials of these channels are determined by the differences $\beta_{Ii,Si} - \beta_{Ii',Si'}$ and $\beta_{Si,Ii} - \beta_{Si',Ii'}$, respectively. The relation $(\beta_{Ii,Si} - \beta_{Ii',Si'}) \cdot (\beta_{Si,Ii} - \beta_{Si',Ii'}) > 0$ then represents a sufficient condition for the profitability of cross-predictability investment strategies. It implies that the mutual shock propagation between S_i and I_i is stronger than between $S_{i'}$ and $I_{i'}$.

Theorems 2, 5, and 6 share a common structure: in each case, the same difference in the strength of linkages, $\beta_i - \beta_{i'}$, drives both the signal asset and investment asset expected excess return differentials, with the spreading channel appearing on the side of the asset that propagates shocks, and the receiving plus-hedging channel on the side that receives them. Cross-predictability is therefore a general implication of directed cash flow linkages in equilibrium, independent of the specific direction of those links.

5 Cross-predictability in our model

5.1 Dissecting the role of risk premia and noise terms

We complete the model-theoretic exposition by demonstrating that *within our model framework* cross-predictability arises under the assumptions of Theorems 2 to 6. To this end, Assumption (A4) implies that realized excess returns can be decomposed into average excess returns and noise, and average excess returns can be proxied by expected excess returns. Defining ε as zero mean noise processes, we have

$$\begin{aligned} R_{Si,t-} &= \mathrm{EER}_{Si,t-} + \varepsilon_{Si,t-}, \\ R_{Si',t-} &= \mathrm{EER}_{Si',t-} + \varepsilon_{Si',t-}, \\ R_{Ii,t+} &= \mathrm{EER}_{Ii,t+} + \varepsilon_{Ii,t+}, \\ R_{Ii',t+} &= \mathrm{EER}_{Ii',t+} + \varepsilon_{Ii',t+}. \end{aligned}$$

The subscripts $t-$ and $t+$ denote the left and right limits in continuous time.

We are interested in the ex-post covariance between the realized return differential of signal assets before time t and the realized return differential of investment assets after

time t. We have

$$\begin{aligned} & \operatorname{Cov}\left(R_{Si,t-} - R_{Si',t-},\, R_{Ii,t+} - R_{Ii',t+}\right) \\ = \; & \underbrace{\operatorname{Cov}\left(\mathrm{EER}_{Si,t-} - \mathrm{EER}_{Si',t-},\, \mathrm{EER}_{Ii,t+} - \mathrm{EER}_{Ii',t+}\right)}_{(i)} \\ & + \underbrace{\operatorname{Cov}\left(\mathrm{EER}_{Si,t-} - \mathrm{EER}_{Si',t-},\, \varepsilon_{Ii,t+} - \varepsilon_{Ii',t+}\right)}_{(ii)} \\ & + \underbrace{\operatorname{Cov}\left(\varepsilon_{Si,t-} - \varepsilon_{Si',t-},\, \mathrm{EER}_{Ii,t+} - \mathrm{EER}_{Ii',t+}\right)}_{(iii)} + \underbrace{\operatorname{Cov}\left(\varepsilon_{Si,t-} - \varepsilon_{Si',t-},\, \varepsilon_{Ii,t+} - \varepsilon_{Ii',t+}\right)}_{(iv)}. \end{aligned} \tag{28}$$

In equilibrium, the existence of directed cash flow linkages in the economy already has implications for expected excess returns. Specifically, the results of Sections 2–4 characterize $\mathrm{EER}^{**}_{Si,t}$ and $\mathrm{EER}^{**}_{Ii,t}$ in closed form. Corollary 2 therefore evaluates the first-order approximation of term (i) in decomposition (28).

Corollary 2. *Under the assumptions of Theorem 2, we have*

$$\begin{aligned} & \operatorname{Cov}\left(EER^{**}_{Si,\,t-} - EER^{**}_{Si',\,t-},\, EER^{**}_{Ii,\,t+} - EER^{**}_{Ii',\,t+}\right) \\ = \; & L\,\mathcal{B}\,\mathcal{A}\,\left(\mathcal{D}_I - \mathcal{C}_I\right)\,\left(\beta_i - \beta_{i'}\right)^2\,\operatorname{Cov}\left(\ell_{t-},\, \ell_{t+}\right) \;\; > \;\; 0. \end{aligned}$$

Analogous corollaries can be stated for Theorems 5 or 6, respectively.

From Corollary 2, we conclude that the sum $\operatorname{Cov}\left(R_{Si,t-} - R_{Si',t-},\, R_{Ii,t+} - R_{Ii',t+}\right)$ is positive as well, implying that cross-predictability arises under the assumptions of Theorem 2 (or 5 or 6, respectively). This is because the terms (ii), (iii), and (iv) are zero under these assumptions. These covariances involve the noise terms, $\varepsilon_{i,t}$, around expected excess returns. In our model, such noise can only arise due to *realized* cash flow shocks, $N_{i,t}$. The Poisson processes $N_{i,t}$ have finite activity, i.e., the number of realized jumps in any positive time interval is almost surely finite. Contemporaneous jumps are excluded with probability one. As a result, the covariance terms (ii), (iii), and (iv) can only be non-zero if there are systematic patterns in realized cash flow shocks which give rise to predictive relations. Assumption (A2) implies that such predictive relations do not exist. We made this assumption for the sake of transparency and focus. We want cross-predictability to arise through cash flow linkages only and not through differences in current levels of jump intensities. If this is the case, the difference $\varepsilon_{Ii,t+} - \varepsilon_{Ii',t+}$ is unpredictable and the difference $\varepsilon_{Si,t-} - \varepsilon_{Si',t-}$ has no predictive power. Therefore, all three terms are zero under the assumptions stated above.

Remark 2. *Under the assumptions of Theorem 2 (or 5 or 6, respectively), we also have*

$$\begin{aligned} & \operatorname{Cov}\left(EER^{**}_{Si,\,t-} - EER^{**}_{Si',\,t-},\, EER^{**}_{Si,\,t+} - EER^{**}_{Si',\,t+}\right) \\ = \;& L^2\,\mathcal{B}^2\,\left(\beta_i - \beta_{i'}\right)^2\,\operatorname{Cov}\left(\ell_{t-},\,\ell_{t+}\right) \;\;>\;\; 0 \end{aligned}$$

and

$$\begin{aligned} & \operatorname{Cov}\left(EER^{**}_{Ii,\,t-} - EER^{**}_{Ii',\,t-},\, EER^{**}_{Ii,\,t+} - EER^{**}_{Ii',\,t+}\right) \\ = \;& \mathcal{A}^2\,\left(\mathcal{D}_I - \mathcal{C}_I\right)^2\,\left(\beta_i - \beta_{i'}\right)^2\,\operatorname{Cov}\left(\ell_{t-},\,\ell_{t+}\right) \;\;>\;\; 0 \end{aligned}$$

By the same arguments as above, we thus have

$$\begin{aligned} \operatorname{Cov}\left(R_{Si,t-} - R_{Si',t-},\, R_{Si,t+} - R_{Si',t+}\right) &> 0, \\ \operatorname{Cov}\left(R_{Ii,t-} - R_{Ii',t-},\, R_{Ii,t+} - R_{Ii',t+}\right) &> 0. \end{aligned}$$

By design, the pure jump model presented in this paper generates a form of "auto-predictability of returns." Similar to cross-predictability, it is explained exclusively via auto-predictability of risk premia.

Corollary 2 also has implications for the cyclical behavior of cross-predictability.

Remark 3. *From Corollary 2, we have*

$$\begin{aligned} & \operatorname{Cov}\left(EER^{**}_{Si,\,t-} - EER^{**}_{Si',\,t-},\, EER^{**}_{Ii,\,t+} - EER^{**}_{Ii',\,t+}\right) \\ = \;& L\,\mathcal{B}\,\mathcal{A}\,\left(\mathcal{D}_I - \mathcal{C}_I\right)\,\left(\beta_i - \beta_{i'}\right)^2\,\operatorname{Cov}\left(\ell_{t-},\,\ell_{t+}\right), \end{aligned}$$

under the assumptions of Theorem 2 (or 5 or 6, respectively). This implies that the magnitude of cross-predictability varies with ℓ_t. When $L < K < 0$, then ℓ_t is countercyclical, meaning that large values of ℓ_t on average coincide with low aggregate consumption growth. When $L > K > 0$, then ℓ_t is procyclical. As shown in Theorems 2 and 4, cross-predictability arises in both cases. Therefore, depending on the sign of K and L, the magnitude of cross-predictability can be procyclical or countercyclical as well.

5.2 Extension to arbitrary finite time windows

All statements so far have been derived under Assumption (A4), implying that average realized returns of signal and investment assets shortly before and after time t equal expected excess returns at time t. In this section, we show in detail what happens when we relax this assumption. Again, we decompose realized excess return differentials into

expected return and noise components.

$$\begin{aligned} R_{Sj,[t-\Delta,\,t]} &= \int_{t-\Delta}^{t} (\mathrm{EER}_{Sj,u} + \varepsilon_{Sj,u})\; du, \\ R_{Ij,[t,\,t+\Delta]} &= \int_{t}^{t+\Delta} (\mathrm{EER}_{Ij,u} + \varepsilon_{Ij,u})\; du, \end{aligned}$$

where $\varepsilon_{Sj,t}$ and $\varepsilon_{Ij,t}$ denote zero mean noise processes for $j = 1, \ldots, n$. The covariance can be decomposed into four terms again:

$$\begin{aligned} & \mathrm{Cov}\left(R_{Si,\,[t-\Delta,\,t]} - R_{Si',\,[t-\Delta,\,t]},\; R_{Ii,\,[t,\,t+\Delta]} - R_{Ii',\,[t,\,t+\Delta]}\right) \\ = \; & \underbrace{\mathrm{Cov}\left(\int_{t-\Delta}^{t} (\mathrm{EER}_{Si,u} - \mathrm{EER}_{Si',u})\; du,\; \int_{t}^{t+\Delta} (\mathrm{EER}_{Ii,u} - \mathrm{EER}_{Ii',u})\; du\right)}_{(i)} \\ & + \underbrace{\mathrm{Cov}\left(\int_{t-\Delta}^{t} (\mathrm{EER}_{Si,u} - \mathrm{EER}_{Si',u})\; du,\; \int_{t}^{t+\Delta} (\varepsilon_{Ii,u} - \varepsilon_{Ii',u})\; du\right)}_{(ii)} \\ & + \underbrace{\mathrm{Cov}\left(\int_{t-\Delta}^{t} (\varepsilon_{Si,u} - \varepsilon_{Si',u})\; du,\; \int_{t}^{t+\Delta} (\mathrm{EER}_{Ii,u} - \mathrm{EER}_{Ii',u})\; du\right)}_{(iii)} \\ & + \underbrace{\mathrm{Cov}\left(\int_{t-\Delta}^{t} (\varepsilon_{Si,u} - \varepsilon_{Si',u})\; du,\; \int_{t}^{t+\Delta} (\varepsilon_{Ii,u} - \varepsilon_{Ii',u})\; du\right)}_{(iv)}. \end{aligned} \tag{29}$$

We can then prove the following theorem.

Theorem 7. *Assume (A1) and (A3), and extend Assumption (A2) into*

(A5) Equations (11) *and* (12) *from Theorem 2 hold at every instant s in both windows (i.e., before and after time t):*

$$\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s} = -\ell_s\, \mathcal{B}\, L\; (\beta_i - \beta_{i'})\,, \tag{30}$$

$$\mathrm{EER}^{**}_{Ii,s} - \mathrm{EER}^{**}_{Ii',s} = -\ell_s\, \mathcal{A}\; (\mathcal{D}_{Ii} - \mathcal{C}_{Ii})\; (\beta_i - \beta_{i'})\,. \tag{31}$$

with a common realizations $\ell_{S1,s} = \ldots = \ell_{Sn,s} = \ell_{I1,s} = \ldots = \ell_{In,s} = \ell_s$.

Then, the first-order approximation of term (i) in the decomposition (29) *is equal to*

$$L\, \mathcal{A}\mathcal{B}\; (\mathcal{D}_I - \mathcal{C}_I)\; (\beta_i - \beta_{i'})^2\; \mathrm{Var}\,(\ell)\; \left(\frac{1 - e^{-\kappa\,\Delta}}{\kappa}\right)^2 \;>\; 0. \tag{32}$$

Proof: See Appendix B.6.

Assumption (A5) postulates that expected excess returns exhibit a one-factor structure with factor loadings being driven solely by cross-sectional differences in the strengths of cash flow linkages. It implies that the basic structure established in Theorem 2 extends to finite horizons. With all jump intensities being equalized exactly at time t (Assumption (A2)) and with the mean reversion speed κ being the same for all intensities, only the occurrence of jumps can cause cross-sectional differences in the levels of $\ell_{i,s}$. Assumption (A5) thus guarantees that we abstract from any cross-sectional variation in expected excess returns arising from the *realization* of jumps and focus only on the equilibrium pricing effects from the *probability* of cash flow jumps.

Theorem 7 states that cross-predictability arising through the risk premia channel in term (i) remains positive for any finite window length Δ. Turning to the other terms in the decomposition (29), Appendix B.6 shows that term (ii) is zero, while term (iii) is negative, and term (iv) is positive under the conditions stated above. Since the signs of terms (iii) and (iv) are opposing, we thus cannot make statements about the sum in (29), nor determine whether term (i) dominates.

To summarize, the equilibrium risk premium channel, term (i), is unambiguously positive for any $\Delta > 0$, but for finite Δ it is confounded with noise terms whose net sign is indeterminate. The limiting case $\Delta \to 0$, which is the focus of the main analysis, isolates this channel in closed form and delivers the clearest characterization of the equilibrium mechanism.

6 Implications of our mechanism for empirical studies

We close the paper with a brief discussion of the implications of our mechanism for empirical studies of cross-predictability. In a nutshell, we argue that empirical researchers studying cross-predictability should control for equilibrium pricing effects before attributing the effect solely to market inefficiencies.

We organize our discussion around the decomposition in Equation (28). As the key contribution of our paper, Corollary 2 specifies conditions under which term (i) is positive in equilibrium. While the empirical literature explicitly acknowledges the existence of fundamental linkages between firms, industries, or countries, it overlooks their implications for expected returns in equilibrium. Broadly speaking, the sources of cross-predictability put forward in the existing literature can be mapped into terms (ii) to (iv).

For instance, "slow information diffusion" has been suggested as a source of cross-predictability. Suppose new *idiosyncratic* information about stock S_i is released prior to time t. The mechanism behind slow information diffusion rests on two assumptions. (A)

The idiosyncratic information is also relevant for stock I_i because of a fundamental link between the two. (B) Investors process the information only with a delay, in general. In Equation (28), such a mechanism would be captured by the terms (iii) and (iv) because "idiosyncratic information about stock S_i" is represented by $\varepsilon_{S_i,t-}$. We do not want to negate that underreaction to news and informational inefficiencies can lead to cross-predictability. However, our paper documents that assumption (A) can already constitute a positive term (i) which empirical researchers should control for.

The fact that our findings are based on a linearized model conceptually simplifies this non-trivial task. Our model features $\beta_i - \beta_{i'}$ as the key quantity that controls the amount of cross-predictability arising through the equilibrium mechanism. Controlling for it comprises two broad steps. First, one needs to quantify the difference between the strengths of directed cash flow links. For instance, one could apply a generalized variance decomposition as in Diebold and Yilmaz (2014) to cash flow data.[11] Second, one needs to include the differences in the strengths of cash flow linkages as a control variable in cross-predictability regressions. The stronger the cross-sectional *differences* between the strengths of cash flow links, the stronger the cross-sectional differences in expected excess returns, and the stronger the cross-predictability arising through the equilibrium mechanism outlined above.

While conceptually clear, this empirical approach comes with a number of challenges which need to be overcome in practice. First, firm-level cash flow data are available at a quarterly frequency at best.[12] Second, directed cash flow links might be time-varying and are estimated with large standard errors which confounds subsequent regressions. Third, linear regressions cannot easily capture non-linear effects. Fourth, the different mechanisms discussed above may also address different dimensions of cross-predictability, for instance, short-term versus long-term horizons.

Alternatively, one could follow the path outlined in Burt and Hrdlicka (2021). As explained in Remark 1, the theoretical mechanism we uncover is related to commonality in momentum. However, the authors connect commonality in momentum to factor momentum for which there is no role in our model.

A further implication of our risk-based mechanism concerns portfolio turnover. In our model, the cross-momentum strategy that exploits cross-predictability in risk premia

[11] An alternative approach could be Granger causalities. Billio, Getmansky, Lo, and Pelizzon (2012) estimate this from return data, but the authors only focus on the existence of links, rather than the strengths of links.

[12] There have been advances in the measurement of cash flow news on a higher frequency recently. For instance, Pettenuzzo, Sabbatucci, and Timmermann (2020) construct a daily measure of cash flow growth for the aggregate market from firm-level dividend announcements. Pettenuzzo, Sabbatucci, and Timmermann (2024) extend this approach and provide a measure of daily aggregate corporate account business activity. However, there are no daily cash flow time series available at a more granular level.

has little turnover, since the constant beta matrix implies that the rankings of signal and investment assets are stable over time. At first glance, this is at odds with the empirical literature.[13] However, the low turnover is not an intrinsic feature of cross-predictability in risk premia per se. It is rather a consequence of deliberate modeling choices that we make for tractability and transparency. The asset pricing literature has shown extensively that expected returns vary over time. We abstract from this time variation in our framework, therefore suppressing any re-ranking of assets and hence turnover. Relaxing this assumption, e.g., by allowing for a time-varying beta matrix, would generate more sizable turnover. Furthermore, with a finite holding period Δ, the noise terms (ii), (iii), and (iv) in decomposition (29) as well as the noise that is eliminated by Assumption (A5) induce further occasional re-ranking among assets. Lastly, extending the model to incorporate diffusive risks would constitute an additional source of turnover. Taken together, the low-turnover prediction of the model should be interpreted as a consequence of its tractable structure, not as a general prediction of equilibrium-based cross-predictability.

7 Conclusion

Cross-predictability of returns has been documented in various empirical settings and sample periods. If markets are informationally efficient and complete, why would the return of one asset predict the return of another economically linked asset? We show that cross-predictability can arise from directed cash flow linkages solely through risk-based equilibrium pricing effects.

We study cross-predictability through average returns of long-short strategies constructed from investment assets, which are pairwise connected to signal assets. Differences in the strengths of cash flow linkages determine both the expected (or average realized) return differentials of signal assets up to time t and the expected (or average realized) return differentials of investment assets after time t. To illustrate this general economic rationale, we rely on a pure jump model which allows us to dissect risk premia in closed form. When signal and investment assets are pairwise connected, their risk premia become interrelated.

To obtain our key result, we do not have to specify any further characteristics of the fundamental link except for the fact that it gives rise to shock propagation at the cash flow level. Our theorems hold irrespective of the direction of the link itself, irrespective of the number of assets in the economy, and irrespective of whether cash flow jumps are positive or negative.

[13] While the high turnover of momentum strategies more generally has been known for a long time, see e.g. Asness, Moskowitz, and Pedersen (2013), the turnover of cross-momentum strategies has not been documented in the literature.

From a broader perspective, our paper connects three previously unconnected strands of the literature: the empirical literature on cross-predictability, the theoretical literature on granular economies and input-output networks, and the equilibrium asset pricing literature. While the empirical literature on cross-predictability explicitly acknowledges the existence of fundamental linkages between firms, industries, or countries, it overlooks their equilibrium implications. Therefore, our paper has implications for empirical studies of cross-predictability. More broadly, empirical researchers should control for equilibrium pricing effects before attributing the effect solely to market inefficiencies. The decomposition of Equation (28) provides a road map for this endeavour.

Our paper is also linked to the literature about granular shocks pioneered by Gabaix (2011). In fact, this granularity is at the heart of our risk-based mechanism as cash flow shocks need to be sufficiently systematic to generate movements in the pricing kernel. The existence of directed cash flow linkages is also a possible candidate for a partial explanation of the empirical performance of principal portfolios constructed by Kelly, Malamud, and Pedersen (2023). Their prediction matrix is agnostic about the source of the underlying fundamental link. Broadly speaking, we suggest a risk-based mechanism through which certain granular cash flow shocks affect the pricing kernel. In this, our findings also touch upon the macro-finance literature that studies the aggregate implications of microeconomic shocks.

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A Sketch of the model solution

In the following, we sketch how to solve the model in Section 2. For further details, we refer the reader to Branger, Konermann, Meinerding, and Schlag (2021) who apply the approach proposed in Eraker and Shaliastovich (2008) to a pure jump model featuring dynamic shock propagation.

A.1 Market prices of jump risk

The vector $X = (y, \ell_{S1}, \ldots, \ell_{Sn}, \ell_{I1}, \ldots, \ell_{In}, y_{S1}, \ldots, y_{Sn}, y_{I1}, \ldots, y_{In})'$ follows the affine jump process

$$dX_t \;=\; \mu\left(X_t\right)\, dt + \xi\, dN_t,$$

where $\mu(X_t) = \mathcal{M} + \mathcal{K}\, X_t$, $\ell_t = (\ell_{S1}, \ldots, \ell_{Sn}, \ell_{I1}, \ldots, \ell_{In}) = l_0 + l_1\, X_t$, and ξ are defined accordingly. The continuous-time version of the Euler equation can be written as $0 = \frac{1}{dt}\, \mathbb{E}_t \left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}} \right]$, where R is the return on the claim to aggregate consumption. The logarithm of the pricing kernel has dynamics $d \ln M_t = -\delta\, \theta\, dt - (1-\theta)\; d \ln R_t - \frac{\theta}{\psi}\, dy_t$. We apply the usual affine conjecture for the log wealth-consumption ratio, $v_t = A + (B_{S1}, \ldots, B_{Sn}, B_{I1}, \ldots, B_{In})\; \ell_t$, and use the Campbell-Shiller approximation for the return on the consumption claim $d \ln R_t = k_{v,0}\, dt + k_{v,1}\, dv_t - (1 - k_{v,1})\; v_t\, dt + dy_t$. Combining the Campbell-Shiller approximation, the affine guess for v_t, the dynamics of the log pricing kernel and the Euler equation, we get a system of equations for A, $B_{S1}, \ldots, B_{Sn}, B_{I1}, \ldots, B_{In}$, $k_{v,0}$ and $k_{v,1}$.

Analogously to Equation (7), we can write the dynamics of the pricing kernel as

$$\begin{aligned} \frac{dM_t}{M_t} &= -r_t\, dt - \left[\mathbb{1} - \varrho\left(-\lambda\right)\right]'\, \left(dN_t - \ell_t dt\right) \\ \text{with} \qquad \lambda &= \Big(\gamma, (1-\theta)\; k_{v,1}\, B_{S1}, \ldots, (1-\theta)\; k_{v,1}\, B_{Sn}, \\ &\qquad (1-\theta)\; k_{v,1}\, B_{I1}, \ldots, (1-\theta)\; k_{v,1}\, B_{In}, 0, \ldots, 0\Big)', \end{aligned}$$

where the jump transform is $\varrho\left(u\right) = \left(\exp\left(u'\, \xi_{S1,t}\right), \ldots, \exp\left(u'\, \xi_{Sn,t}\right), \exp\left(u'\, \xi_{I1,t}\right), \ldots, \exp\left(u'\, \xi_{In,t}\right)\right)'$ since the jump sizes are all constant. We can read off the market prices of risk from these

dynamics:

$$\begin{pmatrix} \text{MPJR}_{S1} \\ \vdots \\ \text{MPJR}_{Sn} \\ \text{MPJR}_{I1} \\ \vdots \\ \text{MPJR}_{In} \end{pmatrix} = [\mathbb{1} - \varrho(-\lambda)] = \begin{pmatrix} 1 - \exp(-\gamma K + k_{v,1}\ (\theta - 1)\ B_{I1}\,\beta_1) \\ \vdots \\ 1 - \exp(-\gamma K + k_{v,1}\ (\theta - 1)\ B_{In}\,\beta_n) \\ 1 - \exp(-\gamma K) \\ \vdots \\ 1 - \exp(-\gamma K) \end{pmatrix}.$$

Since signal asset S_j propagates cash flow shocks to investment asset I_j, S_j's market price of risk contains an additional term, $k_{v,1}\ (\theta - 1)\ B_{Ij}\,\beta_j$. The coefficient B_{Ij} captures the response of the wealth-consumption ratio to changes in I_j's jump intensity ℓ_{Ij}.

A.2 Jump exposures and expected excess returns

To obtain the expected excess returns on the cash flow claims, we follow the same approach as for the consumption claim. The continuous-time Euler equation again reads $0 = \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}}\right]$. Applying this Euler equation, the Campbell-Shiller approximation $d\ln R_{i,t} = k_{i,0}\,dt + k_{i,1}\,dv_{i,t} - (1 - k_{i,1})\ v_{i,t}\,dt + dy_{i,t}$ and the usual affine guess for the log price-cash flow ratio, $v_{i,t} = A_i + (C_{i,S1}, \ldots, C_{i,Sn}, C_{i,I1}, \ldots, C_{i,In})\ \ell_t$, we arrive at a system of equations for the coefficients A_i, $C_i = (C_{i,S1}, \ldots, C_{i,Sn}, C_{i,I1}, \ldots, C_{i,In})$ and the log-linearization constants $k_{i,0}$ and $k_{i,1}$. Analogously to Equation (8), the return of the individual cash flow claim i follows

$$dR_{i,t} = \{\ldots\}\ dt + \{\varrho(\delta_{y,i} + k_{i,1}\,C_i) - \mathbb{1}\}\ dN_t.$$

We define the selection vectors $\delta_{y,i}$ via $dy_{i,t} = \delta'_{y,i}\, dX_t$. From this we can read off the jump exposures:

$$\begin{pmatrix} \mathrm{JEXP}_{S1,S1} & \dots & \mathrm{JEXP}_{S1,Sn} & \mathrm{JEXP}_{S1,I1} & \dots & \mathrm{JEXP}_{S1,In} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathrm{JEXP}_{Sn,S1} & \dots & \mathrm{JEXP}_{Sn,Sn} & \mathrm{JEXP}_{Sn,I1} & \dots & \mathrm{JEXP}_{Sn,In} \\ \mathrm{JEXP}_{I1,S1} & \dots & \mathrm{JEXP}_{I1,Sn} & \mathrm{JEXP}_{I1,I1} & \dots & \mathrm{JEXP}_{I1,In} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathrm{JEXP}_{In,S1} & \dots & \mathrm{JEXP}_{In,Sn} & \mathrm{JEXP}_{In,I1} & \dots & \mathrm{JEXP}_{In,In} \end{pmatrix} = \begin{pmatrix} (\varrho\,(\delta_{y,S1} + k_{S1,1}\, C_{S1}) - \mathbb{1})' \\ \vdots \\ (\varrho\,(\delta_{y,Sn} + k_{Sn,1}\, C_{Sn}) - \mathbb{1})' \\ (\varrho\,(\delta_{y,I1} + k_{I1,1}\, C_{I1}) - \mathbb{1})' \\ \vdots \\ (\varrho\,(\delta_{y,In} + k_{In,1}\, C_{In}) - \mathbb{1})' \end{pmatrix}$$

$$= \begin{pmatrix} \exp\left(L + k_{S1,1}\, C_{S1,I1}\, \beta_1\right) - 1 & \dots & \exp\left(k_{S1,1}\, C_{S1,In}\, \beta_n\right) - 1 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \exp\left(k_{Sn,1}\, C_{Sn,I1}\, \beta_1\right) - 1 & \dots & \exp\left(L + k_{Sn,1}\, C_{Sn,In}\, \beta_n\right) - 1 & 0 & \dots & 0 \\ \exp\left(k_{I1,1}\, C_{I1,I1}\, \beta_1\right) - 1 & \dots & \exp\left(k_{I1,1}\, C_{I1,In}\, \beta_n\right) - 1 & \exp\left(L\right) - 1 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \exp\left(k_{In,1}\, C_{In,I1}\, \beta_1\right) - 1 & \dots & \exp\left(k_{In,1}\, C_{In,In}\, \beta_n\right) - 1 & 0 & \dots & \exp\left(L\right) - 1 \end{pmatrix}$$

The expected excess return can be represented as

$$\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{Sj,t}\,\mathrm{MPJR}_{Sj}\,\mathrm{JEXP}_{i,Sj} + \sum_{j=1}^{n} \ell_{Ij,t}\,\mathrm{MPJR}_{Ij}\,\mathrm{JEXP}_{i,Ij}.$$

A.3 First-order approximations

We reshuffle the nonlinear equations for the coefficients $B_{S1}, \dots, B_{Sn}, B_{I1}, \dots, B_{In}$ mentioned in Appendix A.1, apply the approximation $\exp(x) = 1 + x + O(x^2)$, and expand terms involving Leontief inverse matrices as $(I - \bullet)^{-1} = I + \bullet^1 + \bullet^2 + \dots$ to arrive at the following representation of the market prices of risk vector:

$$\begin{aligned} & & \mathrm{MPJR} &= 1 - \exp\left\{\mathcal{A} + \mathcal{B}\, \mathit{spread} + O\left(\beta^2\right)\right\} \\ & \text{with} & \mathit{spread} &= (\beta_1, \dots, \beta_n, 0, \dots, 0)' \\ & & \mathcal{A} &= -\gamma\, K \qquad (33) \\ & & \mathcal{B} &= \frac{(\theta - 1)\ (1 - \exp\{K\ (1 - \gamma)\})}{\theta\left[(1 - \kappa) - \frac{1}{k_{v,1}}\right]} \qquad (34) \end{aligned}$$

where $O(\beta^2)$ denotes polynomial terms in the β_i of order 2 or higher. We then define

$$\mathrm{MPJR}^{**}_{Sj} \;=\; 1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_j\right\}, \qquad (35)$$

$$\mathrm{MPJR}^{**}_{Ij} \;=\; 1 - \exp\left\{\mathcal{A}\right\}. \qquad (36)$$

For each i, we reshuffle the nonlinear equations for the coefficients $C_{i,S1},\ldots,C_{i,Sn},C_{i,I1},\ldots,C_{i,In}$ mentioned in Appendix A.2, apply the approximation $\exp(x)=1+x+O(x^2)$, and expand terms involving Leontief inverse matrices as $(I-\bullet)^{-1}=I+\bullet^1+\bullet^2+\ldots$ to arrive at the following representation of jump exposures:

$$\text{JEXP}_{Si,j} = \begin{cases} \exp\{L+\mathcal{C}_{Si}\,\beta_i+O(\beta^2)\}-1 & \text{for } j=Si, \\ \exp\{\mathcal{C}_{Si}\,\beta_{i'}+O(\beta^2)\}-1 & \text{for } j=Si',\ i'\neq i, \\ 0 & \text{for } j=Ii',\ i'=1,\ldots,n, \end{cases}$$

$$\text{JEXP}_{Ii,j} = \begin{cases} \exp\{L+O(\beta^2)\}-1 & \text{for } j=Ii, \\ 0 & \text{for } j=Ii',\ i'\neq i, \\ \exp\{\mathcal{D}_{Ii}\,\beta_i+O(\beta^2)\}-1 & \text{for } j=Si, \\ \exp\{\mathcal{C}_{Ii}\,\beta_{i'}+O(\beta^2)\}-1 & \text{for } j=Si',\ i'\neq i, \end{cases}$$

with

$$\mathcal{C}_i = \frac{1-\frac{\theta-1}{\theta}\,[1-\exp\{K\,(1-\gamma)\}]-\exp\{-K\,\gamma\}}{1-\kappa-\frac{1}{k_{i,1}}} \tag{37}$$

$$\mathcal{D}_i = \frac{1-\frac{\theta-1}{\theta}\,[1-\exp\{K\,(1-\gamma)\}]-\exp\{L-K\,\gamma\}}{1-\kappa-\frac{1}{k_{i,1}}} \tag{38}$$

$$\mathcal{D}_i-\mathcal{C}_i = \frac{\exp\{-K\,\gamma\}\,(1-\exp\{L\})}{1-\kappa-\frac{1}{k_{i,1}}}. \tag{39}$$

We then define

$$\text{JEXP}^{**}_{Si,j} = \begin{cases} \exp\{L+\mathcal{C}_{Si}\,\beta_i\}-1 & \text{for } j=Si, \\ \exp\{\mathcal{C}_{Si}\,\beta_{i'}\}-1 & \text{for } j=Si',\ i'\neq i, \\ 0 & \text{for } j=Ii',\ i'=1,\ldots,n, \end{cases} \tag{40}$$

$$\text{JEXP}^{**}_{Ii,j} = \begin{cases} \exp\{L\}-1 & \text{for } j=Ii, \\ 0 & \text{for } j=Ii',\ i'\neq i, \\ \exp\{\mathcal{D}_{Ii}\,\beta_i\}-1 & \text{for } j=Si, \\ \exp\{\mathcal{C}_{Ii}\,\beta_{i'}\}-1 & \text{for } j=Si',\ i'\neq i, \end{cases} \tag{41}$$

B Proofs

B.1 Proof of Theorem 1

B.1.1 First-order approximations of expected excess returns

The expected excess return of asset i is given in Equation (6). Assuming (A2) and using the first-order approximations of the market price of jump risk in Equations (51) and (52) as well as S_i's jump exposures in Equation (53), the expected excess return of signal asset

S_i simplifies to

$$\begin{aligned}\text{EER}_{Si,t} &= \ell_t \left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_i\right\}\right] \left[\exp\left\{L + \mathcal{C}_{Si}\beta_i\right\} - 1\right] \\ &\quad + \ell_t \sum_{j=1, j\neq i}^{n} \left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_j\right\}\right] \left[\exp\left\{\mathcal{C}_{Si}\,\beta_j\right\} - 1\right].\end{aligned}$$

Applying the first-order approximation $\exp(\beta_j) = 1 + \beta_j + O\left(\beta_j^2\right)$, $\exp(\mathcal{A}) \approx 1 + \mathcal{A}$, and $\exp(L) \approx 1 + L$ yield

$$\begin{aligned}\text{EER}_{Si,t} &= -\ell_t \left[\mathcal{A} + \mathcal{B}\,\beta_i + O\left(\beta^2\right)\right] \left[L + O\left(\beta^2\right)\right] \\ &\quad - \ell_t \sum_{j=1}^{n} \left[\mathcal{A} + \mathcal{B}\,\beta_j + O\left(\beta^2\right)\right] \left[\mathcal{C}_{Si}\,\beta_j + O\left(\beta^2\right)\right].\end{aligned}$$

As in Equation (9), we can then define

$$\begin{aligned}\text{EER}^{**}_{Si,t} &= \mathcal{CF}_t + \mathcal{S}_t\,\beta_i + \mathcal{H}_{Si,t} \sum_{j=1}^{n} \beta_j, \\ \text{with} \qquad \mathcal{CF}_t &= -\ell_t\,\mathcal{A}\,L \\ \mathcal{S}_t &= -\ell_t\,\mathcal{B}\,L \\ \mathcal{H}_{Si,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Si}.\end{aligned}$$

Assuming (A2) and using the first-order approximations of the market price of jump risk in Equations (51) and (52) as well as I_i's jump exposures in Equation (54), the expected excess return of investment asset I_i simplifies to

$$\begin{aligned}\text{EER}_{Ii,t} &= \ell_t \left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_i\right\}\right] \left[\exp\left\{\mathcal{D}_{Ii}\,\beta_i\right\} - 1\right] \\ &\quad + \ell_t \sum_{j=1, j\neq i}^{n} \left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_j\right\}\right] \left[\exp\left\{\mathcal{C}_{Ii}\,\beta_j\right\} - 1\right] \\ &\quad + \ell_t \left[1 - \exp\left\{\mathcal{A}\right\}\right] \left[\exp\left\{L\right\} - 1\right].\end{aligned}$$

Applying the first-order approximation $\exp(\beta_j) = 1 + \beta_j + O\left(\beta_j^2\right)$, $\exp(\mathcal{A}) \approx 1 + \mathcal{A}$, and $\exp(L) \approx 1 + L$ yield

$$\begin{aligned}\text{EER}_{Ii,t} &= -\ell_t \left[\mathcal{A} + \mathcal{B}\,\beta_i + O\left(\beta^2\right)\right] \left[\mathcal{D}_{Ii}\,\beta_i + O\left(\beta^2\right)\right] \\ &\quad - \ell_t \sum_{j=1, j\neq i}^{n} \left[\mathcal{A} + \mathcal{B}\,\beta_j + O\left(\beta^2\right)\right] \left[\mathcal{C}_{Ii}\,\beta_j + O\left(\beta^2\right)\right] \\ &\quad - \ell_t \left[\mathcal{A} + O\left(\beta^2\right)\right] \left[L + O\left(\beta^2\right)\right].\end{aligned}$$

As in Equation (10), we can then define

$$\begin{aligned} \mathrm{EER}^{**}_{Ii,t} &= \mathcal{CF}_t + \mathcal{R}_{Ii,t}\,\beta_i + \mathcal{H}_{Ii,t} \sum_{j=1,j\neq i}^{n} \beta_j, \\ \text{with} \qquad \mathcal{CF}_t &= -\ell_t\,\mathcal{A}\,L \\ \mathcal{R}_{Ii,t} &= -\ell_t\,\mathcal{A}\,\mathcal{D}_{Ii} \\ \mathcal{H}_{Ii,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Ii}. \end{aligned}$$

B.1.2 Signs of the coefficients in Equations (9) and (10)

- $\underline{\mathcal{CF}_t > 0}$: We have $L < 0$, $\gamma > 1$ and $K < 0$ by assumption. From this, we can deduce that $\mathcal{A}$, given in Equation (33), is positive.

- $\underline{\mathcal{S}_t > 0}$: From $0 < \kappa < 1$, which is part of assumptions (A1), we know that $\frac{1}{k_{v,1}} > 1-\kappa$ because $\frac{1}{k_{v,1}} = \frac{1+e^{\bar{v}}}{e^{\bar{v}}} > 1$, where $e^{\bar{v}}$ denotes the steady-state wealth-consumption ratio. From this, we conclude that $\mathcal{B} > 0$ in Equation (34) because $\theta < 0$, $K < 0$, and $\gamma > 1$ by assumption. Moreover, we have $L < 0$ by assumption.

- $\underline{\mathcal{H}_{Si,t} < 0}$: We have already shown that $\mathcal{A} > 0$. So we only need to prove that $\mathcal{C}_{Si} > 0$ in Equation (37). To see this, we start from

$$\begin{aligned} \mathcal{C}_{Si} &= \frac{1 - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right] - \exp\{-K\,\gamma\}}{1 - \kappa - \frac{1}{k_{Si,1}}} \\ &= \frac{\exp\{-K\,\gamma\}\,\left[\frac{1}{\theta}\,(\exp\{K\,\gamma\} - \exp\{K\}) + \exp\{K\} - 1\right]}{1 - \kappa - \frac{1}{k_{Si,1}}}. \end{aligned}$$

  Since $0 < \kappa < 1$, we again have $\frac{1}{k_{Si,1}} > 1 - \kappa$ by assumption. Moreover, we have $\exp\{-K\,\gamma\} > 0$ and

$$\frac{1}{\theta}\,(\exp\{K\,\gamma\} - \exp\{K\}) + \exp\{K\} - 1 \;<\; \frac{1}{\theta}\,(\exp\{K\,\gamma\} - 1) + \exp\{K\} - 1 \;<\; 0,$$

  where the first inequality follows from $0 < -\frac{1}{\theta}\exp\{K\} < -\frac{1}{\theta}$. To see the last inequality, define

$$\begin{aligned} f(K) &= \exp\{K\,\gamma\} - 1 - (\exp\{K\} + \gamma)\,(\exp\{K\} - 1) \\ &= \exp\{K\,\gamma\} - 1 - \exp\{2\,K\} - \gamma\exp\{K\} + \exp\{K\} + \gamma \end{aligned}$$

Then $f(0) = 0$ and

$$\begin{aligned} f'(K) &= \gamma \exp\{K\,\gamma\} - 2\exp\{2\,K\} - \gamma \exp\{K\} + \exp\{K\} \\ &= \gamma\,(\exp\{K\,\gamma\} - \exp\{K\}) + \exp\{K\} - 2\exp\{2\,K\} \end{aligned}$$

If $\gamma > 1$ and $-\ln(2) < K < 0$, which is part of assumptions (A1), then $f'(K) < 0$ which implies $f(K) > 0$. In particular,

$$\frac{\exp\{K\,\gamma\} - 1}{\exp\{K\} - 1} \;<\; \exp\{K\} + \gamma \;<\; -\theta.$$

For $\gamma > 2\,\psi - 1$ which is part of assumptions (A1), the latter inequality is satisfied and, thus, $\mathcal{C}_{Si} > 0$ and $\mathcal{H}_{Si,t} < 0$.

- $\underline{\mathcal{H}_{Ii,t} < 0}$: The proof is identical to the one for $\mathcal{H}_{Si,t}$
- $\underline{\mathcal{R}_{Ii,t} > 0}$: We have already shown that $\mathcal{A} > 0$. So we only need to prove that $\mathcal{D}_{Ii} < 0$ in Equation (38). To see this, we start from

$$\begin{aligned} \mathcal{D}_{Ii} &= \frac{1 - \frac{\theta-1}{\theta}\,[1 - \exp\{K\,(1-\gamma)\}] - \exp\{L - K\,\gamma\}}{1 - \kappa - \frac{1}{k_{Ii,1}}} \\ &= \frac{\frac{1}{\theta} + \exp\{-K\,\gamma\}\,\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right]}{1 - \kappa - \frac{1}{k_{Ii,1}}}. \end{aligned}$$

As before, since $0 < \kappa < 1$, we have $\frac{1}{k_{Ii,1}} > 1 - \kappa$ by assumption. Moreover, we have

$$\begin{aligned} \Leftrightarrow \quad & \frac{1}{\theta} + \exp\{-K\,\gamma\}\,\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] > 0 \\ \Leftrightarrow \quad & \exp\{-K\,\gamma\}\,\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] > -\frac{1}{\theta} \\ \Leftrightarrow \quad & \left(1 - \frac{1}{\theta}\right)\exp\{K\} + \frac{1}{\theta}\exp\{K\,\gamma\} - \exp\{L\} > 0 \\ \Leftrightarrow \quad & (\exp\{K\} - \exp\{L\}) + \frac{1}{\theta}\,(\exp\{K\,\gamma\} - \exp\{K\}) > 0. \end{aligned}$$

For $L < K$ which is part of assumptions (A1), $\gamma > 1$ and $\theta < 0$, the last inequality is satisfied and, thus, $\mathcal{D}_{Ii} < 0$. □

B.2 Proof of Theorem 2

Assumption (A3) states that $\mathcal{C}_{Si} = \mathcal{C}_{Si'}$ for all i and i'. Therefore, we have $\mathcal{H}_{S,t} = \mathcal{H}_{Si,t} = \mathcal{H}_{Si',t}$. Consequently, the difference between the expected excess returns of signal assets

S_i and $S_{i'}$ simplifies to

$$\begin{aligned}\mathrm{EER}^{**}_{Si,t} - \mathrm{EER}^{**}_{Si',t} &= \mathcal{CF}_t + \mathcal{S}_t\,\beta_i + \mathcal{H}_{S,t}\sum_{j=1}^{n}\beta_j - \left[\mathcal{CF}_t + \mathcal{S}_t\,\beta_{i'} + \mathcal{H}_{S,t}\sum_{j=1}^{n}\beta_j\right] \\ &= \mathcal{S}_t\,(\beta_i - \beta_{i'})\,,\end{aligned}$$

which coincides with Equation (11) in Theorem 2. Without loss of generality, we assume that signal assets are ordered in such a way that $\mathrm{EER}^{**}_{S1,\,t} > \ldots > \mathrm{EER}^{**}_{Sn,\,t}$. Since $\mathrm{EER}^{**}_{Si,t} > \mathrm{EER}^{**}_{Si',t}$, the left-hand side of Equation (11) is thus positive. From Theorem 1, we know that $\mathcal{S}_t > 0$. We thus conclude that $(\beta_i - \beta_{i'})$ has to be positive.

Assumption (A3) states that $\mathcal{C}_{Ii} = \mathcal{C}_{Ii'}$ for all i, i'. Therefore, we have $\mathcal{H}_{I,t} = \mathcal{H}_{Ii,t} = \mathcal{H}_{Ii',t}$. Assumption (A3) also implies $\mathcal{D}_{Ii} = \mathcal{D}_{Ii'}$ (because $k_{Ii,1} = k_{Ii',1}$) which gives $\mathcal{R}_{I,t} = \mathcal{R}_{Ii,t} = \mathcal{R}_{Ii',t}$. Consequently, the difference between the expected excess returns of investment assets I_i and $I_{i'}$ simplifies to

$$\begin{aligned}\mathrm{EER}^{**}_{Ii,t} - \mathrm{EER}^{**}_{Ii',t} &= \mathcal{CF}_t + \mathcal{R}_{I,t}\,\beta_i + \mathcal{H}_{I,t}\sum_{j=1,j\neq i}^{n}\beta_j \\ &\quad - \left[\mathcal{CF}_t + \mathcal{R}_{I,t}\,\beta_{i'} + \mathcal{H}_{I,t}\sum_{j=1,j\neq i'}^{n}\beta_j\right] \\ &= \mathcal{R}_{I,t}\,(\beta_i - \beta_{i'}) + \mathcal{H}_{I,t}\left(\sum_{j=1,j\neq i}^{n}\beta_j - \sum_{j=1,j\neq i'}^{n}\beta_j\right) \\ &= (\mathcal{R}_{I,t} - \mathcal{H}_{I,t})\,(\beta_i - \beta_{i'})\,,\end{aligned}$$

which coincides with Equation (12) in Theorem 2. From Theorem 1, we know that $\mathcal{R}_{I,t} > 0$ and $\mathcal{H}_{I,t} < 0$, i.e., $(\mathcal{R}_{I,t} - \mathcal{H}_{I,t}) > 0$. Given that $(\beta_i - \beta_{i'}) > 0$, we thus conclude that $\mathrm{EER}^{**}_{Ii,t} - \mathrm{EER}^{**}_{Ii',t} > 0$. □

Remark on the sign of $\mathcal{D}_i - \mathcal{C}_i$. From Equation (39), we have

$$\mathcal{D}_i - \mathcal{C}_i = \frac{\exp\{-K\,\gamma\}\,(1 - \exp\{L\})}{1 - \kappa - \frac{1}{k_{i,1}}}.$$

The numerator is positive: $\exp\{-K\,\gamma\} > 0$ and $1 - \exp\{L\} > 0$ since $L < 0$. The denominator satisfies $1 - \kappa - \frac{1}{k_{i,1}} < 0$, as established above. Therefore $\mathcal{D}_i - \mathcal{C}_i < 0$ under assumptions (A1). Combined with $\mathcal{A} > 0$ and $\ell_t > 0$, this gives

$$\mathcal{R}_{I,t} - \mathcal{H}_{I,t} = -\ell_t\,\mathcal{A}\,(\mathcal{D}_I - \mathcal{C}_I) > 0,$$

since $\mathcal{A} > 0$ and $\mathcal{D}_i - \mathcal{C}_i < 0$ yield $\mathcal{A} \cdot (\mathcal{D}_i - \mathcal{C}_i) < 0$, and the leading minus sign then gives a positive result. The same sign pattern applies in Corollary 2, where $L\,\mathcal{A}\,\mathcal{B}\,(\mathcal{D}_I - \mathcal{C}_I) > 0$ because $L < 0$, $\mathcal{A} > 0$, $\mathcal{B} > 0$, and $\mathcal{D}_I - \mathcal{C}_I < 0$ together yield an even number of negative factors.

B.3 Proof of Theorem 4

To prove the theorem, we need to determine the signs of the coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{D}_i - \mathcal{C}_i$ under the alternative parameter restriction $L > K > 0$. In short, all these coefficients switch sign, implying that the signs of $\mathcal{CF}_t$, $\mathcal{S}_t$ and $\mathcal{R}_{i,t} - \mathcal{H}_{i,t}$ remain unchanged.

To see this, we start from the definitions of the coefficients:

$$\mathcal{A} = -\gamma\, K \tag{42}$$

$$\mathcal{B} = \frac{(\theta - 1)\ (1 - \exp\{K\ (1 - \gamma)\})}{\theta\ \left[(1 - \kappa) - \frac{1}{k_{v,1}}\right]} \tag{43}$$

$$\mathcal{C}_i = \frac{1 - \frac{\theta - 1}{\theta}\ [1 - \exp\{K\ (1 - \gamma)\}] - \exp\{-K\,\gamma\}}{1 - \kappa - \frac{1}{k_{i,1}}} \tag{44}$$

$$\mathcal{D}_i = \frac{1 - \frac{\theta - 1}{\theta}\ [1 - \exp\{K\ (1 - \gamma)\}] - \exp\{L - K\,\gamma\}}{1 - \kappa - \frac{1}{k_{i,1}}} \tag{45}$$

$$\mathcal{D}_i - \mathcal{C}_i = \frac{\exp\{-K\,\gamma\}\ (1 - \exp\{L\})}{1 - \kappa - \frac{1}{k_{i,1}}}. \tag{46}$$

The signs can be determined along the lines of the proof of Theorem 1 outlined in Appendix B.1. When K switches sign and becomes positive, we have that $\mathcal{A} < 0$ and $\mathcal{B} < 0$. The latter inequality follows because the term $(1 - \exp\{K\ (1 - \gamma)\})$ in Equation (43) switches sign when K switches sign, whereas all other terms remain unchanged. From this, we conclude that the signs of $\mathcal{CF}_t$ and $\mathcal{S}_t$ remain unchanged, as compared to Theorem 1.

Similarly, the sign of $\mathcal{D}_i - \mathcal{C}_i$ flips because the term $(1 - \exp\{L\})$ switches sign when imposing $L > K > 0$. As a result, we have that $\mathcal{R}_{i,t} - \mathcal{H}_{i,t} = -\ell_t \mathcal{A}(\mathcal{D}_i - \mathcal{C}_i)$ is again positive, as in Theorem 1. □

B.4 Proof of Theorem 5

The solution of the slightly adjusted model outlined in Section 4.1 is symmetric to the solution presented in Appendix A. The market prices of risk are given by

$$\begin{aligned} \mathrm{MPJR}^{**}_{Sj} &= 1 - \exp\{\mathcal{A}\}, && (47) \\ \mathrm{MPJR}^{**}_{Ij} &= 1 - \exp\{\mathcal{A} + \mathcal{B}\,\beta_j\}. && (48) \end{aligned}$$

The jump exposures are given by

$$\mathrm{JEXP}^{**}_{Si,j} = \begin{cases} \exp\{L\} - 1 & \text{for } j = Si, \\ 0 & \text{for } j = Si',\ i' \neq i, \\ \exp\{\mathcal{D}_{Si}\,\beta_i\} - 1 & \text{for } j = Ii, \\ \exp\{\mathcal{C}_{Si}\,\beta_{i'}\} - 1 & \text{for } j = Ii',\ i' \neq i, \end{cases} \tag{49}$$

$$\mathrm{JEXP}^{**}_{Ii,j} = \begin{cases} 0 & \text{for } j = Si',\ i = 1, \ldots, n, \\ \exp\{L + \mathcal{C}_{Ii}\,\beta_i\} - 1 & \text{for } j = Ii, \\ \exp\{\mathcal{C}_{Ii}\,\beta_{i'}\} - 1 & \text{for } j = Ii',\ i' \neq i. \end{cases} \tag{50}$$

Analogously to Equations (9) and (10), the expected excess returns can then be approximated as

$$\begin{aligned} \mathrm{EER}^{**}_{Si,t} &= \mathcal{CF}_t + \mathcal{R}_{Si,t}\,\beta_i + \mathcal{H}_{Si,t} \sum_{j=1, j\neq i}^{n} \beta_j, \\ \mathrm{EER}^{**}_{Ii,t} &= \mathcal{CF}_t + \mathcal{S}_t\,\beta_i + \mathcal{H}_{Ii,t} \sum_{j=1}^{n} \beta_j, \\ \text{with} \qquad \mathcal{CF}_t &= -\ell_t\,\mathcal{A}\,L \\ \mathcal{R}_{Si,t} &= -\ell_t\,\mathcal{A}\,\mathcal{D}_{Si} \\ \mathcal{H}_{Si,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Si} \\ \mathcal{S}_t &= -\ell_t\,\mathcal{B}\,L \\ \mathcal{H}_{Ii,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Ii}. \end{aligned}$$

Similar to Theorem 2, the expected excess return differentials are then given by

$$\begin{aligned} \mathrm{EER}^{**}_{Si,t} - \mathrm{EER}^{**}_{Si',t} &= (\mathcal{R}_{S,t} - \mathcal{H}_{S,t})\,(\beta_i - \beta_{i'}) \\ \mathrm{EER}^{**}_{Ii,t} - \mathrm{EER}^{**}_{Ii',t} &= \mathcal{S}_t\,(\beta_i - \beta_{i'}). \end{aligned}$$

With the same arguments about the signs of the coefficients as in Theorem 2, the proof then goes through as in Appendix B.2. □

B.5 Proof of Theorem 6

The solution of the extended model outlined in Section 4.2 is a combination of the solutions presented in Appendix A and B.4. Applying the same approximation steps, the market prices of risk are given by

$$\begin{aligned} \text{MPJR}^{**}_{Sj} &= 1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{I_j,S_j}\right\}, && (51)\\ \text{MPJR}^{**}_{Ij} &= 1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{S_j,I_j}\right\}. && (52) \end{aligned}$$

The jump exposures are given by

$$\text{JEXP}^{**}_{Si,j} = \begin{cases} \exp\left\{L + \mathcal{C}_{Si}\,\beta_{I_i,S_i}\right\} - 1 & \text{for } j = Si, \\ \exp\left\{\mathcal{C}_{Si}\,\beta_{I'_i,S'_i}\right\} - 1 & \text{for } j = Si',\ i' \neq i, \\ \exp\left\{\mathcal{D}_{Si}\,\beta_{S_i,I_i}\right\} - 1 & \text{for } j = Ii, \\ \exp\left\{\mathcal{C}_{Si}\,\beta_{S'_i,I'_i}\right\} - 1 & \text{for } j = Ii',\ i' \neq i, \end{cases} \quad (53)$$

$$\text{JEXP}^{**}_{Ii,j} = \begin{cases} \exp\left\{\mathcal{D}_{Ii}\,\beta_{I_i,S_i}\right\} - 1 & \text{for } j = Si, \\ \exp\left\{\mathcal{C}_{Ii}\,\beta_{I'_i,S'_i}\right\} - 1 & \text{for } j = Si',\ i' \neq i, \\ \exp\left\{L + \mathcal{C}_{Ii}\,\beta_{S_i,I_i}\right\} - 1 & \text{for } j = Ii, \\ \exp\left\{\mathcal{C}_{Ii}\,\beta_{S'_i,I'_i}\right\} - 1 & \text{for } j = Ii',\ i' \neq i. \end{cases} \quad (54)$$

The expected excess return of signal assets can then be written as

$$\begin{aligned} \text{EER}_{Si,t} &= \ell_t\,\left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{I_i,S_i}\right\}\right]\,\left[\exp\left\{L + \mathcal{C}_{Si}\beta_{I_i,S_i}\right\} - 1\right] \\ &\quad + \ell_t \sum_{j=1,j\neq i}^{n}\left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{I_j,S_j}\right\}\right]\,\left[\exp\left\{\mathcal{C}_{Si}\,\beta_{I_j,S_j}\right\} - 1\right] \\ &\quad + \ell_t\,\left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{S_i,I_i}\right\}\right]\,\left[\exp\left\{\mathcal{D}_{Si}\,\beta_{S_i,I_i}\right\} - 1\right] \\ &\quad + \ell_t \sum_{j=1,j\neq i}^{n}\left[1 - \exp\left\{\mathcal{A} + \mathcal{B}\,\beta_{S_j,I_j}\right\}\right]\,\left[\exp\left\{\mathcal{C}_{Si}\,\beta_{S_j,I_j}\right\} - 1\right], \end{aligned}$$

resulting in the approximation

$$\text{EER}^{**}_{Si,t} = \mathcal{CF}_t + \mathcal{S}_t\,\beta_{I_i,S_i} + \mathcal{R}_{Si,t}\,\beta_{S_i,I_i} + \mathcal{H}_{Si,t}\sum_{j=1}^{n}\beta_{I_j,S_j} + \mathcal{H}_{Si,t}\sum_{j=1,j\neq i}^{n}\beta_{S_j,I_j},$$

with

$$\begin{aligned} \mathcal{CF}_t &= -\ell_t\,\mathcal{A}\,L \\ \mathcal{R}_{Si,t} &= -\ell_t\,\mathcal{A}\,\mathcal{D}_{Si} \\ \mathcal{H}_{Si,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Si} \\ \mathcal{S}_t &= -\ell_t\,\mathcal{B}\,L. \end{aligned}$$

Symmetrically, the expected excess return of investment assets becomes

$$\begin{aligned}
\mathrm{EER}^{**}_{Ii,t} &= \mathcal{CF}_t + \mathcal{S}_t\,\beta_{S_i,I_i} + \mathcal{R}_{Ii,t}\,\beta_{I_i,S_i} + \mathcal{H}_{Ii,t}\sum_{j=1}^{n}\beta_{S_j,I_j} + \mathcal{H}_{Ii,t}\sum_{j=1,j\neq i}^{n}\beta_{I_j,S_j},\\
\text{with}\qquad \mathcal{R}_{Ii,t} &= -\ell_t\,\mathcal{A}\,\mathcal{D}_{Ii}\\
\mathcal{H}_{Ii,t} &= -\ell_t\,\mathcal{A}\,\mathcal{C}_{Ii}.
\end{aligned}$$

Assuming (A3), the difference between the expected excess returns of two investment assets I_i and $I_{i'}$ (or two signal assets S_i and $S_{i'}$) is given by

$$\mathrm{EER}^{**}_{Ii,\,t} - \mathrm{EER}^{**}_{Ii',\,t} = \mathcal{S}_t\,(\beta_{Si,Ii} - \beta_{Si',Ii'}) + (\mathcal{R}_{I,t} - \mathcal{H}_{I,t})\,(\beta_{Ii,Si} - \beta_{Ii',Si'}) \tag{55}$$

$$\mathrm{EER}^{**}_{Si,\,t} - \mathrm{EER}^{**}_{Si',\,t} = \mathcal{S}_t\,(\beta_{Ii,Si} - \beta_{Ii',Si'}) + (\mathcal{R}_{S,t} - \mathcal{H}_{S,t})\,(\beta_{Si,Ii} - \beta_{Si',Ii'})\,. \tag{56}$$

With the same arguments about the signs of the coefficients as in Theorem 2, the proof then goes through as in Appendix B.2. □

B.6 Proof of Theorem 7

Term $(i) > 0$:

$$\begin{aligned}
&\mathrm{Cov}\left(\int_{t-\Delta}^{t}\left(\mathrm{EER}^{**}_{Si,u} - \mathrm{EER}^{**}_{Si',u}\right)\,du,\ \int_{t}^{t+\Delta}\left(\mathrm{EER}^{**}_{Ii,u} - \mathrm{EER}^{**}_{Ii',u}\right)\,du\right)\\
=\ & L\,\mathcal{A}\,\mathcal{B}\,(\mathcal{D}_I - \mathcal{C}_I)\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}\left(\beta_i\ell_{S_i,s} - \beta_{i'}\ell_{S_{i'},s},\ \beta_i\ell_{I_i,u} - \beta_{i'}\ell_{I_{i'},u}\right)\,ds\,du.
\end{aligned}$$

The integral term is positive under Assumption (A5):

$$\begin{aligned}
&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}\left(\beta_i\,\ell_{S_i,s} - \beta_{i'}\,\ell_{S_{i'},s},\ \beta_i\,\ell_{I_i,u} - \beta_{i'}\,\ell_{I_{i'},u}\right)\,du\,ds\\
=\ & (\beta_i - \beta_{i'})^2\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}\left(\ell_s,\ \ell_u\right)\,du\,ds\\
=\ & (\beta_i - \beta_{i'})^2\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Var}\left(\ell\right)\,e^{-\kappa(u-s)}\,du\,ds\\
=\ & (\beta_i - \beta_{i'})^2\,\mathrm{Var}\left(\ell\right)\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}e^{-\kappa\,(u-s)}\,du\,ds\\
=\ & (\beta_i - \beta_{i'})^2\,\mathrm{Var}\left(\ell\right)\left(\frac{1-e^{-\kappa\,\Delta}}{\kappa}\right)^2\ >\ 0,
\end{aligned}$$

since the process $\{\ell_s\}$ is stationary on $[t-\Delta,\,t+\Delta]$ with the autocovariance function $\mathrm{Cov}\left(\ell_s,\,\ell_u\right) = \mathrm{Var}\left(\ell\right)\,e^{-\kappa\,|u-s|}$ for $s, u \in [t-\Delta,\,t+\Delta]$. □

Term $(ii) = 0$**:** To see this, we make use of Assumption (A5) which implies that the expected excess returns have a factor structure. The EER differential formulas of Theorem 2 hold at every instant s in both windows, with a common scalar factor process $\{\ell_s\}_{s\in[t-\Delta,\,t+\Delta]}$:

$$\begin{aligned}
\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s} &= -L\,\mathcal{B}\,\ell_s\,(\beta_i - \beta_{i'})\,, &(57)\\
\mathrm{EER}^{**}_{Ii,s} - \mathrm{EER}^{**}_{Ii',s} &= -\mathcal{A}\,(\mathcal{D}_I - \mathcal{C}_I)\,\ell_s\,(\beta_i - \beta_{i'})\,. &(58)
\end{aligned}$$

This assumption extend Assumption (A2) from the single portfolio formation date t to every instance throughout both windows. It implies that

$$\begin{aligned}
&\mathrm{Cov}\left(\int_{t-\Delta}^{t}\left(\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s}\right)\,ds,\ \int_{t}^{t+\Delta}\left(\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right)\,du\right)\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}_s\left(\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s},\ \varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right)\,du\,ds\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\Bigg[E_s\left(\left[\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s}\right]\cdot\left[\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right]\right)\\
&\qquad -E_s\left(\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s}\right)\underbrace{E_s\left(\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right)}_{=0}\Bigg]\,du\,ds\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}E_s\left(\left[\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s}\right]\,\left[\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right]\right)\,du\,ds\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}E_s\left(\left[\mathrm{EER}^{**}_{Si,s} - \mathrm{EER}^{**}_{Si',s}\right]\,\underbrace{E_u\left[\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right]}_{=0}\right)\,du\,ds\\
=&\ 0,
\end{aligned}$$

where we make use of the martingale property of ε and the law of iterated expectations. □

Term $(iii) < 0$**:**

$$\begin{aligned}
&\mathrm{Cov}\left(\int_{t-\Delta}^{t}\left(\varepsilon_{Si,s} - \varepsilon_{Si',s}\right)\,ds,\ \int_{t}^{t+\Delta}\left(\mathrm{EER}^{**}_{Ii,u} - \mathrm{EER}^{**}_{Ii',u}\right)\,du\right)\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}\left(\varepsilon_{Si,s} - \varepsilon_{Si',s},\ \mathrm{EER}^{**}_{Ii,u} - \mathrm{EER}^{**}_{Ii',u}\right)\,du\,ds\\
=&\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\mathrm{Cov}\left(\varepsilon_{Si,s} - \varepsilon_{Si',s},\ -\mathcal{A}\,(\mathcal{D}_I - \mathcal{C}_I)\,(\beta_i - \beta_{i'})\,\ell_u\right)\,du\,ds\\
=&\ -\mathcal{A}\,(\mathcal{D}_I - \mathcal{C}_I)\,(\beta_i - \beta_{i'})\int_{t-\Delta}^{t}\int_{t}^{t+\Delta}\underbrace{\mathrm{Cov}\left(\varepsilon_{Si,s} - \varepsilon_{Si',s},\ \ell_u\right)}_{<0}\,du\,ds\\
<&\ 0.
\end{aligned}$$

The last inequality follows from the properties of our cash flow processes. Realized noise shocks ε_{S_i} increase the aggregate jump intensity ℓ_u, and this effect is stronger for S_i than for $S_{i'}$ because $\beta_i > \beta_{i'}$. □

Term $(iv) > 0$**:**

$$\begin{aligned} & \operatorname{Cov}\left(\int_{t-\Delta}^{t} \left(\varepsilon_{Si,s} - \varepsilon_{Si',s}\right) ds,\ \int_{t}^{t+\Delta} \left(\varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right) du\right) \\ = & \int_{t-\Delta}^{t} \int_{t}^{t+\Delta} \underbrace{\operatorname{Cov}\left(\varepsilon_{Si,s} - \varepsilon_{Si',s},\ \varepsilon_{Ii,u} - \varepsilon_{Ii',u}\right)}_{>0} du\, ds \\ > & \ 0 \end{aligned}$$

The last inequality holds for a similar argument as for Term (iii) above. □